

# Gravitational waves from cosmological phase transitions:

## Using LISA to probe particle physics

S@INT seminar - University of Washington

05/05/2022

Jorinde van de Vis

Based on:

R. Jinno, T. Konstandin, H. Rubira, JvdV

F. Giese, T. Konstandin, JvdV

F. Giese, T. Konstandin, K. Schmitz, JvdV

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*JCAP* 12 (2021) 12, 019

*JCAP* 11 (2021) 002

*JCAP* 01 (2021) 072

*JCAP* 07 (2020) 07, 057

arXiv:2108.11947

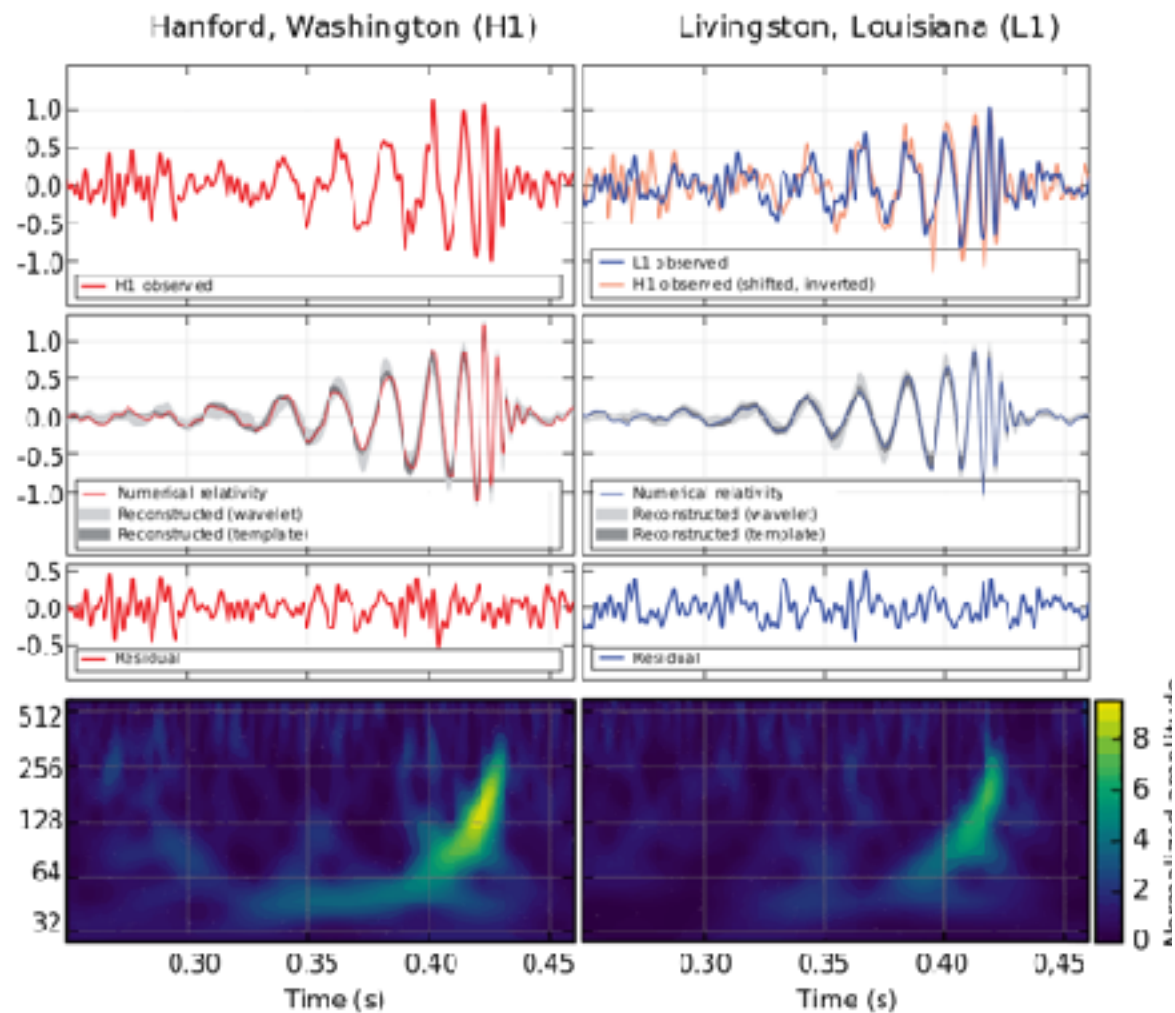
arXiv:2107.06275

arXiv:2010.09744

arXiv:2004.06995

# Gravitational waves

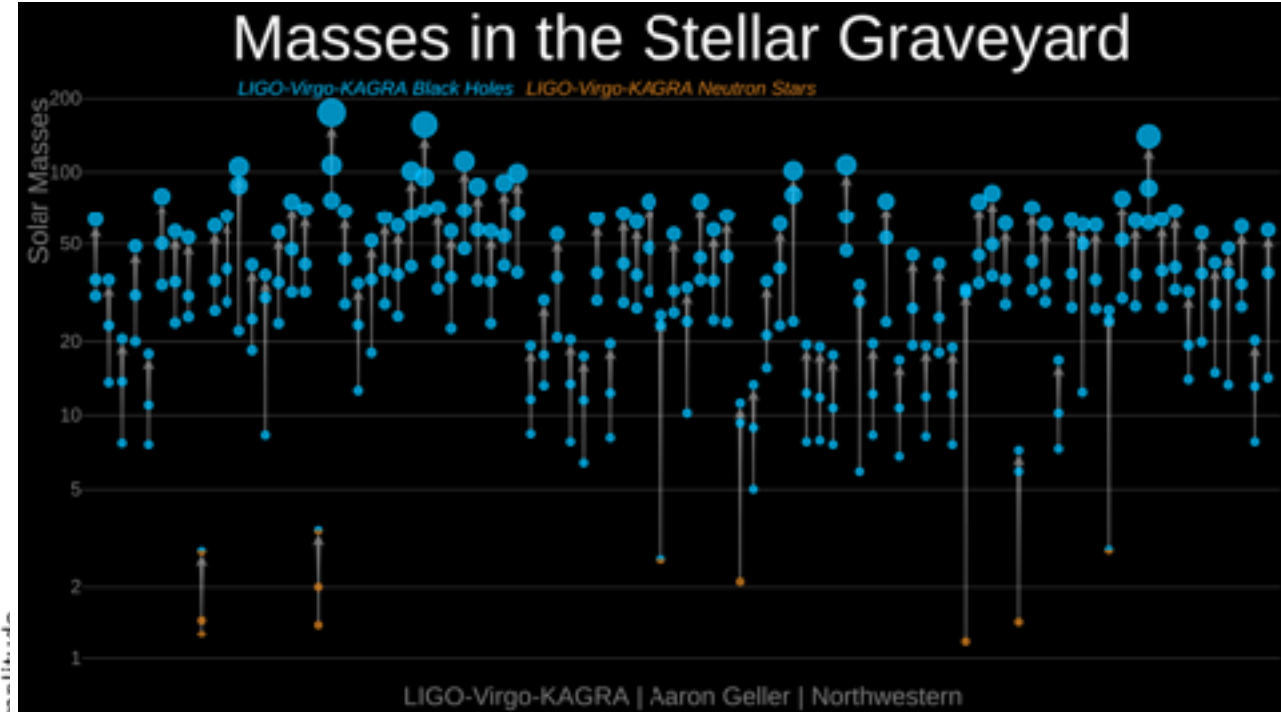
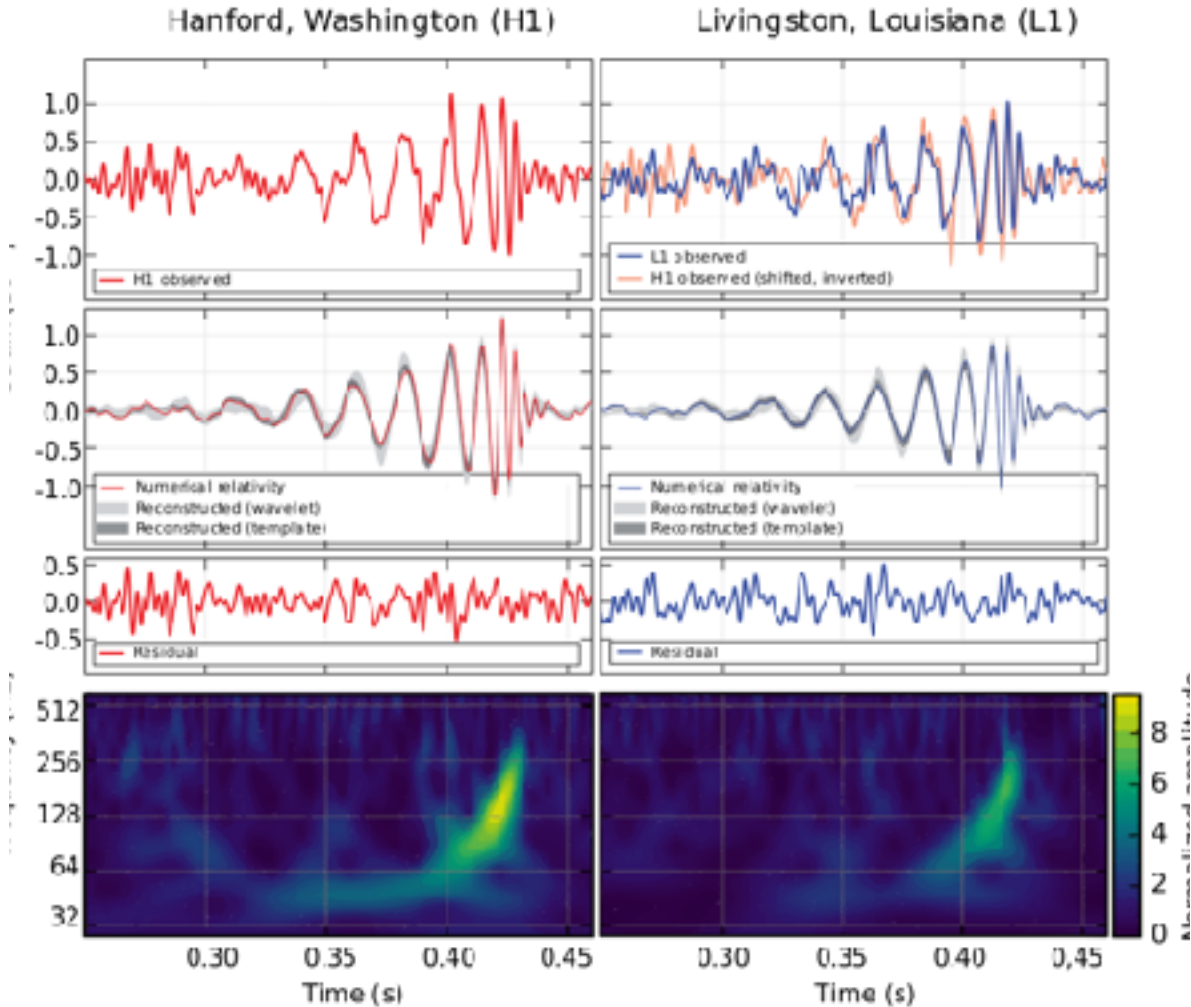
First black hole GW merger event: GW150914



LIGO/Virgo Collab. 2016

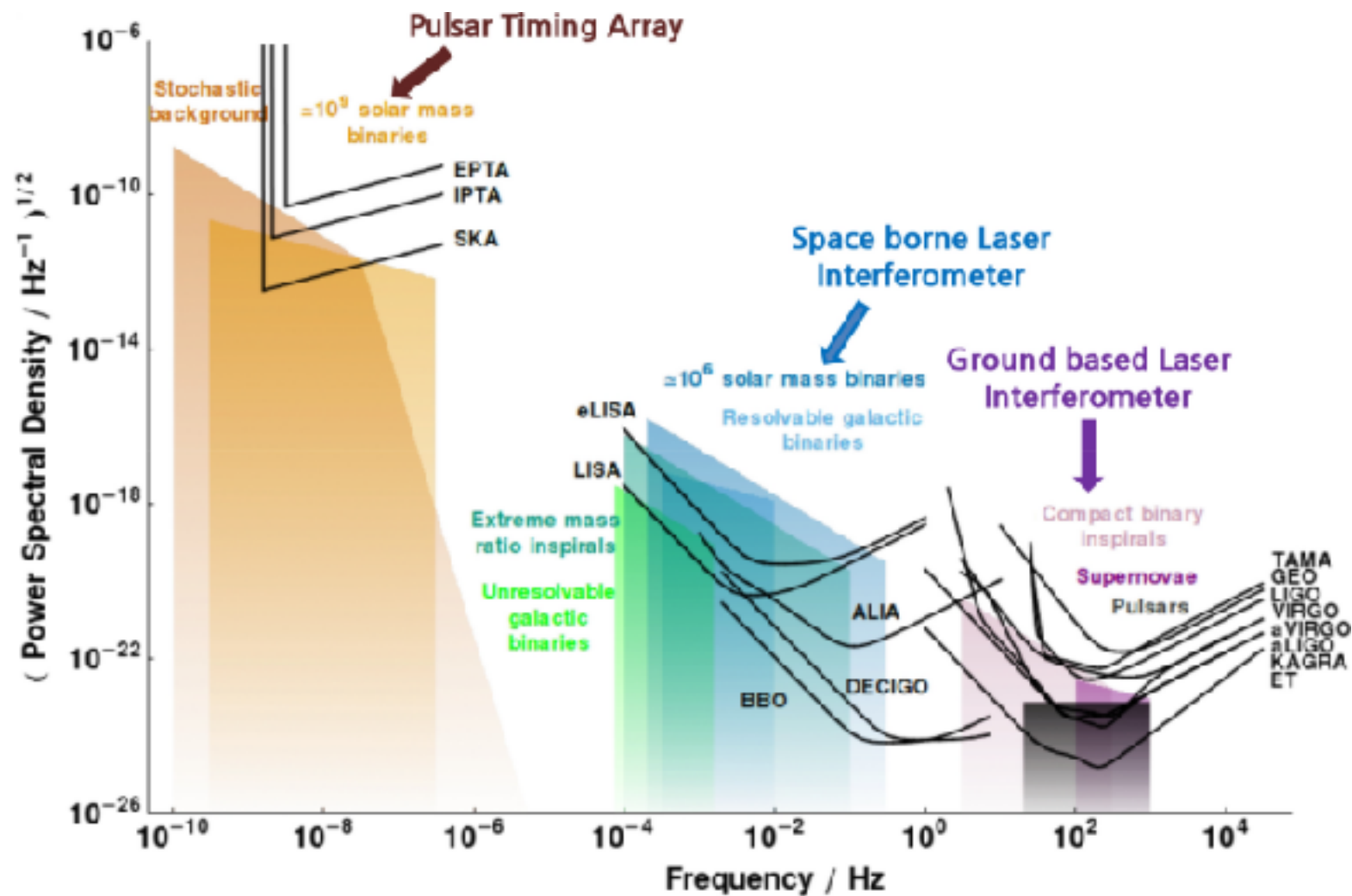
# Gravitational waves

November 2021: 90 events

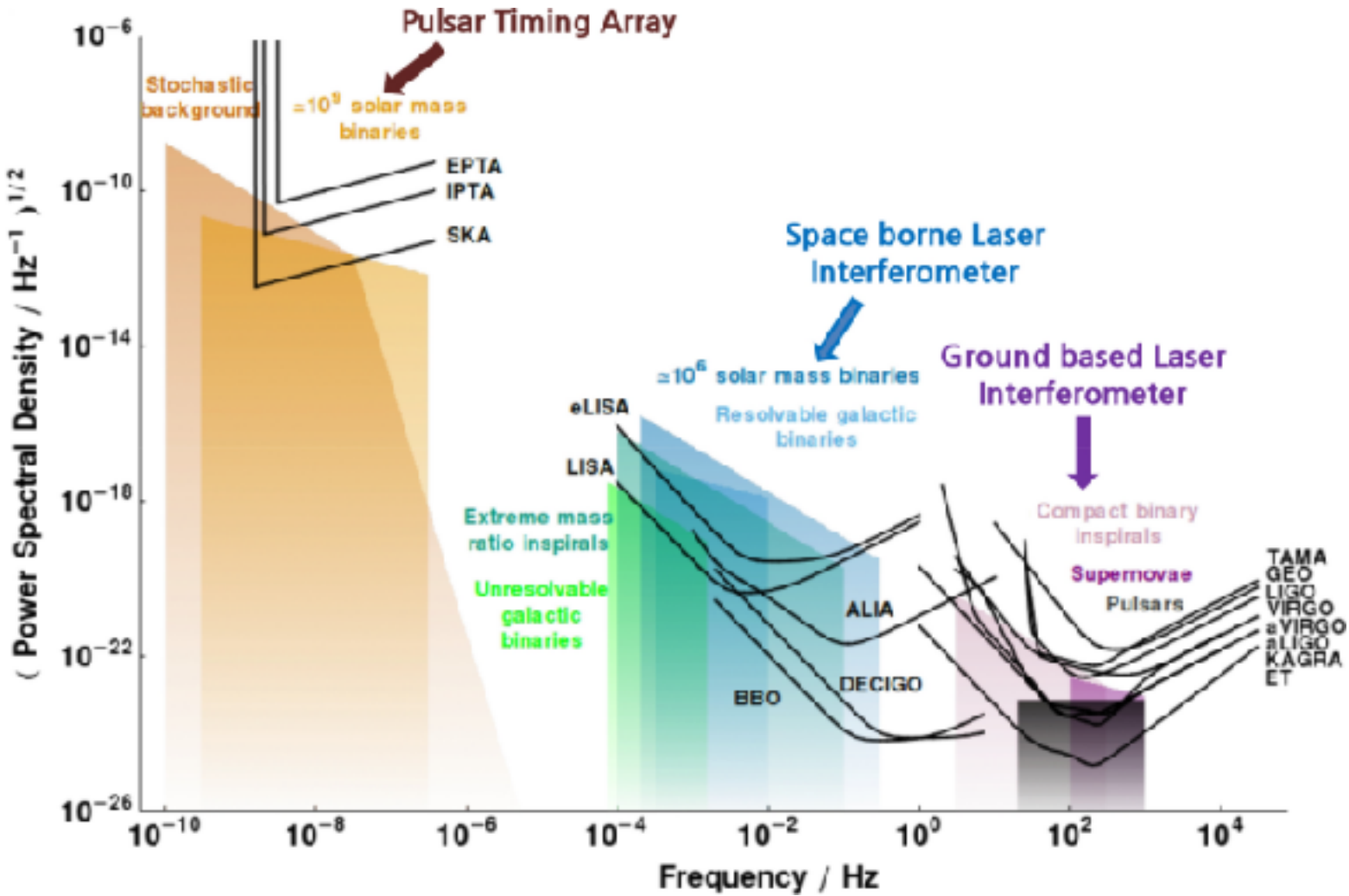


LIGO/Virgo Collab. 2016

# Ongoing and upcoming experiments



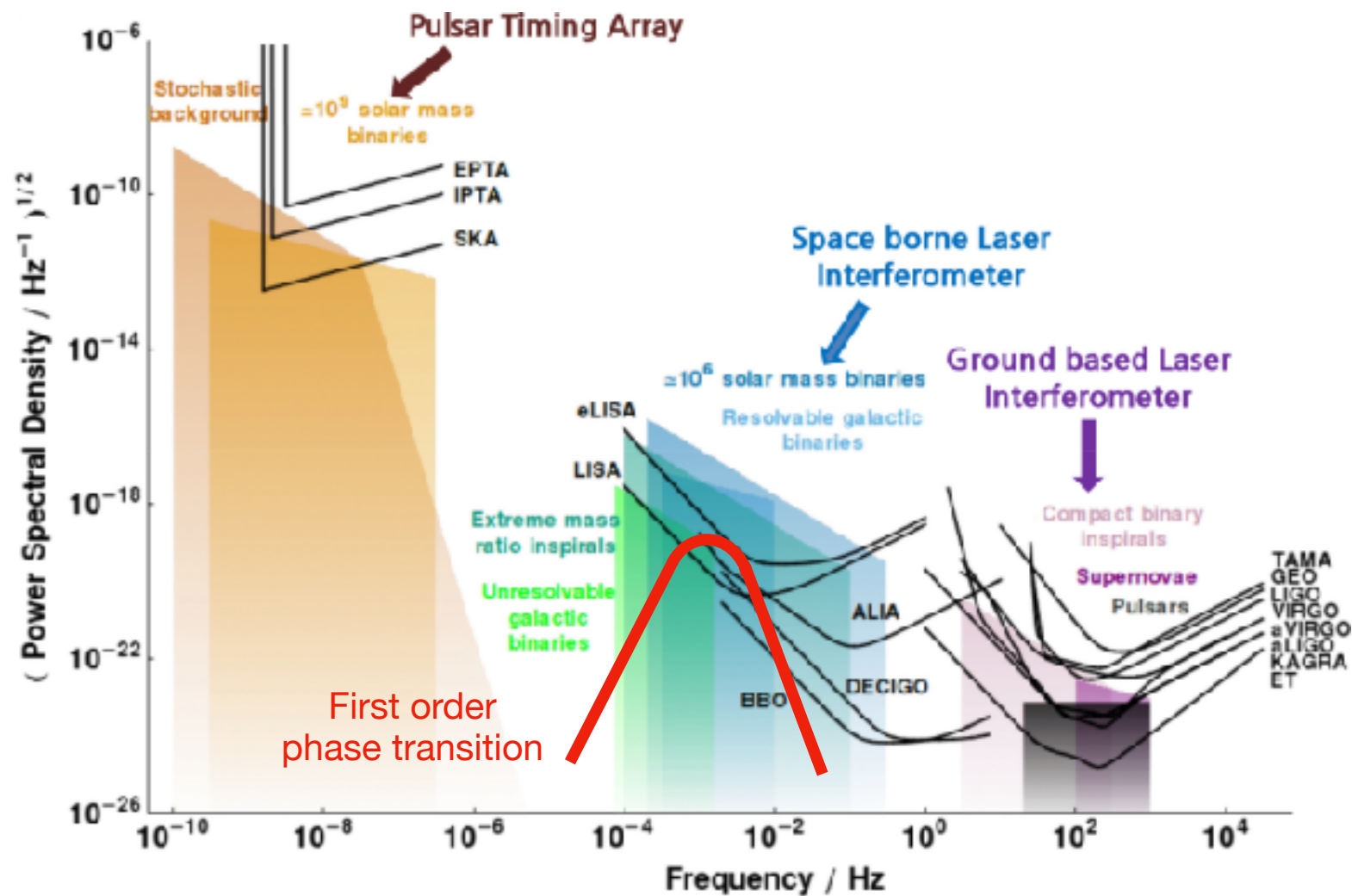
# Ongoing and upcoming experiments



- Size of the detector sets  $f_{\text{detector}}$
- Typical time scale and redshift of source set  $f_{\text{signal}}$

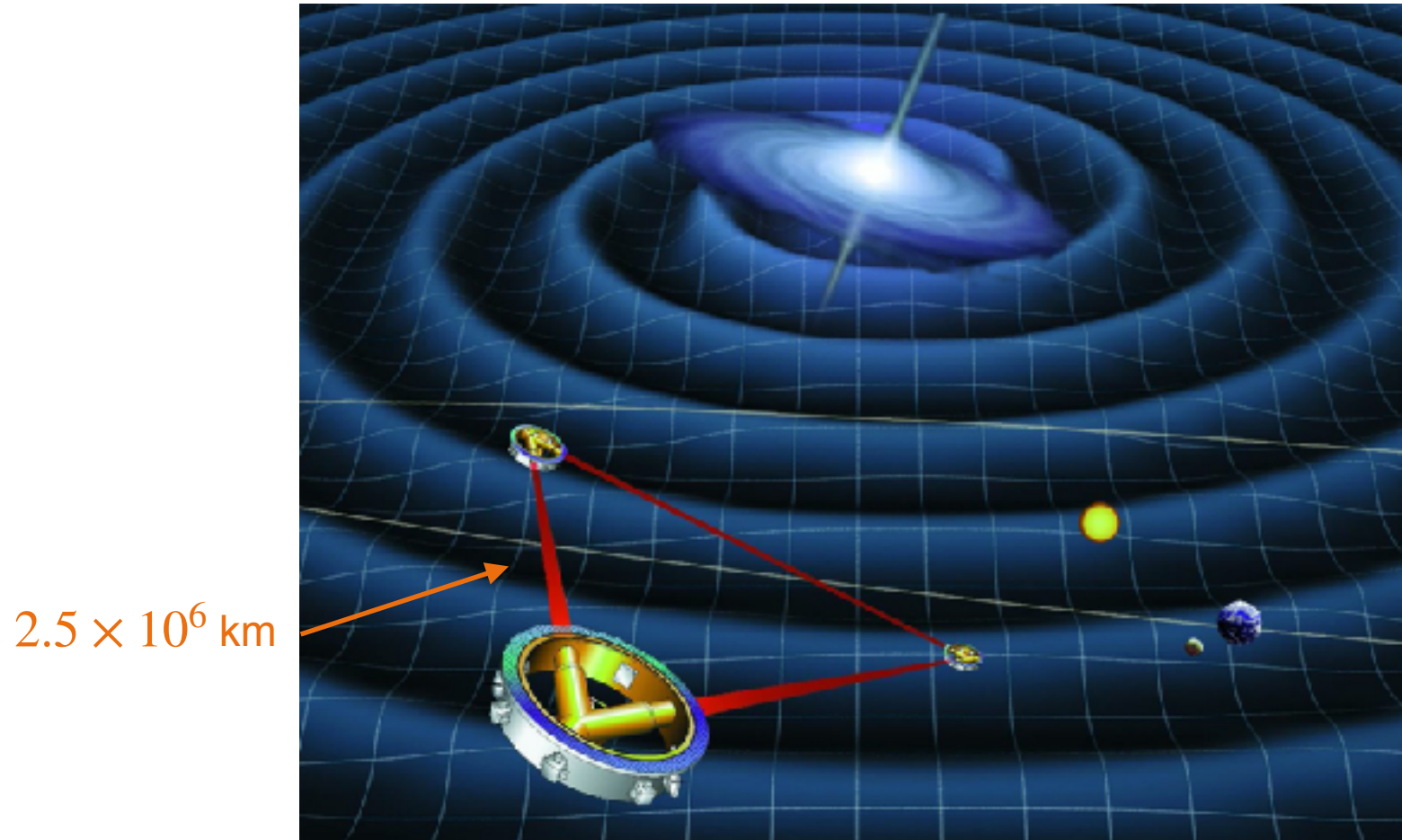
# Ongoing and upcoming experiments

Cosmological phase transition: observable with LISA?



# Laser Interferometer Space Antenna (LISA)

ESA mission, planned in the 2030s



NASA

# Gravitational waves

Expand the metric:  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ ,  $|h_{\mu\nu}| \ll 1$

Minkowski  
↓  
GW



# Gravitational waves

Expand the metric:  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ ,  $|h_{\mu\nu}| \ll 1$

Minkowski  
↓  
GW  
↑

$h_{\mu\nu}$ : only transverse and traceless component

# Gravitational waves

Expand the metric:  $g_{\mu\nu} = \overset{\text{Minkowski}}{\eta_{\mu\nu}} + \underset{\text{GW}}{h_{\mu\nu}}, \quad |h_{\mu\nu}| \ll 1$

$h_{\mu\nu}$ : only transverse and traceless component

Source\*:  $\square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$

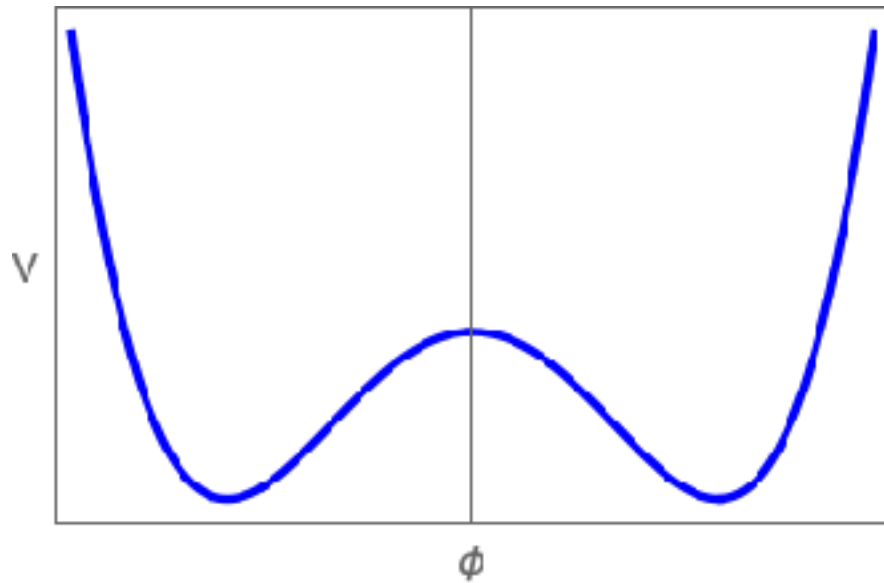
\*In the gauge where  $\partial^\nu \bar{h}_{\mu\nu} = 0$ , with  $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$

**How can a cosmological phase transition source gravitational waves?**

# Cosmological phase transition

Temperature-dependent Higgs\* potential

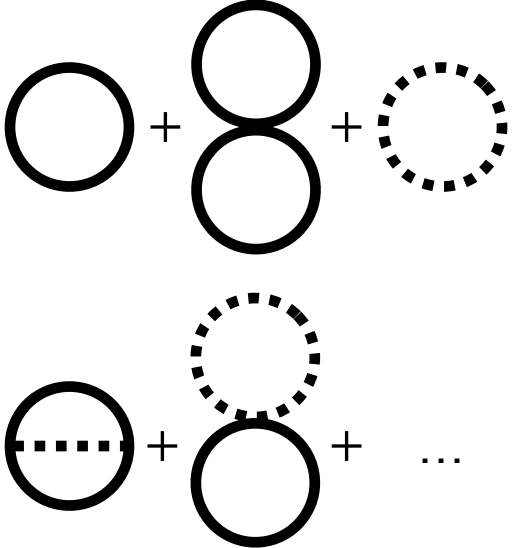
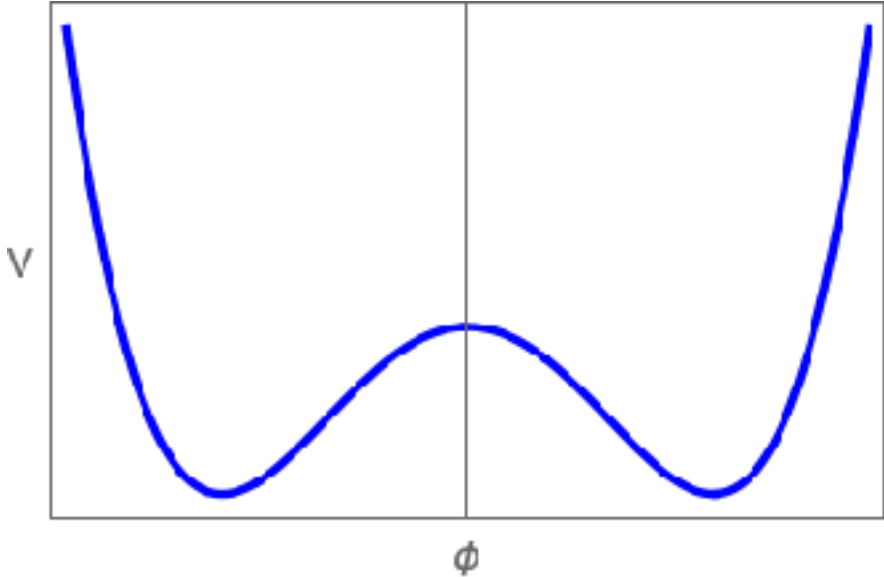
Zero temperature Higgs potential



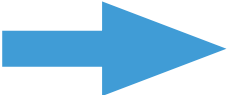
# Cosmological phase transition

## Temperature-dependent Higgs\* potential

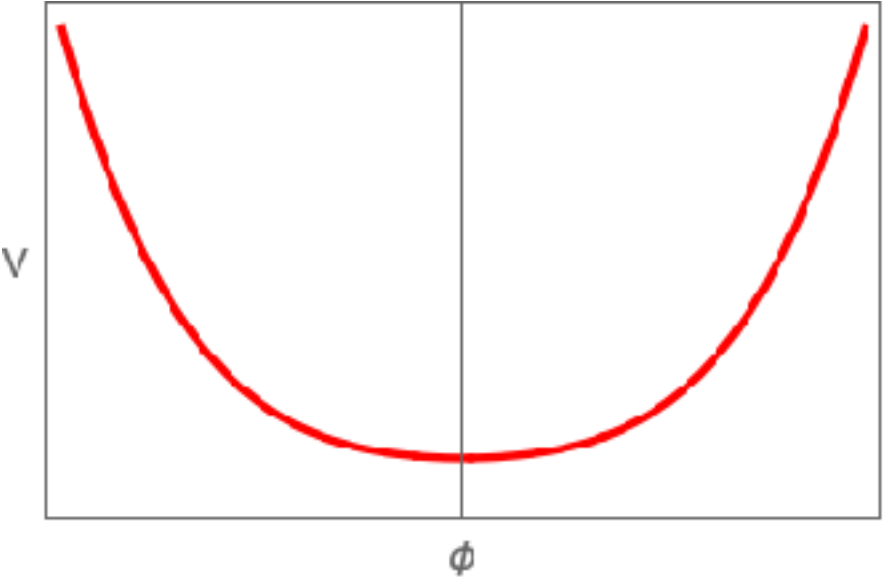
Zero temperature Higgs potential



(Temperature-dependent) vacuum bubbles

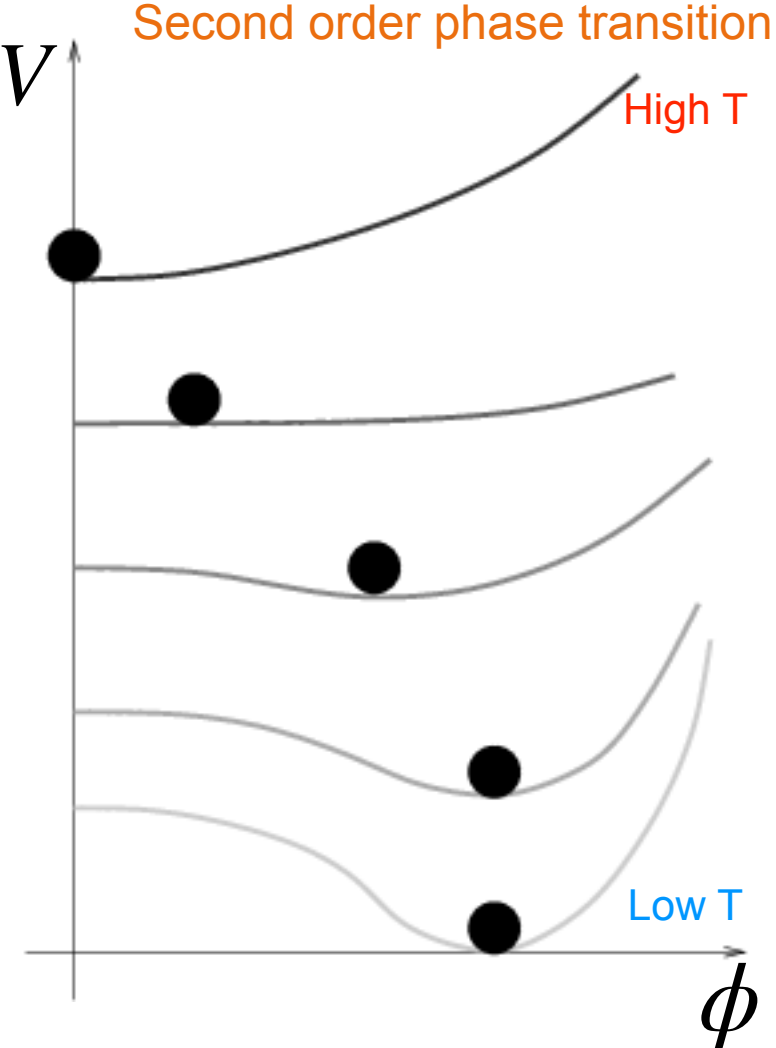
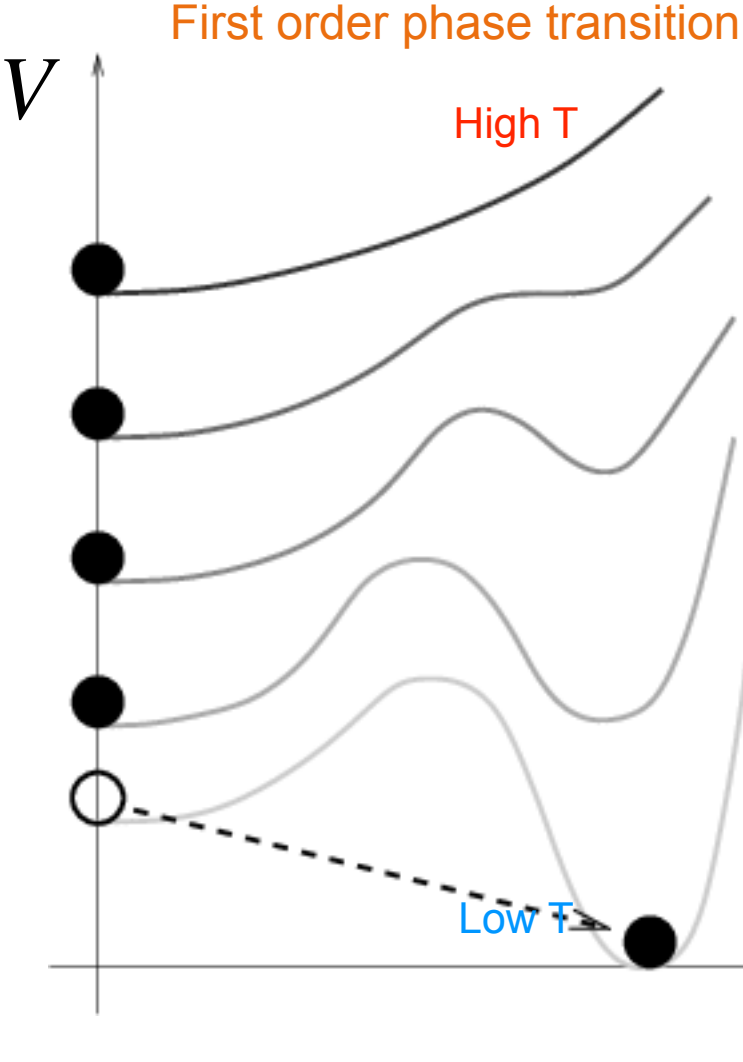


High temperature Higgs potential



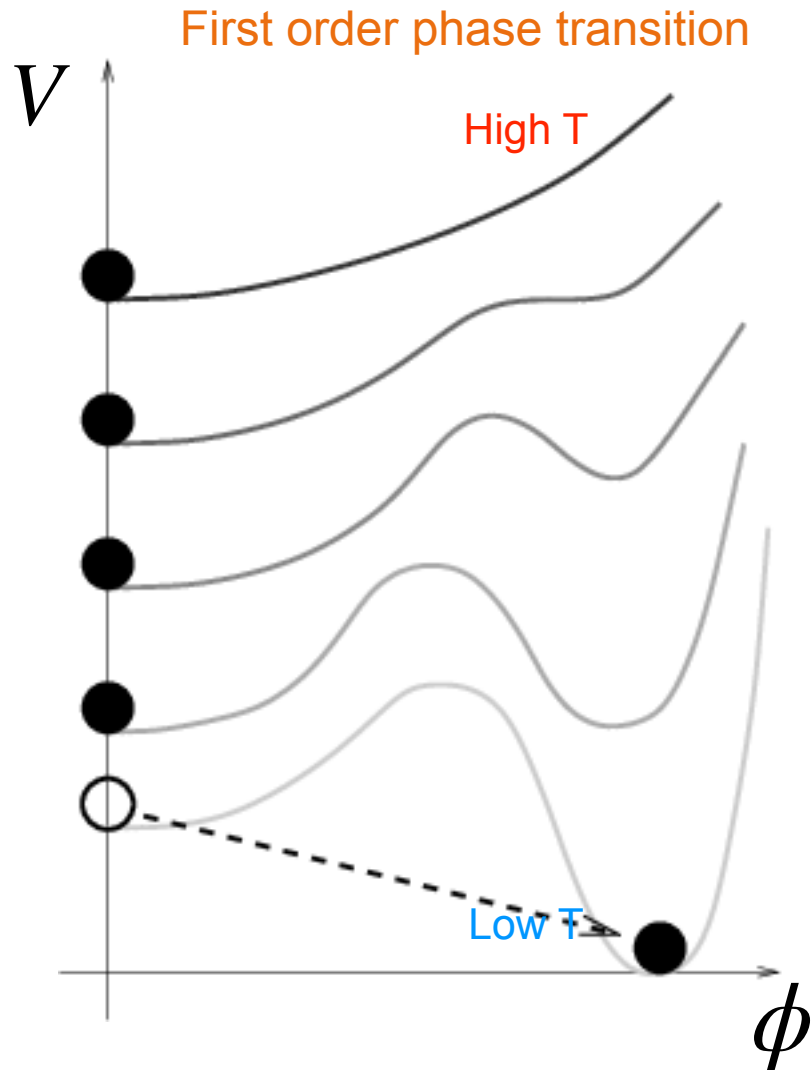
DESY. \*Or some other scalar field

# First order versus second order



# First order phase transition

## Bubble nucleation



# Motion in the primordial fluid sourcing gravitational waves

## Hydrodynamic simulations

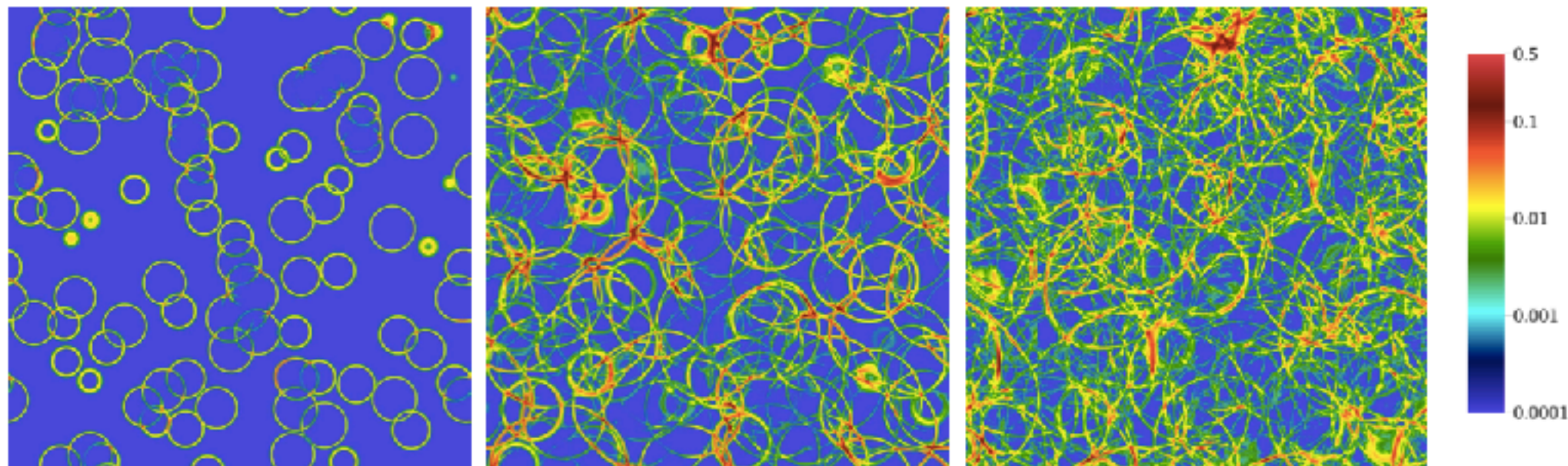


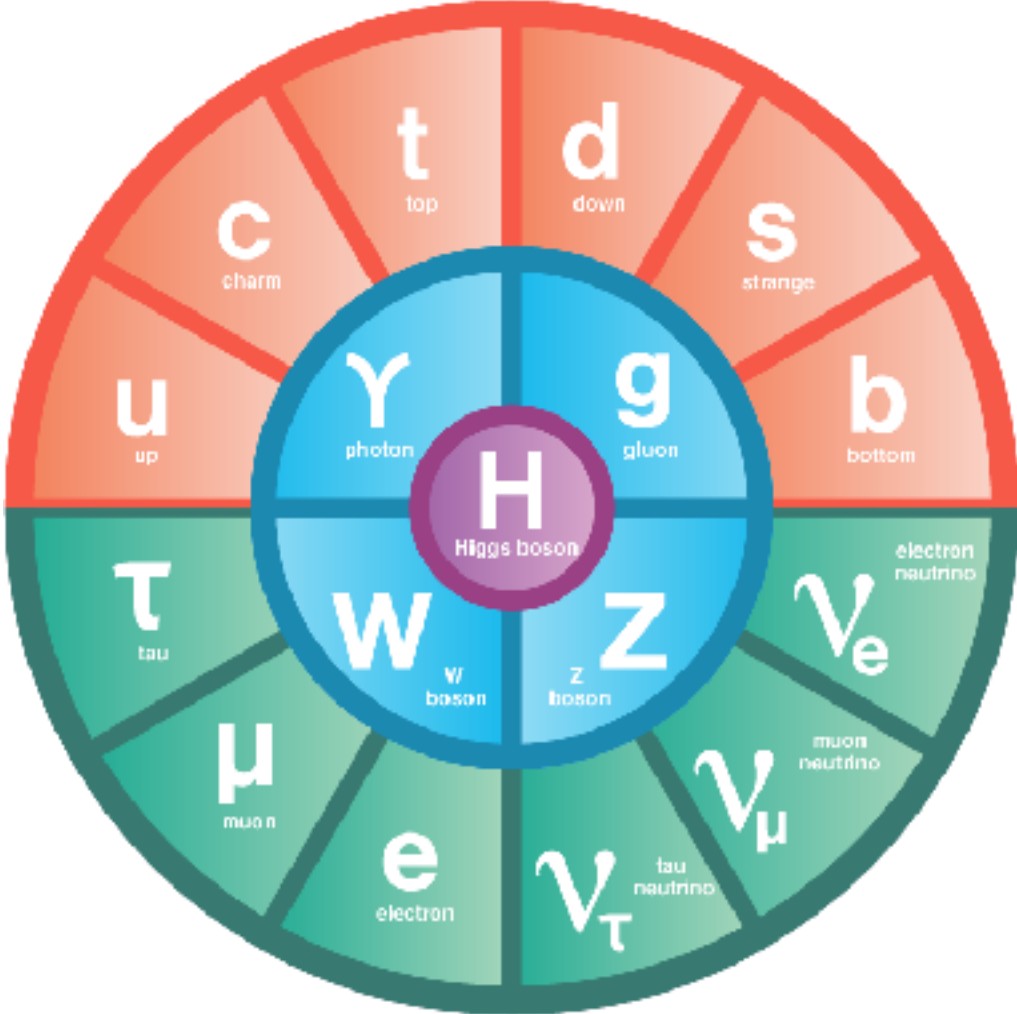
FIG. 4. Slices of fluid kinetic energy density  $E/T_c^4$  at  $t = 500 T_c^{-1}$ ,  $t = 1000 T_c^{-1}$  and  $t = 1500 T_c^{-1}$  respectively, for the  $\eta/T_c = 0.15$ ,  $N_b = 988$  simulation.

Hindmarsh, Huber, Rummukainen, Weir 2015



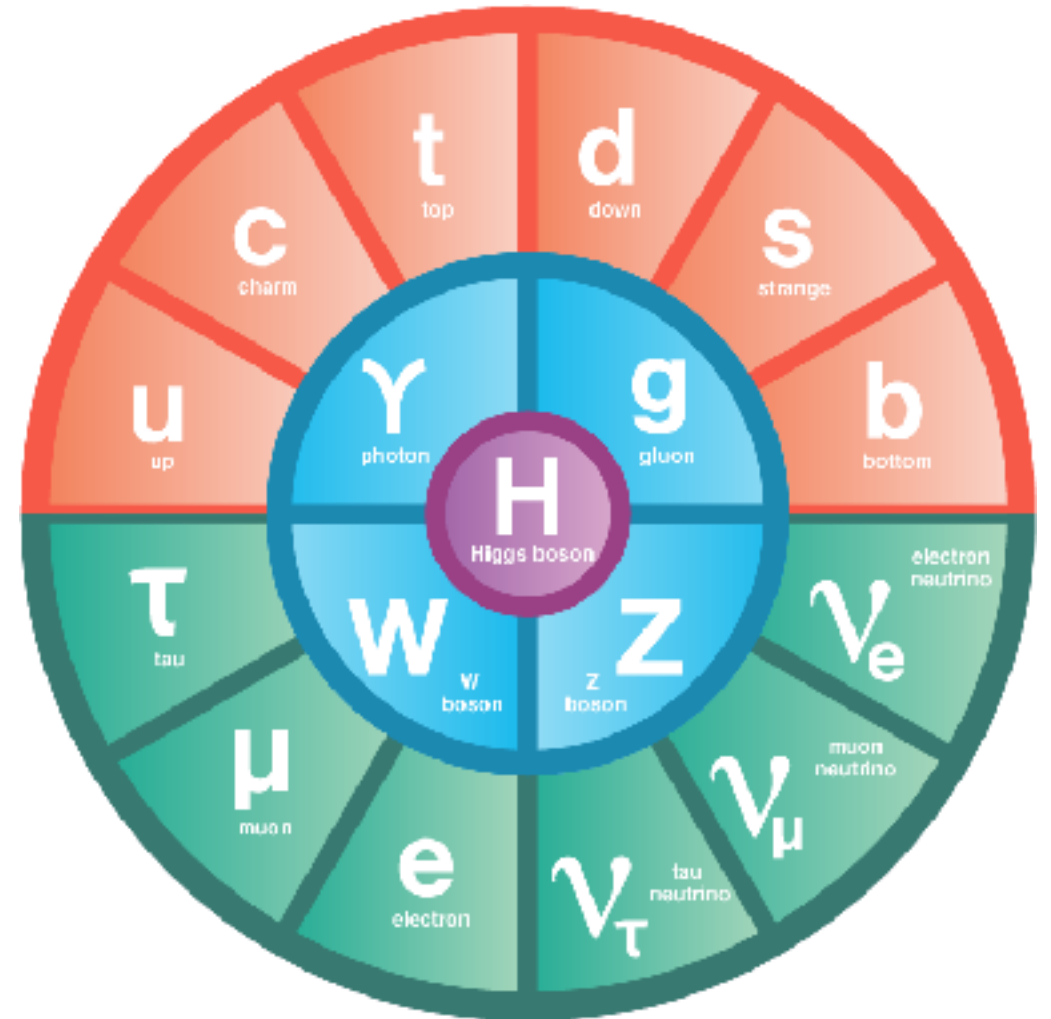
# Phase transitions in the Standard Model of Particle Physics

- Electroweak phase transition around 100 GeV
- QCD phase transition around 150 MeV



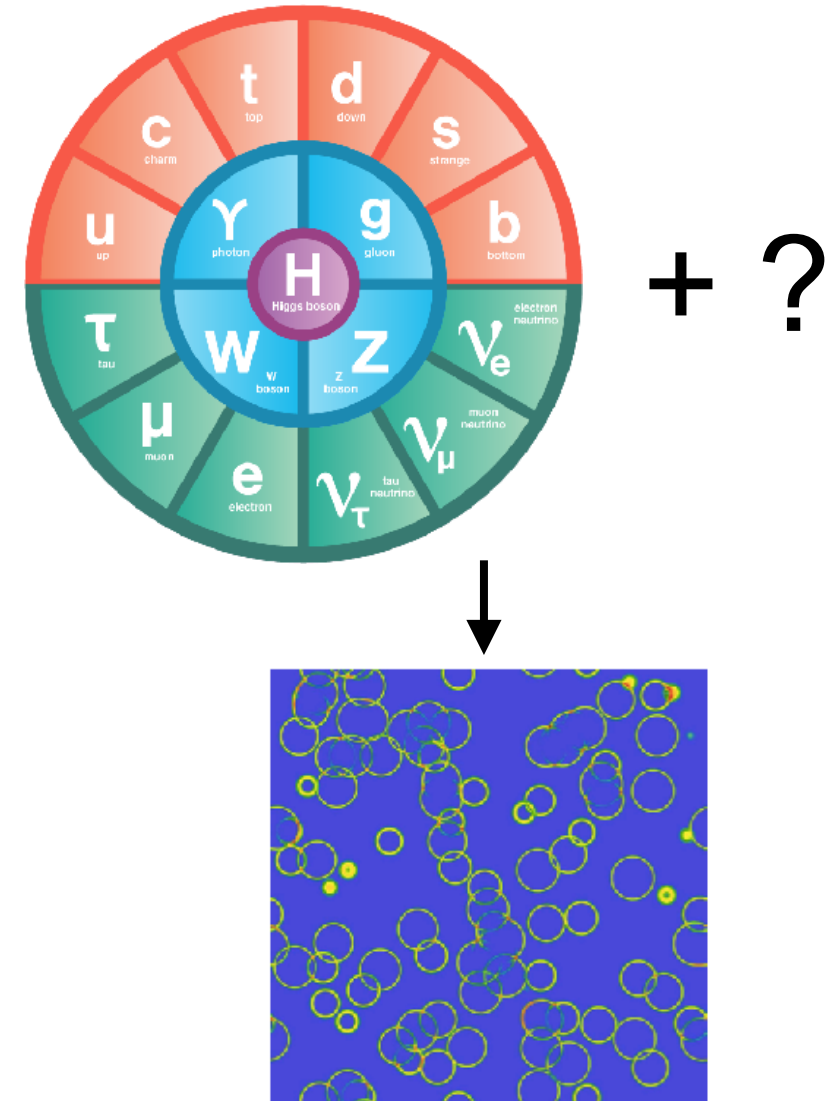
# Phase transitions in the Standard Model of Particle Physics

- Electroweak phase transition around 100 GeV
- QCD phase transition around 150 MeV
- **None of them is first order**

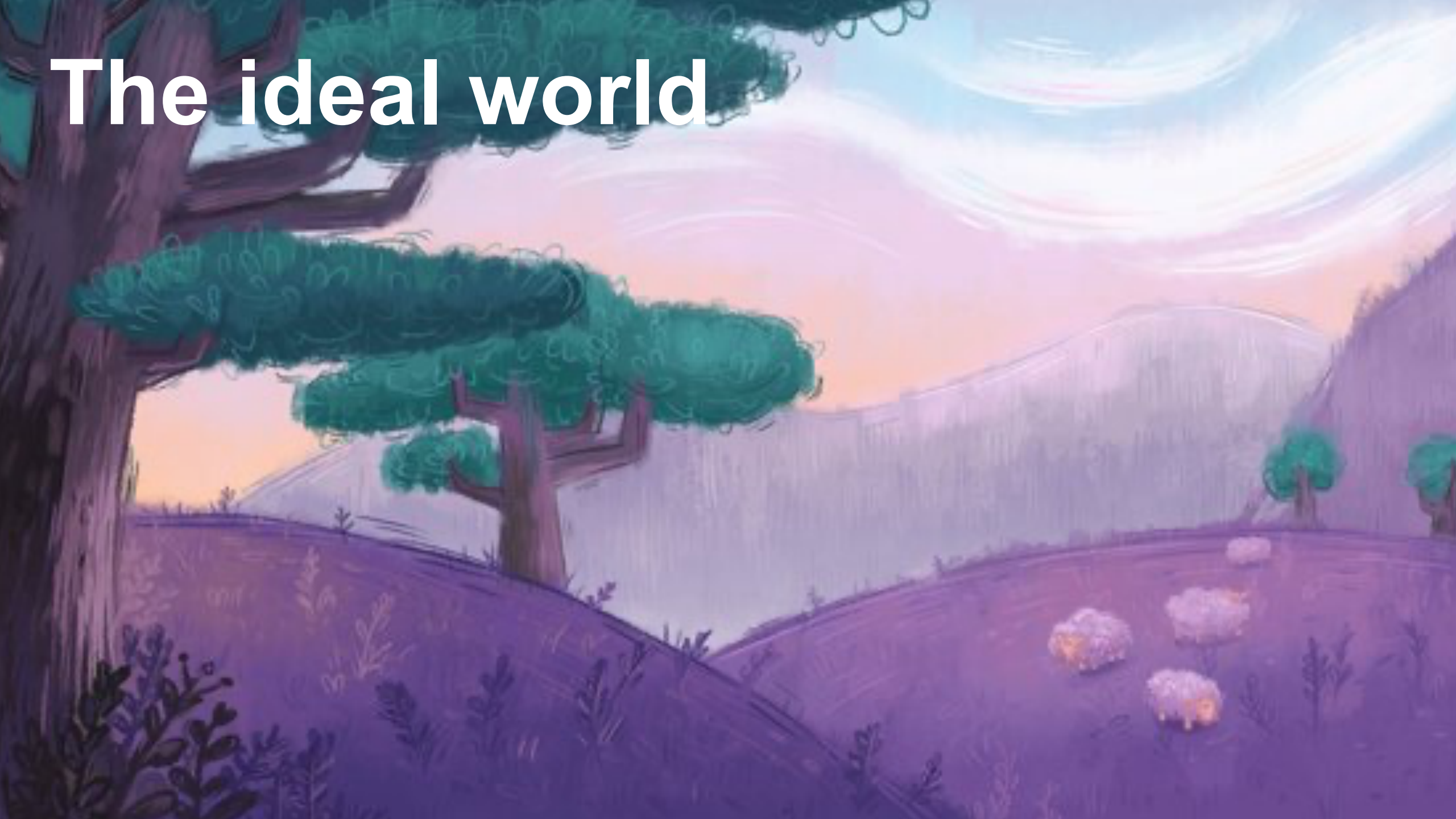


# Gravitational waves from first order phase transition: sign of new particles!

- New particle interacting with the Higgs can make the phase transition first order
- Phase transition in a dark sector?
- Relation to the asymmetry between matter and antimatter (electroweak baryogenesis)?



# The ideal world



# The ideal situation

- We know exactly what the gravitational wave spectrum looks like
- There are enough model-dependent features to distinguish between different BSM models
- The GW signal can be completely reconstructed by LISA
- We can infer the particle physics model from the signal

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Part I

# The ideal situation

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- We can infer the particle physics model from the signal

Part II

**What does the gravitational  
wave signal look like?**



# Three contributions to the gravitational wave signal

- Kinetic energy in the bubble walls

Kosowsky, Turner, Watkins 1992, Kosowsky, Turner 1993, Jinno, Takimoto 2017, Konstandin 2017, Cutting, Hindmarsh, Weir 2018\*

- Sound waves

Hindmarsh, Huber, Rummukainen, Weir 2013, 2015 & 2017, Giblin, Mertens 2013&2014, Cutting, Hindmarsh, Weir 2019

- Turbulence

Caprini, Durrer, 2006, Kahniashvili, Campanelli, Gogoberidze, Maravin, Ratra 2008&2009, Caprini, Durrer, Servant 2009, Kissinger, Kahniashvili 2015

\* A very incomplete list of references

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# Gravitational waves from many bubbles

Hydrodynamic lattice simulations

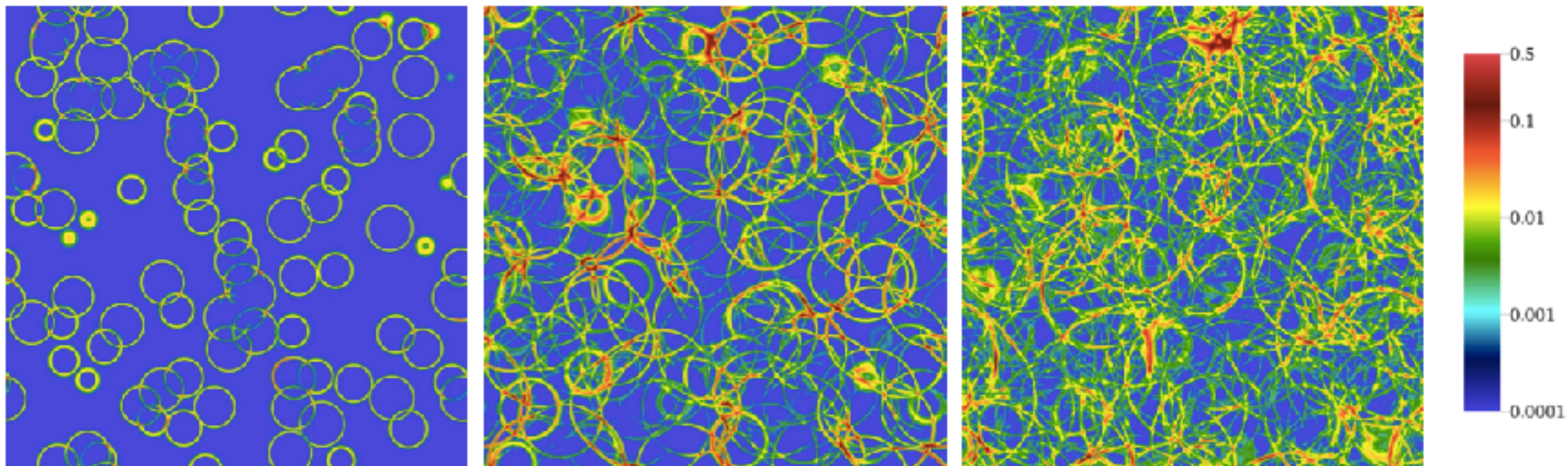


FIG. 4. Slices of fluid kinetic energy density  $E/T_c^4$  at  $t = 500 T_c^{-1}$ ,  $t = 1000 T_c^{-1}$  and  $t = 1500 T_c^{-1}$  respectively, for the  $\eta/T_c = 0.15$ ,  $N_b = 988$  simulation.

Hindmarsh, Huber, Rummukainen, Weir 2015

**Do I have to run a new simulation for every model that predicts a first order phase transition?**

# Fit to lattice result

Gravitational wave power spectrum

$$\frac{d\Omega_{\text{gw}}}{d \ln(f)} = 0.687 F_{\text{gw},0} K^2 H_* R_* / c_s \tilde{\Omega}_{\text{gw}} C \left( f/f_{p,0} \right)$$

LISA cosmology working group, 2019 (based on Hindmarsh, Huber, Rummukainen, Weir 2015 & 2017)

## Fit to lattice result

Redshift (temperature  $T_*$ )

$$\frac{d\Omega_{\text{gw}}}{d \ln(f)} = 0.687 F_{\text{gw},0} K^2 H_* R_* / c_s \tilde{\Omega}_{\text{gw}} C \left( f/f_{p,0} \right)$$

- Relevant parameters:  $T_*$

# Fit to lattice result

Kinetic energy fraction (phase transition strength  $\alpha$ , wall velocity  $v_w$ )

$$\frac{d\Omega_{\text{gw}}}{d \ln(f)} = 0.687 F_{\text{gw},0} K^2 H_* R_* / c_s \tilde{\Omega}_{\text{gw}} C \left( f/f_{p,0} \right)$$

- Relevant parameters:  $T_*$ ,  $\alpha$ ,  $v_w$

# Fit to lattice result

Bubble size at collision ( $v_w$ , phase transition duration  $\beta^{-1}$ )

$$\frac{d\Omega_{\text{gw}}}{d \ln(f)} = 0.687 F_{\text{gw},0} K^2 H_* \boxed{R_*} / c_s \tilde{\Omega}_{\text{gw}} C \left( f/f_{p,0} \right)$$

- Relevant parameters:  $T_*$ ,  $\alpha$ ,  $v_w$ ,  $\beta$



# Fit to lattice result

Sound speed  $\sim 1/\sqrt{3}$

$$\frac{d\Omega_{\text{gw}}}{d \ln(f)} = 0.687 F_{\text{gw},0} K^2 H_* R_* / c_s \tilde{\Omega}_{\text{gw}} C \left( f/f_{p,0} \right)$$

- Relevant parameters:  $T_*$ ,  $\alpha$ ,  $v_w$ ,  $\beta$

# Fit to lattice result

Peak frequency ( $T_*, \beta$ )

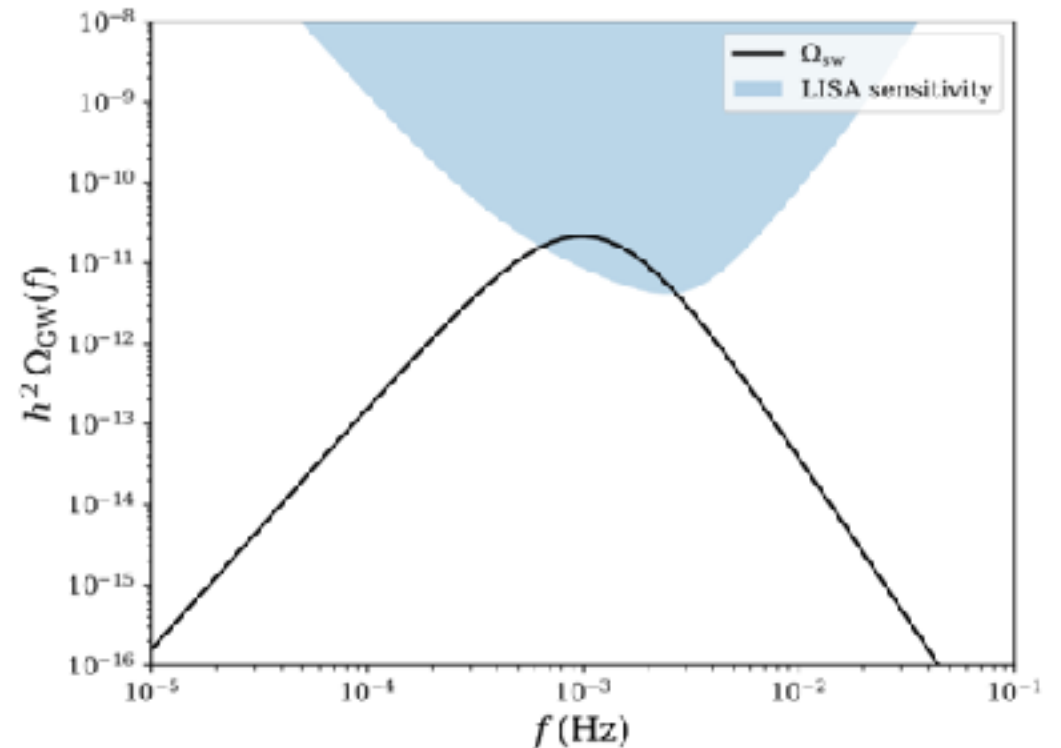
$$\frac{d\Omega_{\text{gw}}}{d \ln(f)} = 0.687 F_{\text{gw},0} K^2 H_* R_* / c_s \tilde{\Omega}_{\text{gw}} C \left( f / f_{p,0} \right)$$

- Relevant parameters:  $T_*, \alpha, v_w, \beta$

# Fit to lattice result

$$\frac{d\Omega_{\text{gw}}}{d \ln(f)} = 0.687 F_{\text{gw},0} K^2 H_* R_* / c_s \tilde{\Omega}_{\text{gw}} C \left( f/f_{p,0} \right)$$

- Relevant parameters:  $T_*$ ,  $\alpha$ ,  $v_w$ ,  $\beta$



# Computation of the power spectrum - common procedure

- Choose a BSM model with a first order phase transition (Higgs + singlet, 2HDM, composite Higgs...)

# Computation of the power spectrum - common procedure

- Choose a BSM model with a first order phase transition
- Analyze at finite temperature and determine  $T_*$ ,  $\alpha$ ,  $\beta$  \*

$$* \alpha \sim \frac{\Delta p}{\rho_{\text{tot}}}$$

# Computation of the power spectrum - common procedure

- Choose a BSM model with a first order phase transition
- Analyze at finite temperature and determine  $T_*$ ,  $\alpha$ ,  $\beta$
- Compute the wall velocity  $v_w$

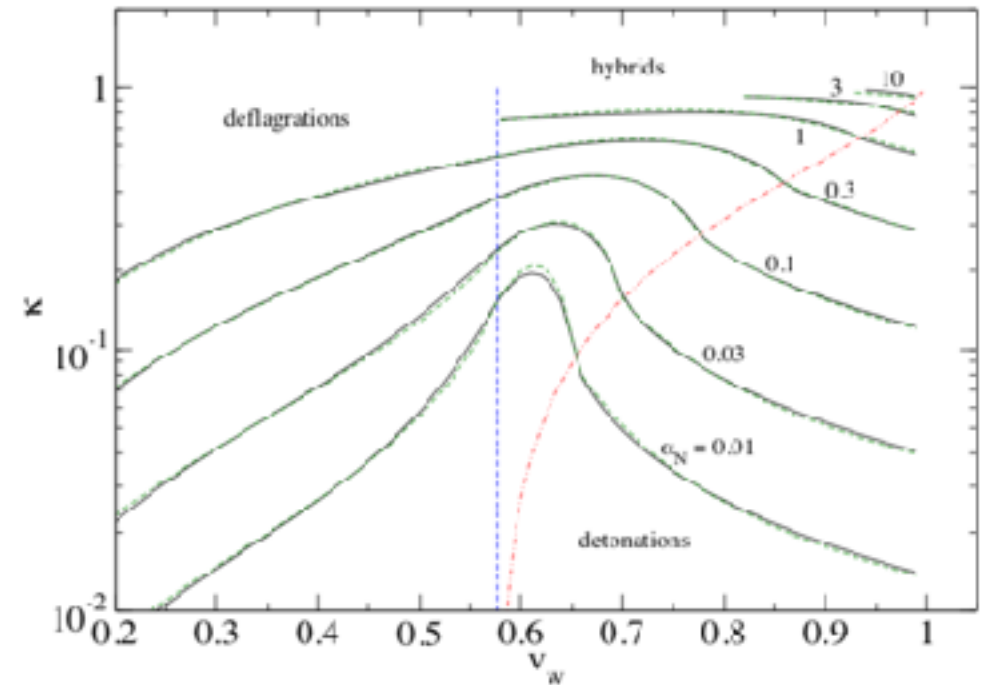
# Computation of the power spectrum - common procedure

- Choose a BSM model with a first order phase transition
- Analyze at finite temperature and determine  $T_*$ ,  $\alpha$ ,  $\beta$
- Compute the wall velocity  $v_w$
- Determine the kinetic energy fraction

# Common determination of kinetic energy fraction

- Obtain from a fit provided by Espinosa et al.
- Fit is based on the bag model

Fit of 'efficiency factor' as function of  $v_w$  and  $\alpha$



J. Espinosa, T. Konstandin, J. No, G. Servant, 2010



# Computation of the power spectrum - common procedure

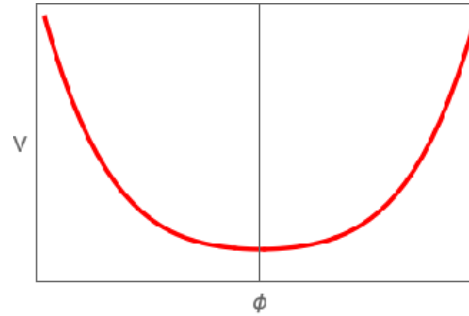
- Choose a BSM model with a first order phase transition
- Analyze at finite temperature and determine  $T_*$ ,  $\alpha$ ,  $\beta$
- Compute the wall velocity  $v_w$
- Determine the kinetic energy fraction
- Plug everything into 
$$\frac{d\Omega_{\text{gw}}}{d \ln(f)} = 0.687 F_{\text{gw},0} K^2 H_* R_* / c_s \tilde{\Omega}_{\text{gw}} C \left( f/f_{p,0} \right)$$

**Improved computation of  $K$ :**  
**(2004.06995, 2010.09744)**

# Hydrodynamics of a single bubble

## Thermodynamic quantities

- Pressure  $p = -F$
- Energy density  $e = T \frac{\partial p}{\partial T} - p$
- Enthalpy  $w = T \frac{\partial p}{\partial T} = e + p$
- Speed of sound  $c_s^2 = \frac{dp/dT}{de/dT}$
- Energy-momentum tensor of the fluid



fluid velocity



$$T^{\mu\nu} = u^\mu u^\nu w + \eta^{\mu\nu} p$$

# Hydrodynamics of a single bubble

## Hydrodynamic equations

$$\bullet \quad 2 \frac{v}{\xi} = \gamma^2 (1 - v\xi) \left[ \frac{\mu^2}{c_s^2} - 1 \right] \partial_\xi v, \quad \frac{\partial_v w}{w} = \left( \frac{1}{c_s^2} + 1 \right) \gamma^2 \mu$$

$v$ : fluid velocity

$\xi$ : radial coordinate

$$\mu = \frac{\xi - v}{1 - \xi v}$$

# Hydrodynamics of a single bubble

## Hydrodynamic equations

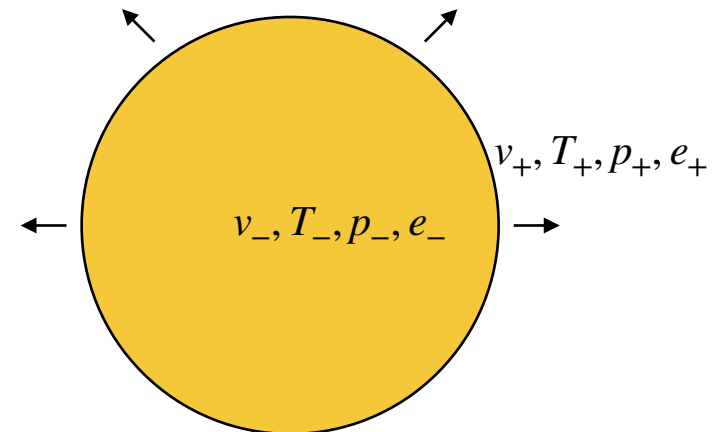
- $$2 \frac{v}{\xi} = \gamma^2 (1 - v\xi) \left[ \frac{\mu^2}{c_s^2} - 1 \right] \partial_\xi v, \quad \frac{\partial_v w}{w} = \left( \frac{1}{c_s^2} + 1 \right) \gamma^2 \mu$$

- Boundary conditions:

$$w(T_n) = w_n$$

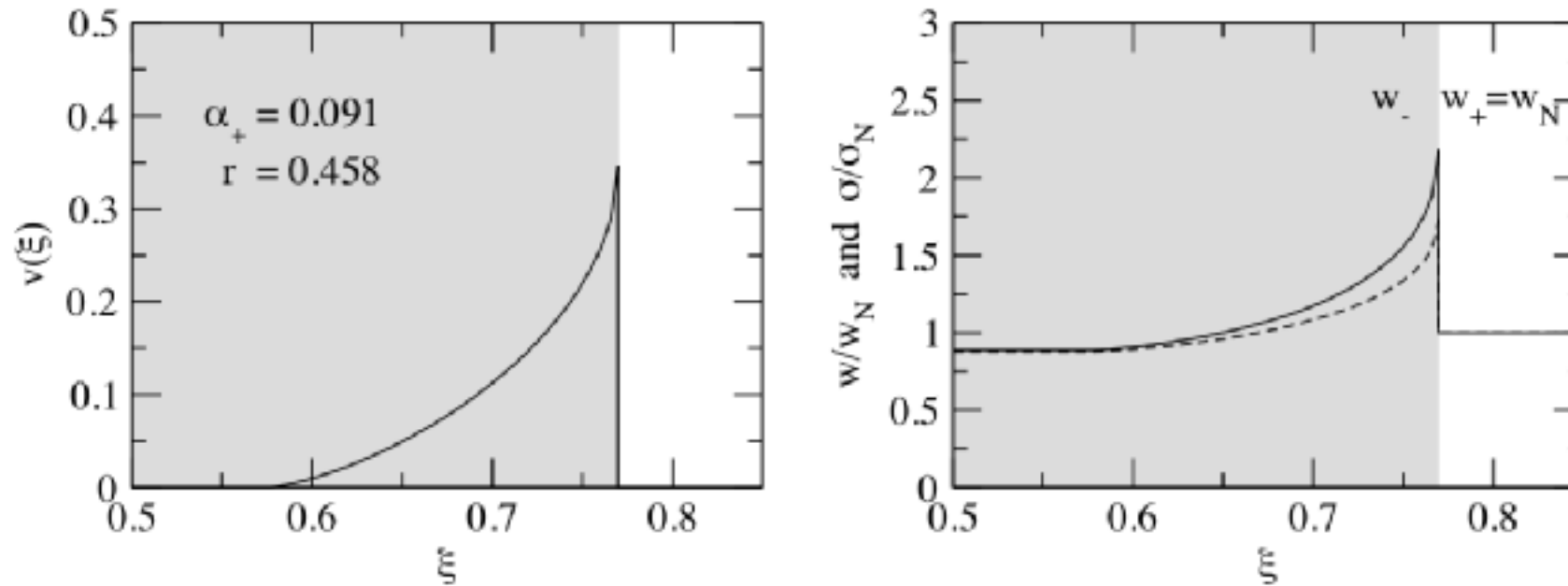
- Matching

$$\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}, \quad v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)}$$



# Hydrodynamics of a single bubble

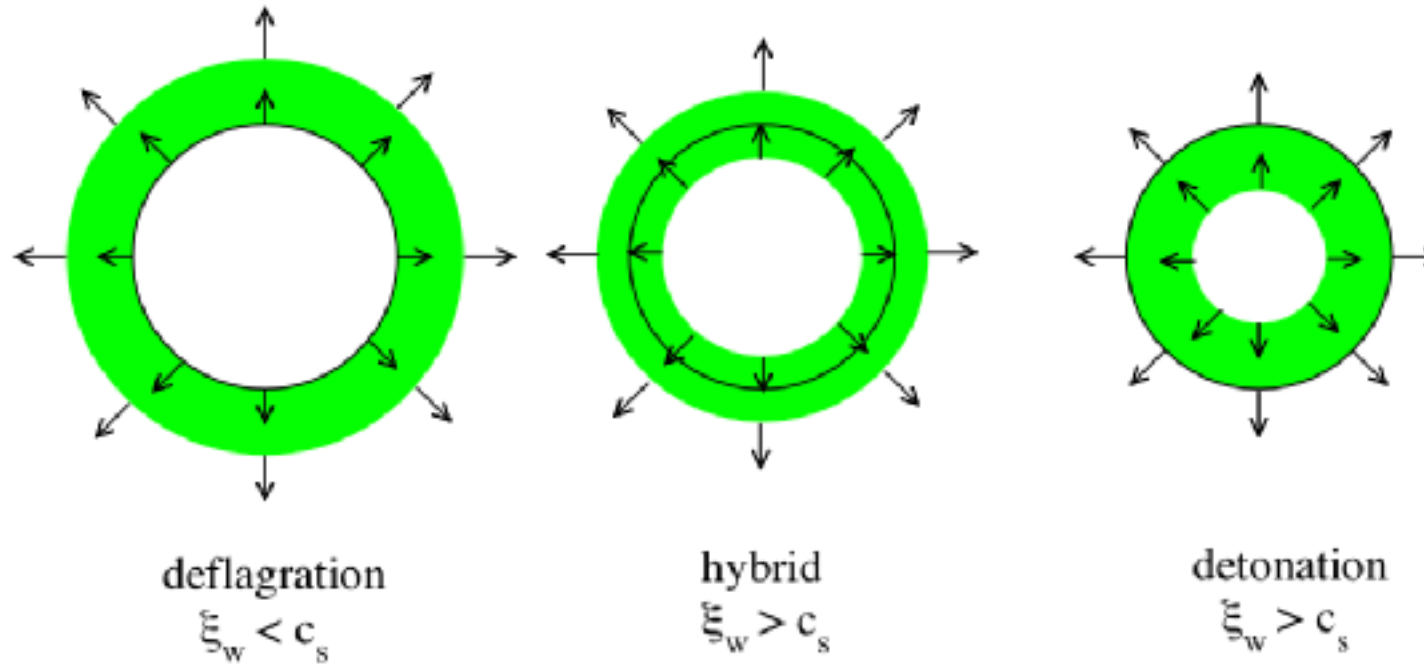
Solution: velocity and enthalpy profiles



J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

# Hydrodynamics of a single bubble

## Three types of solutions



J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

# Kinetic energy fraction

- Kinetic energy in the fluid
- Kinetic energy fraction

$$\rho_{fl} = \frac{3}{v_w^3} \int d\xi \xi^2 v^2 \gamma^2 w$$

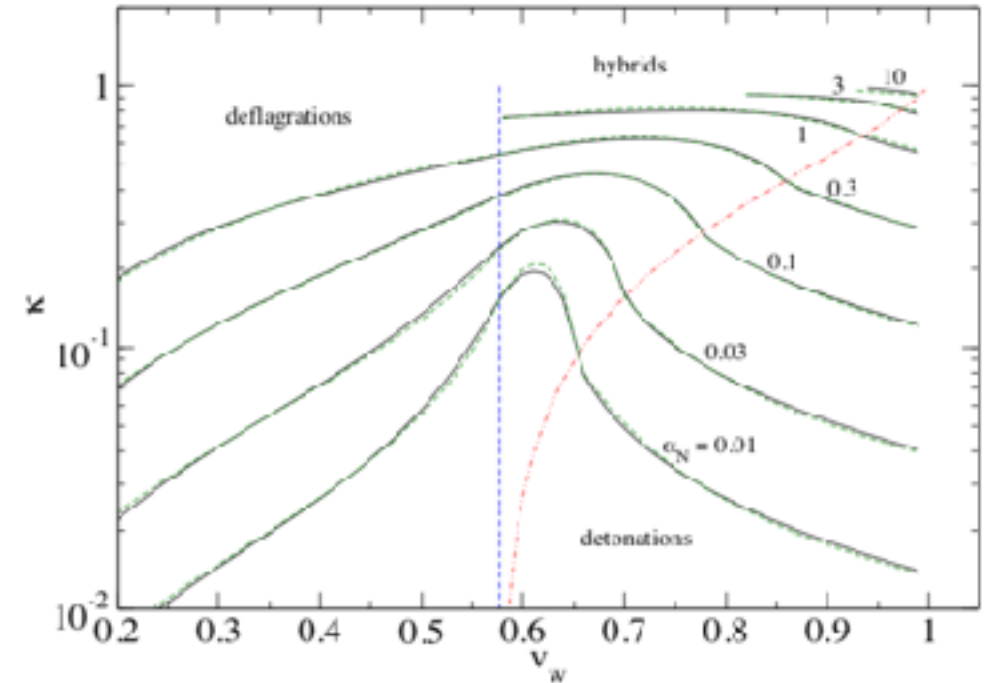
$$K = \frac{\rho_{fl}}{e_n}$$



**Can we solve these equations  
“once and for all”?**

# Solution in the bag model

- $p_s = \frac{1}{3}a_+T^4 - \epsilon$        $e_s = a_+T^4 + \epsilon$
- $p_b = \frac{1}{3}a_-T^4$        $e_b = a_-T^4$
- Bag constant  $\epsilon$  independent of temperature
- Sound speed  $c_s = 1/\sqrt{3}$
- Phase transition strength  $\alpha_\epsilon = \frac{4\epsilon}{3w_n}$
- $K = \frac{\alpha_\epsilon \kappa_\epsilon}{\alpha_\epsilon + 1}$  completely determined by  $\alpha_\epsilon$  and  $v_w$



J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

# Shortcomings in the bag model

$$\begin{aligned} p_s &= \frac{1}{3}a_+T^4 - \epsilon & e_s &= a_+T^4 + \epsilon \\ p_b &= \frac{1}{3}a_-T^4 & e_b &= a_-T^4 \end{aligned}$$

Not a realistic model

- Bag constant  $\epsilon$  independent of temperature

- ~~Sound speed  $c_s = 1/\sqrt{3}$~~

- Phase transition strength  $\alpha_\epsilon = \frac{4\epsilon}{3w_n}$

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- Bag constant  $\epsilon$  independent of temperature

- ~~Sound speed  $c_s = 1/\sqrt{3}$~~

- Phase transition strength

$$\alpha_\epsilon = \frac{4\epsilon}{3w_n}$$

- $K = \frac{\alpha_\epsilon \kappa_\epsilon}{\alpha_\epsilon + 1}$  completely determined by  $\alpha_\epsilon$  and  $\xi_w$

Not a realistic model

How does this get generalized?

# Beyond the bag model

## Model-independent matching

$$\bullet \quad \frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}, \quad v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)}$$

# Beyond the bag model

## Model-independent matching

- $\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}, \quad v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)}$
- Assume  $T_+ \sim T_-$

# Beyond the bag model

## Model-independent matching

- $$\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}, \quad v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)}$$

- Assume  $T_+ \sim T_-$

- $$\frac{v_+}{v_-} \simeq \frac{(v_+ v_- / c_{s,b}^2 - 1) + 3\alpha_{\bar{\theta}}}{(v_+ v_- / c_{s,b}^2 - 1) + 3v_+ v_- \alpha_{\bar{\theta}}}$$

- $$\bar{\theta} \equiv e - \frac{p}{c_{s,b}^2} \quad \alpha_{\bar{\theta}} \equiv \frac{D\bar{\theta}}{3w_n}$$

# Beyond the bag model

## Model-(in)dependent hydrodynamic equations

$$\bullet \quad 2 \frac{v}{\xi} = \gamma^2 (1 - v\xi) \left[ \frac{\mu^2}{c_{s,i}^2} - 1 \right] \partial_{\xi} v, \quad \frac{\partial_v w}{w} = \left( \frac{1}{c_{s,i}^2} + 1 \right) \gamma^2 \mu$$

Take constant in broken and symmetric phase



# Model-independent hydrodynamics

- All model-dependence is captured by  $v_w, \alpha_{\bar{\theta}}, C_{s,b}, C_{s,s}$  \*

- Determine  $\kappa_{\bar{\theta}} = \frac{4\rho_{fl}}{D\bar{\theta}}$  model-independently

- Find  $K = \frac{D\bar{\theta}}{4e_n} \kappa_{\bar{\theta}}$

\* As long as  $T_- \sim T_+$  is reasonable

# Template model

Model with speed of sound  $c_s \neq 1\sqrt{3}$  (but constant)

$$\begin{aligned} \bullet \quad p_s &= \frac{1}{3} a_+ T^\mu - \epsilon & e_s &= a_+ (\mu - 1) T^\mu + \epsilon \\ p_b &= \frac{1}{3} a_- T^\nu & e_b &= \frac{1}{3} a_- (\nu - 1) T^\nu \end{aligned}$$

$$\bullet \quad \nu = 1 + \frac{1}{c_{s,\text{broken}}^2}, \quad \mu = 1 + \frac{1}{c_{s,\text{symm}}^2}$$

L. Leitaó,  
A. Megevand, 2015

# Python snippet (2010.09744) computes efficiency factor

Compute  $\alpha_{\bar{\theta}}$ , both  $c_s$  and choose  $v_w$

```
1 import numpy as np
2 from scipy.integrate import odeint
3 from scipy.integrate import simps
4
5 def m1(a,b):
6     return (a-b)/(1.-a*b)
7
8 def getvw(a,b):
9     return a/(1.-a**2)/b*(1.-b**2)
10
11 def getvm(a1,vw,cs2b):
12     if vw**2<cs2b:
13         return (vw,0)
14     cc = 1.-3.*a1+vw**2*(1./cs2b+3.*a1)
15     disc = -4.*vw**2/cs2b+cc**2
16     if (disc<0.)|(cc<0.):
17         return (np.sqrt(cs2b), 1)
18     return ((cc+np.sqrt(disc))/2.*cs2b/vw, 2)
19
20 def dfdv(xiw, v, cs2):
    ...
    Krf += -wcv*getvcw(vp,vm)
    else:
    Krf = 0
    return (Ksh + Krf)/a1
```

$$K = \frac{D\bar{\theta}}{4e_n} \kappa_{\bar{\theta}}$$

→  $\kappa_{\bar{\theta}}$

**How well does this work?**

# Two toy models

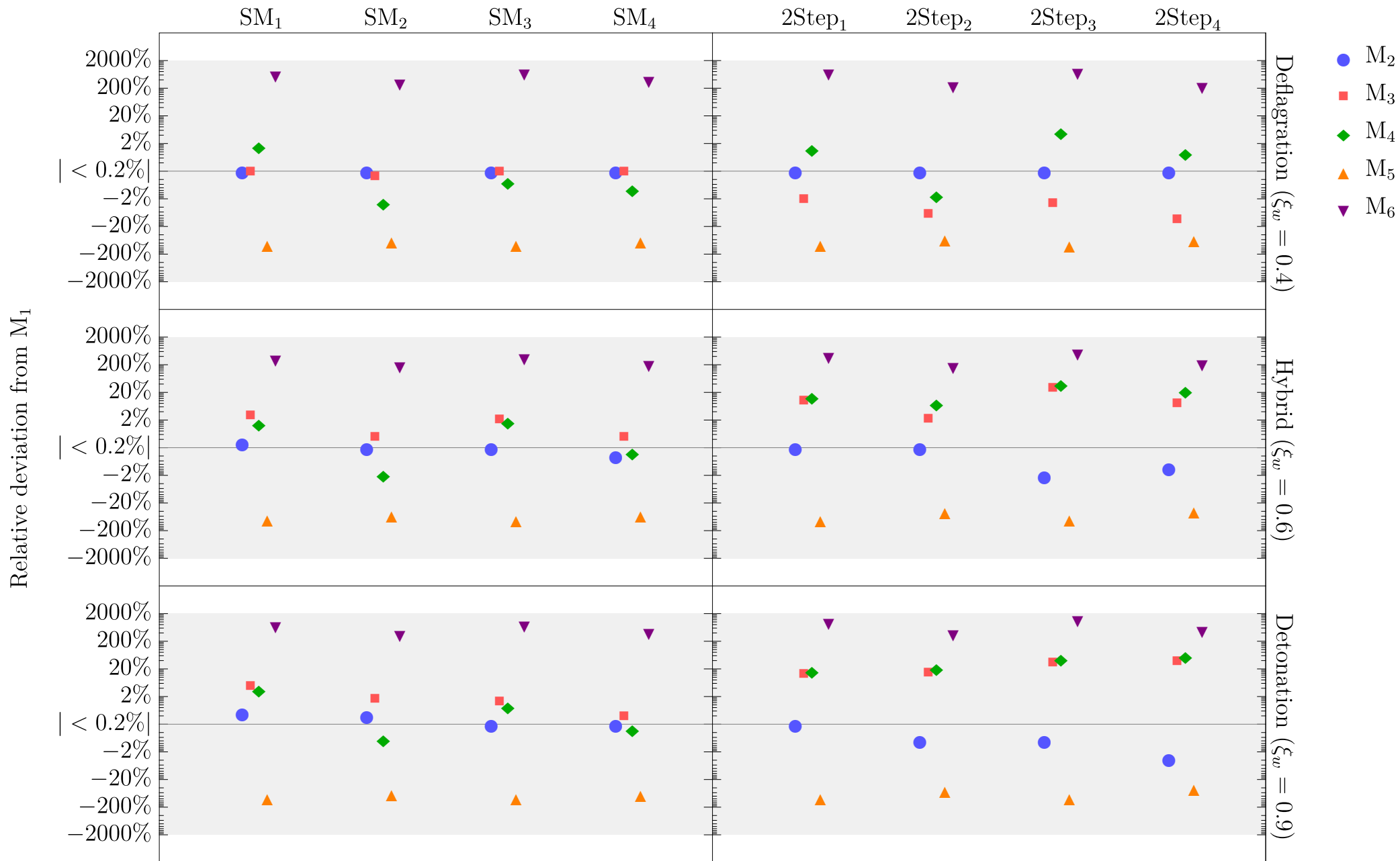
- SM with a light Higgs
- Two-step phase transition
- Choose nucleation temperature manually

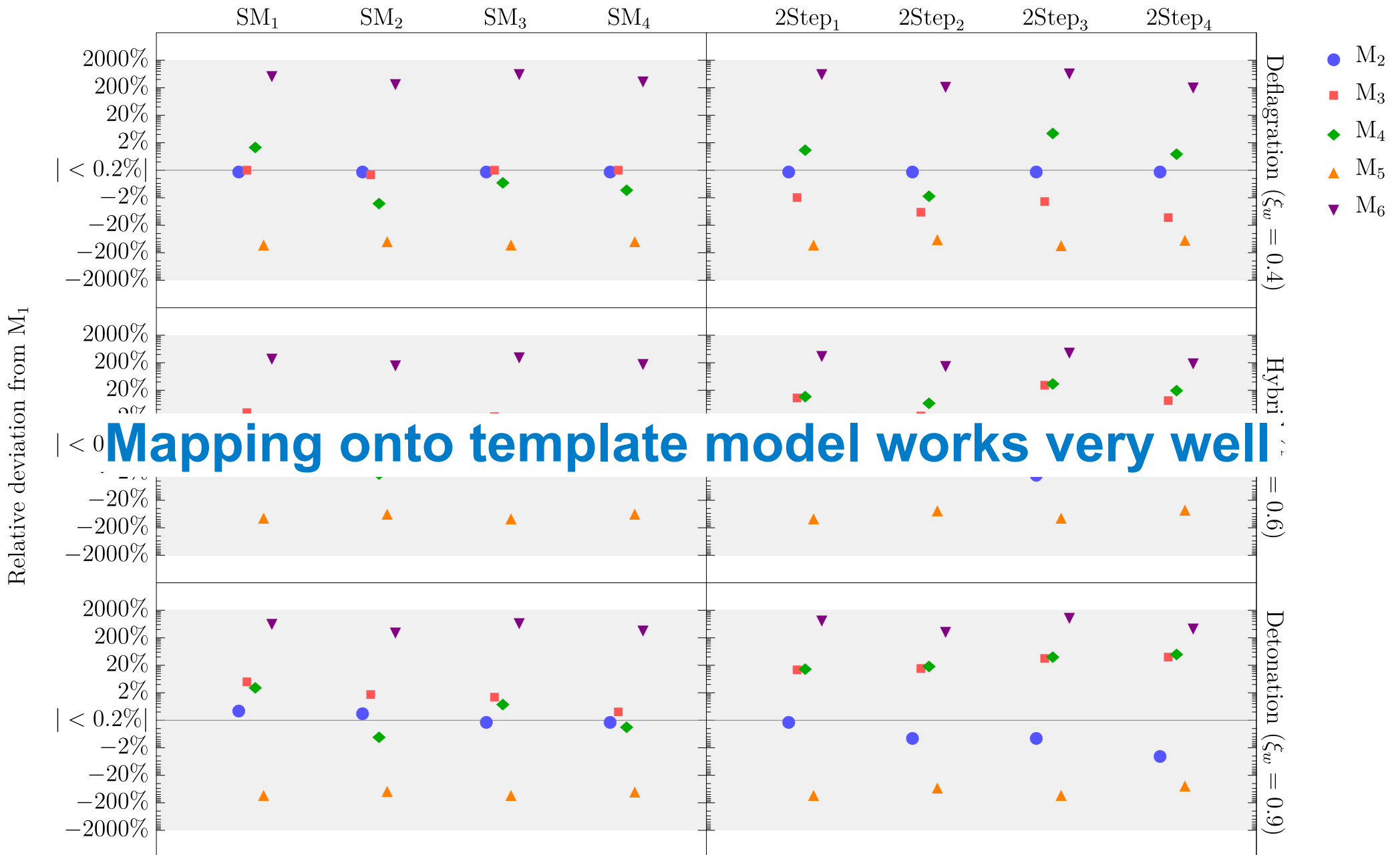
Model	$T_+/T_{\text{cr}}$	$\alpha_{\bar{\theta}}$	$c_s^2$
SM <sub>1</sub>	0.9	0.0297	0.326
SM <sub>2</sub>	0.8	0.0498	0.331
SM <sub>3</sub>	0.9	0.00887	0.331
SM <sub>4</sub>	0.8	0.0149	0.333

Model	$T_+/T_{\text{cr}}$	$\alpha_{\bar{\theta}}$	$c_s^2$
2step <sub>1</sub>	0.9	0.0156	0.312
2step <sub>2</sub>	0.7	0.0704	0.297
2step <sub>3</sub>	0.9	0.0254	0.282
2step <sub>4</sub>	0.7	0.159	0.245

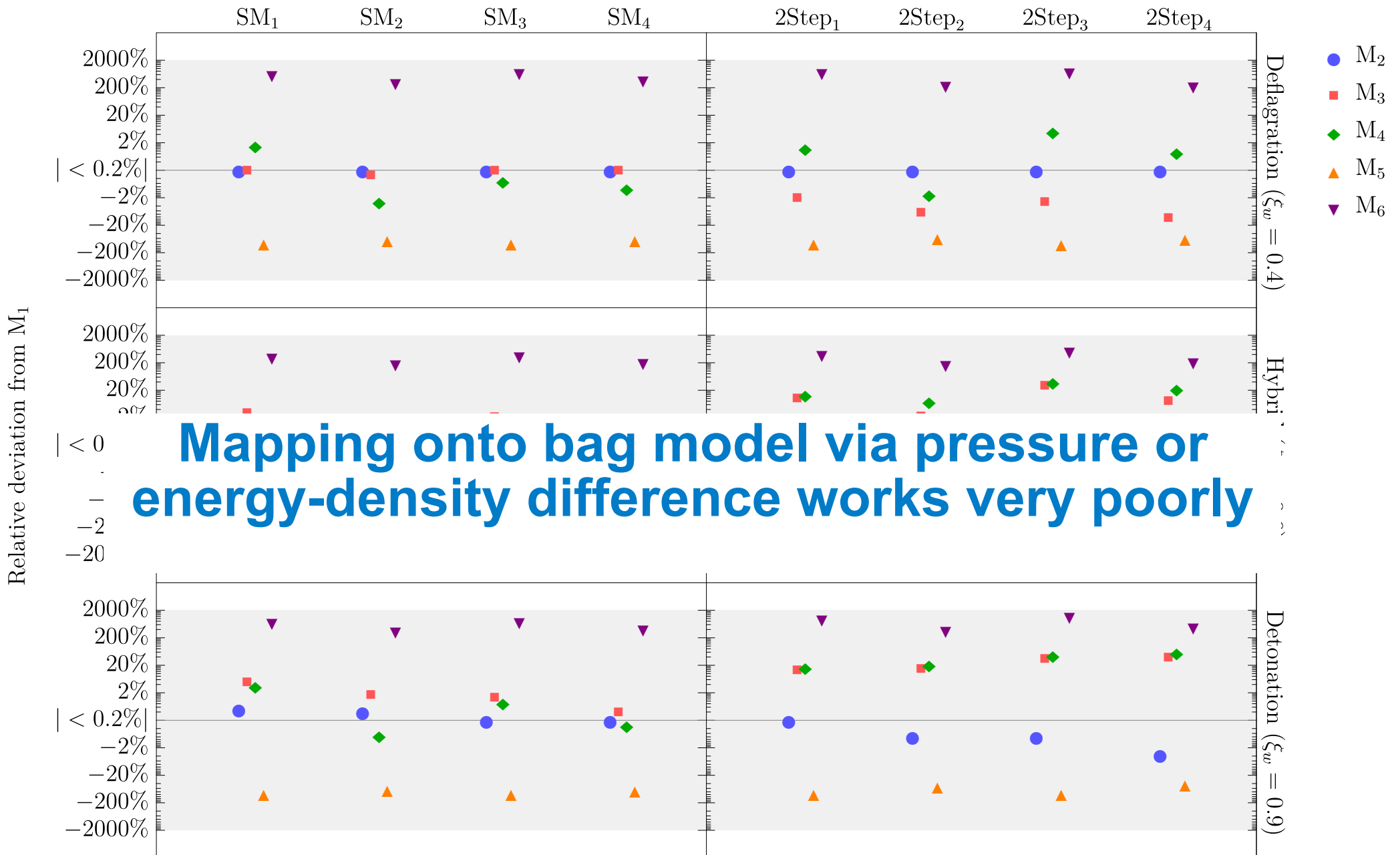
# Six different methods to determine $K$

<b>M1</b>	Full numerical solution
<b>M2</b>	Mapping onto template model
<b>M3 &amp; M4</b>	Mapping onto bag model via trace of energy-momentum tensor
<b>M5</b>	Mapping onto bag model via pressure difference
<b>M6</b>	Mapping onto bag model via energy-density difference









# Beyond the toy model: two-step revisited

What is a realistic value for the sound speed?

- $$V_{\text{tree}}(h, s) = -\frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 - \frac{\mu_s^2}{2}s^2 + \frac{\lambda_s}{4}s^4 + \frac{\lambda_{hs}}{4}h^2s^2 + \Delta V_h$$
- $$V_T(h, s) = \frac{T^4}{2\pi^2} \sum_{\alpha} N_{\alpha} \int_0^{\infty} dx x^2 \log \left[ 1 \pm e^{-\sqrt{x^2 + M_{\alpha}^2(h, s)}/T} \right] + \frac{T}{12\pi} \sum_{\text{bosons } \alpha} N_{\alpha} \left[ M_{\alpha}^3(h, s) - M_{T, \alpha}^3(h, s, T) \right]$$

Heavy particles

Light particles

- $$V_{\delta T} = -\frac{\pi^2}{90} g'_* T^4, \quad g'_* = \frac{345}{4}$$
- $$V_{\text{eff}} = V_{\text{tree}} + V_{cw} + V_{ct} + V_T + V_{\delta T}$$



# Beyond the toy model: two-step revisited

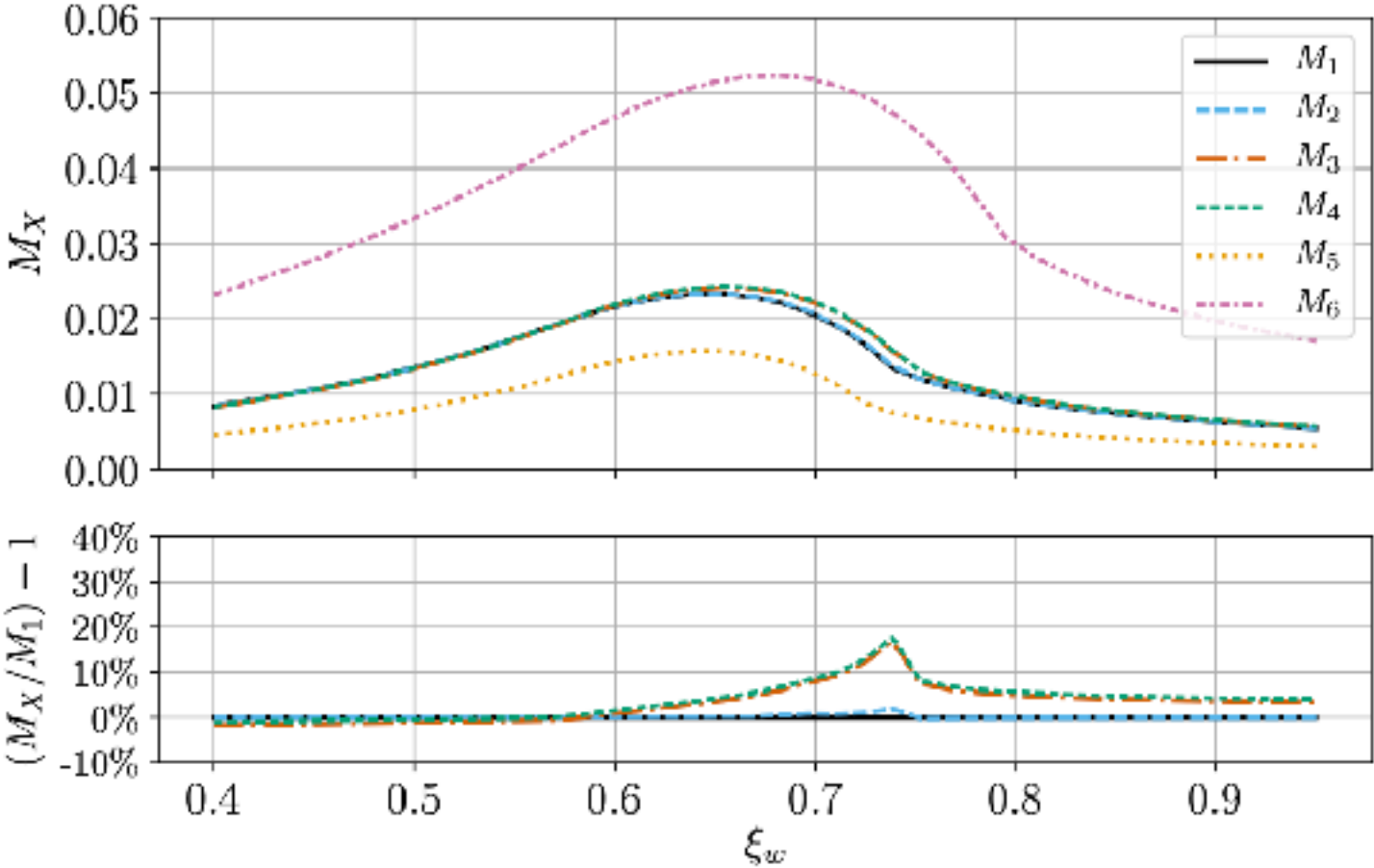
What is a realistic value for the sound speed?

- Find nucleation temperature via  $\frac{S_3}{T} \approx 140$  (one bubble per Hubble)

$m_s$ (GeV)	$\lambda_s$	$\lambda_{hs}$	$T_n$ (GeV)	$\beta/H_*$	$\alpha_e$	$\alpha_{\bar{\theta}_n}$	$c_{s,b}^2$	$c_{s,s}^2$
300	1.90	3.50	87.3	288	0.070	0.035	0.324	0.333
250	2.80	2.80	71.1	152	0.126	0.075	0.325	0.334
250	0.40	2.26	98.9	367	0.051	0.022	0.325	0.333
170	2.80	1.80	69.5	335	0.119	0.065	0.324	0.334

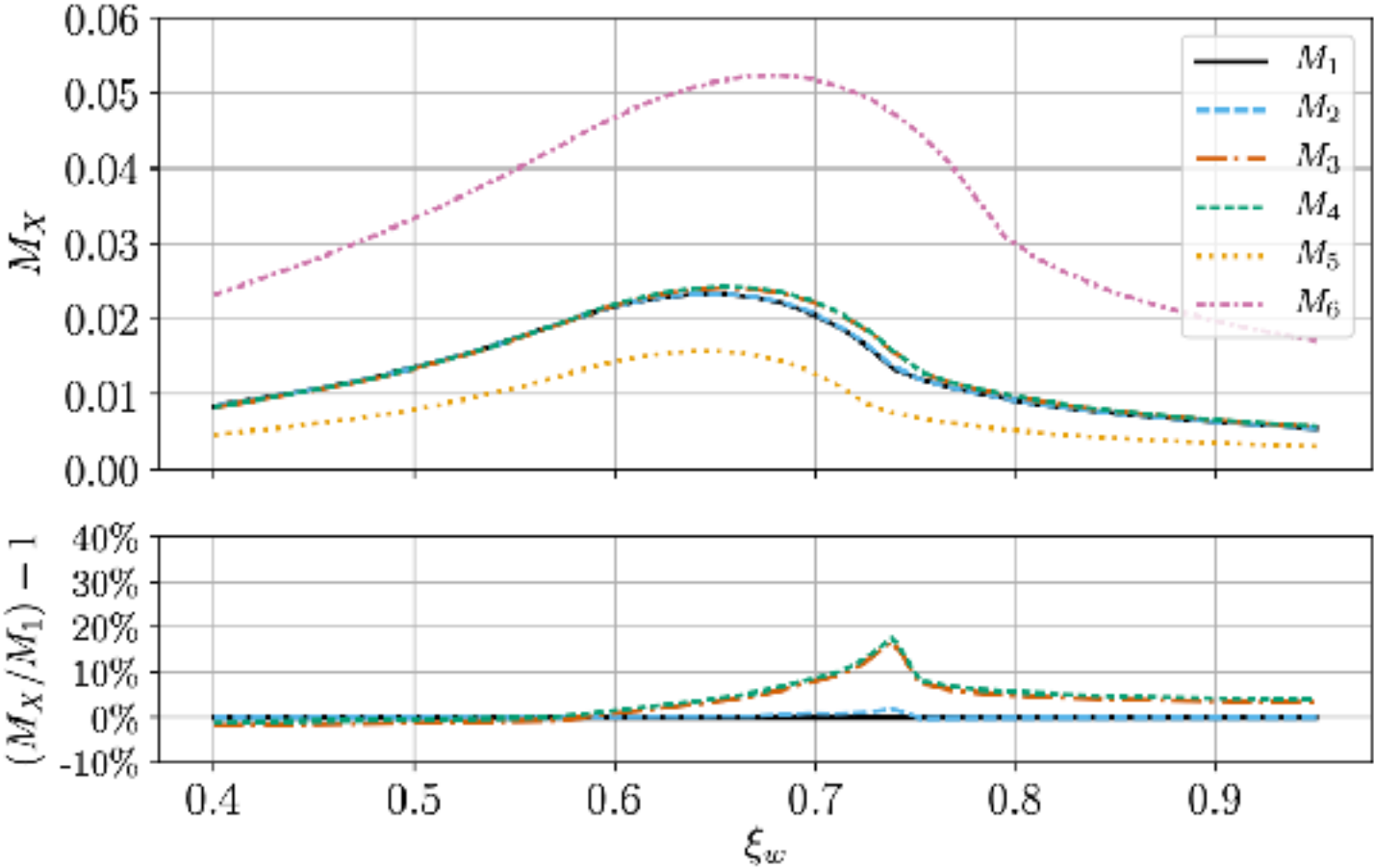
$\sim 1/3\dots$

# Beyond the toy model: two-step revisited



$$m_s = 170 \text{ GeV}$$

# Beyond the toy model: two-step revisited



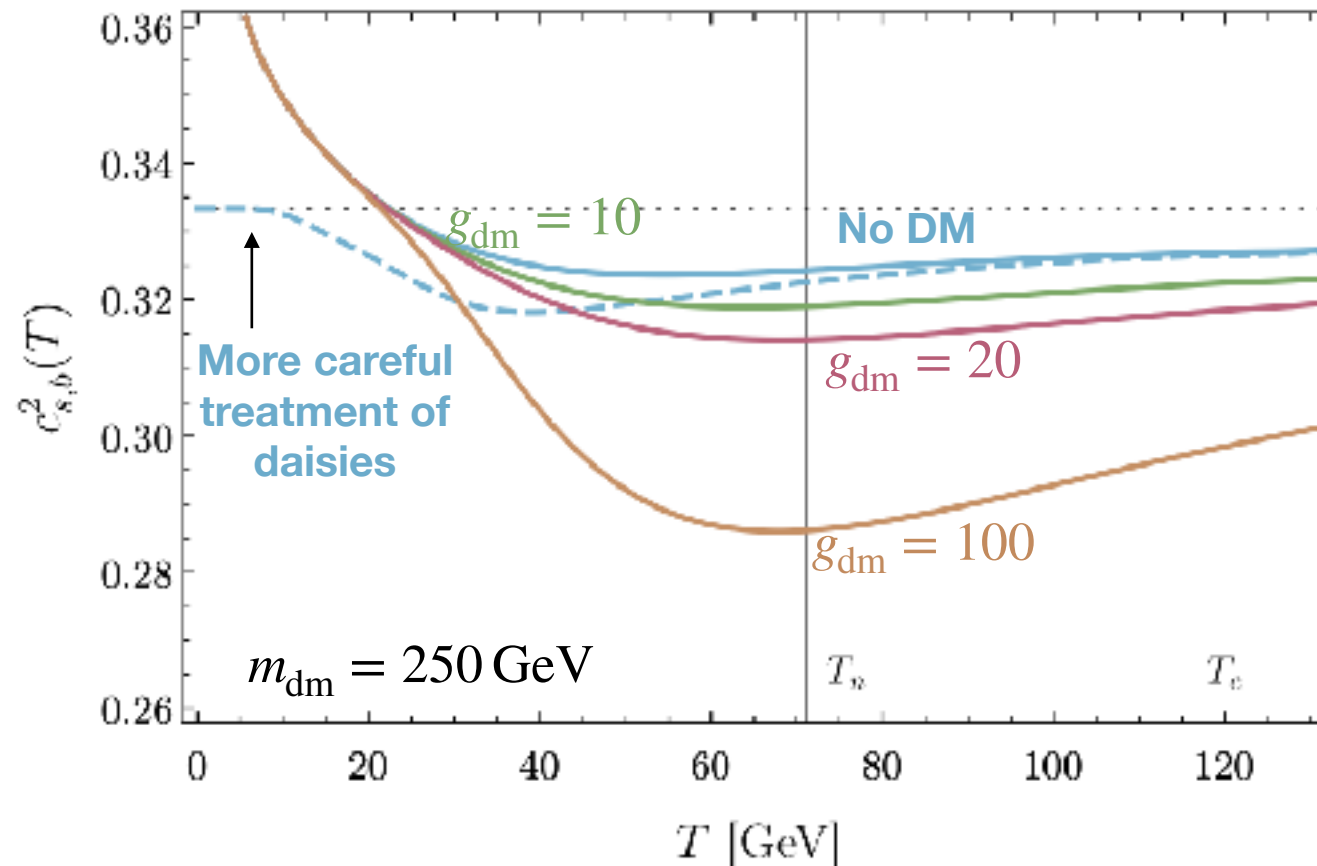
$$m_s = 170 \text{ GeV}$$

Note that  $\Omega_{\text{tot}} \propto K^2$  or  $K^{3/2}$

# Deviations in sound speed from $c_s^2 \sim 1/3$

Sound speed affected by presence of heavy particles

- Many relativistic particles in the SM
- Presence of massive particles suppresses the sound speed



# Beyond the toy model: two-step revisited revisited

## Problems with Daisy resummation!

- Daisy resummation can be very inaccurate (e.g. Croon, Gould, Schicho, Tenkanen, White 2020)
- Ongoing work with T. Tenkanen: computation of the sound speed using Dimensional Reduction

**What does this mean for the  
gravitational wave signal?**



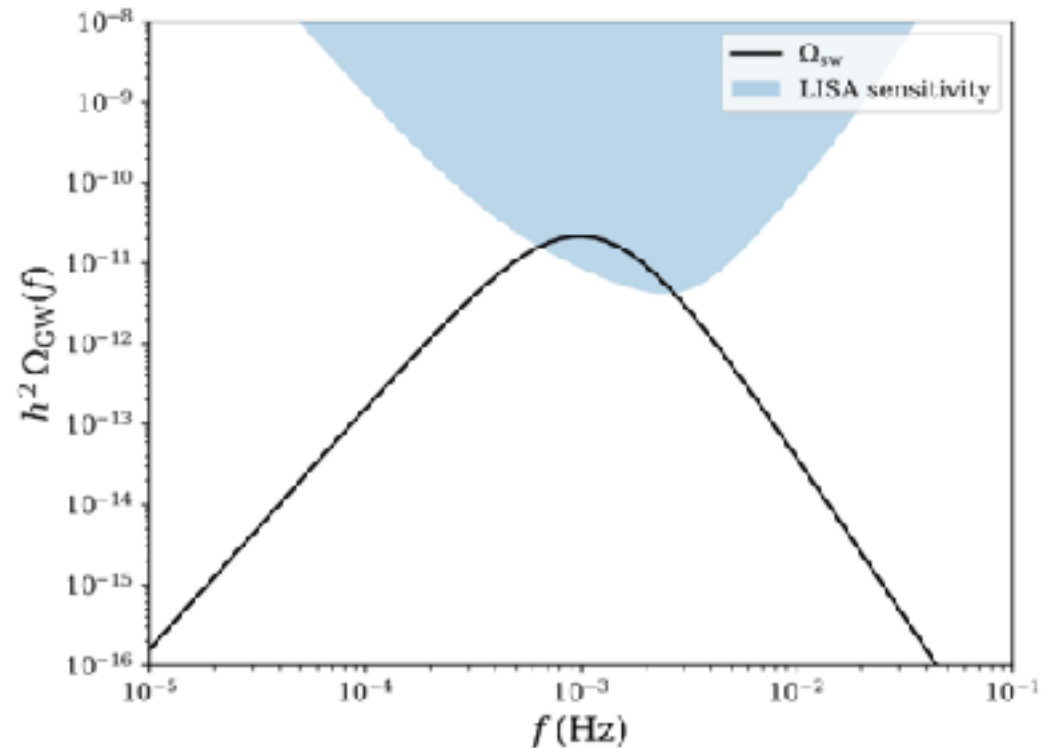
# Fit to lattice result

Effect on amplitude only

$$\frac{d\Omega_{\text{gw}}}{d \ln(f)} = 0.687 F_{\text{gw},0} K^2 H_* R_* / c_s \tilde{\Omega}_{\text{gw}} C \left( f/f_{p,0} \right)$$

Lattice computation assumed

$$c_s = 1/\sqrt{3}$$

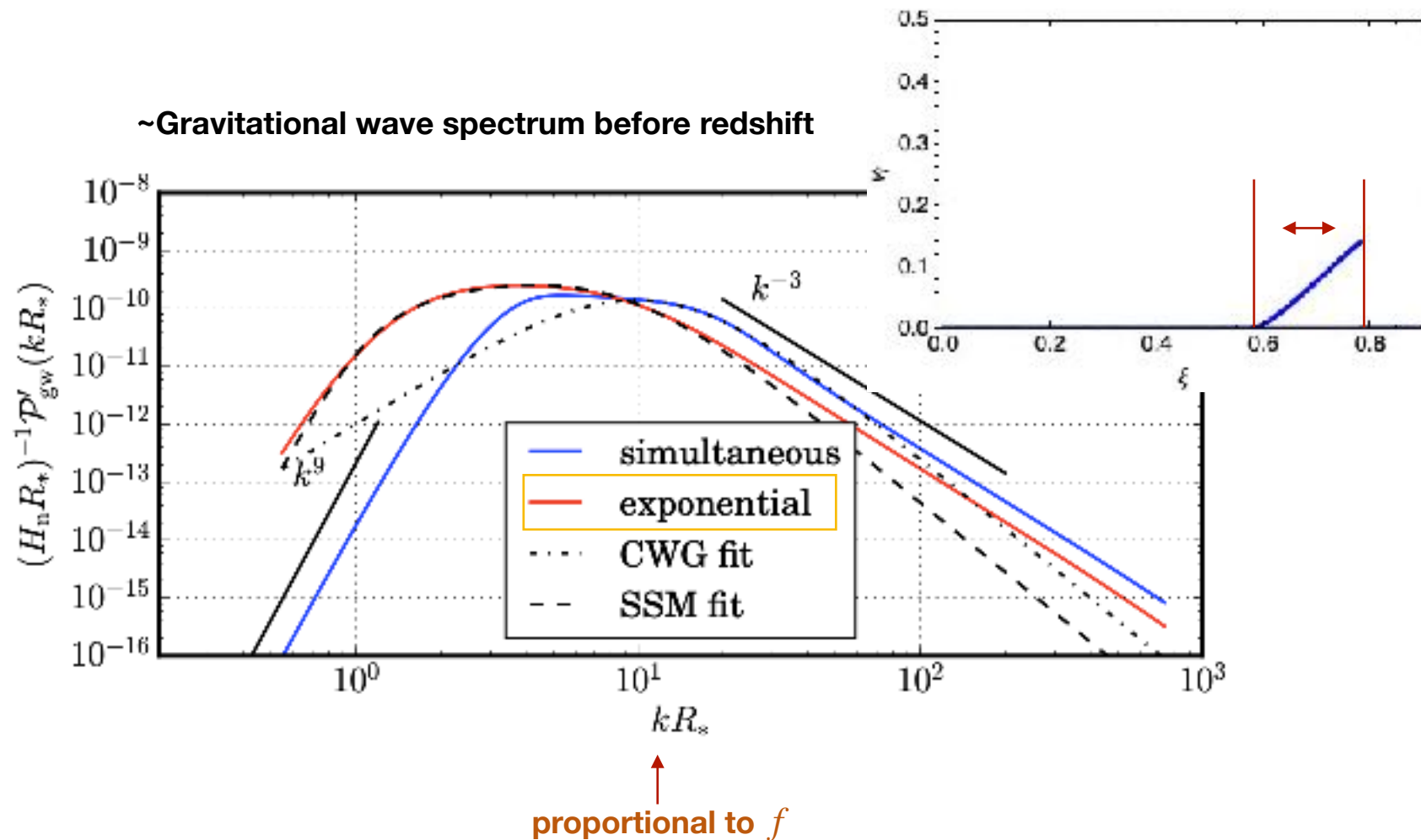


# Sound-shell model (alternative to lattice simulations)

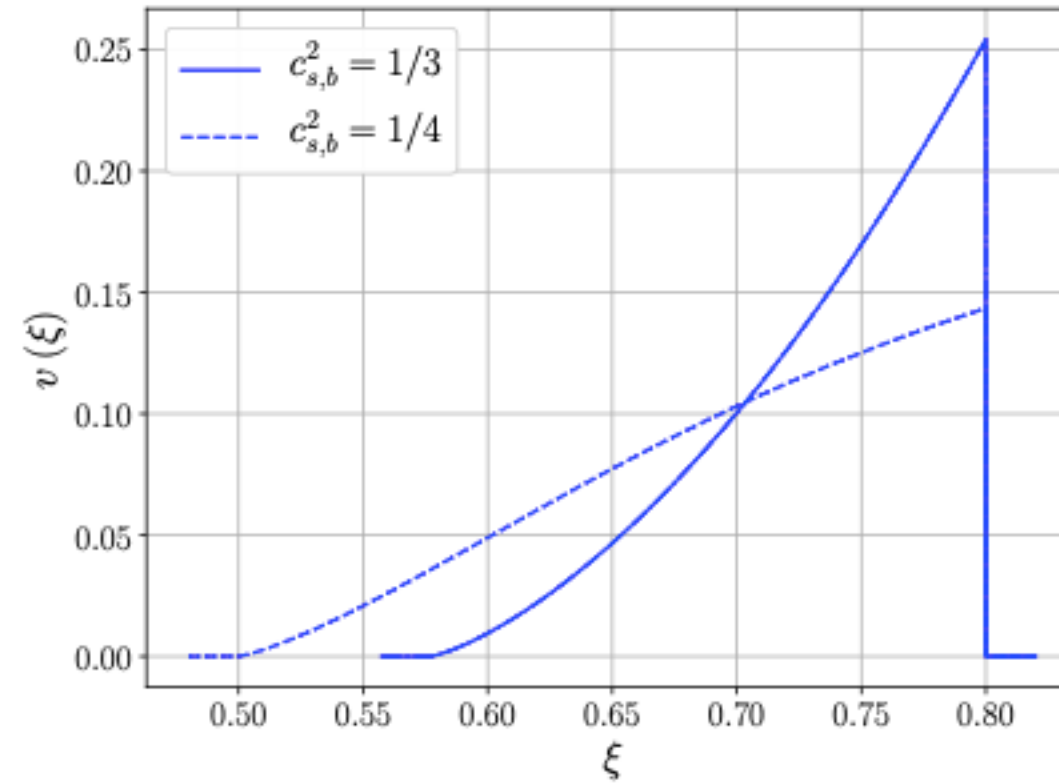
Thickness of fluid profile affects shape of the spectrum

Hindmarsh 2016  
Hindmarsh, Hijazi 2019

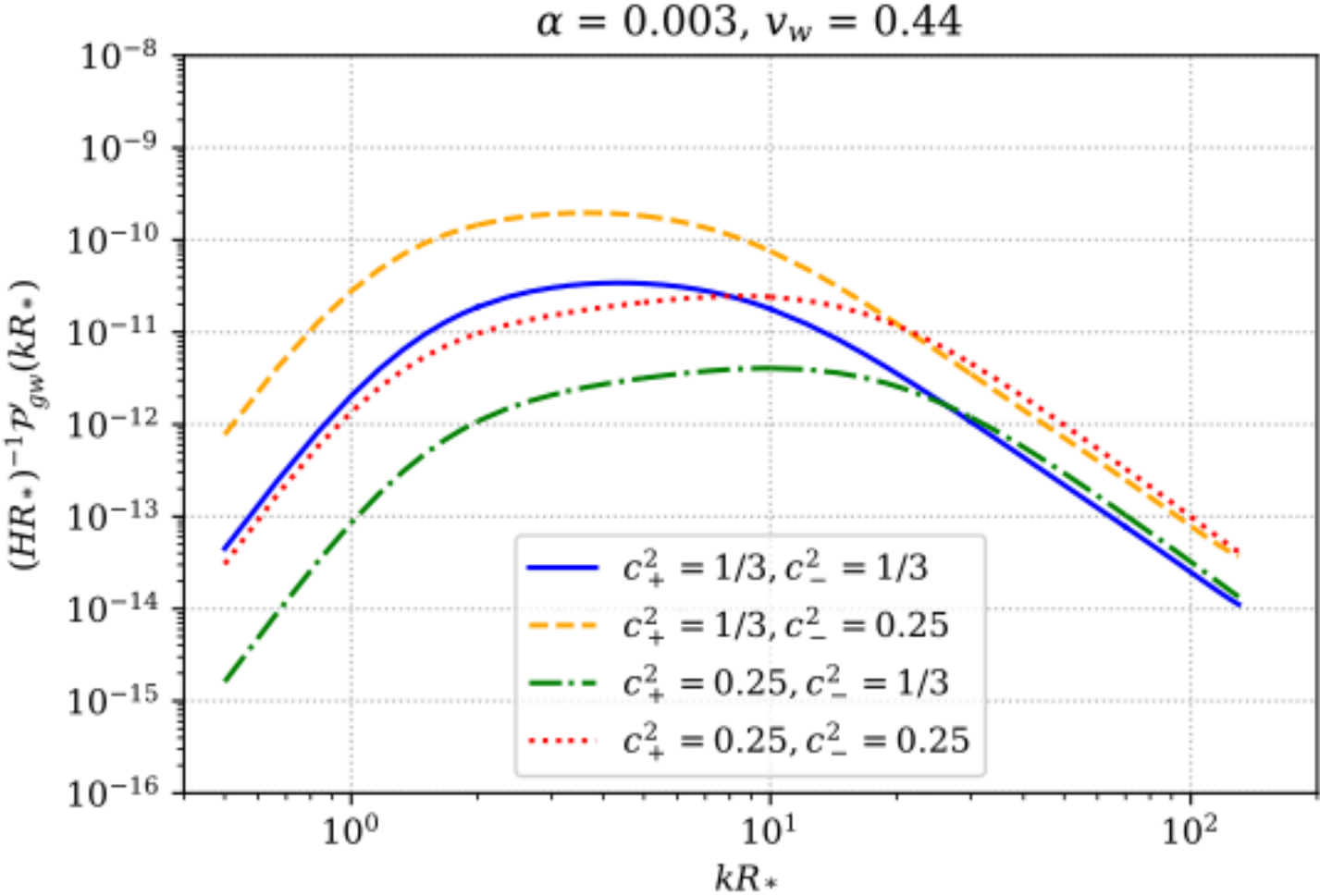
proportional to  
 $h^2 \Omega_{\text{gw}}$  →



# Effect of the sound speed on fluid profile



# Effect of the sound speed on gravitational wave spectrum (sound shell model)



Wang, Huang, Li, 2021

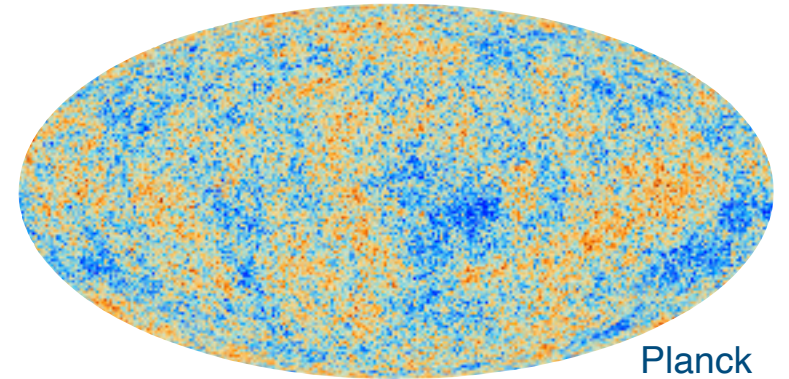
In addition to  $T_*$ ,  $\beta$ ,  $\alpha$ ,  $v_w$ , also  $c_{s,b}$  and  $c_{s,s}$  affect the gravitational wave spectrum.

In addition to  $T_*$ ,  $\beta$ ,  $\alpha$ ,  $v_w$ , also  $c_{s,b}$  and  $c_{s,s}$  affect the gravitational wave spectrum.

But more parameters come into play... (2108.11947)

# Primordial perturbations

- On CMB scales:  $\frac{\delta T}{T} \sim 10^{-4}$
- Maybe  $\frac{\delta T}{T}$  is larger on the scale relevant to the phase transition?
- How does that affect the gravitational wave signal?



# Nucleation rate with perturbations

- Temperature perturbations can enhance/reduce the nucleation rate
- Tunnelling rate  $\Gamma(t) = \Gamma_* \exp \left[ \beta(t - t_*) - \frac{\beta}{H_*} \frac{\delta T}{\bar{T}} \right]$
- Perturbations relevant when  $\left| \frac{\beta}{H_*} \frac{\delta T}{\bar{T}} \right| \equiv |\delta\tilde{T}| \gtrsim 1$
- $\frac{\beta}{H_*} = \mathcal{O}(100) \rightarrow \frac{\delta T}{\bar{T}}$  can be moderate



# Numerics

- Modified version (temperature fluctuation-dependent nucleation rate) of

## **A hybrid simulation of gravitational wave production in first-order phase transitions**

Ryusuke Jinno, Thomas Konstandin and Henrique Rubira

*Deutsches Elektronen-Synchrotron DESY, 22607 Hamburg, Germany*

# Spectrum of the perturbations

- $k$ -modes move with  $c_s$
- Spectrum: top-hat between  $k_*$  and  $k_*/2$

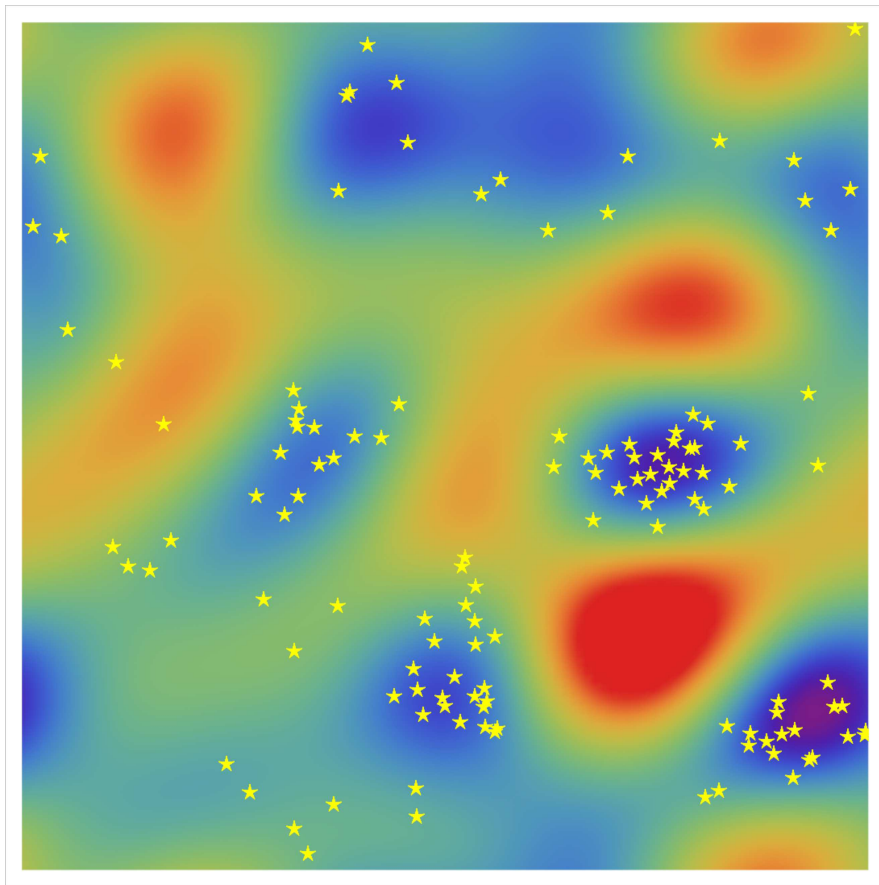
- Power in perturbations  $\sigma^2 = \frac{1}{V} \int d^3x \delta\tilde{T}(x)^2 = \int \frac{d^3k}{(2\pi)^3} \mathcal{P}_{\delta\tilde{T}}(k)$

- Expect strongest effect for  $k_* \sim R_*^{-1} \sim \frac{\beta}{(8\pi)^{1/3}}$   
 $\uparrow$   
 $\delta T = 0$

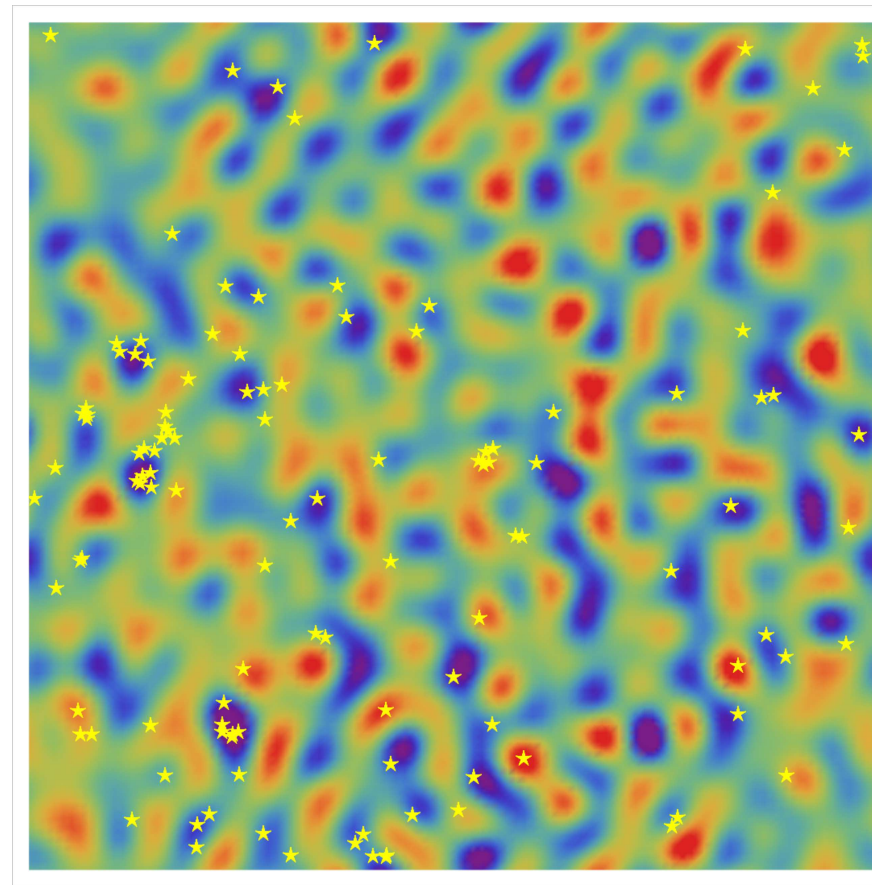
# Results

## Nucleation sites

$$\sigma = 3 \quad L = 40/\beta$$



“IR”:  $4 \times (2\pi/L)$



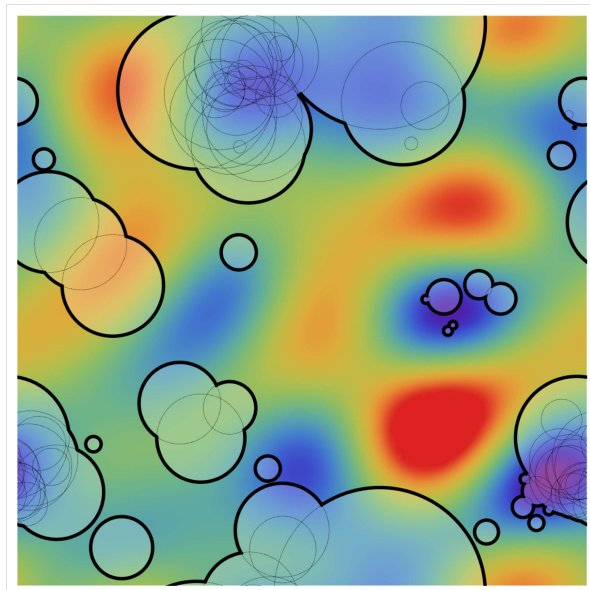
“UV”:  $k_* = 16 \times (2\pi/L)$

$$\Delta z = 2/\beta$$

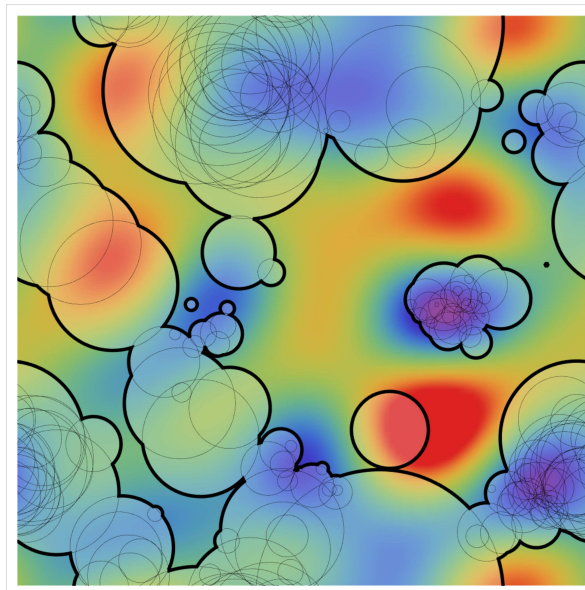
# Results

## Larger effective bubbles

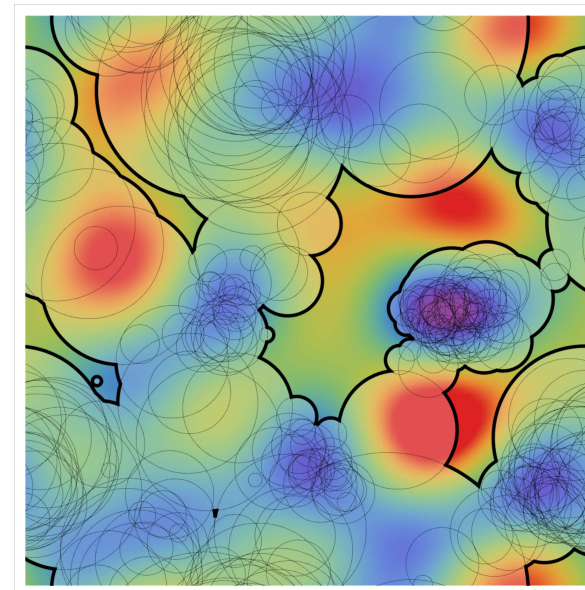
$$\sigma = 3, \quad k_* = 4 \times (2\pi/L)$$



$$t = -6/\beta$$



$$t = -5/\beta$$

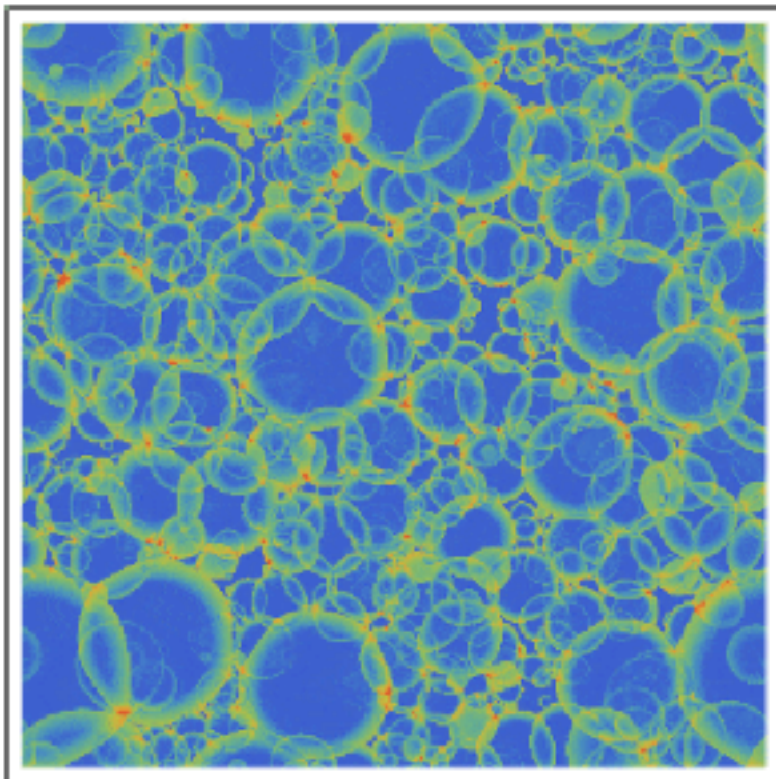


$$t = -4/\beta$$

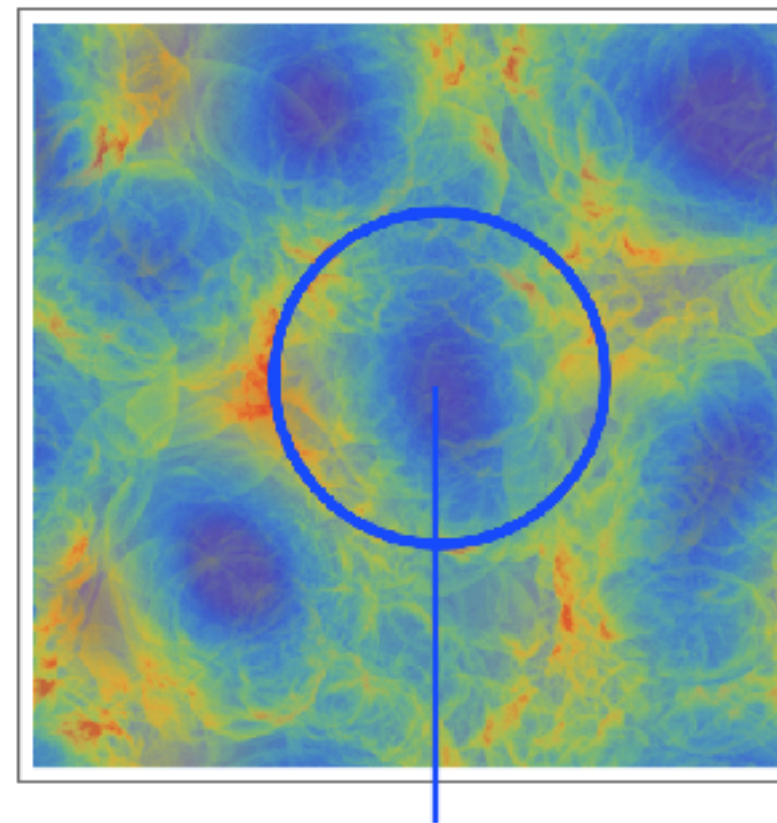
# Results

## Larger effective bubbles

From R. Jinno



No temperature fluctuations



formation of "effective big bubbles"  
around the cold spots

# Results

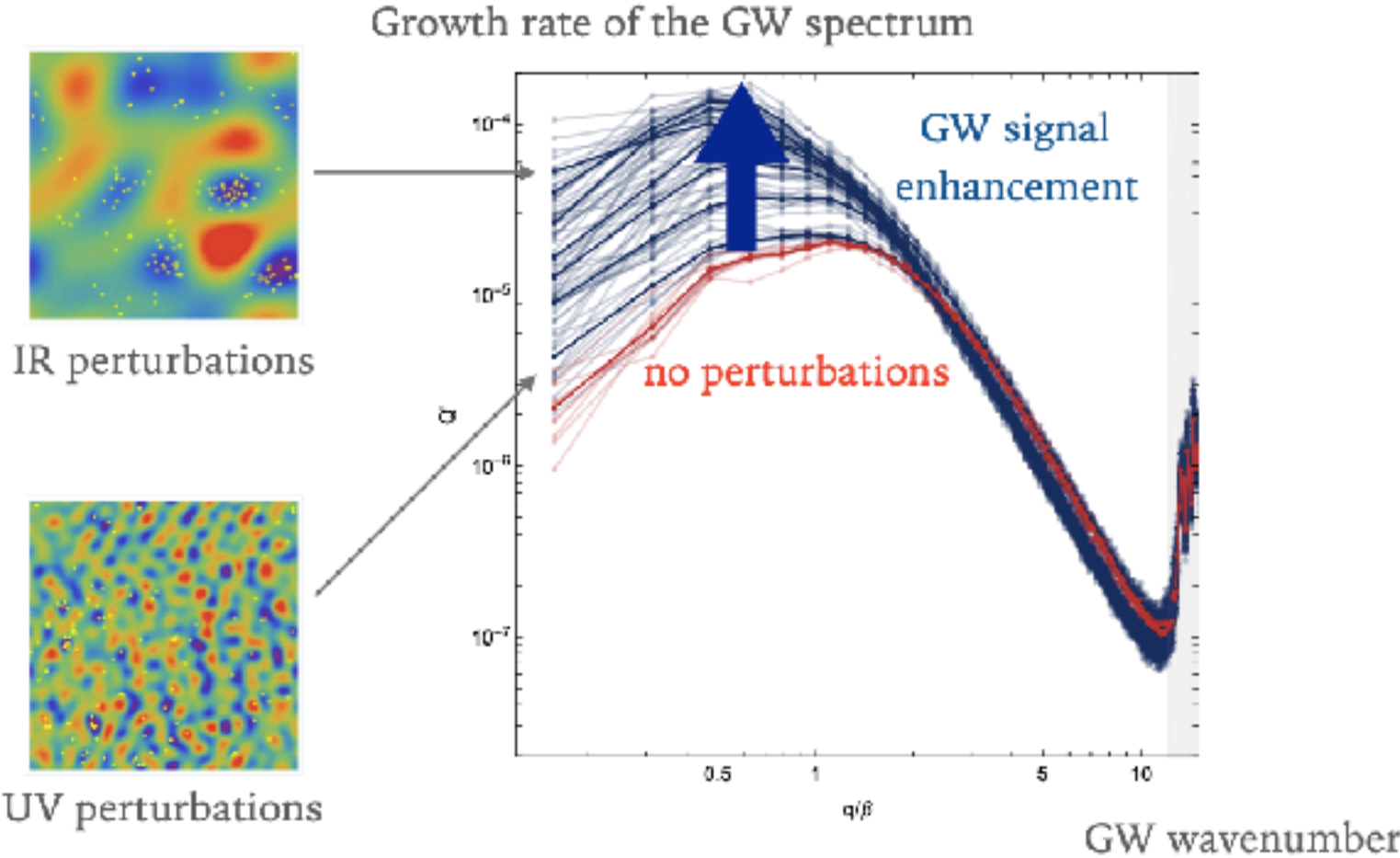
## Gravitational wave signal

- Signal scales as  $\Omega_{\text{gw}} \propto \left( \frac{\kappa\alpha}{1 + \alpha} \right)^2 R_* H_* / v_w$

# Results

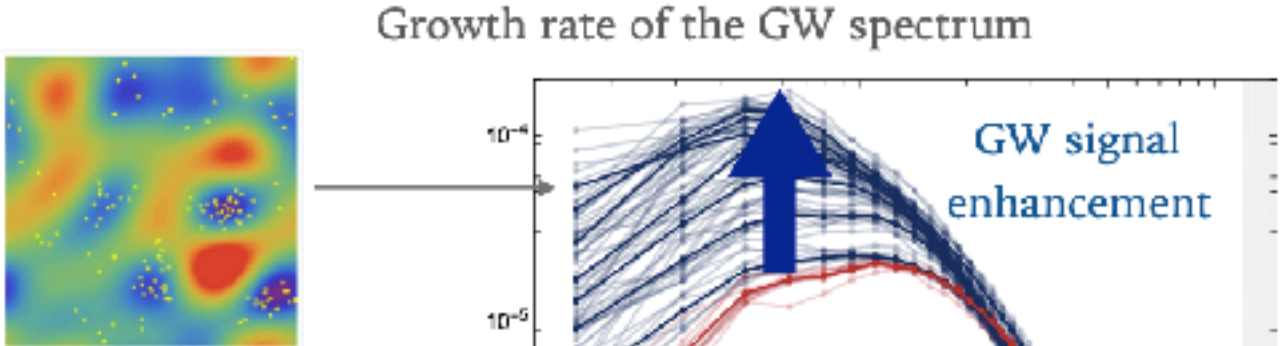
## Gravitational wave signal

From R. Jinno



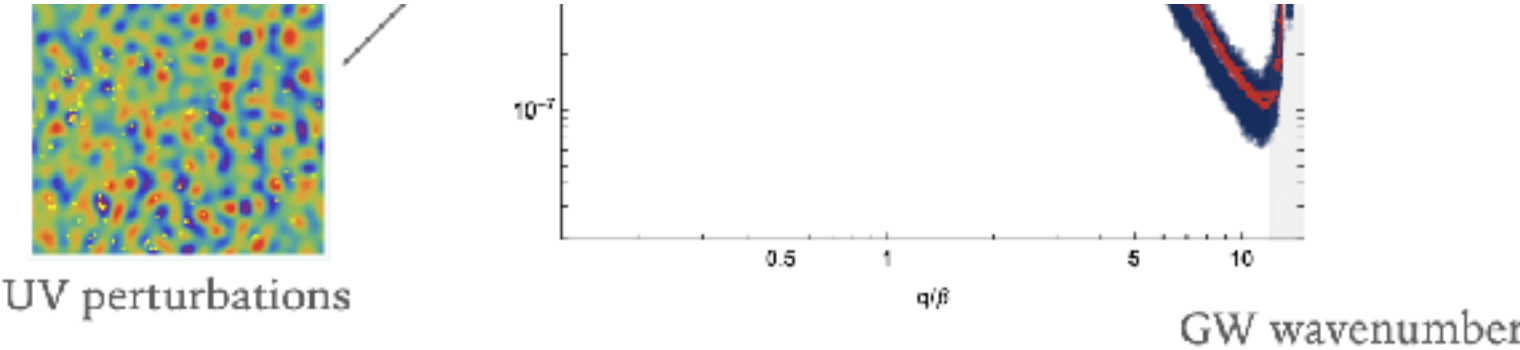
# Results

## Gravitational wave signal



From R. Jin

**The strength and wavenumber of the temperature-perturbation affect the signal too!**





# The ideal situation - how close are we?

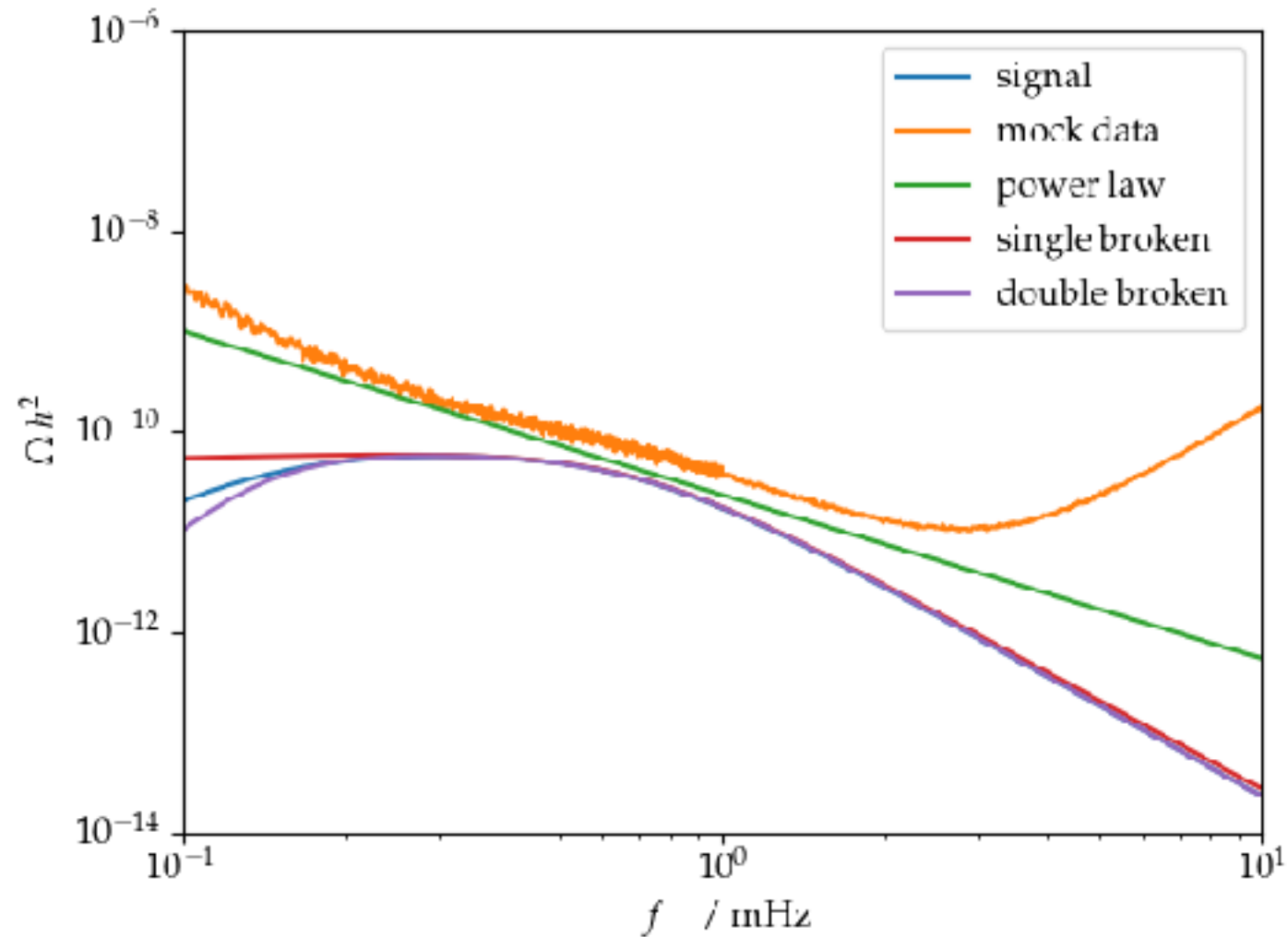
- We know exactly what the gravitational wave spectrum looks like
- There are enough model-dependent features to distinguish between different BSM models
- We are making progress!

# The ideal situation - how close are we?

- We know exactly what the gravitational wave spectrum looks like
- There are enough model-dependent features to distinguish between different BSM models
- Challenges ahead:
- Improved simulations
- Simulations combining different effects
- Accurate computation of phase transition parameters
- Computation of the wall velocity

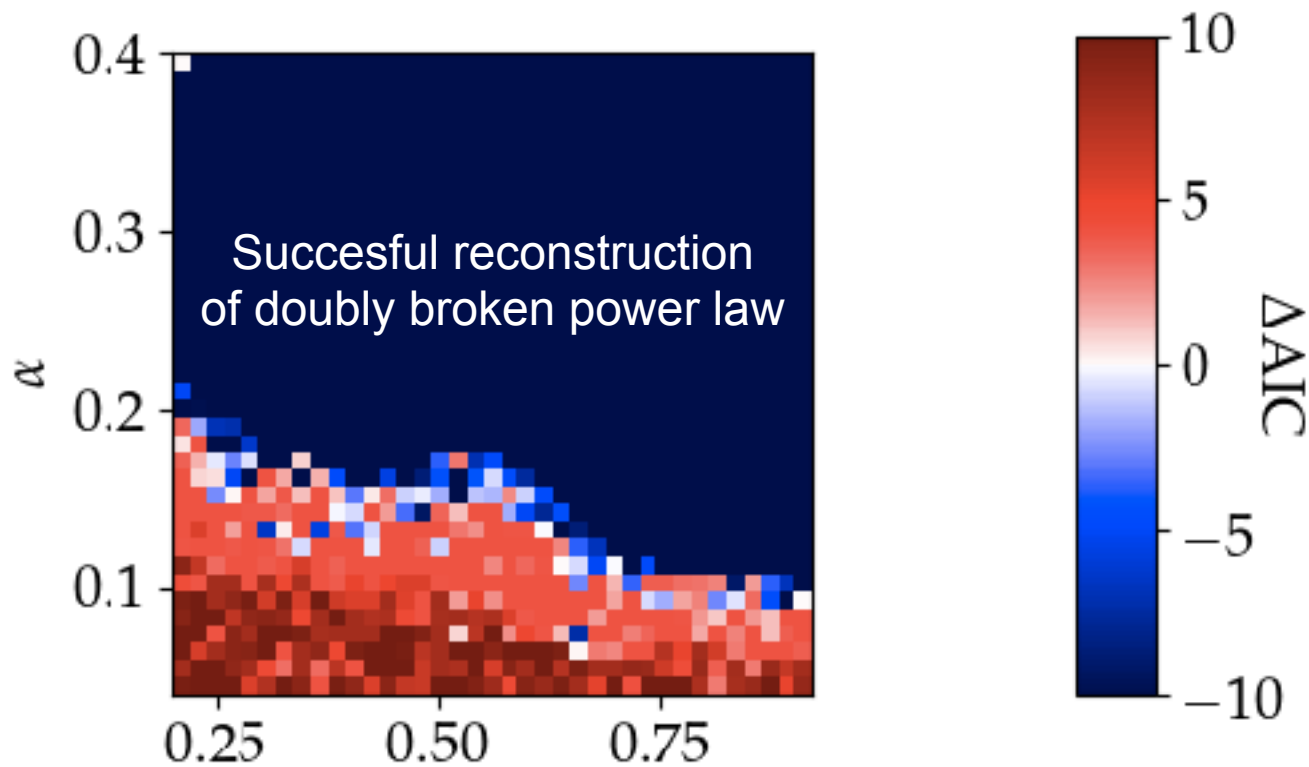
**Can all of this be measured?**

# Can LISA reconstruct a doubly broken power law? 2107.06275



# Reconstruction of the doubly broken power law

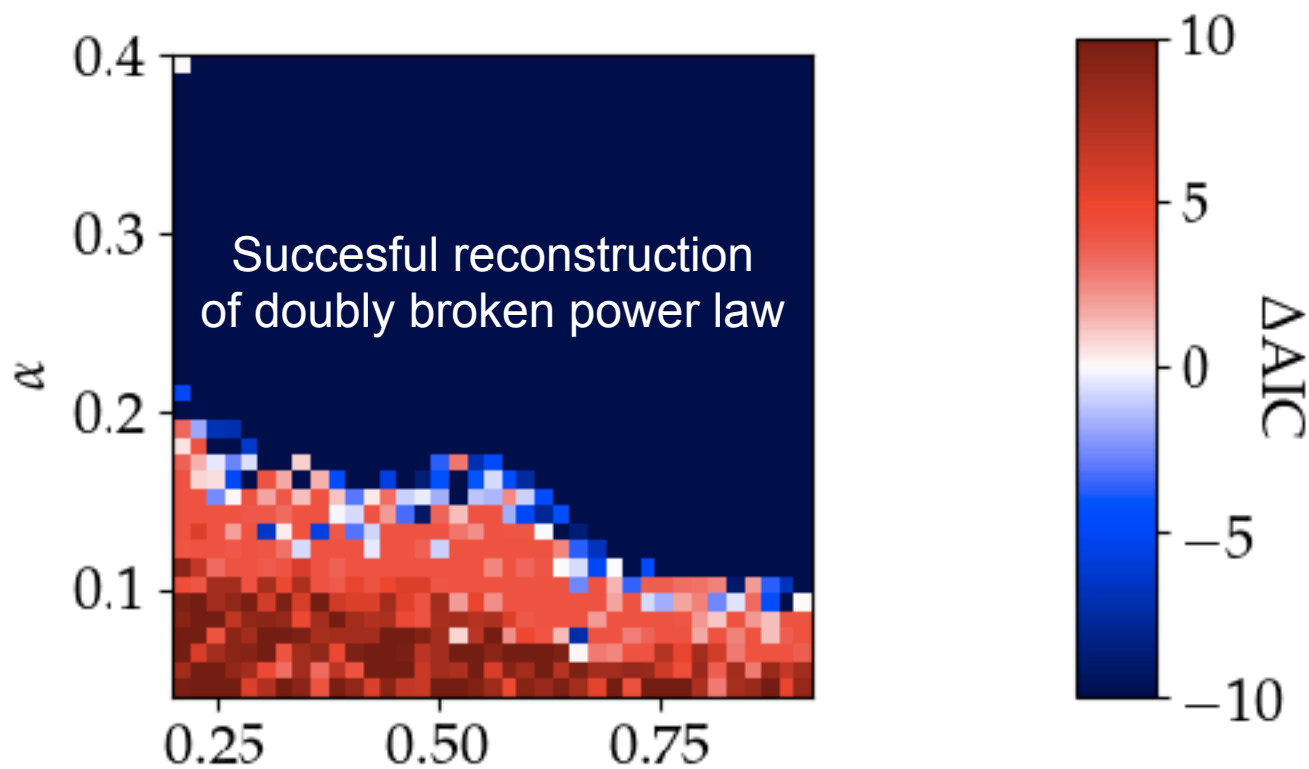
Composite Higgs/gauged lepton models



But for other extensions of the SM, reconstruction is not successful.

# Reconstruction of the doubly broken power law

Composite Higgs/gauged lepton models



But for other extensions of the SM, reconstruction is not successful. Reconstruction of all relevant parameters is not likely with LISA.

# The ideal situation - how close are we?

- The GW signal can be completely reconstructed by LISA
- We can infer the particle physics model from the signal
- Maybe we need a more sensitive detector...
- Need to develop a strategy to infer the model from the GW signal

# Conclusion

- The sound wave gravitational wave spectrum depends on  $T_*, \beta, \alpha, v_w, c_{s,b}, c_{s,s}$ .
- Hydrodynamic equations can easily be solved in the template model.
- Realistic computation of sound speed is work in progress.
- Primordial perturbations can enhance and deform the signal.
- Not all of these effects have been simulated simultaneously.
- If LISA detects a gravitational wave signal, degeneracies likely remain.



# The future looks (and sounds) bright!

