Measurement in Quantum Theory

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Quantum theory (QTh) is the most accurate theory in science - a wealth of high precision results exist. Survives all – often very sophisticated - experimental tests.

Standard (orthodox) QTh is complete in a naïve sense: rules describe experiments completely and correctly. Practitioners (e.g. coupled cluster people!) use them without asking questions about the foundations of QTh. But sometimes they should ask them …

Foundations of quantum mechanics as one of five topics every young physicist leaving academia should know something about (even if, or just because, there is too much pseudoscience around it).
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Large numbers of papers raising questions concerning the foundations of QTh continuously are getting published.
Why?
Because in a less naïve sense

QTh probably is not complete and certainly not final.

Historically, any theory finally turned out to be wrong – valid only within certain limits. A search for weak points of an established theory, thus, makes sense. I personally hope that somebody – by looking into it’s dark corners – may find an effect where QTh fails – and that this is going to happen during my lifetime. Arguments for the non-completeness will be presented soon.

“Doubt everything at least once, even if it is the equation 2x2=4”
“Zweifle an allem wenigstens einmal, und wäre es auch der Satz: zweimal 2 ist 4”

(Georg Christoph Lichtenberg, physicist and satirist, Sudelbücher, around 1794)
Standard (Copenhagen) Rules for the Quantum Theory

(with minimal additions and without mathematical subtleties))

1. The system (= object to be measured) is described by a wave function (WF) (vector) $|\psi_S\rangle$ in Hilbert space. To each observable $O$ find, try out, guess,… a Hermitian operator $O = \sum o_i |o_i\rangle\langle o_i|$. The WF then can be expanded as $|\psi_S\rangle = \sum c_i |o_i\rangle$.

2. Invent, guess, try out,... a (Hermitian) Hamilton operator $H$

3. Before and after the measurement the system moves according to the Schrödinger equation (unitary motion with \( \exp(-i H t) \))

4. A measurement of $O$ puts the system „suddenly“ (not unitarily ?) into one of the eigenstates $|o_i\rangle$ ("collapse of the wave function"). The system then “has” eigenvalue $o_i$ as value of observable $O$. The apparatus is macroscopic, “shows” values $o_i$. That is, it is in the macroscopic ("pointer") state $|a_i\rangle$ related to $|o_i\rangle$.

5. The probability for occurrence of $o_i$ is given by $p_i = |\langle o_i |\psi_S\rangle|^2$ with $|\psi_S\rangle$ as the state of the system before the measurement ("Born probability"). (Density matrices often are used in this context as convenient bookkeeping devices.)
More details about rule 4 (measurement):

1. The apparatus has a complete orthonormal set of macroscopic ("pointer") states $|a_i\rangle$ co-orthogonal to the set $|o_i\rangle$. Widely used assumption: $|a_i\rangle$ are the only states active for measurement. This is oversimplified, but for the moment let me stick to it.

2. Thus: **with this assumption** the WF of system+apparatus is $|\Psi_{SA}\rangle = \sum c'_i |o_i\rangle |a_i\rangle$, with some $c'_i$ coefficients, in general different from the coefficients $c_i$ of the system in $|\psi_S\rangle = \sum c_i |o_i\rangle$ bit they are functions of them: $c'_i = c'_i(c_1...c_N)$. Such “Schmidt expansion” always exists if one basis is provided, but it is not always unique. Switching on the apparatus the state moves into this expansion, from now on named “premeasurement WF”:

$$e^{-iHt} |\psi_S\rangle |a_0\rangle = \sum c'_i(t) |o_i\rangle |a_i\rangle \equiv |\Psi'_{prem}\rangle$$

($|a_0\rangle$=initial state of apparatus). With $c'_i$ replaced by the expansion coefficients $c_i$ of the system WF it is the more common von Neumann’s form of the premeasurement WF.

3. The apparatus must be “objective”: it weights all states $|o_i\rangle$ in the same way (ideal measurement). Any differences in the results must come from the state measured, i.e. from the coefficients $c_i$ in the system WF $|\psi_S\rangle = \sum c_i |o_i\rangle$

4. “Not knowing” the Born rule, but accepting that the collapse into a certain state $|a_i\rangle$

$$|\Psi'_{prem}\rangle = \sum c'_i |o_i\rangle |a_i\rangle \rightarrow |o_k\rangle |a_k\rangle$$

occurs with some probability $p_k$, then it must depend on the $c'_k$: $p_k = p(c'_1...c'_N)$
5. The **apparatus is a quantum many body system**. At least outside the collapse it moves with $\exp(-iHt)$, where $H$ is a Hamiltonian, containing many degrees of freedom and known forces. How then can it produce the premeasurement WF? QTh at most allows that the exact many body Hamiltonian leads to this WF pre-measurement WF $e^{-iHt}\psi_s|a_0\rangle = \sum c_i|o_i\rangle|a_i\rangle$ as an exact result, at least if the pointer states are truly the only active ones. Some people even consider this WF as something which does not require any proof. The fact that the piggy-back like structure of this WF guarantees that the apparatus is in the pointer state $|a_i\rangle$, if the system is in the state $|o_i\rangle$ supports their belief. I shall return to this topic later.

**Simple example: photographic plate and a particle beam.**
Measuring the coordinate of a particle. A wave function $\int dkf(k)e^{i(kx-\omega t)}$ describes the behaviour everywhere inclusive possible absorption on the surface.

Then: collapse into the state $|x\rangle$ (“particle at coordinate $x$”)

$$\int dkf(k)e^{i(kx-\omega t)}|a_0\rangle_{\text{surface}} \rightarrow |x\rangle_{\text{black dot at } x\rangle_{\text{pointer state}}}$$

The collapse is not an ordinary absorption process controlled solely by $\exp(-iHt)$ (i.e. by a wave equation alone). It is **not** describable as a shrinking process from the wave to the state $|x\rangle$ driven by $\exp(-iHt)$ alone. No unitary transformation can generate such shrinking if combined with selecting - with some probability $p(x)$ - one of the $|x\rangle$. On the other hand, a shrinking via unitary motion to a preselected state $|x\rangle$ is possible in principle, but it is no measuring process.
II. More about Collapse

*It is common folklore* that collapse is not due to \( \exp(-iHt) \) It is considered as fundamental – not derivable from other physical laws. (Of course, the Hamiltonian participates in the collapse.)

But we cannot be quite sure:

It *could be* something like a irreversible thermodynamic process within the many body system, governed *only and only* by \( \exp(-iHt) \) where \( H \) contains all degrees of freedom, including interaction with the system. This would be something which happens locally inside the apparatus. However, as long as one is not able to make this explicit, one is forced to consider the collapse as a phenomenological (practically fundamental) process.

EPR (Einstein-Podolski-Rosen) experiment: Two identical apparatuses, each for one particle. After measuring locally the spin of particle 1 apparatus 1 has switched to a pointer state corresponding to spin of particle 1. Particle 2 then found far away with spin opposite to spin of particle 1 with apparatus 2 in the corresponding pointer state. The two apparatuses measure locally, and the entanglement of the WF mediates globally between them.
A more detailed account of the EPR – Experiment

Entangled (zero spin) WF of the two spin1/2- particles

\[ |\psi_S\rangle = \frac{1}{\sqrt{2}} \phi_1(x_1)\phi_2(x_2)(|+_1\rangle_2 - |-_1\rangle_2 + |+_1\rangle_2 - |-_1\rangle_2) \]

Premeasurement WF:

\[ |\Psi'_{prem}\rangle \equiv e^{-iHt}|\psi_S\rangle |a(x_1,0)\rangle |a(x_2,0)\rangle = \phi_1(x_1)\phi_2(x_2)^* \]

\[ \begin{pmatrix} c'_{+-}|+_1\rangle_2 |a(x_1,+)\rangle |a(x_2,-)\rangle + c'_{--}|-_1\rangle_2 |a(x_1,-)\rangle |a(x_2,+)\rangle \end{pmatrix} \]

where \( |a(x_k,i)\rangle \) = states of apparatus \( k (=1,2) \). The measurement on particle 1 leads for instance to

\[ |\Psi'_{prem}\rangle \rightarrow \begin{pmatrix} |+_1\rangle_1 |a(x_1,+)\rangle |a(x_2,-)\rangle \phi_1\phi_2 \end{pmatrix} \]

Measuring particle 2 (collapsing wave function of particle 2) merely confirms the state \( |-_2\rangle \) via \( |a(x_2,-)\rangle \). The reason is that in \( |\Psi'_{prem}\rangle \) both apparatuses are coupled to both particles. Thus the first collapse to \( |+_1\rangle_1 \) happens if and only if particle 2 is in \( |-_2\rangle \). The apparatuses measure locally, the WF is global.

Thus: Collapse

- **practically** is fundamental (cannot be derived from known physical laws)
- **and global** (possible influence from everywhere)
- and **cannot be disentangled from Schrödinger equation of motion**
- (apparatus is a many body system, driven by known Hamiltonians !)
III Environment

Three arguments for the inclusion of the environment:

1. **Globality of measurement process**: total environment must be included (local environment does not suffice)

2. The Schmidt expansion $|\Psi_{\text{prem}}\rangle = e^{-iHT}\psi_S|a_0\rangle = \sum c'_i|o_i\rangle|a_i\rangle$ is not unique: unitary transformations of the two sets $|o_i\rangle$ and $|a_i\rangle$ can be performed such that the role of system and apparatus are exchanged (basis ambiguity).

   **Simple solution**: Adding a third system with a complete set $|e_i\rangle$ - named “environment” – one has the Schmidt expansion for the “extended premeasurement WF”

   $$|\Psi_{\text{prem}}\rangle = e^{-iHt}\psi_S|a_0\rangle|e_o\rangle = \sum c'_i|o_i\rangle|a_i\rangle|e_i\rangle \rightarrow \text{some } |o_i\rangle|a_i\rangle|\psi_{\text{collapse}}\rangle$$

   Due to a theorem about the uniqueness of the triorthogonal expansion* adding the environment makes the Schmidt expansion of the premeasurement WF unique. Everything presented so far goes through just by adding the states of the environment. However, this approach raises a number of questions which will be taken up later.

   * It does not always exist.

3. **Absurd macroscopic superpositions** occur in closed quantum systems with chaotic Hamiltonians – they must be treated as open systems: environment comes in. (I use here Zurek’s example)
Chaotic motion of macroscopic system requires open system:

Closed chaotic macroscopic systems necessarily show macroscopic quantum effects ("loss of quantum-classical correspondence")

Reason: for chaotic systems small patches in phase space stretch as $\exp(+\Lambda t)$ in instable and shrink as $\exp(-\Lambda t)$ in stable directions with Liapunov exponents $\Lambda > 0$.

A shrinking in the $p$-direction generates an immediate stretching in the $x$-direction due to the Heisenberg uncertainty principle and the distribution rapidly fills the whole space.

- A shrinking in the $p$-direction generates a QM-stretching in the $x$-direction
- Volume (size $L$) filled after time.
- If $P =$total momentum range
  
  $(PL$ "classical action).

(Spread due to Schrödinger eq. governed by $PL/\hbar$ and not by $\ln(PL/\hbar)$)
Example:
Hyperion, potato shaped moon of Saturn.
Orientation tumbles around in a chaotic manner due to interaction with tidal wave of Saturn.
“Wave function of Hyperion”
Would develop after ~ 20 years, if QTh “open”.
Example:

\[ \Psi = \frac{1}{\sqrt{2}} ( \text{angle 1} + \text{angle 2} ) \]

If one looks at it one would observe one of the two orientations with probability 1/2 and hyperion also would have this orientation in reality. This “Schrödinger cat state” develops within the human life span.

Actually the wave function would spread in some way over all angles. Thus one may see one of many orientations and this orientation is real.
Environment generates new Symmetries

W,H.Zurek (2003) found and named a new invariance, the “environment assisted invariance” (“einvariance”) for Schmidt expansions of this type.

Definition: a state is called einvariant under the unitary transformation $U_{SA}$ acting on S+A only if it can be counteracted by a unitary transformation $U_E$ acting on E only. With $U_{SA} \equiv u_{SA} \otimes 1_E$ and $U_E \equiv 1_{SA} \otimes u_E$

( $1_{SA}$ and $1_E$ = unit vectors; $|i\rangle_{SA} \equiv |i\rangle_S \otimes |i\rangle_A$ ) one has einvariance if

$$U_{SA} U_E |\Psi'_{\text{prem.}}\rangle = |\Psi'_{\text{prem.}}\rangle$$

Zurek found two such unitary transformations, against which $|\Psi'_{\text{prem.}}\rangle$ is einvariant

A phase transformation, $u_{SA}^{\text{phase}}$, with countertransformation $u_{E}^{\text{phase}}$:

$$u_{SA}^{\text{phase}} (\zeta_i, \zeta_j) \equiv e^{i\zeta_i} |i\rangle_{SA} \langle i| + e^{i\zeta_j} |j\rangle_{SA} \langle j|$$

$$u_{E}^{\text{phase}} (\zeta_i, \zeta_j) \equiv e^{-i\zeta_i} |i\rangle_{EE} \langle i| + e^{-i\zeta_j} |j\rangle_{EE} \langle j|$$

and a swap transformation, $u_{SA}^{\text{swap}}$, with countertransformation $u_{E}^{\text{swap}}$:

$$u_{SA}^{\text{swap}} (\zeta_{ij}, \zeta_{ji}) \equiv e^{i\zeta_{ij}} |i\rangle_{SA} \langle j| + e^{i\zeta_{ji}} |j\rangle_{SA} \langle i|$$

$$u_{E}^{\text{swap}} (\zeta_{ij}, \zeta_{ji}) \equiv e^{-i\zeta_{ij}} |i\rangle_{EE} \langle j| + e^{-i\zeta_{ji}} |j\rangle_{EE} \langle i|$$

(and generalizations to more than 2 states).
Idea of Zurek’s proof of Born probability

1. Irrelevance of phases: $u_{S+A}^{\text{phase}}$ exchange phases between S+A and E, and, thus phases are not the “property of the system”; they are shared between S+A and E

Simple picture of the reasoning leading to this result (in the figure SA means system + apparatus)

"identical " implies: consequences are the same

$\rightarrow$ S+A unaffected by $u_{S+A}^{\text{phase}}$

$\rightarrow$ phases irrelevant (cannot influence probabilities )

additional result:

only $|c_i'|$ relevant
2. $|c'_i|^2$ are probabilities

Use swap transformation $U_{\text{swap}}$. Swaps the labels $i$ of the states. Can be removed by counterswapping.

In the same way as for the phase transformation one arrives at: $S+A$ is not affected by the swap.

Zurek discusses first the case of equal $|c'_i|$ (with necessarily $|c'_i|^2 = 1/N$, $N=$ dimension of Hilbert space).

Since $S+A$ is not affected by swaps, all states are equivalent and all their consequences are the same. Thus they occur with equal probability. This implies the probability $1/N$ for each state. Here is the concept of probability assumed. Trivial?

Generalization to non equal $|c'_i|$: argue as above within each subset of equal $|c'_i|$, leading to probabilities as rational numbers. Then use some fine graining techniques from probability theory to generalize to non rational numbers.

Thus in the equation of motion description of the transition into the general Schmidt expansion

$$e^{-iHt}|\psi_S\rangle\langle a_0|e_o\rangle = \sum c'_i o_i \langle a_i|e_i\rangle$$

(exact if only the pointer states contribute) the $|c'_i|^2$ are the probabilities for the occurrence of the $|o_i\rangle$. Knowing that the Born probabilities are given by the $|c_i|^2$ (with $c_i$ from the system WF $|\psi_S\rangle = \sum c_i \langle o_i|$ nature somehow must specialize the general Schmidt exact expansion to the “v. Neumann premeasurement WF”:

$$e^{-iHt}|\psi_S\rangle\langle a_0|e_o\rangle = \sum c_i o_i \langle a_i|e_i\rangle$$
Premeasurement and environment

1. What happens:
   
i) The time development leading to the final premeasurement WF $e^{-iH\Delta t}\Psi_s|a_0\rangle|e_o\rangle = \sum c_i|o_i\rangle|a_i\rangle|e_i\rangle$ is driven by a Hamiltonian $H = H_S + H_A + H_E + H_{SA} + H_{SE} + H_{AE}$, describing known particles and known interactions. These Hamiltonians, thus, are quite common objects. Yet they perform a special job by generating the premeasurement WF (piggy-back structure with only macroscopic states $|a_i\rangle$ of the apparatus, and states $|e_o\rangle$ of the environment plus incorporating the coefficients $c_i$ of the system WF) before the collapse, i.e. before the genuine measurement.

   ii) The system under investigation and the environment have to be accepted as they are. Merely the apparatus can be engineered by man. He may select the apparatus Hamiltonian from known Hamiltonians. Except for this, the only place where he can manipulate things is the initial state $|a_0\rangle$ of the apparatus. It is the job of the experimentalist to find or construct such a state. This typically is routine: In the former example $|a_0\rangle$ represents the blank photographic plate.

   iii) The experimentalist also will screen off the system from the environment, but, of course not from the apparatus. After all he wants to investigate an isolated system. Thus, if done carefully, there is no direct interaction $H_{SE}$ between system and environment. But via the premeasurement WF there is still such indirect interaction.

   iv) On the other hand, the macroscopic pointer states typically interact directly with the environment: Straying phonons will scatter off the black dots of the photographic plate. Thus, the need for the inclusion of the environment seems to be quite natural.
2. Questions and some Answers

i) Knowing the entanglement with the environment, why does it not matter where we perform the measurement, i.e. how the near or far environment looks like or what it consists of?
The answer can only be: besides the ordinary interaction with the environment there is something more fundamental at work. This must be some “precollapse” mechanism – thus part of the collapse postulate.

ii) How do the ordinary well known Hamiltonians generate the Schmidt expansion with only the macroscopic pointer states?
A suggestive answer is: the experimentalist constructs his apparatus such that it works this way, i.e. he makes sure that no other states of the apparatus interfere. This answer is not quite satisfactory, since a realistic premeasurement WF may well contain other (undesirable) states, and still the collapse may work as it should. Again, a precollapse mechanism may be invoked.

iii) Are we sure that we must incorporate the environment exactly in this way? Whereas the biorthogonal Schmidt expansion certainly exists, the addition of a third orthogonal set is arbitrary and merely motivated by the wish or necessity to incorporate the environment.
iv) Isn’t it better to give up altogether any derivation of the premeasurement WF and consider it part of the collapse rule? One probably should not to dismiss the premeasurement idea with the pointer states completely, since it successfully explains the Born probability.

This is what most people seem to prefer: They consider the premeasurement WF, with the coefficients taken from the system WF, as something to be taken for granted: a derivation is not necessary.

Note:

In the literature there exist various models for the apparatus and environment to achieve the desired form of the premeasurement WF with a model Hamiltonian $H_M$, $e^{-iH_Mt}|\psi_S\rangle|a_0\rangle|e_0\rangle = \sum c_i |o_i\rangle|a_i\rangle|e_i\rangle$ and, thus the Born probability results.
Summary
As a practitioner one may accept and use the rules of QTh as they occur in the textbooks, but still it is good to keep the following facts in the back of one’s mind:

1. **System and apparatus are entangled with the environment.** This entanglement is *not restricted to the neighbourhood*. It is not (only) an ordinary interaction with the nearby environment, for instance responsible for the decoherence into chiral states in light molecules. This fact possibly has unexpected consequences.

2. **Switching on the apparatus, there develops from some initial states a “premeasurement WF” exhibiting this entanglement with the environment.**

3. **There exists a widely accepted explicit premeasurement WF as a tri-diagonal Schmidt expansion** with the “piggy-back” structure described above, including the entanglement with the environment and the macroscopic pointer states of the apparatus. It *plays a fundamental role* in the recent literature.

4. **Apparatus and environment are many body systems described by known Hamiltonians.** The time development of system, apparatus and environment is driven by the full Hamiltonian. The required reduction to the tri-diagonal structure with the pointer states is plausible, but not proven: no mechanism is known how the many quite different realistic Hamiltonians for system and apparatus and their interactions manage this.
5. One may as well avoid this problem by **considering the tri-diagonal structure as a fundamental pre-collapse rule** – most people seem to tend in this direction.

6. This tri-diagonal premeasurement WF **leads to new symmetries**, exhibiting the fact that the phases are common property of system and environment. The absolute values of the expansion coefficients of this WF are probabilities.

7. If these expansion coefficients are the same as those from the system WF one has a **proof of the Born probability** and may erase it from the list of rules for QTh.

8. One may instead consider the Born probability rule as fundamental. Then the tri-diagonal WF is compatible with it, or possibly even can be derived from it.