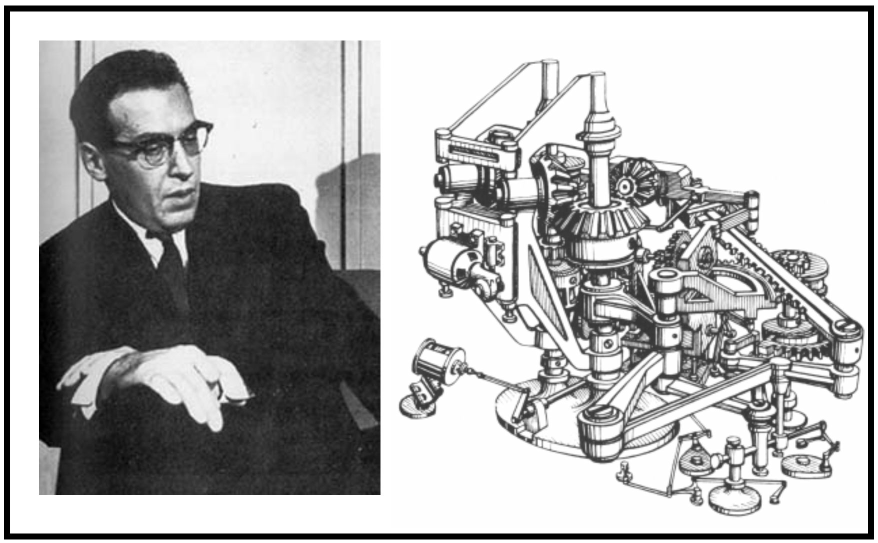
The Schwinger Mechanism and Graphene



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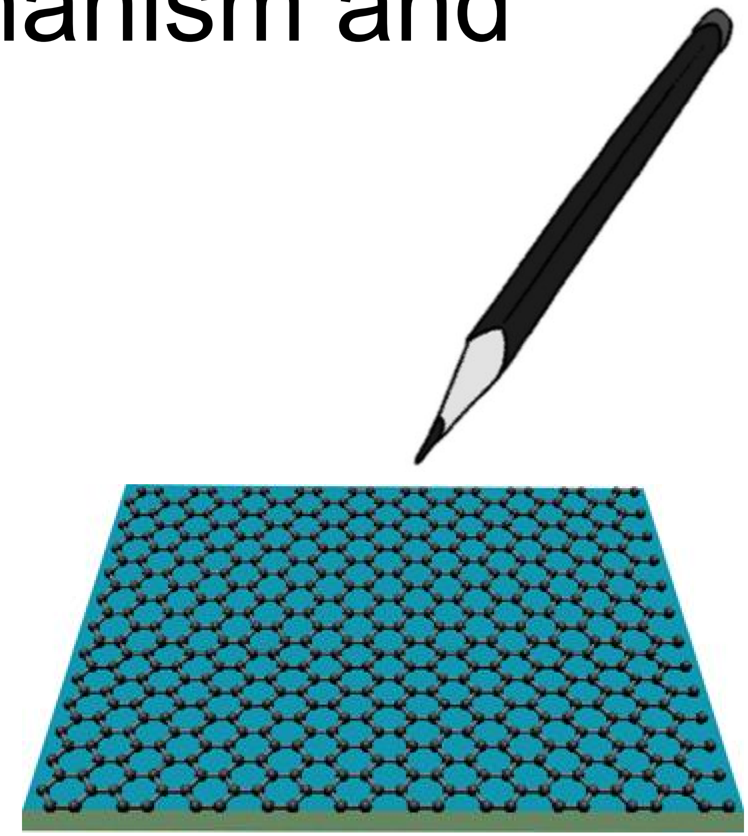

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Outline

- What is the Schwinger Mechanism ?
  - Why is it worth worrying about?
  - How is it derived?
  - Are there subtle issues for which an independent test is needed?
- Why Graphene?
- An idealized experiment
- Complications
- Can the Schwinger Mechanism be tested in Graphene?

What is the Schwinger Mechanism?

- The Schwinger mechanism refers to the production of charged fermion---anti-fermion pairs out of the vacuum by a static external electric field.
- This is essentially the dielectric breakdown of the vacuum
  - Pairs are produced at a **fixed rate per unit volume per unit time** depending **only** on the **field strength**.

$$\Gamma_{Schwinger} = \frac{(eE)^2}{4\pi^3} \sum_{n=1}^{\infty} n^{-2} \exp\left(\frac{-n\pi m^2}{eE}\right)$$

  - Formula holds provided the variation of E over both space and time is slow compared to all relevant scales.

Why should you care?

- Schwinger's 1951 paper introducing it is one of the most influential physics papers in history: there are **2441 citations** in SPIRES (which only goes back to 1975---two and ½ decades after the paper was written), including 95 in 2007.
- It has motivated analysis in many parts of particle physics and quantum field theory:
  - Models of string breaking in QCD
  - Insights into Hawking radiation near black holes
- Despite its importance it has never been tested experimentally; graphene presents an opportunity to do this for an analogous system.

Related to the $Z\alpha>1$ problem---“sparking the vacuum”

- Recent interest in newly created negative energy states for $Z\alpha>1$ in graphene.
- Note as in nuclear case similar effect for extended charges (~*R*).
- For case of large *Z* and fixed *R* (with many bound states) Schwinger formula used as a LDA should describe a critical part of the **dynamics** after the charge is put into place, i.e. the rate for particle emission

Derivation of Schwinger Formula

Essential for understanding what goes thru in graphene

- Key assumptions:
  - Fermions coupled to external field.
  - Quantum effects associated with exchange of virtual photons between fermions or emission of real photons are neglected. (These are order α)
    - Corresponds to the formal limit $e \to 0$, $E \to \infty$ with eE fixed
  - Field remains constant in space & time for relevant scales.
    - Back reaction can be neglected
- In one sense this is more like relativistic quantum mechanics then field theory--- it doesn't involve interacting fields. However key field theory issue of particle creation is central.
  - Analogous to Hawking radiation in this sense.

Schwinger's 1951 Derivation

Based on calculation of effective potential due to external fields---effective potential rather effective action reflects constant fields:

- Elegant "proper time formalism" introduced which in general applicable to full effective action.
- Application to constant electromagnetic field relatively straightforward.

In units with $\hbar = 1,\ c = 1$

Proper time integration

$$\Delta\mathcal{L} = -\frac{1}{8\pi^2}\int_0^\infty ds\, s^{-3} e^{-m^2 s}\left(\frac{(es)^2 \mathcal{G}\,\mathrm{Re}(\cosh(\mathrm{esX}))}{\mathrm{Im}(\cosh(\mathrm{esX}))} - 1 - \tfrac{2}{3}(es)^2 \mathcal{F}\right)$$

$$\mathcal{F} \equiv \tfrac{1}{2}\left(B^2 - E^2\right) \qquad \mathcal{G} \equiv \vec{E}\cdot\vec{B} \qquad X^2 \equiv 2\mathcal{F} + i\mathcal{G}$$

From renormalization

For pure magnetic field take limit as E goes to zero

$$\Delta\mathcal{L} = -\frac{1}{8\pi^2}\int_0^\infty ds\, s^{-3} e^{-m^2 s}\left(eBs\coth(eBs) - 1 - \tfrac{1}{3}(B^2)\right) = \frac{2\alpha^2 B^4}{45 m^4} + \ldots$$

Physically corresponds to the difference in energy of filled relativistic Landau levels as compared to freely propagating electrons.

Pure electric field case: naively one can just exploit covariance. The effective Lagrangian is Lorentz invariant and can only depend on $\mathcal{F} = (B^2 - E^2)/2$ and $\mathcal{G} = \mathbf{E}\cdot\mathbf{B}$. $\mathcal{G}=0$ for pure electric or magnetic field. $\mathcal{F}$ Is unchanged if $B \rightarrow iE$. If one takes the final result and makes substitution:

$$\Delta\mathcal{L} = \frac{2\alpha^2 E^4}{45 m^4} + \ldots$$

However look at integral with $B \to iE$

$$\Delta \mathcal{L} = -\frac{1}{8\pi^2}\int_0^\infty ds\, s^{-3} e^{-m^2 s}\left(eEs\cot(eEs) - 1 - \tfrac{1}{3}(E^2)\right)$$

Note this integral is not well defined: there are poles at

$$s = s_n = \frac{n\pi}{eE} \qquad n = 1, 2, 3, \ldots$$

Schwinger's *ad hoc* proposal to deal with this: shift the poles by $i\varepsilon$ (*i.e.* the replacement $(s-s_n)^{-1} \to (s-s_n-i\varepsilon)^{-1} = P[(s-s_n)^{-1}] + i\pi\,\delta(s-s_n)$. This renders the integral finite and well-defined. However, it leads to an imaginary contribution to the Lagrangian from the residues at the poles.

$$2\,\mathrm{Im}(\mathcal{L}) = \frac{1}{4\pi}\sum_{n=1}^{\infty} s_n^{-2} e^{-m^2 s_n} = \frac{(eE)^2}{4\pi^3}\sum_{n=1}^{\infty} n^{-2}\exp\left(\frac{-n\pi m^2}{eE}\right)$$

- Schwinger interpreted the imaginary part of the Lagrangian density as an indication of the breakdown of the vacuum.
  - Derivation of effective potential assumed no physical particles created.
  - By analogy to standard treatment of unstable systems probability system remains with no pairs produced at time t is

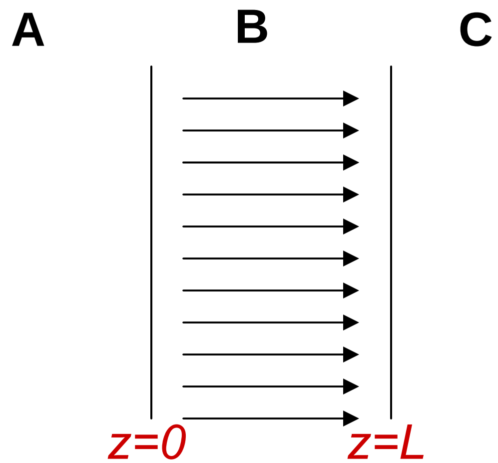
$$P(t) = \left| \exp\left(\int d^3x' \int_0^t dt'\, \mathcal{L}(\vec{x}', t') \right) \right|^2 = \exp\left(-\int d^3x' \int_0^t dt'\, 2\,\mathrm{Im}\left[\mathcal{L}(\vec{x}', t')\right] \right)$$

Thus, the rate of pair production per unit time per unit volume is

$$\Gamma_{Schwinger} = 2\,\mathrm{Im}(\mathcal{L}) = \frac{1}{4\pi} \sum_{n=1}^{\infty} s_n^{-2} e^{-m^2 s_n} = \frac{(eE)^2}{4\pi^3} \sum_{n=1}^{\infty} n^{-2} \exp\left(\frac{-n\pi m^2}{eE} \right)$$

- Some general comments
  - There is an *ad hoc* quality to derivation. It depends on system infinite in both space and time but is incompatible with it. Clearly the phenomenon is transient in some sense. A derivation taking into account boundary conditions is clearly useful in clarifying physics.
    - A derivation including boundary conditions using proper time formalism is quite cumbersome.
  - The exponential suppression factors are suggestive of quantum tunneling.
    - A formalism based on tunneling can give real insight
    - The result is clearly non-linear in the tunneling amplitude---it's the sum of exponential factors.

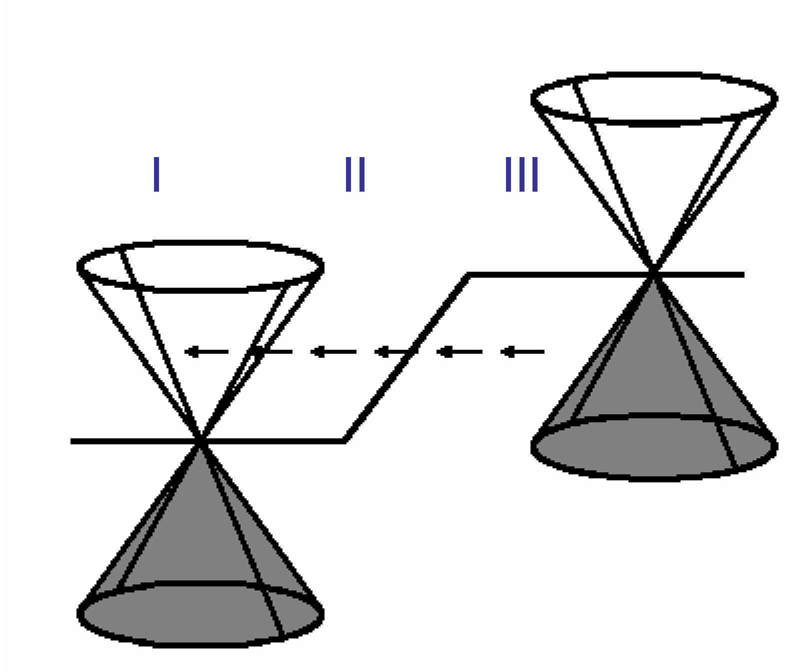
- Formalism of Casher, Neuberger & Nussinov (1979).
  - Variant based on Wang & Wong (1988)
  - Consider an infinite parallel plate capacitor



Use Dirac equation with gauge chosen as a time-independent scalar potential

$$\left[\gamma^{\mu}\left(-i\partial_{\mu}-A_{\mu}\right)+m\right]\psi=0 \qquad A_{\mu}=\delta_{\mu 0}\,E\,z\,\theta(z)\,\theta(L-z)$$

In regions A & C the system satisfies the ordinary free particle Dirac Equation; in region III the energy is simply shifted up by ***V=LE***



In region B the system satisfies the ordinary Dirac Equation with a potential. In general one can solve this equation and numerically match boundary conditions.

The key issue is tunneling from occupied levels to unoccupied ones as these tunnel events act as pair creation. Provided L is much larger than all other length scales in the problem one can accurately compute tunneling using WKB. Condition for WKB: $EL >> m \qquad EL^2 >> 1$

In solving the Dirac equation and implementing WKB, it is useful to note that the (admittedly unrealistic) problem is translation invariant in x & y and hence the transverse momentum p_T is a good quantum number. The WKB tunneling probability for a particle from the right or a hole from the left, when allowed by the Pauli principle is

$$P_{WKB} = \exp\left(-2 \int_{z_1}^{z_2} dz \sqrt{(\varepsilon - e E z)^2 - p_T^2 - m^2} \right)$$

Turning points given by

$$(\varepsilon - e E z_1)^2 - p_T^2 - m^2 = 0 \qquad (\varepsilon - e E z_2)^2 - p_T^2 - m^2 = 0$$

Integral dominated by max of exponential and insensitive to turning point (except for rare cases where turning points are very close to each other) so that integral can be taken to infinity and explicitly evaluated.

$$P_{WKB} = \exp\left(\frac{-2\pi\left(m^2 + p_T^2\right)}{2eE} \right)$$

Same result obtained by exactly solving Dirac equation, matching boundaries of regions and taking L→∞.

Key physical idea (which is often taken implicitly) is that the potential was turned on sometime in the past which then affects the levels as indicated previously. After some transient behavior the system is effectively in an "in-state vacuum" in which right moving levels on the left and left moving levels on the right have not been affected by the potential as they come from infinity and know nothing about the potential. The outgoing levels *are* affected and can tunnel if allowed by Pauli principle.

Consider the *"vacuum persistence probability"*, the probability that system remains in the vacuum---i.e. no tunneling has happened. Now to formulate this first treat space and time as discrete and then take continuum limit.

Pauli principle implies at most one tunnel event per mode

$$P_{vac} = \prod_{spin} \prod_{z} \prod_{t} \prod_{\vec{p}_T} \left(1 - P_{WKB}(p_T)\right)$$

$$= \exp\left(\sum_{spin} \sum_{z} \sum_{t} \sum_{\vec{p}_T} \log\left(1 - P_{WKB}(p_T)\right) \right)$$

Cell sizes for various quantities:

$$\Delta p_x = \frac{2\pi}{L_x} \qquad \Delta p_y = \frac{2\pi}{L_y} \qquad \Delta t = \frac{2\pi}{\omega} = \frac{2\pi}{(2E_T)}$$

Macroscopic Transverse size

Frequency of tunneling "trials"; it is given by the energy of the produced pair.

Converting sums to integrals then yields

$$P_{vac}(T) = \exp(-\underbrace{L_x L_y L}_{\text{volume}}\ T\,\Gamma)$$

Pair production rate per unit volume

$$\Gamma = -\frac{eE}{4\pi^2}\int d(p_T^2)\log\left(1-\exp\left[\frac{\pi\left(p_T^2+m^2\right)}{eE}\right]\right)$$

Evaluating Integral for Γ yields:

$$\Gamma = -\frac{eE}{4\pi^2}\int d(p_T^2)\log\left(1-\exp\left[\frac{\pi\left(p_T^2+m^2\right)}{eE}\right]\right)$$

$$= \frac{(eE)^2}{4\pi^3}\sum_{n=1}^{\infty} n^{-2}\exp\left(\frac{-n\pi m^2}{eE}\right)$$

Same result as obtained by Schwinger

This derivation---

•Shows the relevant “vacuum” state is the in-state vacuum.

• Makes manifest the nonlinear dependence on tunneling probability

•Allows the computation of the distribution of transverse momenta of the created pairs.

•Provides a natural formalism with which to include the effects of regions finite in space or time.

- As with Schwinger's original derivation there is an ad hoc quality about this WKB based one.
  - Based on infinite transverse size
  - Based on assumption system is in the in-state vacuum which is clearly unstable and cannot be maintained indefinitely without dynamical description of how it is formed.
- Given this situation it would be very useful to explicitly verify the validity of the Schwinger formula for pair production in experiment and to establish experimentally the circumstances where it is valid.
- **However the Schwinger formula has never been tested experimentally**.

Nature of the problem:

$$\Gamma = \frac{(eE)^2}{4\pi^3} \sum_{n=1}^{\infty} n^{-2} \exp\left(\frac{-n\pi m^2}{eE} \right)$$

Exponentially suppressed unless $eE \sim \pi m^2$ or bigger--- this corresponds to the potential difference over a Compton wavelength being of order m.

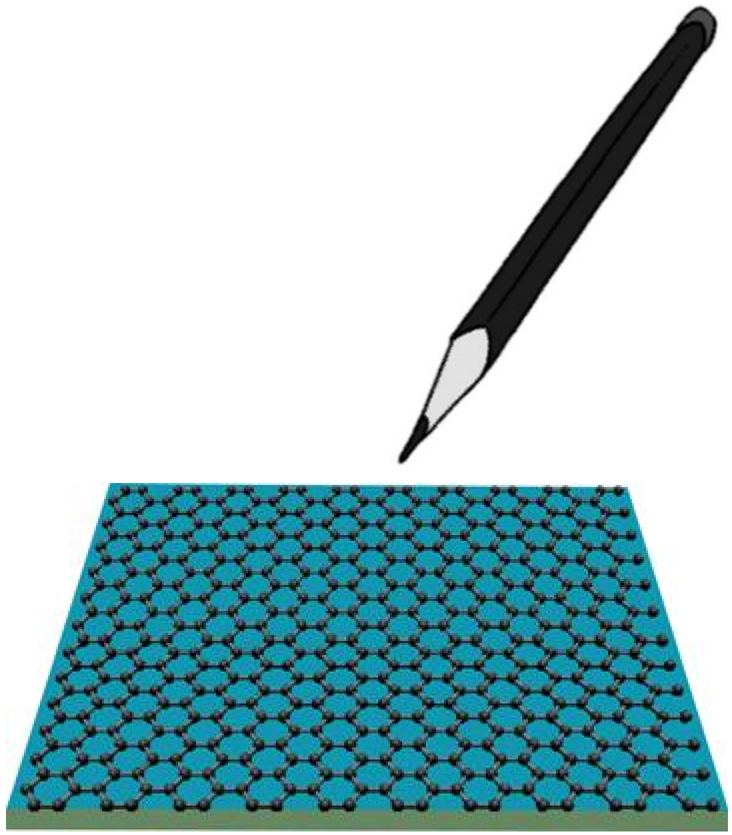
Inserting Planck's constant and speed of light yields

$$eE \sim \frac{m^2 c^3}{\hbar}$$

This corresponds to an electric field of approximately 10^{16} V/cm.

$$\text{for } E \sim 10^6 \text{ V/m}, \quad \exp\left(-\frac{\pi m^2 c^3}{e E \hbar} \right) \sim \exp(-4 \times 10^{10})$$

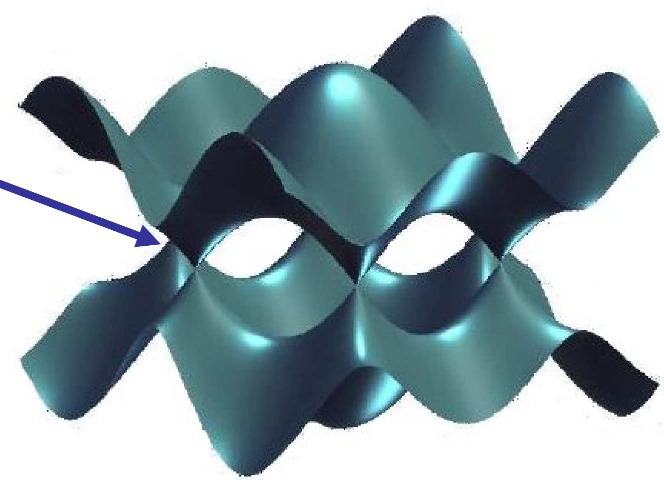
Why Graphene?



- All practical static laboratory E-fields are too small for the Schwinger mechanism to produce pairs at a measurable rate.
  - There has been significant attention as whether the E-field in intense laser pulses but observables depend on complicated dynamics.
- The key problem is the fact in nature the electron is the lower bound for the ratio m^2c^3/q.
  - If there existed massless fermions the exponential suppression would be absent and the Schwinger mechanism simple to observe.
  - Given the absence of massless (or very light) particles in nature, one might hope to find condensed matter systems whose spectrum *acts* like relativistic massless or light charged fermions.
  - In essence system would act as an analog computer to test underlying field theory.

- In this context, graphene represents an ideal way to probe the dynamics of the Schwinger mechanism in an experimentally realizable context.
  - It has long been known for more that the single particle spectrum for fermions in two spatial dimensions moving in a potential with hexagonal symmetry will have two points in the momentum space with “Dirac-cones”

$$\varepsilon^2 = \tilde{c}^2\left(\Delta p_x{}^2 + \Delta p_y{}^2\right) + \ldots$$



College Park Metro Station, College Park, Maryland

Station is tiled in hexagons----quasi-particles associated with motion along station floor have a Dirac cone in the band-structure.

– Graphene also has hexagonal symmetry. Thus to the extent one has a flawless infinite sheet of graphene and
  - A single particle description is valid for the dynamics
  - The ground state has single particle levels filled to the Dirac points
  - All particle & hole excitations of dynamical relevance are sufficiently low as to be in the Dirac cone region

  the system "looks like" the vacuum of charged massless fermions in a world with two spatial dimensions and a speed of light given by $\tilde{c}$.

– As it happens undoped graphene has the Fermi energy exactly at the Dirac point and thus the system should act analogously to the QED for vacuum structure of electrons..

Graphene represent the possibility to study *"QED in a pencil trace"*

- This slogan is a bit of a misnomer
  - the "electrons" do act correctly as 2+1 dimensions massless charged fermions.
  - the photons however live in 3+1 dimensions.
  - The theory corresponds to QED in neither 2+1 or 3+1 dimensions.
- Fortunately from the perspective of the Schwinger mechanism it is close enough for government work:
  - Schwinger mechanism depends on interaction with an external potential which acts the same in 2+1 and 3+1 dimensions

The Schwinger Mechanism in 2+1 dimensions

- As a first step we need the Schwinger formula for 2+1 dimensions.
  - One can repeat either of the two preceding derivations in 2+1 dimensions.
  - The only substantive differences are the lack of a spin degree of freedom in 2+1 dimensions, the different structure of the γ matrices and the fact there is only one quadradic Lorentz form (E^2-B^2) .
- The result is:

$$\Gamma_{2+1} = f\,\frac{(eE)^{3/2}}{4\pi^3}\sum_{n=1}^{\infty}\frac{\exp\left(\frac{-n\pi m^2}{eE}\right)}{n^{3/2}}$$

$$= f\,\frac{(eE)^{3/2}}{4\pi^3}\,\zeta\left(\tfrac{3}{2}\right)$$

# of independent "flavors" of fermions

Riemann Zeta function $\zeta(3/2) \approx 2.612$

For $m=0$

Inserting factors of Planck's constant and "speed of light"

$$\Gamma_{2+1}^{massless} = f \frac{(eE)^{3/2}}{4\pi^3 \hbar^{3/2} \tilde{c}^{1/2}} \zeta\left(\tfrac{3}{2}\right)$$

For graphene $f=4$; 2 for each spin and 2 for each Dirac point

One chief characteristic is that the rate scales as $(e\,E)^{3/2}$. Note that this can be obtained entirely by dimensional anlysis---in units where Planck's constant and the speed of light are taken to be unity the *only* scale in the problem is *E*.

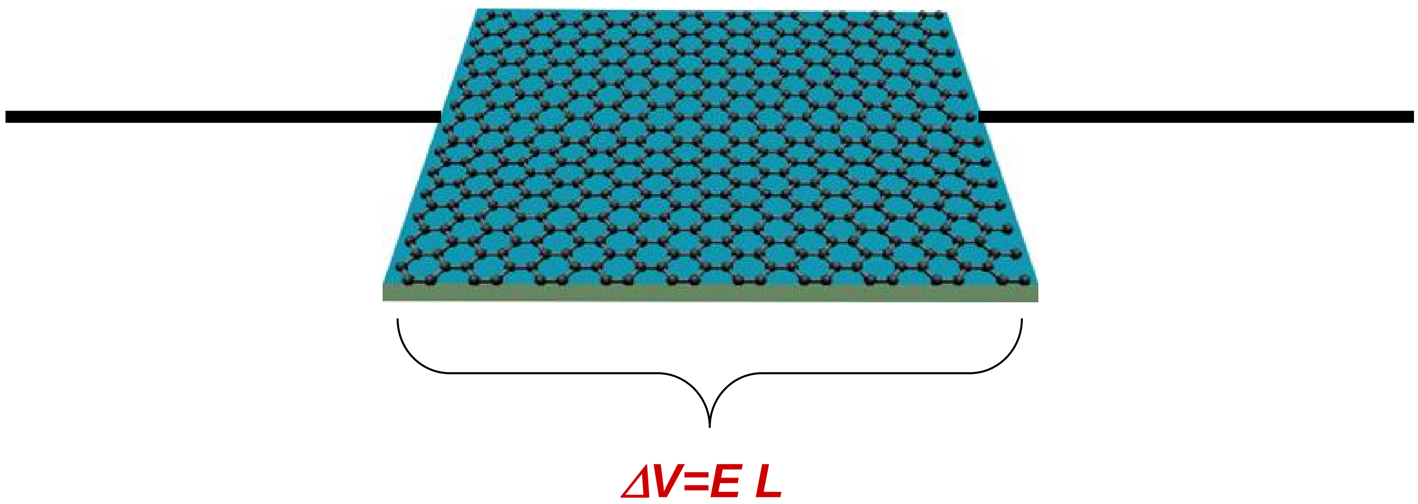
Thus a real test of the Schwinger mechanism requires a checking the prefactor. Recall that the ζ(3/2) follows from the Fermi-Dirac statistics and multiple scattering attempts.

An idealized Experiment

- Assume at the outset atomically perfect, flat graphene in its ground state.
  - Effects due to impurities considered later
- How would you test Schwinger Formula experimentally?
  - Forces one to focus in more detail on how the Schwinger mechanism works
    - Finite spatial and temporal extent are critical
  - System will be rather different then typical cond-mat set up

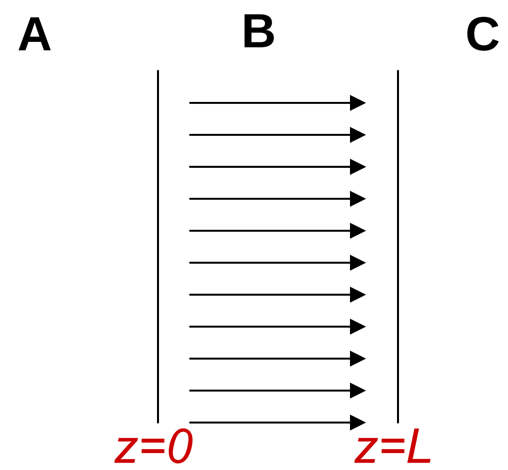
At first glance this looks like a non-linear conductance; the number of pairs created per unit time per unit width will correspond to a current in steady state.

Apparently current created by Schwinger pairs goes as $I \sim (e\, \Delta V/L)^{3/2}\, L\, W$



Unfortunately, the pairs once created, need not go down the wires.

There appears to be no steady state way to probe Schwinger mechanism in any finite system.

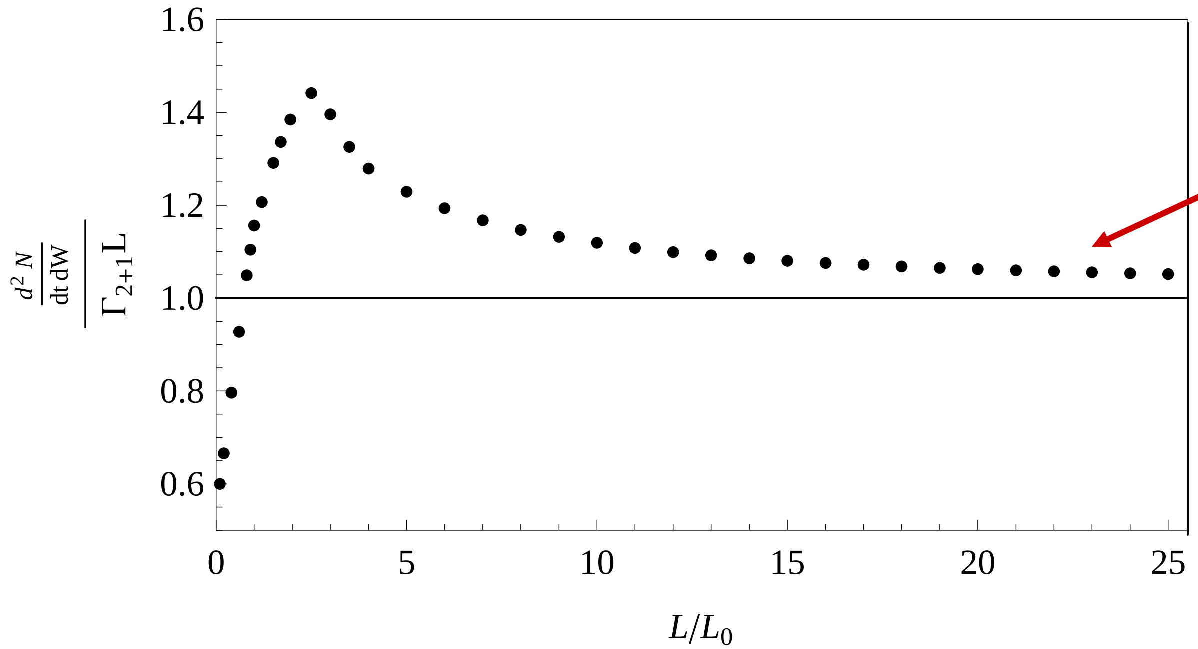


System considered in derivation was infinite in transverse extent and had a "vacuum" (i.e. graphene sheet) which stretched off to infinity in both directions . If such a system existed, one could simply turn on the E-field, and count the rate at which particle holes appear in regions A & C

Only relevant issue would be whether *L* is large enough for Schwinger formula to apply. On simple dimensional ground this condition is

$$L >> L_0 \equiv \sqrt{\frac{\hbar \tilde{c}}{e E}}$$

If this condition is not satisfied one can explicitly solve the Dirac equation in the three regions and match at boundaries to determine tunneling probality; integrating over transverse momenta as before yields pair production rate



Convergence is rather slow but there are no issues of principle

where $L_0 \equiv \dfrac{\hbar \tilde{c}}{\sqrt{e E}}$

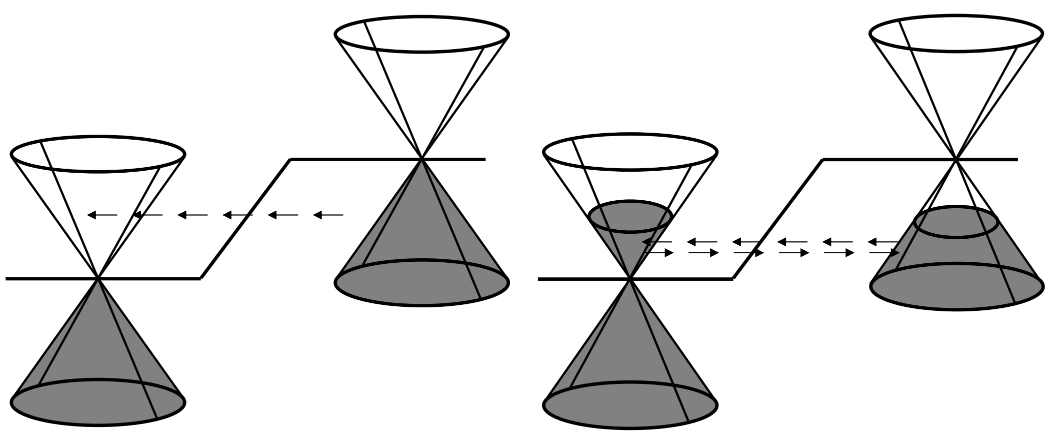
However, no system is infinite in transverse direction. Suppose the field region was finite in the transverse direction (but the graphene sheet were infinite). In that case at spatial infinity the two potential is zero on all sides.

The idea of an "in-state" vacuum" is meaningless. There are a finite (but large) number of single particle levels which are unoccupied and shifted below zero or occupied and shifted above zero when the potential is turned on. Thus tunneling only goes on for a finite time. Eventually the system equilibrates, even in the absence of backreaction.

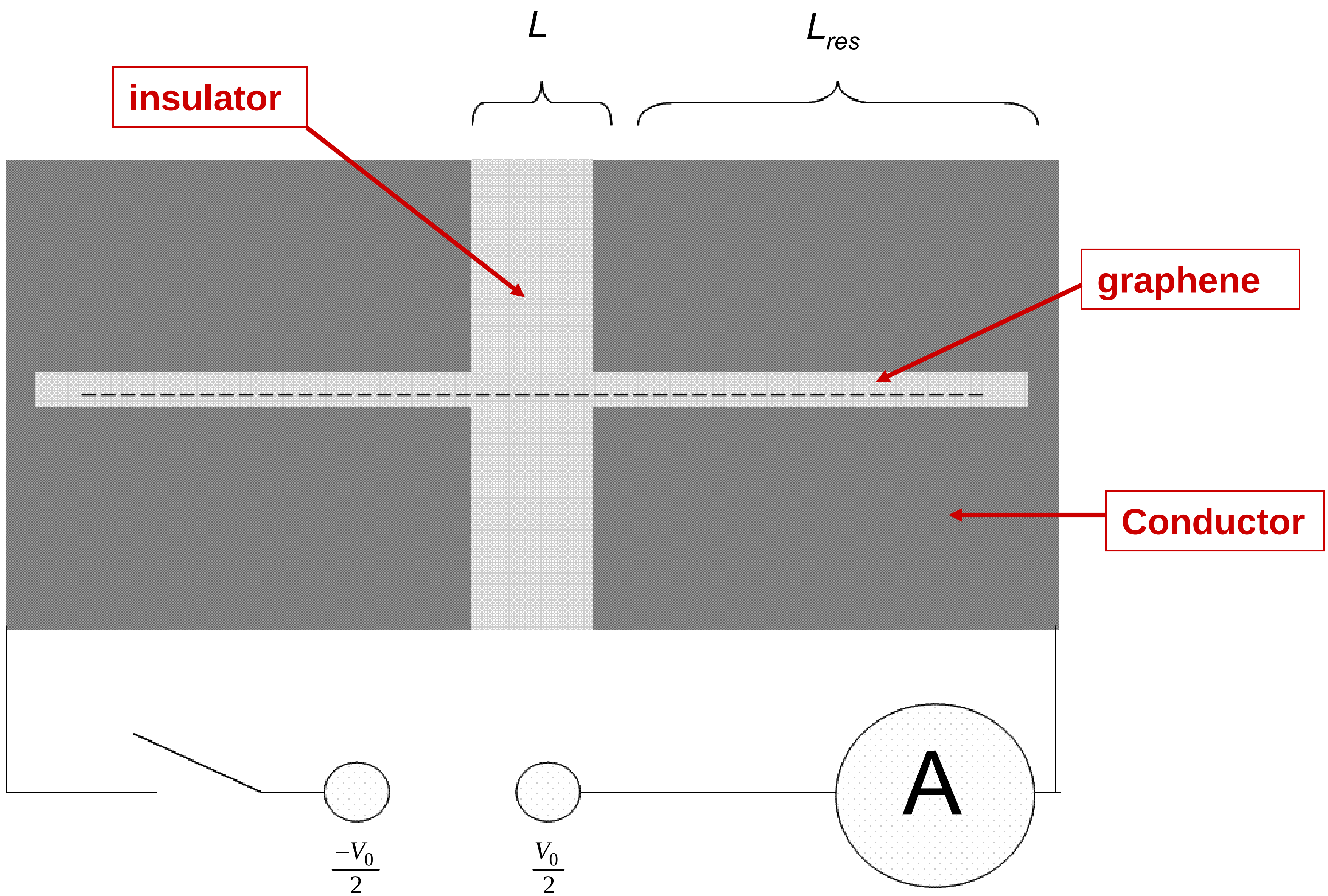
Thus Schwinger mechanism for any finite system is transient, *even when back reaction of fields are negligible.* This true in both 2+1 and 3+1 dimensions.

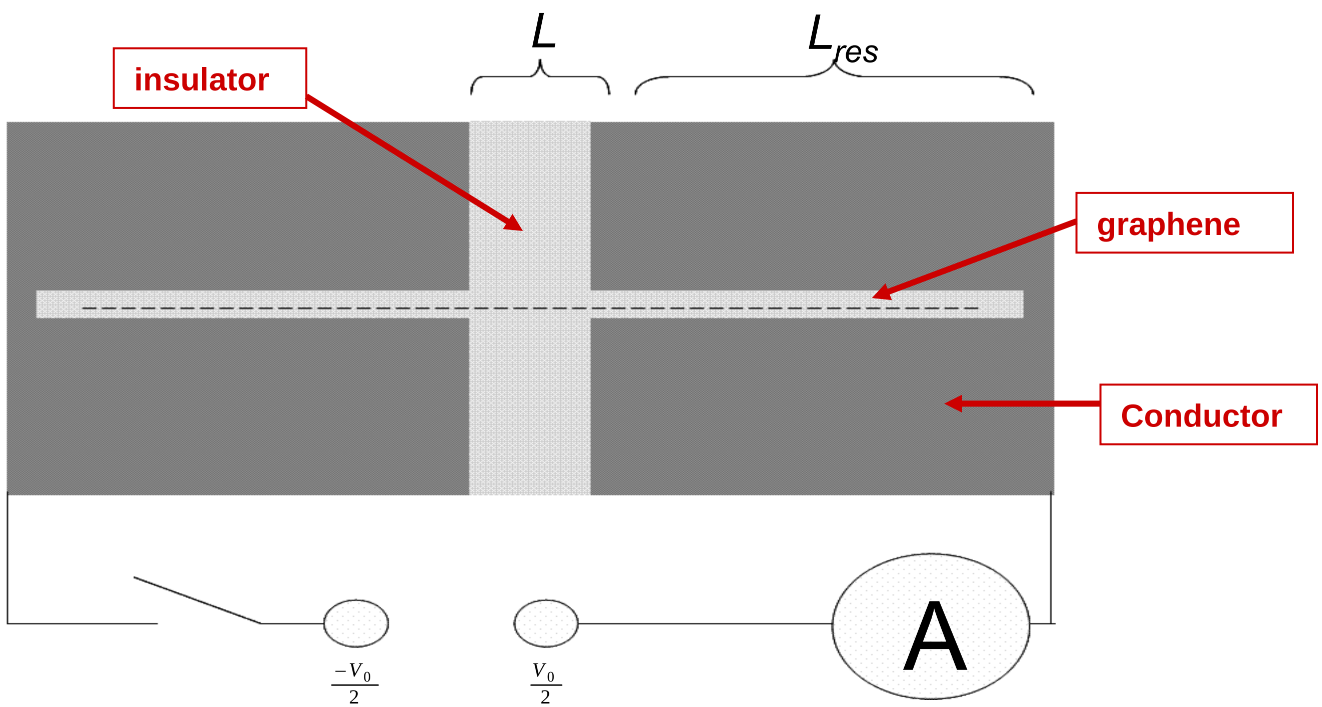
Note that while the Schwinger formula is local, the transient nature of the problem depends on global properties of the system.

For graphene, there is another effect---the "universe" corresponds to a sheet of graphene which is finite. Thus, there can never be an "in-state" vacuum with an infinite supply of states coming in form infinity. There are always a finite number of levels shifted. Schwinger mechanism is transient. **Ideally one will have a large "reservoir" of levels so the transient behavior will last a long time**



Idealized experimental setup---cross-sectional view





Note that the voltage is held fixed, thus "back reaction" does not play a role.

- **When switch is thrown there is transient behavior for time of order τ_{trans}, after which time the Schwinger formula should apply.**
- **The Schwinger formula should apply until the "reservoirs" fills substantially---a time of order τ_{fill}.**
- **Produced particles in reservoirs must be canceled to keep voltage fixed; induced currents can be monitored.**

for $\tau_{trans} << t << \tau_{fill}$

$$I_{Sch} = e L W \Gamma_{2+1} = \frac{e \zeta\left(\frac{3}{2}\right)(eV_0)^{3/2} W}{\pi^2 \hbar^{3/2} \tilde{c}^{1/2} L^{1/2}}$$

Dimensional analysis

$$\tau_{trans} = \sqrt{\frac{\hbar}{\tilde{c}\, e E}} = \sqrt{\frac{\hbar L}{\tilde{c}\, e V_0}}$$

\# of levels ~ L_{res}

$$\tau_{fill} = \sqrt{\frac{e E}{\tilde{c}^3 \hbar}} L L_{res} = \sqrt{\frac{e V_0 L}{\tilde{c}^3 \hbar}} L_{res}$$

Conditions for validity

Apart from time conditions given above:

• L,W must be enough so that finite volume effects are unimportant.

• The system must be clean enough so that it is well approximated as having the Dirac cone structure of relevant scales.

•Finite temperature effects must be small

•V_0 small enough to be in the "Dirac Cone region"

$$\text{Only length scale asssociated with Schwinger production rate}: L_0 \equiv \sqrt{\frac{\hbar \tilde{c}}{e\,E}}$$

Finite volume effects are small provided that

$$L, W >> L_0 \qquad W >> \sqrt{\frac{L\hbar\tilde{c}}{eV_0}} \qquad \mathrm{L} >> \frac{\hbar\tilde{c}}{eV_0}$$

A natural way to ensure effects of impurities (including effects of phonons) is small:

$$l_{mfp}^{eff} >> L_0 \equiv \sqrt{\frac{L\hbar\tilde{c}}{eV_0}}$$

There is evidence from transport measurements of the Columbia group (Kim, Stormer) with analysis from Maryland group (Das Sarma) that the dominant mechanism of impurity scattering is due to Coulomb impurities. PRL 99 246803 (2007)

Estmate of l_{mfp}^{eff} for Coulomb scattering :

The distance a quasiparticle must travel in a medium until the direction changes substantially is given by the distance needed to come to a point where the kinetic energy is comparable to the Coulomb potential

$\vec{k}$

$$\varepsilon = \tilde{c}\hbar k$$

$$U = \frac{e^2}{\kappa R}$$

Dielectric constant of insulator

Equating ε and U yields $R = \dfrac{c}{\tilde{c}} \dfrac{4\pi\alpha}{\kappa} \dfrac{1}{k}$

$$R = \frac{c}{\tilde{c}} \frac{\alpha}{\kappa} \frac{1}{k} \qquad l_{Coul}^{eff} = \frac{1}{n_{imp} R} = \frac{\tilde{c}}{\underbrace{\alpha c}_{.45}} \frac{\kappa k}{n_{imp}}$$

$$\hbar \tilde{c} k \sim eV_0 \qquad l_{Coul}^{eff} \sim \frac{\kappa e V_0}{\hbar c \alpha n_{imp}}$$

Finite Temp effects

Due to Boltzmann fluctuations the reservoirs are not empty when the switch is thrown. Thus there is a current due to these pre-existing thermal particles or holes, randomly moving into the field region and being transported across.

- This effect is much smaller than the current generated by the Schwinger mechanism provided that

$$T << \left(\frac{\hbar \tilde{c} \left(eV_0 \right)^3}{L k_B^4} \right)^{1/4}$$

Dirac cone region

General dimensional analysis: $V_0 << \frac{\hbar \tilde{c}}{e\,a} \approx 2.6\,eV$

Lattice spacing

In practice the Dirac cone structure works with modest corrections for $V_0 \leq 1$ eV.

Will ordinary conductivity mask the Schwinger Mechanism?

In principle the two effects are quite different both in the transient nature of the Schwinger mechanism and its non-linearity.

However, if ordinary conductivity produced currents on the same scale it would be hard to pull out contribution of Schwinger mechanism.

$$I_{ordinary} = \underbrace{\left(\frac{W}{L}\right)}_{\substack{Aspect \\ Ratio}} \underbrace{\sigma}_{comductivity} V$$

$$\frac{I_{ordinary}}{I_{Schwinger}} = \sigma \underbrace{\left(\frac{h}{4e^2}\right)}_{\mathcal{O}(1)} \underbrace{\left(\frac{2\pi}{\zeta\left(\frac{3}{2}\right)}\right)}_{\mathcal{O}(1)} \underbrace{\left(\frac{\hbar^{1/2}\,\tilde{c}^{1/2}}{(eV_0)^{1/2} L^{1/2}}\right)}_{<<1} << 1$$

Typical value of σ empirically is ~ 4 e^2/h

Condition on L required for Schwinger formula to be Kosher

Current due to Schwinger pair production will dominate.

Can the Schwinger Mechanism be tested in Graphene?

In principle, it can provided that samples which are sufficiently large, clean and cold can be studied. This is issue is whether it is practical to meet the various conditions for the validity.

Consider "realistic" numbers:

Size of sample~ 100 μm
Coulomb Impurity density† ~ 2×10^{11} cm^{-2}

† Best sample reported from Columbia group

Typical Parameters:

$L, L_{res} \sim 30\ \mu m \quad W \sim 100\ \mu m \quad V_0 \sim 1\ V$

$$L_0 \sim 150\ nm \qquad l_{Coul}^{eff} \sim \kappa\, 350\ nm$$

$$\underbrace{L, L_{res}, W \gg L_0}_{\text{well satisfied}} \qquad \underbrace{l_{Coul}^{eff} \gg L_0}_{\text{moderately well satisfied for large } \kappa}$$

$$\underbrace{T \ll \left(\frac{\hbar\, \tilde{c}\, (eV_0)^3}{L k_B^4}\right)^{1/4} \sim 800^\circ K}_{\text{easily satisfied}}$$

$$\tau_{trans} = \sqrt{\frac{\hbar L}{\tilde{c}\, e V_0}} = 1.5 \times 10^{-13} s$$

$$\underbrace{\tau_{trans} << \tau_{fill}}_{\text{Very well satisfied}}$$

$$\tau_{fill} = \sqrt{\frac{e V_0 L}{\tilde{c}^3 \hbar}} L_{res} = 6.5 \times 10^{-9} s$$

N.Ishii

The condition t << τ_{fill} requires very rapid measurements but it does not appear to be inconceivable experimentally

The Punchline

A test of the Schwinger mechanism in graphene looks marginal given present samples. The key issue is getting a long enough effective mean free path

As samples get bigger and cleaner, it should be possible.

Summary

- The Schwinger mechanism is an important piece of QED which has never been tested.
  - Focusing on how it can be measured clarifies the underlying physics and emphasizes the intrinsically transient nature of the effect.
- Graphene is an ideal place to test the Schwinger mechanism due to the lack of exponential suppression
  - While present samples may be somewhat marginal for a stringent test, increases in sample sizes and purities would allow for viable tests