Baryons in Holographic QCD

"From Strings to Things", INT-08-1

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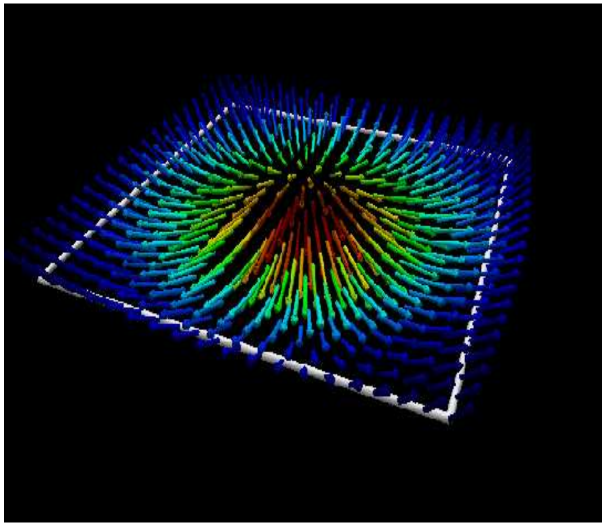


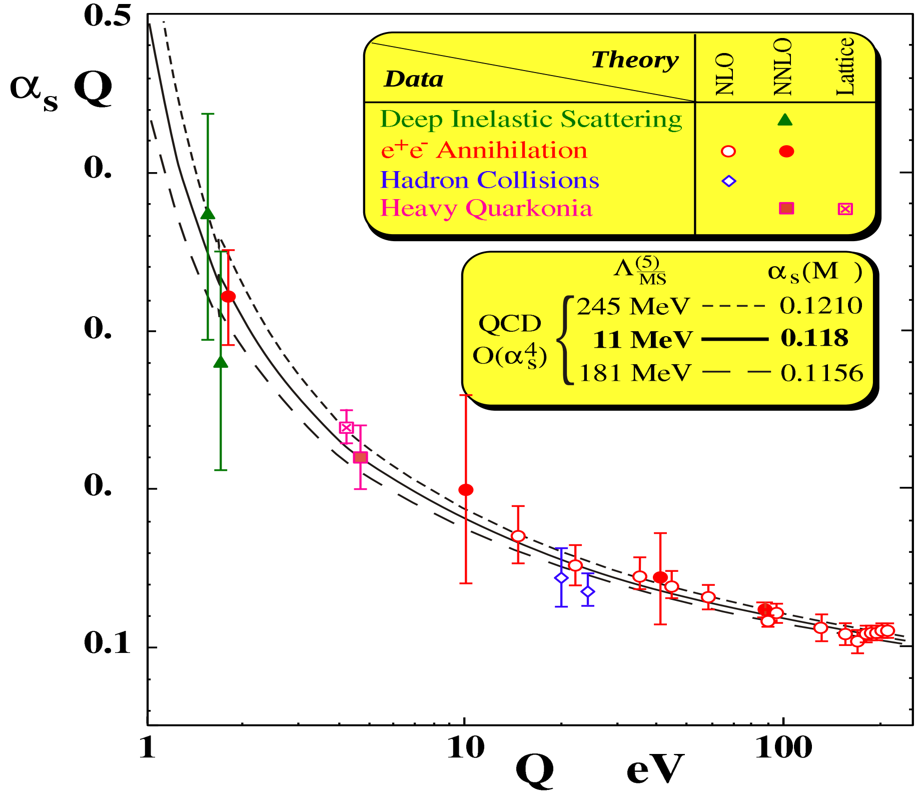
- Top-down with M. Rho, H.-U. Yee, P. Yi, K. Lee, and C. Park
  hep-th/0701276, arXiv:0705.2632, arXiv:0710.4615, arXiv:0804.1326
- Bottom-up with T. Inami, H.-U. Yee, H. Kim, S. Siwach:
  hep-ph/0609270, arXiv:0709.0314

I. Introduction

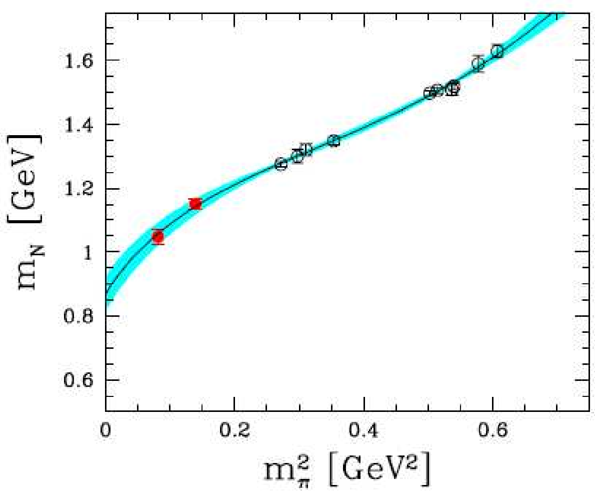
- QCD is believed to be the theory of strong interaction:

$$\mathcal{L} = -\frac{1}{4} {F^a_{\mu\nu}}^2 + \bar{q}_i \left(i\not{D} - M_q \right) q_i + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \, . \qquad (1)$$

- Evidences?



- Solving QCD is hard, since it's strongly coupled and has no expansion parameter.
- Lattice (ICHEP06)



Stat. error $\lesssim 3\%$

QCDSF

- Recent discovery of AdS/CFT correspondence provides a new scheme to solve strongly coupled gauge theories like QCD.

  $\longrightarrow$ Holographic QCD or AdS/QCD, 5D gravity dual to QCD.

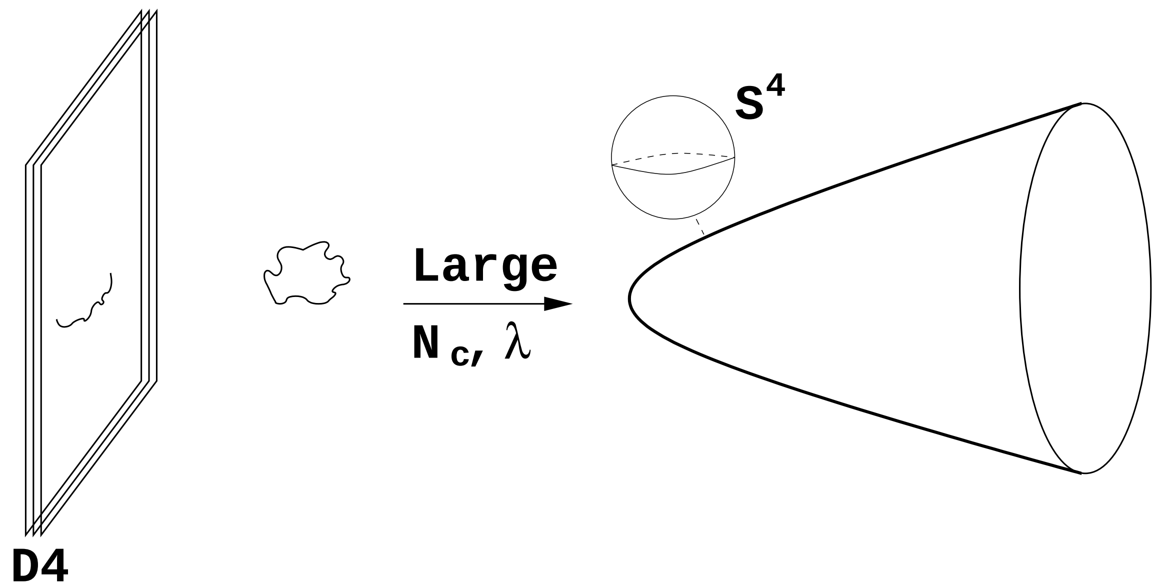
- There are several models on holographic QCD, top-down or bottom-up, as good as or better than other models ($< 30\% \sim 1/N_c$).

- But, what's more important is its model-independent features, insensitive to $1/N_c$ corrections, in contrast with other models:
  - New relations among couplings $g_A \sim \mu_{\rm an}, g_{\omega NN} \approx 3 g_{\rho NN}, \cdots$
  - Baryons as 5D instanton solitons: universal new VMD.
  - New sum rules:

$$\mu^p_{\rm an} + \mu^n_{\rm an} = 0 \;\; (1.79 - 1.91 = 0.12\mu_N, \;\text{experiments}),$$
$$d_p + d_n = 0 \;\; (0.26 - 0.21 = 0.05\bar{\theta}\, e\cdot \text{fm}, \;\text{Shintani et al 07})\,.$$

II. Holographic QCD

- N_c stack of $D4$ brane over $R^3 \times S^1$ describes pure $SU(N_c)$ YM. (Witten '98)

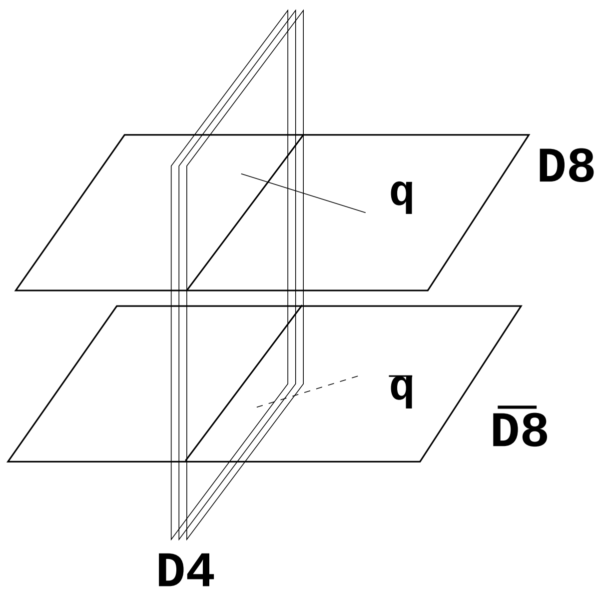


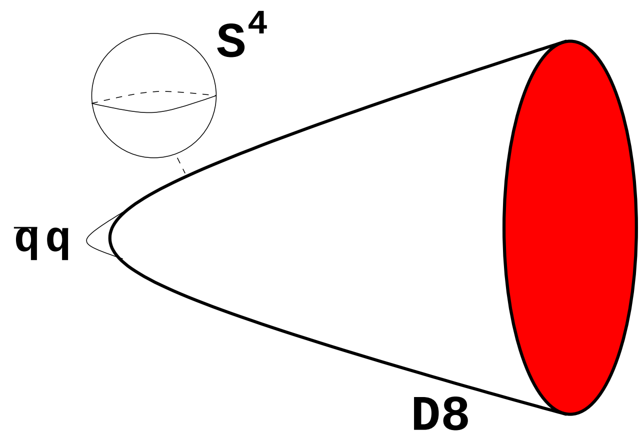
$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2\right) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

with $R^3 = \pi g_s N_c l_s^3$ and $f(U) = 1 - U_{KK}^3/U^3$

- The ratios of glueball mass agree with the lattice data rather well. (Csaki et al ’99; Brower et al ’99)

- Adding flavors was done by Sakai-Sugimoto (2004) (Cf. Karch and Katz in $D3$-$D7$, probe approximation,’02).





- Spontaneous chiral symmetry breaking is geometrically realized:

$$SU(N_F)_L \times SU(N_F)_R \mapsto SU(N_F)_V \; . \qquad (2)$$

- Effective action on D8 is a $U(N_F)$ gauge theory,

$$
\begin{aligned}
S_{D8} &= -\mu_8 \int \mathrm{d}^9 x\, e^{-\phi} \sqrt{-\det\left(g_{MN} + 2\pi\alpha' F_{MN}\right)} \\
&\quad + \mu_8 \int \sum C_{p+1} \wedge \mathrm{Tr}\, e^{2\pi\alpha' F} \,,
\end{aligned}
$$

- The gauge fields contain pions and whole tower of vector mesons:

$$
A_\mu(x,z) = \alpha_\mu(x)\psi_0(z) + \beta_\mu(x) + \sum_{n\geq 1} B_\mu^{(n)}\psi_n(z)\,, \tag{3}
$$

where with $\xi = \exp(i\pi(x)/f_\pi)$

$$
\alpha_\mu = \left\{\xi^\dagger, \partial_\mu \xi\right\}\,, \quad \beta_\mu = \left[\xi^\dagger, \partial_\mu \xi\right]\,. \tag{4}
$$

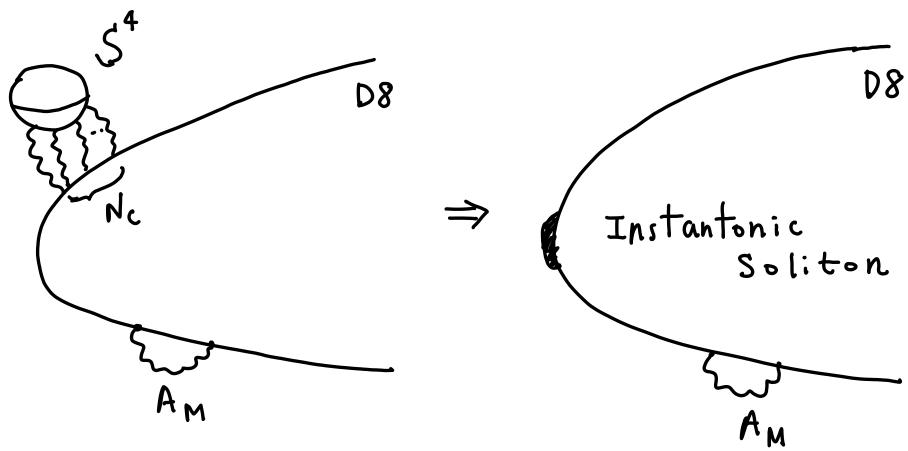
- The effective action successfully describes all the interactions of pions and vector mesons, including the Skyrme term.

III. Baryons as 5D Instanton Solitons

- What are baryons in hQCD? It must be solitons:

$$m_{\text{baryon}} \sim N_c \,. \tag{5}$$

- In SS model, D4 brane wrapping S^4 is the baryon vertex (Witten).
- $D4$ brane becomes instanton in $D8$ (Douglas '95).



- In 5D YM there is a topologically conserved current, $d\,{}^*J = 0 = DF$,

$$J^M = \frac{1}{24\pi^2}\epsilon^{MNLPQ}\,\mathrm{tr}F_{NL}F_{PQ}\,. \tag{6}$$

- One can define the baryon current

$$B^{\mu} = \frac{1}{8\pi^2} \int \mathrm{d}z \epsilon^{\mu\nu\rho\sigma} \mathrm{tr} F_{\nu\rho} F_{\sigma z} \,. \tag{7}$$

- In the gauge $A_z = 0$ one may write $U = \exp(2i\pi/f_\pi)$

$$A_\mu(x,z) = U^{-1}\partial_\mu U \psi_0(z) + \sum_{n\geq 1} B_\mu^{(n)} \psi_n(z) \,. \tag{8}$$

Then the baryon current becomes the Skyrme current

$$B^{\mu} = \frac{1}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{tr} U^{-1}\partial_\nu U U^{-1}\partial_\rho U U^{-1}\partial_\sigma U \tag{9}$$

- Unlike Skyrme model we know that it has to be a baryon current, since the baryon carries the N_c unit of quark number in SS model

$$S_{CS}^{D4} = \int_{R\times S^4} C \wedge e^{F/2\pi} \sim N_c \int_R A \,. \tag{10}$$

coupling to the instanton number density,

$$S_{CS}^{D8} = \frac{N_c}{24\pi^2} \int_{M^4 \times R} \omega_5(A)\,, \quad \rho(x) = \frac{\delta S}{\delta A_0(x)} = \frac{N_c}{24\pi^2} \int \mathrm{d}z F\tilde{F}\,.$$

Small but hairy instantons

- In the SS model the DBI action tends to shrink the solitons but the Coulomb repulsion stabilizes them.
- In the conformally flat metric, the energy density becomes

$$-\int_{x,w} \frac{1}{4e^2(w)} \mathrm{tr} F_{mn} F^{mn} + \mathrm{h.o.}, \quad \frac{1}{e^2(w)} \equiv \frac{\lambda N_c}{108\pi^3} \frac{M_{KK} U(w)}{U_{KK}}\,. \tag{11}$$

- A point-like instanton that is localized at $w = 0$ would have the energy

$$m_B^{(0)} \equiv \frac{4\pi^2}{e^2(0)} = \frac{\lambda N_c}{27\pi} M_{KK}\,. \tag{12}$$

- Since the instanton carries $U(1)$ charge, there is Coulomb repulsion that prevents the instanton from collapsing:
(Rho+Yee+Yi+DKH '07; Hata+Sakai+Sugimoto+Yamato '07)

$$\rho_{baryon} \sim \frac{9.6}{M_{KK}\sqrt{\lambda}}\,, \tag{13}$$

where $M_{KK} \simeq 1$ GeV is the UV cut-off of SS model.

- At low energy the baryons are described as point-like bulk spinors,

$$\int d^4x dw \left[-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - i m_b(w)\bar{\mathcal{B}}\mathcal{B} + g_5(w)\frac{\rho^2_{baryon}}{e^2(w)}\bar{\mathcal{B}}\gamma^{mn}F_{mn}\mathcal{B}\right]$$
$$- \int d^4x dw \frac{1}{4e^2(w)}\,\mathrm{tr}\,F_{mn}F^{mn} + \cdots\,, \tag{14}$$

- Hairy instantons: the spinor sources YM fields

$$\nabla^2 A_m^a = 2g_5(0)\rho_{baryon}^2 \bar{\eta}_{mn}^a \partial_n \delta^{(4)}(x) \,, \tag{15}$$

whose solution goes as

$$A_m^a = -\frac{g_5(0)\rho_{baryon}^2}{2\pi^2} \bar{\eta}_{mn}^a \partial_n \frac{1}{r^2 + w^2} \tag{16}$$

to compare with the 't Hooft ansatz

$$A_m^a = -\bar{\eta}_{mn}^a \partial_n \log\left(1 + \frac{\rho^2}{r^2 + w^2}\right) \simeq -\rho^2 \bar{\eta}_{mn}^a \partial_n \frac{1}{r^2 + w^2} \,, \tag{17}$$

- Including the quantum fluctuations to match the long-range instanton tail (Adkins+Nappi+Witten),

$$g_5(0) = \frac{2\pi^2}{3} \tag{18}$$

- The Lagrangian is unique up to operators with two derivatives in the large N_c and large $\lambda = g_s^2 N_c$ and valid for $E < M_{KK}$.

- Though the coefficient of the Pauli term might be model dependent, the fact that it contains only the nonabelian part of the flavor symmetry is model-independent!
  $\longrightarrow$ The $U(1)$ coupling does not have the Pauli term.

- One immediate consequence of this is that the Pauli form factor

$$F_2^p(q^2) = -F_2^n(q^2) + \text{h.o.}. \tag{19}$$

- Especially for instance $\mu_{\text{an}}^p + \mu_{\text{an}}^n = 0$, which is very close to the experimental value, $(\mu_{\text{an}}^p + \mu_{\text{an}}^n)_{\text{exp}} = 1.79\mu_N - 1.91\mu_N = -0.12\mu_N$

Holographic Monopole Catalysis of Baryon Decay

arXiv:0804.1326 with K.-M. Lee, C. Park, and H.-U. Yee

- Baryon number is not conserved under magnetic monopole. (Rubakov '81; Callan '82)

- The instanton number is also not conserved in the presence of monopole:

$$DF \neq 0 \longrightarrow d^*J \neq 0\,. \tag{20}$$

- In SS model the magnetic monopole background gives a BC,

$$A(+\infty) = A(-\infty) = QA^{EM}, \quad Q = \mathrm{diag}\,(2/3, -1/3)\ . \tag{21}$$

- In a general background A_L and A_R with $\xi_\pm^{-1} = P\exp(-\int_0^{\pm\infty} A_z)$ and $\xi_+^{-1}\xi_- = U$ we write

$$A_\mu(x,z) = A_{L\mu}^{\xi_+}(x)\psi_+(z) + A_{R\mu}^{\xi_-}(x)\psi_-(z) + \text{(excited modes)},$$

where $A_{L\mu}^{\xi_+} = \xi_+ A_{L\mu}\xi_+^{-1} + \xi_+\partial_\mu\xi_+^{-1}$, $A_{R\mu}^{\xi_-} = \xi_- A_{R\mu}\xi_-^{-1} + \xi_-\partial_\mu\xi_-^{-1}$.

- Then the baryon current becomes

$$B^\mu = \frac{1}{24\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\,\left(U^{-1}\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U\right)$$
$$-\frac{1}{8\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\,\partial_\nu\left(U^{-1}A_{L\alpha}\partial_\beta U + A_{R\alpha}U^{-1}\partial_\beta U - U^{-1}A_{L\alpha}U A_{R\beta}\right)$$
$$-\frac{1}{8\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\left(\partial_\nu A_{L\alpha}\,A_{L\beta} + \frac{2}{3}A_{L\nu}A_{L\alpha}A_{L\beta} - (L\leftrightarrow R)\right)\,.$$

- We find a unified formula for the baryon number violation

$$\partial_\mu B^\mu = \frac{1}{32\pi^2}\left(\mathrm{Tr}F_L\tilde{F}_L - \mathrm{Tr}F_R\tilde{F}_R\right) + \frac{i\delta^{(3)}(\vec{x})}{2\pi}\int_{-\infty}^{+\infty}dz\,\mathrm{Tr}\,(QF_{tz})\,,$$

- For the monopole catalysis of instanton-baryon decay, $U = \exp(2i\pi/f_\pi)$ we have

$$\frac{dB}{dt} = \frac{1}{\pi f_\pi}(\partial_t\pi^0)\,. \qquad (22)$$

IV. Phenomenology: Static properties of baryons

- Once you are given the holographic action, you can get various couplings of baryons after KK reduction.
- Vector couplings of baryons,

$$
\begin{aligned}
g^{(n)}_{\rm min} &= \int_{-w_{max}}^{w_{max}} dw \, |f_L(w)|^2 \, \psi_{(n)}(w) \,, \\
g^{(n)}_{\rm mag} &= 2C \int_w dw \left(\frac{g_5(w) U(w)}{g_5(0) U_{KK} M_{KK}} \right) |f_L(w)|^2 \, \partial_w \psi_{(n)}(w) \,.
\end{aligned}
\tag{23}
$$

- For SS model in the large N_c,

$$
C = \frac{6}{\pi^2} \frac{\lambda N_c}{108\pi^3} (\rho_{baryon} M_{KK})^2 \simeq 0.18 N_c \,. \tag{24}
$$

For bottom-up, C can be fixed by the anomalous magnetic moment.

- The axial coupling for the SS model with $\lambda N_c = 50$

$$g_A \approx 1.30 - 1.31, \quad g_A^{\rm exp} = 1.2670 \pm 0.0035 \tag{25}$$

- **The ρNN and ωNN coupling constants:**

1. In the large λ limit

$$|g_{\omega^{(k)}NN}| \simeq N_c \times |g_{\rho^{(k)}NN}| \tag{26}$$

2. For $\lambda N_c = 50$ in the SS model the couplings get corrections from the subleading Pauli term

$$g_{\rho NN} \approx 3.6, \quad g_{\omega NN} \approx 12.6 \tag{27}$$

Thus the relation (26) is modified to

$$\mathcal{R} \equiv \frac{g_{\omega NN}}{3 g_{\rho NN}} \approx 1.2 \tag{28}$$

$$g_{\rho NN}^{\rm emp} \approx 4.2 - 6.5, \;\; \mathcal{R} \approx 1.1 - 1.5. \tag{29}$$

- Magnetic moments:

$$\frac{\mu^{an}_{proton}}{e_{EM}} = \frac{0.18 N_c}{M_{KK}}, \qquad \frac{\mu^{an}_{neutron}}{e_{EM}} = -\frac{0.18 N_c}{M_{KK}}. \tag{30}$$

- With the shift $N_c \to N_c + 2$ and $m_B \simeq M_{KK}$

$$\mu_p = 1 + 1.08 \left(\frac{N_c + 2}{3} \right) \simeq 2.8, \; \mu_n = -1.08 \left(\frac{N_c + 2}{3} \right) \simeq -1.8$$

The experimental values, $\mu_p = 2.79 \mu_N$ and $\mu_n = -1.91 \mu_N$.

- Using AdS/CFT correspondence we compute the form factors,

$$\langle p'|\, J^\mu(x)\,|p\rangle = e^{iqx}\,\bar{u}(p')\,\mathcal{O}^\mu(p,p')\,u(p), \quad q = p' - p \qquad (31)$$

$$\mathcal{O}^\mu = \gamma^\mu \left[\frac{1}{2}F_1^S(q^2) + F_1^a(q^2)\tau^a\right] + \frac{\gamma^{\mu\nu}}{2m_B}q_\nu \left[F_2^S(q^2) + F_2^a(q^2)\tau^a\right] ,$$

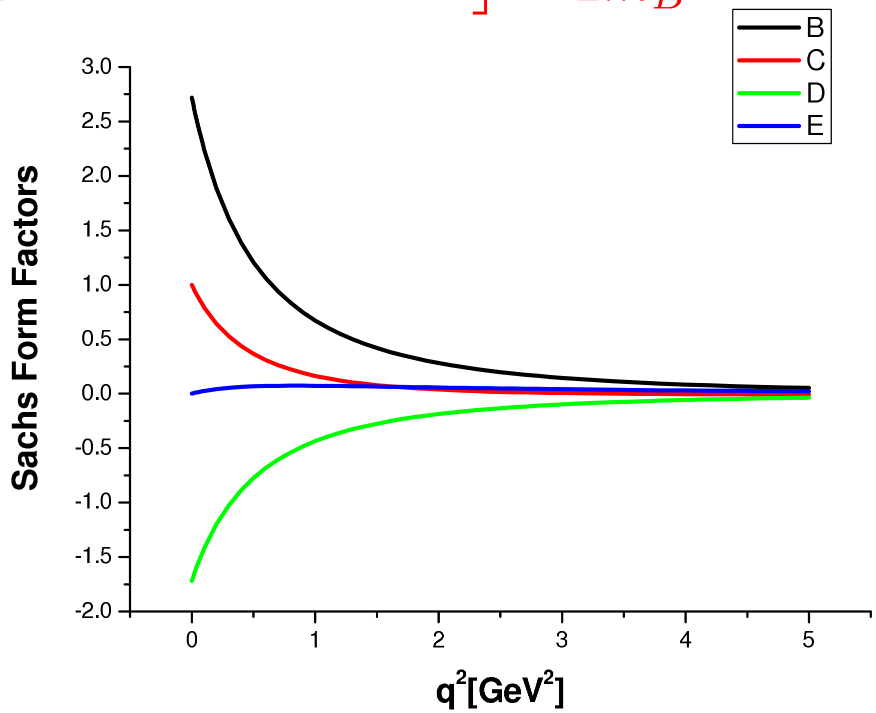
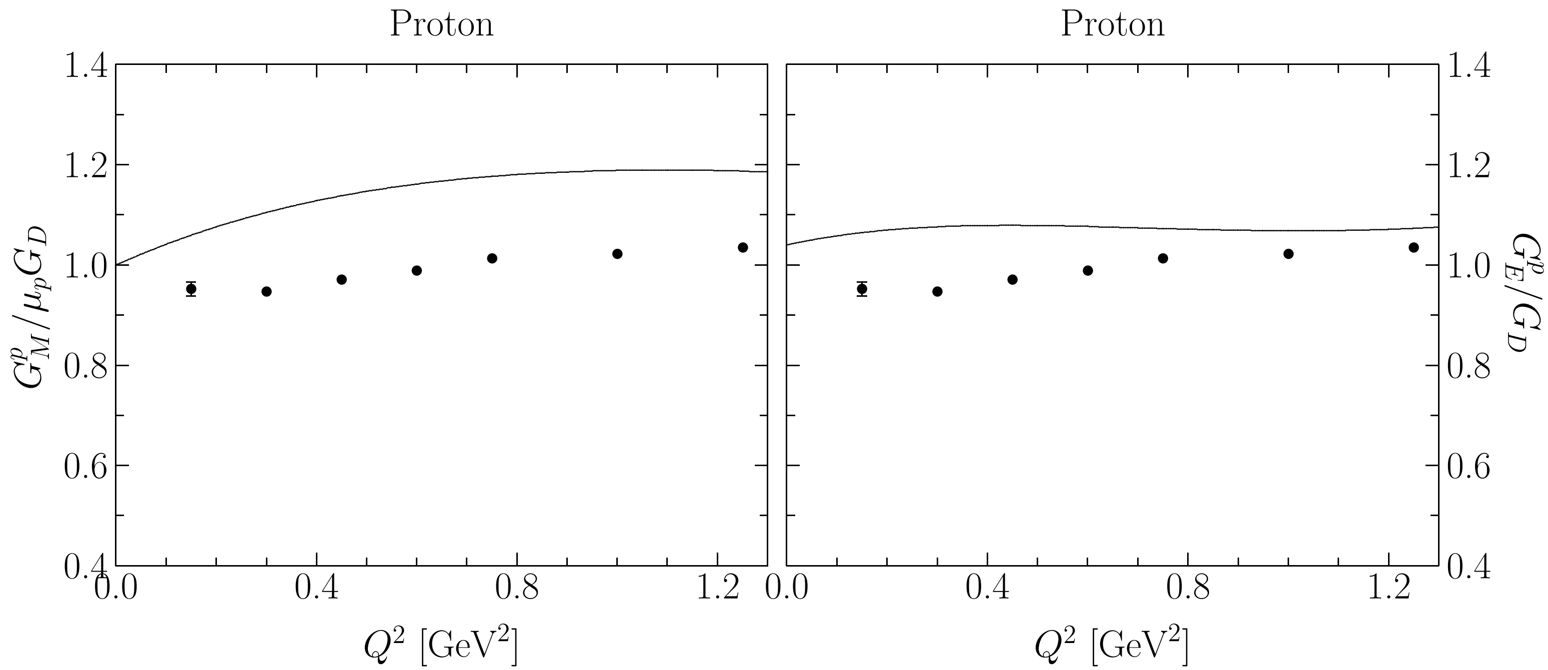


Figure 1: The Sachs form factors: B=G_M^p, C=G_E^p, D=G_M^n, and E=G_E^n

- To see how well our form factors fit the experimental data (R. C. Walker *et al.* (1994) and Jones et al (2000)), we plot the ratio with the dipole form factors, $G_D = 1/(1+q^2/0.71)^2$ and the ratio of the magnetic and electric form factors: (Rho+Yee+Yi+DKH, arXiv:0710.4615)



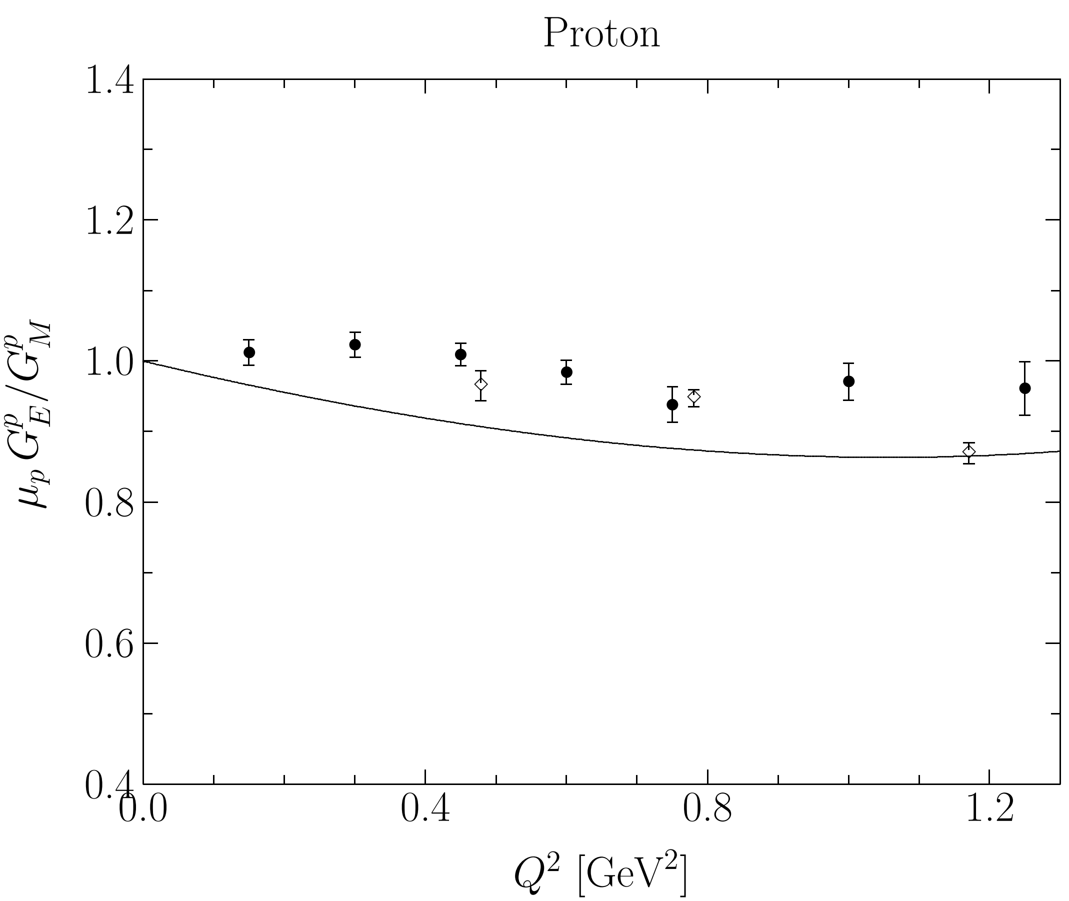


Figure 2: The open cicles are the polarization measurements at JLab and the filled circles are the data taken from Walker et al. (1994). The solid line is the prediction in the SS model.

New Vector Meson Dominance

- If we expand the nonnormalizable photon field in terms of the normalizable vector meson ψ_{2k+1} of mass m_{2k+1} as

$$A(q,w) = \sum_k \frac{g_{v^{(k)}} \psi_{(2k+1)}(w)}{Q^2 + m_{2k+1}^2} \,, \tag{32}$$

- EM form factors then take the form

$$F_1(Q^2) \; = \; \sum_{k=1}^{\infty} \left(g_{V,min}^{(k)} Q_{\rm em} + \frac{5}{3} g_{V,mag}^{(k)} \, \tau^3 \right) \frac{\zeta_k m_{2k+1}^2}{Q^2 + m_{2k+1}^2} \,, \tag{33}$$

$$F_2(Q^2) \; = \; \sum_{k=1}^{\infty} \frac{g_2^{(k)} \zeta_k m_{2k+1}^2}{Q^2 + m_{2k+1}^2} \,, \tag{34}$$

where

$$g_{V,min}^{(k)} = \int_{-w_{max}}^{w_{max}} dw\, |f_L(w)|^2\, \psi_{(2k+1)}(w) \tag{35}$$

$$g_{V,mag}^{(k)} = 2\int_{-w_{max}}^{w_{max}} dw\, \kappa(w)\, |f_L(w)|^2\, \partial_w \psi_{(2k+1)}(w)\,,$$

$$g_2^{(k)} = 4m_N \int_{-w_{max}}^{w_{max}} dw\, \kappa(w) f_L^*(w) f_R(w) \psi_{(2k+1)}(w)\,. \tag{36}$$

- The charge and magnetic moment sum rules are saturated for protons (and similarly for neutrons) within a few %:

$$F_1^p(0) \equiv 1 \simeq \sum_{k=1}^{4} \left(g_{V,min}^{(k)} + \frac{5}{6} g_{V,mag}^{(k)} \right) \zeta_k = 1.04,$$

$$F_2^p(0) \equiv \mu_p - 1 \simeq \frac{5}{6} \sum_{k=1}^{4} g_2^{(k)} \zeta_k = 1.66\,. \tag{37}$$

V. Nucleon Electric Dipole Moments

arXiv:0709.0314 with H.-U. Yee, H. Kim, S. Siwach

- The physical strong CP-violation parameter

$$\bar{\theta} = \theta + \mathrm{Arg}\,\mathrm{Det} M_q \ . \tag{38}$$

- The electric dipole moment of neutron

$$d_n = \left(\frac{\mathrm{const.}}{m_N}\right)\left(\frac{m_q\,\bar{\theta}}{m_N}\right) < 2.9 \times 10^{-26}\, e \cdot \mathrm{cm} \tag{39}$$

- Since in the SS model $m_q = 0$, we consider the bottom-up approach. (cf. Aharony+Kutasov '08; Hashimoto+Hirayama+Lin+Yee '08)

Hong-Inami-Yee model (PLB 2006):

- We need to introduce two bulk spinors:

$$S_{\rm kin} = \int_{z,x} \left[i\bar{N}_1 \Gamma^M D_M N_1 + i\bar{N}_2 \Gamma^M D_M N_2 - \frac{5}{2}\bar{N}_1 N_1 + \frac{5}{2}\bar{N}_2 N_2 \right] ,$$

$$S_m = \int_{z,x} \left[-g\bar{N}_1 X N_2 - g\bar{N}_2 X^\dagger N_1 \right] ,$$

- 5d bulk metric and the covaraint derivative:

$$ds^2 = \frac{1}{z^2}\left(-dz^2 + \eta^{\mu\nu} dx_\mu dx_\nu \right) \quad \epsilon \leq z \leq z_m , \qquad (40)$$

$$D_M = \partial_M + \frac{i}{4}\omega_M^{AB}\Gamma_{AB} - i(A^a)_M t^a , \qquad (41)$$

- For two flavors, anomaly matching requires baryons to be massless when chiral symmetry is unbroken.

- Mass should come from the Yukawa coupling for spin 1/2:

$$\mathcal{L}_{\rm int} \ni -g\left(\bar{N}_2 X N_1 + {\rm h.c.}\right) \ , \tag{42}$$

where $X = \frac{1}{2} m\, z + \frac{1}{2}\sigma\, z^3$, $(\sigma = \langle \bar{q}\, q\rangle)$.

- To allow a left-handed zero mode for N_1, we impose

$$\lim_{\epsilon\to 0} N_{1L}(\epsilon) = 0 \ ({\rm normalizability}) \quad \text{and} \quad N_{1R}(z_m) = 0. \tag{43}$$

- The remaining boundary conditions for $N_{1L}(z_m)$ and $N_{1R}(\epsilon)$ are then determined by the equations of motion.

- The boundary conditions for N_2 are similar except for interchanging $L \leftrightarrow R$.

- We now Fourier-transform the bulk spinor as

$$f_{1L,R}(p,z)\,\psi_{1L,R}(p) = \int \mathrm{d}^4x\, N_{1L,R}(x,z)e^{ip\cdot x}\,, \tag{44}$$

where the 4D spinors satisfy with $\psi_{1L} = \gamma^5\psi_{1L}$ and $\psi_{1R} = -\gamma^5\psi_{1R}$

$$\not{p}\,\psi_{1L,R}(p) = |p|\,\psi_{1R,L}(p) \tag{45}$$

- The K-K mode equations become with $M = 0$ and $\Delta = 9/2$

$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{1}{2}g\sigma z^2 \\ -\frac{1}{2}g\sigma z^2 & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} = -|p| \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix},$$

$$\begin{pmatrix} \partial_z - \frac{4-\Delta}{z} & \frac{1}{2}g\sigma z^2 \\ \frac{1}{2}g\sigma z^2 & \partial_z - \frac{\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} = |p| \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix},$$

- We find the nucleon spectra:

$z_m(\mathrm{GeV}^{-1})$	g	(p,n)(GeV)	N(1440)	N(1535)	3rd	4th
$(0.33)^{-1*}$	8.67	0.94*	2.14	2.24	3.25	3.30
$(0.205)^{-1}$	14.4	0.94*	1.44*	1.50	2.08	2.12

Table 1: Numerical result for spin-$\frac{1}{2}$ baryon spectrum. $*$ indicates an input and we used $\sigma = \frac{\sqrt{2}\xi}{g_5\, z_m^3}$ with $3.4 \leq \xi \leq 4$.

- To give correct anomalous magnetic moments, we have to introduce

$$S_{\rm dipole} = iD\int_{x,z}\left[\bar{N}_1\Gamma^{MN}(F_L)_{MN}N_1 - \bar{N}_2\Gamma^{MN}(F_R)_{MN}N_2\right]\ ,$$

- The Pauli term is parity-invariant and should not have $U(1)$ part!

- The above Pauli term also gives NEDM, since they are related by $U(1)_A$ rotation.

- In HIY model of holographic baryons, we know how much the baryon wavefunction rotates in the presence of $\bar{\theta}$:

$$\frac{d_e}{\mu_m^{\rm ano}} = -\frac{g}{4}\frac{m}{m_N}\bar{\theta}\,, \qquad (46)$$

- The bifundamental bulk scalar X has a solution when $\bar{\theta}$ is nonzero but small

$$\langle X(z)\rangle = \left[\frac{1}{2}(mz+\sigma z^3) + \frac{i}{4}m\bar{\theta}\, z\right]\mathbf{1} \tag{47}$$

- The first order in $\bar{\theta}$ effect of $i\delta v(z)$ on the wave functions is a simple phase factor in the eigenfunctions

$$\begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} = e^{i\alpha}\begin{pmatrix} f_{1L}^{(0)} \\ f_{2L}^{(0)} \end{pmatrix}, \quad \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} = e^{i\beta}\begin{pmatrix} f_{1R}^{(0)} \\ f_{2R}^{(0)} \end{pmatrix}, \tag{48}$$

with

$$(\alpha-\beta) = \frac{g}{m_N}\left(\frac{\delta v(z)}{z}\right) = \frac{g}{4}\frac{m}{m_N}\bar{\theta} \quad , \tag{49}$$

- We find the NEDM with $g = 9.18$ from HIY model

$$d_e \;=\; -1.8\mu_N \cdot \frac{g}{4}\frac{m}{m_N}\bar{\theta} = -1.08\times 10^{-16}\;\bar{\theta} \quad (e\cdot \mathrm{cm})\,. \tag{50}$$

Pion-Nucleon couplings

- Pion fields are a linear combination of 5th component of $A \equiv \frac{1}{2}(A_L - A_R)$ and the phase of $X = \langle X \rangle\, e^{-Q+iP} = [v_0 + i\delta v]e^{-Q+iP}$.
- After getting the pion profile and KK-reduction, we get

$$\bar{g}_{\pi NN} = -(\alpha-\beta)\int_0^{z_m} dz \left[\frac{A(z)}{z^4} f_{1L}^{(0)} f_{2L}^{(0)} + \frac{g v_0 p(z)}{2z^5}\left[(f_{1L}^{(0)})^2 + (f_{2L}^{(0)})^2 \right]\right]$$
$$+\ \frac{g}{2}\int_0^{z_m} dz \left[\frac{1}{z^5}\Big(v_0(z) q(z) + \delta v(z) p(z) \Big)\left[(f_{1L}^{(0)})^2 - (f_{2L}^{(0)})^2 \right]\right],$$

where $(\alpha-\beta) = \frac{g}{4}\frac{m}{m_N}\bar{\theta}$ and $\delta v(z) = \frac{m\bar{\theta}}{4} z$.

- For the model of $z_m = (330\,\mathrm{MeV})^{-1}$ and $v_0(z) = \frac{1}{2}(mz + \sigma z^3)$ with $m = 2.34$ MeV, $\sigma = (311\,\mathrm{MeV})^3$, we have $g = 9.18$ and

$$\bar{g}_{\pi NN} \;=\; +0.017\,\bar{\theta}\,. \tag{51}$$

VI. Conclusion and Outlook

- Baryons are realized as 4D instanton solitions in holographic QCD, which are made of pions and whole towers of vector mesons.
- The effective chiral Lagrangian for baryons is uniquely determined up to the Pauli term.
- New VMD is a key feature of holgraphic QCD: Form factors, $\cdots$.
- As a model to QCD, holographic QCD is as good as other models, $\sim 1/N_c$.
- Since it is based on principle, it gives relations to low energy parameters of hadrons. Low energy parameters of hadrons are unified into a few parameters in 5D:
  1. Mass spectrum.
  2. Magnetic moments of baryons and g_A: $g_A \sim \mu_{\rm an}$
  3. Various couplings with vector mesons: $g_{\omega NN} \approx N_c\, g_{\rho NN}, \cdots$.

- Furthermore, it has model-independent predictions, insensitive to $1/N_c$ corr: New sum rules due to the instanton nature of baryons:

$$\mu^p_{\rm an} + \mu^n_{\rm an} = 0, \quad d_n + d_p = 0. \tag{52}$$

- We estimate the CP-violating effects in QCD in Hong-Inami-Yee model (with $g = 9.18$), giving more stringent bound on $|\bar{\theta}| < 3 \times 10^{-10}$:
  1. Nucleon electric dipole moments

$$d_e = -\mu^p_{\rm an} \cdot \frac{g}{4}\frac{m}{m_N}\bar{\theta} = -1.08 \times 10^{-16}\ \bar{\theta} \quad (e \cdot {\rm cm})\,. \tag{53}$$

  2. CP-violating nucleon-pion coupling

$$\bar{g}_{\pi NN} = +0.017\ \bar{\theta}\,. \tag{54}$$

- In hQCD we have a unified and new formula for baryon number violation in the presence of magnetic monopole:

$$\partial_\mu B^\mu = -\frac{i\delta^{(3)}(\vec{x})}{2\pi}{\rm Tr}\left[QU^{-1}\partial_t U + QU^{-1}A_{Lt}U - QA_{Rt}\right]\,. \tag{55}$$