INT Program 09-1: "Effective field theories and the many-body problem"

Introduction:
Knowns and Unknowns in Many-body theory

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These slides are intended as an introduction and guide to the educated non-expert (for example, an EFTer)

What are we calling "many-body theories"?

In general, these are theories (or, better, *methods*) that
(a) are *aimed* at systems with $A > 4$
(b) explicitly treat all or some of the many-particle correlations

The main examples we consider include:
* Configuration-interaction (CI) or configuration-mixing shell model and variants (Monte Carlo Shell Model etc)
* Coupled-cluster which is closely related to the CI shell model
* Green's-function Monte Carlo

I will focus on the CI shell model as a test case and discuss briefly the other methods

The CI shell model, part I: How it works:

Given some Hamiltonian...

in coordinate space: $$\hat{H} = \sum_i -\frac{\hbar^2}{2m}\nabla_i^2 + \sum_{i<j} V(r_i, r_j) + \sum_{i<j<k} V(r_i, r_j, r_k) + \ldots$$

in occupation space: $$\hat{H} = \sum_i \varepsilon_i \hat{a}_i^+ \hat{a}_i + \sum_{i<j,k<l} V_{ijkl} \hat{a}_i^+ \hat{a}_j^+ \hat{a}_l \hat{a}_k + \ldots$$

...find (mostly low-lying) eigenstates by diagonalizing in a finite basis

$$|\Psi\rangle = \sum_\alpha c_\alpha |\alpha\rangle \qquad |\alpha\rangle = \prod_i \hat{a}_i^+ |0\rangle \qquad \mathbf{H}_{\alpha\beta} = \langle \alpha | \hat{H} | \beta \rangle$$

The CI shell model, part I: How it works:

The *many-body* basis states are Slater determinants built from orthonormal single-particle states.

This is for convenience. The many-body basis is trivially orthonormal and many-body matrix elements are "easy" to calculate.

Any single-particle basis can be used. (Typical are h.o..) The choice affects convergence, spurious states, etc., but not the basic algorithms.

The input two-body matrix elements are integrals that are *computed externally and read in through a file.* Thus there is ***no*** limitation on the kind of two-body interaction used. (The choice of single-particle wfns will affect the *values* of the matrix elements.)

The CI shell model, part I: How it works:

3-body forces (and higher) are computationally *much* more intensive, requiring an order of magnitude more memory, CPU time, etc.

Short-range correlations cause difficulties. We often renormalize interactions with strong repulsive core via Lee-Suzuki or other.

CI shell model is best for detailed, microscopic spectroscopy – excited states. Can also be used for detailed response functions.

Depending on truncations, one can do
-- *ab initio,* including 3-body forces, up through *A* = 16
-- semi-phenomenological up into the *pf*-shell + selected beyond

The CI shell model, part II: The Central Mystery

The configuration-interaction shell model is *both*
-- very complicated! and yet...
-- very simple!

We have seen much success in *ab initio* calculations.
Successes = binding energies, spectra in light nuclei ($A < 12$), spin-orbit splitting.

But such calculations require:

- many configurations to converge
- strong renormalization of the interaction
- 3-body forces

and some things fail like B(E2s), 4p-4h states in upper *p*-shell.

→ very complicated!

The CI shell model, part II: The Central Mystery

The configuration-interaction shell model is *both*
-- very complicated! and yet...
-- very simple!

On the other hand...

Semi-phenomenological shell-model calculations work extremely well:

One can start with a "realistic" 2-body *only* interaction
and tweak just a few matrix elements
(mostly "monopole" parts related to the mean-field);
furthermore for operators often need simple effective charges

and get very good agreement with data over a major shell

→ very simple!

Can we understand how to get
from
the “complicated” *ab initio* shell model
to
the “simple” semi-phenomenological shell model?

Can theory (EFT or other)

- Make the connection more rigorous?
- If not eliminate then at least better guide the fitting?
- Help us understand and control effective charges?
- Allow us to construct effective operators for less accessible systems (e.g. $0\nu\beta\beta$-decay)?

Can we understand how to get
from
the “complicated” *ab initio* shell model
to
the “simple” semi-phenomenological shell model?

This workshop will shape these concerns into “more useful” questions.

Other methods

A number of methods are related:
-- "Shell-model Monte Carlo" (auxiliary-field path integral)
-- Coupled clusters

These use *exactly* the same input as CI shell-model.
The method of solution is different but can be compared directlyto
CI shell model.
Can tackle much large spaces; trade-off is, excited states more difficult.

Other methods

Green's function Monte Carlo:

-- Starts with variational wavefunction: Slater determinant + correlation functions on top (e.g. "Jastrow functions")

This makes orthonormality less simple; integrals become highly complex.

Can handle short-range correlations well– use "bare" interaction with strong repulsive core.

Works in coordinate space; most at ease with local interactions.

Excited states can be difficult.

Scattering

EFTs often constrained by scattering data (I think)

Scattering is difficult for CI-shell model (especially when one uses renormalized interactions); one approach is RGM – see S. Quaglioni's talk tomorrow – with impressive results.

Stetcu and van Kolck have also tackled scattering in shell-model framework.

Other shell-model approaches include continuum shell-model and Gamow shell-model.

Scattering is "easier" for GFMC, in part because of using bare interaction

Summary

EFTs tell us how to rigorously do physics with a certain cut-off.

MBT phenomenology demonstrates we *can* do many-body calculations with "cut-offs" – but mostly done by trial and error.

Can we learn from the former and make the latter more rigorous?

Issues:
- Scattering is not the most "natural" constraint; spectroscopy is
- Non-local forces OK for CI shell model, CC; hard for GFMC
- 3-body/density-dependent forces are difficult
- Phenomenology suggests 3-body forces imbedded in effective 2-body
- Need to construct effective operators alongside interactions; again, phenomenology suggests this is (mostly) simple.