Understanding magnetic instability in gapless superconductors: loss of phase coherence driven by mismatch

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Warning: it is still very preliminary, and might be nonsense (hope not totally).

Based on hep-ph/0509177, not a formal paper.
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Key point: phase fluctuation plays the role of quantum disordering the superconducting phase when mismatch is large.
I. A brief review on instabilities in gapless SCs

Pairing with mismatch

Gapless quasi-particles

\( \delta \mu > \Delta \)

\( g_{2SC} \): Shovkovy, MH, PLB564:205,2003

\( g_{CFL} \): Alford, Kouvaris, Rajagopal, PRL92:222001,2004


Rajagopal's talk

Liu's talk
(Chromo)Magnetic instability

\[ \psi^2/\xi^2 \]

\[ g_{2SC} \]

\[ g_{CFL} \]

MH, I. Shovkovy, PRD70:051501, 2004; 094030, 2004

K. Fukushima, hep-ph/0506080

Resolving magnetic instability

\[ < \psi(\vec{r}) \gamma_5 \lambda_2 \tau_2 \psi_C(\vec{r}) > = \Phi e^{2i\vec{q}\cdot\vec{r}} \quad (2SC) \]

Giannakis, Ren, PLB611:137-146, 2005; NPB723:255-280, 2005

\[ < \psi_{\alpha}\gamma_5 \psi_{\beta j} > = \sum_{l=1}^{3} \Delta_l(t) e^{\alpha M_{Lij}} \quad (CFL) \]

Casalbuoni, Gatto, Ippolito, Nardulli, Ruggieri, hep-ph/0507247

Nardulli’s talk

More discussion on LOFF, see Yang’s talk.
Many ways go to the LOFF-like state

2. Goldstone current: Hong’s talk, or hep-ph/0506097
Kryjevski, hep-ph/0508180; Schaefer’s talk, or hep-ph/0508190
4. More ...

1 and 2 offer a Doppler shift superfluid velocity for the quasi-particles.

What’s really happening? Rischke’s remarks
phase decoherence (this talk)

Previous treatment for g2SC, gCFL and BP:

At fixed mismatch, looking for the possibility of BCS Cooper pairing. This is a story of balance between energy gain and loss, e.g., Liu’s talk.

Another way of thinking:

Starting from conventional BCS state without mismatch, asking how this BCS state will be eventually destroyed by increasing mismatch.
II. How a superconductor will be destroyed?

Firstly, what is a superconductor?

\[ \Delta(x) = |\Delta| e^{i\varphi(x)} \]

| Thermodynamic variable \[ |\Delta| \] | Dynamic variable \[ \frac{1}{2} \rho_s (\nabla \varphi - e A)^2 \] |
| --- | --- |
| \( T_c \) | stiffness |

Two energy scales

\[ |\Delta| \rightarrow E^{BCS} \]
\[ \rho_s \rightarrow E^{phase} \]

The energy scale for pairing established
The energy scale for phase coherence established

The system is governed by the lower energy scale

\[ E = \min \{ E^{BCS}, E^{phase} \} \]

e.g. if stiffness is small (soft) but gap magnitude is large, the system is governed by phase fluctuation.

**BCS Superconducting phase:**

\[ \langle \Delta \rangle \neq 0 \]

\[ E^{BCS} < E^{phase} \]

Strongly coherent, ordered, rigid (large superfluid density), phase fluctuation is absent, BCS MF good

**Pseudogap phase:** long-range phase decoherence

\[ |\Delta| \neq 0, \langle \Delta \rangle = |\Delta| \langle e^{i\varphi(x)} \rangle = 0 \]

\[ E^{BCS} > E^{phase} \]

“Doping” green fermions

\[ n_g > n_r \]

Loss of long-range phase coherence, quantum disordered, BCS MF cannot describe strong phase fluctuation
Normal phase: \[ |\Delta| = 0 \]

No order at all, very soft (zero superfluid density)

How a superconductor can be destroyed?

1. Drop magnitude to zero, BCS-like
2. Gradually loss phase coherence, BKT-like

Further drop magnitude to zero?

SC

NM

PG
How a superconductor will be destroyed by mismatch?

1. Definitely non-BCS-like


2. Possibly BKT-like

Berezinskii-Kosterlitz-Thouless

Mismatch Increases, superfluid density decreases, phase fluctuation becomes more important

III. The role of phase fluctuation

1. BCS at mean-field approximation

The minimal model for gapless phase

\[ SU(2)_c \times U(1)_{EM} \times SU(2)_f \]

\[ L_q = \bar{q} (i \not{\partial} + \hat{\mu} \gamma_0) q + G \Delta \left[ (\bar{q}^C i \varepsilon \gamma_5 q)(\bar{q} i \varepsilon \gamma_5 q^C) \right] \]

\[ \mu_u = \bar{\mu} - \delta \mu, \mu_d = \bar{\mu} + \delta \mu \]

\[ D_\mu = \partial_\mu - ie A_\mu \]

\[ q \in u, d \] Original fermions
Introducing auxiliary field, bosonization:

\[ \mathcal{L}_q = \bar{q}(i\not\! \! \! \partial + \hat{\mu}\gamma^0)q - \frac{1}{2} \Delta [\bar{u}q \varepsilon \gamma_5 q^C] - \frac{1}{2} [\bar{u}q^C \varepsilon \gamma_5 q] \Delta^* - \frac{\Delta^2}{4G_{\Delta}} \]

\[ \Delta = |\Delta| e^{i\varphi} \]

BCS MF: neglecting phase fluctuation \[ \Delta = |\Delta| \]

Comments on BCS MF

1. It is fine with small mismatch when the system is rigid;
2. It is not a good approximation for large mismatch when the system is “soft”;
3. In all the papers regarding the instability in gapless or BP phases, phase fluctuation has been totally neglected.

2. Formulating the role of phase fluctuation

A. Longitudinal phase fluctuation

B. Transverse phase fluctuation

\[ \nabla \times \nabla \varphi = 0 \]

\[ \nabla \times \nabla \varphi = 2\pi \hat{z} \delta(\vec{r}) \]

Topological defects in the phase order: Abrikosov-Nielsen-Olesen vortex(2D) / string(3D)
In order to couple the phase fluctuation to quasi-particles, one has to isolate the uncertain charge carried by q, similar to the “charge-spin separation” in HTSC.

We use: Franz-Tesanovic (FT) singular gauge transformation


\[
\bar{\psi}_u = e^{i\varphi_u} \bar{q}_u, \\
\bar{\psi}_d = e^{i\varphi_d} \bar{q}_d, \\
\varphi_u + \varphi_d = \varphi
\]

\[
\nabla \times \nabla \varphi_u(d) = 2\pi i \sum_i Q_i \delta(\vec{r} - \vec{r}_i^{(d)})
\]

A new set of charge neutral quasi-particles

Two emergent gauge fields:

\[v_\mu = \frac{1}{2} \partial_\mu \varphi, \quad \varphi_u + \varphi_d = \varphi\]

\[a_\mu = \frac{1}{2} (\partial_\mu \varphi_u - \partial_\mu \varphi_d)\]

\[v_\mu\] Doppler gauge field / superflow field, couples with charges

\[a_\mu\] Berry gauge field / Topological gauge field, couples with charge neutral quasiparticles (isospin)

Massive in SC, massless in PG state
\[ \mathcal{L}_\psi = \mathcal{L}_\psi^{q_p} + \mathcal{L}_0^{a,v}, \]
\[ \mathcal{L}_\psi^{q_p} = \psi \left( \hat{\mathcal{D}} + \hat{\mu} \gamma_0 \right) \psi - \frac{1}{2} \Delta \left[ i \bar{\psi} \varepsilon \gamma_5 \psi^C \right] + c.c. - \frac{\Delta_0^2}{4u^2} \]
\[ \hat{D} = (\partial_\mu + i2a_\mu) + i(v_\mu - eA_\mu) \]

**Contribution of gauge fields:**

\[ \mathcal{L}_0 \to \frac{K_\mu}{2} (∂ × v)_\mu^2 + \frac{K_\mu}{2} (∂ × a)_\mu^2 \]

**Another way, introducing dual disorder field:**

\[ \mathcal{L}_0 [v, a] \to \frac{1}{4\pi^2 |\Phi|^2} (∂ × v)^2 + \frac{1}{4\pi^2 |\Phi|^2} (∂ × a)^2 \]

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**IV. Expected phase diagram**

- **Loss of phase coherence**
- **Gapless modes**
- **LOFF-like:**

A. Melikyan, Z. Tesanovic, cond-mat/0408344
### V. Conclusion and discussion

1. When mismatch is large, the system cannot be described very well using BCS at MF;

2. With the increase of mismatch, the phase fluctuation plays more and more important role, it softens the superconducting phase.

3. At some critical mismatch, the strong phase fluctuation destroys the long-range phase coherence, turns the system to a phase decoherent pseudogap phase, while the gap amplitude is still finite.

4. Further increase of mismatch will drive the gap amplitude to zero (normal phase) or other possible pairing state (spin-1).

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1. The existence of the PG state is dependent on the assumption that at large mismatch, the amplitude fluctuation is not as important as phase fluctuation.

2. The topological defects might be different for different systems (g2SC,gCFL,BP).

3. Further studies are needed on the topological excitation.
Open for any criticism, comments, and suggestions !!!