The Roles of Nuclear Physics and the Maximum Mass in Constraining the Neutron Star Radius

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Bayesian Methods in Nuclear Physics
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The Dense Matter Equation of State and Neutron Star Structure
  - General Causality, Maximum Mass and GR Limits
  - Neutron Matter and the Nuclear Symmetry Energy
  - Theoretical and Experimental Constraints on the Symmetry Energy

Extrapolating to High Densities with Piecewise Polytropes

Constraints on Neutron Star Radii

Universal Relations

Observational Constraints on Radii
  - Photospheric Radius Expansion Bursts
  - Thermal Emission from Quiescent Binary Sources
  - Effects of Systematic Uncertainties

Further Observations of Masses and Radii
Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G (mc^2 + 4\pi pr^3)(\varepsilon + p)}{c^4 r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi \frac{\varepsilon}{c^2} r^2$$

Equation of State

Observations
The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).

\[ p = \varepsilon - \varepsilon_0 \]

\[ w = \varepsilon / \varepsilon_0 \]

\[ y = p / \varepsilon_0 \]

\[ x = r \sqrt{G \varepsilon_0 / c^2} \]

\[ z = m \sqrt{G^3 \varepsilon_0 / c^2} \]

\[ \varepsilon_0 \] is the only EOS parameter.

The TOV solutions scale with \( \varepsilon_0 \).

\[ p = 0 \]

\[ \varepsilon_0 \]
A lower limit to the maximum mass sets a lower limit to the radius for a given mass.

Similarly, a precision upper limit to $R$ sets an upper limit to the maximum mass.

\[ R_{1.4} > 8.15 M_\odot \text{ if } M_{\text{max}} \geq 2.01 M_\odot. \]

\[ M_{\text{max}} < 2.4 M_\odot \text{ if } R < 10.3 \text{ km}. \]

If quark matter exists in the interior, the minimum radii are substantially larger.
Although simple average mass of w.d. companions is 0.23 M⊙ larger, weighted average is 0.04 M⊙ smaller.

Demorest et al. 2010
Fonseca et al. 2016
Antoniadis et al. 2013
Barr et al. 2016
Champion et al. 2008
Mass-Radius Diagram and Theoretical Constraints

GR: $R > 2GM/c^2$

$P < \infty$: $R > (9/4)GM/c^2$

causality: $R \gtrsim 2.9GM/c^2$

- normal NS
- SQS

$R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$
Radii are highly correlated with the neutron star matter pressure around \( n_s - 2n_s \approx (0.16 - 0.32) \text{ fm}^{-3} \).

(Lattimer & Prakash 2001)

Neutron star matter is nearly purely neutrons, \( x \sim 0.04 \).

Nuclear symmetry energy

\[
S(n) \equiv E(n, x = 0) - E(n, 1/2)
\]

\[
E(n, x) \approx E(n, 1/2) + S_2(n)(1 - 2x)^2 + S_4(n)(1 - 2x)^4 \ldots
\]

\[
S(n) \approx S_2(n) \approx S_v + \frac{L}{3n_s}(n - n_s) + \frac{K_{\text{sym}}}{18} \left( \frac{n - n_s}{n_s} \right)^2 \ldots
\]

\( S_v \sim 32 \text{ MeV}; L \sim 50 \text{ MeV} \) from nuclear systematics.

Neutron matter energy and pressure at \( n_s \):

\[
E(n_s, 0) \approx S_v + E(n_s, 1/2) = S_v - B \sim 16 \text{ MeV}
\]

\[
p(n_s, 0) = \left( n^2 \frac{\partial E(n, 0)}{\partial n} \right)_{n_s} \approx \frac{Ln_s}{3} \sim 2.5 \text{ MeV fm}^{-3}
\]
The liquid droplet model is a useful frame of reference. Its symmetry parameters $S_v$ and $S_s$ are related to $S_v$ and $L$:

$$\frac{S_s}{S_v} \simeq \frac{aL}{r_o S_v} \left[ 1 + \frac{L}{6S_v} - \frac{K_{sym}}{12L} + \ldots \right].$$

- Symmetry contribution to the binding energy:

$$E_{sym} \simeq S_v A l^2 \left[ 1 + \frac{S_s}{S_v A^{1/3}} \right]^{-1}.$$

- Giant Dipole Resonance (dipole polarizability)

$$\alpha_D \simeq \frac{AR^2}{20S_v} \left( 1 + \frac{5}{3} \frac{S_s}{S_v A^{1/3}} \right).$$

- Neutron Skin Thickness

$$r_{np} \simeq \sqrt{\frac{3}{5}} \frac{2r_o l}{3} \frac{S_s}{S_v} \left( 1 + \frac{S_s}{S_v A^{1/3}} \right)^{-1} \left( 1 + \frac{10}{3} \frac{S_s}{S_v A^{1/3}} \right).$$
Theoretical Neutron Matter Calculations

Nuclei provide information for matter up to $n_s$.

Theoretical studies, beginning from fitting low-energy neutron scattering data and few-body calculations of light nuclei, can probe higher densities.

- **Auxiliary Field Diffusion Quantum Monte Carlo** (Gandolfi & Carlson)
- **Chiral Lagrangian Expansion** (Drischler, Hebeler & Schwenk)

Gandolfi et al. (2015)

Drischler et al. (2015)
Theoretical and Experimental Constraints

H: Chiral Lagrangian

G: Quantum Monte Carlo

$S_v - L$ constraints from Hebeler et al. (2012)

Neutron matter constraints are compatible with experimental constraints.
Neutron Star Crusts

The evidence is overwhelming that neutron stars have crusts.

- Neutron star cooling, both long term (ages up to millions of years) and transient (days to years), supports the existence of $\sim 0.5 - 1$ km thick crusts with masses $\sim 0.02 - 0.05 M_\odot$.

- Pulsar glitches are best explained by $n\,^1S_0$ superfluidity, largely confined to the crust, $\Delta I/I \sim 0.01 - 0.05$.

The crust EOS, dominated by relativistic degenerate electrons, is very well understood.
Piecewise Polytropes

Crust EOS is known: \( n < n_0 = 0.4n_s \).

Read, Lackey, Owen & Friedman (2009) found high-density EOS can be modeled as piecewise polytropes with 3 segments. They found universal break points \( n_1 \approx 1.85n_s, n_2 \approx 3.7n_s \) optimized fits to the entire family of modeled EOSs.

For \( n_0 < n < n_1 \), assume neutron matter EOS. Arbitrarily choose \( n_3 = 7.4n_s \).

For a given \( p_1 \) (or \( \Gamma_1 \)):
0 \( < \Gamma_2 < \Gamma_{2c} \) or \( p_1 < p_2 < p_{2c} \).
0 \( < \Gamma_3 < \Gamma_{3c} \) or \( p_2 < p_3 < p_{3c} \).

Minimum values of \( p_2, p_3 \) set by \( M_{\text{max}} \); maximum values set by causality.
Where EOS gives $c_s > c$, force $c_s = c$. 
Maximum Mass and Causality Constraints

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- $p_3 < p_2$
- $c_s^2 < 1$ throughout the star

Graph showing the relationship between $p_2$, $c_s^2$, and $M_{\text{max}}$.
Radius - $p_1$ Correlation

$c_s^2 < 1$ throughout star

$M_{\text{max}}$
- $2.50M_\odot$
- $2.30M_\odot$
- $2.10M_\odot$
- $2.01M_\odot$
- $1.90M_\odot$

$R_{1,4}$ (km)

$p_1$ (MeV fm$^{-3}$)
Mass-Radius Constraints from Causality

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\[ P(\mathcal{M}|\mathcal{D}) \propto P(\mathcal{D}|\mathcal{M})P(\mathcal{M}) \]

\[ \mathcal{M} \rightarrow (p_1, p_2, p_3), \quad \mathcal{D} \rightarrow M(R) \]

\[ P(p_i) = \int dp_j dp_k P(\mathcal{D}|\mathcal{M})P(\mathcal{M}) \]

\[ P(\hat{R}|\hat{M}) = \int dp_1 dp_2 dp_3 P(\mathcal{D}|\mathcal{M})P(\mathcal{M})\delta[R(\hat{M}, p_1, p_2, p_3) - \hat{R}] \]

\[ P(\hat{R}) = \int dM dp_1 dp_2 dp_3 P(M)P(\mathcal{D}|\mathcal{M})P(\mathcal{M})\delta[R(M, p_1, p_2, p_3) - \hat{R}] \]
Piecewise-Polytrope $R_{M=1.4}$ Distributions

$c_s^2 < 1$ throughout star
$c_s^2 = 1$ arbitrarily enforced

frequency

$M_{\text{max}}$
- 2.50$M_{\odot}$
- 2.30$M_{\odot}$
- 2.10$M_{\odot}$
- 2.01$M_{\odot}$
- 1.90$M_{\odot}$
Assumes $P(M)$ from observed pulsar-timing masses

- $c_s^2 < 1$ throughout star
- $c_s^2 = 1$ arbitrarily enforced

- $M_{\text{max}}$
  - 2.50$M_\odot$
  - 2.30$M_\odot$
  - 2.10$M_\odot$
  - 2.01$M_\odot$
  - 1.90$M_\odot$
Upper Limits to Maximum Mass

The Roles of Nuclear Physics and the Maximum Mass in Constraining the Neutron Star Radius
Universal Relations

With the assumptions

- Known crust EOS
- Bounded neutron matter EOS \( (p_{\text{min}} < p_1 < p_{\text{max}}) \)
- Two piecewise polytropes for \( p > p_1 \)
- Causality is not violated
- \( M_{\text{max}} \) is limited from below

tight correlations among the compactness, moment of inertia, binding energy and tidal deformability result.
Moment of Inertia - Compactness Correlations

\[ M_{\text{max}} > 1.90 \, M_\odot \]
\[ \text{error}_{\text{mean}} = 0.011 \]
\[ \text{error}_{\text{max}} = 0.069 \]

\[ M_{\text{max}} > 2.01 \, M_\odot \]
\[ \text{error}_{\text{mean}} = 0.010 \]
\[ \text{error}_{\text{max}} = 0.048 \]

\[ M_{\text{max}} > 2.10 \, M_\odot \]
\[ \text{error}_{\text{mean}} = 0.009 \]
\[ \text{error}_{\text{max}} = 0.038 \]

\[ M_{\text{max}} > 2.30 \, M_\odot \]
\[ \text{error}_{\text{mean}} = 0.006 \]
\[ \text{error}_{\text{max}} = 0.026 \]
\[ \frac{I}{M^3} = \alpha_0/\beta^2 + \alpha_1/\beta + \alpha_2 \]
Binding Energy - Compactness Correlations

\[ M_{\text{max}} > 1.90 \, M_{\odot} \]
error mean = 0.021
error max = 0.197

\[ M_{\text{max}} > 2.01 \, M_{\odot} \]
error mean = 0.019
error max = 0.163

\[ M_{\text{max}} > 2.10 \, M_{\odot} \]
error mean = 0.016
error max = 0.144

\[ M_{\text{max}} > 2.30 \, M_{\odot} \]
error mean = 0.012
error max = 0.077
Binding Energy - Mass Correlations

\[ M_{\text{max}} > 1.90 \, M_\odot \]
\[ \text{error}_{\text{mean}} = 0.020 \]
\[ \text{error}_{\text{max}} = 0.072 \]

\[ M_{\text{max}} > 2.01 \, M_\odot \]
\[ \text{error}_{\text{mean}} = 0.019 \]
\[ \text{error}_{\text{max}} = 0.060 \]

\[ M_{\text{max}} > 2.10 \, M_\odot \]
\[ \text{error}_{\text{mean}} = 0.019 \]
\[ \text{error}_{\text{max}} = 0.049 \]

\[ M_{\text{max}} > 2.30 \, M_\odot \]
\[ \text{error}_{\text{mean}} = 0.015 \]
\[ \text{error}_{\text{max}} = 0.051 \]
Tidal Deformatibility - Moment of Inertia

$M_\text{max} > 1.90 \, M_\odot$
$error_{\text{mean}} = 0.003$
$error_{\text{max}} = 0.039$

$M_\text{max} > 2.01 \, M_\odot$
$error_{\text{mean}} = 0.002$
$error_{\text{max}} = 0.018$

$M_\text{max} > 2.10 \, M_\odot$
$error_{\text{mean}} = 0.002$
$error_{\text{max}} = 0.015$

$M_\text{max} > 2.30 \, M_\odot$
$error_{\text{mean}} = 0.002$
$error_{\text{max}} = 0.010$
Simultaneous Mass/Radius Measurements

- Measurements of flux $F_\infty = \left(\frac{R_\infty}{D}\right)^2 \sigma T_{\text{eff}}^4$ and color temperature $T_c \propto \lambda_{\text{max}}^{-1}$ yield an apparent angular size (pseudo-BB):

  $$\frac{R_\infty}{D} = \frac{(R/D)}{\sqrt{1 - 2GM/Rc^2}}$$

- Observational uncertainties include distance $D$, interstellar absorption $N_H$, atmospheric composition. Best chances are:
  - Isolated neutron stars with parallax (atmosphere ??)
  - Quiescent low-mass X-ray binaries (QLMXBs) in globular clusters (reliable distances, low $B$ H-atmospheres)
  - Bursting sources (XRBs) with peak fluxes close to Eddington limit (gravity balances radiation pressure)

$$F_{\text{Edd}} = \frac{cGM}{\kappa D^2}\sqrt{1 - 2GM/Rc^2}$$
PRE $M - R$ Estimates

Ozel & Freire (2016)
QLMXB $M - R$ Estimates

Ozel & Freire (2016)
Combined $R$ fits

Assumed $P(M)$ is that measured from pulsar timing ($\bar{M} = 1.4M_{\odot}$).

Ozel & Freire (2015)
Folding Observations with Piecewise Polytropes

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Bayesian Analyses

Ozel et al. (2015)

Steiner et al. (2012)
Role of Systematic Uncertainties

Apparent tension between nuclear physics expectations and astronomical observations for the value of the mean radius.

Possible explanations:

- Non-uniform temperature distributions

\[ R^2 T^4 = R_1^2 T_1^4 + R_2^2 T_2^4 \]

\[ R^2 \left( 3 - \frac{\hbar \nu_0}{kT} \right) \simeq 0 \]

\[ R_1^2 \left( 3 - \frac{\hbar \nu_0}{kT_1} \right) e^{-\hbar \nu_0/kT_1} + R_2^2 \left( 3 - \frac{\hbar \nu_0}{kT_2} \right) e^{-\hbar \nu_0/kT_2} \simeq 0 \]

\[ R_1 = R_2 = Ry/\sqrt{2}, \quad x = T_2/T_1, \quad z = T/T_1 \]

\[ z^4 = y^2(1 + x^4)/2, \quad z = 1 + (1 - z/x)e^{-3z/x} e^{3z} \]

For \( 0 < x < 1, z < 1 \) and \( y > 1 \), so that

\[ R_1^2 + R_2^2 = y^2 R^2 > R^2 \]
- Interstellar absorption
- Atmospheric composition: In quiescent sources, He or C atmospheres can produce about 50% larger radii.
- Non-spherical geometries: In bursting sources, improper to use spherically-symmetric Eddington flux formula.
- Disc shadowing: In burst sources, leads to underprediction of \( A = f_c^{-4}(R_\infty/D)^2 \), overprediction of \( \alpha \propto 1/\sqrt{A} \), and underprediction of \( R_\infty \propto \sqrt{\alpha} \).
Conclusions

▶ Neutron matter calculations and nuclear experiments are consistent with each other and set reasonably tight constraints on symmetry energy behavior near the nuclear saturation density.

▶ These constraints, together with assumptions that neutron stars have hadronic crusts and are causal, predict neutron star radii $R_{1.4}$ in the range $12.0 \pm 1.0$ km.

▶ Astronomical observations of photospheric radius expansion X-ray bursts and quiescent sources in globular clusters suggest $R_{1.4} \sim 10.5 \pm 1$ km, unless maximum mass and EOS priors are implemented.

▶ Should observations require smaller or larger neutron star radii, a strong phase transition in extremely neutron-rich matter just above the nuclear saturation density is suggested. Or should GR be modified?