Twist expansion of the nucleon structure functions

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(JB, K.Golec-Biernat, K.Peters)

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Introduction

Question of interest:
Signals of saturation in deep inelastic $eA$-scattering.

Candidate No 1: nucleon structure functions at low $Q^2$, small $x$.
Believers: geometric scaling, saturation models are successful, constant ratio in diffraction.
Disbelievers: DGLAP (with small-$x$ improvements) works well. Diffraction?

Need to take a closer look at corrections to DGLAP. Example: higher twist.

This talk:
a little bit of theory (higher twist vs.BK-equation),
umerical analysis of HERA data, based upon improved GBW model
(JB, Golec-Biernat, Kowalski)
Theory: higher twist vs. saturation

Leading twist DGLAP: single ladder (single chain) structure, at low $x$ dominance of gluons:

\[ \text{tr} \left( F^{\mu_1 \alpha} D^{\mu_2} \ldots D^{\mu_{n-1}} F^{\mu_n}_\alpha g_{\mu_1 \mu_n} \right) + \text{perm} \]

Corrections: Higher twist vs. fan diagrams (BK-equation)
A) Higher twist: $2n$-gluon operator (Bukhvostov, Frolov, Kuraev, Lipatov)
In the following: discuss 4 gluon operator $\rightarrow$ twist 4

$$tr \left(F^{\alpha \mu_1} ... F^{\mu_i \alpha} ... F^{\mu_j \beta} ... F^{\mu_n \beta}\right), \ tr \left(F^{\alpha \mu_1} ... F^{\mu_i \beta} ... F^{\mu_j \alpha} ... F^{\mu_n \beta}\right), \ tr \left(F^{\alpha \mu_1} ... F^{\mu_i \beta} ... F^{\mu_j \beta} ... F^{\mu_n \alpha}\right)$$

At large $N_c$ evolution simplifies: “double DGLAP”
Coupling to photon ('coefficient function'):
16 diagrams, symmetric in color and momentum, calculated in double-log approximation, project on $1/Q^4$ contribution.

Most remarkable property:

- transverse photon: positive sign, no logarithmic enhancement
- longitudinal photon: negative sign, logarithmic enhancement

Potential cancellation in $F_2 = F_T + F_L$

Remark on higher $n$: eikonalization of ladder exchange $\rightarrow$ saturation models

Numerics see below
B) Summation over $\ln 1/x$ contributions with arbitrary number of $t$-channel gluons.

Target: nucleus or collection of large number of color neutral scattering centers inside the proton:

Diagrams sum up to:

Sum over fan diagrams: BK-equation.

Physical picture: recombination of gluons inside the proton (nucleus): **Saturation picture**
Two comments on this result:

1) only two gluons couple to the quark loop (although graphs with 2,3,4 gluons are included):
reggeization of the gluon + property of quark loop

Leads to the simple form of dipole cross section (need only two-gluon correlator).

No eikonalization.

Note these special features of the quark loop; in general one needs higher correlators:
Drell-Yan or inclusive jet cross section.
2) Twist decomposition of the fan diagrams (BK equation):

there are no twist 4, twist 6,.. terms: 'leading-twist shadowing'.

From the point of view of operator product expansion:
special mixing pattern of quasi-partonic and non-quasipartonic operators:

No transition (in LO) from quasipartonic to non-quasipartonic operator (BFKL).
Zero in the triple Pomeron vertex (JB, Kutak)

BK equation: “wipes out the twist expansion of the eikonal graphs”
Two alternative scenarios for corrections to leading-twist DGLAP:

- **Twist expansion**
  - Eikonalization, saturation models
  - *This talk*

- **Log 1/x summation**
  - BK equation, (saturation models)
For numerics: the BGK\textsuperscript{1} (GBW) model

BGK and GBW models are eikonal models:

\[
\sum (\sigma(x, r) = \sigma_0 \left\{ 1 - \exp \left( -\Omega(x, r^2) \right) \right\})
\]

\[
\Omega(x, r^2) = \frac{\pi^2 r^2 \alpha_s(x^2 \mu^2(r)) g(x, x^2 \mu^2(r))}{3\sigma_0}
\]

Correct structure of higher twist corrections, with evolution in the large-$N_c$ limit.

\textsuperscript{1}JB,Golec-Biernat,Kowalski
Twist expansion: twist-2n in \( n \)-DGLAP exchange.

Remark on technical details: Mellin-transform, singularities in \( s \)-plane:

\( n \)-ladder exchange → branchpoint at \( s = -n \)

\[
\begin{align*}
  f(r^2) &= \int_C \frac{ds}{2\pi i} (r^2)^{-s} \tilde{f}(s) \\
  \sigma_{L,T}^\gamma_p(x, Q^2) &= \int_{C_s} \frac{ds}{2\pi i} \left( \frac{Q_0^2}{Q^2} \right)^{-s} \tilde{\sigma}(x, s) \tilde{H}_{T,L}(-s)
\end{align*}
\]

Encoded in the photon wave function:

negative sign and logarithmic enhancement of twist-4 correction to \( F_L \).
Numerical results

Twist ratios: twist 4/ total at \( x = 4 \cdot 10^{-5} \)

\( F_T \): twist 4 corrections are positive

\( F_L \): twist 4 corrections are negative

In the sum: almost complete cancellation
Consistent with old MRST estimate:

Twist 4 corrections \( \leq 10\% \)
Twist ratios: twist 4 / twist 2

$F_T$: twist 4 corrections are positive

$F_L$: twist 4 corrections are negative

In the sum: almost complete cancellation
How to apply to $eA$ - scattering:

first extract saturation scale. A recent estimate:

$$Q_s^2 = Q_0^2 \left( \frac{x_0}{x} \right)^\lambda$$

with

$$x_0 = 10^{-4}, \quad Q_0^2 = 0.8 \text{ GeV}^2, \quad \lambda = 0.287$$

gives

$$Q_s^2 = 1 \text{ GeV}^2 \ @ x = 4 \cdot 10^{-5}$$
$$Q_s^2 = 3 \text{ GeV}^2 \ @ x = 10^{-6}$$

Then apply the scaling rule

$$Q_s^2 \sim A^{1/3}$$
Application to $F_L$: Higher twist corrections should be larger than in $F_2$. HERA results: errors are large in the region of interest. Comparison (within the BGK model) of twist 2 and all-twist:
Conclusions

There is something special about the “detector“ in DIS (fermion loop):

- higher twist: opposite sign between $\Delta F_L$ and $\Delta F_T$
- small-$x$ summation: fan structure (BK equation)
- does not need to be true in other environments (Drell-Yan, inclusive jets):
  - higher twist (saturation) effects should be larger
- Needed: analogous numerical analysis of BK description for $F_L$ and $F_T$ at low $Q^2$ and small $x$.

Conclusions for $eA$ colliders:

- essential to measure $F_L$
- $\sigma_{tot}(\gamma^* A)$ may not be the best quantity to look for saturation:
  - this workshop