Conductance of fully equilibrated quantum wires

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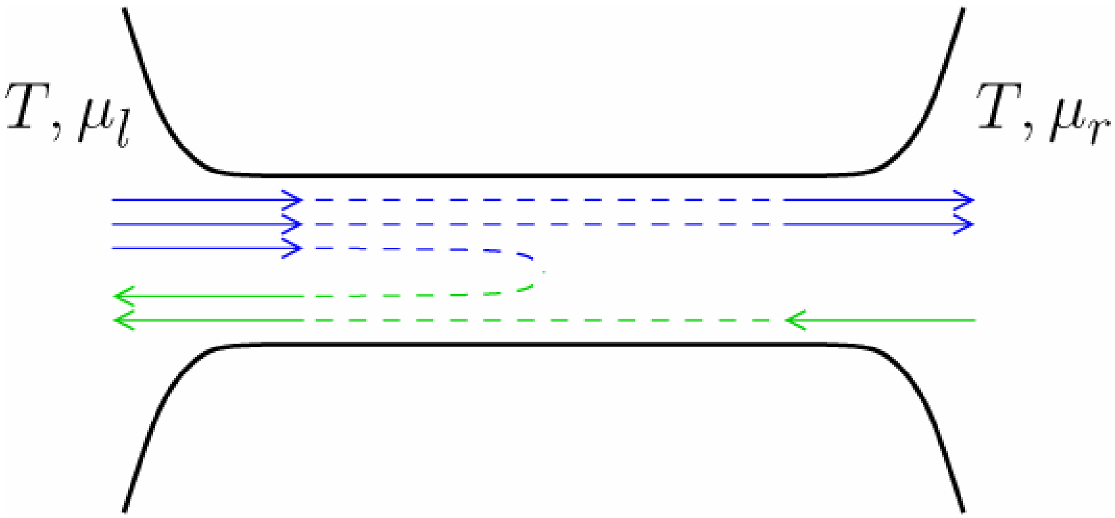
In collaboration with J. Rech and T. Micklitz



$\mu_l \neq \mu_r$

no equilibrium

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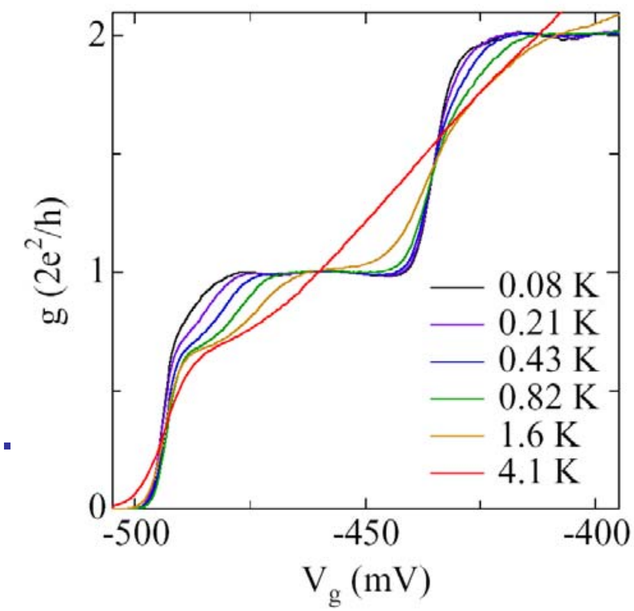
Argonne National Laboratory

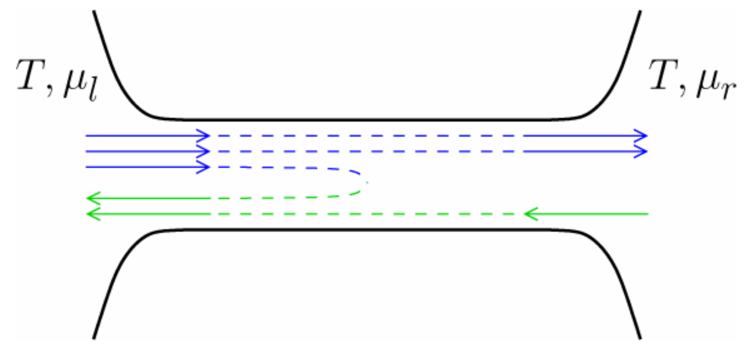
Seattle, 8/07/09

Outline

- Motivation: experiments with quantum wires
  - quantization of conductance,
  - temperature-dependent corrections.

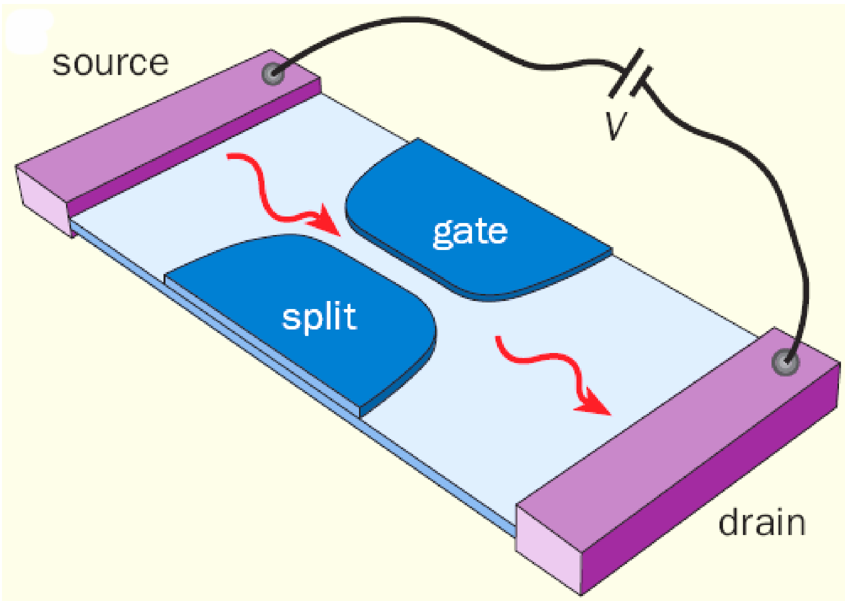


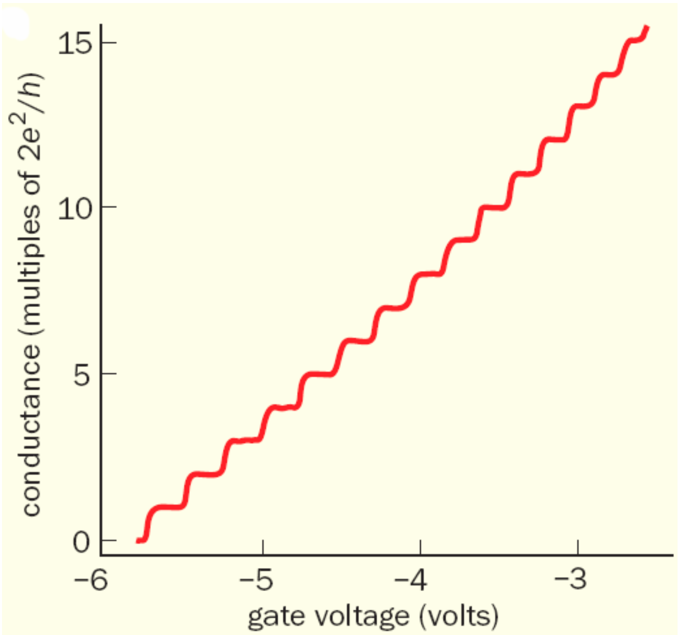
- Theory: Temperature-dependent correction to quantized conductance due to the equilibration processes.



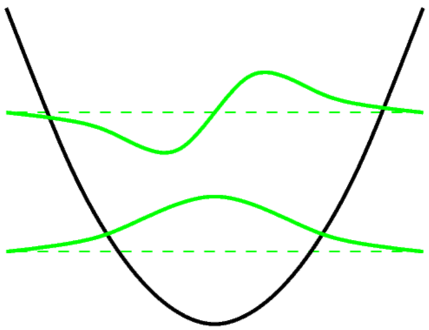
GaAs Quantum wires

From Berggren & Pepper, 2002

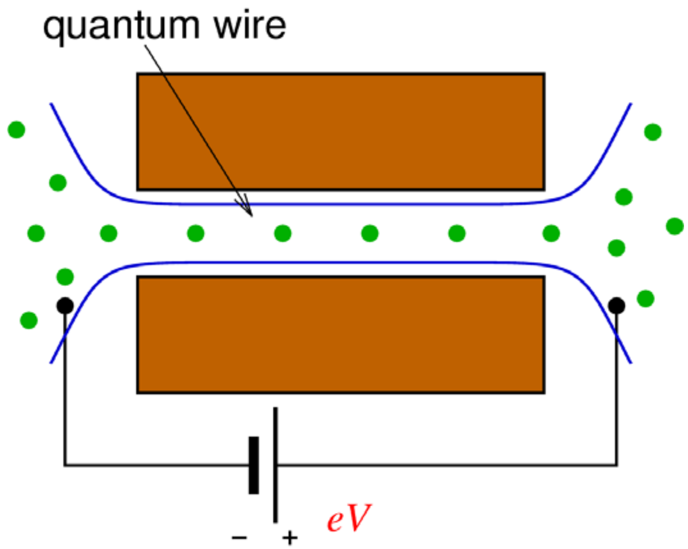


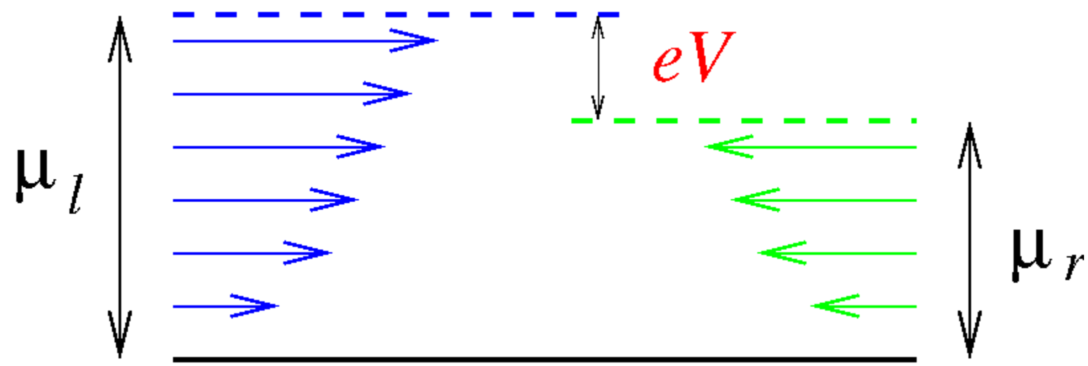


The plateaus of conductance are due to the population of multiple subbands in the wire



Conductance of a single-channel Quantum Wire





Current:

$$
\begin{aligned}
I &= 2e \int_0^\infty \frac{dp}{h} v_p [n_F(\epsilon_p - \mu_l) - n_F(\epsilon_p - \mu_r)] \\
&= \frac{2e}{h}(\mu_l - \mu_r) \int_0^\infty d\epsilon \left(-\frac{\partial n_F}{\partial \epsilon}\right) \\
&= \frac{2e^2}{h} V n_F(0)
\end{aligned}
$$

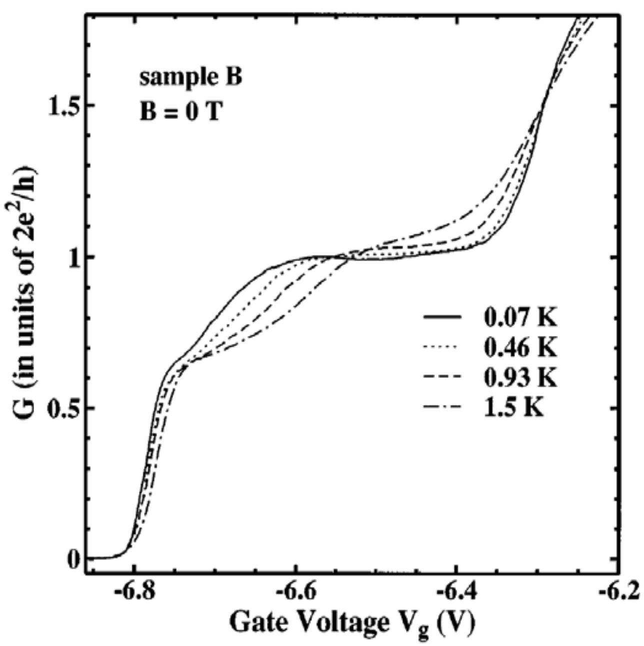
Conductance:

$$G = \frac{2e^2}{h} \frac{1}{1 + e^{-\mu/T}}$$

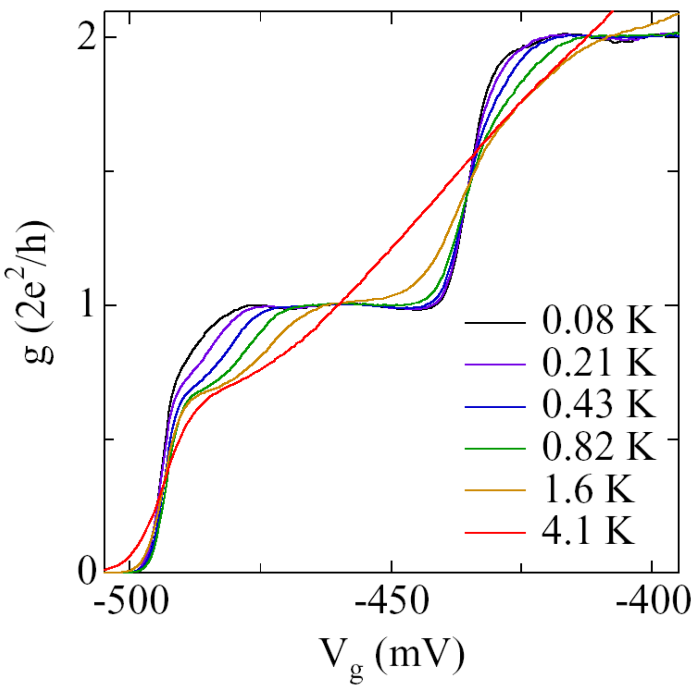
$$G = \frac{2e^2}{h} \text{ at } \mu \gg T$$

0.7 structure

Conductance vs. gate voltage at different temperatures:



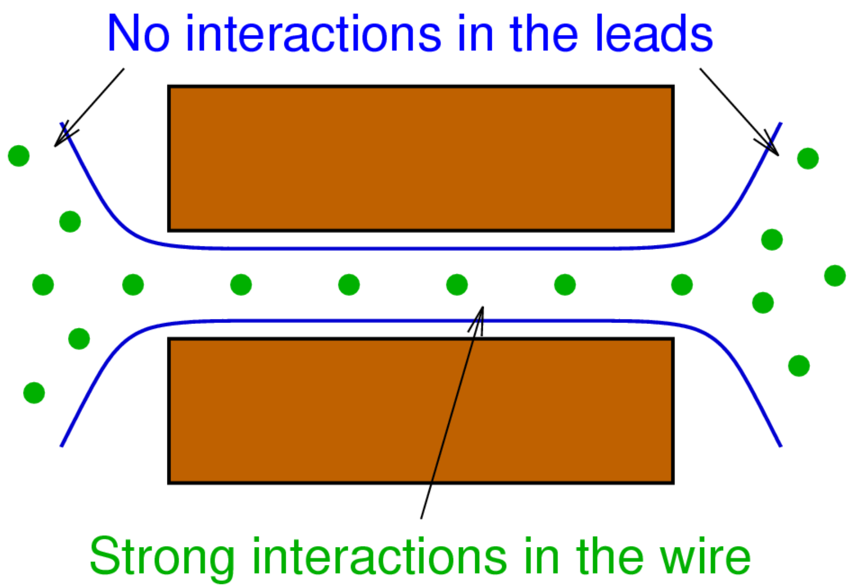
Thomas *et al*., 1996



Cronenwett *et al*., 2001

As the temperature grows, the conductance develops a shoulder near $0.7 \times \frac{2e^2}{h}$

Conductance of a Luttinger liquid



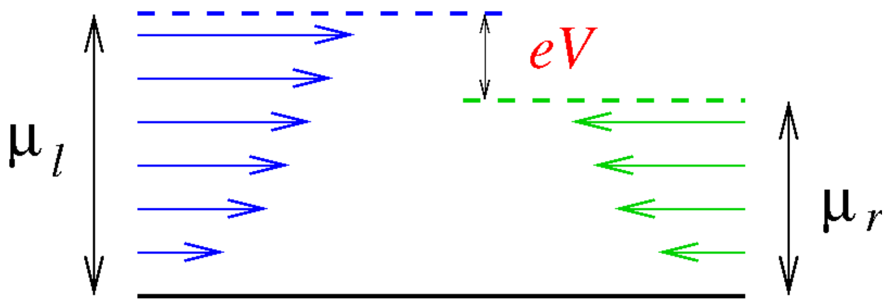
One-dimensional model
with non-interacting leads

[Maslov & Stone;
Ponomarenko;
Safi and Schulz, 1995]

Conductance is controlled by the leads, $G = \frac{2e^2}{h}$

No sign of 0.7 structure!

Almost non-interacting electrons



Distribution function

$$f_p^{(0)} = \frac{\theta(p)}{e^{(\epsilon_p - \mu_l)/T} + 1} + \frac{\theta(-p)}{e^{(\epsilon_p - \mu_r)/T} + 1}$$

Quadratic spectrum $\epsilon_p = \frac{p^2}{2m}$ ⇨ Gallilean invariance

Equilibrium in a frame moving with the drift velocity $v_d = \frac{I}{en}$

$$f_p = \frac{1}{e^{(\epsilon_p - v_d p - \mu)/T} + 1}$$

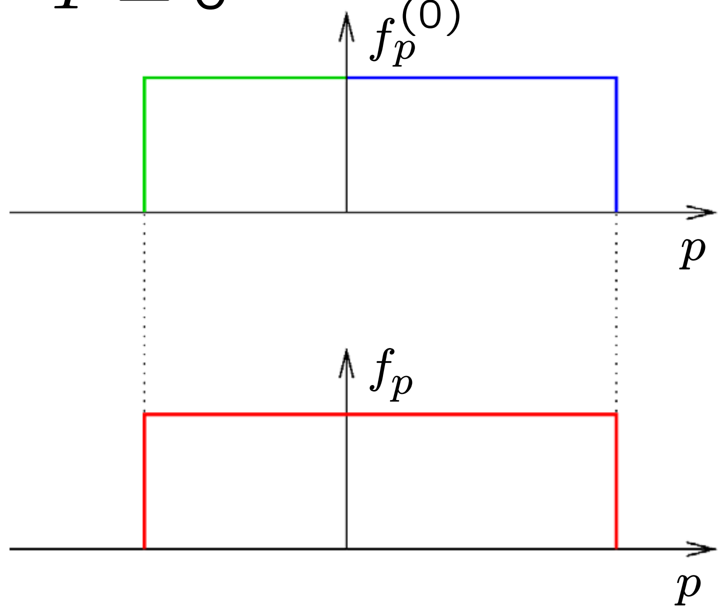
[cf. Pustilnik *et al*., 2003]

In an infinitely long wire we expect $f_p^{(0)} \to f_p$

Compare the distributions

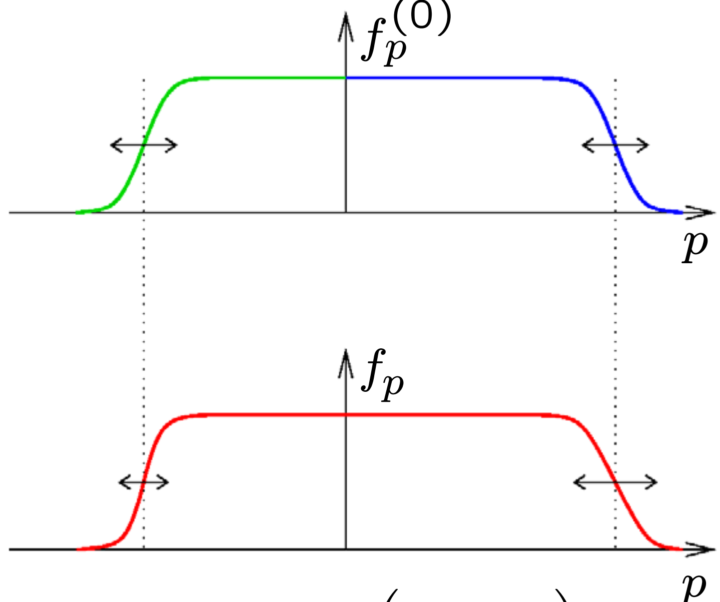
$$f_p^{(0)} = \frac{\theta(p)}{e^{(\epsilon_p - \mu_l)/T} + 1} + \frac{\theta(-p)}{e^{(\epsilon_p - \mu_r)/T} + 1} \quad \text{vs.} \quad f_p = \frac{1}{e^{(\epsilon_p - v_d p - \mu)/T} + 1}$$

$T = 0$



$$\mu_{l,r} = \mu \pm v_d p_F$$

$T > 0$



$$T^{R,L} = T\left(1 \pm \frac{v_d}{v_F}\right)$$

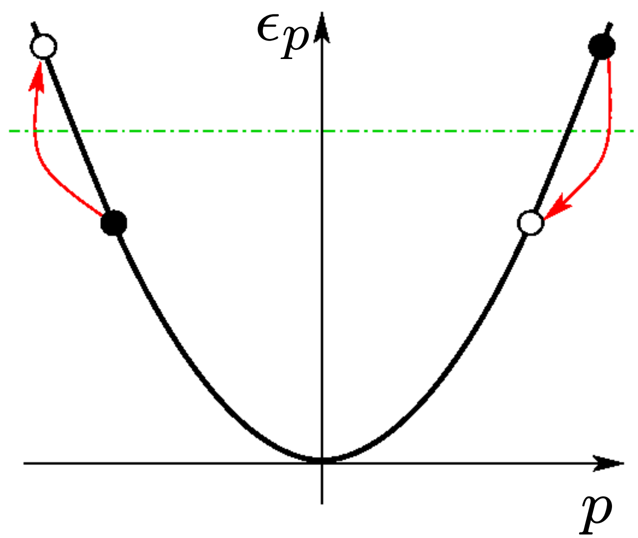
How does electron equilibration affect conductance of the wire?

Relaxation mechanism

Two-particle collisions

Conservation laws:

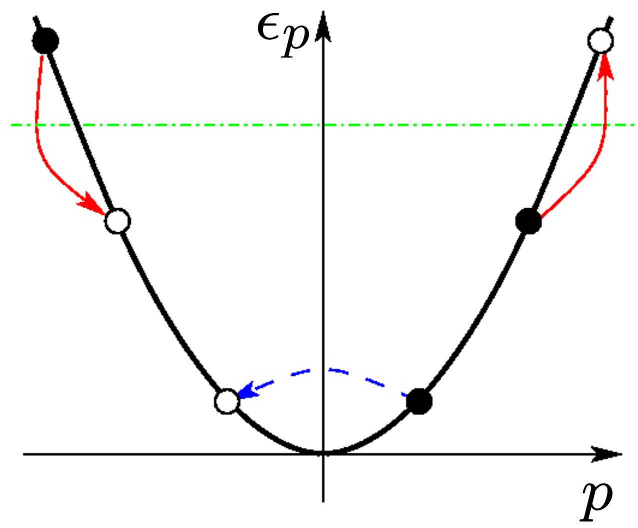
$$p_1 + p_2 = p_1' + p_2'$$
$$\epsilon_{p_1} + \epsilon_{p_2} = \epsilon_{p_1'} + \epsilon_{p_2'}$$



$p_1 = p_1',\ p_2 = p_2'$ or $p_1 = p_2',\ p_2 = p_1'$

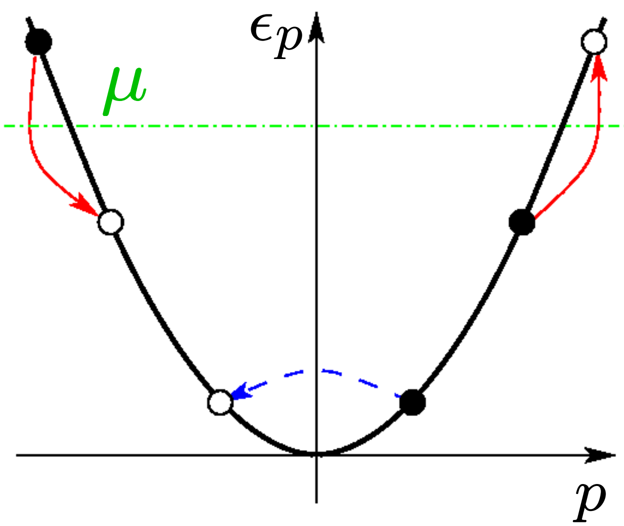
(no change of the distribution function)

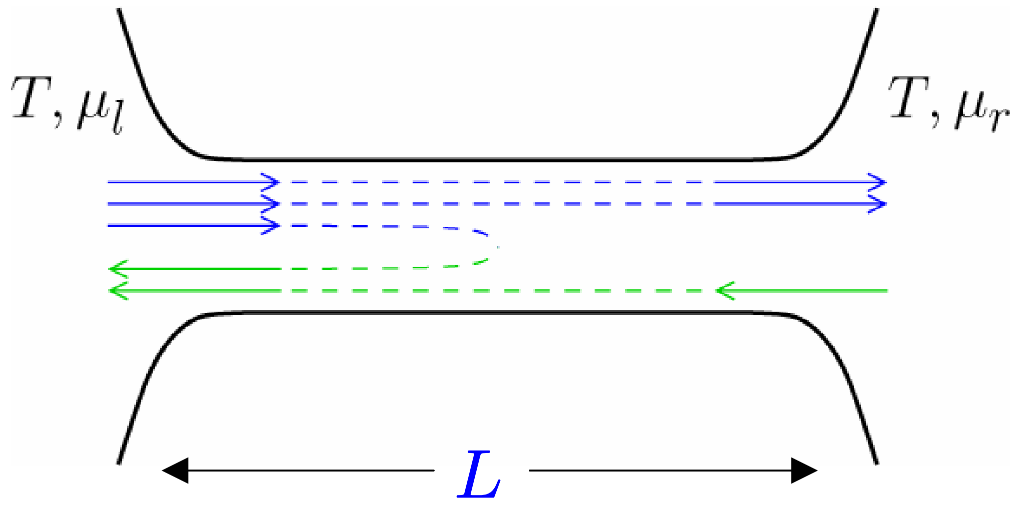
Three-particle collisions



Enough freedom to affect the distribution function

Correction to the conductance of a short wire





Three-particle collisions backscatter some electrons.
In a short wire the correction to the conductance is small:

$$\delta G = -\frac{2e^2}{h}\,\frac{L}{l_{eee}}\,e^{-\mu/T} \qquad \frac{1}{l_{eee}} \propto \left(\frac{T}{\mu}\right)^7$$

[Lunde, Flensberg & Glazman, 2007]

Short vs. long wire

Non-interacting electrons: $$f_p^{(0)} = \frac{\theta(p)}{e^{(\epsilon_p - \mu_l)/T} + 1} + \frac{\theta(-p)}{e^{(\epsilon_p - \mu_r)/T} + 1}$$

Weakly interacting electrons

Short wire: Small correction to the distribution function

$$f_p^{(0)} \to f_p^{(0)} + \delta f_p$$

Long wire: Full equilibration

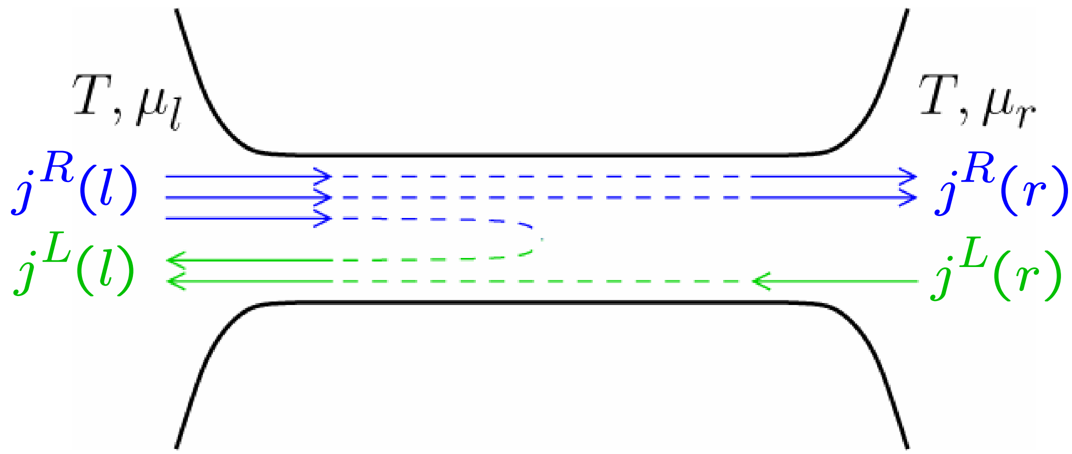
$$f_p^{(0)} \to f_p = \frac{1}{e^{(\epsilon_p - v_d p - \mu)/T} + 1}$$

Conductance of a long wire can be found by analyzing the conservation laws

Conservation of particle number

1. Consider the current of right-moving electrons

$$j^R(l) - j^R(r) = -\dot{N}^R$$



2. The total current is conserved: $j^R(r) + j^L(r) = j$

3. Exclude the unknown current $j^R(r)$

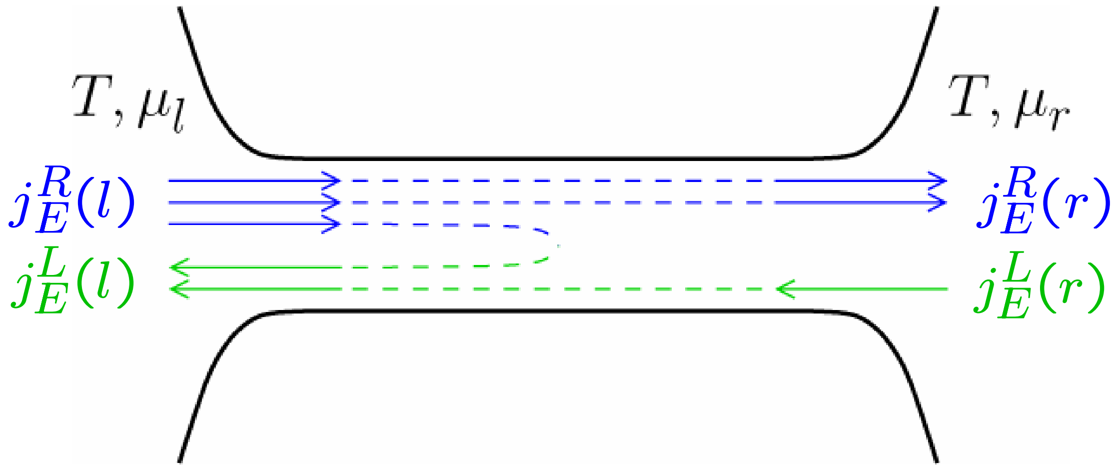
$$j^R(l) + j^L(r) = j - \dot{N}^R$$

4. Apply Landauer formula to the left-hand side

$$\frac{2e^2}{h} V = I - e\dot{N}^R$$

Conservation of energy

Repeat the first three steps for the energy currents



$$j_E^R(l) + j_E^L(r) = j_E - \dot{E}^R$$

The unperturbed current in the left-hand side is known

$$j_E^R(l) + j_E^L(r) = \frac{2e}{h} V \mu \qquad \text{i.e., } j_E^{(0)} = \mu \frac{I}{e}$$

The energy current in the wire is proportional to the electric current

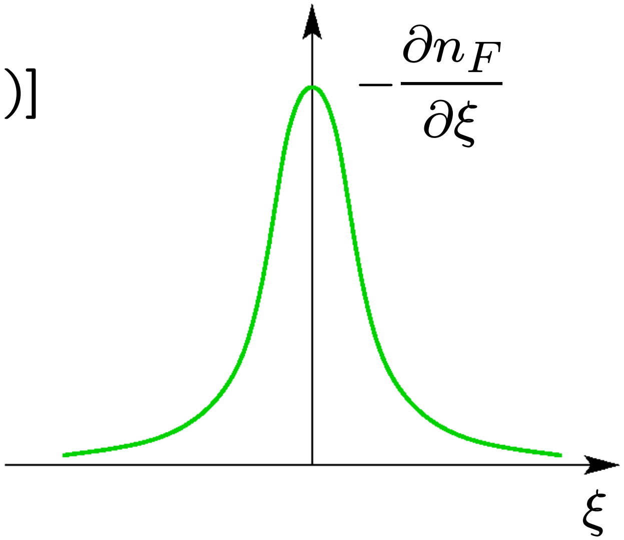
$$j_E = \mu \frac{I}{e} \left[1 + \frac{\pi^2}{6} \left(\frac{T}{\mu} \right)^2 \right]$$

$$\frac{2e^2}{h} V = I \left[1 + \frac{\pi^2}{6} \left(\frac{T}{\mu} \right)^2 \right] - \frac{e}{\mu} \dot{E}^R$$

Evaluation of the energy currents

Energy current supplied by the leads

$$
\begin{aligned}
j_E^{(0)} &= 2\int_0^\infty \frac{dp}{h}\, v_p \epsilon_p [n_F(\epsilon_p-\mu_l) - n_F(\epsilon_p-\mu_r)] \\
&= \frac{2}{h}(\mu_l-\mu_r)\int_0^\infty d\epsilon\,\epsilon \left(-\frac{\partial n_F}{\partial \epsilon}\right) \\
&\simeq \frac{2e}{h}V\int_{-\infty}^{\infty} d\xi\,(\mu+\xi)\left(-\frac{\partial n_F}{\partial \xi}\right) \\
&= \frac{2e}{h}V\mu = \mu\frac{I}{e}
\end{aligned}
$$



Energy current inside the wire

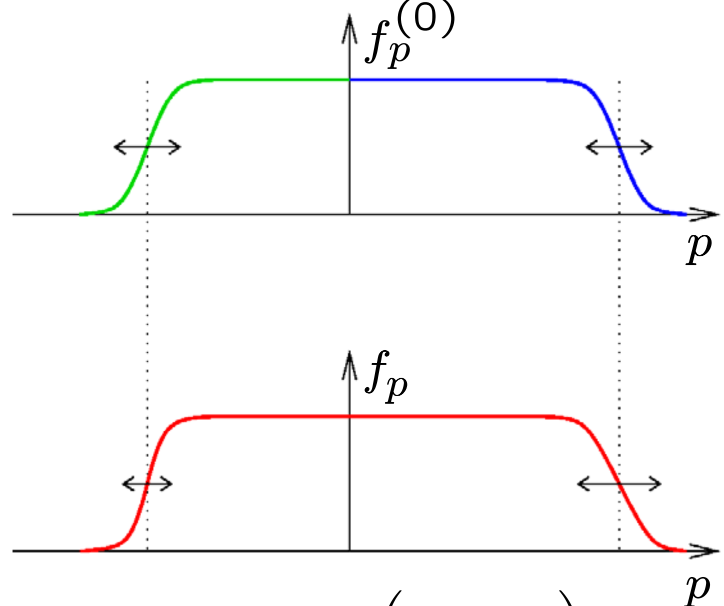
$$
\begin{aligned}
j_E &= 2\int_0^\infty \frac{dp}{h}\, v_p\, \epsilon_p [n_F(\epsilon_p - v_d p - \mu) - n_F(\epsilon_p + v_d p - \mu)] \\
&= \frac{4}{h} v_d \int_0^\infty d\epsilon\,\epsilon\, p(\epsilon) \left(-\frac{\partial n_F}{\partial \epsilon}\right) \simeq \frac{4}{h} v_d \sqrt{2m}\int_{-\infty}^{\infty} d\xi\,(\mu+\xi)^{3/2}\left(-\frac{\partial n_F}{\partial \xi}\right) \\
&= \ldots \simeq \mu\frac{I}{e}\left[1+\frac{\pi^2}{6}\left(\frac{T}{\mu}\right)^2\right]
\end{aligned}
$$

Illustration

$$f_p^{(0)} = \frac{\theta(p)}{e^{(\epsilon_p - \mu_l)/T} + 1} + \frac{\theta(-p)}{e^{(\epsilon_p - \mu_r)/T} + 1}$$

$$f_p = \frac{1}{e^{(\epsilon_p - v_d\, p - \mu)/T} + 1}$$

$T > 0$

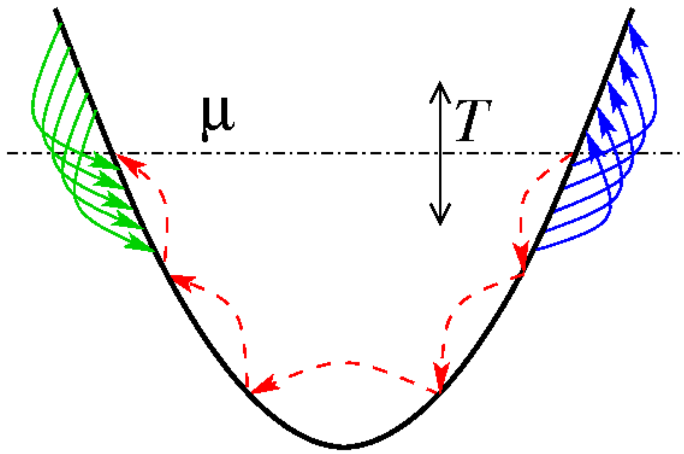


$$T^{R,L} = T\left(1 \pm \frac{v_d}{v_F}\right)$$

Right-movers carry more energy, left-movers carry less energy

$$j_E^{(0)} = \mu \frac{I}{e} \to j_E = \mu \frac{I}{e}\left[1 + \frac{\pi^2}{6}\left(\frac{T}{\mu}\right)^2\right]$$

Conservation of momentum



$$\Delta N^R = -1, \ \Delta E^R = \ ?$$

$$\frac{2e^2}{h}V = I - e\dot{N}^R$$

$$\frac{2e^2}{h}V = I\left[1 + \frac{\pi^2}{6}\left(\frac{T}{\mu}\right)^2\right] - \frac{e}{\mu}\dot{E}^R$$

Momentum change: $\Delta p^L + \Delta p^R - 2p_F = 0$

Energy change: $-v_F \Delta p^L + v_F \Delta p^R = 0$

$$\Delta p^L = \Delta p^R = p_F$$

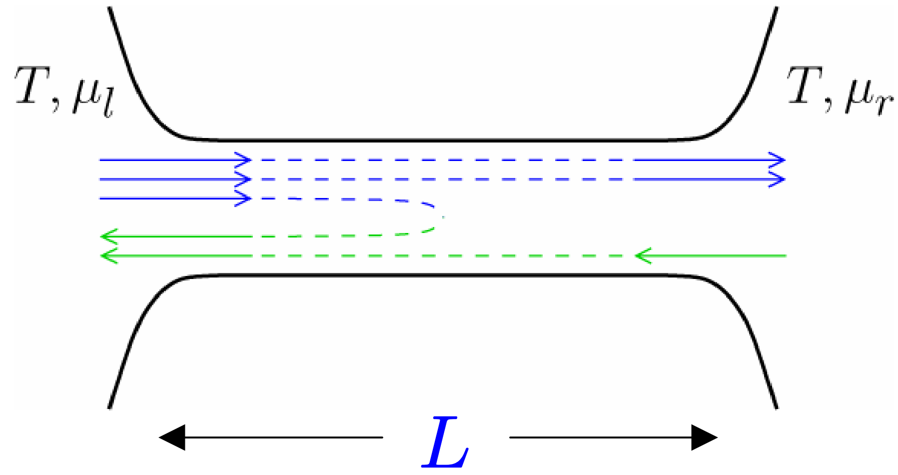
The energy of the right-movers changes by $\Delta E^R = -\mu + v_F \Delta p^R = \mu$

$$\dot{E}^R = -\mu \dot{N}^R$$

Conductance of a long wire

At $L \to \infty$

$$G \simeq \frac{2e^2}{h}\left[1 - \frac{\pi^2}{12}\left(\frac{T}{\mu}\right)^2\right]$$



Compare with

Non-interacting wire: $$G \simeq \frac{2e^2}{h}\left(1 - e^{-\mu/T}\right)$$

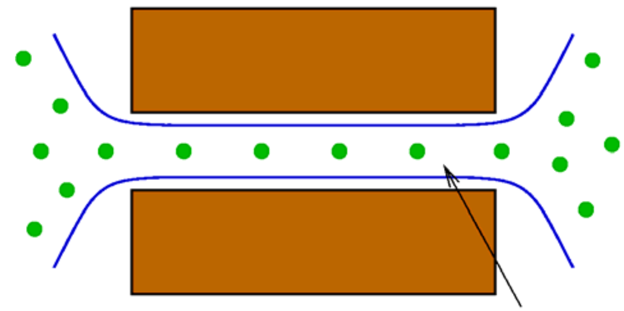
Short wire: $$\delta G \simeq \frac{2e^2}{h}\left(1 - \frac{L}{l_{eee}} e^{-\mu/T}\right)$$

[Lunde, Flensberg & Glazman, 2007]

Thermoelectric effects in a long wire

1. Seebek effect: $V = -S\Delta T\big|_{I=0}$

2. Peltier effect: $\dot{Q} = \Pi I\big|_{\Delta T=0}$



At full equilibration $$j_E = \mu\frac{I}{e}\left[1+\frac{\pi^2}{6}\left(\frac{T}{\mu}\right)^2\right]$$

Heat current $$\dot{Q} = j_E - \mu\frac{I}{e} = \frac{\pi^2T^2}{6e\mu}I \Rightarrow \Pi = \frac{\pi^2T^2}{6e\mu}$$

Onsager relation $$S = \frac{\Pi}{T} \Rightarrow S = \frac{\pi^2 T}{6e\mu}$$

Cf. non-interacting wire: $S = 0$

Thermal conductance of a long wire

Definition $\dot{Q} = K \Delta T \big|_{I=0}$

In the absence of electric current the distribution function

$$f_p = \frac{1}{e^{(\epsilon_p - v_d\, p - \mu)/T} + 1} \qquad v_d = \frac{I}{en}$$

becomes the standard Fermi-Dirac distribution. Thus $\dot{Q} = 0$ and thermal conductance $K = 0$

Figure of merit $Z = \frac{GS^2}{K} \to \infty \quad (Z \propto L)$

Conclusions

1. When electric current flows through a long quantum wire, the electrons come to equilibrium in a moving frame

$$f_p = \frac{1}{e^{(\epsilon_p - v_d p - \mu)/T} + 1} \qquad v_d = \frac{I}{en}$$

2. As a result, the quantized conductance acquires a power-law correction

$$G \simeq \frac{2e^2}{h}\left[1 - \frac{\pi^2}{12}\left(\frac{T}{\mu}\right)^2\right]$$

3. The thermal conductance of the wire vanishes, but the thermopower does not.