Third cumulant in Quantum Point Contacts

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Experiment: **G. Gershon, Y. Bomze, Technion**
Theory: **E. Sukhorukov, Un. of Geneva**

- Predictions for $\ll J^2 \gg$ and $\ll J^3 \gg$
- Experimental setup
- Nontrivial relation between $\ll V^3 \gg$ and $\ll J^3 \gg$
  - Importance of coulomb interactions -- environmental corrections
  - Nonlinear contributions
- Experimental results
- Source of confusion in naïve calculations

How one can characterize fluctuations?

Noise: $$S(\omega) = \int_{-\infty}^{+\infty} \langle J(t)J(0)\rangle \exp(-i\omega t)dt$$ emission

Statistics of a charge transmitted during some time τ

$$\tau \gg 1/\max(T, eV)$$

$P(n) - ?$ Probability for n particles to be transmitted during some time τ

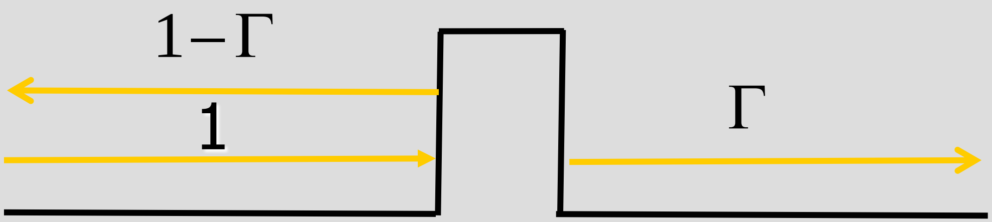
Alternatively we can discuss cumulants:

$$\langle\langle n\rangle\rangle = \langle n\rangle, \ \langle\langle n^2\rangle\rangle = \langle \delta n^2\rangle, \ \langle\langle n^3\rangle\rangle = \langle \delta n^3\rangle$$

$$\langle\langle J\rangle\rangle = \frac{e\langle n\rangle}{\tau} = I \quad \langle\langle J^2\rangle\rangle = \frac{e^2\langle \delta n^2\rangle}{\tau} = S(0) \quad \langle\langle J^3\rangle\rangle = \frac{e^3\langle \delta n^3\rangle}{\tau}$$

Simplest case – number of attempts fixed
$T = 0$

N – number of attempts $\quad\quad$ Γ -- probability to pass a barrier



Binomial process!

$$\langle n \rangle = N\Gamma \qquad \langle\langle J \rangle\rangle = \frac{e\langle n \rangle}{\tau} = \frac{eN\Gamma}{\tau} = I$$

$$\langle \delta n^2 \rangle = N\Gamma(1-\Gamma) \qquad \langle\langle J^2 \rangle\rangle = eI(1-\Gamma)$$

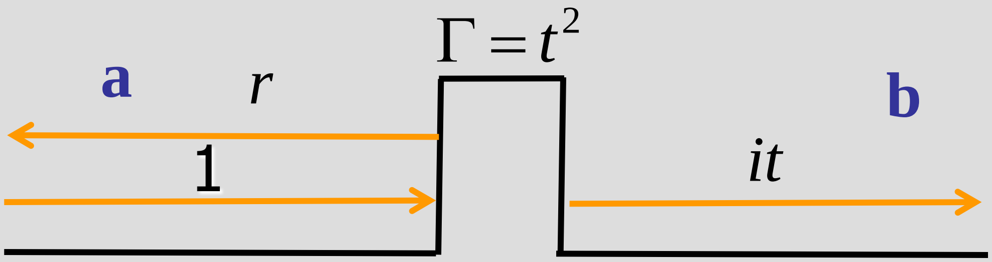
$$\langle \delta n^3 \rangle = N\Gamma(1-\Gamma)(1-2\Gamma) \qquad \langle\langle J^3 \rangle\rangle = e^2 I(1-\Gamma)(1-2\Gamma)$$

Motivation: $\ll J^3 \gg$ contains e^2 and starts from zero at $I = 0$

$\ll J^2 \gg$ -- Khlus(1987) Lesovik(1989) Yurke&Kochansky(1989)

$\ll J^3 \gg$ -- Levitov&Lesovik(1993) Levitov&Lee(1996)

Naïve calculations



$$q(\tau) = \left\langle \int_{o}^{\tau} \hat{J}(t)\,dt \right\rangle$$

$$\left\langle \delta q^{k} \right\rangle = \left\langle \left(\int_{o}^{\tau} \delta \hat{J}(t)\,dt \right)^{k} \right\rangle$$

$$I \propto (a^{+} \ \mathrm{b}^{+}) \begin{pmatrix} t^{2} & irt \\ -irt & -t^{2} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

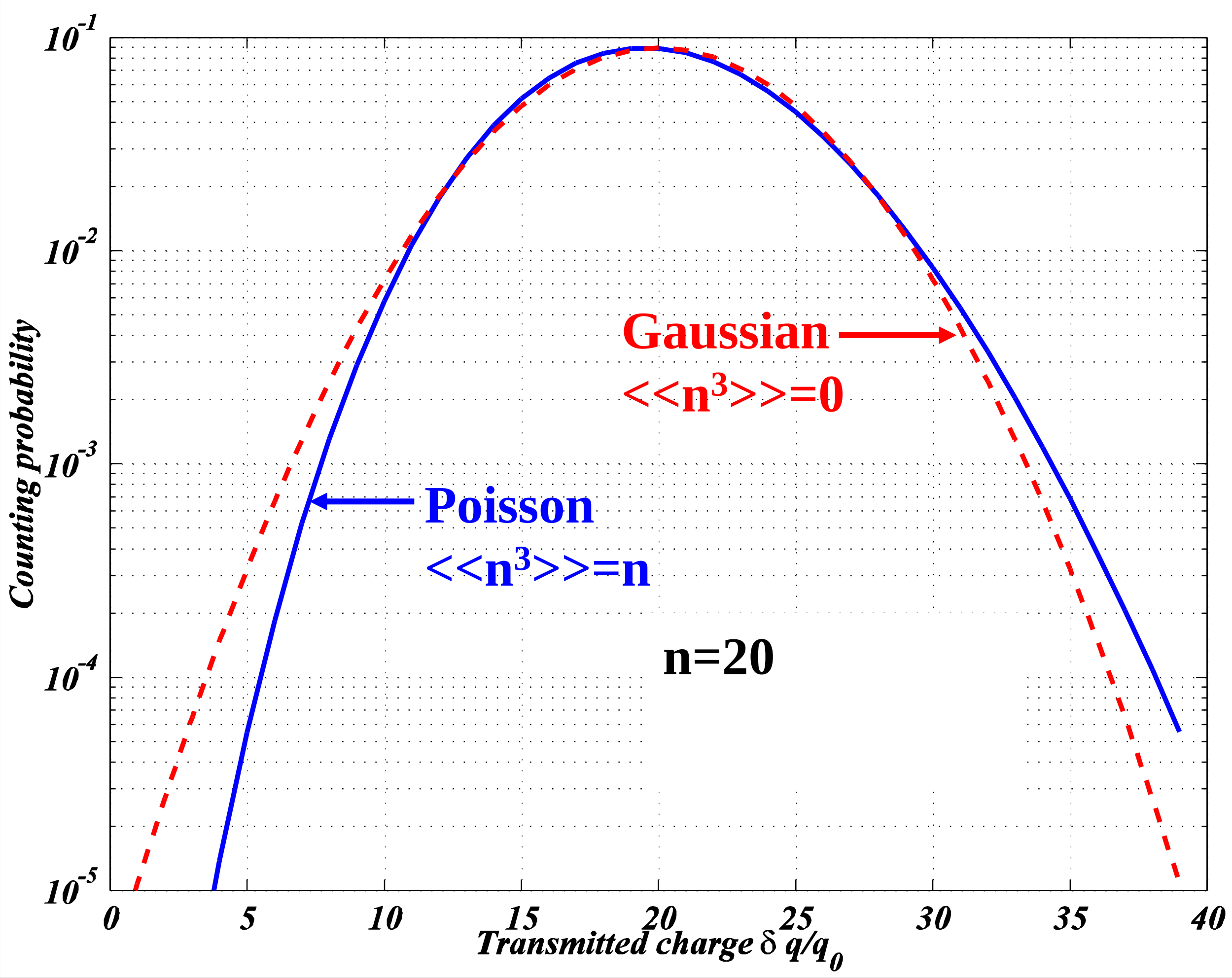
$$\langle\langle J^{3} \rangle\rangle = \frac{\delta q^{3}}{\tau} = g_{0} V \Gamma^{2} (1-\Gamma)$$

$$\Updownarrow$$

$$\langle\langle J^{3} \rangle\rangle = \frac{\delta q^{3}}{\tau} = g_{0} V \Gamma (1-\Gamma)(1-2\Gamma)$$

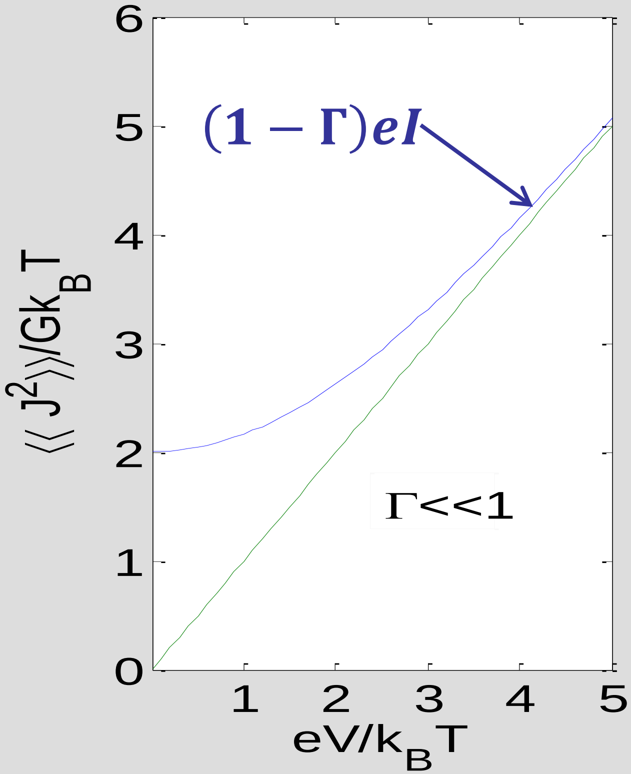
Technical resolution -- Bachmann, Graf, Lesovik (2010)

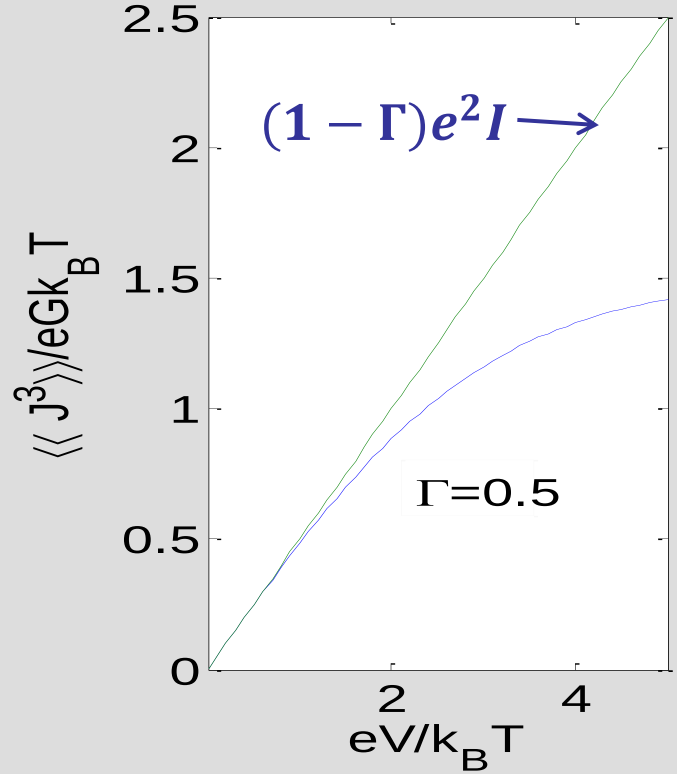
Gaussian vs Poisson distributions



Our measurements:
n~1000

<<J^3>> at finite temperature





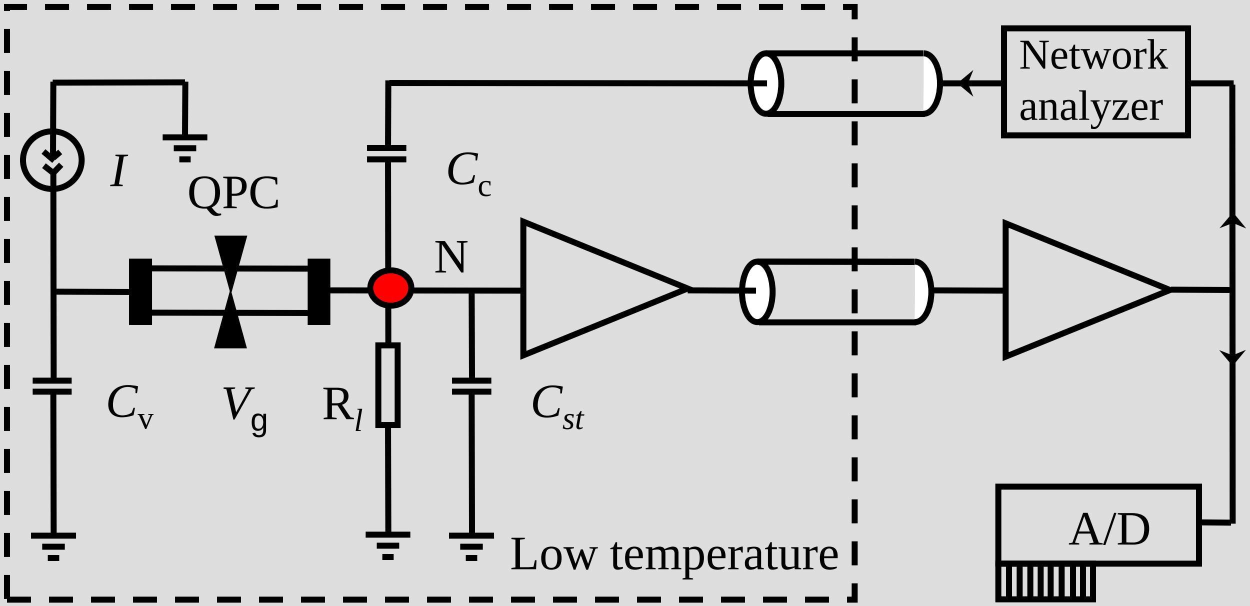
$$\langle\langle J^2\rangle\rangle = g_0\left[2k_B T\Gamma^2 + k_B T U\coth\left(\frac{U}{2}\right)\Gamma(1-\Gamma)\right]$$

$U = eV/k_BT$

$$\langle\langle J^3\rangle\rangle = e g_0 K_B T\cdot\Gamma(1-\Gamma)\left[6\Gamma\frac{\sinh(U)-U}{\cosh(U)-1} + U(1-2\Gamma)\right]$$

Levitov&Lesovik(1993) Levitov&Lee(1996)

Experimental Setup



$$P(V_N) \Rightarrow \langle\langle V_N^2 \rangle\rangle \text{ and } \langle\langle V_N^3 \rangle\rangle$$

What is being measured?

number of photons reaching an amplifier during a period $2\pi/\omega$:

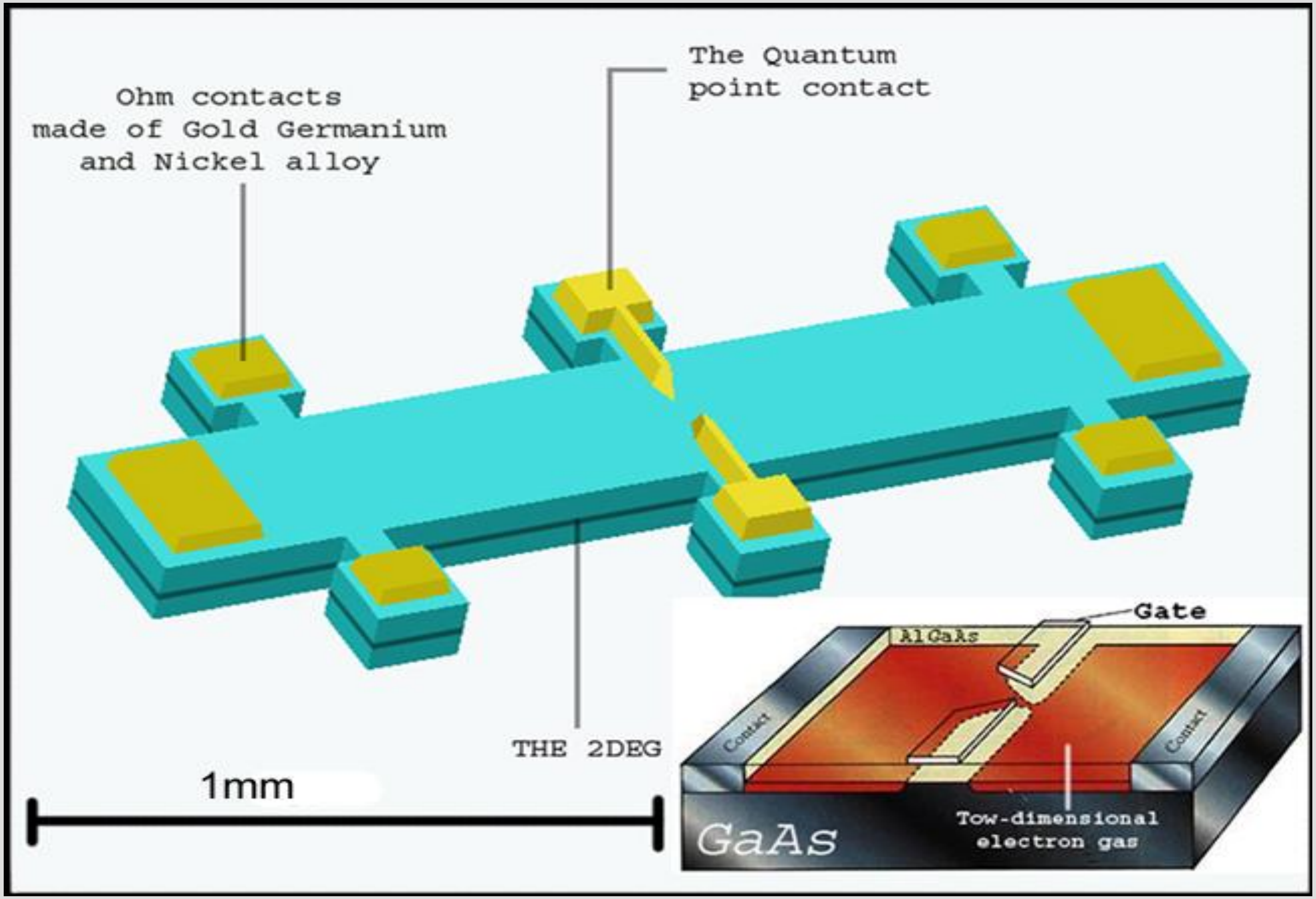
$$n_p \sim \frac{eIR\Delta\omega}{\hbar\omega\cdot\omega} = \frac{eV}{\hbar\omega}\frac{\Delta\omega}{\omega}$$

When $n_p \gg 1$ – classical case. Then a transistor amplifier measures charge

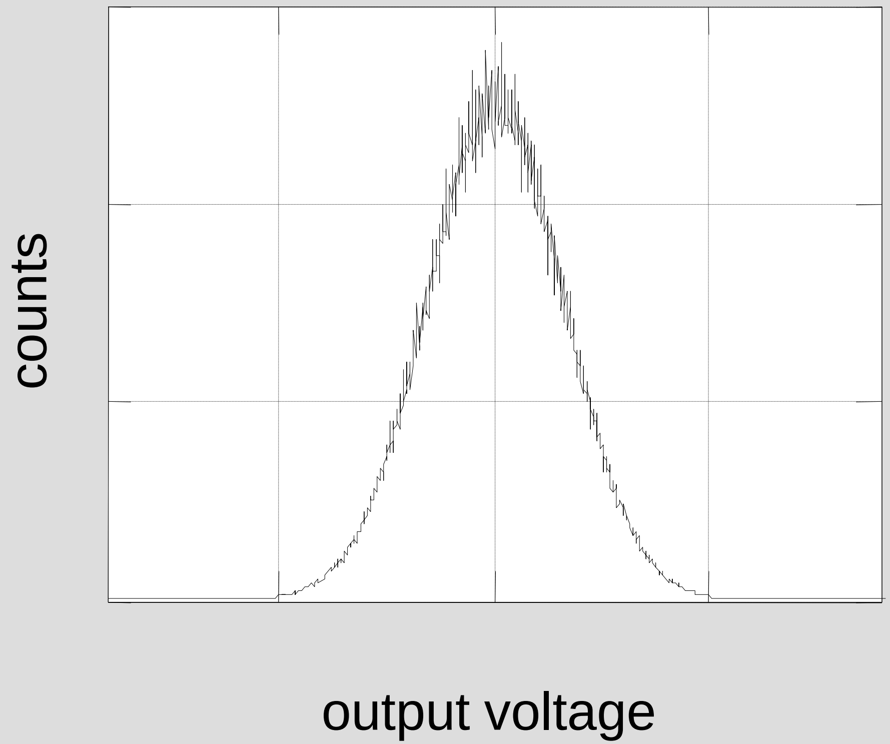
Opposite case, $n_p \ll 1$, when $\hbar\omega \sim eV$ – B. Reulet

$$P(V_N) \Rightarrow \langle\langle V_N^2 \rangle\rangle \text{ and } \langle\langle V_N^3 \rangle\rangle$$

The QPC

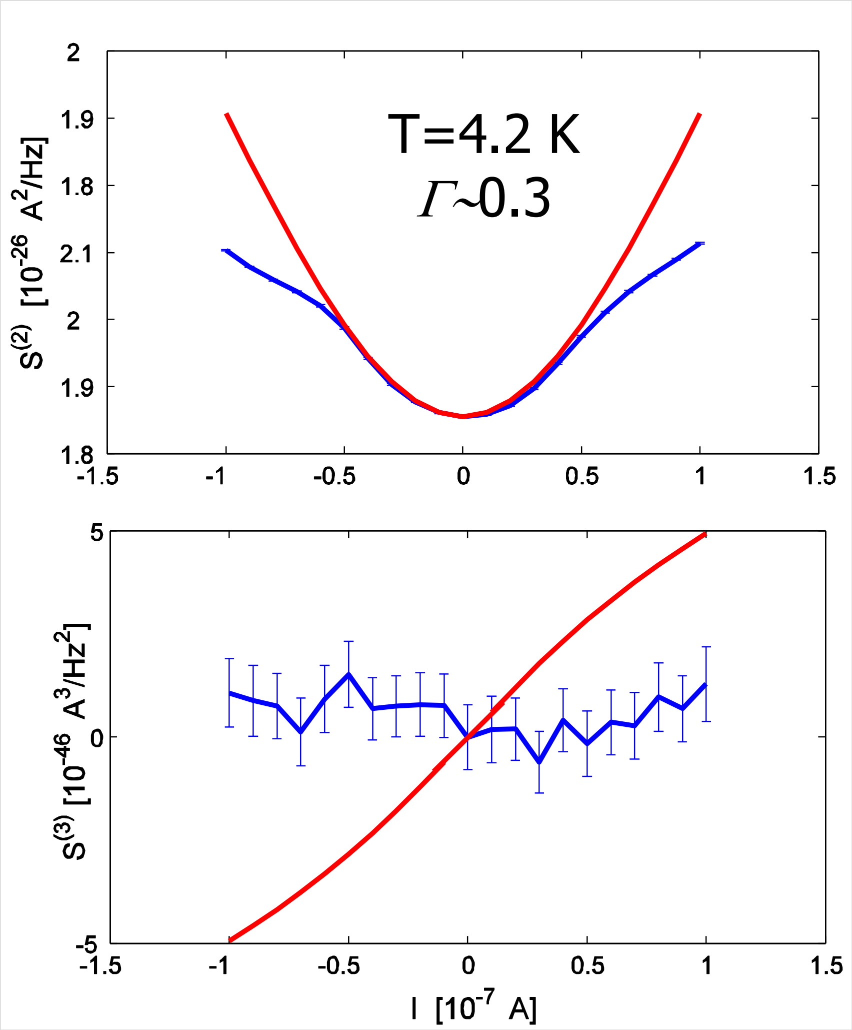

Ohm contacts
made of Gold Germanium
and Nickel alloy
The Quantum
point contact
THE 2DEG
1mm
Gate
AlGaAs
Contact
Contact
GaAs
Tow-dimensional
electron gas

The Probability Distribution Function

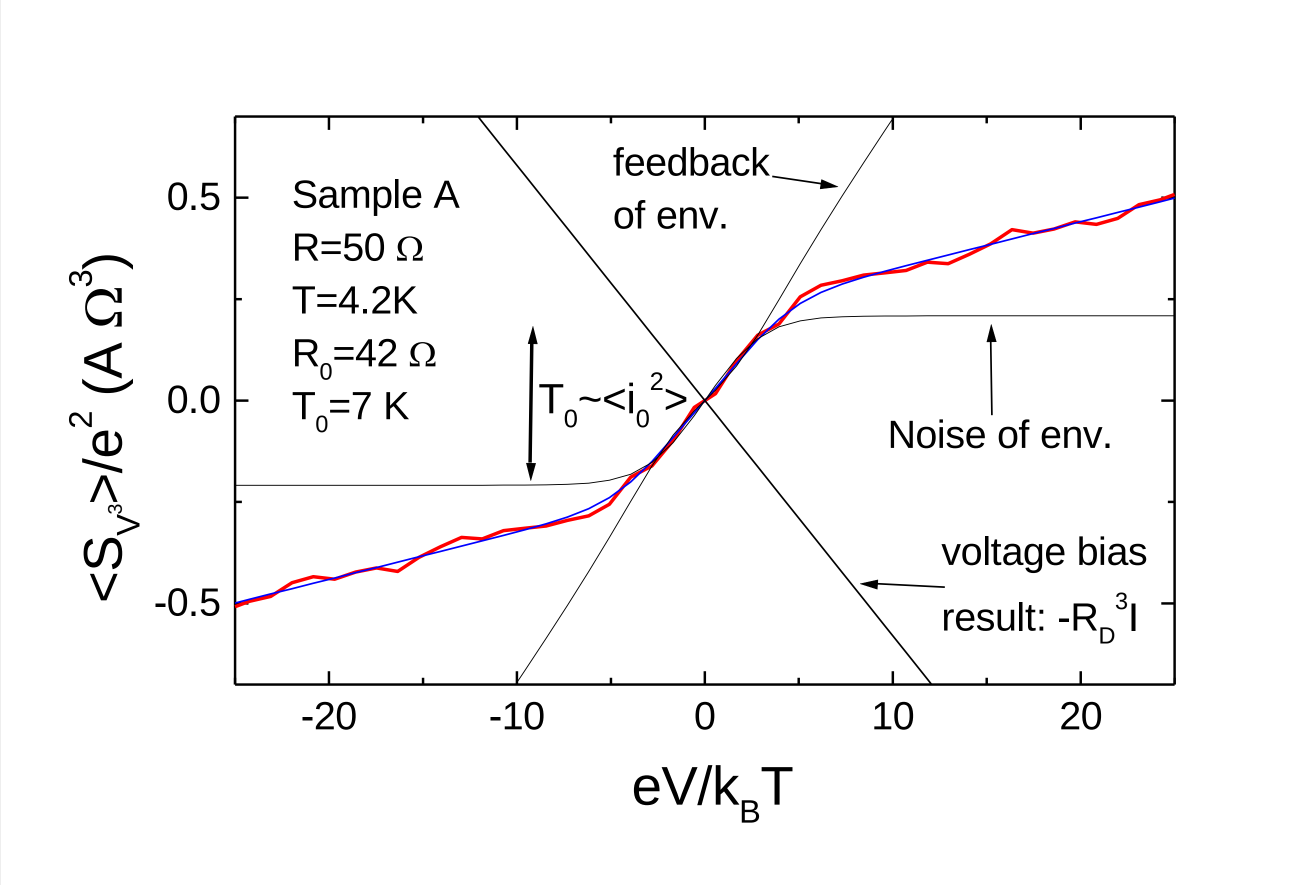


Typically 1000 electrons during 30 ns

$\ll J^2 \gg$ and $\ll J^3 \gg$ in QPC at ν=4

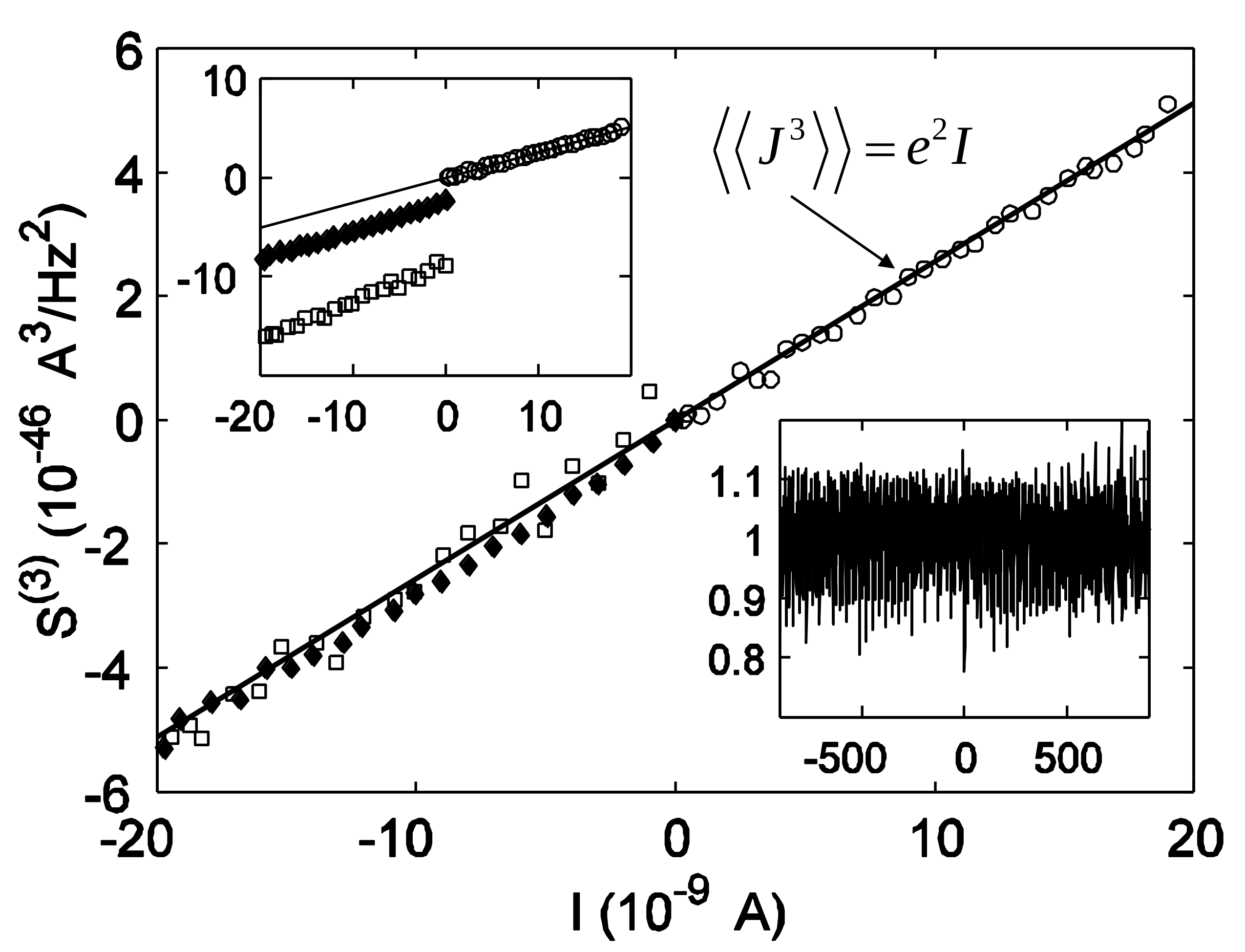


Experimental results from Yale



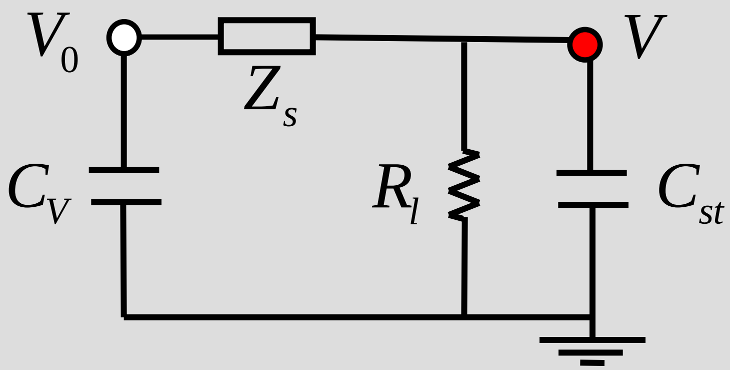
B. Reulet, J. Senzier and D. E. Prober, 2002

<<J3>> in a tunneling junction



PRL (2005)

$Z_s \gg R_l$ -- voltage bias, zero temperature



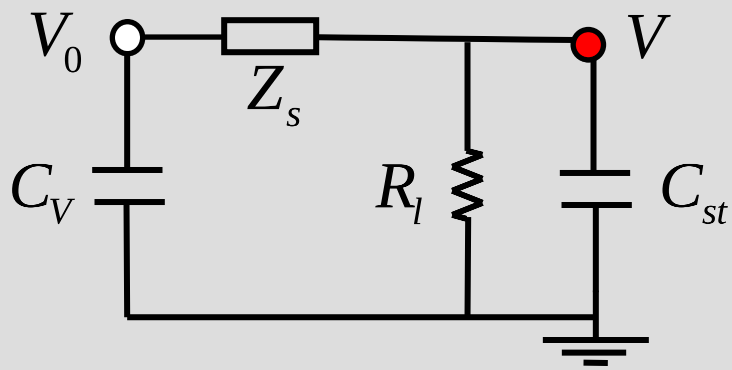
number of attempts fixed: $N = V_0 \tau e / h$

measured: fluctuations of $q = en$

Binomial statistics of charge:

$$P_N(n) = \begin{pmatrix} n \\ N \end{pmatrix} \Gamma^n (1-\Gamma)^{N-n}$$

$Z_s \ll R_l$ – current bias, zero temperature



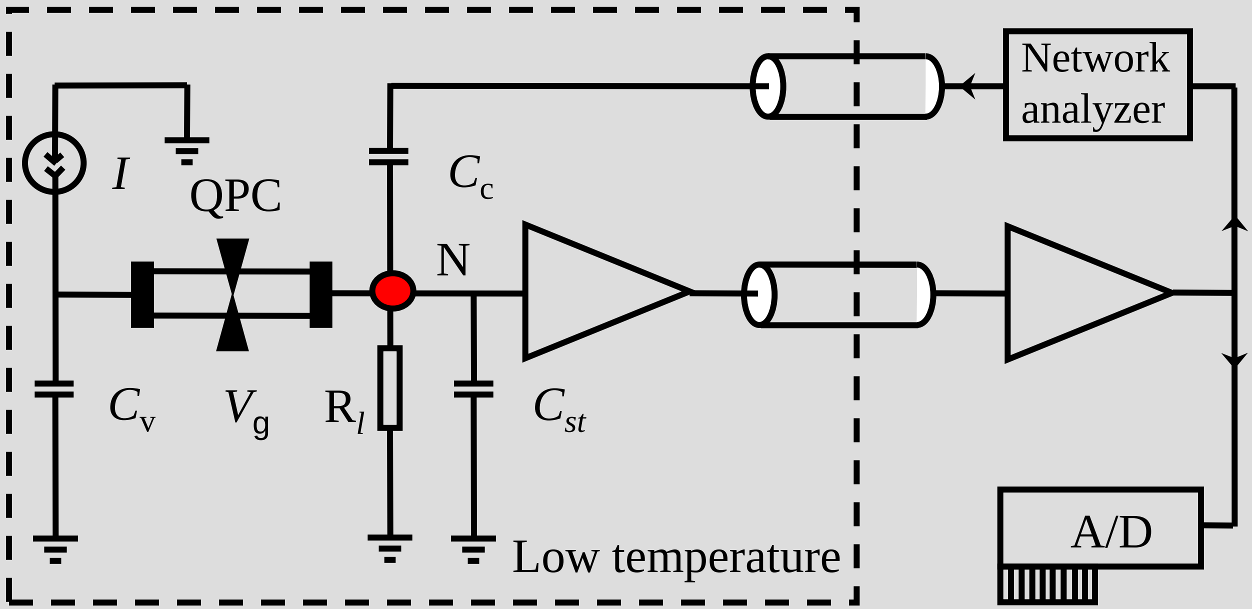
Current and therefore transmitted charge $q = ne$ is fixed

Measured: fluctuations of attempts $N = \frac{e}{h}\int_0^{\tau} dt V(t)$

Pascal distribution of N: $P_n(N) = \binom{n-1}{N-1}\Gamma^n (1-\Gamma)^{N-n}$

Y. Nazarov

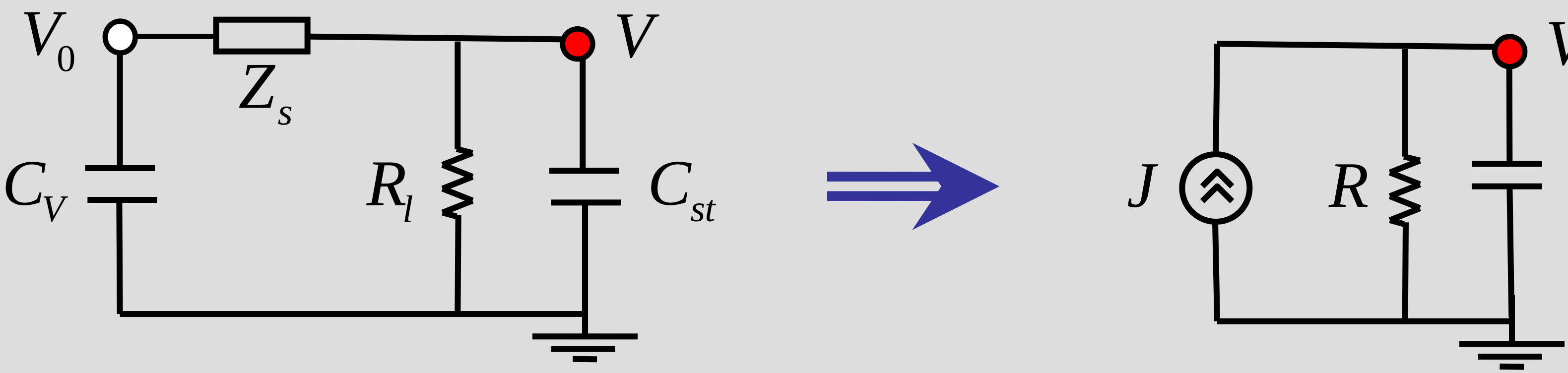
Experimental Setup



Probability distribution $P(V)$ is measured and from it cumulants are computed

$$P(V) \Rightarrow \langle\langle V^2 \rangle\rangle \text{and } \langle\langle V^3 \rangle\rangle$$

Environmental corrections



$$\langle\langle V^3\rangle\rangle = R^3\left(\langle\langle J^3\rangle\rangle + \langle\langle J^3\rangle\rangle_{env}\right)$$

$$\langle\langle J^3\rangle\rangle_{env} = -3R\langle\langle J^2\rangle\rangle \frac{d\langle\langle J^2\rangle\rangle}{dV}$$

This can be done when there is time separation:
$\ll J(t_1)j(t_2) \gg$ are correlated
on a time scale $\tau_c \ll 1/\omega$

$$\langle\langle J^2\rangle\rangle \approx \frac{2T}{R} + g_0\Gamma T\frac{(eV)^2}{6T^2}$$

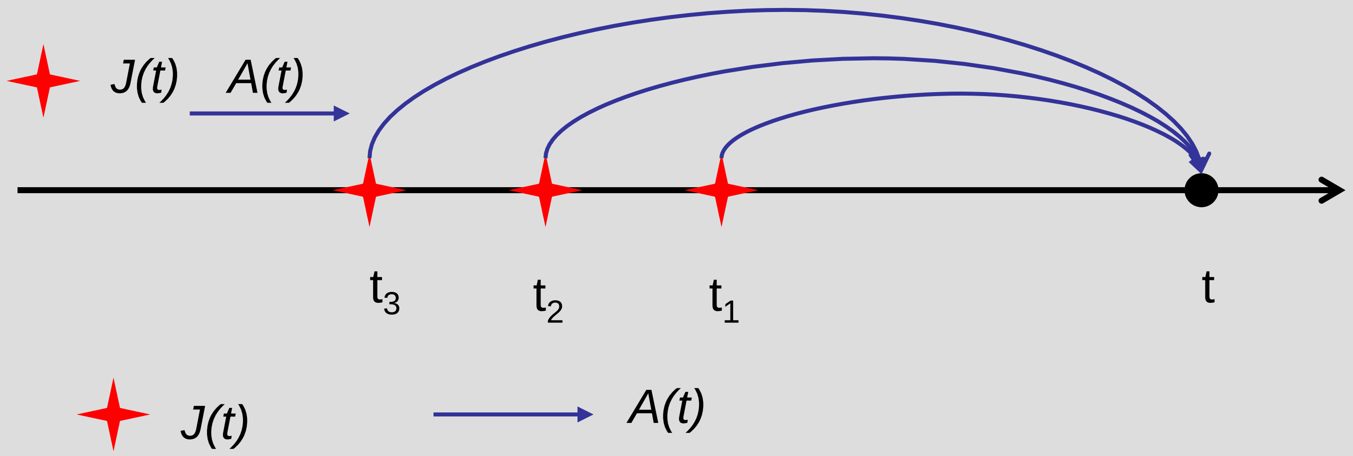
$$\langle\langle J^3\rangle\rangle_{env} \approx -2g_0 e^2\Gamma V = -2e^2 I = -2\langle\langle J^3\rangle\rangle$$

Low transmission tunneling junction
$\Gamma \ll 1$, $eV \ll T$

Even for $R << Z_s$ environmental corrections are not small! What is required
Is $R_l = 0$ and $T_l = 0$

Kindermann, Nazarov, Beenakker (2002)

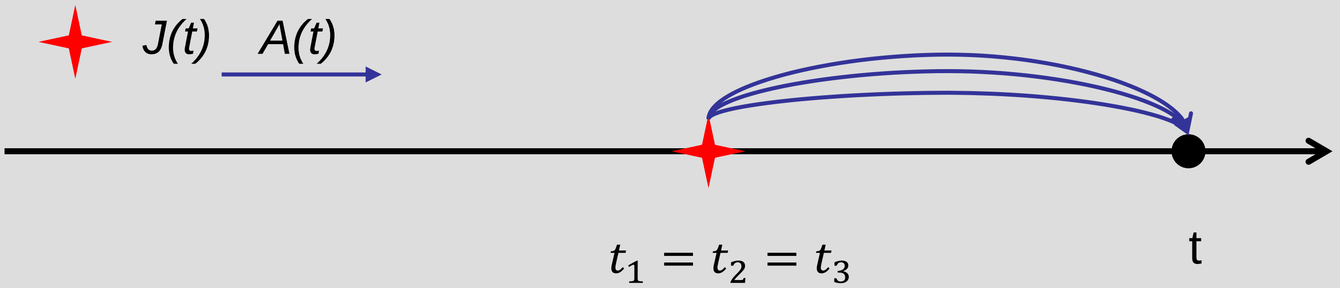
Different contributions to $\langle \delta V^3 \rangle$



Voltage at a time *t* is the response on the current at earlier times $t' < t$

$$V^3(t) = \int dt_1 dt_2 dt_3 A(t-t_1) J(t_1) A(t-t_2) J(t_2) A(t-t_3) J(t_3)$$

"Intrinsic" contribution to $\langle \delta V^3 \rangle$

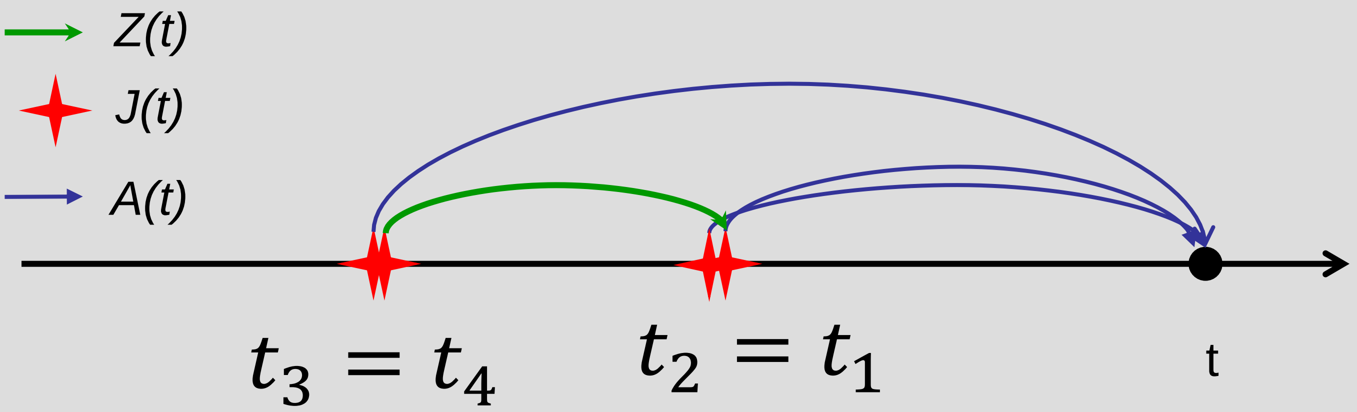


$$V^3(t) = \int dt_1 dt_2 dt_3 A(t-t_1) J(t_1) A(t-t_2) J(t_2) A(t-t_3) J(t_3)$$

Averaging gives "Intrinsic" (at constant voltage) contribution

$$\langle\langle V^3 \rangle\rangle_{\text{int}} = \langle\langle J^3 \rangle\rangle \int dt_1 A^3(t-t_1)$$

"Environmental" contribution



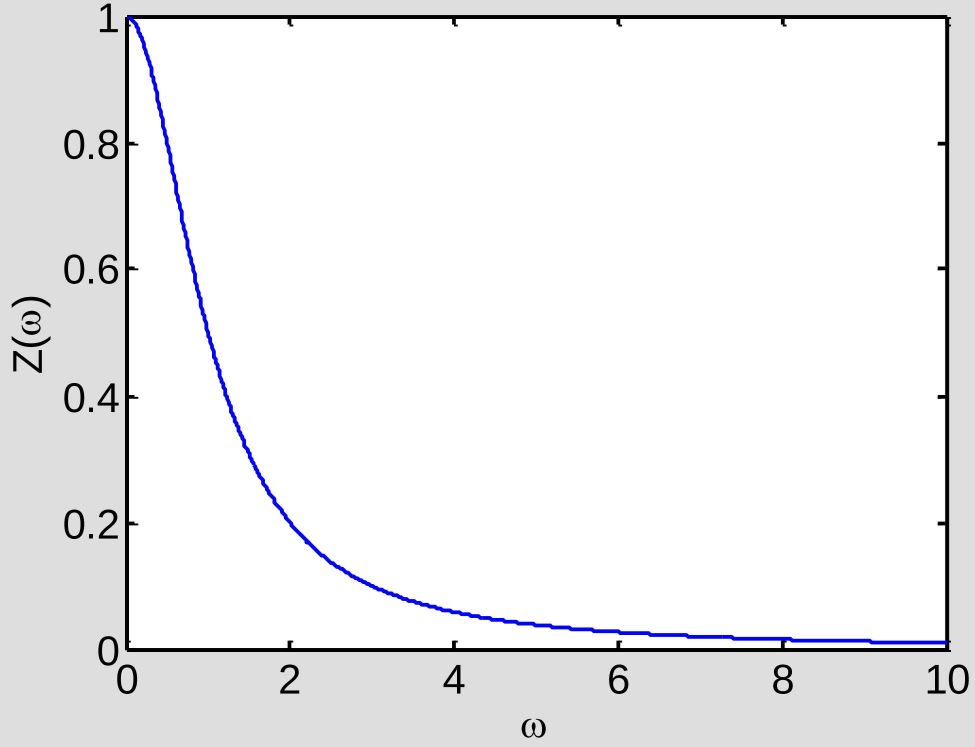
Another contribution arises due to voltage fluctuations over the sample at a time t_3 affecting its noise at a later time t_1:

$$\langle\langle V^3 \rangle\rangle_{env} = 3\langle\langle J^2 \rangle\rangle \frac{d\langle\langle J^2 \rangle\rangle}{dV} \int dt_1 dt_2 A^2(t-t_1) A(t-t_3) Z(t_1-t_3)$$

This can be done when there is time separation: $\ll j(t_1)j(t_2) \gg$ are correlated on a time scale $\tau_c \ll 1/\omega$

“environmental” contribution

For a particular case of Z set by an RC circuit $Z(t) = R\exp(-t/\tau)$ and frequency-independent amplification $A(t) = \Theta(t)$

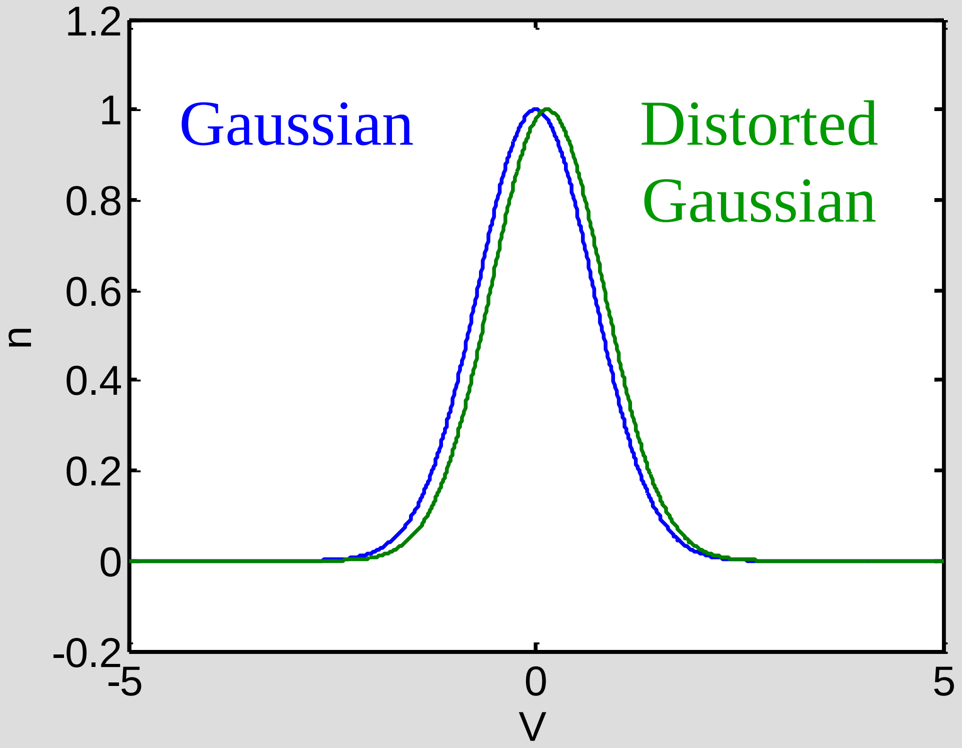


for QPC at $eV << T$, and $R_l \ll Z_s$

$$\langle\langle J^3 \rangle\rangle_{env} = -\frac{3}{2} R \langle\langle J^2 \rangle\rangle \frac{d\langle\langle J^2 \rangle\rangle}{dV}$$

$$\langle\langle J^3 \rangle\rangle_{env} = -\langle\langle J^3 \rangle\rangle$$

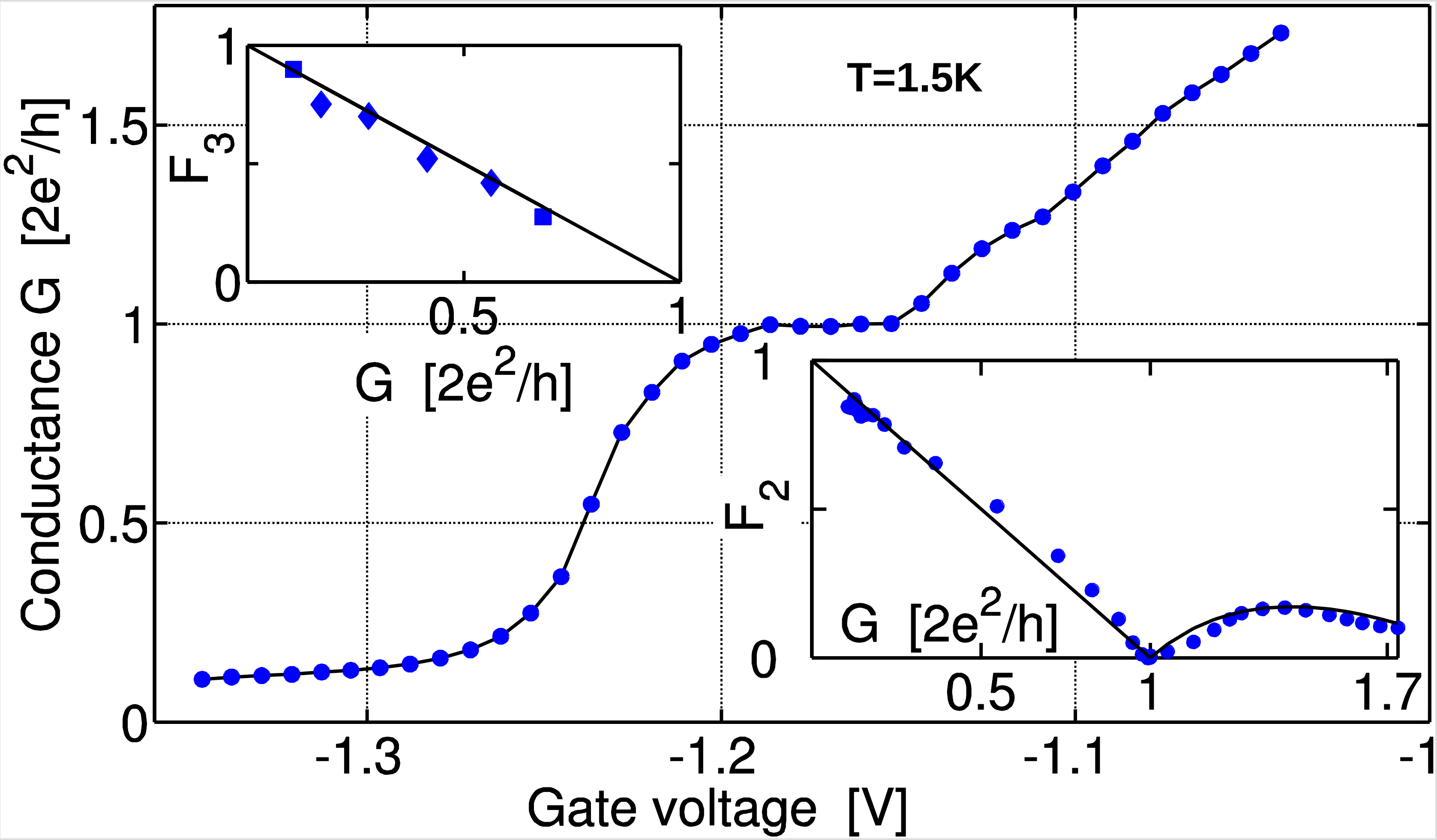
“nonlinear” contribution



Comes both from amplifiers and the sample

$$\left\langle\left\langle V^3 \right\rangle\right\rangle_{nl} = 3R_{\parallel} \left\langle\left\langle J^2 \right\rangle\right\rangle \frac{d^2 I}{dV^2} \int dt_1 dt_2 A(t-t_1)\left(A(t-t_2)Z(t_1-t_2)\right)^2$$

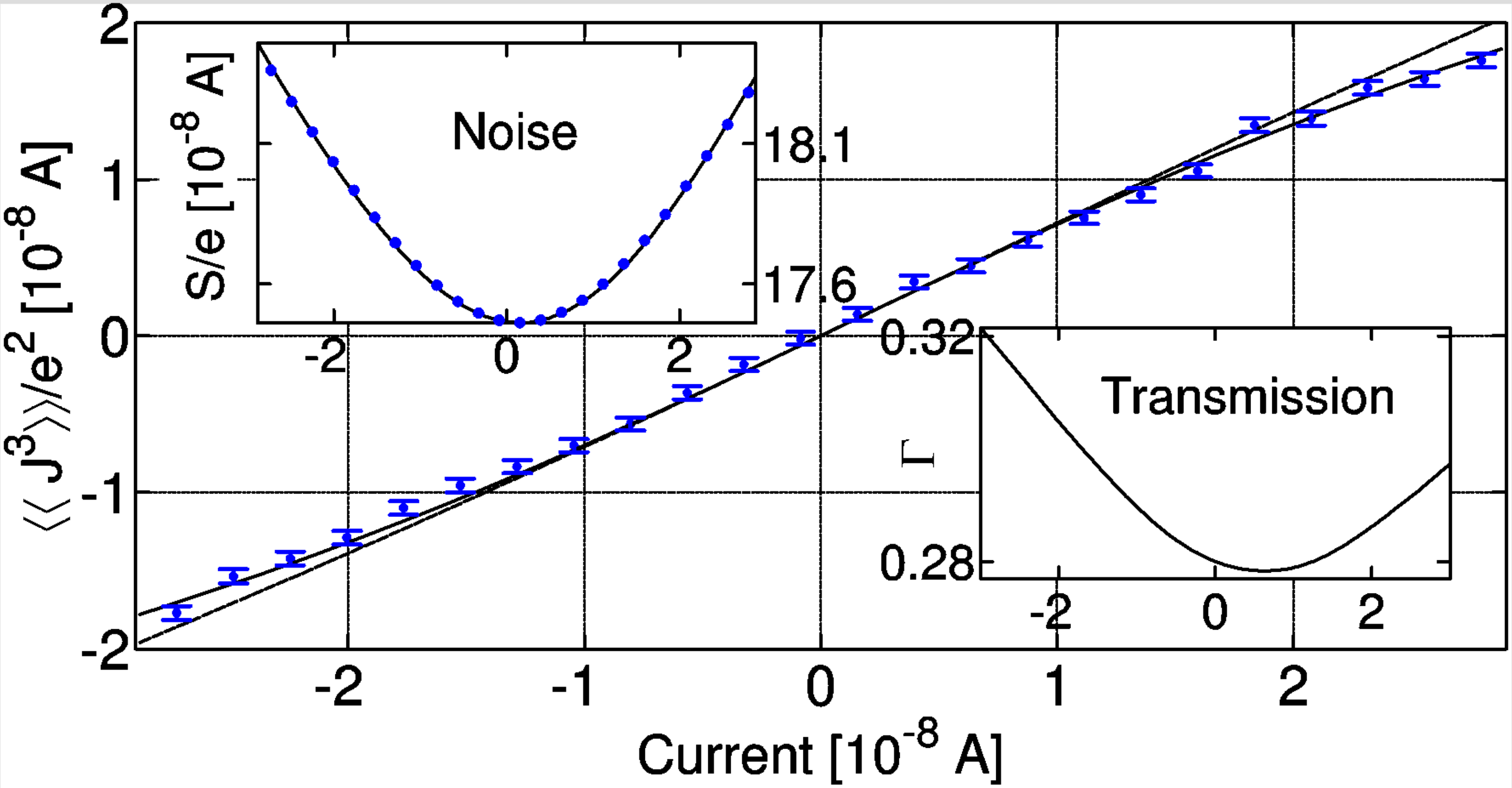
QPC characterization



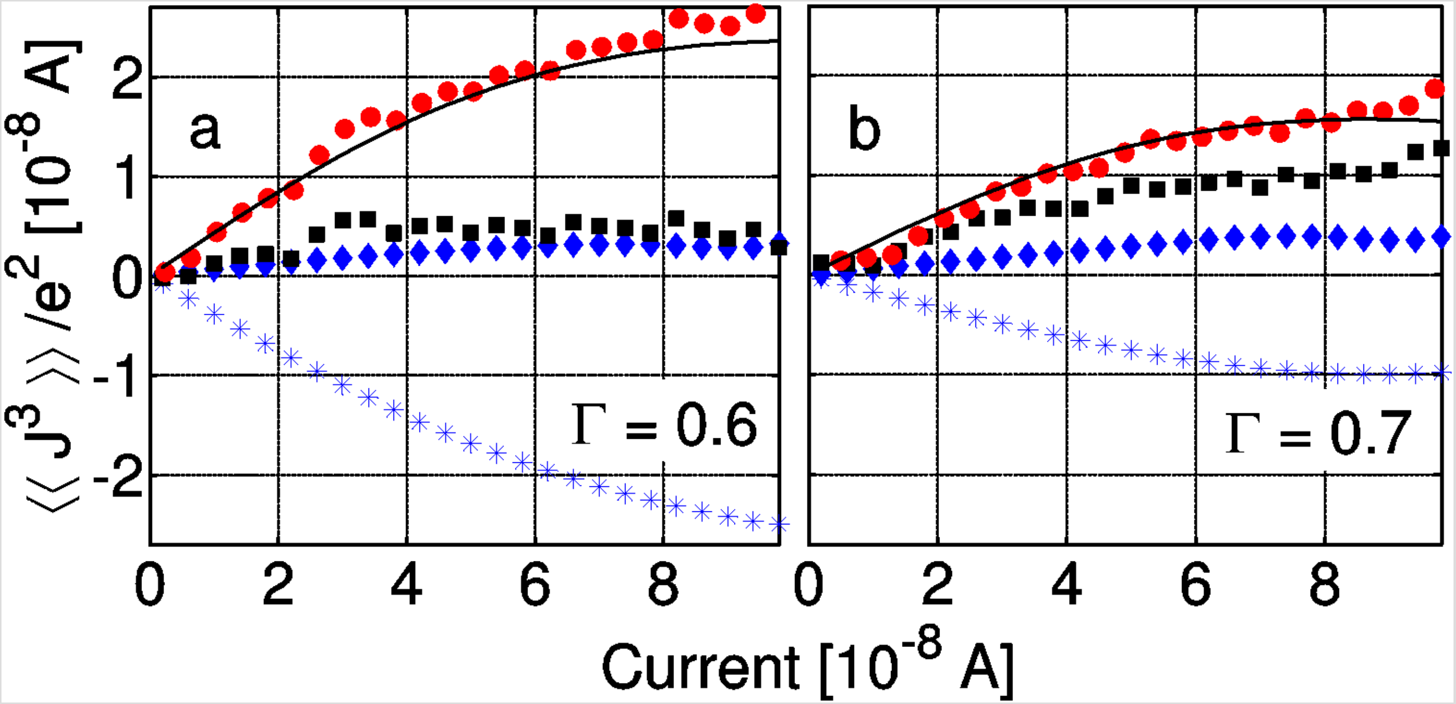
PRL (2008)

$\ll J^2 \gg$ and intrinsic $\ll J^3 \gg$ at $\Gamma \approx 0.3$

$$\Gamma = \frac{I}{g_0 V} \qquad T \approx 5K$$

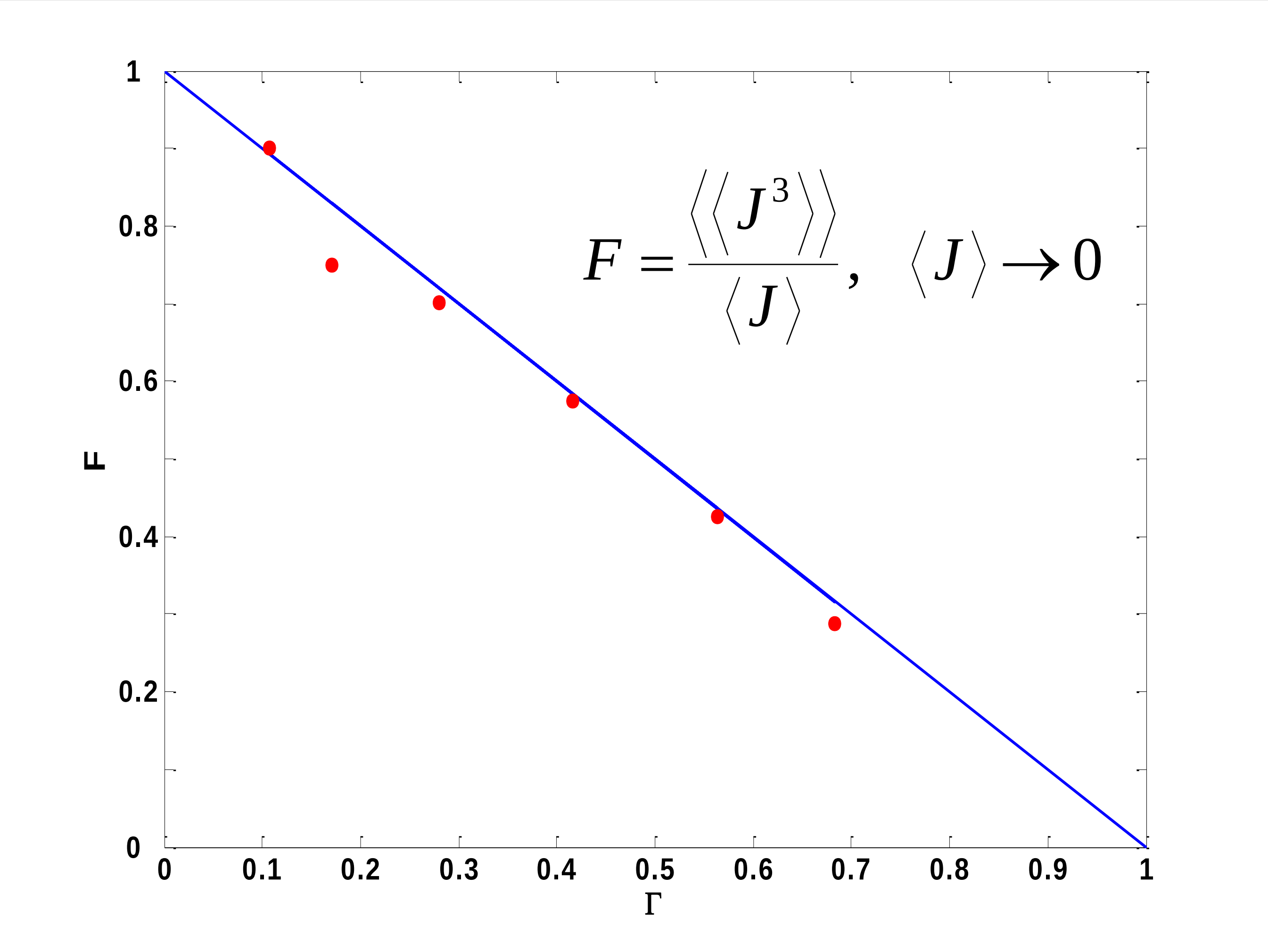


Different contributions to $\langle\langle J^3 \rangle\rangle$



- ■ As calculated from the $\langle\langle V^3 \rangle\rangle$
- * Environmental contribution
- ♦ Nonlinear contribution
- ● "Intrinsic" contribution

"Fano" factor for $\langle\langle J^3 \rangle\rangle$ at eV $\ll$ T



Conclusions

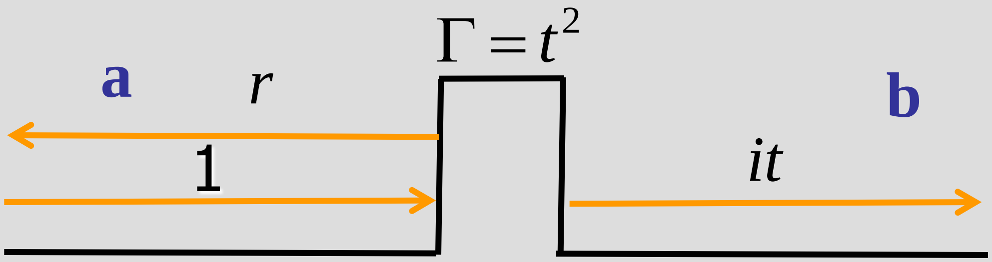
- **Predictions for $\ll J^3 \gg$ in QPC are verified**

- **A-meter much stronger affects higher cumulants than lower**

- **An ideal A-meter has zero resistance and temperature**

- **Impact/ Effort – small parameter.**

Naïve calculations



$$q(\tau) = \left\langle \int_o^\tau \hat{J}(t)\,dt \right\rangle$$

$$\left\langle \delta q^k \right\rangle = \left\langle \left(\int_o^\tau \delta \hat{J}(t)\,dt \right)^k \right\rangle$$

$$I \propto (a^+ \;\; \mathrm{b}^+) \begin{pmatrix} t^2 & irt \\ -irt & -t^2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\left\langle\left\langle J^3 \right\rangle\right\rangle = \frac{\delta q^3}{\tau} = g_0 V \Gamma^2 (1-\Gamma)$$

$\Updownarrow$

$$\left\langle\left\langle J^3 \right\rangle\right\rangle = \frac{\delta q^3}{\tau} = g_0 V \Gamma (1-\Gamma)(1-2\Gamma)$$

Technical resolution -- Bachmann, Graf, Lesovik (2010)

Calculation of the counting statistics

$$\chi(\lambda) = \sum_{i,f} \exp(i\lambda(q_f - q_i))P(f \leftarrow i)P(i)$$

$$\chi(\lambda) = \sum_{i,f} \exp(i\lambda(q_f - q_i))U^*_{f,i}U_{f,i}\rho_i =$$

$$\sum_{i,f} U^*_{i,f} \exp(i\lambda q_f)U_{f,i} \exp(-i\lambda q_i))\rho_i =$$

$$= \left\langle \exp(i\lambda q(t)) \exp(-i\lambda q(0)) \right\rangle = T \left\langle \exp(i\lambda (q(t) - q(0)) \right\rangle$$

Assumption: $\hat{q}(0)$ and $\hat{\rho}(0)$ commute

Calculation of the counting statistics

$$\chi(\lambda) = \left\langle \exp(i\lambda\hat{q}(t))\exp(-i\lambda\hat{q}(0)) \right\rangle$$
$$= T\left\langle \exp(i\lambda(\hat{q}(t) - \hat{q}(0)) \right\rangle$$

T-ordering is to put $\hat{q}(0)$ to the right of $\hat{q}(0)$

Using e.g. wave packet approach one can get the “binomial” statistics (Levitov, Lesovik, 1993)

How to express it through the integral of currents?

$$\left\langle \delta q^3 \right\rangle = T \left\langle (q(t) - q(0))^3 \right\rangle$$

Consider a slightly different object

$$Q^3(t_1, t_2, t_3) = T \left\langle (q(t_1) - q(0))(q(t_2) - q(0))(q(t_3) - q(0)) \right\rangle$$

Properties: $Q^3=0$ if one of $t_i=0$. Therefore it can be expressed as:

$$Q^3(t_1, t_2, t_3) = \int_0^t dt'_1 \, dt'_2 \, dt'_3 \, \frac{d^3 Q^3}{dt'_1 \, dt'_2 \, dt'_3}$$

Time ordering is crucial to ensure $Q^3=0$ for $t_i=0$!!!

Bachman, Graf, Lesovik (2009)

"Contact" terms

$$\frac{d^3 Q^3}{dt_1 dt_2 dt_3} \neq Tj(t_1)\, j(t_2)\, j(t_3)$$

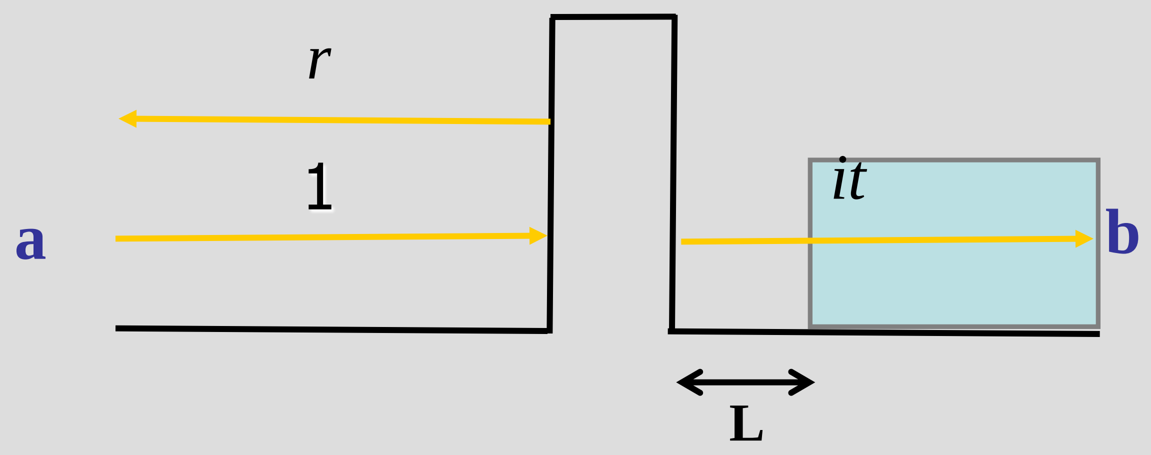
$$\frac{dQ^3}{dt_3} = T\left\langle (q(t_1) - q(0))(q(t_2) - q(0))\, j(t_3) \right\rangle$$

$$\frac{dQ^3}{dt_2 dt_3} = T\left\langle (q(t_1) - q(0))\, j(t_2)\, j(t_3) \right\rangle +$$

$$+T\left\langle (q(t_1) - q(0))[q(t_2),\, j(t_2)] \right\rangle \delta(t_2 - t_3)$$

Differentiation over t_1 would generate two more δ-functions, provided $[q(t), j(t)] \neq 0$. So, there are additions to the term Accounted for in naïve calculations: $\int_0^t d^3 t'\, j(t'_1) j(t'_2) j(t'_3)$

My favorite choice of $\hat{J}$



$$\hat{q} = \begin{pmatrix} t^2 & itr \\ -itr & r^2 \end{pmatrix} \theta(x-L) + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \theta(x+L)$$

$$\hat{J} = \begin{pmatrix} t^2 & itr \\ -itr & r^2 \end{pmatrix} \delta(x-L) - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \delta(x+L)$$

$$\left[\hat{J}, \hat{q}\right] = 0$$

Compare with: $$\hat{J} = \begin{pmatrix} t^2 & itr \\ -itr & -t^2 \end{pmatrix} \delta(x)$$