Stability of the Freeze-Out Process in Quark-Gluon Plasma Evolution

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Outline

- Stability, viscosity and Navier-Stokes equation
- Space-Time geometry of the Freeze-Out (FO)
- Kinetic Freeze-Out pictures: gradual and discontinuous
- Summary, questions and future plans
Stability of Fluid Dynamics

From Boltzmann transport equation, by using Chapman-Enskog method we get solution:

- **0th order:** **Euler equation.** Perfect fluid. In dimensionless form, reads as:
  \[
  \frac{\partial \tilde{u}}{\partial \tilde{t}} + S(\tilde{u} \tilde{\nabla})\tilde{u} = -S \tilde{c}^2 \frac{\tilde{\nabla} \tilde{\rho}}{\tilde{\rho}}
  \]
  Any perturbation to the equation leads to turbulence.

- **1st order:** **Navier-Stokes equation.** Includes viscosity, but has problem of causality.
  \[
  \frac{\partial \tilde{u}}{\partial \tilde{t}} + S(\tilde{u} \tilde{\nabla})\tilde{u} = -S \tilde{c}^2 \frac{\tilde{\nabla} \tilde{\rho}}{\tilde{\rho}} - \frac{S}{Re} \left[ \tilde{\nabla} \tilde{u} + (q + 1/3) \tilde{\nabla} (\tilde{\nabla} \tilde{u}) \right]
  \]
  Where \( S \) is Strouhal number, describing oscillating flow mechanisms.
  \( Re \) is the Raynolds number - the measure of stability.
Reynold’s number

\[ Re = \frac{l_1 u_1}{\nu} \]

\[ \nu = \frac{\eta}{\rho} \] - kinematic viscosity:

\( \rho \) - density:

\( \eta \) - shear viscosity:

\( l_1 \) - length

\( u_1 \) - speed

A criterion of fluid flow: is it absolutely \textbf{steady} or on the average steady with small unsteady fluctuations (turbulent).

Solving Euler eq. \textbf{numerically}, solution is influenced by \textbf{numerical viscosity}.

Whenever the Reynolds number is less than about 2,000, flow in a pipe is generally laminar, whereas, at values greater than 2,000, flow is turbulent.
Phase Transition of the water

L.P.Csernai, J.I.Kapusta, L.McLerran, nucl-th/0604032
Viscosity and phase transition

\[ \frac{\eta}{s} = \frac{15 f^4}{16\pi T^4} \]

No realistic model to describe that region

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Most interesting region
Evolution

Two nuclei collide semi-transparently
System thermalizes
System is too dilute to be in equilibrium
Detected particle spectra

0.2-1fm/c
Evolution

Two nuclei collide semi-transparently → System thermalizes → System is too dilute to be in equilibrium → Detected particle spectra

Initial stage | Fluid dynamical evolution | Freeze-Out

0.2-1fm/c

Particle emission

Pressure gradients develop via expansion → Phase transition, hadronization → Resonance decays

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Multi Module Model

• Initial State – phenomenological approach for pre-equilibrium
• Computational Fluid Dynamics: Hydrodynamical evolution
• Freeze-Out (FO) process.
Initial state

Initial state by V. Magas, L.P. Csernai and D. Strottman
Phys. Rev. C64 (01) 014901

NexSpherio by F. Grassi, Y. Hama, T. Kodama, B. Tavares
Relativistic Fluid Dynamics

From kinetic theory. Boltzmann Transport Equation for the evolution of phase-space distribution:

\[ P_k^{\mu} f_{k,\mu} = \sum_{l=1}^{N} C_{kl}(x, p_k) \]

Then using microscopic conservation laws in the collision integral \( C \):

\[ T^{\mu\nu}_{,\mu} = \sum_{k=1}^{N} T^{\mu\nu}_{k,\mu} = 0. \quad N^\mu_{,\mu} = \sum_{k=1}^{N} N^\mu_{k,\mu} = 0. \]

These conservation laws are valid for any, eq. or non-eq. distribution, \( f(x,p) \). These cannot be solved, more information is needed!

Boltzmann H-theorem: (i) for arbitrary \( f \), the entropy increases,

(ii) for stationary, eq. solution the entropy is maximal, \( \Rightarrow \exists \ EoS \)

\[ P = P(e,n) \]

Solvable for local equilibrium!
Computational Fluid Dynamics

• Viscosity \textit{exists} in the CFD, because of the mesh - local equilibrium is assumed in every cell.

• Fluid Dynamics does not work for systems away from local \textit{equilibrium}!
• FD is applicable at the middle stages of a heavy ion reaction.
3-Dim Hydro code - PIC
Hydro evolution and FO surface

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Gradual Freeze Out in a layer

‘Pre’: All particles interact in the thermal equilibrium. FD is applicable.

‘Post’: Fully frozen out matter. Particles move freely towards detectors.

\[ N^\mu = \int \frac{d^3p}{p^0} p^\mu f(x, p) \]

\[ N^\mu(x) = N^{\mu}_i(x) + N^{\mu}_f(x) , \]
\[ \partial_\mu N^\mu(x) = 0 , \]
\[ \partial_\mu N^{\mu}_i(x) = -\partial_\mu N^{\mu}_f(x) \]

Fig.1 Schematic view of the FO layer, described by two 3-dim hypersurfaces in space-time.

\[ N^\mu_i |_{S_1} = N^\mu \]
\[ N^\mu_i |_{S_2} = 0 \]
Reference frames

\[ d\sigma_\mu = \gamma_0 (\nu_\sigma, 1, 0, 0)_{\text{RFG}} \]

\[ d\sigma_\mu = (0, 1, 0, 0)_{\text{RFF}} \]
Boltzmann Transport Equation

Assumptions of the BTE:

1. Only binary collisions.

2. Molecular chaos: number of collisions at $x$ is proportional to $f(x,p_1) \times f(x,p_2)$.

3. $f(x,p)$ is a smoothly varying function compared to the mean free path.

$$p^\mu \partial_\mu f(p) = \frac{1}{2} \int 12D_4 f(p_1)f(p_2)W_{p_1p_2}^{pp_4} - \frac{1}{2} \int 2D_{34} f(p)f(p_4)W_{pp_2}^{p_3p_4}$$

Where: $12D_3 \equiv \frac{d^3p_1}{p_1^0} \frac{d^3p_2}{p_2^0} \frac{d^3p_3}{p_3^0}$
Loss and Gain terms and FO probability

\[ f(x, p) = f^i(x, p) + f^f(x, p) \]

\[
p^\mu \partial_\mu f^f = \frac{1}{2} \int 12 \mathcal{D} q_1 f_1^i f_2^i \mathcal{P}_f W_{12}^{p^4} \quad \text{No Loss component in } f^f
\]

\[
p^\mu \partial_\mu f^i = -\frac{1}{2} \int 12 \mathcal{D} q_1 f_1^i f_2^i \mathcal{P}_f W_{12}^{p^4} + p^0 \frac{f_{eq}^i - f^i}{\tau_{\text{rel}}}
\]

- FO of the particles from interacting component
- Redistributing particles in momentum space

Collisions can be following:

\[ f_1^i \rightarrow f^f \]
\[ f_2^i \rightarrow f^f \]
\[ f_4^i \rightarrow f^f \]
\[ f_4^f \rightarrow f^f \]
Can BTE handle FO process in the layer?

Density gradient of interacting particles is high in the normal direction, thus:

1. “Molecular chaos” assumption is **not** valid and the number of collisions is **not** proportional to \( f(x, p_1) \times f(x, p_2) \)
   It must be **delocalized**! i.e.
   proportional to \( f(x_1, p_1) \times f(x_2, p_2) \)

2. M.f.p. is infinite on the \( S_1 \) hypersurface – all particles are frozen.

**Assumptions in BTE must be improved!**

Fig.2. FO layer in \( xy \) plane. \( L \) is length of the layer, \( d\sigma_\mu \) is the direction of density gradient.
Space-Time change in FO

We use Enskog-Chapman method:

\[ f_k(x_k, p_k) = f_k^{(0)}(x, p_k) [1 + \phi(x_k, p_k)] \]

\[ f_k^{(0)}(x, p_k) \] is locally thermalized distribution

where

\[ \phi_k(x_k, p_k) \approx \phi_k(x, p_k) \]

\[ + (x_k^\mu - x^\mu) \partial_\mu \phi_k(x, p_k) \]

\[ + c_p \phi p_k^\mu d\sigma_\mu \]

Describes the gradual decrease of interacting density

Asymmetry of the momentum distribution due to larger momentum flux from the pre-FO direction.

Particles with momentum \( p_k^\mu \) propagates from \( x_k^\mu \) to \( x^\mu \), so that

\( (x_k^\mu - x^\mu) = -c_x \phi p_k^\mu \), then

\[ \phi_k(x_k, p_k) \approx \phi_k(x, p_k) + c_\phi p_k^\mu d\sigma_\mu \]
Covariant Escape Probability

\[ d\sigma^\mu \partial_\mu f^f(x,p) = f^i(\tilde{x}_1,p) \, P_{esc}^* \]

\[ P_{esc}^* = \frac{1}{\lambda(\tilde{x}_1)} \left( \frac{L}{L - x^\mu d\sigma_\mu} \right)^a \left( \frac{p^\mu d\sigma_\mu}{p^\mu u_\mu} \right)^a \Theta(p^\mu d\sigma_\mu) \]

\( a \) is influencing the FO profile

\( a<1 \) no complete physical FO

\( a=1 \) complete FO

\( a>1 \) power like complete FO

For simplicity one can use:

\[ \left( \frac{p^\mu d\sigma_\mu}{p^\mu u_\mu} \right) \approx \cos \Theta_p \]

\( P_{esc}^* \) is Lorentz invariant and works for space-like and for time-like hypersurface!
Escape Probability in Momentum Space

Escape probability factors for different points on FO hypersurface in the rest frame of the gas. Momentum values are in units of $[mc]$.

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Change of Temperature

Space-Like
Just the interacting component has temperature

Time-Like

MOLNÁR, CSERNAI, MAGAS, NYÍRI, AND TAMOSIUNAS
PHYSICAL REVIEW C 74, 024907 (2006)
The FO layer might be approximated as a discontinuity
The post FO distribution in the RFF

Time-like FO - equilibrium distribution

Space-like FO - non-equilibrium.

Just particles with higher momentum Freezes Out
Negative contribution in the Cooper-Frye formula

\[ N = \int_S N^\mu d\sigma_\mu \quad N^\mu = \int \frac{d^3 p}{p^0} p^\mu f(x, p; T, n, u') \]

\[ E \frac{dN}{d^3 p} = \int_S f(x, p; T, n, u') p^\mu d\sigma_\mu \]

\( p^\mu d\sigma_\mu \) for space-like hypersurface can be positive or negative

**The way out:** modifying the distribution function

1. Cut-Juttner distribution: \( f(x, p) = f^{\text{Juttner}}(x, p) \Theta(p^\mu d\sigma_\mu) \)


Making The Canceling Juttner distribution

In the Reference Frame of the Front (RFF)

\[ f^\text{cl} = (f^\text{Juttner}_R - f^\text{Juttner}_L) \Theta(p^\mu d\sigma_\mu) = \]

\[ = \frac{1}{(2\pi\hbar)^3} \left( \exp \frac{\mu - p^\mu u^R_\mu}{T} - \exp \frac{\mu - p^\mu u^L_\mu}{T} \right) \Theta(p^\mu d\sigma_\mu) \]

where \( u^R_\mu = (\gamma, \gamma v, 0, 0) \) and \( u^L_\mu = (\gamma, -\gamma v, 0, 0) \).
The CJ Distribution Function With Different Initial Velocities

\[ v_0 = 0.7 \, [1/c], \Lambda_B = 200 \, \text{MeV}, \, T_0 = 60 \, \text{MeV}, \, n_0 = 1 \, [\text{fm}^{-3}] \]

\[ v_0 = 0.85 \, [1/c], \Lambda_B = 200 \, \text{MeV}, \, T_0 = 60 \, \text{MeV}, \, n_0 = 1 \, [\text{fm}^{-3}] \]

Distribution curves are in arbitrary units ( \( T = 1, \, m = 1 \) )
Summary and Questions:

• Kinetic description provides deeper understanding of the non-equilibrium Freeze-Out process.

• Cancelling Juttner distribution resembles disribution obtained by kinetic description.

• The signature of turbulent FO might be seen in fluctuations of momentum distribution.

• Physical “numbers” (macroscopic parameters) are real just in case of equilibrium, for non-equilibrium they are parameters of distribution function.

• HowtTo connect experimentally measurable $v_2$ with with transport coefficients of Navier-Stokes equation (Re)?
Freeze out influence for $v_2$

$p_t$ from hydro for different particles underestimated with right chemistry.

$v_2$ of $\Omega$ is suppressed at high $p_t$. Nu Xu

$v_2$ is building up in this region. Particles with high enough momentum can escape.