Many-body localization-delocalization transition for 1D bosons in the quasiperiodic potential

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- Introduction.
- Many-body localization-delocalization transition
- Classical and degenerate bosons
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- Phase diagram
- Conclusions

Collaborations B.L. Altshuler (Columbia Univ.), V. Michal (LPTMS, Orsay)

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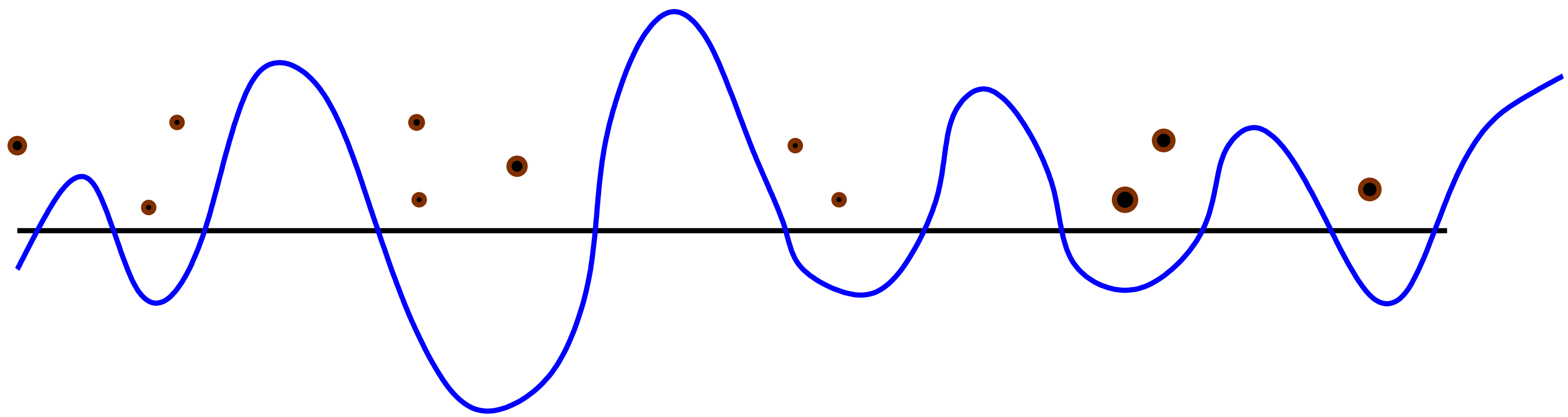
Quantum gases in disorder

One-dimensional disordered bosons at finite temperature



DOGMA $\rightarrow$ No finite temperature phase transitions in 1D
as all spatial correlations decay exponentially

There is a non-conventional phase transition between two distinct states

Fluid and Insulator

I.I. Aleiner, B.L. Altshuler, GS, (2010)

Old question

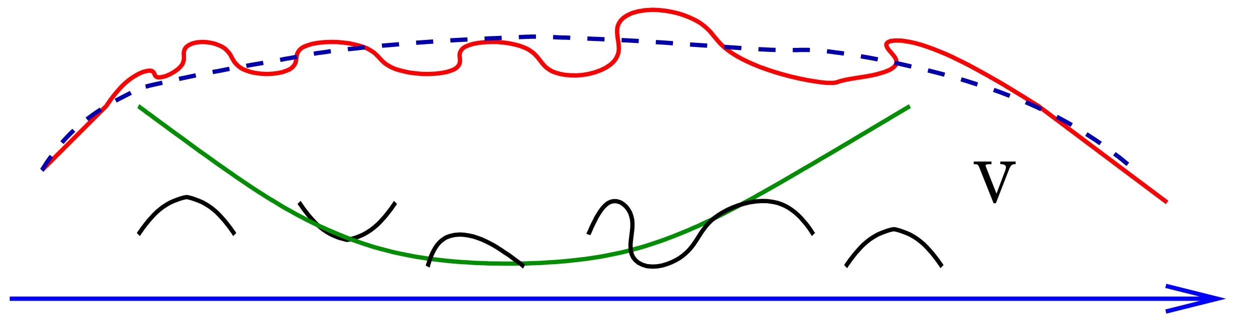
How does the interparticle interaction influences Anderson localization?

Crucial for charge transport in electronic systems

Appears in a new light for disordered ultracold bosons

Palaiseau, LENS, Rice, Urbana experiments. More underway

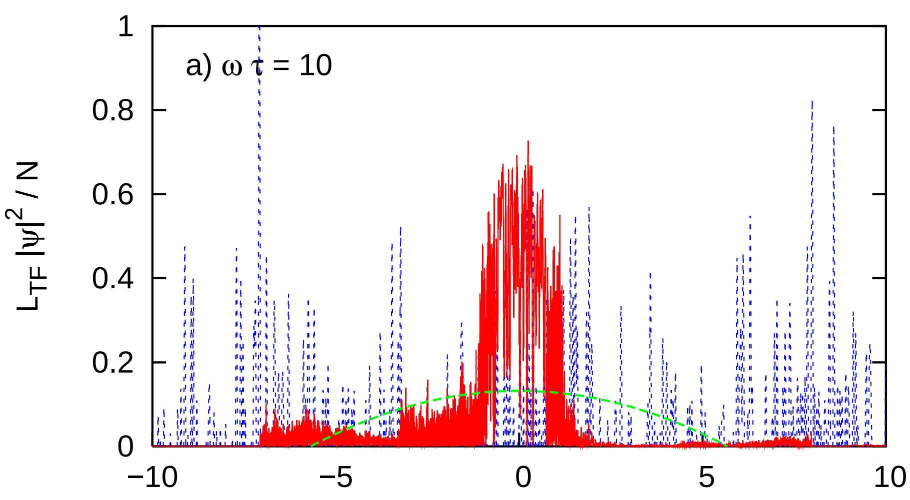
Experiments



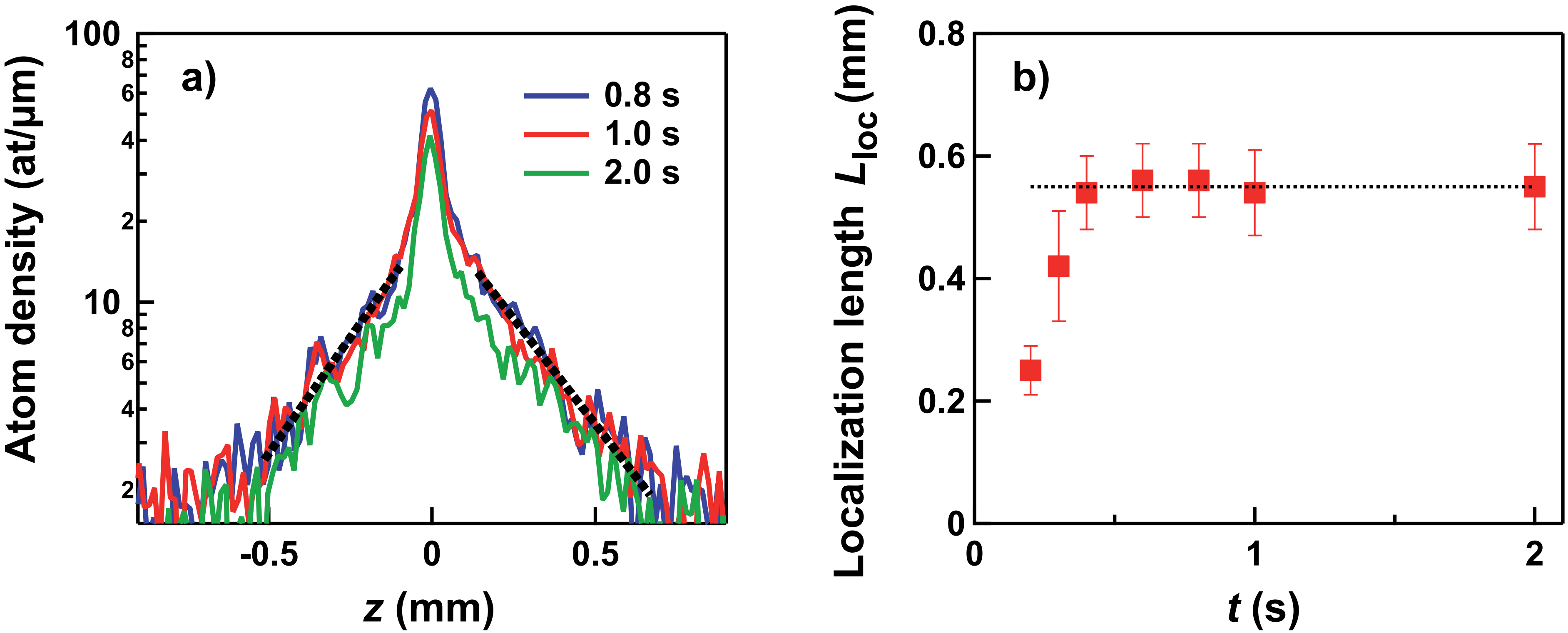
BEC in a harmonic $+$ weak random potential $|V(z)| \ll ng \Rightarrow$ small density modulations of the static BEC. Switch off the harmonic trap, but keep the disorder $\Rightarrow$ What happens?

First experiments (Orsay, LENS, Rice)

The expansion stops and BEC gets stacked in between 2 large peaks

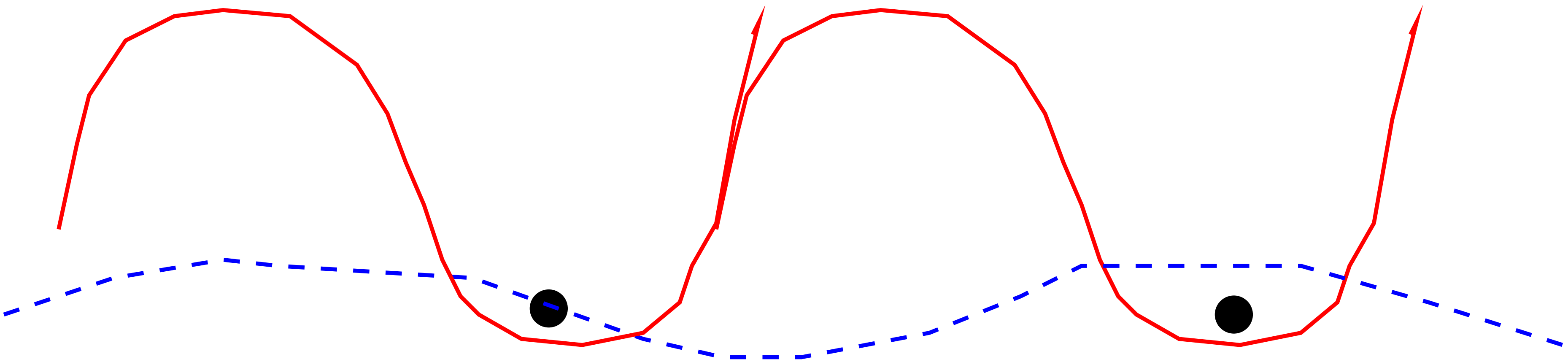


Orsay experiment



LENS experiment

1D quasiperiodic potential



Single-particle state

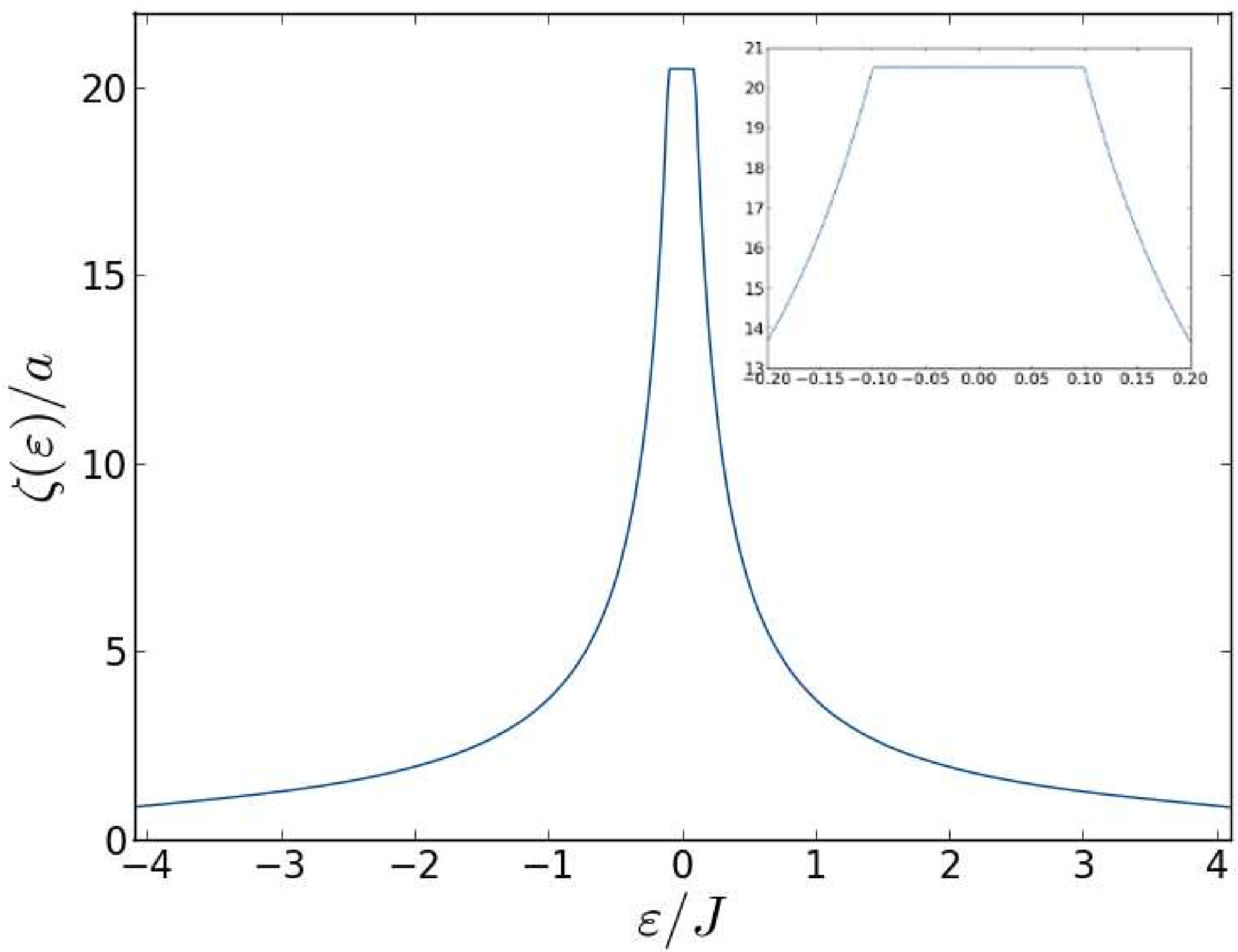
$$J(\psi_{n+1} + \psi_{n-1}) + V\cos(2\pi\beta n)\psi_n = \varepsilon\psi_n$$

$V > 2J \rightarrow$ all single-particle states are localized

AAH model for interacting bosons

$$H_{int} = U \sum_j n_j(n_j - 1)/2$$

Localization length $\zeta_{min} = a; \quad \zeta_{max} = aV/(V - 2J)$

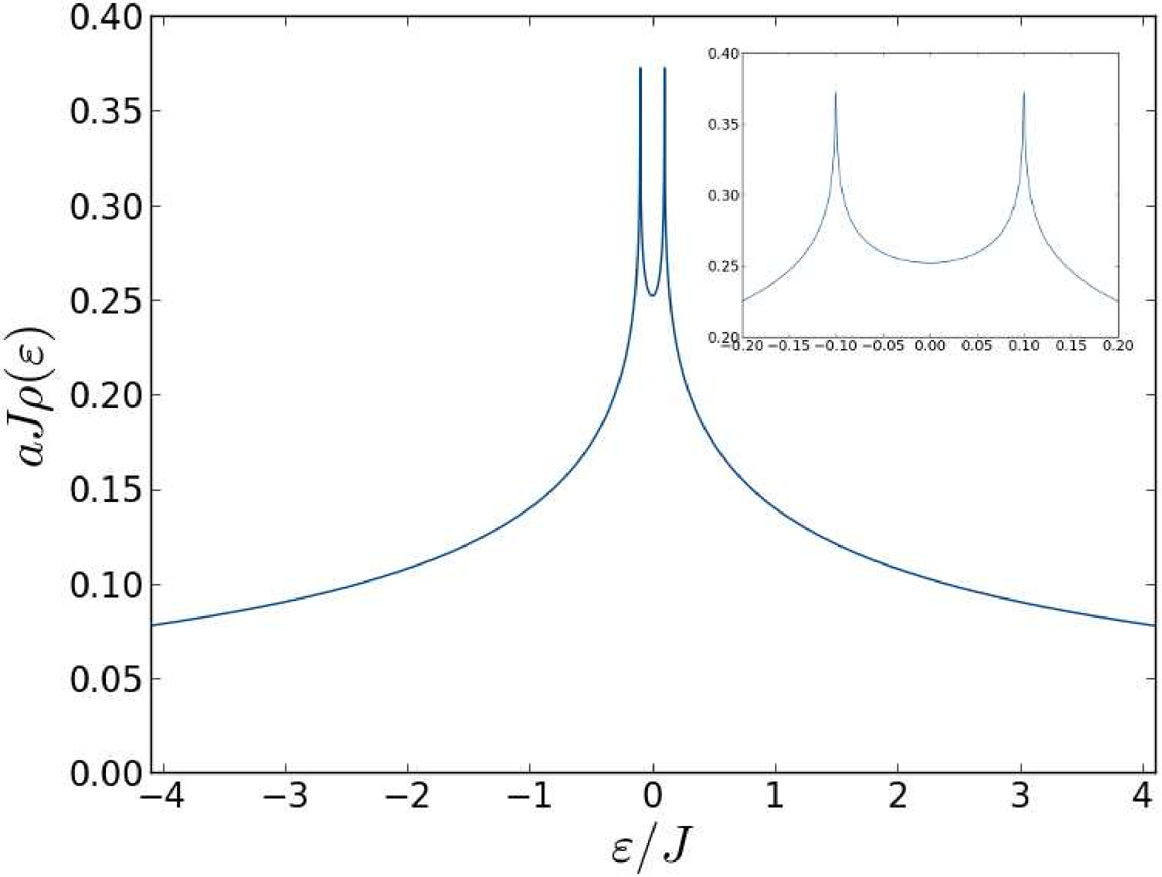


AAH model. Density of states

$\beta \ll 1 \Rightarrow$ quasiclassical DOS

$$\rho(\varepsilon) = \frac{1}{L}\int_{-\pi}^{\pi}\frac{dk}{2\pi}\int_{-L/2}^{L/2} dx\,\delta(\varepsilon - 2J\cos ka - V\cos 2\beta x)$$

$$\rho(\varepsilon) = \frac{2}{\pi^2 V a}\ln\left(\frac{2\sqrt{2}V}{\varepsilon - V + 2J}\right); \quad V \gg \varepsilon > (V - 2J)$$



Many-body localization-delocalization transition

(Aleiner, Altshuler, Basko 2006-2007)

Analyze how different states of two particles $|\alpha,\beta\rangle$ hybridize due to the interaction

The probability $P(\varepsilon_\alpha)$ that for a given state $|\alpha\rangle$ there exist $|\beta\rangle, |\alpha'\rangle, |\beta'\rangle$

such that $|\alpha,\beta\rangle$ and $|\alpha',\beta'\rangle$ are in resonance:

$\langle\alpha,\beta|H_{int}|\alpha',\beta'\rangle$ exceeds $\Delta_{\alpha\beta}^{\alpha'\beta'} \equiv |\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_{\alpha'} - \varepsilon_{\beta'}|$

MBLDT criterion $P(\varepsilon_\alpha) \sim 1$

MBLDT

Matrix element for large occupation numbers

$$\langle \alpha, \beta | H_{int} | \alpha', \beta' \rangle = U \sum_j \sqrt{|N_\beta N_{\alpha'} + N_\beta N_{\beta'} - N_{\alpha'} N_{\beta'}|}\, \psi_\alpha(j)^* \psi_\beta(j)^* \psi_{\alpha'}(j) \psi_{\beta'}(j)$$

$$\varepsilon_\alpha \approx \varepsilon_{\alpha'}; \;\; \varepsilon_\beta \approx \varepsilon_{\beta'} \Rightarrow \sqrt{|N_\beta N_{\alpha'} + N_\beta N_{\beta'} - N_{\alpha'} N_{\beta'}|} \approx N_\beta$$

$$\psi_\alpha(j) \approx \zeta_\alpha^{-1/2}; \qquad |j - j_\alpha| < \zeta_\alpha / 2$$

$$\psi_\alpha(j) \approx 0 \qquad |j - j_\alpha| > \zeta_\alpha / 2$$

$$\langle \alpha, \beta | H_{int} | \alpha', \beta' \rangle = U N_\beta \frac{a}{\zeta_{max}}$$

Energy mismatch

$$\delta_\alpha \equiv |\varepsilon_\alpha - \varepsilon_{\alpha'}| \rightarrow \; [\zeta_\alpha \rho(\varepsilon_\alpha]^{-1}; \;\; \delta_\beta \approx \zeta_\beta \rho(\varepsilon_\beta]^{-1}$$

$$\Delta_{\alpha\beta}^{\alpha'\beta'} = |\delta_\alpha - \delta_\beta| \approx \left| \frac{1}{\zeta_\alpha \rho(\varepsilon_\alpha)} - \frac{1}{\zeta_\beta \rho(\varepsilon_\beta)} \right| \approx \frac{1}{(\zeta\rho)_{min}}$$

MBLDT criterion

The probability that $\langle\alpha,\beta|H_{int}|\alpha',\beta'\rangle$ exceeds $\Delta_{\alpha\beta}^{\alpha'\beta'}$

$$P_{\alpha\beta}^{\alpha'\beta'} \approx U N_\beta \frac{a(\zeta\rho)_{min}}{\zeta_{max}}$$

$$P(\varepsilon_\alpha) = \sum_{\beta,\alpha',\beta'} P_{\alpha\beta}^{\alpha'\beta'} = U\int d\varepsilon_\beta \rho(\varepsilon_\beta)\zeta_\beta N_\beta \frac{a(\zeta\rho)_{min}}{\zeta_{max}}$$

Critical coupling strength $U_c \approx \left[\int d\varepsilon_\beta \rho(\varepsilon_\beta)\zeta_\beta N_\beta \frac{a(\zeta\rho)_{min}}{\zeta_{max}}\right]^{-1}$

$T \ll 2J \rightarrow$ disregard deeply localized low-energy states

$\rho(\varepsilon)$ and $\zeta(\varepsilon)$ decrease with increasing energy

$$U_c \approx \min_\alpha \left\{ \left[\int_{-V+2J}^{\varepsilon_\alpha} d\varepsilon_\beta N_\beta \rho(\varepsilon_\beta)\rho(\varepsilon_\alpha) a\zeta_\alpha + \int_{\varepsilon_\alpha}^{\infty} d\varepsilon_\beta N_\beta \rho^2(\varepsilon_\beta)\frac{a\zeta_\beta^2}{\zeta_\alpha}\right]^{-1} \right\}$$

Classical regime

Weak interactions $U \ll J$ $\quad n = \int_0^\infty \frac{\rho(\varepsilon) d\varepsilon}{\exp[(\varepsilon - \mu)/T] - 1}$

,

$$T > T_d \quad \| \mu| = T \ln \left(\frac{T}{T_d} \right); \quad T_d = \frac{n}{\rho(T_d)}$$

$\rho^2(\varepsilon_\beta \zeta_\beta^2 \varepsilon_\beta$ peaks at low energies $\varepsilon \sim |V - 2J| \ll T$

$$U_c \tilde{\nu} \approx T \frac{\pi^2}{2} \frac{\ln(V/T_d)}{\ln^2[V/(V - 2J)]}$$

Increasing T favors localization!

Quantum degenerate regime

$$T_* < T < T_d \Rightarrow \quad \text{decoherent system}$$

$$T_* \rightarrow \quad \left(\frac{\delta n}{n}\right)^2 \sim \int_0^\infty \frac{T\varepsilon\rho(\varepsilon)}{\varepsilon^2 + 2\varepsilon U\tilde{\nu}} d\varepsilon \sim 1$$

$$T_* \simeq T_d \frac{\ln(V/T_d)}{\ln(V/U\tilde{\nu})}$$

$$|\mu| = T \ln\left[\frac{1}{1 - \exp(-T_d/T)}\right]$$

Multiple occupation of the states

$$U_c\tilde{\nu} \approx (V - 2J) + |\mu|)\frac{T_d}{T}\frac{\pi^2}{2}\frac{\ln(V/T_d)}{\ln^2[V/(V-2J)]}$$

Normal behavior

Low-temperature regime

$$T < T_*$$

$T = 0 \Rightarrow$ the boson density is fragmented into lakes of size ζ_* if $\varepsilon_* > U\tilde{\nu}$

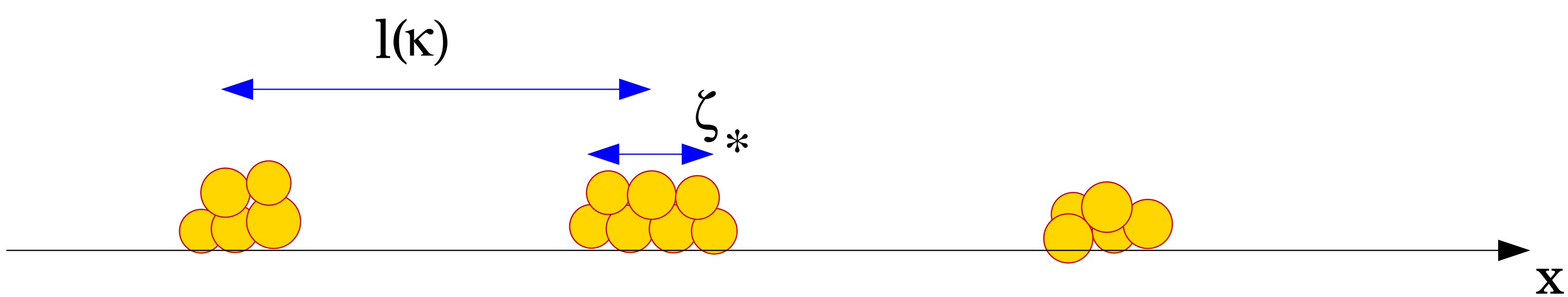
$$\varepsilon_* \sim [\zeta_* \rho(\varepsilon_*)]^{-1} \simeq \frac{\pi^2}{2} \frac{V - 2J}{\ln(V/\varepsilon_*)}$$

Energy cost of bringing a boson to lake i is $E_i = \epsilon_i + gN_i/\zeta_* = \mu$. Occupation $\sim \zeta_*(\mu - \epsilon_i)/Ua$

$$n = \mu^2/2gE_* \rightarrow \quad \mu \simeq \sqrt{2\varepsilon_* U\tilde{\nu}}$$

$U\tilde{\nu} < \varepsilon_* \rightarrow$ small fraction of low-energy states is occupied

Interlake distance $l > \zeta_* \rightarrow$ insulator

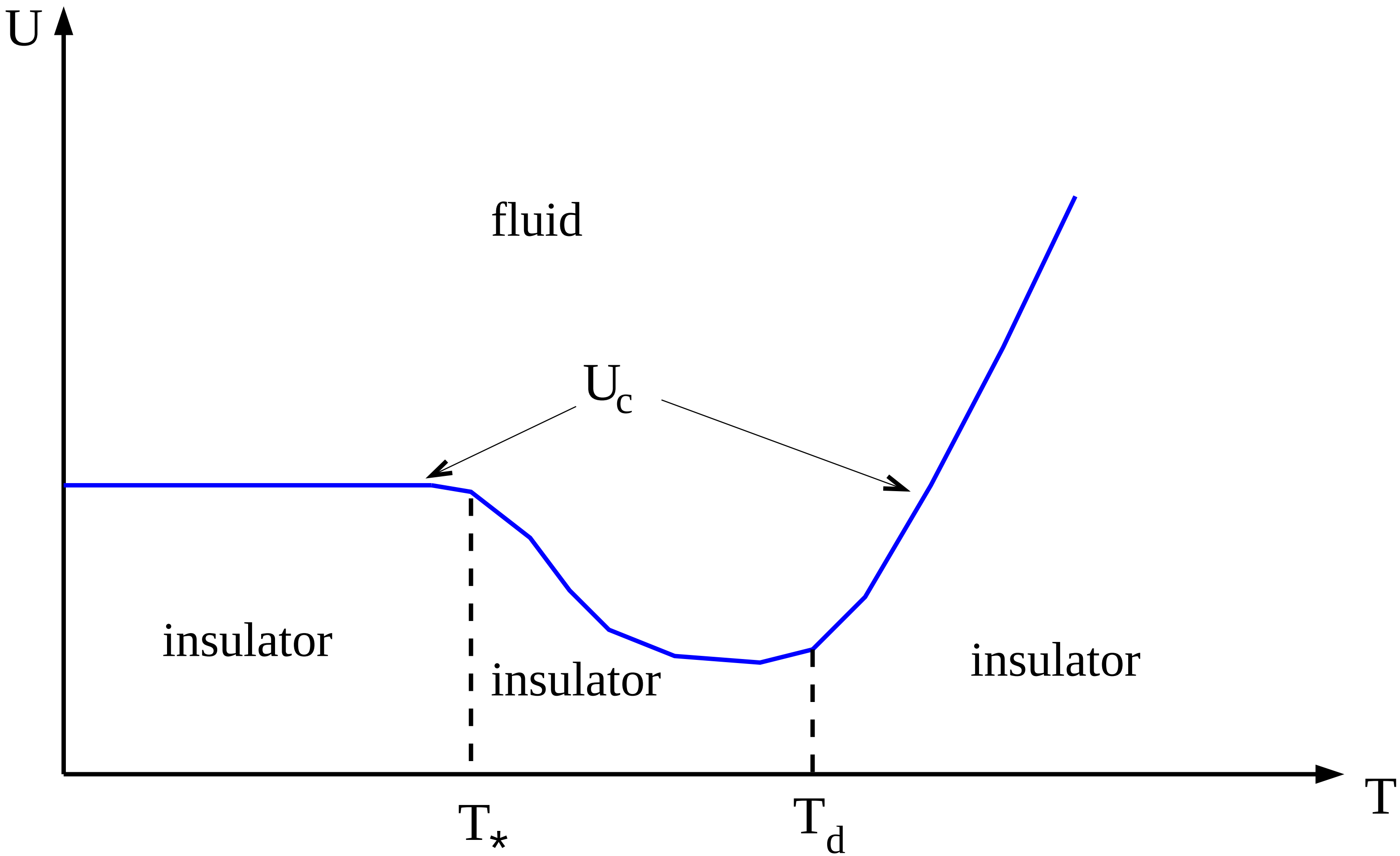


$U\tilde{\nu} > \varepsilon_* \rightarrow$ the interlake coupling drives the system to superfluid state

$$\varepsilon_* \sim U\tilde{\nu} \Rightarrow \quad \frac{\pi^2}{2} \frac{V - 2J}{\ln[V/(V - 2J)]} \sim U\tilde{\nu}$$

Phase diagram

Low-T criterion coincides with that in the quantum decoherent regime at $T \simeq T_*$



Conclusions

Atoms in quasiperiodic potentials $\rightarrow$ interesting system to study

Increasing temperature may favor localization

What I did not show $\rightarrow$ high-temperature regime ($T \gg 2J$)

What we did not yet do $\rightarrow$ strongly interacting regime

Thank you for attention!