Universality at small $x$ : from DIS to heavy ion collisions

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Outline

1. Gluon saturation at small $x$
2. Factorization in Deep Inelastic Scattering
3. Nucleus-Nucleus collisions
1. Gluon saturation at small x
   Saturation domain
   Multiple scatterings
   CGC effective theory
Saturation domain

\[ \log(x^{-1}) \quad \Lambda_{\text{QCD}} \]

\[ \log(Q^2) \]

\[ \log(\frac{Q^2}{x-1}) \]
Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area:

\[ \rho \sim \frac{x G_A(x, Q^2)}{\pi R_A^2} \]

Recombination cross-section:

\[ \sigma_{gg \rightarrow g} \sim \frac{\alpha_s}{Q^2} \]

Recombination happens if \( \rho \sigma_{gg \rightarrow g} \gtrsim 1 \), i.e. \( Q^2 \lesssim Q_s^2 \), with:

\[ Q_s^2 \sim \frac{\alpha_s x G_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} \frac{1}{x^{0.3}} \]

Note: At a given energy, the saturation scale is larger for a nucleus (for \( A = 200 \), \( A^{1/3} \approx 6 \))
1 Gluon saturation at small $x$

Saturation domain
Multiple scatterings
CGC effective theory
Multiple scatterings

- **Power counting:**

\[
\frac{2 \text{ scatterings}}{1 \text{ scattering}} \sim \frac{Q_s^2}{M_\perp^2} \quad \text{with} \quad Q_s^2 \sim \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2}
\]

- When this ratio becomes \( \sim 1 \), all the rescattering corrections become important

  ▶ one must resum all \([Q_s/P_\perp]^n]\)

- These effects are not accounted for in DGLAP or BFKL
Gluon saturation at small $x$
Saturation domain
Multiple scatterings
CGC effective theory
• Main difficulty: How to treat collisions involving a large number of partons?
Requirements

- **Main difficulty**: How to treat collisions involving a large number of partons?

- **Dilute regime**: one parton in each projectile interact (what the standard perturbative techniques are made for)
Requirements

- Main difficulty: How to treat collisions involving a large number of partons?

- Dense regime: **multiparton processes** become crucial
  - new techniques are required
  - multi-parton distributions are needed
CGC: Degrees of freedom

CGC = effective theory of small x gluons

- The fast partons \((k^+ > \Lambda^+)\) are frozen by time dilation
  ▶ described as static color sources on the light-cone:

\[
J^\mu = \delta^{\mu+} \rho(x^-, \vec{x}_\perp) \quad (0 < x^- < 1/\Lambda^+)
\]

- Slow partons \((k^+ < \Lambda^+)\) cannot be considered static over the time-scales of the collision process
  ▶ must be treated as standard gauge fields
  ▶ eikonal coupling to the current \(J^\mu : A_\mu J^\mu\)

- The color sources \(\rho\) are random, and described by a distribution \(W_{\Lambda^+}[\rho]\), with \(\Lambda^+\) the longitudinal momentum that separates “soft” and “hard”
CGC: renormalization group evolution

Independence w.r.t. $\Lambda^+$ → evolution equation (JIMWLK):

$$\frac{\partial W_{\Lambda^+}}{\partial \ln (\Lambda^+)} = \mathcal{H} \ W_{\Lambda^+}$$

$$\mathcal{H} = \frac{1}{2} \int_{\vec{x}_\perp, \vec{y}_\perp} \frac{\delta}{\delta \alpha (\vec{y}_\perp)} \eta (\vec{x}_\perp, \vec{y}_\perp) \frac{\delta}{\delta \alpha (\vec{x}_\perp)}$$

where

$$-\partial^2_\perp \alpha (\vec{x}_\perp) = \rho (1/\Lambda^+, \vec{x}_\perp)$$

- $\eta (\vec{x}_\perp, \vec{y}_\perp)$ is a non-linear functional of $\rho$
- Resums all the powers of $\alpha_s \ln (1/x)$ and of $Q_s/\rho_\perp$ that arise in loop corrections
- Simplifies into the BFKL equation when the source $\rho$ is small (expand $\eta$ in powers of $\rho$)
1. **Gluon saturation at small x**
   - Saturation domain
   - Multiple scatterings
   - CGC effective theory

2. **Factorization in Deep Inelastic Scattering**
   - Leading Order
   - Next to Leading Order
   - Leading Log resummation

3. **Nucleus-Nucleus collisions**
   - Stages of AA collisions
   - Energy-Momentum tensor
   - Rapidity correlations
Factorization in Deep Inelastic Scattering

Leading Order
Next to Leading Order
Leading Log resummation
Inclusive DIS at Leading Order

- CGC effective theory with cutoff at the scale $\Lambda_0^-$:

  \[ \quad \cdots \quad \text{fields} \quad \rightarrow\leftarrow \quad \text{sources} \quad \rightarrow \quad k^- \]

  \[ \Lambda_0^- \quad P^- \]

- At **Leading Order**, DIS can be seen as the interaction between the target and a $q\bar{q}$ fluctuation of the virtual photon:

  ![Diagram](attachment:diagram.png)
Inclusive DIS at Leading Order

- Forward dipole amplitude at leading order:

\[
T_{\text{LO}}(\vec{x}_\perp, \vec{y}_\perp) = 1 - \frac{1}{N_c} \text{tr} \left( U(\vec{x}_\perp) U^\dagger(\vec{y}_\perp) \right) \]

Wilson lines

\[
U(\vec{x}_\perp) = \text{P exp} i g \int \frac{dz^+}{x P^-} A^-(z^+, \vec{x}_\perp)
\]

\[
[D_\mu, F^{\mu\nu}] = \delta^{\nu^+} \rho(x^+, \vec{x}_\perp)
\]

▷ at LO, the scattering amplitude on a saturated target is entirely given by classical fields

- Note: the \( q\bar{q} \) pair couples only to the sources up to the longitudinal coordinate \( z^+ \lesssim (x P^-)^{-1} \). The other sources are too slow to be seen by the probe
Factorization in Deep Inelastic Scattering

- Leading Order
- Next to Leading Order
- Leading Log resummation
Inclusive DIS at NLO

- Consider now quantum corrections to the previous result, restricted to modes with $\Lambda_1^- < k^- < \Lambda_0^-$ (the upper bound prevents double-counting with the sources):

  ![Diagram showing fields and sources](attachment:image.png)

  $k^-$

  $\Lambda_1^- \quad \Lambda_0^- \quad P^-$

- At NLO, the $q\bar{q}$ dipole must be corrected by a gluon, e.g.:
At leading log accuracy, the contribution of the quantum modes in that strip is:

\[
\delta T_{\text{NLO}}(\vec{x}_\perp, \vec{y}_\perp) = \ln \left( \frac{\Lambda_0^-}{\Lambda_1^-} \right) \mathcal{H} \ T_{\text{LO}}(\vec{x}_\perp, \vec{y}_\perp)
\]
Inclusive DIS at NLO

- These NLO corrections can be absorbed in the LO result,

\[ \left( T_{\text{LO}} + \delta T_{\text{NLO}} \right)_{\Lambda_0^-} = \left( T_{\text{LO}} \right)_{\Lambda_1^-} \]

provided one defines a new effective theory with a lower cutoff \( \Lambda_1^- \) and an extended distribution of sources \( W_{\Lambda_1^-}[\rho] \):

\[
W_{\Lambda_1^-} \equiv \left[ 1 + \ln \left( \frac{\Lambda_0^-}{\Lambda_1^-} \right) \mathcal{H} \right] W_{\Lambda_0^-}
\]

(JIMWLK equation for a small change in the cutoff)
2 Factorization in Deep Inelastic Scattering
   Leading Order
   Next to Leading Order
   Leading Log resummation
Inclusive DIS at Leading Log

- Iterate the previous process to integrate out all the slow field modes at leading log accuracy:

\[
\sigma_{\gamma^* T} = \int_0^1 dz \int d^2 \vec{r}_\perp |\psi(q|z, \vec{r}_\perp)|^2 \sigma_{\text{dipole}}(x, \vec{r}_\perp)
\]

\[
\sigma_{\text{dipole}}(x, \vec{r}_\perp) \equiv 2 \int d^2 \vec{x}_\perp \int [D\rho] W_{xP^\perp} [\rho] T_{LO}(\vec{x}_\perp, \vec{y}_\perp)
\]

- One does not need to evolve down to \( \Lambda^- \to 0 \): the DIS amplitude becomes independent of \( \Lambda^- \) when \( \Lambda^- \lesssim xP^- \)

\begin{center}
\begin{tikzpicture}
\draw[thick,->] (0,0) -- (4,0) node[right] {\( k^- \)};
\draw[thick] (0.5,-0.5) -- (0.5,0.5); \draw[thick] (1.5,-0.5) -- (1.5,0.5); \draw[thick] (2.5,-0.5) -- (2.5,0.5);
\draw[red, thick] (0,0) -- (0.5,0) node[above] {$\Lambda^- xP^\perp$}; \draw[red, thick] (1,0) -- (1.5,0) node[above] {$\Lambda^- \Lambda_0^-$}; \draw[red, thick] (2,0) -- (2.5,0) node[above] {$P^-$};
\end{tikzpicture}
\end{center}

\begin{center}
\begin{tikzpicture}
\draw[blue, thick] (0,0) -- (0.5,0) node[above] {$\Lambda^- \to 0$}; \draw[blue, thick] (1,0) -- (1.5,0) node[above] {$\to$}; \draw[blue, thick] (2,0) -- (2.5,0) node[above] {$\to$};
\end{tikzpicture}
\end{center}
Nucleus-Nucleus collisions

Stages of AA collisions

Energy-Momentum tensor

Rapidity correlations
The Color Glass Condensate provides a framework to describe nucleus-nucleus collisions up to a time $\tau \sim Q_s^{-1}$.

Subsequent stages are described as fluid dynamics.
Reminder on hydrodynamics

Equations of hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0$$

Additional inputs:

- EoS: $p = f(\epsilon)$
- Transport coefficients: $\eta, \zeta, \cdots$

- Required initial conditions: $T^{\mu\nu}(\tau = \tau_0, \eta, \vec{x}_\perp)$
3 Nucleus-Nucleus collisions
Stages of AA collisions
Energy-Momentum tensor
Rapidity correlations
From CGC: power counting

- CGC effective theory with cutoff at the scale $\Lambda_0^+$:

\[ T_{\mu\nu} = \frac{Q_s^4}{g^2} \left[ c_0 + c_1 g^2 + c_2 g^4 + \cdots \right] \]
Leading Order

\[ T_{\mu\nu}^{\text{LO}} = \sum \text{trees} \]

(all propagators retarded)

Leading Order contribution given by classical fields:

\[ T_{\mu\nu}^{\text{LO}} \equiv c_0 \frac{Q_s^4}{g^2} = \frac{1}{4} g_{\mu\nu} F^{\lambda\sigma} F_{\lambda\sigma} - F^{\mu\lambda} F_{\nu\lambda} \]

\[ \left[ D_\mu, F^{\mu\nu} \right] = J_1^\nu + J_2^\nu \]

Yang–Mills equation

\[ \lim_{t \to -\infty} A_\mu(t, \vec{x}) = 0 \]
Leading Logs  [FG, Lappi, Venugopalan (2008)]

- The $c_n$ are not numbers of order one: large logarithms of the CGC cutoff appear in $c_{1,2,\ldots}$
- Like in DIS, the coefficients of the logs are given by the action of the JIMWLK Hamiltonian on the LO observable:

$$\delta T_{\text{NLO}}^{\mu\nu} = \left[ \ln \left( \frac{\Lambda_0^-}{\Lambda_1^-} \right) \mathcal{H}_1 + \ln \left( \frac{\Lambda_0^+}{\Lambda_1^+} \right) \mathcal{H}_2 \right] T_{\text{LO}}^{\mu\nu}$$

- By iterating this process, one arrives at:

$$\langle T_{\mu\nu}(\tau, \eta, \vec{x}_\perp) \rangle_{\text{LLog}} = \int \left[ D\rho_1 \ D\rho_2 \right] W_1[\rho_1] \ W_2[\rho_2] \ \underbrace{T_{\text{LO}}^{\mu\nu}(\tau, \vec{x}_\perp)}_{\text{for fixed } \rho_{1,2}}$$
Why factorization works: causality

The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$
Why factorization works: causality

- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  ▶️ it must happen (long) before the collision
Why factorization works: causality

- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  - it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  - the logarithms are intrinsic properties of the projectiles, independent of the measured observable
Factorization and causality

\[ T_{\mu\nu}^{\text{LO}} = \sum_{\text{trees}} \text{ (all propagators retarded)} \]

\[ \delta T_{\mu\nu}^{\text{NLO}} = \sum_{\text{trees}} \]
Factorization and causality

\[ T_{\text{LO}}^{\mu \nu} = \sum \text{trees} \]

(all propagators retarded)

\[ \delta T_{\text{NLO}}^{\mu \nu} = \sum \text{trees} \]

\[ \ln \left( \frac{\Lambda^0}{\Lambda^1} \right) \mathcal{H}_1 + \ln \left( \frac{\Lambda^0}{\Lambda^1} \right) \mathcal{H}_2 \]

\[ T_{\text{LO}}^{\mu \nu} \]

- Note: this would not work if the graphs were made of Feynman propagators instead of retarded ones
3 Nucleus-Nucleus collisions

Stages of AA collisions
Energy-Momentum tensor
Rapidity correlations
Correlations in $\eta$ and $\vec{x}_\perp$

- The factorization valid for $\langle T^{\mu\nu} \rangle$ can be extended to multi-point correlations:

$$
\langle T^{\mu_1\nu_1}(\tau, \eta_1, \vec{x}_{1\perp}) \cdots T^{\mu_n\nu_n}(\tau, \eta_n, \vec{x}_{n\perp}) \rangle_{\text{LLog}} = \\
= \int \left[ D\rho_1 \ D\rho_2 \right] \ W_1[\rho_1] \ W_2[\rho_2] \\
\times T^{\mu_1\nu_1}_{\text{LO}}(\tau, \vec{x}_{1\perp}) \cdots T^{\mu_n\nu_n}_{\text{LO}}(\tau, \vec{x}_{n\perp})
$$

- Note: at Leading Log accuracy, all the correlations come from the distributions $W[\rho_{1,2}]$
  - they are a property of the pre-collision initial state
Color flux tubes  [Lappi, McLerran (2006)]

- At $\tau = 0^+$, the chromo-$\vec{E}$ and $\vec{B}$ fields form longitudinal “flux tubes” extending between the projectiles:

- Correlation length in the transverse plane: $\Delta r_\perp \sim Q_s^{-1}$

- Correlation length in rapidity: $\infty$ at LO, $\alpha_s^{-1}$ after the leading logs have been resummed
Origin of long range rapidity correlations

Long range rapidity correlations are created early

From causality, the latest time at which a correlation between two particles can be created is:

\[ t_{\text{correlation}} \leq t_{\text{freeze out}} \ e^{-\frac{1}{2}|y_A - y_B|} \]

▷ long range rapidity correlations in the final state probe the rapidity dependence of \( W[\rho_{1,2}] \)
Summary

- Gluon saturation is enhanced in nuclei, and can be reached at higher $x$ (compared to nucleons)

- Saturation plays an important role in the description of the initial stages of nucleus-nucleus collisions

- In the saturated non-linear regime, there exist some universal distributions $W[\rho]$ that describe the dense projectiles both in DIS and AA collisions
  - Resums the logs of $\sqrt{s}$ at leading log accuracy
  - Applies to sufficiently inclusive observables
  - Causality plays an important role in this factorization
  - Ordinary $k_t$-factorization is broken in AA collisions