Quasielastic e/ν Scattering and Two-Body Currents

- Nuclear interactions and electroweak currents: a review
- Role of two-body currents in inclusive e/ν scattering: the enhancement of the one-body response
- Connection between the short-range structure of nuclei and the excess strength induced by two-body currents
- Summary

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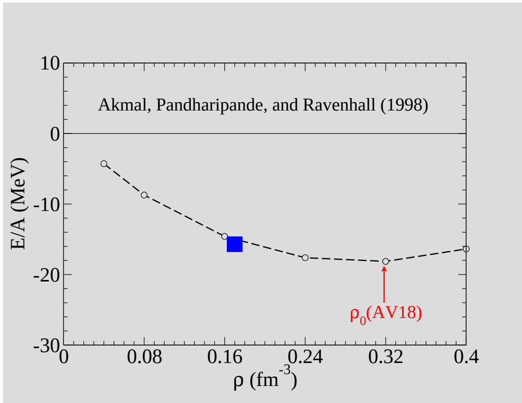
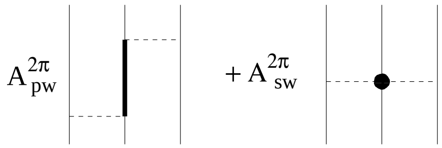
Nuclear Interactions

- $v = v_0(\text{static}) + v_p(\text{momentum dependent}) \rightarrow v(\text{OPE})$ fits large NN database with $\chi^2 \simeq 1$

- NN interactions alone fail to predict:
  1. spectra of light nuclei
  2. Nd scattering
  3. nuclear matter $E_0(\rho)$



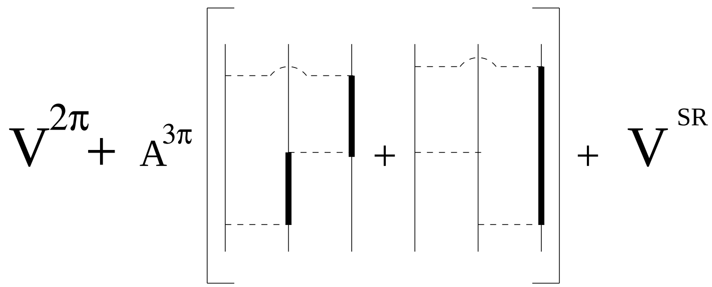
- 2π-NNN interactions:



NNN Interactions: Beyond 2π-Exchange

Pieper and Wiringa, private communication

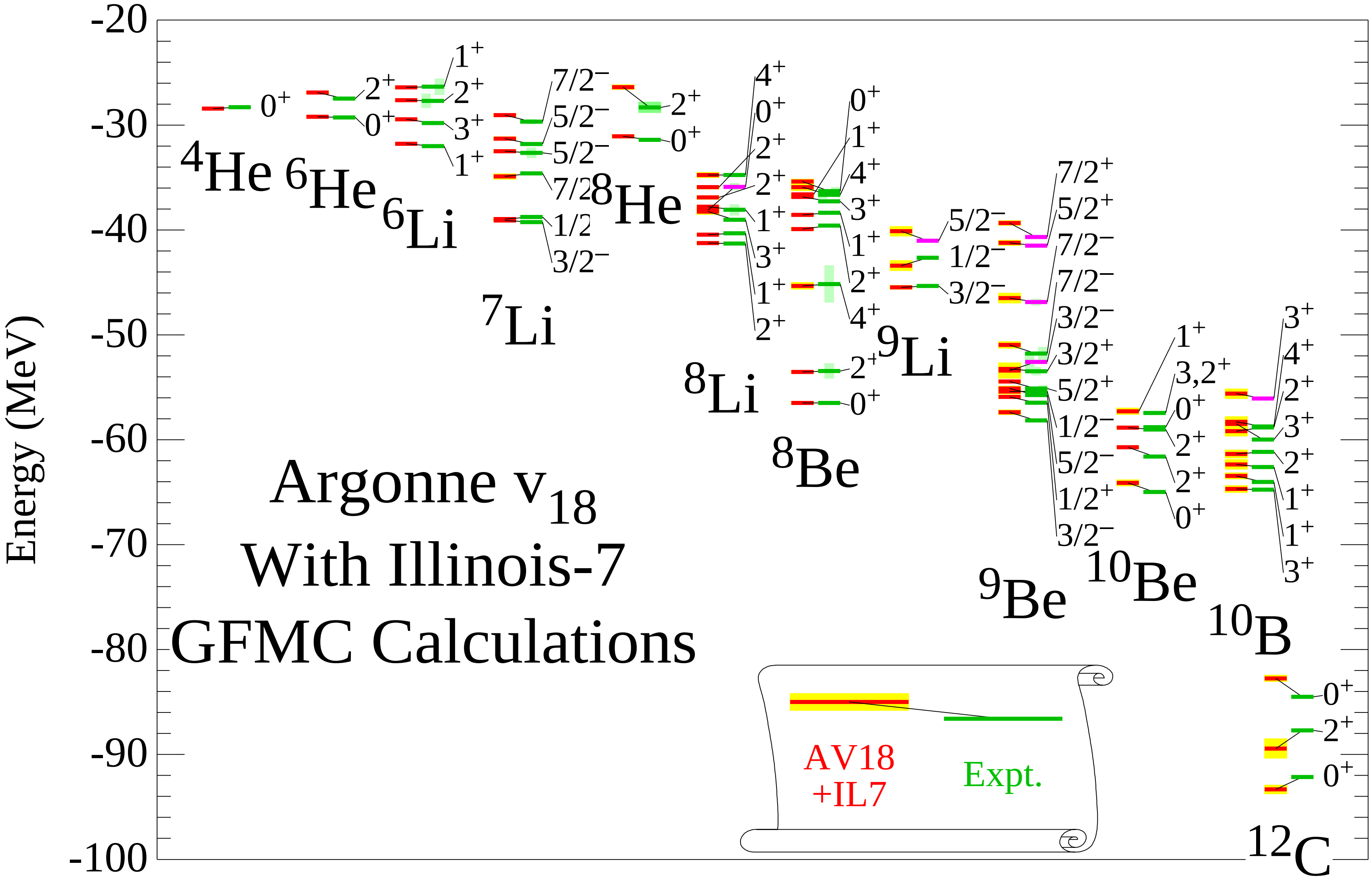
IL7 model has important $T = 3/2$ terms



parameters (~ 4) fixed by a best fit to the energies of low-lying states (~ 17) of nuclei with $A \leq 10$

AV18/IL7 Hamiltonian reproduces well:

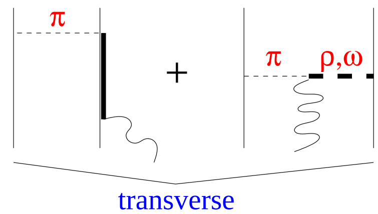
- spectra of A=9–12 nuclei (attraction provided by IL7 in $T = 3/2$ triplets crucial for p-shell nuclei)
- low-lying p-wave resonances with J^π=$3/2^-$ and $1/2^-$ respectively, as well as low-energy s-wave ($1/2^+$) scattering


Argonne v_{18}
With Illinois-7
GFMC Calculations
Energy (MeV)
-20
-30
-40
-50
-60
-70
-80
-90
-100
^{4}He
^{6}He
^{6}Li
^{7}Li
^{8}He
^{8}Li
^{8}Be
^{9}Li
^{9}Be
^{10}Be
^{10}B
^{12}C
AV18
+IL7
Expt.

EM Current Operators I

Marcucci *et al.* (2005)

$\mathbf{j} = \mathbf{j}^{(1)}$

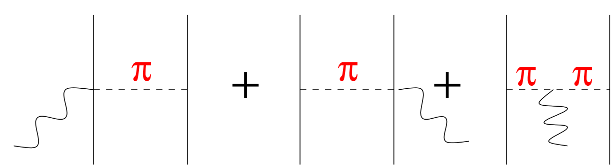
$+ \mathbf{j}^{(2)}(\mathrm{v}) +$ 

$+ \mathbf{j}^{(3)}(\mathrm{V}^{2\pi})$

- Static part v_0 of v from π-like (PS) and ρ-like (V) exchanges
- Currents from corresponding PS and V exchanges, for example

$$\mathbf{j}_{ij}(v_0; PS) = \mathrm{i}\, G_E^V(Q^2)\, (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z\, v_{PS}(k_j) \Big[\boldsymbol{\sigma}_i - \frac{\mathbf{k}_i - \mathbf{k}_j}{k_i^2 - k_j^2} (\boldsymbol{\sigma}_i \cdot \mathbf{k}_i) \Big] (\boldsymbol{\sigma}_j \cdot \mathbf{k}_j) + i \rightleftharpoons j$$

with $v_{PS}(k) = v^{\sigma\tau}(k) - 2\, v^{t\tau}(k)$ projected out from v_0 terms

$\mathbf{j}^{(2)}(\mathrm{v}) \xrightarrow{\text{long range}}$ 

EM Current Operators II

- Currents from v_p via minimal substitution in i) explicit and ii) implicit p-dependence, the latter from

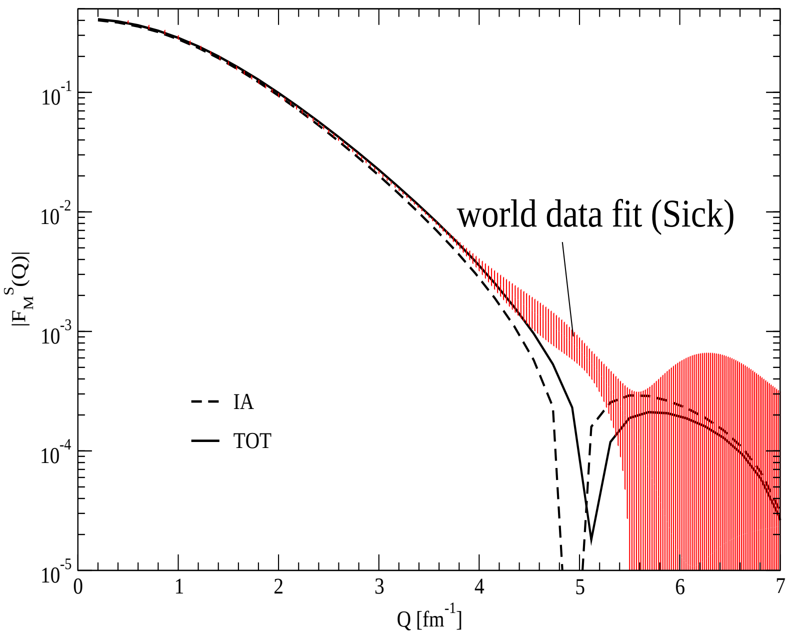
$$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = -1 + (1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\, \mathrm{e}^{\mathrm{i}(\mathbf{r}_{ji}\cdot\mathbf{p}_i + \mathbf{r}_{ij}\cdot\mathbf{p}_j)}$$

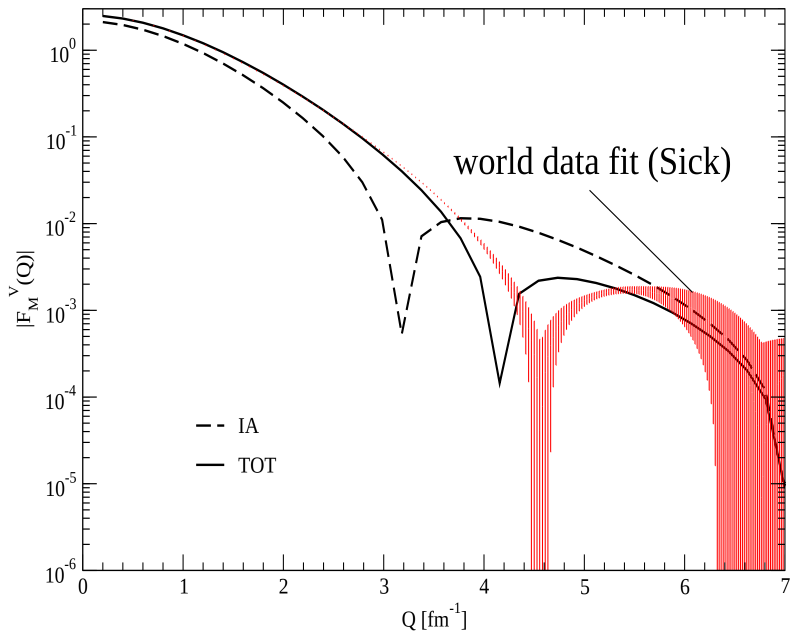
- Currents are conserved, contain no free parameters, and are consistent with short-range behavior of v and $V^{2\pi}$, but are not unique

$$\mathbf{q} \cdot \left[\mathbf{j}^{(1)} + \mathbf{j}^{(2)}(v) + \mathbf{j}^{(3)}(V^{2\pi})\right] = \left[T + v + V^{2\pi}\,,\, \rho\right]$$

- EM current (and charge) operators also derived in χEFT up to one loop (Pastore *et al.* 2009-2013; Kölling *et al.* 2009-2011)

Isoscalar and Isovector Magnetic Form Factors of ^{3}He/^{3}H

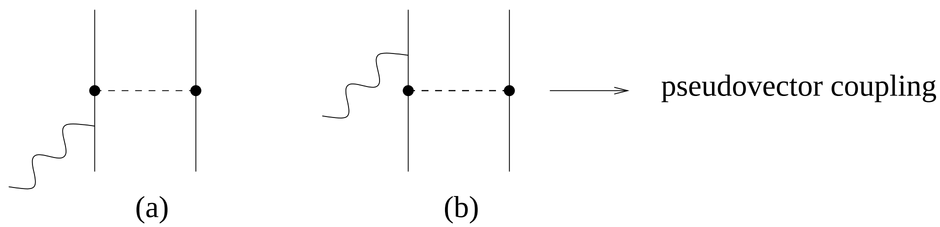




- Isoscalar two-body current contributions small
- Leading isovector two-body currents from OPE

EM Charge Operators

Leading two-body charge operator derived from analysis of the virtual pion photoproduction amplitudes:



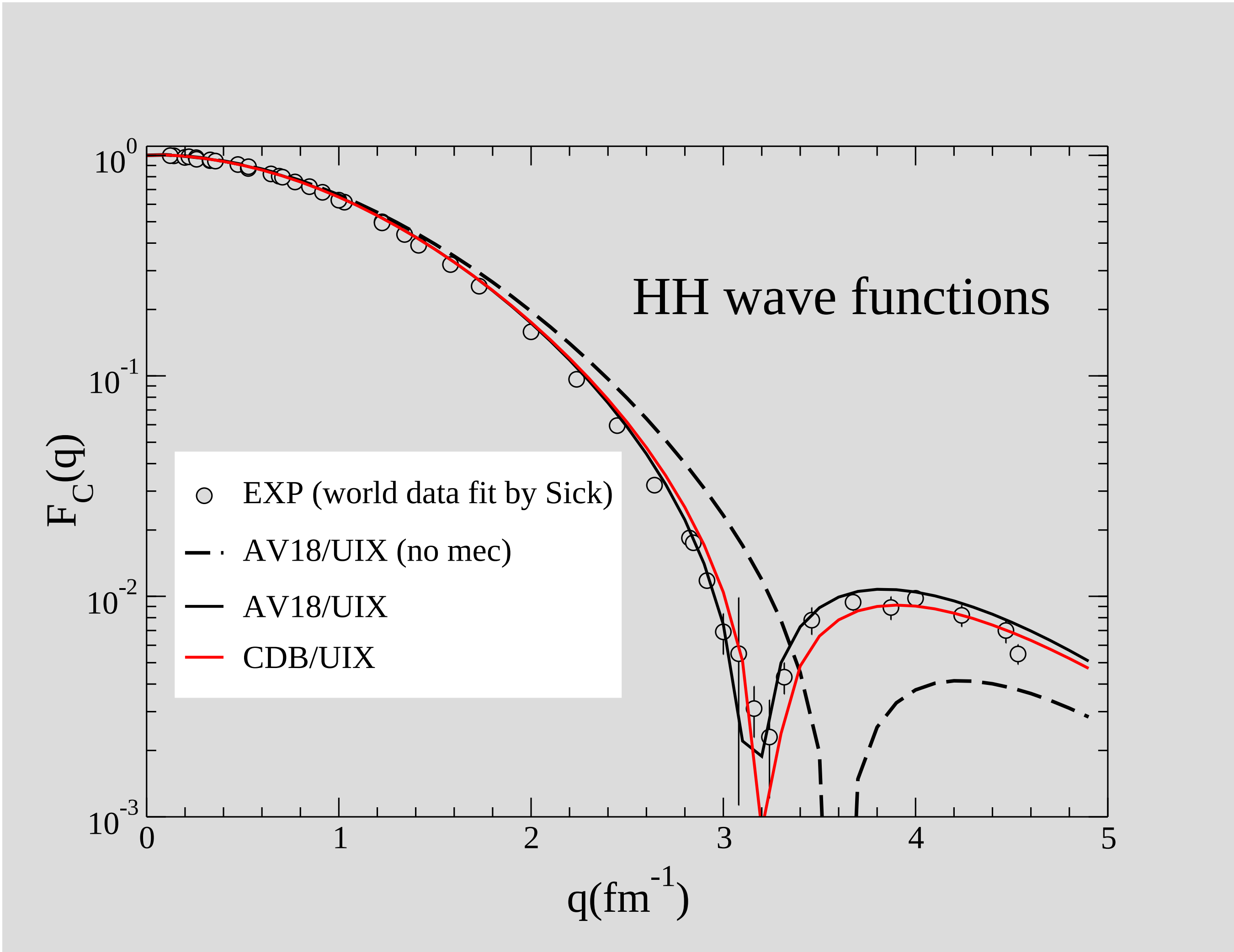
$$\text{diagram (a)} = v^{\pi}_{ij}\,\frac{1}{E_i - E}\,\frac{F_1^S + F_1^V\tau_{i,z}}{2} \rightarrow \text{included in IA}$$

$$- \frac{v_{PS}(k_j)}{2\,m}\,\boldsymbol{\sigma}_i\cdot\mathbf{q}\,\boldsymbol{\sigma}_j\cdot\mathbf{k}_j\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\,\frac{F_1^S + F_1^V\tau_{i,z}}{2} + \mathcal{O}(E_i - E)$$

- Crucial for predicting the charge f.f.'s of ^{2}H, ^{3}H, ^{3}He, and ^{4}He
- Additional (small) contributions from vector exchanges as well as transition mechanisms like $\rho\pi\gamma$ and $\omega\pi\gamma$

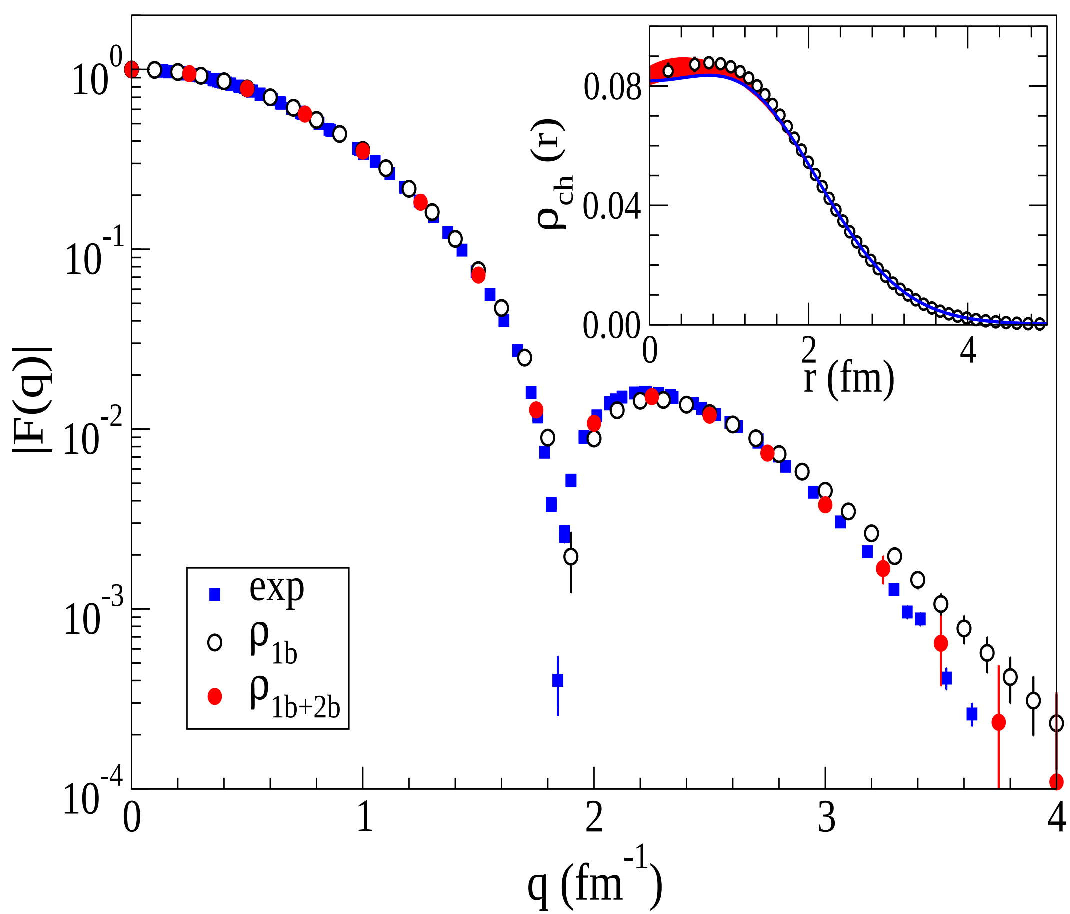
^{4}He Charge Form Factor

Viviani *et al.* (2007)



^{12}C Charge Form Factor

Lovato *et al.* (2013)



Weak Current Operators

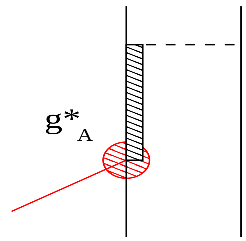
- Charge-changing (CC) and neutral (NC) weak currents (ignoring s-quark contributions)

$$j^{\mu}_{CC} = j^{\mu}_{\pm} + j^{\mu 5}_{\pm}$$

$$j^{\mu}_{NC} = -2\sin^2\theta_W \, j^{\mu}_{\gamma,S} + (1 - 2\sin^2\theta_W)\, j^{\mu}_{\gamma,z} + \, j^{\mu 5}_{z}$$

with $j_{\pm} = j_x \pm i\, j_y$ and the CVC constraint $\left[\, T_a \, , \, j^{\mu}_{\gamma,z} \,\right] = i\, \epsilon_{azb}\, j^{\mu}_{b}$

- Contributions to two-body axial currents from π and ρ exchange, $\rho\pi$ transition, and Δ-excitation



- Axial currents in χEFT at N^{3}LO depend on a single LEC d_R
- Common strategy: fix g^*_A or $d_R(\Lambda)$ in χEFT by fitting the GT m.e. in ^{3}H β-decay

Predictions for μ-Capture Rates on ^{2}H and ^{3}He

Marcucci *et al.* (2011–2012)

- Including radiative corrections from Czarnecki, Marciano, and Sirlin (2007)

	$\Gamma_0(^3\text{He})\ \text{s}^{-1}$
EXP	1496(4)
SNPA(AV18/UIX)	1496(8)
χEFT*(AV18/UIX)	
$\Lambda = 500$ MeV	1497(8)
$\Lambda = 600$ MeV	1498(9)
$\Lambda = 800$ MeV	1498(8)

- Chiral potentials (N3LO/N2LO) and currents lead to *conservatively* $\Gamma(^2\text{H})$=399(3) sec^{-1} and $\Gamma(^3\text{He})$=1494(21) sec^{-1}

Inclusive e/ν Scattering

- Inclusive $\nu/\overline{\nu}$ $(-/+)$ cross section given in terms of five response functions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\epsilon'\mathrm{d}\Omega} = \frac{G^2}{8\,\pi^2}\,\frac{k'}{\epsilon}\left[v_{00}\,R_{00} + v_{zz}\,R_{zz} - v_{0z}\,R_{0z} + v_{xx}\,R_{xx} \mp v_{xy}\,R_{xy}\right]$$

$$R_{\alpha\beta}(q,\omega) \sim \overline{\sum_i}\sum_f \delta(\omega{+}m_A{-}E_f)\langle f\,|\,j^\alpha(\mathbf{q},\omega)\,|i\rangle^*\langle f\,|\,j^\beta(\mathbf{q},\omega)\,|i\rangle$$

- In (e,e') scattering, interference $R_{xy}=0$, current conservation implies $j^z_\gamma \sim (\omega/q)j^0_\gamma$, and only $R_{00}{=}R_L$ and $R_{xx}{=}R_T$ are left

- Theoretical analysis via:
  1. Sum rules
  2. “Explicit” calculations of $R_{\alpha\beta}$ (EM only in ^{4}He for now)

Ab Initio Approaches to Inclusive Scattering (IS)

Response functions require knowledge of continuum states: hard to calculate for $A \geq 3$

- Sum rules: integral properties of response functions
- Integral transform techniques

$$E(q,\tau) = \int_0^\infty \mathrm{d}\omega \, K(\tau,\omega) \, R(q,\omega)$$

  and suitable choice of kernels (i.e., Laplace or Lorentz) allows use of closure over $| f \rangle$, thus avoiding need of explicitly calculating nuclear excitation spectrum

- While in principle exact, both these approaches have drawbacks

Sum Rules

Schiavilla *et al.* (1989); Carlson *et al.* (2002–2003)

$$S_\alpha(q) = C_\alpha \int_{\omega_{\rm th}^+}^{\infty} {\rm d}\omega \, \frac{R_\alpha(q,\omega)}{G_{Ep}^2(q,\omega)}$$

$$= C_\alpha \left[\langle 0 | \, O_\alpha^\dagger(\mathbf{q}) \, O_\alpha(\mathbf{q}) \, | 0 \rangle - | \, \langle 0 | \, O_\alpha(\mathbf{q}) \, | 0 \rangle \, |^2 \right]$$

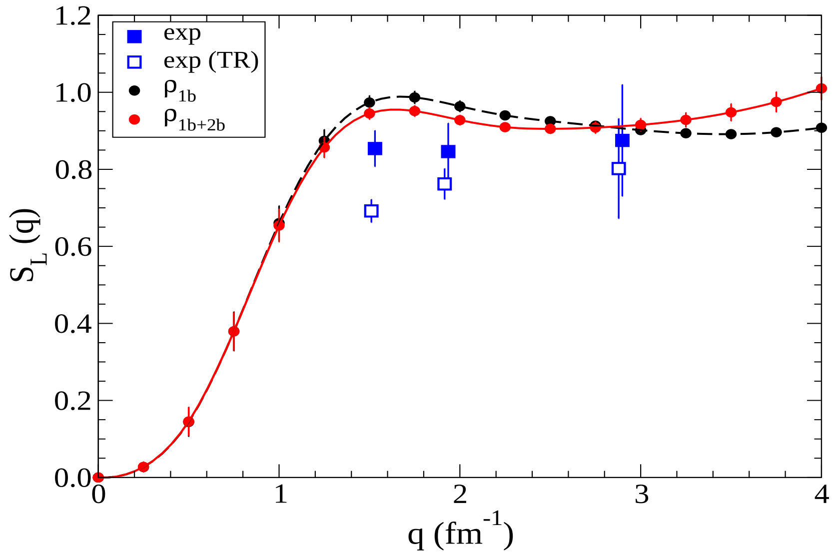
- $O_\alpha(\mathbf{q}) = \rho_\gamma(\mathbf{q})$ or $\mathbf{j}_\gamma^\perp(\mathbf{q})$ for $\alpha = L$ or T (divided by G_{Ep})
- C_α are normalization factors so as $S_\alpha(q \to \infty) = 1$ when only one-body are retained in ρ_γ and $\mathbf{j}_\gamma^\perp$
- $S_\alpha(q)$ only depend on ground state and can be calculated exactly with quantum Monte Carlo (QMC) methods
- Direct comparison between theory and experiment problematic:
  1. $R_\alpha(q,\omega)$ measured by (e,e') up to $\omega_{\rm max} \leq q$
  2. Present theory ignores explicit pion production mechanisms, crucial in the Δ-peak region of R_T

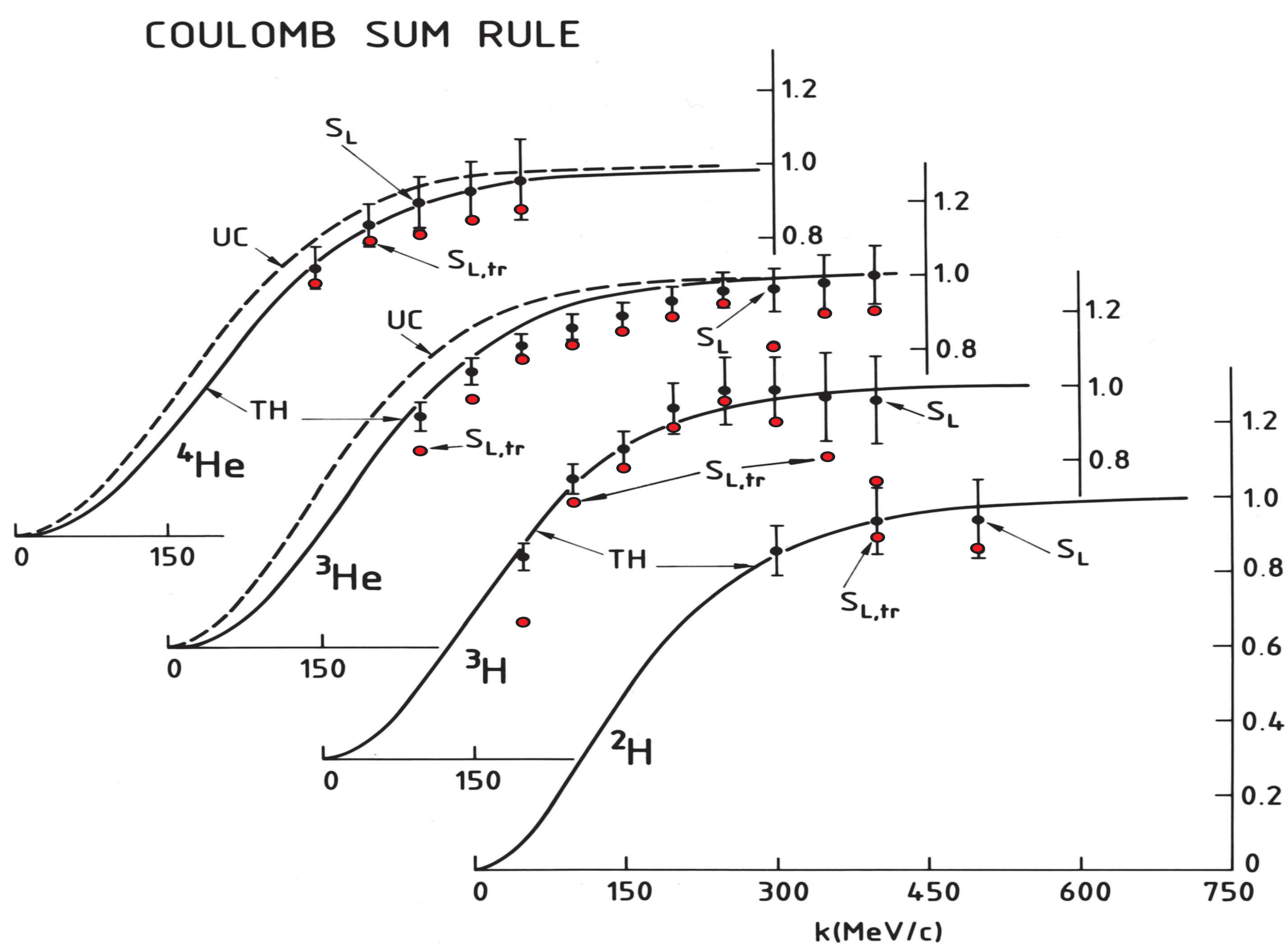
The Coulomb Sum Rule in ^{12}C

Lovato *et al.* (2013)

- Theory and experiment in reasonable agreement (when using free G_{Ep})
- Contribution for $\omega > \omega_{\rm max}$ estimated by assuming

$$R_L(q, \omega > \omega_{\rm max}; A) \propto R_L(q, \omega; \text{deuteron})$$





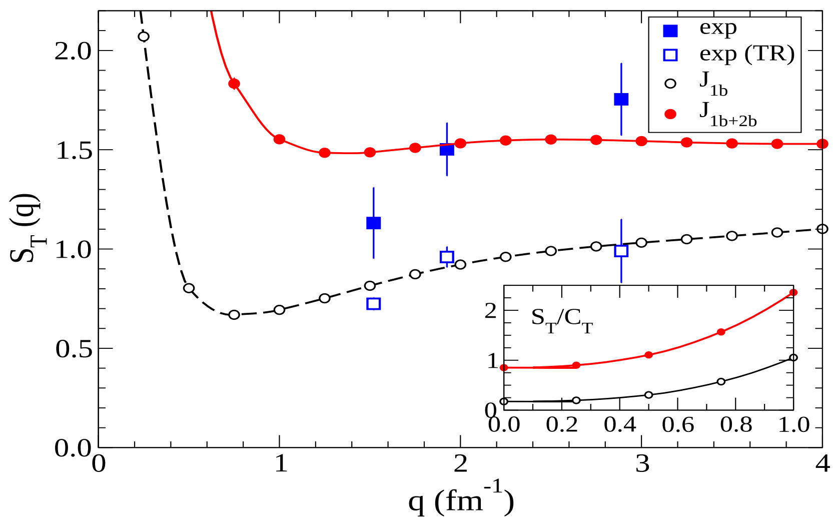
COULOMB SUM RULE
S_L
S_L,tr
UC
TH
4He
3He
3H
2H
1.2
1.0
0.8
0.6
0.4
0.2
0
150
300
450
600
750
k(MeV/c)

The Transverse Sum Rule in ^{12}C

Lovato *et al.* (2013)

- Large contribution from two-body currents
- Comparison with experiment problematic
- Divergence at small q fictitious due to normalization factor

$$C_T = \frac{2}{Z\,\mu_p^2 + N\,\mu_n^2}\,\frac{m^2}{q^2}$$



Response Functions

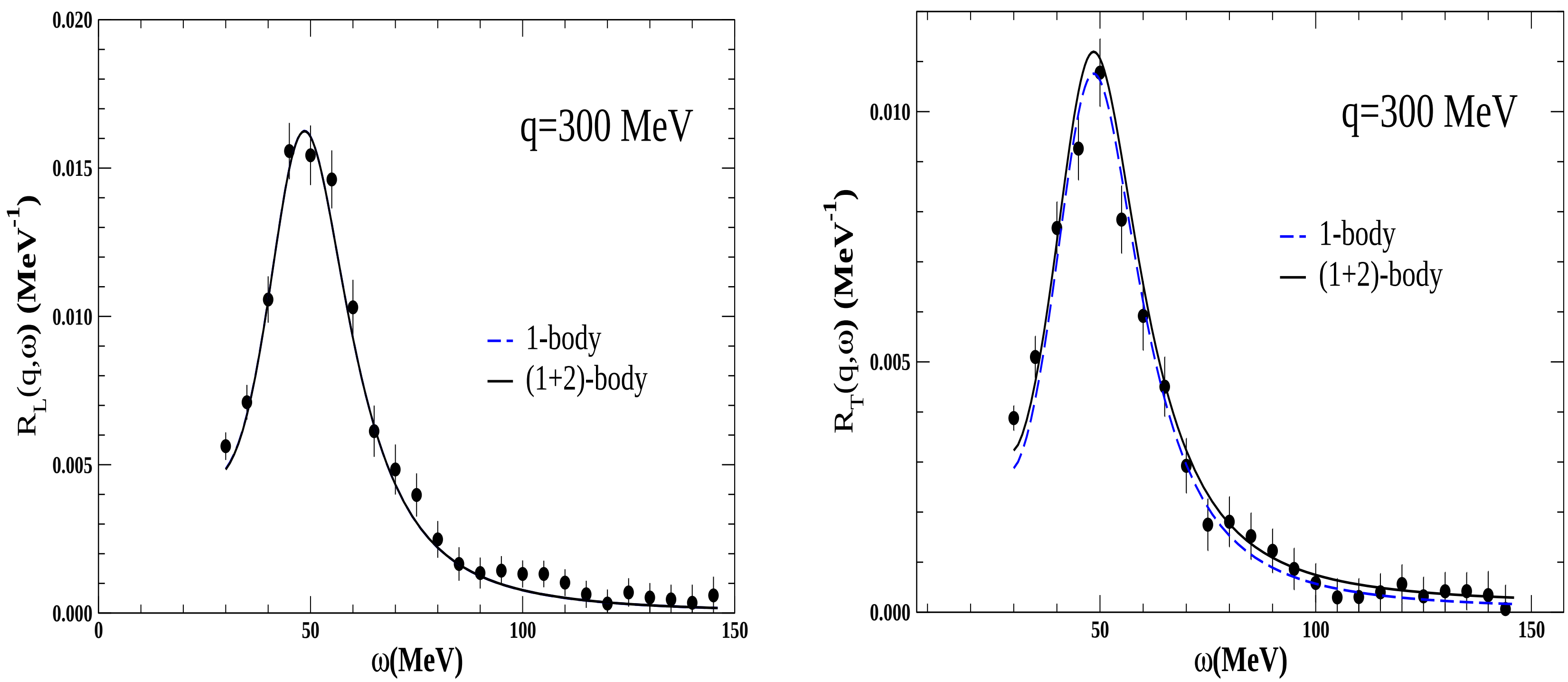
Carlson and Schiavilla (1992,1994)

- Direct calculation in ^{2}H; calculation of Euclidean response functions in $A \geq 3$

$$\begin{aligned}
\widetilde{E}_\alpha(q,\tau) &= \int_{\omega_{\rm th}^+}^{\infty} {\rm d}\omega\, {\rm e}^{-\tau(\omega-E_0)}\, \frac{R_\alpha(q,\omega)}{G_{Ep}^2(q,\omega)} \\
&= \langle 0 |\, O_\alpha^\dagger(\mathbf{q}) {\rm e}^{-\tau(H-E_0)} O_\alpha(\mathbf{q})\, | 0 \rangle - (\text{elastic term})
\end{aligned}$$

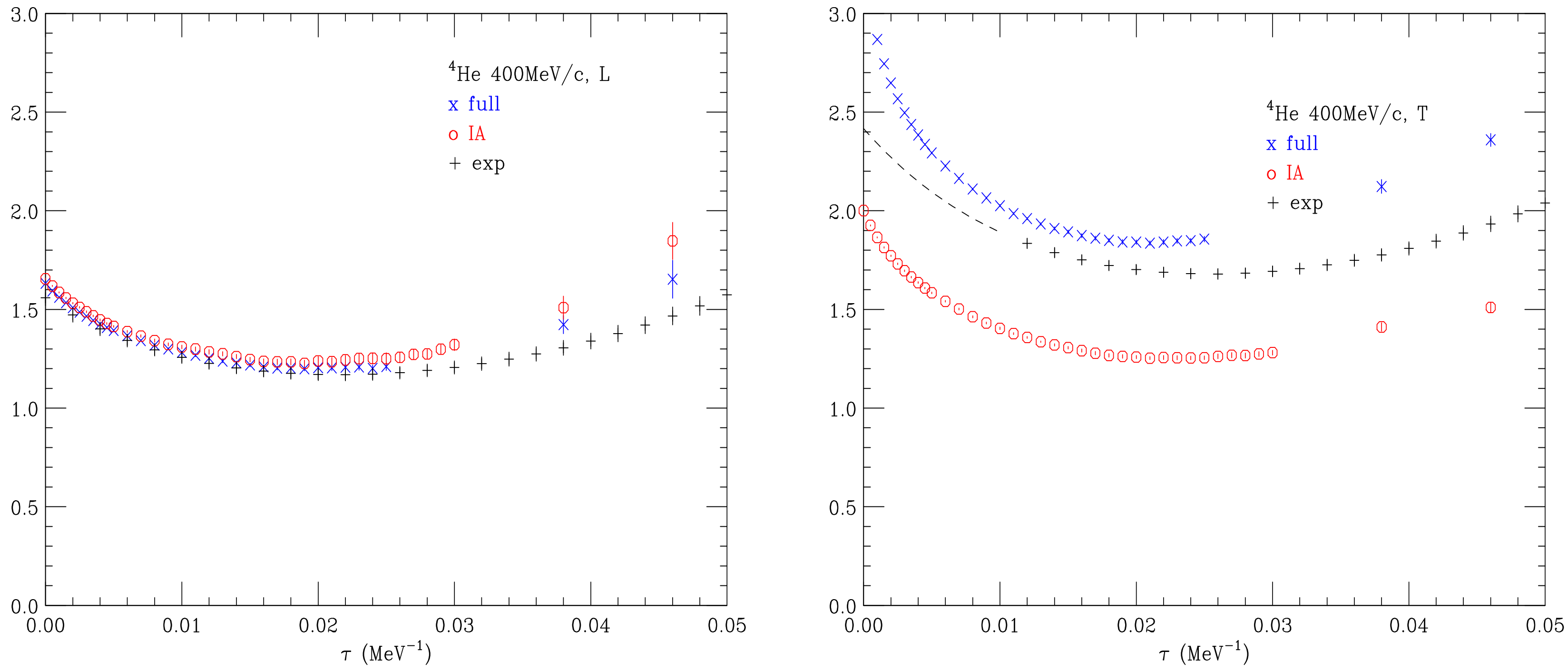
- ${\rm e}^{-\tau(H-E_0)}$ evaluated stochastically with QMC
- No approximations made, exact
- At $\tau=0$, $\widetilde{E}_\alpha(q;0) \propto S_\alpha(q)$; as τ increases, $\widetilde{E}_\alpha(q;\tau)$ is more and more sensitive to strength in QE region
- Inversion of $\widetilde{E}_\alpha(q;\tau)$ is a numerically ill-posed problem; Laplace-transform data instead

^{2}H Longitudinal and Transverse Response Functions



A few % increase due to two-body currents at the top of the QE peak in R_T, much larger as ω increases

^{4}He Longitudinal and Transverse Euclidean Response Functions



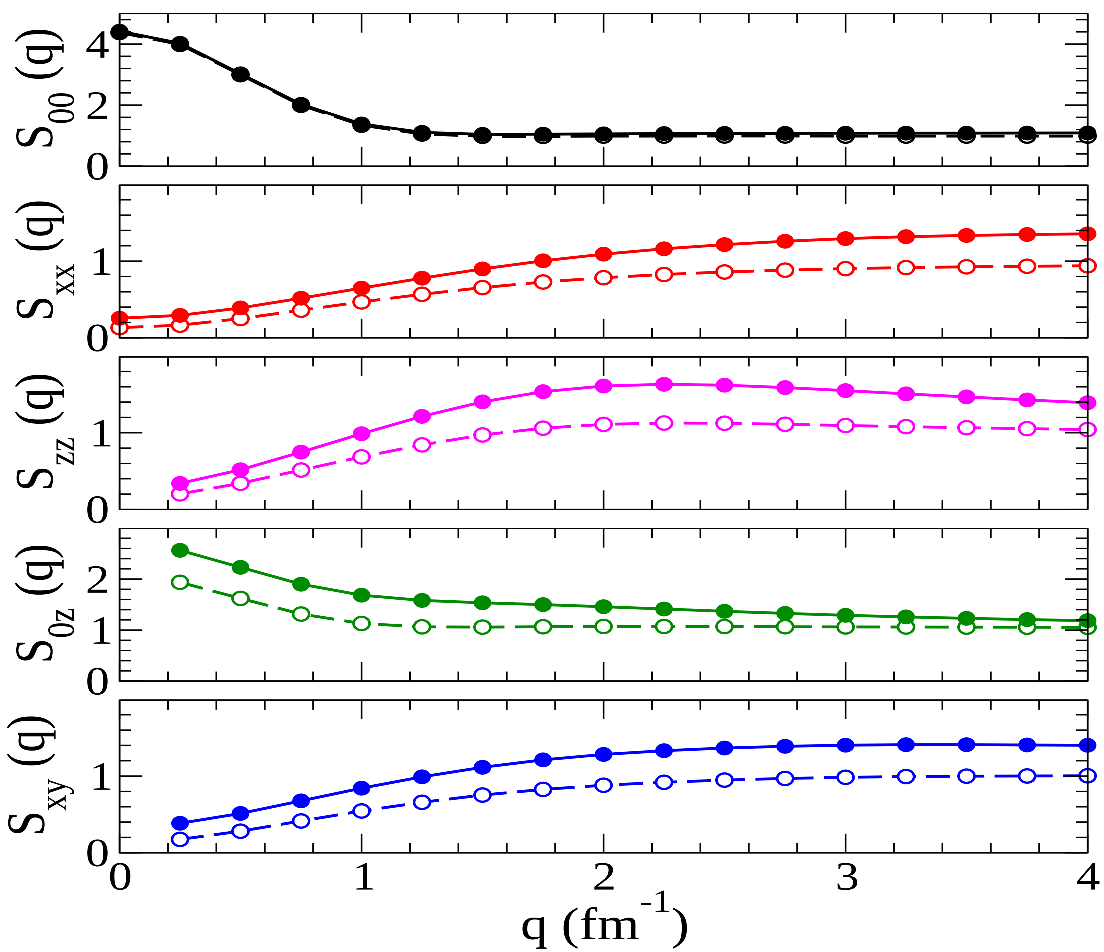
$$E_\alpha(q,\tau) = \exp\left[\tau\, q^2/(2\, m)\right]\ \widetilde{E}_\alpha(q,\tau)$$

and $E_L(q,\tau) \rightarrow Z$ for a collection of protons initially at rest

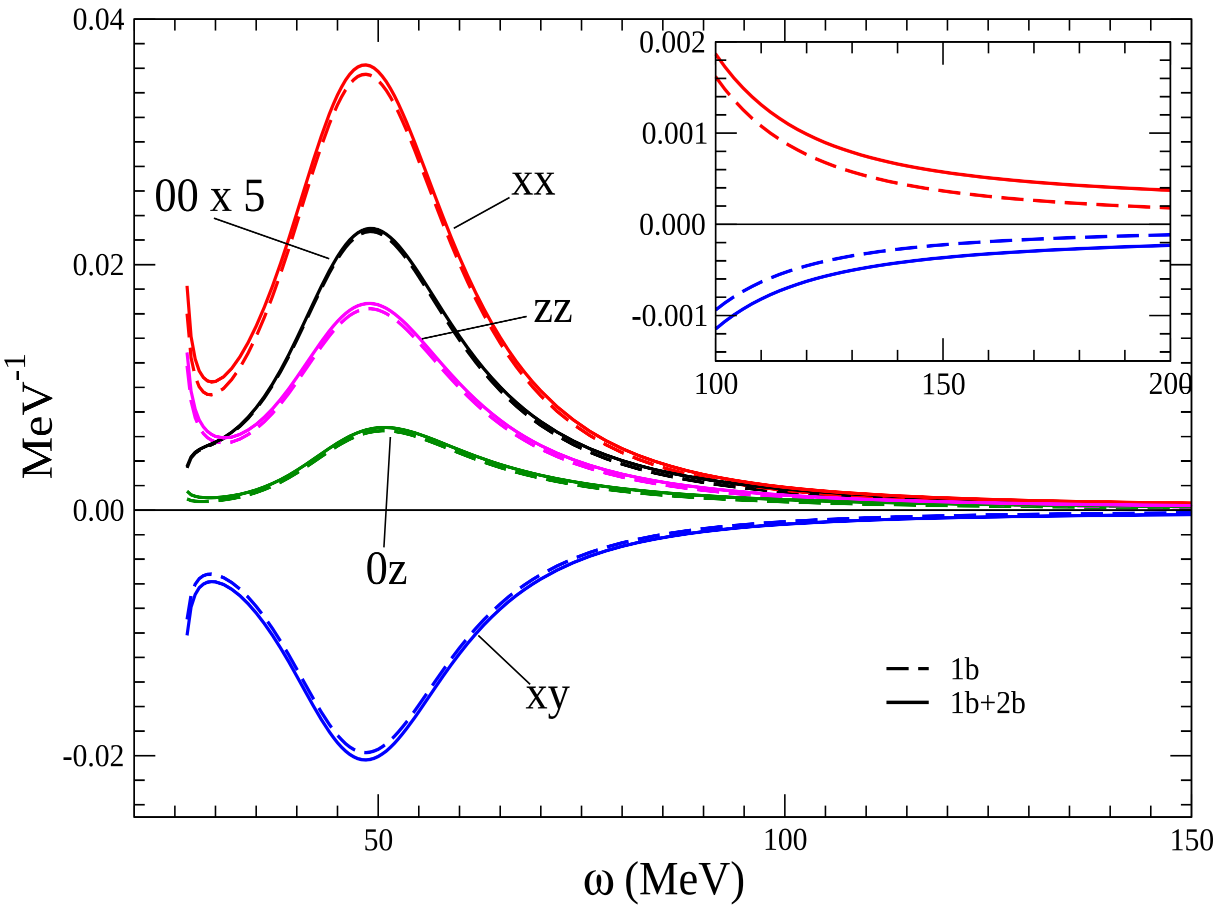
- The $\tau \gtrsim 0.015$ MeV^{-1}region is sensitive to QE strength; R_T enhancement much larger than in ^{2}H

Sum Rules of NC Weak Response Functions in ^{12}C

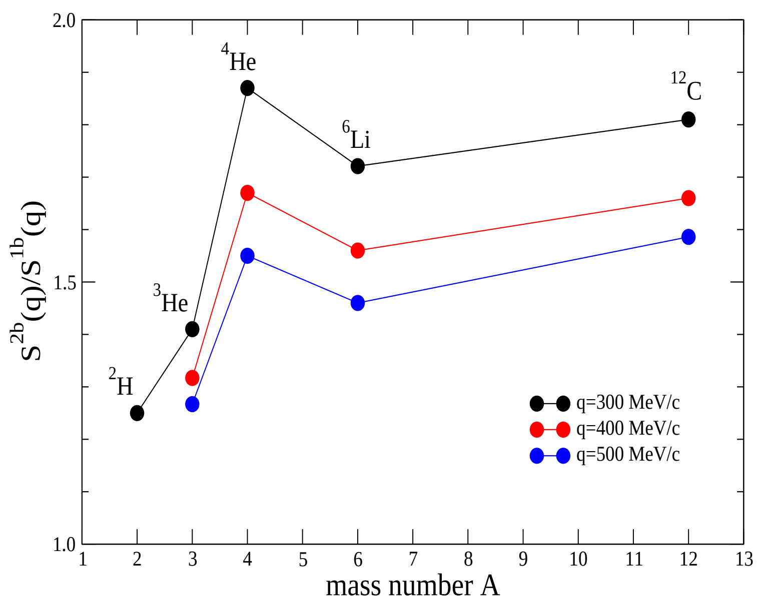
Lovato *et al.*, in preparation (2013)



NC Weak Response Functions in ^{2}H



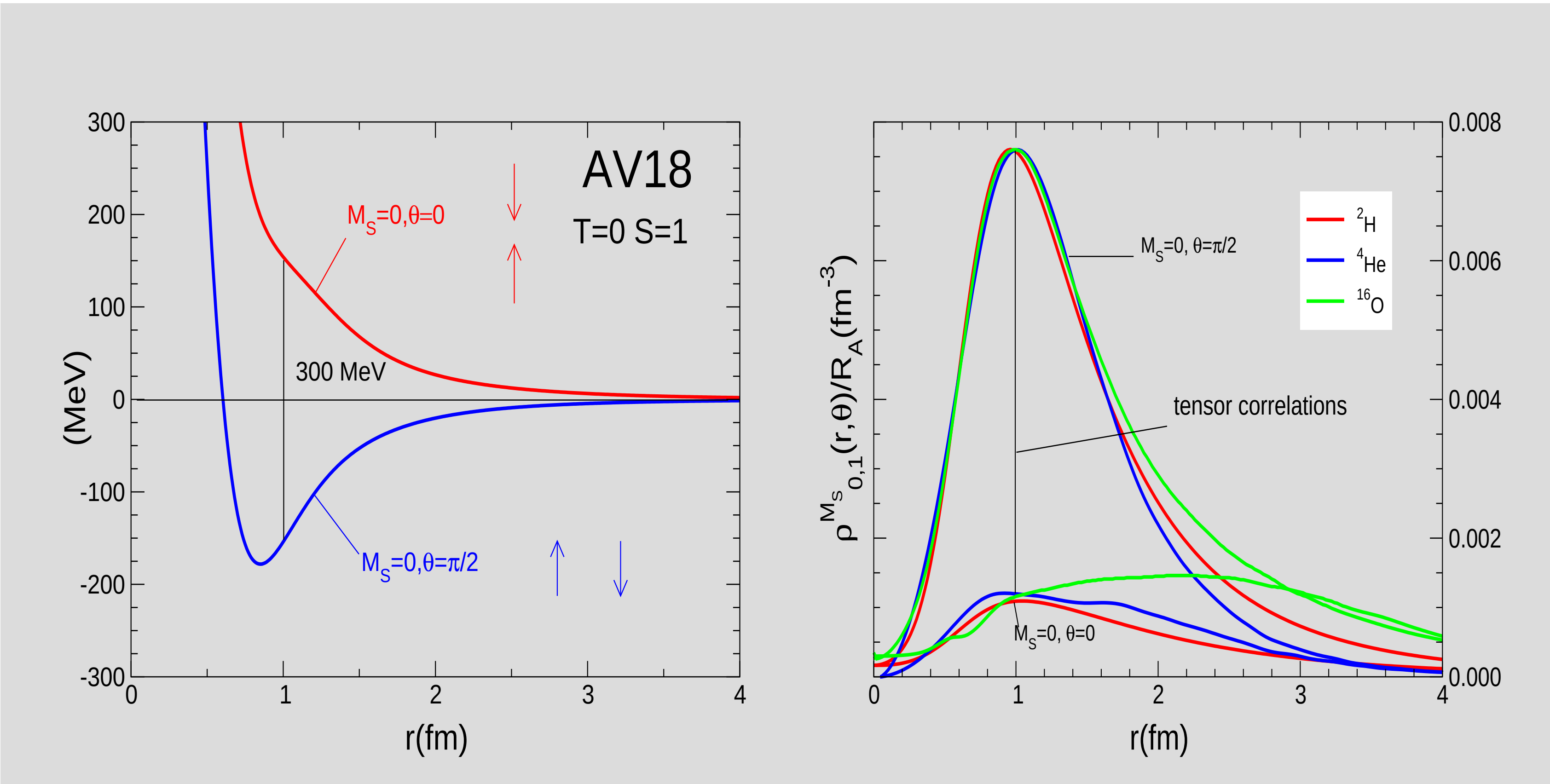
Excess Transverse Strength Systematics



- What portion of the excess strength $\Delta S_T = S_T - S_T^{1b}$ is in the QE region?
- Is the A-dependence of ΔS_T understood?

Short-Range Structure of $T, S = 0, 1$ Pairs in Nuclei

- short-range repulsion of v_{NN} (common to many systems)
- tensor character of v_{NN} (unique to nuclei)



Forest *et al.*, PRC**54**, 646 (1996)

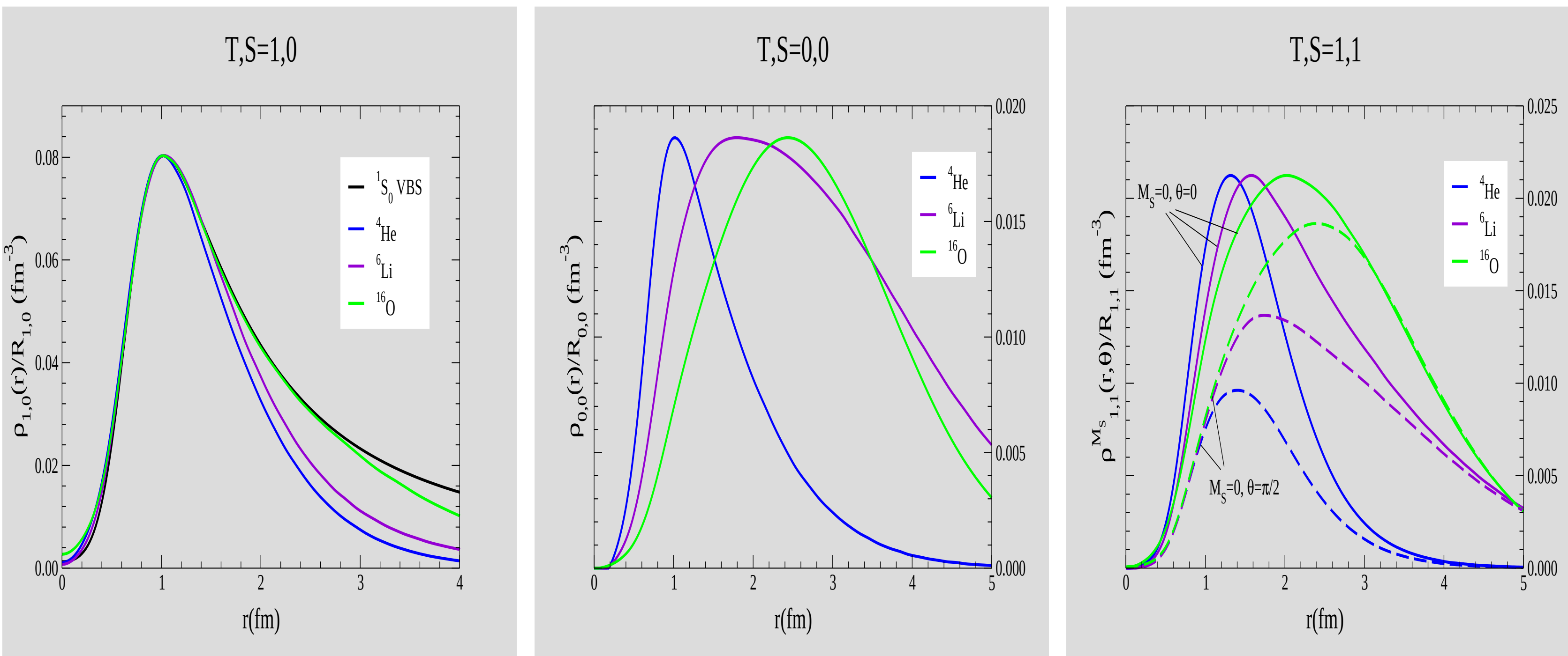
- $\langle O_{ij} \rangle_A \simeq R_A \langle O_{ij} \rangle_d$, where O_{ij} is any short-range operator effective in the $T = 0, S = 1$ channel (like the electroweak O_{ij})

Scaling

	R_A	$N^{\mathrm{IP}}_{T=0,S=1}$	$\langle v^\pi \rangle_A / \langle v^\pi \rangle_d$	$\sigma^\pi_A / \sigma^\pi_d$	$\sigma^\gamma_A / \sigma^\gamma_d$
^{3}He	2.0	1.5	2.1	2.4(1)	$\simeq 2$
^{4}He	4.7	3	5.1	4.3(6)	$\simeq 4$
^{6}Li	6.3	5.5	6.3		
^{7}Li	7.2	6.75	7.8		$\simeq 6.5(5)$
^{12}C	18.5	18			
^{16}O	18.8	30	22	17(3)	16(3)

Two-Nucleon Density Profiles in $T,S \neq$0,1 States

- Scaling persists in T,S=1,0 channel (^{1}S$_0$ state) for $r \leq 2$ fm
- But no scaling occurs in remaining channels (interaction either repulsive or weakly attractive)



A-Systematics of ΔS_T

Carlson *et al.* (2002)

$$\Delta S_T \propto \langle 0 | \sum_{l<m} \left[(j_l^\dagger + j_m^\dagger) j_{lm} + \text{h.c.} \right] + \sum_{l<m} j_{lm}^\dagger j_{lm} + \ldots | 0 \rangle$$

- Neglecting 3- and 4-body terms (represented by $\ldots$)

$$\Delta S_T{}^A(q) \simeq C_T \int_0^\infty \mathrm{d}x \, \mathrm{tr} \left[F(x;q) \, \rho^A(x;pn) \right]_{\sigma\tau} \equiv \int_0^\infty \mathrm{d}x \, I^A(x)$$

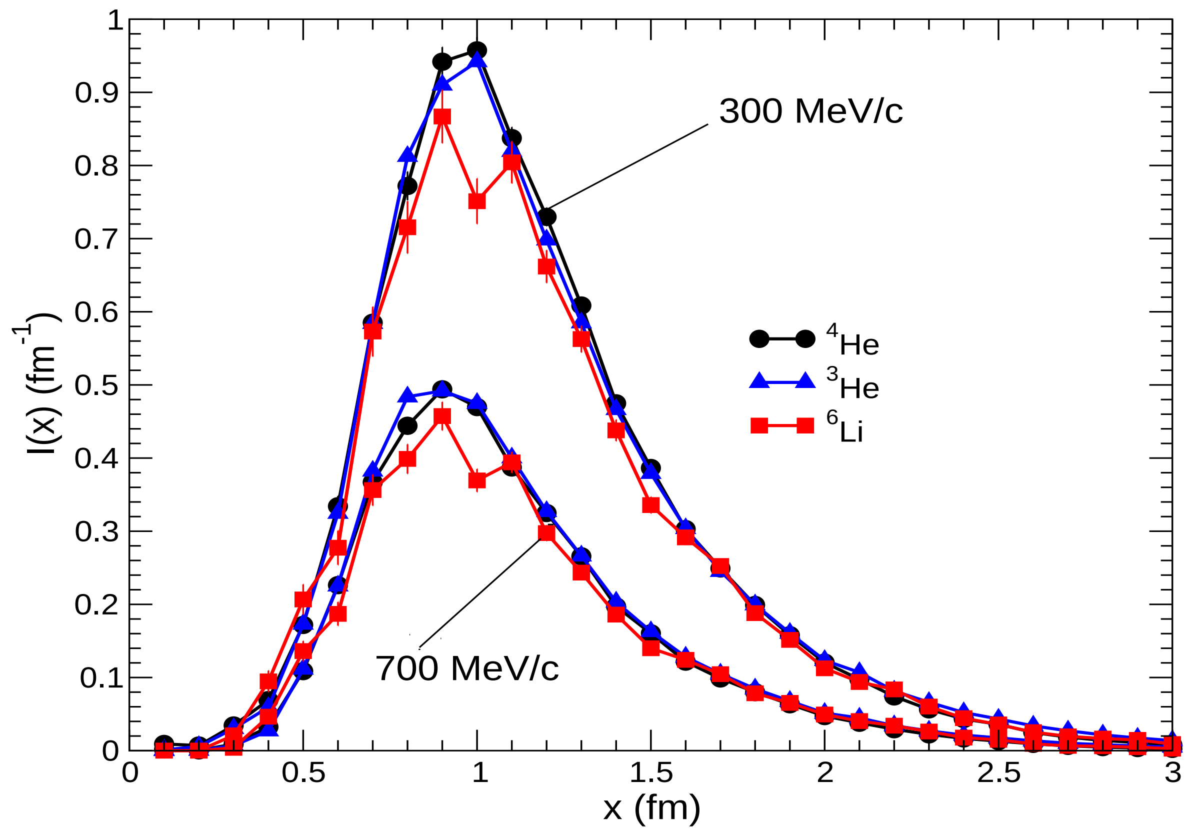
  F=matrix in NN $\sigma\tau$-space depending on j_{lm} (range $x \lesssim 1/m_\pi$)

  ρ^A=A-dependent NN density matrix in $\sigma\tau$-space

- Scaling property $\rho^A(x;pn,T=0) \simeq R_A \, \rho^d(x)$ and similarly for $T=1$ pn pairs with $\rho^d \rightarrow \rho^{qb}$; hence

$$I^A(x) \text{ scales as } \frac{R_A}{Z\,\mu_p^2 + N\,\mu_n^2}$$

A-Scaling Property



After rescaling by $R_A / \left(Z\,\mu_p^2 + N\,\mu_n^2 \right)$, the integrand $I^A(x)$ is $\sim$ the same in all nuclei

Tensor Correlations and Two-Nucleon Momentum Distributions

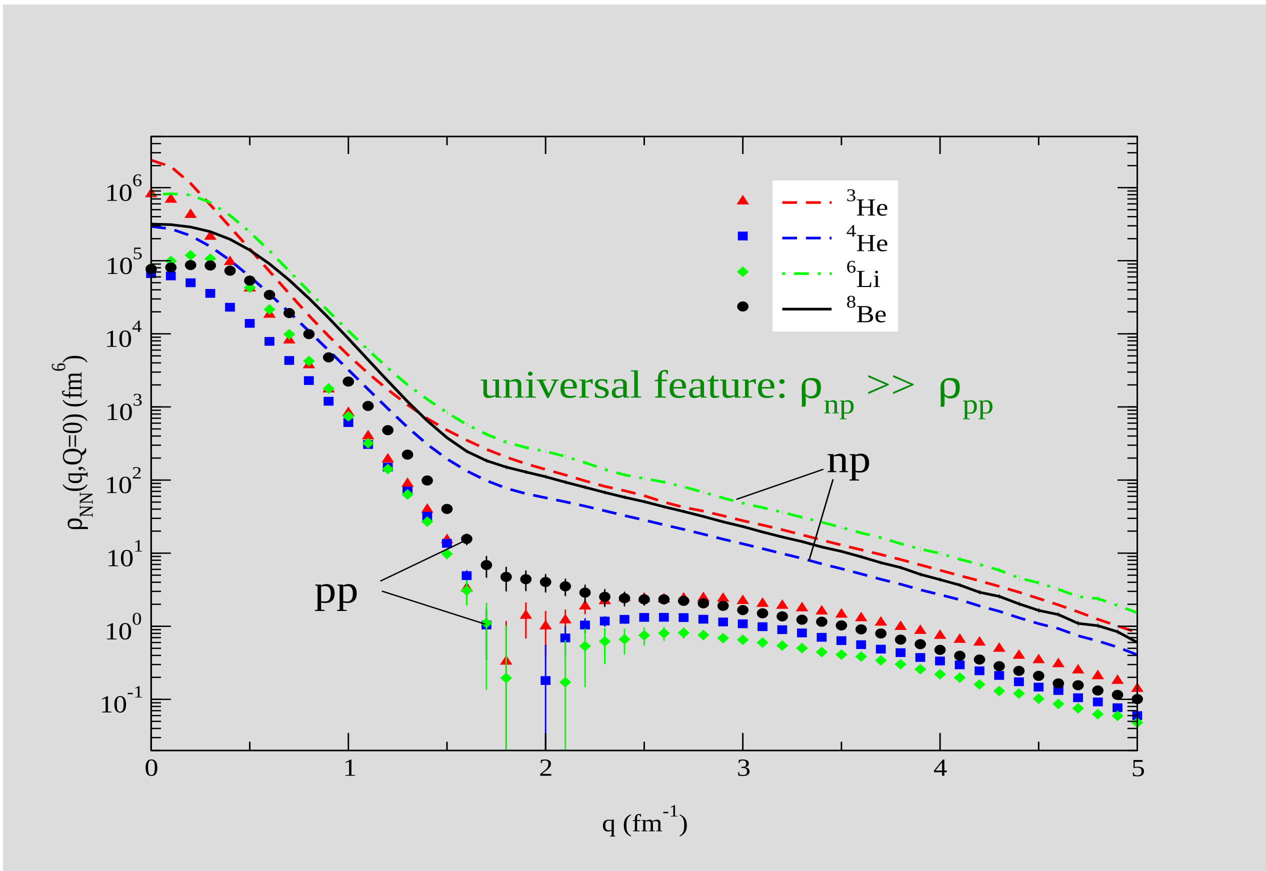
$$\rho^{NN}(\mathbf{q},\mathbf{Q}) = \frac{1}{2J+1}\sum_{M_J}\langle\psi_{JM_J} \mid \sum_{i<j} P_{ij}^{NN}(\mathbf{q},\mathbf{Q}) \mid \psi_{JM_J}\rangle$$

where $\mathbf{q}$ and $\mathbf{Q}$ are respectively the relative and total momenta of the NN pair, and

$$P_{ij}^{NN}(\mathbf{q},\mathbf{Q}) \equiv \delta(\mathbf{k}_{ij}-\mathbf{q})\delta(\mathbf{K}_{ij}-\mathbf{Q})\, P_{NN}(ij)$$

- np (pp) pairs predominantly in $T{=}0$ deuteron-like ($T{=}1$ $^1\mathrm{S}_0$) state $\longrightarrow$ large differences between ρ^{np} and ρ^{pp}
- Pair-momentum distributions useful for estimates of NN-knockout x-sections
- ρ^{NN} can be calculated exactly with QMC

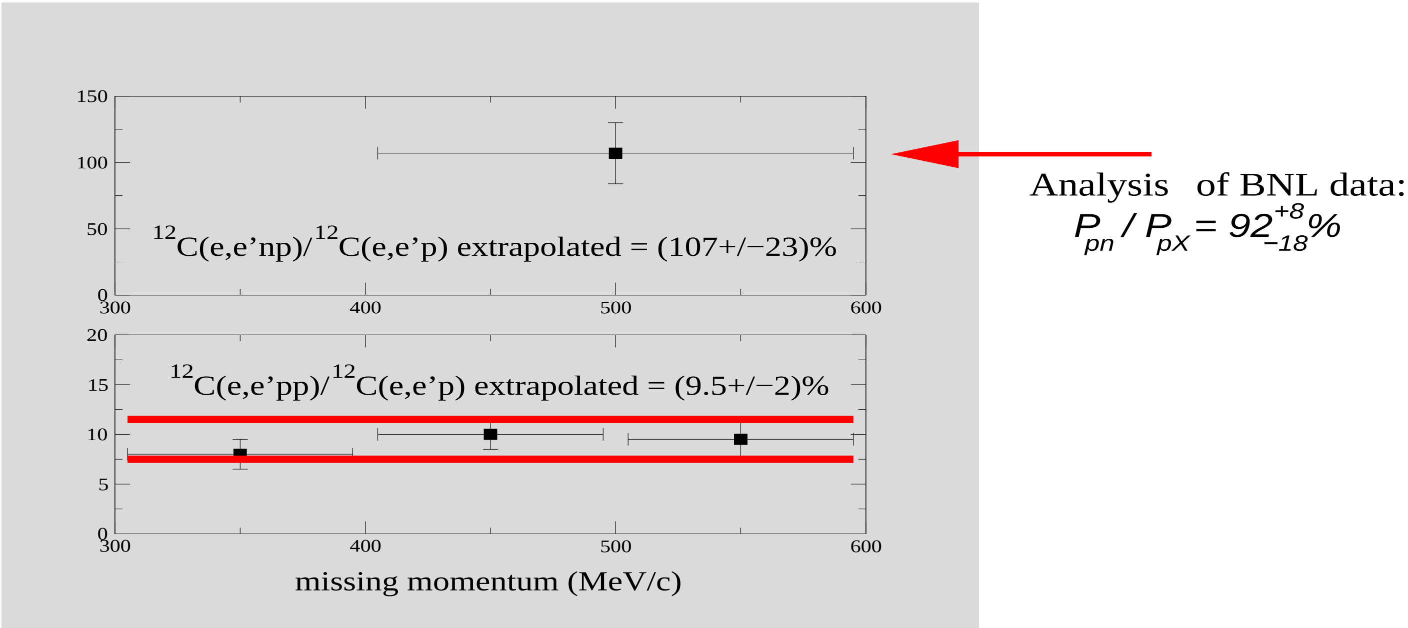
NN momentum distributions at Q=0 (back-to-back)



Schiavilla, Wiringa, Pieper, and Carlson, PRL **98**, 132501 (2007)

Effects of Tensor Correlations on NN Knock-Out Processes

- JLab measurements on ^{12}C$(e, e'pp)$[a] and $(e, e'np)$[b]
- Analysis of ^{12}C(p, pp) and (p, ppn) BNL data[c]
- Possibly also seen in π-absorption: $\sigma(\pi^-, np)/\sigma(\pi^+, pp) \ll 1$[d]



[a] Shneor *et al.*, PRL**99**, 072501 (2007); [b] Subedi *et al.*, Science **320**, 1476 (2008); [c] Piasetzky *et al.*, PRL**97**, 162504 (2006); [d] Ashery *et al.*, PRL**47**, 895 (1981)

Summary

- Large enhancement due to two-body electroweak currents in the sum rules of electromagnetic and weak response functions
- There is a direct connection between this enhancement and the short-range structure of *np* pairs in nuclei
- This short-range structure (presumably!) also drives the increase of the one-body response due to two-body currents
- Calculations of ^{4}He (Euclidean) EM response functions show that excess strength may be as large 20–30% in QE region
- Similar enhancement of the NC (and CC) one-body response functions is expected for ^{12}C (next stage of calculations)