Foundation of Proton Model

\[ F_2(x, Q^2) = x \sum \frac{e q^2}{Q^2} q(x, Q^2) \]

\( q(x, Q^2) \) determined by target wavefunction \( \Psi_{g1p} \)

All interactions after photon acts in consequence

\[ \rightarrow \text{Phase or power-law suppressed!} \]

BHAPS \( \Rightarrow \) QCD net proton model + DGLAP
Conventional Interpretation of DIS

\[ F_2(x, Q^2) = \sum_{\text{parton types}} \sum_{\text{momentum fraction}} q^2 \times q(x, Q^2) \]

\[ q(x, Q^2) = \sum_{n \geq 2} \left[ \int d^2 k_n \int dx_1 \left| \psi_n(x_1, k_n, x) \right|^2 \right] \delta(x_n - x) \]

\[ \psi_n(x_1, k_n, x) \text{ eigenfunctions of } H_{\text{LF}} \]

\[ q(x, Q^2) \text{ determined by target LFUFTs + DGLAP} \]

FSE: higher-twist + irrelevant phase
Light-Cone Wavefunctions

encode all helicity, transversity distributions

\[ q(x, \lambda) \]

\[ = \sum_{n, \xi} \int \left| \psi_{n, \xi}(x; \lambda) \psi(x, \lambda) \right|^2 d\xi dx d^3 \lambda \]

\[ \Theta(\Delta^2 - m^2) \]

\( q \lambda \lambda \) \( \lambda \lambda \lambda \)
Unexpected Role of Final State Interactions in Deep Inelastic Scattering

- Single-spin asymmetry \( S_\rho \cdot \bar{q} \times \bar{P}_2 \)
- Diffraction at Leading Twists
- Nuclear Shadowing (interference of diff channels)
- Energy Loss \( P_T \) Broadening

Diffraction, Nuclear Shadowing, Pomeron not in nuclear wavefunction!

\[ \Delta L_{\text{offe}} = \frac{2u}{Q^2} = \frac{1}{M_X B_Y} \]
Explicit calculation of $F_2$ SSA

Collins, X.D.

Overlap of wave functions with $\Delta x_2 = 1$

\[ \text{Re}(x_1 - x_2) \]

$x_1 - x_2$: IR Finite

\[
i S_P = \overrightarrow{q} \times \overrightarrow{P_q} = i S_P \: \overrightarrow{q} \times \overrightarrow{r}
\]

\[
\overrightarrow{P_q} = \overrightarrow{q} + \overrightarrow{r}
\]

\[
\bar{P}_g = A_n \sim \frac{\alpha_s(r_g^2) \: x_{by} \: M \: l_1 \: l_2 \: r_g^2}{r_g^2 + M^2}
\]

 بيانكين سكلىينغ في $\text{finite } r_g$

Same matrix element as $\alpha_P = F_2 (x)$
\( \sigma = \epsilon_{\mu \nu \lambda \sigma} P_{\mu} S_{\lambda} P_{\nu} \tau_{\sigma} = M \hat{s} \cdot \vec{q} \times \vec{r} \)

\[ A_{\mu} = P_{\mu} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \]

\[ = C_T \alpha_s (1 - \alpha^2) (\Delta M + m) \frac{r_x}{(\Delta M + m)^2 + r_1^2} \]

\[ \times \left[ \frac{r_1^2 + \Delta (1 - \Delta) (-M^2 + \frac{m^2}{\Delta^2} + \frac{\lambda^2}{1 - \Delta})}{\Delta (1 - \Delta) (-M^2 + \frac{m^2}{\Delta^2} + \frac{\lambda^2}{1 - \Delta})} \right] \]

\[ \times \left[ \frac{1}{r_1^2} \ln \frac{r_1^2 + \Delta (1 - \Delta) (-M^2 + \frac{m^2}{\Delta^2} + \frac{\lambda^2}{1 - \Delta})}{\Delta (1 - \Delta) (-M^2 + \frac{m^2}{\Delta^2} + \frac{\lambda^2}{1 - \Delta})} \right] \]

\[ A = x g \beta, \quad \frac{\lambda^2}{\Delta^2} = \langle e^{-\beta \Delta} (k_z - \vec{r}_1)^2 \rangle \]

BHS
Pauli Form Factor $F_2(q^2) \quad \kappa = F_2(0)$

Resolved overlap of LCWFs with $\Delta L_2 = 1$

\[ \begin{align*}
\psi^{1/2} & \quad -1/2 - 1/2 - 1/2 \quad L_2 = 2 \\
\psi^{-1/2} & \quad -1/2 - 1/2 - 1/2 \quad L_2 = 1 \\
\psi^{1/2} & \quad 1/2 1/2 -1/2 \quad L_2 = 0 \\
\psi^{-1/2} & \quad 1/2 1/2 -1/2 \quad L_2 = -1
\end{align*} \]

Same matrix elements appear in SSA

\[ \kappa_p = \sum q \kappa_{g/p} \]

\[ A_{\text{un}} = \text{SSA} \quad \alpha \sum q^2 \kappa_{g/p} \alpha \]
Longitudinally Pol. Target: SSA for $\pi^+$

Target SSA: CLAS (4.3 GeV, and 5.7 GeV) consistent with HERMES (27.5 GeV)

Sivers only interpretation require large $A_{UT}$

$A_{UL}$ z-dependence also consistent both in magnitude and sign with predictions based on Collins mechanism

$$A_{UL} \propto \sin \theta_\gamma \times A_{UT} \propto \sin \theta_\gamma \frac{f_{1 T} u(x)}{u(x)}$$

H. Avakian CIPANP 2003 May 21
Single-Spin Asymmetry in Semi-Inclusive DIS

Distinguish Sivers vs Collins Effects

- **Sivers:** \[ \vec{S} \cdot \vec{q} \times \vec{P}_{jet} \]
  - No hadronization necessary
  - Observe quark direction \( \vec{P}_q = \vec{P}_{jet} \)

- **Collins:** T-odd fragmentation necessary
  \[ H^+ : \vec{S}_q \cdot \vec{P}_H \times \vec{P}_q \]

- **Sivers:** \( \sin (\phi_H^t - \phi_S^t) \) mod. of \( \phi_S^t \)
- **Collins:** \( \sin (\phi_H^t + \phi_S^t) \)

- **Sivers:** \( A_{UL} \), \( A_{UT} \) same in \( \nu p \rightarrow H \rightarrow \text{charged current} \)

- **Collins:** \( A_{UL} \), \( A_{UT} = 0 \)

\[ \frac{Z_{\text{CLCR}}}{Z_{\text{CL+EN}}} \approx 20 \text{ natural current} \]
New perspectives on Final-State-Interactions

\[ \frac{f_{q/N}(x_B, Q^2)}{f_{q/N}(x_B, Q^2)} = \frac{1}{2\pi} \int d\gamma^+ e^{-ix\cdot p + y^-} \]

\[ \langle N(p) | \bar{q}(y^-) \delta^+(\not{p} - \not{q}) | N(p) \rangle \]

Usual argument \( A^+ = 0 \) gauge

no effect from FSI!

phase irrelevant

\[ \rho_{q/N}(x_B, Q^2) = \sum \int d\alpha^+ d^n k_2 \left| N_+(x, k_2) \right|^2 \sum \delta(x - x_B) \]

Identify \( f_{q/N} \) with LC Prob. Dist.
Explicit Calculation of FSI

\[ q^+ \leq 0 \text{ frame} \]

LFT OPN: gluon exchanged after photon acts
Find non-zero leading-twist FSI effect
Leading twist diffraction, Eikonal form
Shadowing correction, not Coulomb effect
Gauge-independent:

Checked: Feynman, LCC \( [A^+ = 0] \) (ML \( \rightarrow \) PV prescription)

Lesson from LCC:

\[ g_{\mu \nu} - \frac{n \cdot H + k \cdot n \times H}{n \cdot k} \]

At pole: \( n \cdot k = k^+ = O(\frac{1}{\Lambda}) \).
Consider

In Feynman gauge, must keep

\[ |M|^2 = \left| \sin \left[ \frac{g^2 W(r_1, r_2)}{2} \right] \right| \frac{g^2 W(r_1, r_2)/2}{m_0^2} \]

\[ W(r_1, r_2) = \int d^2k_1 \frac{1-e^{ik_1 \cdot r_1}}{k_1^2} e^{ik_1 \cdot r_2} \]

= \frac{1}{2 \pi} \log \left( \frac{m^2 + m_0^2}{m^2} \right)
In light-cone gauge ("kouchegos" prescription) must keep

These graphs are suppressed in Feynman gauge but in l.c.g.

\[ \sigma_{lc}^{\mu\nu} = \frac{i}{k^2 i t} \left[ -g^{\mu\nu} + \frac{\pi^{\mu\nu} k^\mu + \pi^{\mu\nu} k^\nu}{n \cdot k} \right] \]

and \( n \cdot k = k^+ = 0(\frac{1}{t}) \) for on-shell states!

* Result: identical answer as Feynman gauge
  * Not included in l.c.g. wavefunctions!

m-L, PV prescriptions differ by n.s. D. Y. Y. stones
Effect of Rescattering on the DIS Cross Section

\[ q^+ p \rightarrow x \rightarrow p_1, \quad P_i : \text{aligned jet} \]
\[ \rightarrow \ldots P_i : \text{spectator jet} \]
\[ p \rightarrow p' \]

\[ Q^4 \frac{d\sigma}{dQ^2 dx_2} = \frac{\alpha}{16\pi^2} \frac{1-y}{y^2} \frac{1}{2m} \int \frac{dP_{1-}}{P_2} \frac{d^2 r_1}{d^2 r_2} \frac{d^2 r_2}{d^2 r_2} |M|^2 \]

\[ |M| = \left| \sin \left[ \frac{g^2 W(r_2^2)}{2} \right] \frac{m_{\text{Boon}}(P_1, \bar{r}_1, \bar{r}_2)}{g^2 W(r_1^2, r_2^2)/2} \right| \]
\[ \Rightarrow \quad \alpha < 1 \text{ for all } \bar{r}_1, \bar{r}_2 \]

Eqn. to sum q cuts of forward virt. comp. amp.

Find shadowing
- only arises from diagrams involving
- \( P_1 \) virtual to \( P_1 \)

Cuts give Glueball-Glueball shadowing

Some result in Feynman, e.g. \( (\alpha L) \)
\[ \uparrow \text{ from } \frac{\not{P}_1 \cdot \not{P}_2}{4\pi} \text{ term} \]
Non-universal Pomeron Coupling

DIS Diffraction

2nd gluon occurs after $g^*$ interaction exchange

Cut gives imaginary phase!

Coupling depends on coin diph. moment $19$

Pomeron not part of proton $\psi$

Not universal!

$Pg/p(x), Pa/p(x), Pa1/p(x), Pom/p(x)$
Diffractive Dissociation (large rapidity gaps) is leading twist in QCD.

\[ \frac{d\sigma}{dM^2} \sim \left( \frac{1}{M^2 + Q^2} \right)^2, \]

\[ \sigma_T \sim \frac{1}{Q^2} \]

aligned jet regime

large color dipole

- Coherence for \( \text{L_{coh}} = \frac{2V}{Q^2} > R_N \)
- Shadowing in nuclei for \( \text{L_{ene}} = \frac{2V}{Q^2} > R_A \)

Review by Hebecker Kopeliovich, Niedermayer

By +kogut

Hoyer, Nagler, 8de
Diffraction leads to nuclear shadowing:

Destructive interference leads to shadowing at low $x$.

Shadowing of quark, gluon distributions

Reggeon exchange leads to out-of-shadowing

Lu, 81b

Senat, Yang, 81b
Proposed by Ji and Yu (also Collins)

\[ \gamma \rightarrow q L \]
complex phase

Augment LFWF (lcg) with phase

\[ L_1 = \mathcal{P} \exp \left[ i g \int_0^\infty d^2 \gamma \cdot A_1^2 (\gamma^- = \infty, \gamma^+) \right] \]

where

\[ A_1^2 = -\frac{\alpha}{2\pi} \Theta(\gamma^-) \nabla_1 \ln \mathcal{R}_1 \]

\[ A^+ = A^- = 0 \]

- corresponds to Coulomb field \( A \) charged particle moving at \( v = c \)

Equivalently

\[ D^{\mu\nu}(q) = -\frac{i}{q^2} \left( g_{\mu\nu} - q_{\mu} q_{\nu} + q_{\mu} q_{\nu} \right) \]

- non-causal b.c. (BHMPs)

- process specific - not universal
\[ \Psi_{LF} \leftrightarrow \text{FSIs} \Rightarrow \Psi_{LF}(x, k, \lambda_0) \]

Phase: \[ e^{iW_a} e^{iW_b} = e^{iW} \]

Augment LFWF:

\[ \Psi_{LF} \Rightarrow \Psi_{LF} \frac{e^{iW} - 1}{iW} \]

\[ F_2 \Rightarrow F_2 \left| \frac{\sin \frac{W}{2}}{\frac{W}{2}} \right|^2 \]

Two Wilson lines! Like external field.

* Shadowing not contained in \( \Psi_{LFWF} \)
  - not in LQCD or \( \Psi_{BG} \)
  - not in local operators

\[ T++ \]
Conclusions

- T-odd Physics
  - new insight into QCD
  - role of ISI, FSI

- Sivers + Collins effects both present in QCD
- BHS, Collins
- mechanisms understood at perturbative level
- $\alpha_s (\mu^2)$?

- SSA related to Diffractive DIS, Gedeweg
- $k_T$ distributions, energy loss
- from ISI, FSI

- Bodwin, Lepage, Sidis; Boer, Hwa, Sidis
- Metz, Polyakov et al; Boer, Mulders

- DIS structure functions versus Binosi
- augmented LEPTOS
  - Ji, Tang
  - Subre QCD in external fields

- Many questions, principle
  - Sum rules; factorization (process depend!); DGL in nuclei
When is $\Psi_{LC}$ maximal?

$$\Psi_n = \frac{\Gamma_n}{M^2 - M_n^2}, \quad M_n^2 = \sum_i m_i^2$$

Denominator minimum when

$$X_i = X_c = \frac{m_{2i}}{\sum_j m_{2j}}$$

and

$$M_n \to \tilde{M}_n = \sum_j m_{2j}$$

In rest frame: $x_i = \frac{k_i^0 + k_i^j}{p^+} \Rightarrow x_i$

when $k_i^0 = 0$!

Thus maximal w.f. is equal $\gamma_i$.

Heavy particles have most of $x_i$.

Isgur - Wise symmetry

$$B \to \pi^+ \pi^- \pi^0, \quad x_{\pi^+} \approx \frac{m_{\pi^+}}{m_B}, \quad \Psi_{18}(x_i, \vec{k}_i, x_c)$$
Intuition on LC Wavefunctions

\[ \Phi \rightarrow x_i, k_i, \lambda_i \quad a < x_i < 1 \]
\[ \sum x_i = 1, \sum k_i = 0 \]

\[ \Phi = \frac{\Gamma(x_i, k_i)}{M^2 - \sum \frac{k_i^2 + m^2}{x_i} + i\epsilon} \]
\[ m^2 = (\xi, k_0^2) \]

\[ \Phi \text{ peaks at } x_0 = \frac{m_1 + i}{\sum M_{1i}} \quad (m_i^2 = k_i^2 + m^2) \]

"equal velocity" \( \Rightarrow \) minimum mu. mass

\[ x_i = \frac{k_i^+}{P^+} = \frac{k_i^0 + k_i^\tau}{P^0 + P^\tau} \]

\[ \Delta y_i = \ln x_i = y_P - y_i \]

Non-relativistic:

\[ x_i = \frac{k_i^0 + k_i^\tau}{M} = \frac{m_i + l_i^2}{M} \quad \text{peaks at } \frac{m}{M} \]

\[ x = 0 \text{ is } k^\tau \geq 0 \text{ for } M \geq 0. \]
Other Applications of LF Quantization

Light-Front Thermodynamics

Set boundary conditions at fixed $\tau = t + x/c$, not $t$

$$Z_{LF} = \sum_n \exp \left( -\frac{m_n^2}{T_{LF}} \right)$$

Light-Front Lippmann-Schwinger

$$T = H_x + H_c \frac{1}{m^2 - \sum \frac{m_n^2}{x} + c\omega}$$

Variational Solutions to Bound-Stack Problem:

$$\text{minimize} \quad \langle \psi_{\text{trial}} | H_{LF} | \psi_{\text{trial}} \rangle$$

Construct $\langle \psi_{\text{trial}} | h \rangle = \psi_{\text{trial}}(k_i, k_f, x)$

Using normal from L2 ladder relation's
Variational Method:

Minimize \( \langle p | H_{LF} | p \rangle \)
by varying trial LFWFS!

Construct model

\[
\psi(M^2) = \frac{f_2}{3\pi} \left( \frac{M^2 + \epsilon^2}{(M^2 + \epsilon^2)^2} \right) P(x_i)
\]

\[
M^2 = \frac{3}{\xi_i} \left( \frac{4\sin^2 \theta}{x_i} \right)
\]

Construct higher Fock states, orbital excitations via \( \frac{1}{D} H^{\text{L}} \psi(\Psi) \).

Calculate observables, \( F_i(t), q(x_i), \ldots \)
\[ H_{\text{lc}} |\Phi\rangle = m^2 |\Phi\rangle \]

\[ \langle m | H_{\text{lc}} | n \rangle \langle n | \Phi \rangle = m^2 \langle m | \Phi \rangle \]

general solutions obtained: mass spectrum, wavefunctions

- QED (1+1), QED (1+1)
- QCD (1+1) adjoint matter
- QED (2+1)

Given \( Y^j(x, \xi, \eta, \kappa) \), compute

- Form Factors
- Structure Functions, helicity structure
- Decay constants
- Exciton Amplitudes
- High \( k^2, x \sim 1 \) from QCD
Summary

Light-Front Quantization and Gauge Theory

* "Rigorous" repn of quantum field theory

$$H_{LF} |\Psi\rangle = M^2 |\Psi\rangle \Rightarrow$$

$$|\Psi_n\rangle = \Psi(n)(x, k^+, \lambda)$$

Boost-invariant, ghost-free repn of hadrons in terms of $Q$, $G$

* LFQ $\rightarrow$ Stochastic Model $\Rightarrow$ zero mode of $\phi_H$

$$\Rightarrow$$ new formulation of SSB, Higgs, W, G

* XSB $\rightarrow$ QCD $\rightarrow$ Effective Theory

$\Rightarrow$ needs fundamental QCD basis

* New solns of $H_{LF} |\Psi\rangle = M^2 |\Psi\rangle$

renormalization, models, DLCQ, TL

* FSI in DIS $\Rightarrow$ SSA, Diff., Shadowing

not in $H_{LF}$
Some advantages of Light-Front-Quantized QCD

- No fermion doubling
- Multiple fermion flavors
- Minkowski space
- Gauge-fixed: Physical Degrees of Freedom
  - Manifest Frame Independence: $p^+, \vec{p}$ arbitrary
  - $J^z$ kinematical
  - LF Vacuum Trivial + Zero modes
- Vanishing anomalous gravitomagnetic moment
  \[ B(0) = 0 \quad \text{as} \quad M \to 0 \]
- DLCQ discretization retains symmetries
  - Continuum limit: $k \to \infty$
- LF wavefunctions, continuum solus:
  - Amplitudes, phases as well as spectra
- Solus for QCD (1+1), SUSY (1+1)
  - QCD (3+1): Challenging!