Baryon Resonances in a Coupled Analysis of Meson and Photon induced Reactions

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Introduction: Baryon spectrum in experiment and theory

- above 1.8 GeV much more states are predicted than observed, “Missing resonance problem”

Lattice calculation (single hadron approximation):

![Graph showing $N^*$ and $\Delta^*$ spectrum](graph)

- only about half of the states have **** or *** status
- PDG listing: major part of the information from $\pi N$ elastic 
  (Exception: BnGa multi-channel PWA)

$\Rightarrow$ large coupling to inelastic channels?
Experimental studies of hadronic reactions: major progress in recent years

**Photoproduction:** e.g. from JLab, ELSA, MAMI, GRAAL, SPring-8

- enlarged data base with high quality for different final states
- (double) polarization observables
  → alternative source of information besides $\pi N \rightarrow X$
  → towards a complete experiment: unambiguous determination of the amplitude (up to an overall phase)

**Electroproduction:** e.g. from JLab, MAMI, MIT/Bates

- electroproduction of $\pi N$, $\eta N$, $KY$, $\pi\pi N$
- access the $Q^2$ dependence of the amplitude, information on the internal structure of resonances
Complete Experiment


\[ \hat{M} = iF_1 \vec{\sigma} \cdot \vec{\epsilon} + F_2 \vec{\sigma} \cdot \hat{q} \vec{\sigma} \cdot (\hat{k} \times \vec{\epsilon}) + iF_3 \vec{\sigma} \cdot \hat{k} \hat{q} \cdot \vec{\epsilon} + iF_4 \vec{\sigma} \cdot \hat{q} \hat{q} \cdot \vec{\epsilon} + iF_5 \vec{\sigma} \cdot \hat{k} \hat{k} \cdot \vec{\epsilon} + iF_6 \vec{\sigma} \cdot \hat{q} \hat{k} \cdot \vec{\epsilon} \]

\( F_i \): complex functions of \( \theta, W \), constructed from multipoles \( E_{L\pm}, M_{L\pm} \)

\( \Rightarrow \) 16 polarization observables: asymmetries composed of beam, target and/or recoil polarization measurements

\( \Rightarrow \) Complete Experiment: unambiguous determination of the amplitude


\[ \hat{M} = iF_1 \vec{\sigma} \cdot \vec{\epsilon} + F_2 \vec{\sigma} \cdot \hat{q} \vec{\sigma} \cdot (\hat{k} \times \vec{\epsilon}) + iF_3 \vec{\sigma} \cdot \hat{k} \hat{q} \cdot \vec{\epsilon} + iF_4 \vec{\sigma} \cdot \hat{q} \hat{q} \cdot \vec{\epsilon} + iF_5 \vec{\sigma} \cdot \hat{k} \hat{k} \cdot \vec{\epsilon} + iF_6 \vec{\sigma} \cdot \hat{q} \hat{k} \cdot \vec{\epsilon} \]

\( F_i \) = \( F_i(W, \theta, Q^2) \), multipoles \( E_{L\pm}, M_{L\pm}, L_{L\pm} \) (or \( E_{L\pm}, M_{L\pm}, S_{L\pm} \))

\( \Rightarrow \) 36 polarization observables
Different analyses frameworks: a few examples

- **GWU/SAID approach**: PWA based on Chew-Mandelstam $K$-matrix parameterization
- **unitary isobar models**: unitary amplitudes + Breit-Wigner resonances
  - MAID, Yerevan/JLab, KSU
- **multi-channel $K$-matrix**: BnGa (mostly phenomenological Bgd, N/D approach), Gießen (microscopic Bgd)
- **dynamical coupled-channel (DCC)**: 3-dim scattering eq., off-shell intermediate states
  - ANL-Osaka (EBAC), Dubna-Mainz-Taiphe, Jülich-Bonn

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The Jülich-Bonn DCC approach
Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial-wave basis

\[
\langle L'S'p'|T_{\mu\nu}^{ij}|LSp\rangle \quad = \quad \langle L'S'p'|V_{\mu\nu}^{ij}|LSp\rangle + \\
\sum_{\gamma,L''S''} \int_0^\infty dq \; q^2 \quad \langle L'S'p'|V_{\mu\gamma}^{ij}|L''S''q\rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q|T_{\gamma\nu}^{ij}|LSp\rangle
\]

- **potentials** $V$ constructed from effective $\mathcal{L}$
- **$s$-channel diagrams**: $T^P$
  - genuine resonance states
- **$t$- and $u$-channel**: $T^{NP}$
  - dynamical generation of poles
  - partial waves strongly correlated
Dynamical coupled-channels (DCC): \textit{simultaneous} analysis of different reactions

The scattering equation in partial-wave basis

\[
\langle L'S' p'| T_{\mu\nu}^{ll} | LSp \rangle = \langle L'S' p'| V_{\mu\nu}^{ll} | LSp \rangle + \sum_{\gamma, L'' S''} \int_{0}^{\infty} dq \ q^2 \langle L'S' p'| V_{\mu\gamma}^{ll} | L'' S'' q \rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L'' S'' q | T_{\gamma\nu}^{ll} | LSp \rangle
\]

- \textbf{free parameters} fitted to data:

\textbf{s-channel: resonances} \( (T^P) \)

\textbf{t- and u-channel exchange: "background"} \( (T^{NP}) \)

\[
m_{bare} + f_{\pi NN^*}
\]

\textbf{cut offs} \( \Lambda \) in form factors \( \left( \frac{\Lambda^2 - m_{ex}^2}{\Lambda^2 + q^2} \right)^{n} \)

(couplings fixed from SU(3))
Resonance states: Poles in the $T$-matrix on the 2\textsuperscript{nd} Riemann sheet

- pole position $E_0$ is the same in all channels
- residues $\rightarrow$ branching ratios

$\text{Re}(E_0)$ = “mass”, $-2\text{Im}(E_0)$ = “width”

- (2-body) unitarity and analyticity respected
- 3-body $\pi\pi N$ channel:
  - parameterized effectively as $\pi\Delta$, $\sigma N$, $\rho N$
  - $\pi N/\pi\pi$ subsystems fit the respective phase shifts
  $\downarrow$ branch points move into complex plane
### Multipole amplitude

\[
M^{lj}_{\mu \gamma} = V^{lj}_{\mu \gamma} + \sum_\kappa T^{lj}_{\mu \kappa} G_\kappa V^{lj}_{\kappa \gamma}
\]

(partial wave basis)

\[
V_{\mu \gamma} \approx \tilde{\gamma}_\mu^a (q) P_{\mu i}^{NP} (E) + \sum_i \gamma_{\mu; i}^a (q) P_i^P (E) \frac{m_{N_i}}{E - m_{i_i}^b}
\]

\[m = \pi, \eta, B = N, \Delta\]

\[T_{\mu \kappa}: \text{Jülich hadronic } T\text{-matrix} \quad \rightarrow \text{Watson's theorem fulfilled by construction} \]

\[\rightarrow \text{analyticity of } T: \text{extraction of resonance parameters}\]

**Photoproduction potential:** approximated by energy-dependent polynomials
Data analysis and fit results
Combined analysis of pion- and photon-induced reactions

Fit parameters:

- $\pi N \rightarrow \pi N$
  $\pi^- p \rightarrow \eta n, K^0 \Lambda, K^0 \Sigma^0, K^+ \Sigma^-$
  $\pi^+ p \rightarrow K^+ \Sigma^+$

$\Rightarrow$ 128 free parameters

  - 11 $N^*$ resonances $\times$ (1 $m_{\text{bare}}$ + couplings to $\pi N$, $\rho N$, $\eta N$, $\pi \Delta$, $K \Lambda$, $K \Sigma$))
  - 10 $\Delta$ resonances $\times$ (1 $m_{\text{bare}}$ + couplings to $\pi N$, $\rho N$, $\pi \Delta$, $K \Sigma$))

- $\gamma p \rightarrow \pi^0 p, \pi^+ n, \eta p, K^+ \Lambda$

$\Rightarrow$ $\sim$ 500 free parameters

  - couplings of the polynomials

- $\sim$ 40,000 data points

$\downarrow$ calculations on the JURECA supercomputer: parallelization in energy ($\sim 300 - 400$ processes)
Preliminary: $K^+\Lambda$ photoproduction in the JüBo model

simultaneous fit of $\gamma p \rightarrow \pi^0 p$, $\pi^+ n$, $\eta p$, $K^+\Lambda$ and $\pi N \rightarrow \pi N$, $\eta N$, $K\Lambda$, $K\Sigma$

$\gamma p \rightarrow K^+\Lambda$:

- **Differential cross section**

  ![Differential cross section graph](#)


- **Recoil polarization**

  ![Recoil polarization graph](#)


- **Beam asymmetry**

  ![Beam asymmetry graph](#)


- **Target asymmetry**

  ![Target asymmetry graph](#)

  - LL09: Lleres EPJA 39 (2009)
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\[ \gamma p \rightarrow K^+\Lambda: \]

- $C_x$
  - BR07: Bradford PRC 75 (2007)
  - 1838 MeV
  - 2169 MeV

- $O_x$
  - LL09: Lleres EPJA 39 (2009)
  - 1649 MeV
  - 1883 MeV

- $C_z$
  - BR07: Bradford PRC 75 (2007)
  - 1987 MeV
  - 2169 MeV

- $O_z$
  - LL09: Lleres EPJA 39 (2009)
  - 1728 MeV
  - 1808 MeV
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Impact of new polarization data
Recent new data on $\gamma p \rightarrow \pi N$:

- $E, G, H, P, T$ in $\gamma p \rightarrow \pi^0 p$ from ELSA Thiel et al. PRL 109, 102001 (2012); Gottschall et al. PRL 112, 012003 (2014); Hartmann et al. PLB 748, 212 (2015); Thiel et al. arXiv:1604.02922
- $\Sigma$ in $\gamma p \rightarrow \pi^0 p$ and $\gamma p \rightarrow \pi^+ n$ from JLab Dugger et al. PRC 88, 065203 (2013) 89, 029901(E) (2014)
- $\Sigma$ in $\gamma p \rightarrow \pi^0 p$ from MAMI Hornidge et al. PRL 111, 062004 (2013)

⇒ included in the SAID, BnGa, JüBo fits

- compare multipoles before and after the inclusion of the new data
- conversion to a common solution?
The SAID, BnGa and JüBo approaches

<table>
<thead>
<tr>
<th>All three approaches:</th>
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<tbody>
<tr>
<td>• coupled channel effects</td>
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<td>• unitarity (2 body)</td>
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<td>• amplitudes are analytic functions of the invariant mass</td>
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**SAID PWA**

Based on Chew-Mandelstam $K$-matrix

- $K$-matrix elements parameterized as energy-dependent polynomials
- Resonance poles are dynamically generated (except for the $\Delta(1232)$)
- Masses, width and hadronic couplings from fits to pion-induced $\pi N$ and $\eta N$ production

**Bonn-Gatchina (BnGa) PWA**

Multi-channel PWA based on $K$-matrix (N/D)

- Mostly phenomenological model
- Resonances added by hand
- Resonance parameters determined from large experimental data base: pion-, photon-induced reactions, 3-body final states

**Jülich-Bonn (JüBo) DCC model**

Based on a Lippmann-Schwinger equation formulated in TOPT

- Hadronic potential from effective Lagrangians
- Photoproduction parameterized by energy-dependent polynomials
- Resonances as $s$-channel states (dynamical generation possible)
- Resonance parameters determined from pion- and photon-induced data
Fig. 1. Selected data and the predictions from the four different PWAs: black solid line: BnGa2011-02, blue dashed: JüBo2015B, green dotted: MAID2007, red dash-dotted: SAID CM12. The predictions are based on fits which did not yet use these new data. The new data are shown for the beam asymmetry $\Xi$ for $\gamma p \rightarrow \Lambda n$ [81] (1st row), for the beam asymmetry $\Xi$ in the low-energy region [80] and at higher energies (2nd row) for $\gamma p \rightarrow \Lambda^0 p$, (2nd and 3rd row). The next three rows show $T$ [82], $G$ [38,83], and $E$ [37,84] for $\gamma p \rightarrow \Lambda^0 p$. Note that the data from refs. [80] and [81] are included in the fits of JüBo2015 B and SAID CM12.


Predictions: black solid lines: BnGa, red dash-dotted: SAID, blue dashed: JüBo, green dotted: MAID
Fig. 2. The new fit results of the different PWAs in comparison with the new data: black solid line: BnGa, blue dashed: JüBo, red dash-dotted: SAID. New data are shown for the beam asymmetry \( \Xi \) for \( \gamma p \rightarrow \Lambda + n \) [81] (1st row), for the beam asymmetry \( \Xi \) in the low-energy region [80] and at higher energies (2nd row) for \( \gamma p \rightarrow \Lambda^0_p \), (2nd and 3rd row). The next three rows show \( T \), \( G \) [38,83], and \( E \) [37,84] for \( \gamma p \rightarrow \Lambda^0_p \). The BnGa fit did not yet use the data on the beam asymmetry \( \Xi \) for \( \gamma p \rightarrow \Lambda^0_p \) in the low-energy region [80]. Nevertheless, the new fit is fully consistent with the new data.

The \( M_{1-} \) multipole (fig. 3(c), (d)) drives the excitation of the \( J^P = 1/2^+ \) partial wave containing the Roper \( N(1440)1/2^+ \) resonance, the three-star \( N(1710)1/2^+ \) resonance, the one-star \( \Sigma(1750)1/2^+ \), and the four-star \( \Sigma(1910)1/2^+ \). The imaginary part of the \( M_{1-} \) multipole evidences clearly \( N(1440)1/2^+ \), the contributions from the higher-mass resonances are small. The new data lead to a small improvement of the consistency of the results for the imaginary part of the multipole. In the real part a significant improvement can be observed.


Fits: black solid lines: BnGa, red dash-dotted: SAID, blue dashed: JüBo
Comparison of multipoles before & after including the new data: Selected examples

- **Before** and **After**
- **E\textsubscript{0\textregistered}(\pi^0\textregistered p)**
- **E\textsubscript{0\textregistered}(\pi^0\textregistered p)**
- **E\textsubscript{2\textregistered}(\pi^0\textregistered p)**
- **E\textsubscript{2\textregistered}(\pi^0\textregistered p)**

- **M\textsubscript{1\textregistered}(\pi^0\textregistered p)**
- **M\textsubscript{1\textregistered}(\pi^0\textregistered p)**
- **M\textsubscript{2\textregistered}(3/2)p**
- **M\textsubscript{2\textregistered}(3/2)p**

- **Re A [mfm]**
- **Im A [mfm]**

- **W [MeV]**

- **Black solid lines:** BnGa, **red dash-dotted:** SAID, **blue dashed:** JüBo, **green dotted:** MAID
Consistency of the results

Pairwise variances between two PWAs:

\[
\text{var}(1, 2) = \frac{1}{2} \sum_{i=1}^{16} (\mathcal{M}_1(i) - \mathcal{M}_2(i)) (\mathcal{M}_1^*(i) - \mathcal{M}_2^*(i))
\]

\(\mathcal{M}: \gamma p \rightarrow \pi^0 p\) multipoles up to \(L = 4\)

- beyond 1.7 GeV: BnGa, SAID, JüBo multipoles now in closer agreement
- 1.5 to 1.7 GeV:
  - BnGa agrees well with SAID and with JüBo
  - larger discrepancies between SAID and JüBo
Summary

• Progress in experimental and theoretical study of the baryon spectrum

• Jülich-Bonn model:
  - DCC approach that respects analyticity and (2 body) unitarity
  - simultaneous analysis of pion- and photon-induced reactions
  - preliminary results for $K^+\Lambda$ photoproduction

• Impact of new polarization data for pion photoproduction from ELSA, CLAS, MAMI:
  - joint analysis of the BnGa, SAID and JüBo groups
  - comparison of the multipoles before and after the inclusion of the new data

  → agreement between the three analyses is improved!
Thank you for your attention!