Similarity Renormalization Group for Nuclear Decays and Reaction Processes

INT 20-78W

Renormalization Group Approaches to the Many-Body Problem

Sofia Quaglioni
Goal:
Predictive understanding of nuclei and their interactions

Helium burning

s-process

r-process

Thermonuclear Fusion

Fission

Neutron stars

\[ ^{4}\text{He} \rightarrow ^{8}\text{Be}^{*} (10^{-16} \text{ s}) \rightarrow ^{8}\text{Be} \rightarrow ^{4}\text{He} \]

\[ ^{12}\text{C} \rightarrow ^{16}\text{O} \]

\[ ^{0}\text{νββ Decay} \]

\[ ^{132}\text{Sn} \rightarrow \text{Neutron stars} \]
Reactions ‘R’ Us

The diagram illustrates a red giant star with a shell labeled as "Carbon and Oxygen." Inside the shell, there is a process labeled as "Helium Burning." The diagram also shows a reaction involving helium nuclei, labeled as $^4\text{He}$, and isotopes, with time labels such as $(10^{-16}\text{ s})$. The reactions involve transitions between helium, carbon, and oxygen isotopes, with symbols indicating gamma rays ($\gamma$).
At stellar energies, reaction probabilities are very small!

\[
\sigma(E) = \frac{S(E)}{E} \exp \left( -\frac{2\pi Z_1 Z_2 e^2}{\hbar \sqrt{2E/m}} \right)
\]

Astrophysical S-factor: nuclear contribution

Coulomb contribution (tunneling)
We need reliable theory to accurately evaluate the S-factor at stellar energies.
Nuclei provide us with a window into physics beyond the Standard Model.

\[
\frac{1}{T_{1/2}} = G^{0\nu} m_{\beta\beta}^2 |M^{0\nu\beta\beta}|^2
\]
Currently best path to fundamental understanding combines effective field theory and ab initio methods.

Quantum Chromodynamics

Chiral Effective Field Theory

Ab initio many-body calculations

All active (pointlike) nucleons, non-relativistic quantum mechanics

Two & three-nucleon (NN+3N) forces
Many powerful methods for the description of bound states, static properties

Example: Configuration Interaction
No Core Shell Model (NCSM)

B. R. Barrett et al (2013), Prog. Part. Nucl. Phys. 69, 131
Resonances, scattering and reactions require unified treatment of bound and unbound states

$$\Psi = \sum_\lambda c_\lambda | \lambda \rangle + \sum_\nu \int dr u_\nu(r) | \nu \rangle$$

Static solutions for aggregate system, describe all nucleon close together

Navratil P et. al (2016), Physica Scripta 91, 053002
Resonances, scattering and reactions require unified treatment of bound and unbound states

\[ \Psi = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} \text{Cluster} \\ \text{States} \end{array} \right\rangle + \sum_{\nu} \int dr u_{\nu}(r) \left| \begin{array}{c} \text{Separated} \\ \text{Projectiles} & \text{Targets} \end{array} \right\rangle \]

Continuous microscopic cluster states, describe separated projectiles & targets

Navratil P et. al (2016), Physica Scripta 91, 053002
Resonances, scattering and reactions require unified treatment of bound and unbound states

\[
\Psi = \sum_{\lambda} c_{\lambda} | \Psi_{\lambda} \rangle + \sum_{\nu} \int dr u_{\nu}(r) | \Phi_{\nu} \rangle
\]

No Core Shell Model with continuum (NCSMC)

Similarity renormalization group (SRG) evolution enables and simplifies ab initio many-body calculations.

\[ \tilde{H}_\lambda = \mathcal{U}_\lambda H \mathcal{U}^+_{\lambda} \]

\[ \frac{d\tilde{H}_\lambda}{d\lambda} = -4 \left[ \eta(\lambda),\tilde{H}_\lambda \right] \]

\[ H_{\lambda=\infty} = H \]

\[ \langle k | \tilde{H}_\lambda^{(2)} | k' \rangle \]

(two-body)

\( \lambda_0 > \lambda_1 > \lambda_2 \ldots \)

Bogner S et al. (2010), Prog. Part. Nucl. Phys. 65 (2010)

See talk by Dick Furnsthal
SRG evolution in A-nucleon space induces many-body terms (up to A-body) \`
\[ \tilde{H}_\lambda = \tilde{H}_\lambda^{[1]} + \tilde{H}_\lambda^{[2]} + \tilde{H}_\lambda^{[3]} + \cdots \tilde{H}_\lambda^{[A]} \]
\`

Determined in A=2 system

Determined in A=3 system
SRG evolution in A-nucleon space induces many-body terms (up to A-body)

\[ \tilde{H}_\lambda = \tilde{H}_\lambda^{[1]} + \tilde{H}_\lambda^{[2]} + \tilde{H}_\lambda^{[3]} + \cdots + \tilde{H}_\lambda^{[A]} \]

Varying \( \lambda \) provides diagnostic tool to assess contribution of omitted many-body terms, tests unitarity.
SRG evolution in A-nucleon space induces many-body terms (up to A-body)

\[ \tilde{H}_\lambda = \tilde{H}_\lambda^{[1]} + \tilde{H}_\lambda^{[2]} + \tilde{H}_\lambda^{[3]} + \cdots \tilde{H}_\lambda^{[4]} \]

Nomenclature:

**NN only**: Start with initial T+V\textsubscript{NN} and keep \( \tilde{H}_\lambda^{[1]} + \tilde{H}_\lambda^{[2]} \)

**NN+3N-induced**: Start with initial T+V\textsubscript{NN} and keep \( \tilde{H}_\lambda^{[1]} + \tilde{H}_\lambda^{[2]} + \tilde{H}_\lambda^{[3]} \)

**NN+3N**: Start with initial T+V\textsubscript{NN}+V\textsubscript{3N} and keep \( \tilde{H}_\lambda^{[1]} + \tilde{H}_\lambda^{[2]} + \tilde{H}_\lambda^{[3]} \)
For lightest nuclei, SRG-evolved NN+3N forces yield unitarily equivalent results in smaller spaces

Jurgenson et al. (2009), Phys. Rev. Lett. 103, 082501
SRG yields faster convergence, consistent forces for scattering and reactions

SRG-evolved forces yield unitarily equivalent results for scattering in light nuclei

Rimantas L (2018), Phys. Rev. C 97, 044002
Induced 3N forces are a small price to pay. 3N forces essential for quantitative description!
Starting from SRG-evolved chiral NN+3N forces, can predict nucleon, deuterium scattering on $^4\text{He}$. . .

Proton elastic recoil off helium

Elastic d–$^4\text{He}$ scattering

... as well as polarized deuterium-tritium fusion

Hupin G et al. (2019), Nat. Comm. 10, 351
Scattering & reactions in heavier systems with general configuration-interaction implementation

\[ \Psi = \sum_\lambda c_\lambda \left| \chi_\lambda \right\rangle + \sum_\nu \int dr u_\nu(r) \left| \phi_\nu \right\rangle + \text{additional terms} \]

New formalism enables the description of $^4\text{He} + ^4\text{He}$ scattering, first stage of helium burning

Kravvaris K et al. (2020), in preparation
Advanced (CPU+GPU) architectures are enabling significant speed-ups, higher-fidelity calculations.  

Average 20x speed-up factor for dynamic solutions

Memory of GPU cards present limiting factor in moving beyond $A \approx 12$

Kravvaris K et al. (2020), in preparation
Ab initio theory can now provide predictions for neutron standard light-nuclei cross sections.

Elastic and transfer reactions simultaneously

NN-only

Kravvaris K et al. (2020), in preparation
Main challenges going forward are growing number of reaction channels, basis sizes

1) Can Tensor Network Renormalization Group (TRG) methods be used to push present ab initio reaction theory even further?

2) Can some form of in-medium SRG be used to decouple desired and unwanted (higher energy) reaction channels?

3) Can such a decoupling of reaction channels be used to derive effective nucleus-nucleus interactions (optical potentials)?
Also want to predict capture cross sections. Need SRG evolution of general observables!

No experimental data! Still missing theory UQ

Kravvaris K et al. (2020), in preparation
A somewhat undesirable side effect of SRG is that observables must be evolved consistently.

\[ \tilde{O}_\lambda = \tilde{U}_\lambda \circ \tilde{U}_\lambda^* \]

Final-state transformation \hspace{2cm} Initial-state transformation
A somewhat undesirable side effect of SRG is that observables must be evolved consistently

\[ \tilde{\mathcal{O}}_\lambda = \tilde{U}_\lambda \mathcal{O} \tilde{U}_\lambda^* \]

1) Solve SRG flow equation for \( \tilde{\mathcal{O}}_\lambda \)

2) Compute

\[ \tilde{U}_\lambda = \sum_\alpha |\psi_\alpha(\lambda)\rangle \langle \psi_\alpha(\lambda = \infty)| \]

\[ \begin{align*}
\text{After the evolution} & \\
\text{Before the evolution} & 
\end{align*} \]

A somewhat undesirable side effect of SRG is that observables must be evolved consistently

\[ \tilde{O}_\lambda = \tilde{U}_\lambda O \tilde{U}_\lambda^* \]

Best) Magnus formulation \[ \tilde{U}_\lambda = e^{\tilde{\Omega}_\lambda} \]

\[ \frac{d\tilde{\Omega}_\lambda}{d\lambda} = -\frac{4}{\lambda^5} \left\{ \eta(\lambda) + \frac{1}{2} [\eta(\lambda), \tilde{\Omega}_\lambda] + \cdots \right\} \]

As SRG drives Hamiltonian to band diagonal, observables can grow more off-diagonal.

Electric dipole transition

\[
\hat{D} = \sqrt{\frac{4\pi}{3}} \sum_{i=1}^{A} \frac{\tau_i^z}{2} r_i Y_{10}(\hat{r}_i)
\]

\( ^3S_1(0) \rightarrow ^3P_2(1) \)

Schuster MD et al. (2015), Phys. Rev. C 92, 014320
SRG evolution in A-nucleon space induces many-body terms (up to A-body)

\[ \tilde{\mathcal{O}}_\lambda = \tilde{\mathcal{O}}_\lambda^{[1]} + \tilde{\mathcal{O}}_\lambda^{[2]} + \tilde{\mathcal{O}}_\lambda^{[3]} + \ldots \tilde{\mathcal{O}}_\lambda^{[A]} \]

Evolution implemented in momentum space, harmonic oscillator space, ...
SRG evolution in A-nucleon space induces many-body terms (up to A-body)

\[ \tilde{\mathcal{O}}_{\lambda} = \tilde{\mathcal{O}}^{[1]}_{\lambda} + \tilde{\mathcal{O}}^{[2]}_{\lambda} + \tilde{\mathcal{O}}^{[3]}_{\lambda} + \ldots \tilde{\mathcal{O}}^{[A]}_{\lambda} \]

Varying \( \lambda \) provides diagnostic tool to assess contribution of omitted many-body terms, tests unitarity
For the lightest nuclei, SRG-evolved operators yield unitarily equivalent results for observables.

Omitted induced two-body terms

Omitted induced three-body terms

Induced many-body contributions tend to become progressively smaller.

Omitted induced two-body terms

Omitted induced three-body terms

Omitted induced four-body terms

The extent of the renormalization varies with the range of the observable

\[ \hat{O}(\vec{r}_1, \vec{r}_2) = A \exp \left( -\frac{(\vec{r}_1 - \vec{r}_2)^2}{a_0^2} \right) \]

Varying range

Three-body contribution relatively more important for longer range observables

\[ \hat{O}(\vec{r}_1, \vec{r}_2) = A \exp \left( - \frac{(|\vec{r}_1 - \vec{r}_2|^2)}{a_0^2} \right) \]

Varying range

Convergence of SRG-evolved observables not necessarily faster than for bare operators

\[ {^4}\text{He electric dipole polarizability} \]

Schuster MD et al. (2014), Phys. Rev. C 90, 011301(R)
Gamow-Teller transitions are a form of $\beta$-decay, sensitive to renormalizations due to nuclear medium.
Gamow-Teller decays in light nuclei serve as a test for SRG evolution, 2BC effects.

\[ \Gamma^{GT}(1) + \Gamma^{GT}(2) + \text{MEC} \]

\[ {}^3\text{H} \to {}^3\text{He} \]

$\lambda = 1.6$

$\lambda = 1.8$

$\lambda = 2.0$

N4LO500 NN ($c_D = -1.8$ in MEC), $\hbar\Omega = 20\text{MeV}$
Gamow-Teller decays in light nuclei serve as a test for SRG evolution, 2BC effects.
For the most part, in light nuclei 2-body currents improve agreement with experiment

Gysbers PH et al. (2019), Nat. Phys. 15, 428
Discrepancy between experimental & theoretical $\beta$-decay rates was resolved from first principles ...

$\beta$-decay in $pf$ shell nuclei

$|M_{GT}|$ Theory (unquenched)

$|M_{GT}|$ Experiment

$\bar{\omega}_{NN+3N}$

$1B+2BC$

$\bar{\omega}_{NN+3N}$

$1B+2BC$

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$1B+2BC$
... now the same approach is being applied to $0\nu\beta\beta$ matrix elements

Gysbers PH, Miyagi T. et al., in preparation
... now the same approach is being applied to $0\nu\beta\beta$ matrix elements

Gysbers PH, Miyagi T. et al., in preparation
... now the same approach is being applied to $0\nu\beta\beta$ matrix elements

$^{48}\text{Ca} \rightarrow ^{48}\text{Ti} \ 0\nu\beta\beta$

Ab initio

Challenges going forward

Inclusion of two- and three-body transition operators for capture reactions, nuclear decays

1) Efficient configuration-interactions techniques combined with Tensor Network Renormalization Group (TRG) methods?

2) Normal ordering of operators. How do we quantify uncertainty?

3) How do we quantify effect of neglected higher body terms?