Towards a microscopic theory for low-energy heavy-ion reactions

Role of internal degrees of freedom in low-energy nuclear reactions

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1. Introduction: Environmental Degrees of Freedom
2. Application of RMT to subbarrier fusion
3. Discussions: Towards a microscopic theory for low-energy heavy-ion reactions
4. Summary
Fusion: compound nucleus formation

E.R. \( (A_{CN} < 170) \)

E.R. + F \( (220 > A_{CN} > 170) \)

F \( (A_{CN} > 220) \)

Recent review:
K. Hagino and N. Takigawa,

courtesy: Felipe Canto
the simplest approach to fusion cross sections: potential model

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) P_l(E)$$
Subbarrier fusion reactions

Potential model:
Reproduces the data reasonably well for $E > V_b$
Underpredicts $\sigma_{\text{fus}}$ for $E < V_b$

cf. seminal work:
R.G. Stokstad et al., PRL41(’78)465
PRC21(’80)2427
Strong target dependence at $E < V_b$

low-lying collective excitations
Subbarrier fusion: strong interplay between reaction and structure

coupled-channels equations

\[ \sigma_{\text{fus}}(E) = \int_{0}^{1} d(\cos \theta) \sigma_{\text{fus}}(E; \theta) \]

Def. Effect: enhances \( \sigma_{\text{fus}} \) by a factor of 10 ~ 100

Fusion: interesting probe for nuclear structure
Coupled-Channels method

Coupling between rel. and intrinsic motions

\[ H = -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + H_0(\xi) + V_{\text{coup}}(r, \xi) \]

\[ \psi(r, \xi) = \sum_k \psi_k(r) \phi_k(\xi) \]

Entrance channel

Excited channel

Excited states

Ground state

\( H_0(\xi) \phi_k(\xi) = \epsilon_k \phi_k(\xi) \)
**Coupled-channels framework**

Coupling between rel. and intrinsic motions

- $0^+$ entrance channel
- $0^+$ excited channel
- $2^+$ excited channel
- $4^+$ excited channel

- Quantum theory which incorporates excitations in the colliding nuclei
- A few collective states (vibration and rotation) which couple strongly to the ground state + transfer channel
- Several codes in the market: ECIS, FRESCO, CCFULL......

Has been successful in describing heavy-ion reactions

However, many recent challenges in C.C. calculations
Scattering processes:

Double folding potential
Woods-Saxon ($a \sim 0.63$ fm)

successful

Fusion process: not successful

$\rightarrow a \sim 1.0$ fm required (if WS)
Deep subbarrier fusion data

C.L. Jiang et al., PRL93(’04)012701
“steep fall-off of fusion cross section”

K. H., N. Rowley, and M. Dasgupta, 
PRC67(’03)054603
potential inversion with deep subbarrier data

K.H. and Y. Watanabe,
PRC76 ('07) 021601(R)

cf. Earlier work on potential inversion:
A.B. Balantekin, S.E. Koonin, and
J.W. Negele, PRC28('83)1565

cf. Density-Constrained TDHF:
A.S. Umar and V.E. Oberacker,
Euro. Phys. J. A39 ('09)243
energy dependence of surface diffuseness parameter

M. Dasgupta et al., PRL99(’07)192701

- dynamical effects not included in C.C. calculation?
- energy and angular momentum dissipation?
- weak channels?

this talk
typical excitation spectrum: electron scattering data

low-lying collective excitations

GDR/GQR

$E_{\text{GDR}} \sim 79A^{-1/3} \text{ MeV}$

$E_{\text{GQR}} \sim 65A^{-1/3} \text{ MeV}$

M. Sasao and Y. Torizuka,
PRC15('77)217

low-lying non-collective excitations

- Giant Resonances: high $E_x$, smooth mass number dependence
  \[ \text{adiabatic potential renormalization} \]
- Low-lying collective excitations: barrier distributions,
  strong isotope dependence
- Non-collective excitations: either neglected completely or
  implicitly treated through an absorptive potential
IS Octupole response of $^{48}$Ca (Skyrme HF + RPA calculation: SLy4)

collective state: $|coll\rangle \sim \sum_{ph} X_{ph} a_p^\dagger a_h |0\rangle$

strong coupling

single-particle (non-collective) state weak, but many $|s.p.\rangle \sim a_p^\dagger a_h |0\rangle$
Our interest: couplings to (relatively) low-lying single-particle levels
e.g., collective levels in $^{116}$Sn

model space in a typical C.C. calculation
Our interest: couplings to (relatively) low-lying single-particle levels

112 levels up to 4.1 MeV
(93 single-particle levels)
nearly “complete” level scheme

S. Raman et al.,
PRC43(‘91)521

role of these s.p. levels in reaction dynamics?
Indications of non-collective excitations: a comparison between \(^{20}\text{Ne}+^{90}\text{Zr}\) and \(^{20}\text{Ne}+^{92}\text{Zr}\)

\[ D_{\text{qel}}(E) = -\frac{d}{dE} \left( \frac{\sigma_{\text{qel}}(E, \pi)}{\sigma_R(E, \pi)} \right) \]

QEL = elastic + inelastic + transfer

- C.C. results are almost the same between the two systems
- Yet, quite different barrier distribution and Q-value distribution

E. Piasecki et al., PRC80(‘09)054613
A problem: the nature of non-collective states is poorly known (the energy, spin, parity only)
i.e., no information on the coupling strengths
Random Matrix Model

Coupled-channels equations:

\[
\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \epsilon_k - E\right] \psi_k(r) + \sum_{k'} \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \psi_{k'}(r) = 0
\]

\[|\phi_k\rangle \text{: complicated single-particle states}
\]

coupling matrix elements \( V_{kk'} = \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \) are random numbers generated from a Gaussian distribution:

\[
\overline{V_{ij}(r)} = 0,
\]

\[
\frac{V_{ij}(r)V_{kl}(r')} = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \frac{w_0}{\sqrt{\rho(\epsilon_i)\rho(\epsilon_j)}}
\]

\[
\times e^{-\frac{(\epsilon_i-\epsilon_j)^2}{2\Delta^2}} \cdot e^{-\frac{(r-r')^2}{2\sigma^2}} \cdot h(r)h(r')
\]

D. Agassi, C.M. Ko, and H.A. Weidenmuller, Ann. of Phys. 107(‘77)140
RMT model for H.I. reactions:
- originally developed by Weidenmuller et al. to analyze DIC
- similar models have been applied to discuss quantum dissipation
  - M. Wilkinson, PRA41('90)4645
  - A. Bulgac, G.D. Dang, and D. Kusnezov, PRE54('96)3468
  - S. Mizutori and S. Aberg, PRE56('97)6311

D. Agassi, H.A. Weidenmuller, and C.M. Ko, PL 73B('78)284
Application to $^{20}\text{Ne} + ^{90,92}\text{Zr}$ reactions

- **Internuclear potential**
  
  Woods-Saxon potential
  
  \[ V_0 = 55 \text{ MeV (}^{90}\text{Zr}), 62.3 \text{ MeV (}^{92}\text{Zr}), \]
  \[ r_0 = 1.2 \text{ fm}, a=0.65 \text{ fm} \]

- **Coupling form factor** $h(r)$
  
  derivative of Woods-Saxon

- **Non-collective couplings**
  
  up to 5.7 MeV, only from the ground state
  
  \[ \Rightarrow 38 \text{ channels for } ^{90}\text{Zr}, 75 \text{ channels for } ^{92}\text{Zr} \]

  energy and radial coherence lengths: Weidenmuller et al.
  
  \[ \Delta = 7 \text{ MeV}, \sigma = 4 \text{ fm} \]

  the overall coupling strength: adjustable parameter
  
  (the same value between $^{90}\text{Zr}$ and $^{92}\text{Zr}$)

- **Collective couplings**
  
  rot. states of $^{20}\text{Ne}$ up to $6^+ + 2^+$ and $3^-$ two-phonons in Zr
Quasi-elastic cross sections

$E_{\text{eff}} = 2E \frac{\sin(\theta_{\text{c.m.}}/2)}{1 + \sin(\theta_{\text{c.m.}}/2)}$

Q-value distributions

cf. Q-value distribution from backward scattering:

M. Evers et al.,
PRC78('08)034614

C.J. Lin et al.,
PRC79('09)064603

(elastic + collective) peaks + non-collective bumps
These states are excited during nuclear reactions in a complicated way.

Nuclear intrinsic d.o.f. act as environment for nuclear reaction processes

"intrinsic environment"
How much do we know about “friction”?  

Fusion model → friction free: strong absorption inside the barrier

coupling to environment ←→ dissipation & friction
DIC also in light systems?

Transfer processes with large negative Q-value?

\[
\sigma_{\text{fus}}^{(\text{exp})}(E) = S \cdot \sigma_{\text{capt}}^{(\text{th})}(E; a = 0.63)
\]

\[
S \equiv \frac{\sigma_{\text{fus}}^{(\text{exp})}(E)}{\sigma_{\text{fus}}^{(\text{exp})}(E) + \sigma_{\text{DIC}}^{(\text{exp})}(E)}
\]

J.O. Newton, K.H. et al., PRC70(’04)024605
How much do we know about “friction”? 

Fusion model → friction free: strong absorption inside the barrier

The topic of energy dissipation in fusion should be re-visited
- re-analyses of DIC data: maybe helpful
- Consistent theoretical model (dissipative quantum tunneling)
Non-collective excitations in isolated nuclei

$^{20}\text{Ne} + ^{92}\text{Zr}$

- Random matrix model?

Unified quantum theory for fusion (subbarrier, deep subbarrier) & DIC?

Single-particle (non-collective) excitations in H.I. reactions quantum mechanical model for Wall-Window friction?
(Big) open question:

- Construction of a microscopic nuclear reaction model applicable at low energies?

    → many-particle tunneling

cf. nuclear structure calculations

- 2-body nn interaction ➔ mean-field ➔ RPA
  residual interaction ➔ TDHF

advantage: non-empirical
disadvantage: difficult to control a mean-field

guiding principle
complementary

deep understanding of results

- mean-field pot. ➔ residual interaction ➔ RPA
  TDHF
• 2-body nn interaction → mean-field → RPA
  residual interaction → TDHF

many reaction theories correspond to this type

• mean-field pot. → residual interaction → RPA
  TDHF
Microscopic nuclear reaction theories

TDHF, QMD, AMD not applicable to low-energy fusion (classical nature)

Cluster approach (RGM) only for light systems
H.O. wave function (separation of cm motion)

Double Folding approach surface region: OK, but inside?
role of antisymmetrization?
validity of frozen density approximation?

Full microscopic theory: ATDHF, TD-GCM, ASCC ?
imaginary-time TDHF?

how to understand quantum tunneling from many-particle point of view?
Another issue Is reaction fast or slow?

Many-body (N-particle system) Hamiltonian

\[ H = \sum_i t_i + \sum_{i<j} v_{i,j} \]

Large Amplitude Collective Motion

\[ H = H_{rel} + H_{s.p.} + H_{coup} \]

✧ Sudden approach (fast collision)
  - Double Folding Model
  - Optical Model
  - Coupled-channels model
  - Resonating Group Method (RGM) \( \{ \) const. reduced mass \( \mu \) \( \} \)

✧ Adiabatic approach (slow collision)
  - Liquid-drop model (+ shell correction)
  - Adiabatic TDHF
  \( \leftarrow \) Coordinate dependent mass \( \mu(r) \)
cannot discriminate one of them at present

sudden approach (frozen density)  adiabatic approach

S. Misić and H. Esbensen,
PRL96(‘06)112701

T. Ichikawa, K.H., A. Iwamoto,
PRC75(‘07)057603

✓ need further studies from several perspectives
✓ construction of dynamical model without any assumption on adiabaticity
Summary

Heavy-ion subbarrier fusion reactions

✓ strong interplay between reaction and structure
✓ quantum tunneling with several kinds of environment

Open questions

✓ how do we understand many-particle tunneling?
  - related topics: fission, alpha decays, two-proton radioactivities
    Large amplitude collective motions
✓ role of noncollective excitations?
  - dissipation, friction
✓ microscopic understanding of subbarrier fusion?
✓ unified theory of fusion and DIC?