Finite density QCD with dynamical staggered quarks by Taylor expansion (II)

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Introduction

Assuming three flavours, u, d, and s quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \ \prod_{f=u,d,s} \mathrm{Det} \ M(m_f, \mu_f) \quad . \tag{1}$$

Defining $\mu_0 = \mu_u + \mu_d + \mu_s$ and $\mu_3 = \mu_u - \mu_d$, baryon and isospin density/susceptibilities can be obtained as :
(Gottlieb et al. '87, '96, '97, Gavai et al. '89)

$$n_i = \frac{T}{V} \frac{\partial \ln \mathcal{Z}}{\partial \mu_i}$$

and $$\chi_{ij} = \frac{T}{V} \frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j} \quad .$$

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Higher order susceptibilities are defined by

$$\chi_{fg\cdots} = \frac{T}{V}\frac{\partial^n \log Z}{\partial\mu_f\partial\mu_g\cdots} = \frac{\partial^n P}{\partial\mu_f\partial\mu_g\cdots} . \tag{2}$$

These are Taylor coefficients of the pressure P in its expansion in μ.

Defining

$$\frac{\mu_2^*}{T} = \sqrt{\frac{12\chi_{uu}/T^2}{|\chi_{uuuu}|}} , \tag{3}$$

and similarly, μ_i^* (using i^{th} term $/(i+2)^{th}$ term), the Taylor series expansion for $\Delta P = P(\mu) - P(\mu = 0)$, can be re-organized as,

$$\frac{\Delta P}{T^4} = \left(\frac{\chi_{uu}}{T^2}\right)\left(\frac{\mu}{T}\right)^2\left[1 + \left(\frac{\mu}{\mu_2^*}\right)^2\left[1 + \left(\frac{\mu}{\mu_4^*}\right)^2\left[1 + \dots\right]\right]\right]. \tag{4}$$

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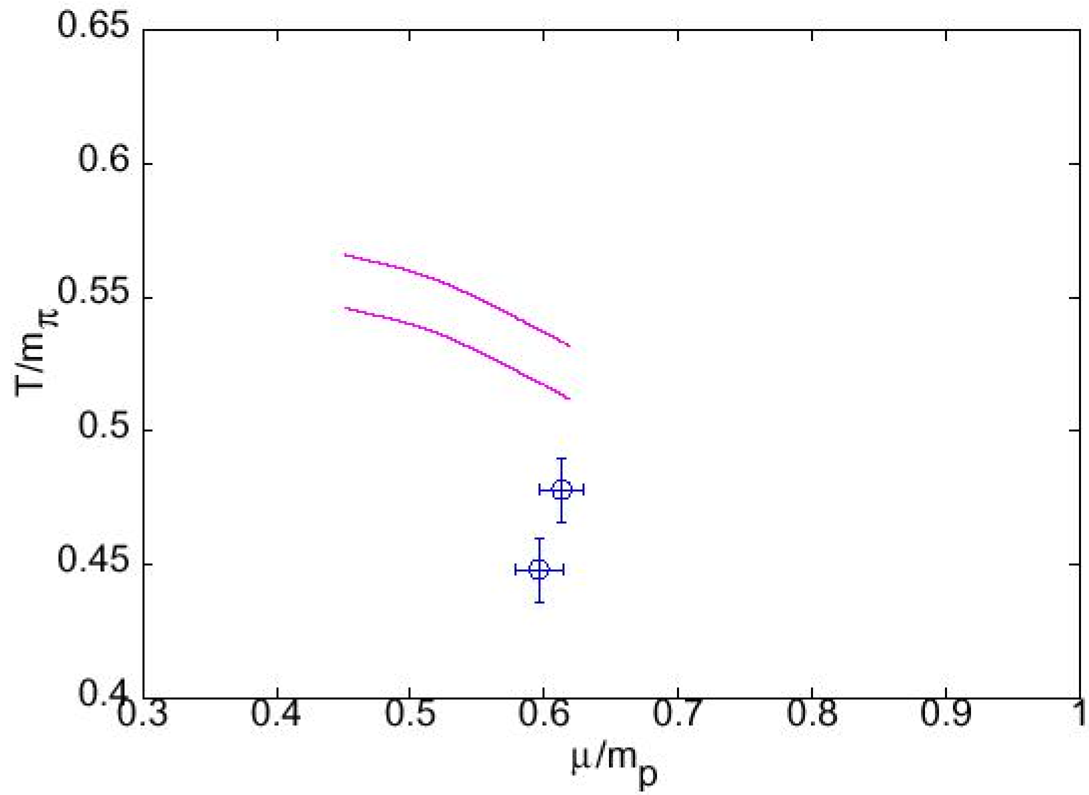
In Part I, Sourendu Gupta showed

- the details of our method to obtain μ_i^*
- our results for i =2, 4, 6 for various temperatures
- and their limiting behaviour for large i and our estimate for the critical point.



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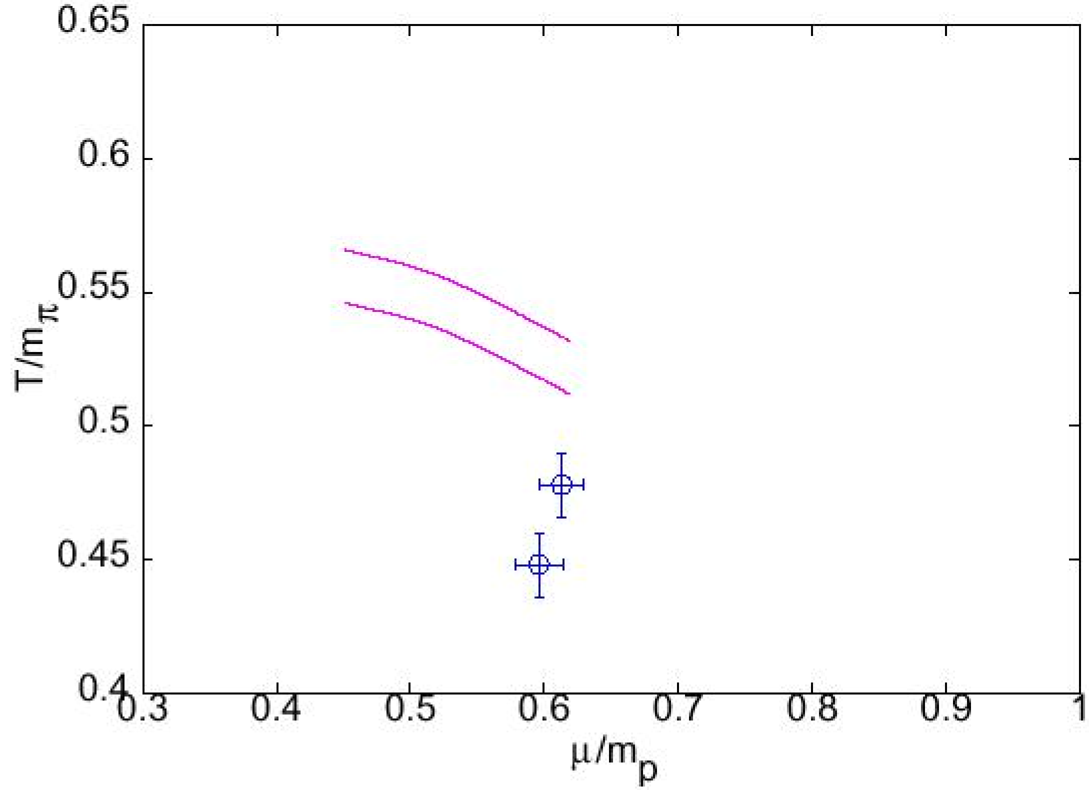

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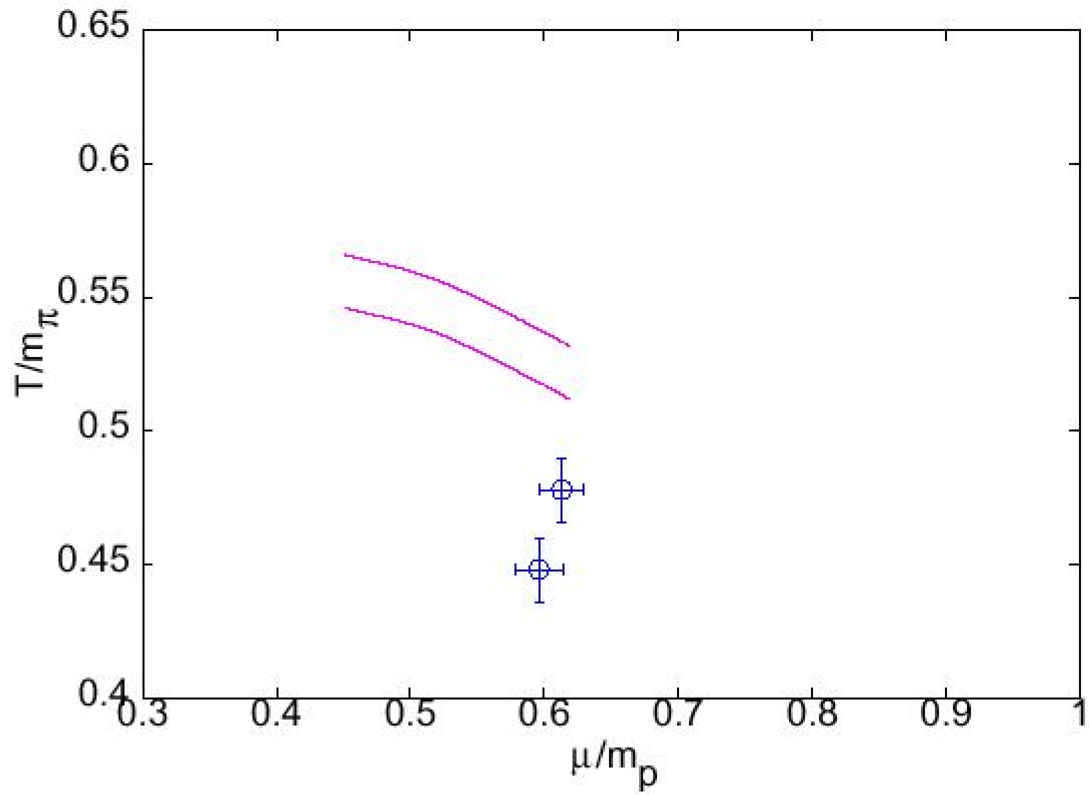

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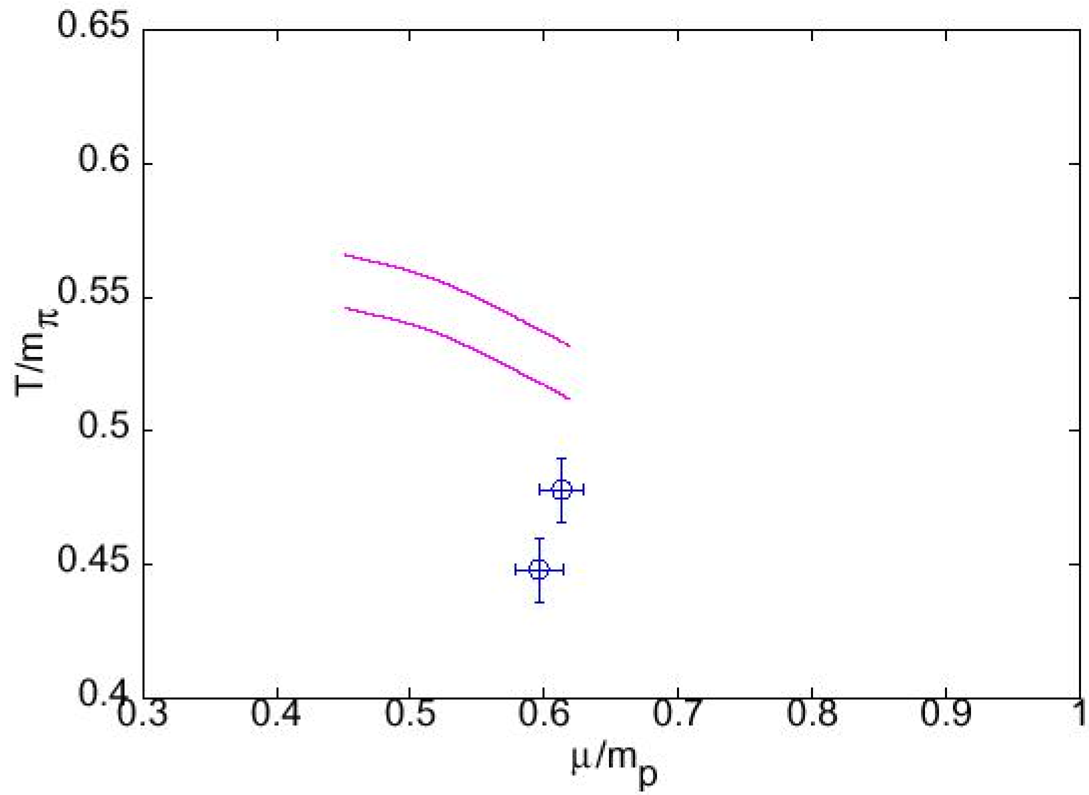

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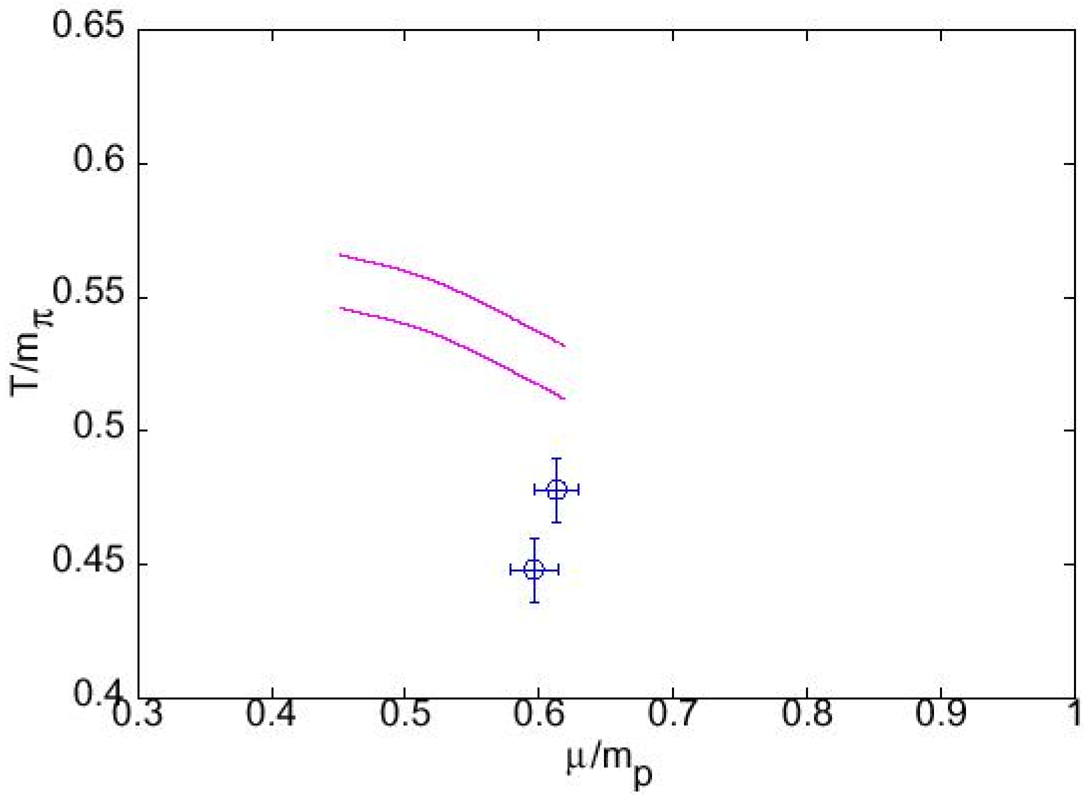

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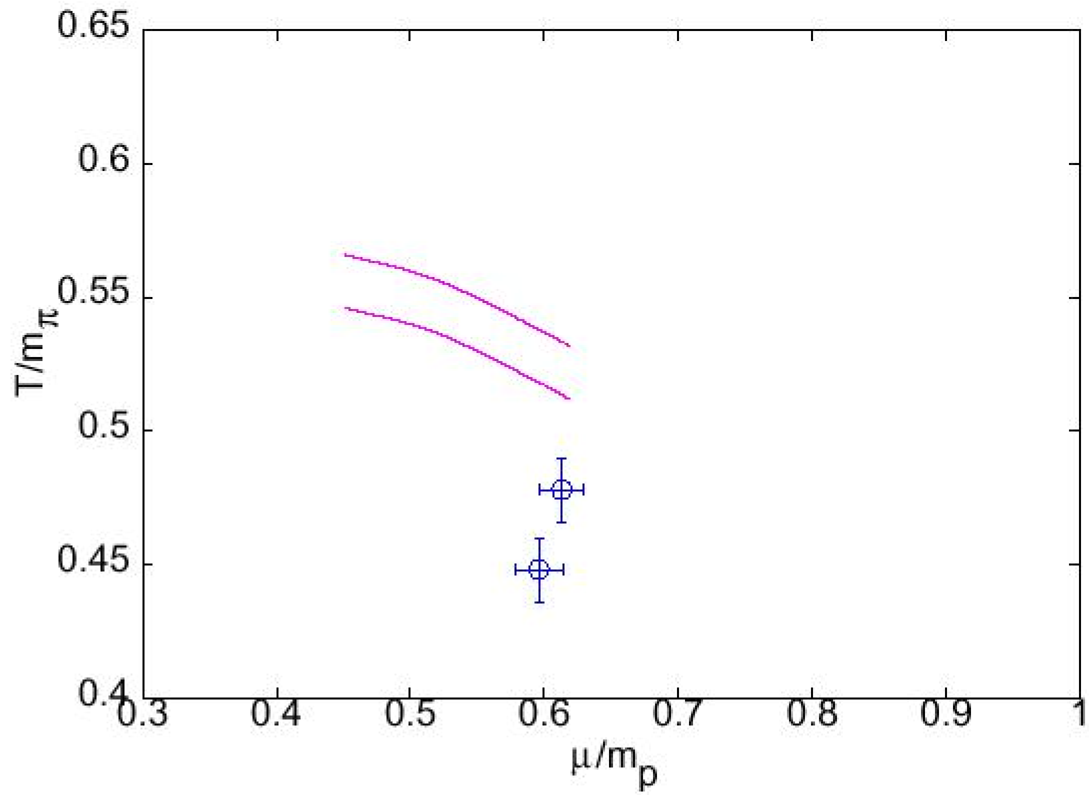

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Motivation

- Quark-Gluon Plasma in Heavy Ion Collisions.
- Reliable signals needed to establish it.
- Enhancement of strangeness production as a promising signal of QGP (Rafelski-Müller, Phys. Rev. Lett '82, Phys. Rept '86..).
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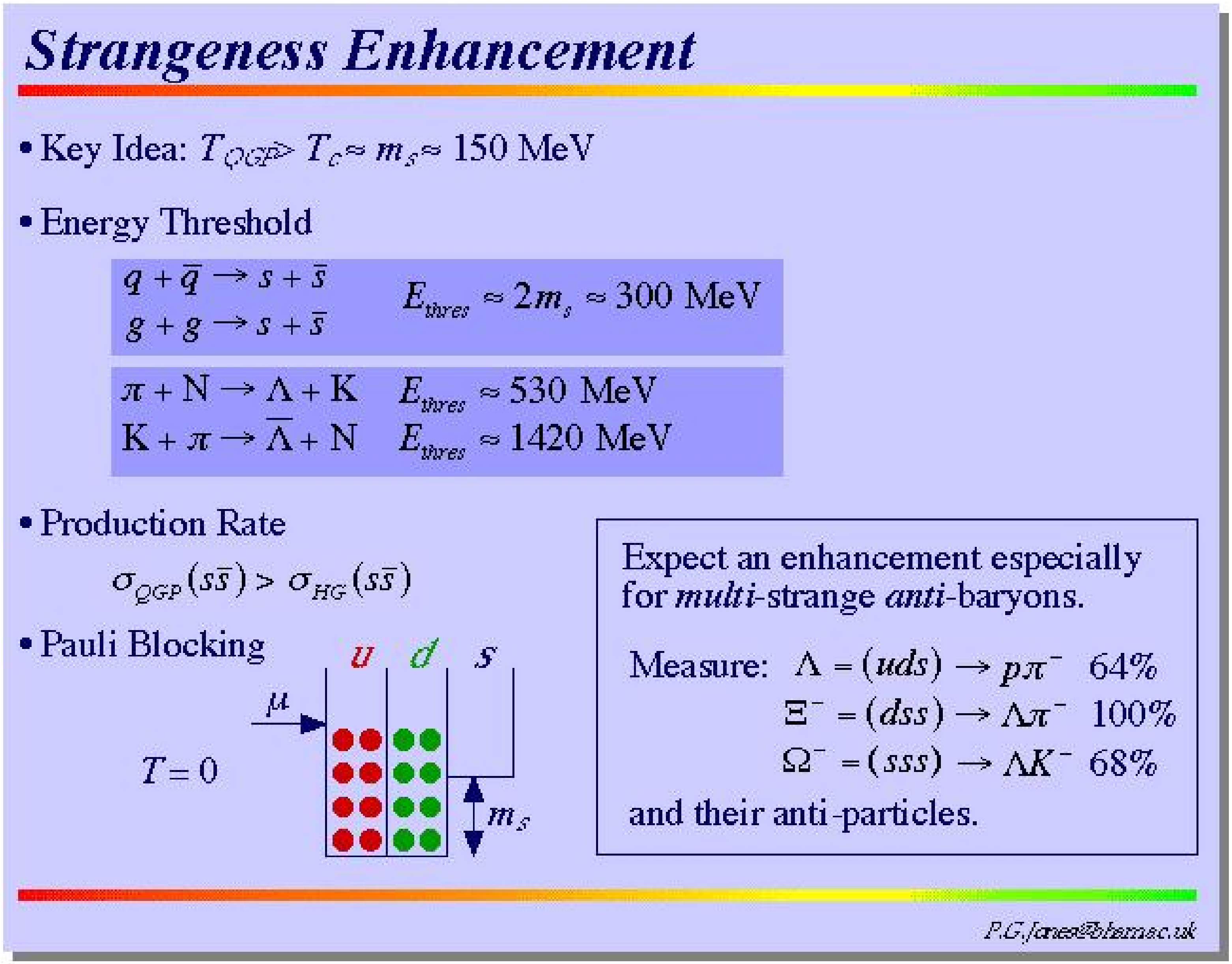
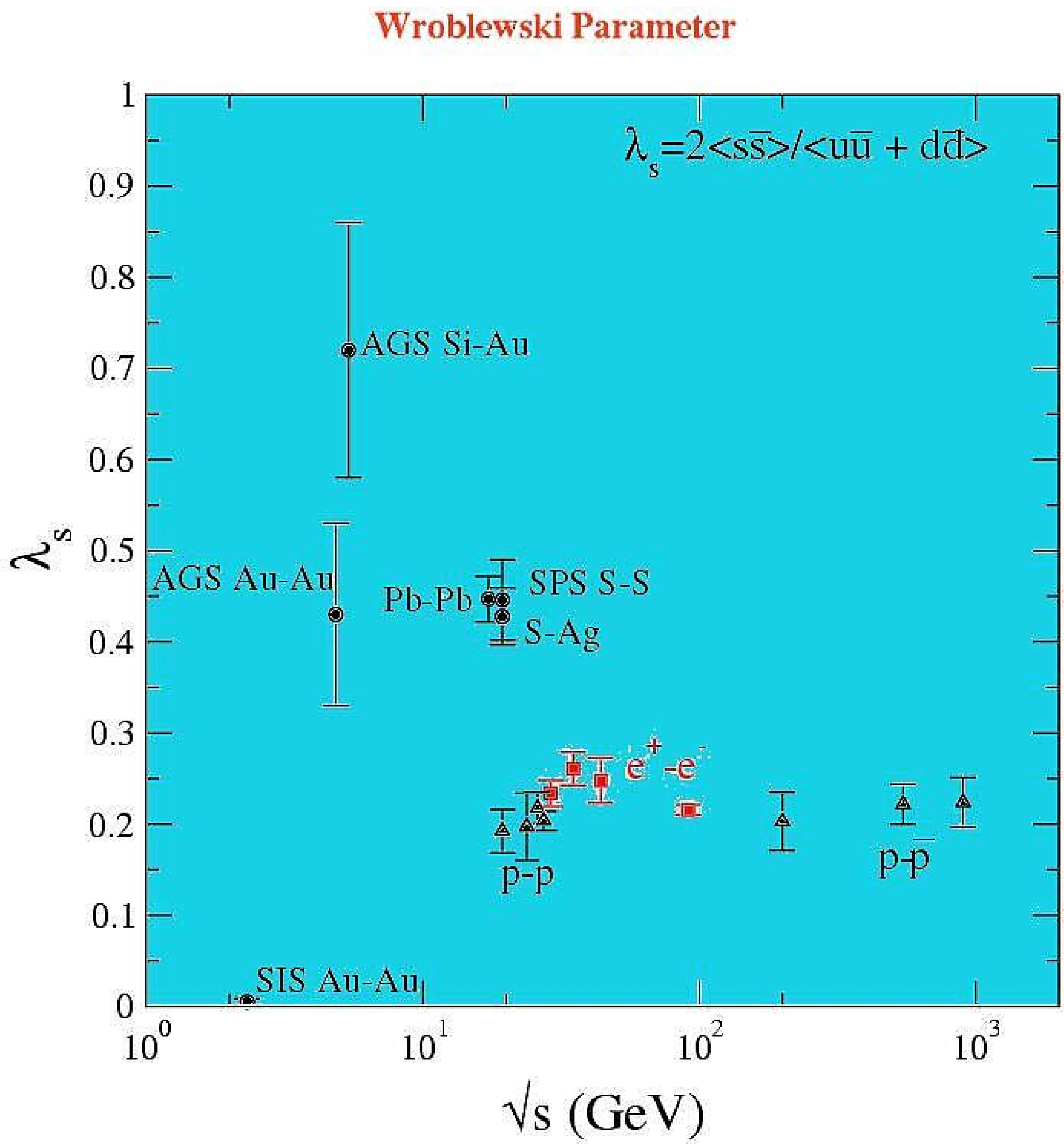


From STAR Webpage

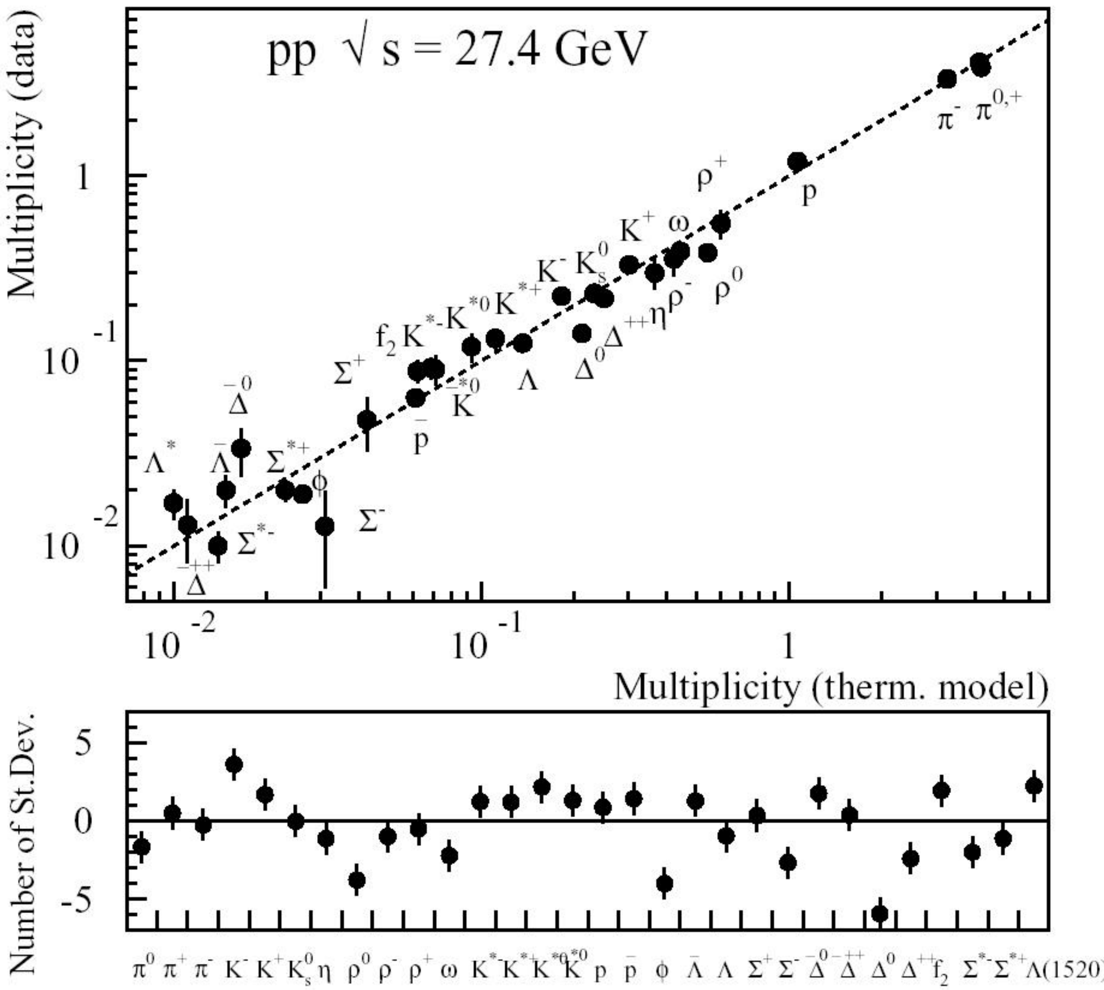


Ratio of newly created strange quarks to light quarks :

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u} + d\bar{d}\rangle} \quad (5)$$

Wroblewski Parameter

- Hadron gas fireball model (Becattini-Heinz '97).
- 3 Free parameters : T, V, and $N_{s\bar{s}}$.
- Fit many hadron abundances.
- Obtain λ_s from data.
- Find $\lambda_s \sim$ 0.4 (0.2) for AA (pp).



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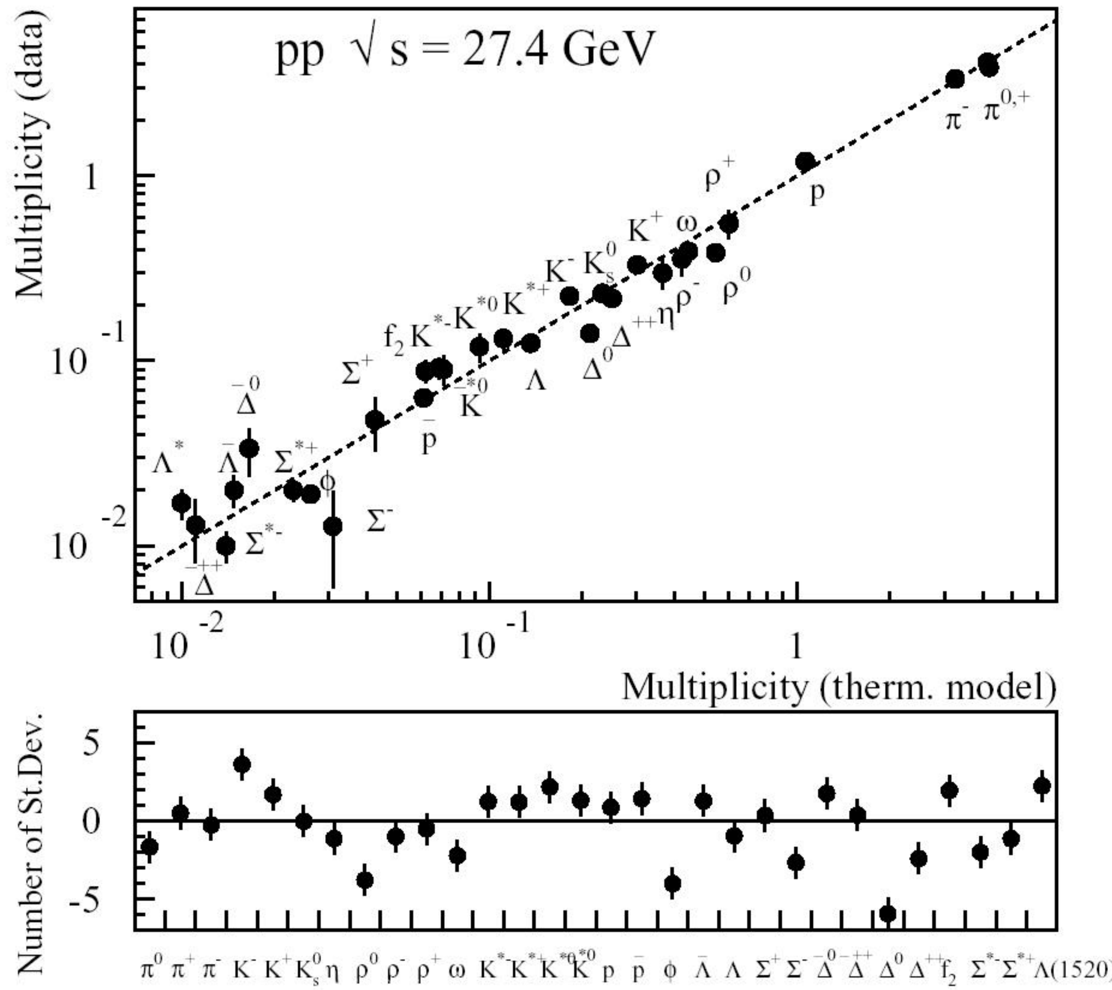

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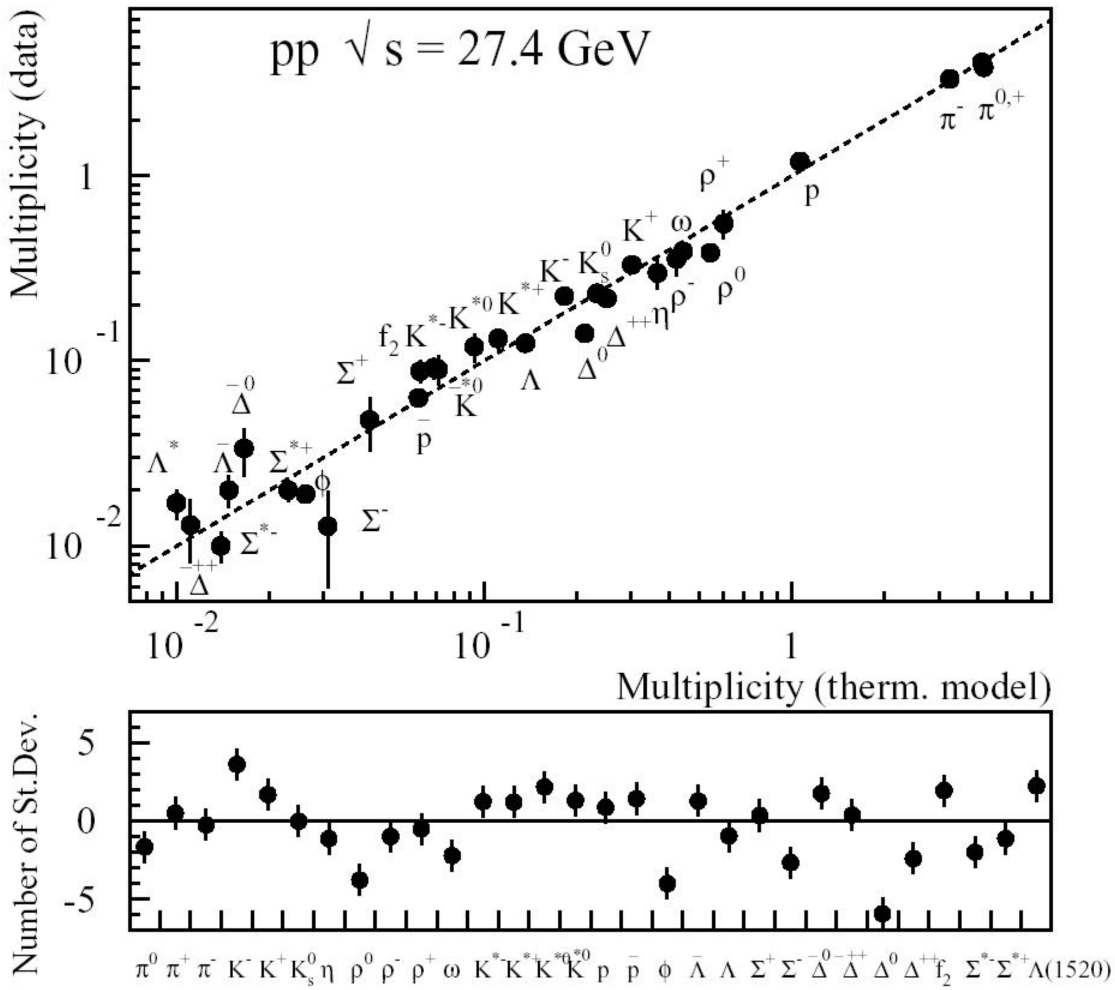

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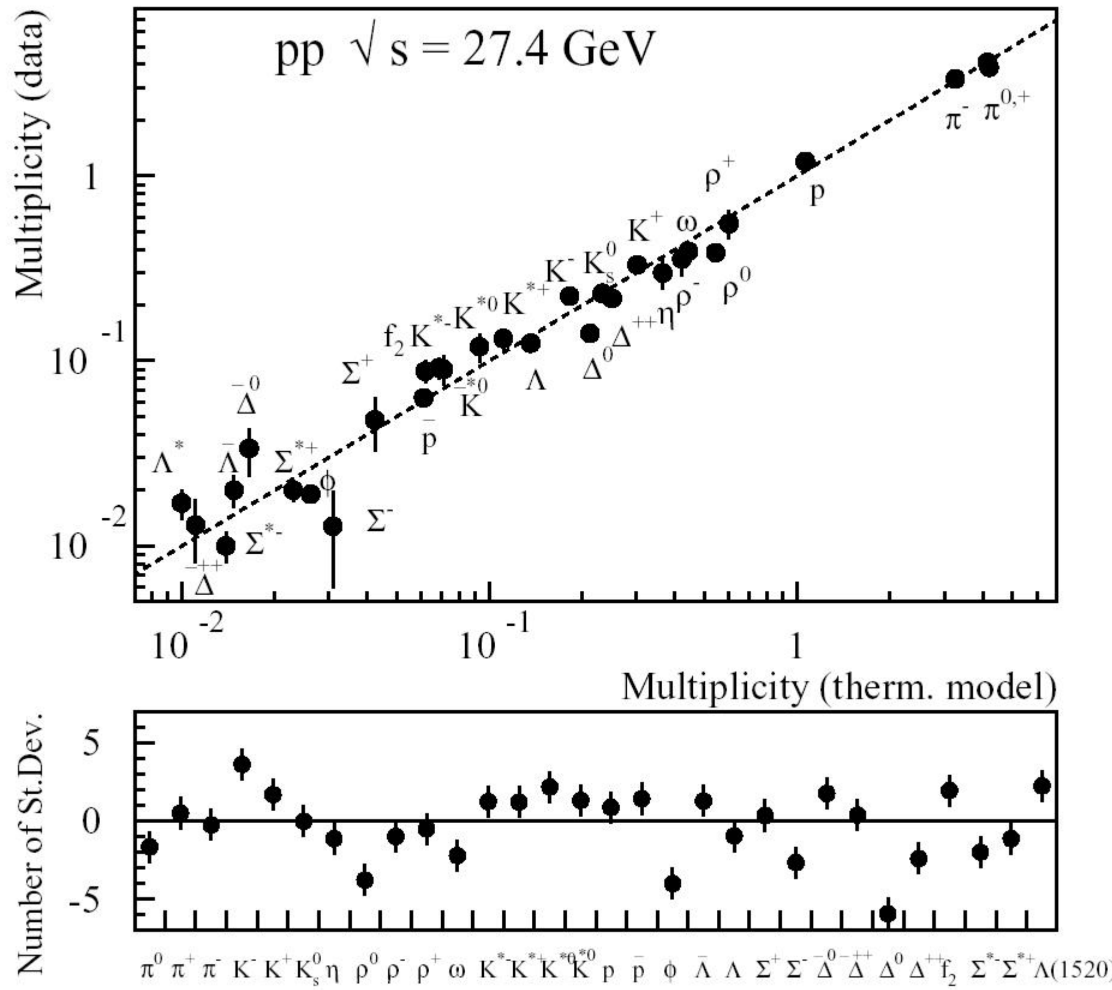

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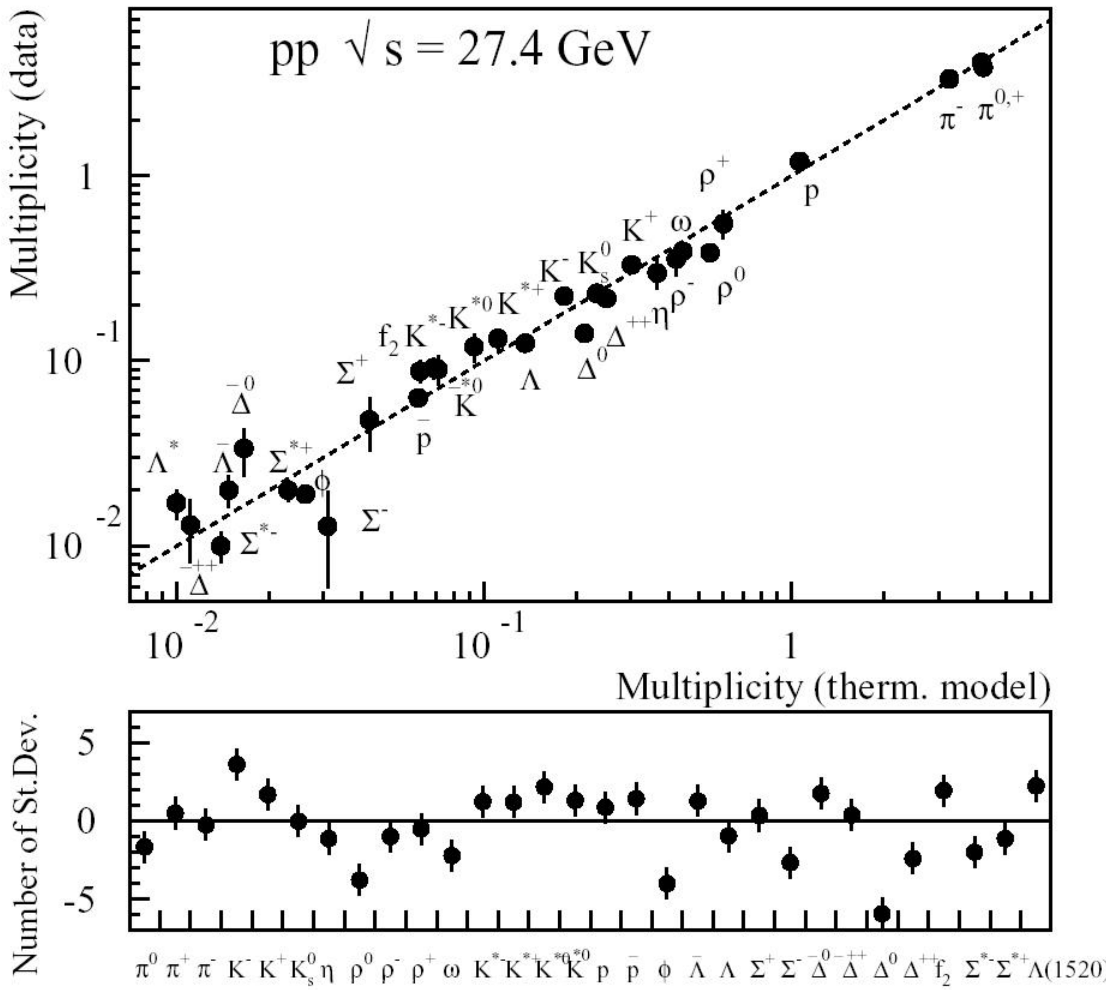

Quark Number Susceptibility

♠ We have argued that

$$\lambda_s = \frac{2\chi_s}{\chi_u + \chi_d} . \qquad (6)$$

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♠ Our improvement: Constant Physics (i.e., Fixed m_q/T_c), Continuum limit...

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Here $\mathcal{O}_2 = \mathrm{Tr}\ M_u^{-1}M_u'' - \mathrm{Tr}\ M_u^{-1}M_u'M_u^{-1}M_u'$, and $\mathcal{O}_{11}(m_u) = (\mathrm{Tr}\ M_u^{-1}M_u')^2$, and the traces are estimated by a stochastic method:
$\mathrm{Tr}\ A = \sum_{i=1}^{N_v} R_i^\dagger A R_i/2N_v$, and $(\mathrm{Tr}\ A)^2 = 2\sum_{i>j=1}^{L}(\mathrm{Tr}\ A)_i(\mathrm{Tr}\ A)_j/L(L-1)$,
where R_i is a complex vector from a set of N_v subdivided in L independent sets.

Comparing Full and Quenched QCD

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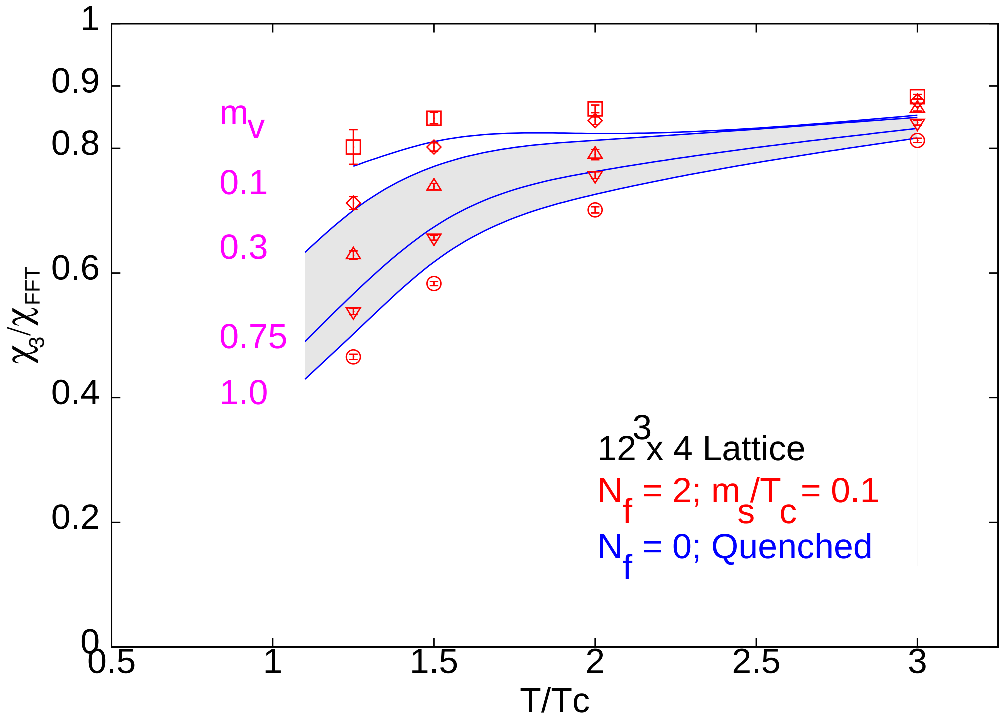

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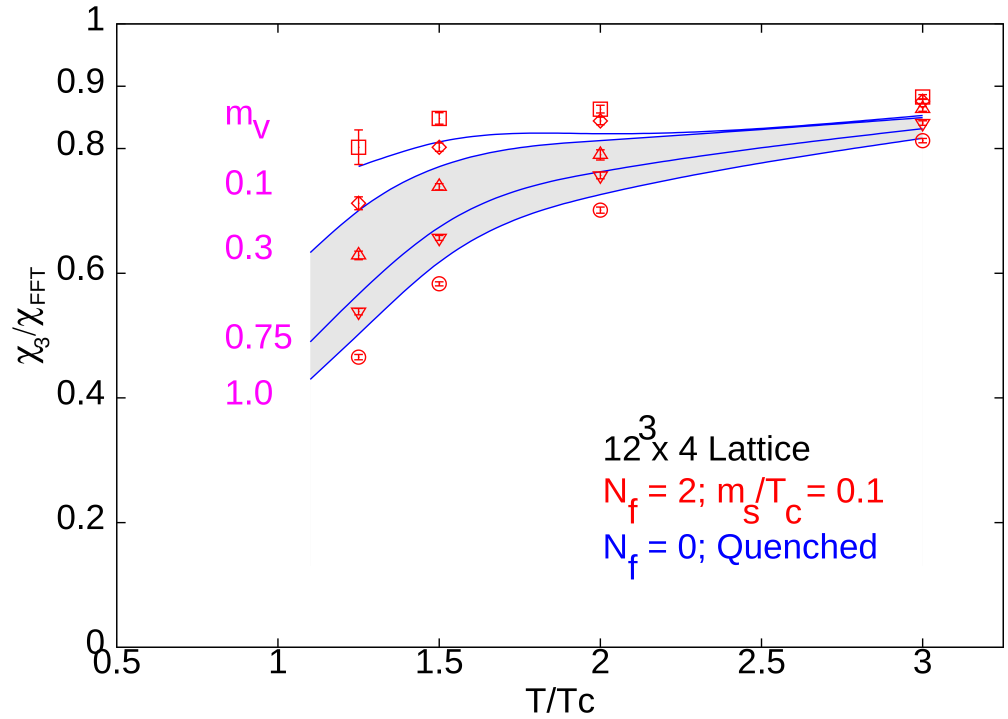

Note :

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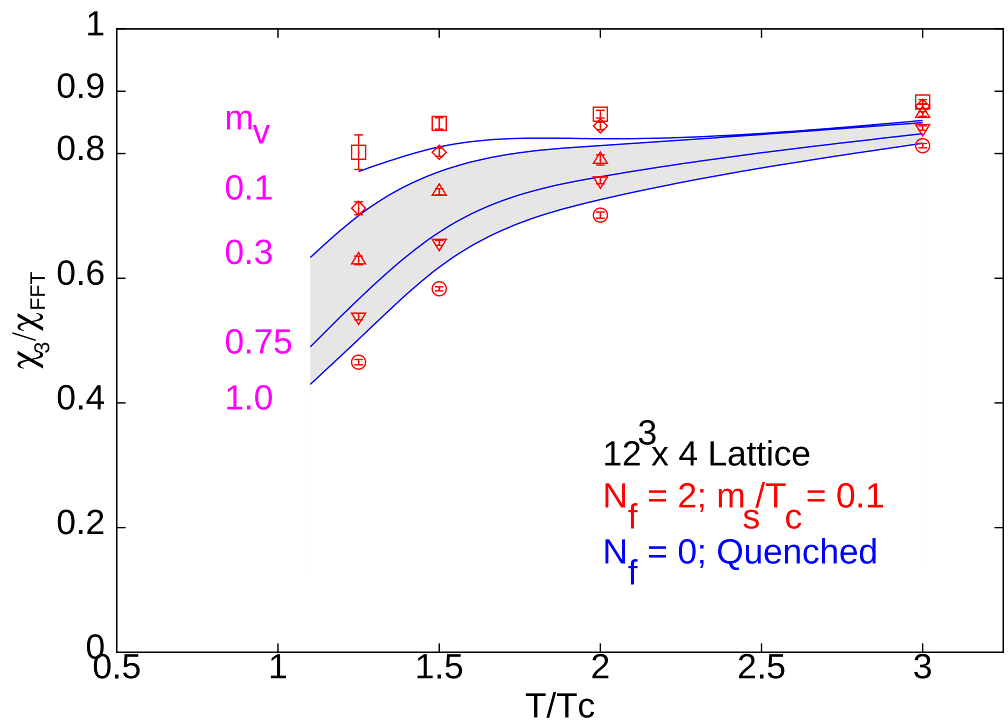

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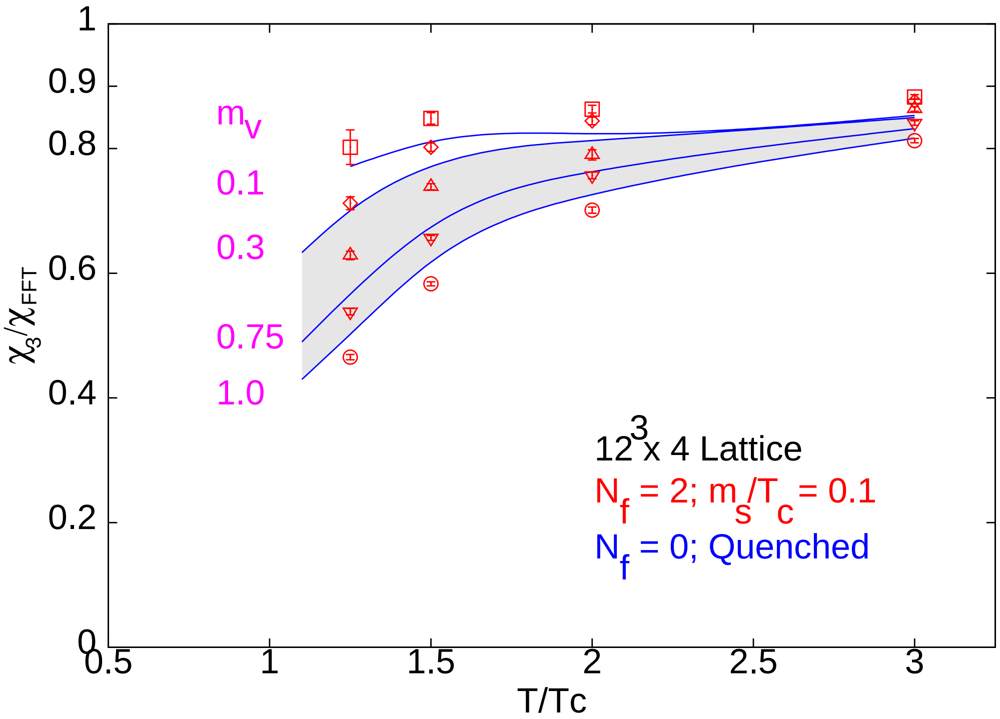

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3) PDG values for strange quark mass $\Longrightarrow m_v^{strange}/T_c \simeq$ 0.3-0.7 (N_f=0);
0.45-1.0(N_f=2).

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Weak coupling expansion gives:
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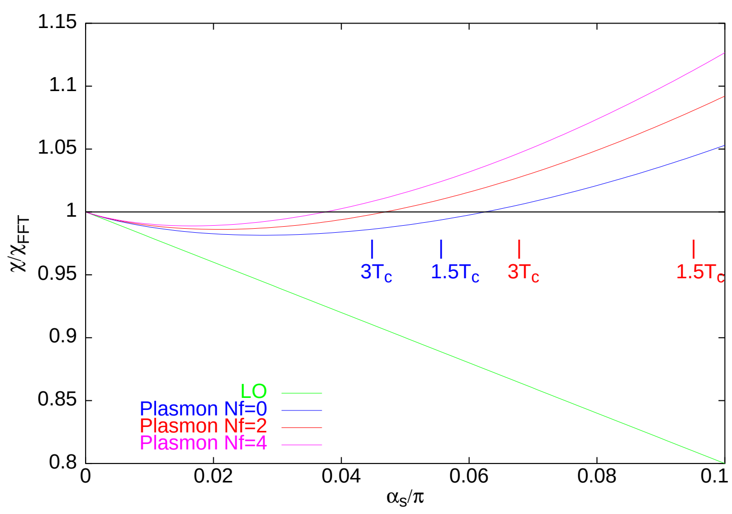

♣ Minm 0.981 (0.986) at 0.03 (0.02) for $N_f = 0$ (2).

♣ For $1.5 \leq T/T_c \leq 3$ pert. theory $\longrightarrow$ 0.99-0.98 (1.08=1.03) for $N_f = 0$ (2).

Resummed Perturbation Theory

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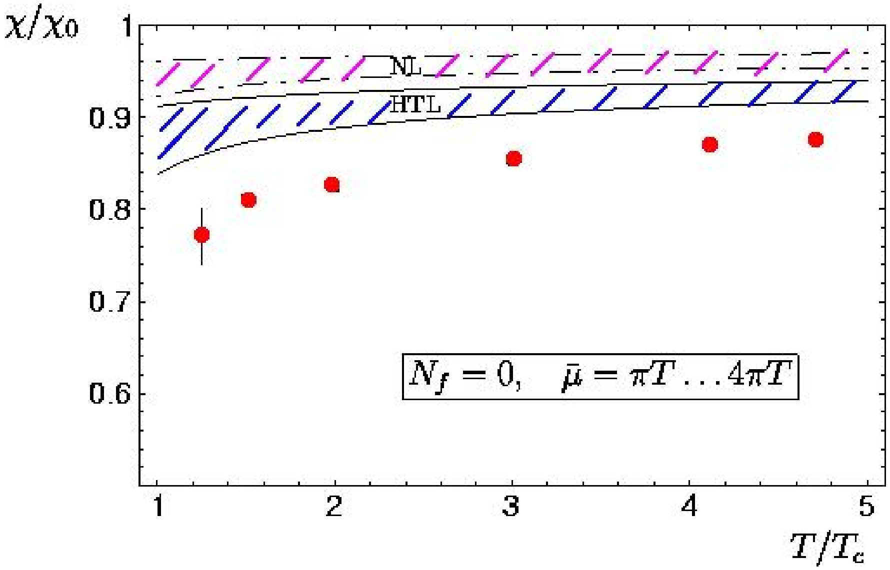

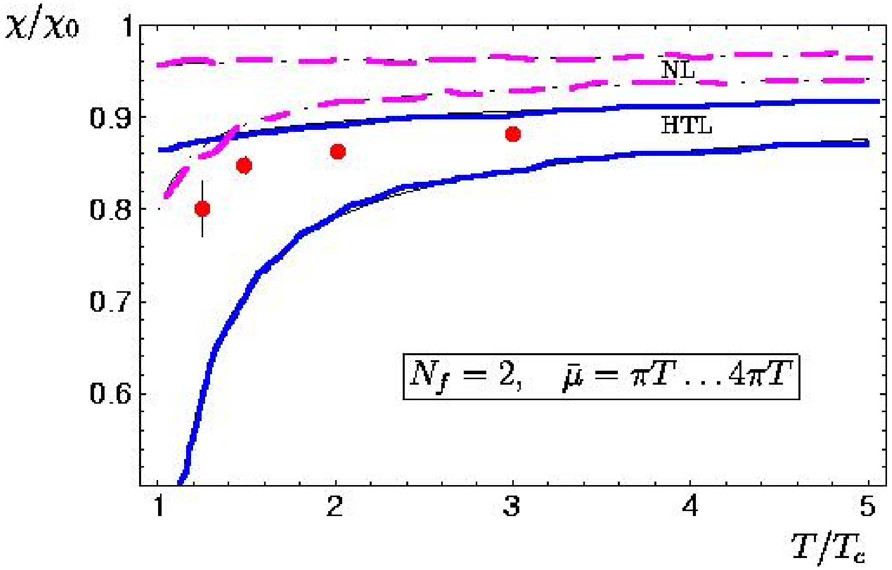

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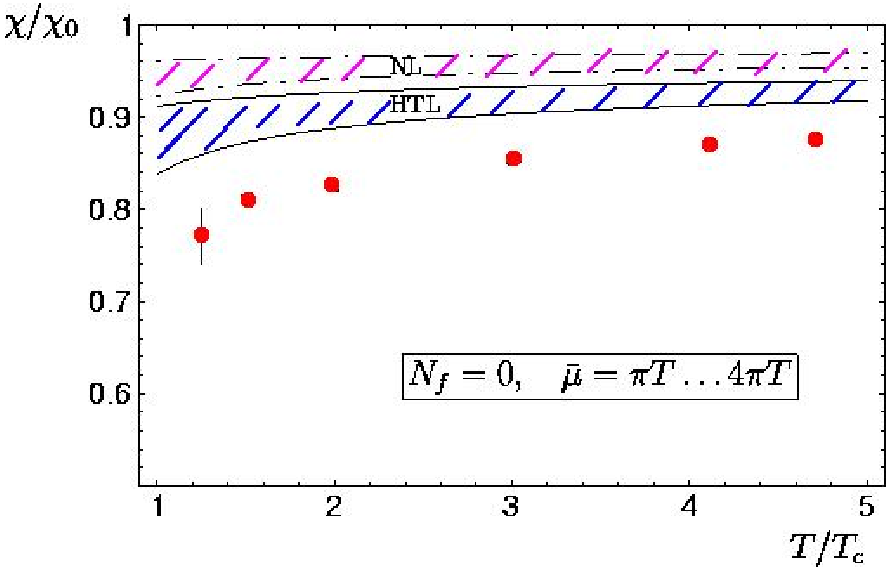

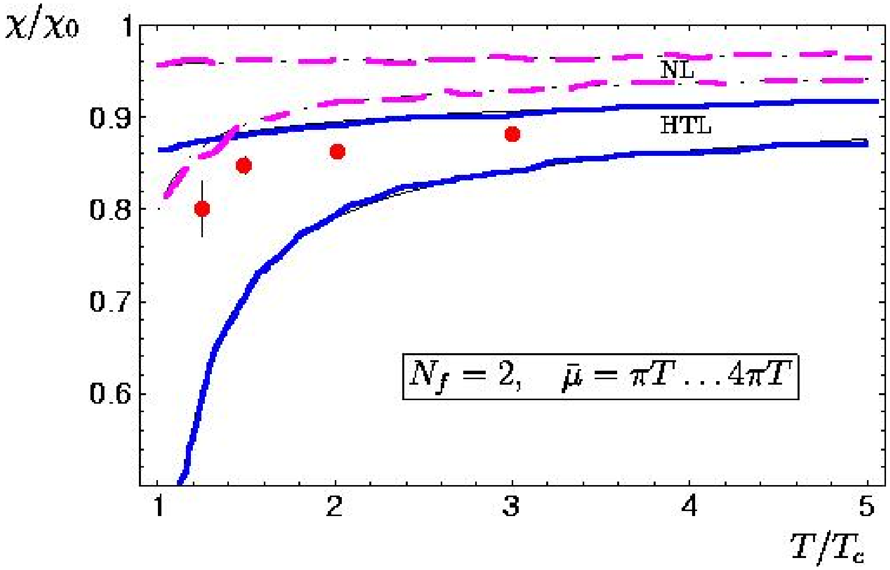

Our results for $N_t = 4 \rightsquigarrow$ Lattice artifacts ?
Check for larger N_t and improved actions.

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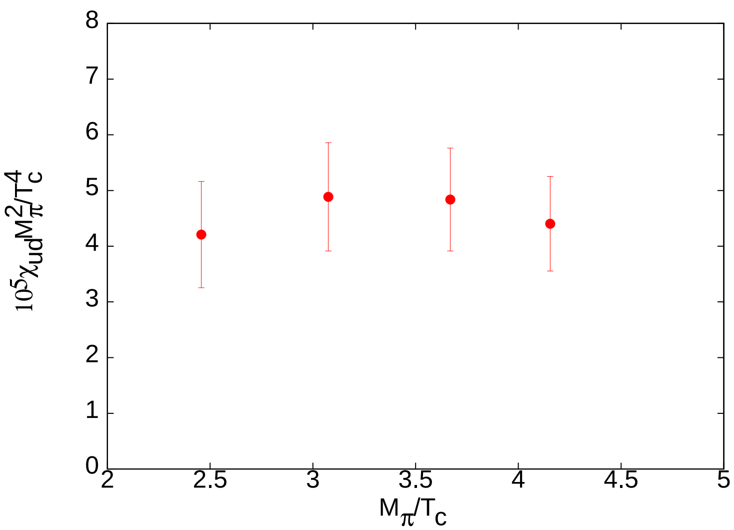

♣ $12^3 \times 4$ Lattice; Quenched.
♣ $T = 0.75 T_c$
♣ Gavai, Gupta & Majumdar, PR D 2002

Taking Continuum Limit

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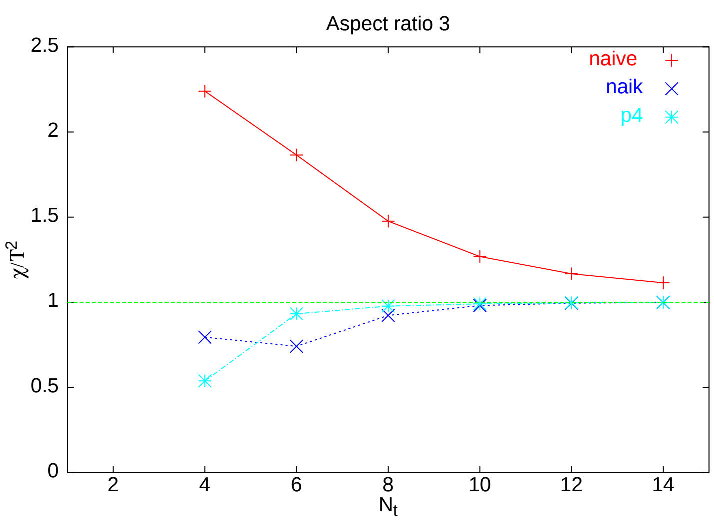
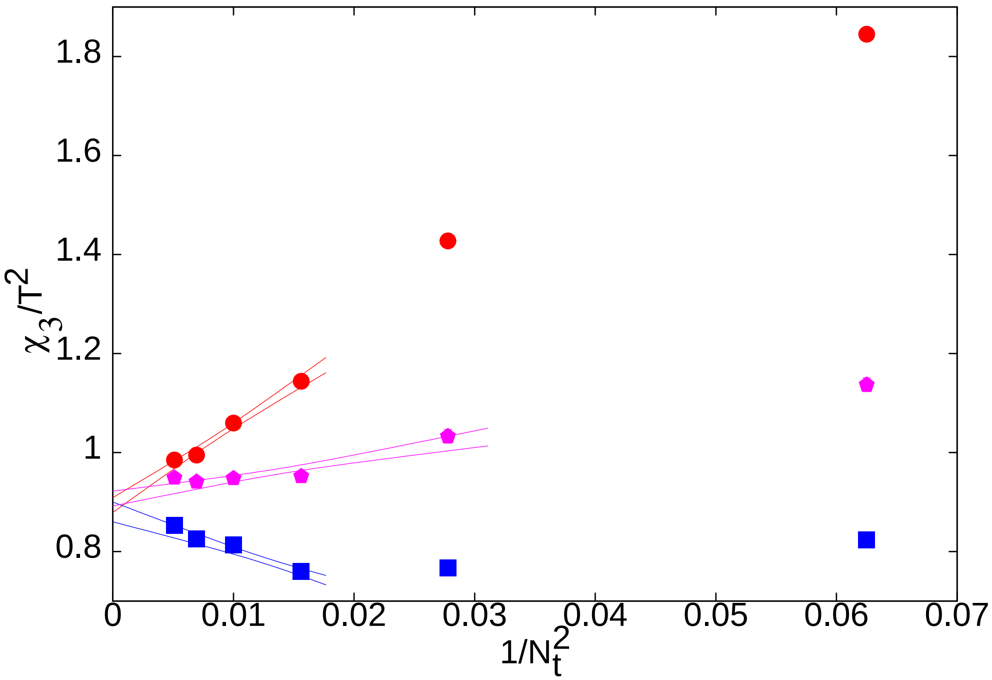


♠ Does improve the N_t-dependence of the free fermions for $N_t \geq 6$.

Results at $2T_c$:



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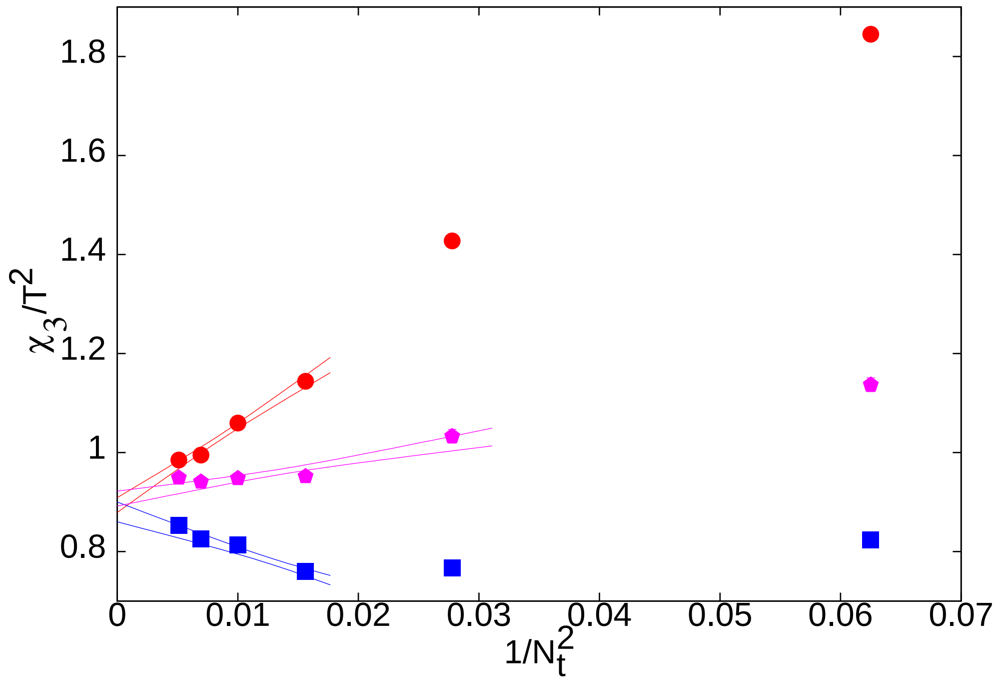

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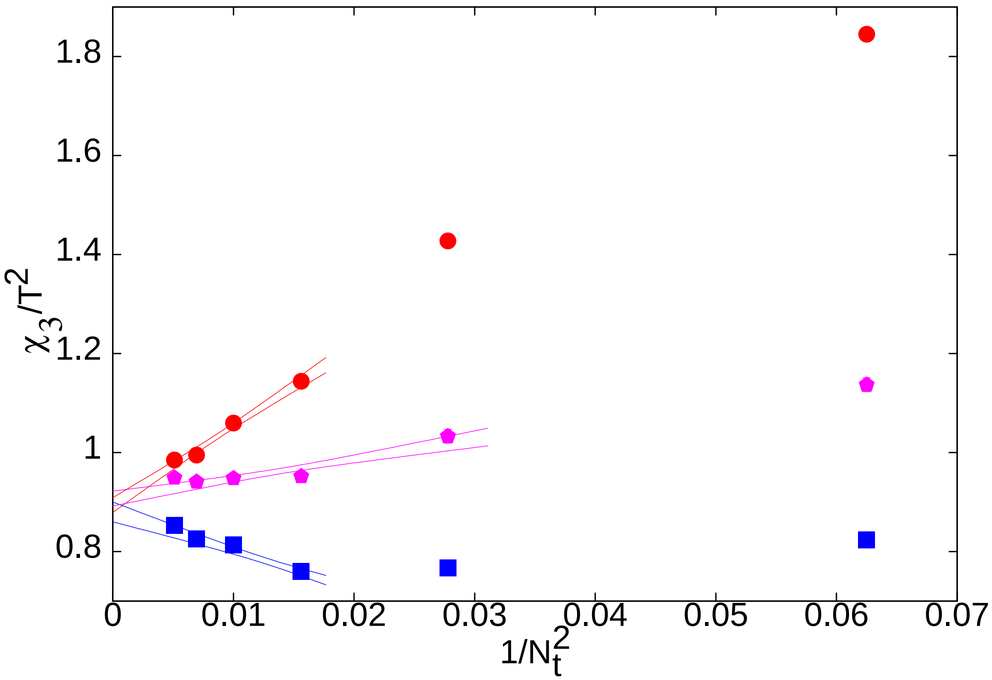

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◇ Milder $N_t^{-2} \sim a^2$-dependence for Naik fermions.

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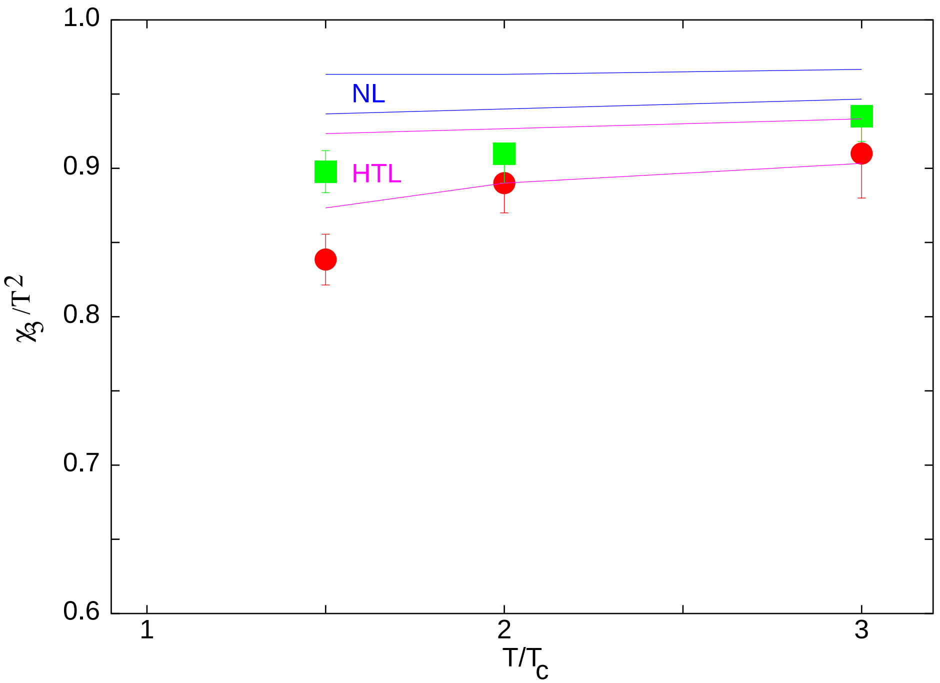

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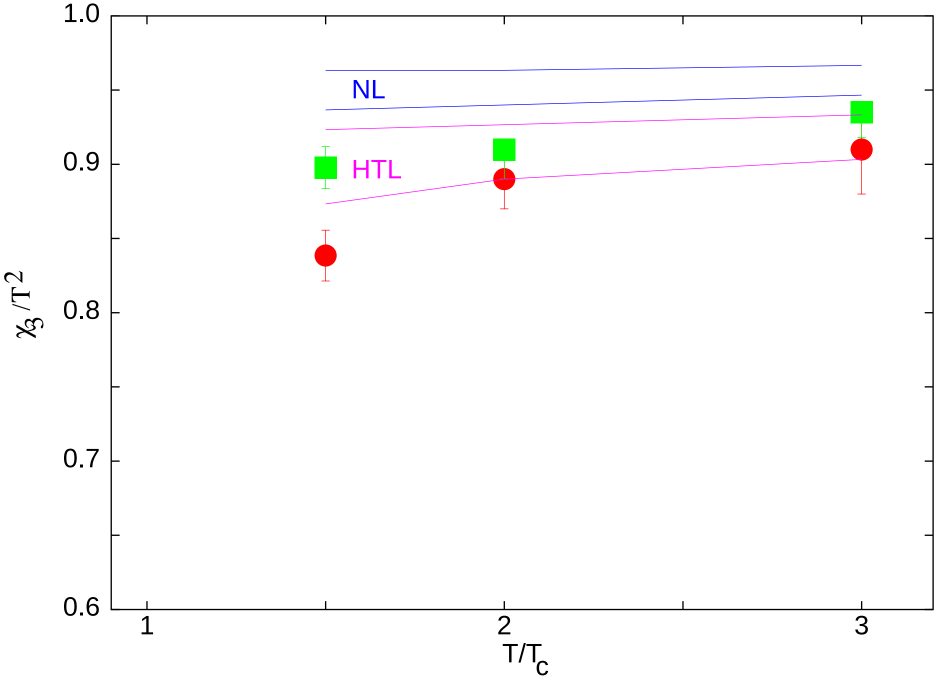

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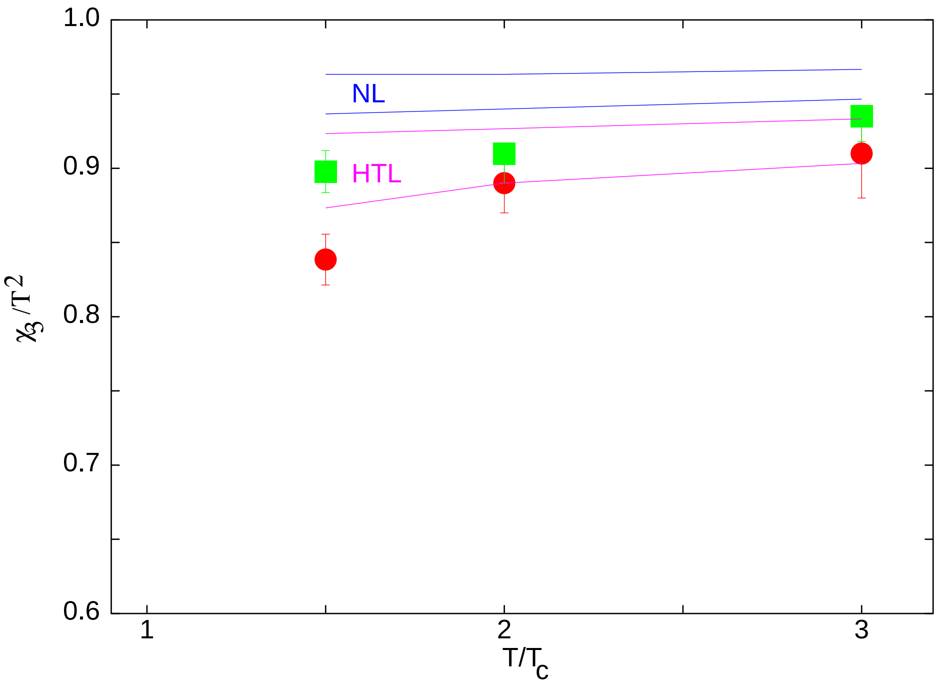

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♡ Note that χ_{ud} behaves the same way for ALL N_t and both fermions, leading to the same $O(10^{-6})$ values in continuum too.

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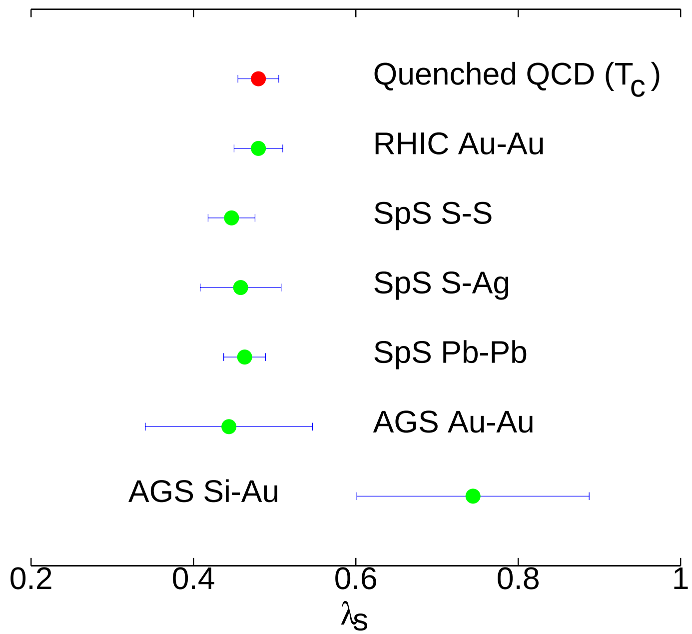

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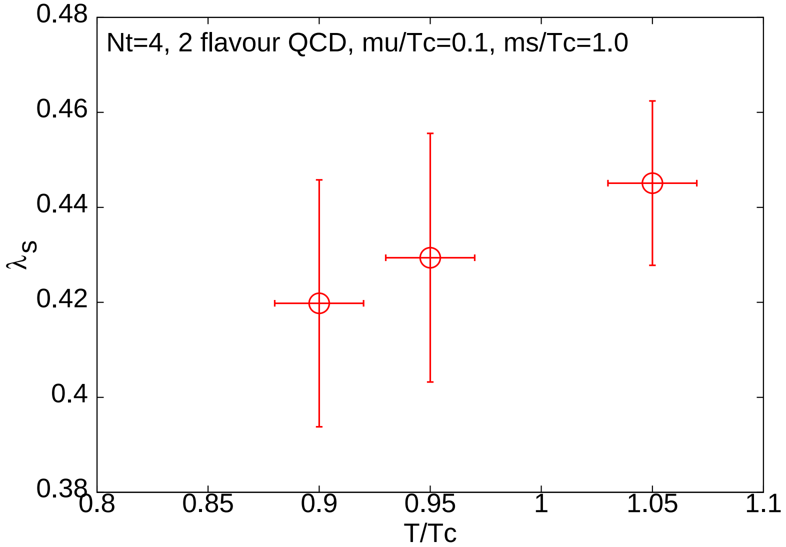

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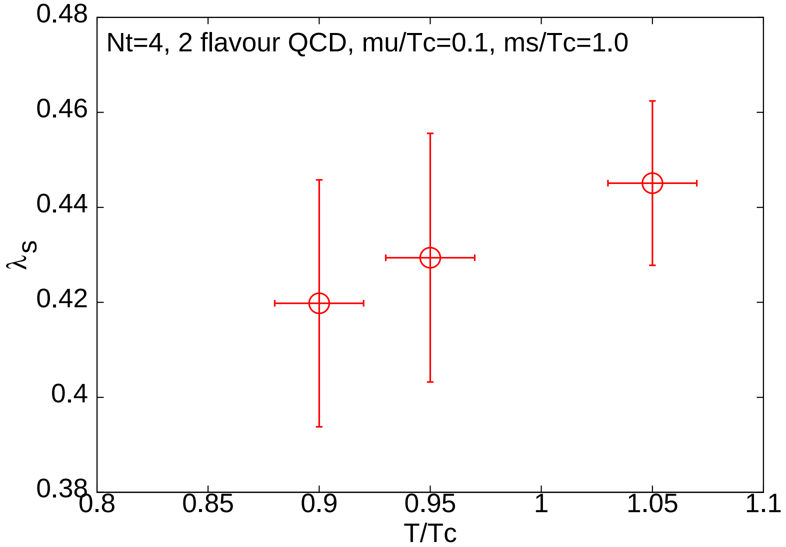

♣ Large finite volume effects below T_c
♣ Up to 12^3 Lattices used.
♣ Strong dependence on m_s expected.
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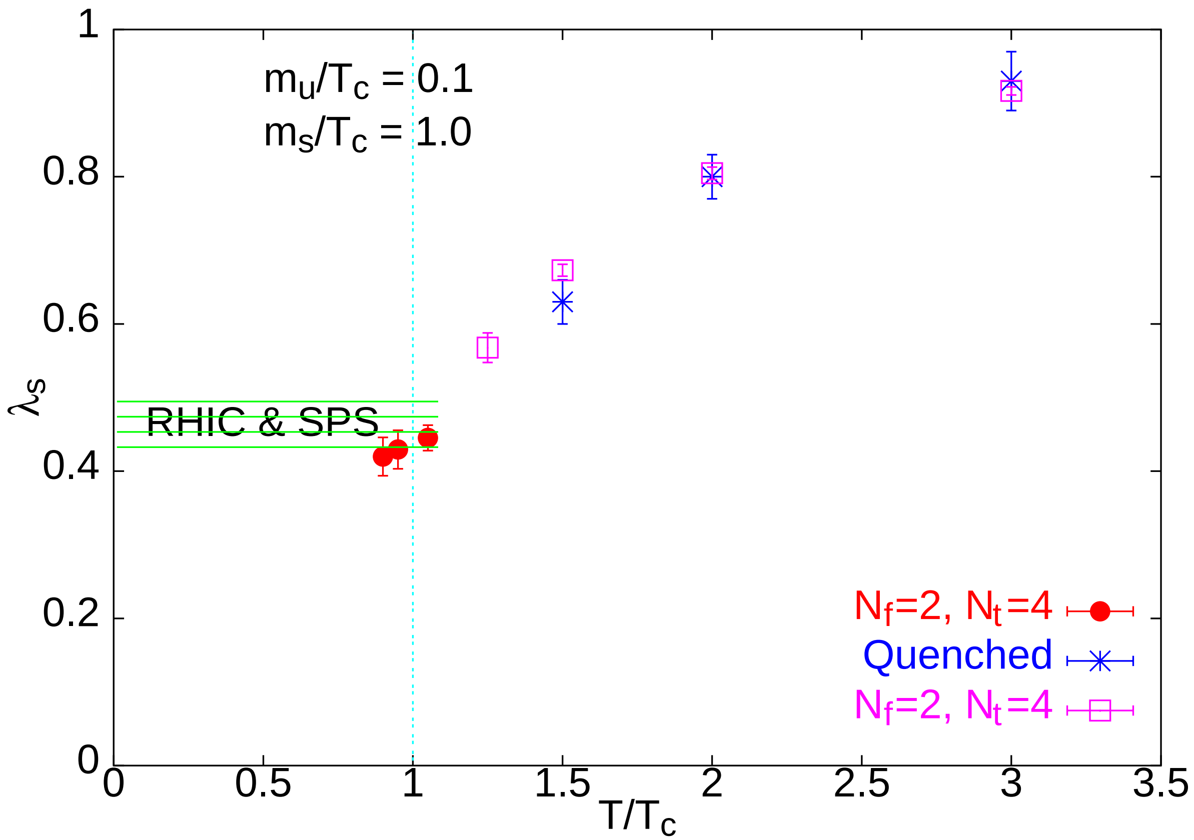
λ_s as a function of T



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Using the χ's upto 8th Order, $\Delta P(\mu, T)$ and $\chi(\mu, T)$ can be obtained.

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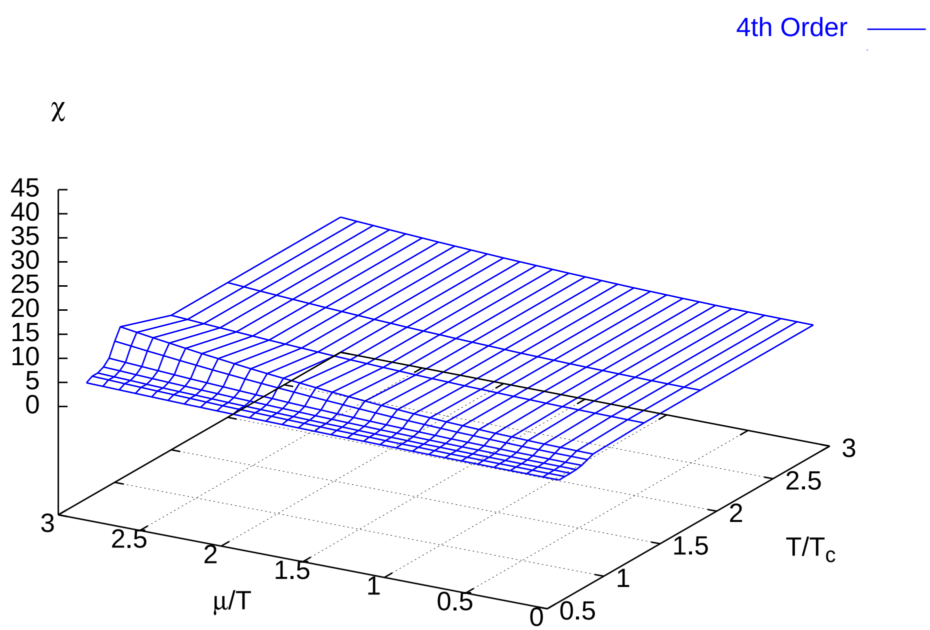

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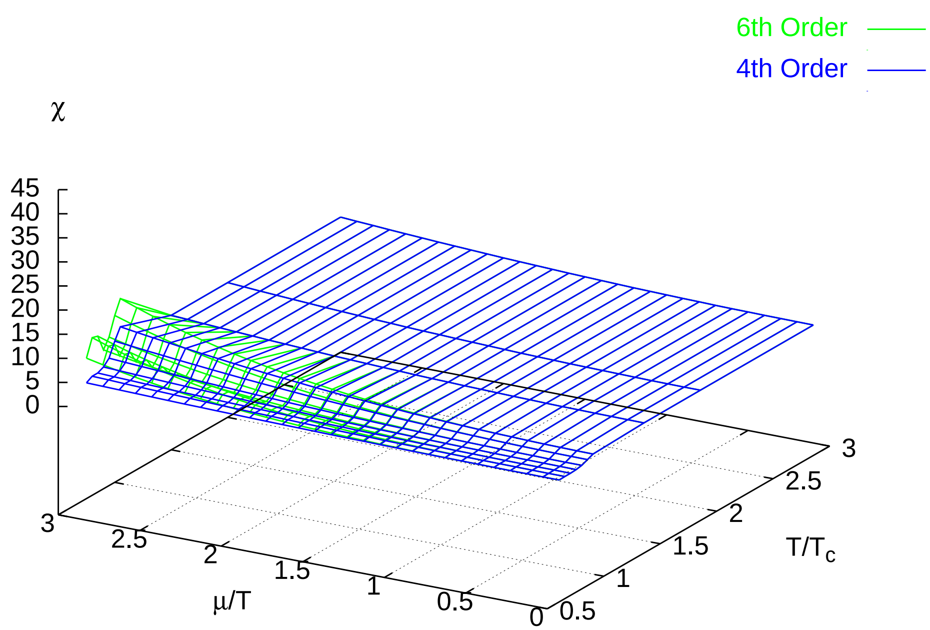

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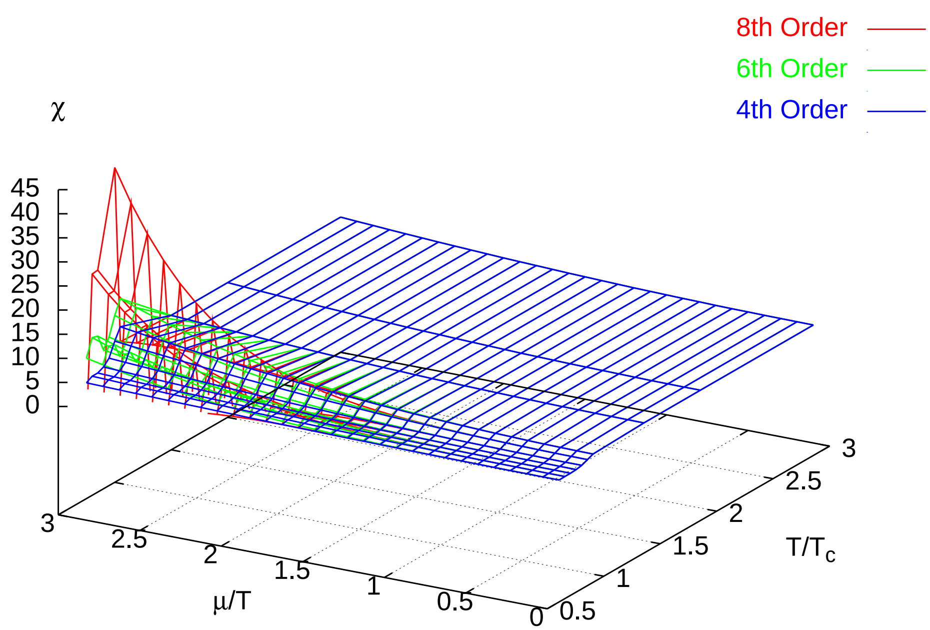

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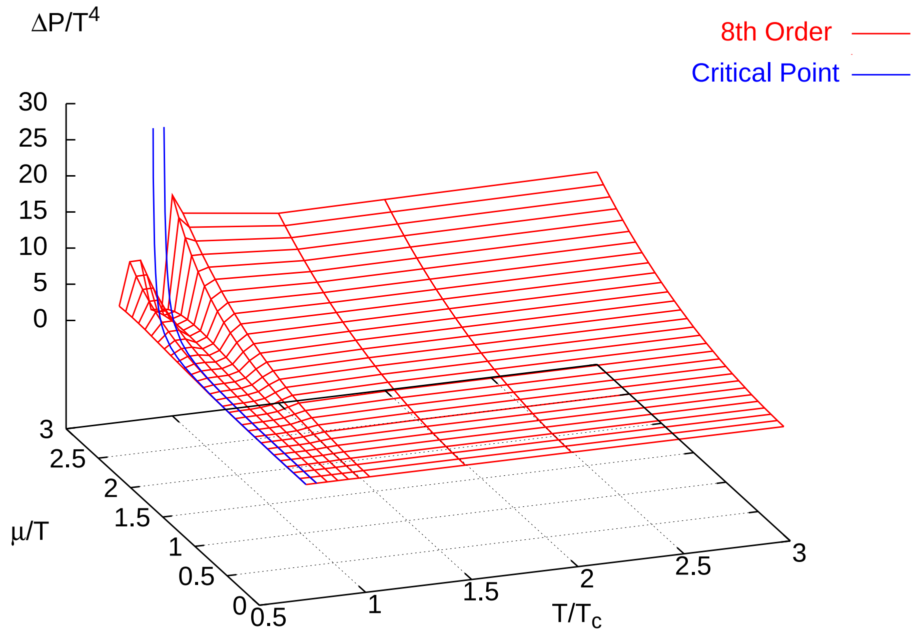

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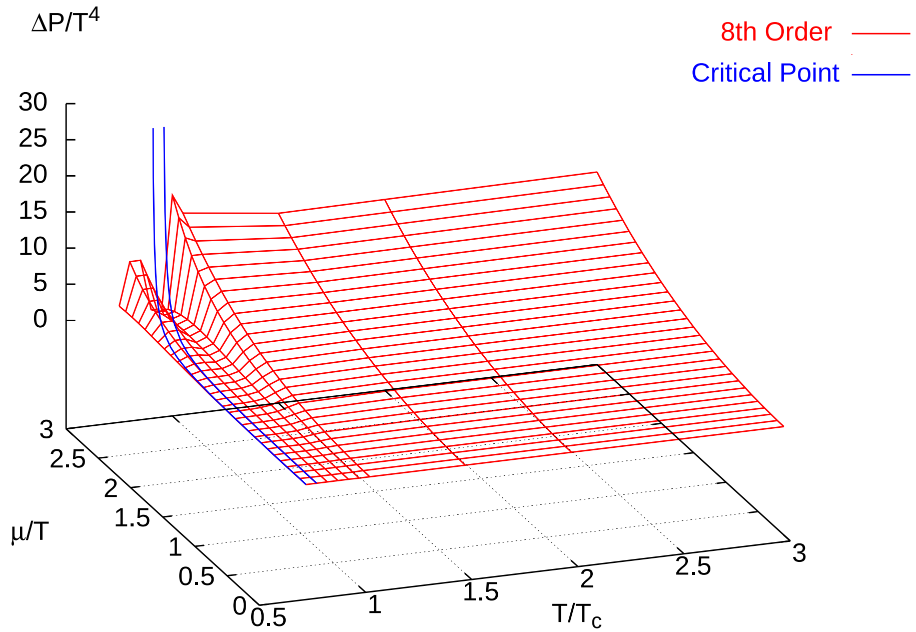

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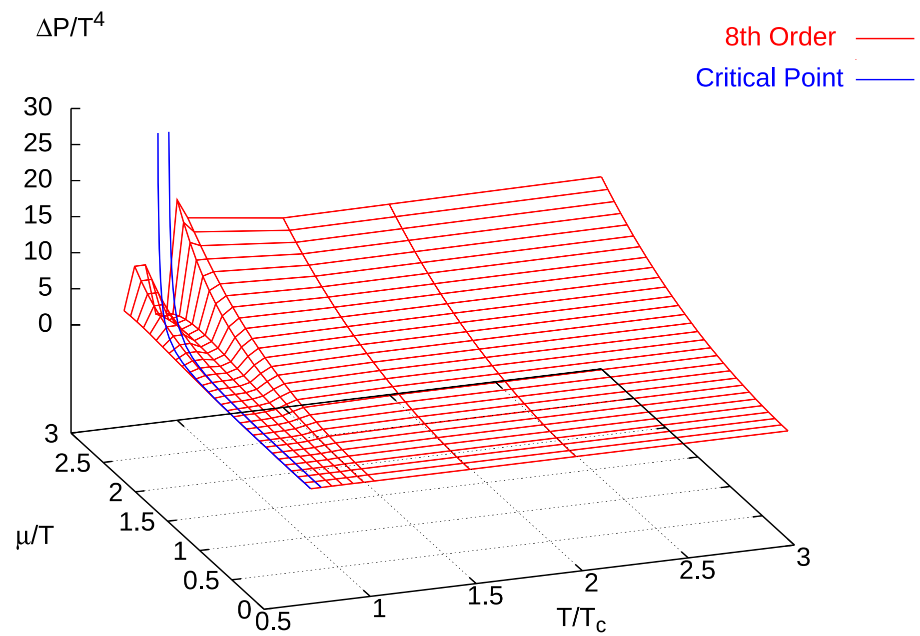

Continuum Limit

Recall,

- Chemical potential on lattice : Multiply each $U_4(x)$ by $f(a\mu)$ and $U_4^\dagger(x)$ by $1/f(a\mu)$, where $f(a\mu) = 1 + a\mu + \mathcal{O}(a^2)$. (Gavai, PRD '85)

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In continuum, $f(a\mu) = 1 + a\mu \rightarrow f''(0) = 0$.
On lattice, in general, all derivatives exist and depend on the nature of function : prescription dependence !

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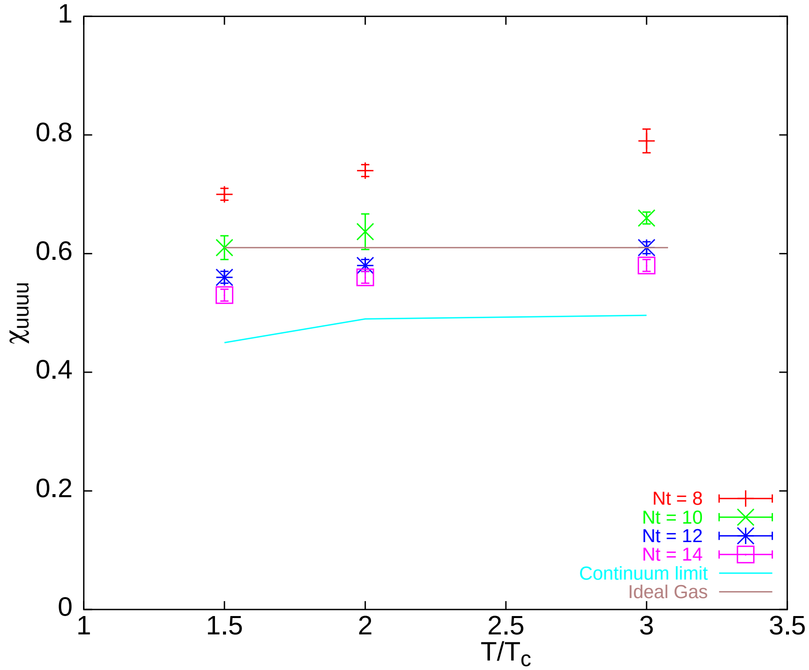
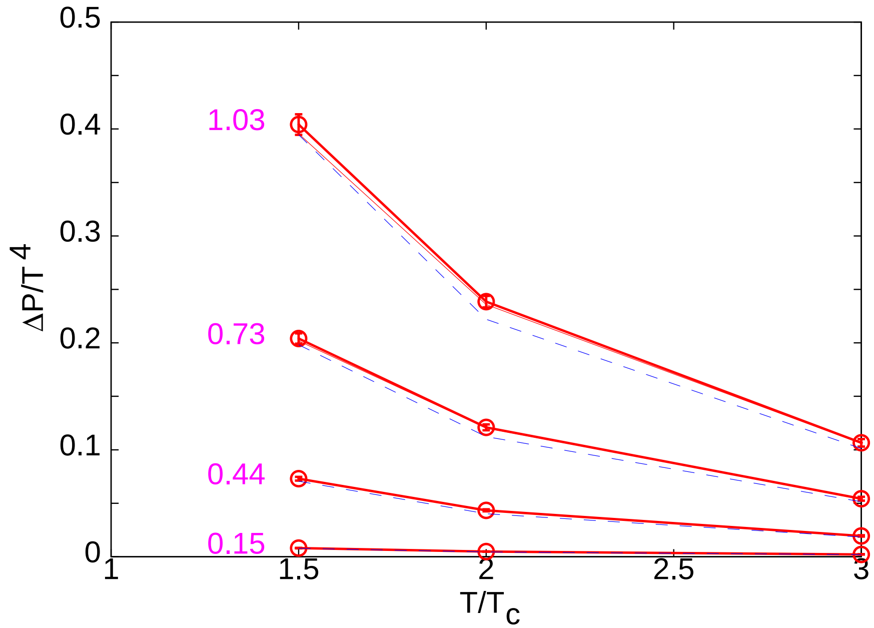
Our Results

Our results for χ_{uuuu} and ΔP: Gavai and Gupta, PR D68, '03





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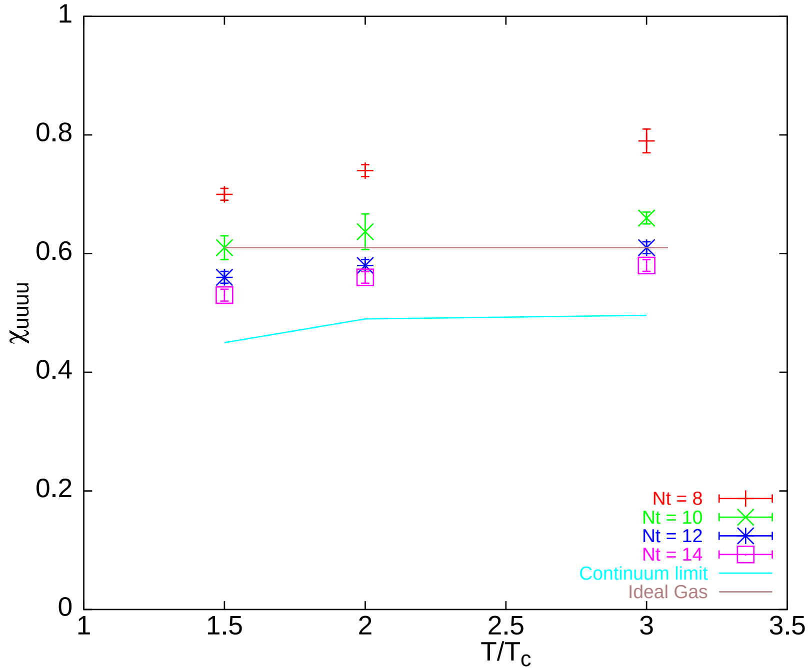

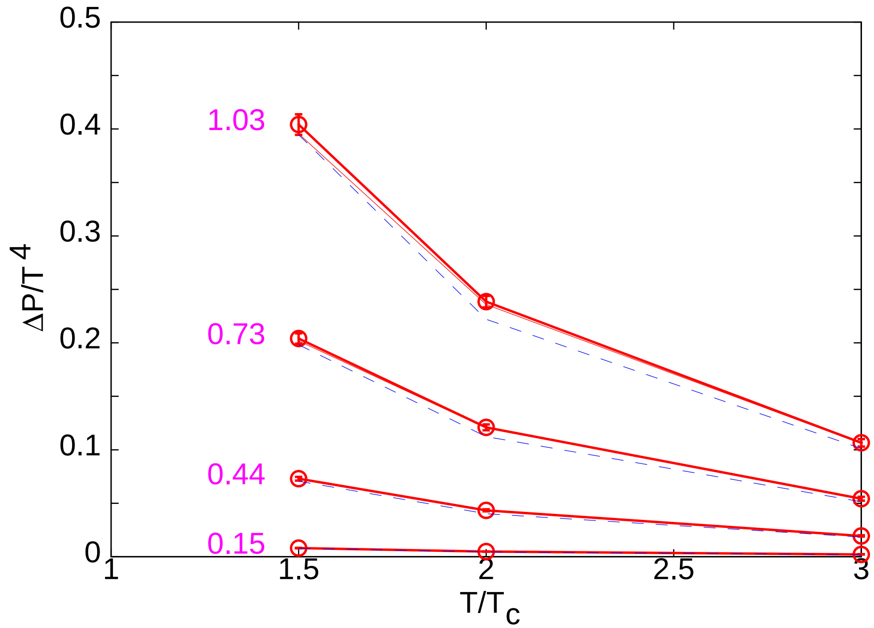

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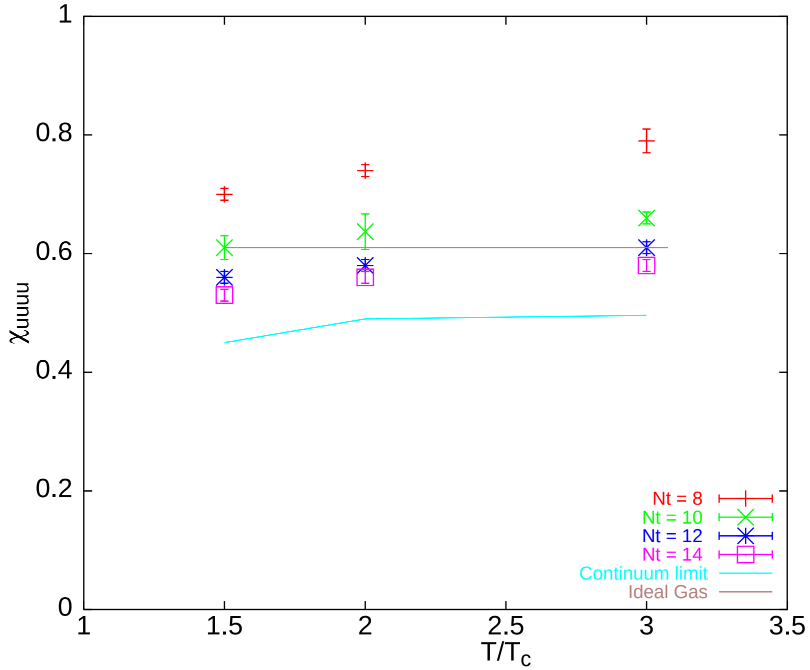

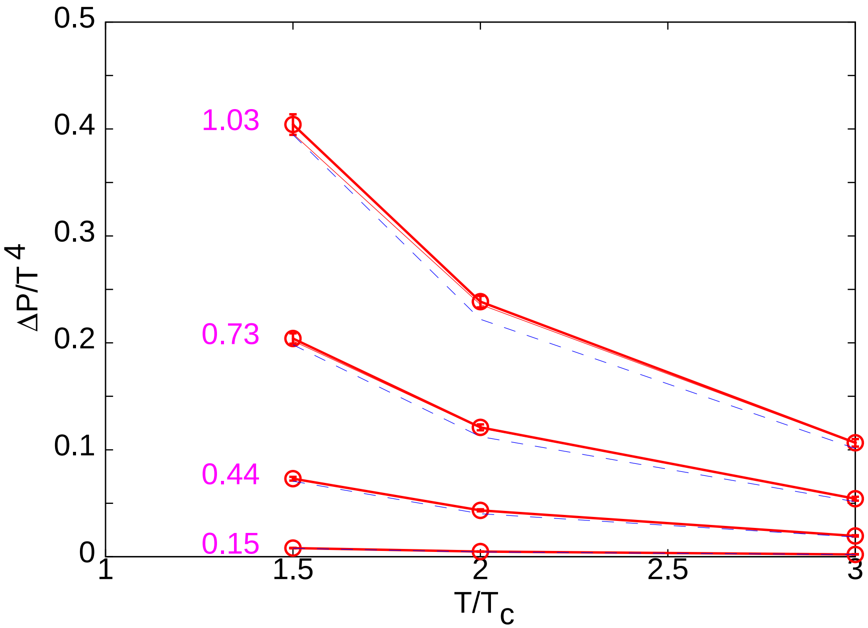

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♡ Our results for P agree with Fodor-Katz (PL B568, '03) and the recent Bielefeld results (PR D68, '03).

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