Upper and Lower “Bounds” for Pairing Gap at Unitarity

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Lattice Methods: K. Schmidt (ASU) & Shiwei Zhang (W&M)
Original work w/ K. Schmidt (ASU), V. Pandharipande, S.Y. Chang (Ill)

Simple (Universal) Interaction
Highly Tunable
Fundamental Studies of strongly-paired systems (nuclei and QCD)
`Benchmark’ for Strongly-Coupled Fermions

\[ \mathcal{H} = \sum_{k=1}^{A} \left( -\frac{\hbar^2}{2m_k} \nabla_k^2 \right) + \sum_{i<j} v(r_{ij}) \]

<table>
<thead>
<tr>
<th>Cold Fermi Atoms</th>
<th>Neutrons</th>
</tr>
</thead>
<tbody>
<tr>
<td>scattering length</td>
<td>tunable</td>
</tr>
<tr>
<td>effective range</td>
<td>0</td>
</tr>
</tbody>
</table>
Rich Set of Experimental Results

Radial Density

Polarization

Vortices

RF response
Neutron Matter

Neutron-Neutron interaction - dominantly s-wave (spin 0) at low energy
Large scattering length ~ -18 fm
Modest effective range ~ 2.7 fm

Zero Temperature Equation of State Difficult to get wrong --- at low density
Even if no new phases, parameters including Superfluid gap $\Delta$ are important

Superfluid gap for low-density neutron matter affects cooling

Benchmark for pairing in the strong-coupling QCD

QCD at high densities

Neutron star cooling curves
Superfluid (Pairing) Gap

Pairing Gap (apparently) difficult to get right!
Situation now worse than shown

Dean and Hjorth-Jenson
RMP (2003)
Cold (T=0) Fermions vs. Polarization

Zero Temperature

Boson-Fermion Mixture

Polarization

Coupling

Isolated Fermi Surfaces

BCS

Weak Interactions

$\alpha < 0$
Method I: Diffusion (Green’s function) Monte Carlo

Fixed Node - Variational Upper Bound

Vary parameters in nodal surfaces ~ different ‘phases’ (superfluid or normal)

Transient Estimation

Comparisons to Lattice Methods at Equal Populations

\[ \psi(\tau \to \infty) = \lim_{\tau \to \infty} e^{-(\mathcal{H} - E_T)\tau} \psi_V \]

Variational wavefunction

\[ \psi_V(R) = \prod_{i,j'} f(r_{ij'}) \Phi_{BCS}(R) \]
66 particles
Caution: states are metastable for potential range > 0
Transient Estimation
Releasing fixed-node constraint

$E / E_{FG}$

$\tau N E_{FG}$

$a/r0 = 1/6$
Lattice Methods

Auxiliary Field QMC - evolve single particle orbitals
(Hirsch, Scalapino, Koonin, ...)

Continuum Limit = Limit of large # particles and dilute system

Fixed Particle number
BCS-like trial state used for importance sampling

Largest system: 38 particles on a 20x20x20 lattice: 0.25% filling

Exact (no sign problem) for zero polarization
Lattice Results at Unitarity

Unitarity Limit

- $N = 14$, $L = \text{odd}$
- $N = 14$, $L = \text{even}$
- $N = 38$
### Measurements and EOS at $a = \infty$

<table>
<thead>
<tr>
<th>Value</th>
<th>Reference</th>
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<tbody>
<tr>
<td>0.51 (4)</td>
<td>Kinast, et al., Science (2005)</td>
</tr>
<tr>
<td>0.32 (+.13,-.1)</td>
<td>Bartenstein, et al., PRL (2004)</td>
</tr>
<tr>
<td>0.36(15)</td>
<td>Bourdel, et al., PRL (2004)</td>
</tr>
<tr>
<td>0.46(5)</td>
<td>Partridge, et al., PRL (2004)</td>
</tr>
<tr>
<td>0.45(5)</td>
<td>Stewart, et al., PRL (2006)</td>
</tr>
<tr>
<td>0.41(15)</td>
<td>Tarruell, et al., cond-mat/0701181</td>
</tr>
</tbody>
</table>

![Graphs](image-url)
Momentum Distribution

Pair Distributions

At unitarity
Very different from
Fermi Liquid

Strongly
Peaked
Pair distribution
At Unitarity, expect large gap for unpaired particles
Excitation Energy/ $E_{(FG)}$ vs. $k$

In weak coupling (BCS): minimum near $k_f$
In strong coupling (BEC), minimum at $k=0$
If background (unpolarized) superfluid is correct, get upper ‘bound’ for gap

JC & Sanjay Reddy
PRL 95, 060401 (2005)
Polarized Systems

Up to $\sim$20% polarization, quasi-particles are nearly non-interacting
EOS vs. Polarization at Unitarity

At Unitarity, expect large gap for unpaired particles
Large Polarization

\[ E_{N+1}(k) - \frac{3}{5} \frac{k_f^2}{(2m)} = \eta(k/k_f) \frac{k_f^2}{(2m)} \]

1 down spin in a sea of up spins

QMC calculation
\[ \eta(0) = -0.60(03) \]
Normal Phase at high Polarization

Checked for p-wave superfluidity,
Superfluid generalized from Fermi Gas,
Neither clearly preferred

See results by
Lobo, Recati, Giorgini, Stringari
PRL 2006
Polarization vs. Radius: MIT data

MIT data $P=0.41$

unpolarized superfluid

$N^\uparrow - N^\downarrow$

Fully polarized Normal state
At $T = 0$, assume 1st order phase transition at a local polarization of $\sim 45\%$

Calculated gap $\approx 0.5 \ (0.05) \ Ef$

If experiments say there is no polarization in the superfluid at $T=0$:

Equilibrium (chemical potentials, pressure) implies gap $> 0.40(0.02) \ Ef$

Very close to Sarma phase at unitarity and $T=0$
MIT Data ($P = 0.41$)

Thermally populated particles in superfluid
Is this consistent w/ RF response?
measurement of 0.2 EF claimed

Tune RF to specific transition: flip a minority spin to a 3rd (strongly-interacting) state – zero momentum transfer

Decreasing Temperature

Decreasing Polarization
Entire Response Difficult to Calculate:  
2 Simple Quantities: Sum Rule and ‘Threshold’

Sum Rule = \( \langle V_{13} \rangle - \langle V_{12} \rangle \) goes to zero as \( a_{13} \Rightarrow a_{12} \)

Threshold = \( \text{BE} (a_{13}) - \text{BE} (a_{12}) \) for normal

Width decreases as \( v_{13} \) becomes similar to \( v_{12} \)
Sum Rule decreases also

Roughly consistent w/ experiment
Conclusions / Future Directions

After a few years, we know the pairing gap at Unitarity much better than we know the neutron superfluid gap

\( \Delta / EF = 0.5 \) (0.1)

Fully Polarized state cannot exist in the bulk at finite polarization

Even small temperatures will polarize the superfluid state near the transition, but not in the trap center

Experiment:

Experiments which measure both \( n, n^{↑} - n^{↓} \) vs. \( r \)

for different Geometries, Polarizations and Temperatures

Theory

Calculations in different geometries
More accurate calculations of Gap and dispersion
Calculations of different possible phases