

University of Washington  
Institute for Nuclear Theory

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# Radiative corrections in neutrino physics



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LA-UR-23-20009

# Questions in neutrino physics

number of species

are there sterile neutrinos?

PMNS matrix

CP violation in  
lepton sector

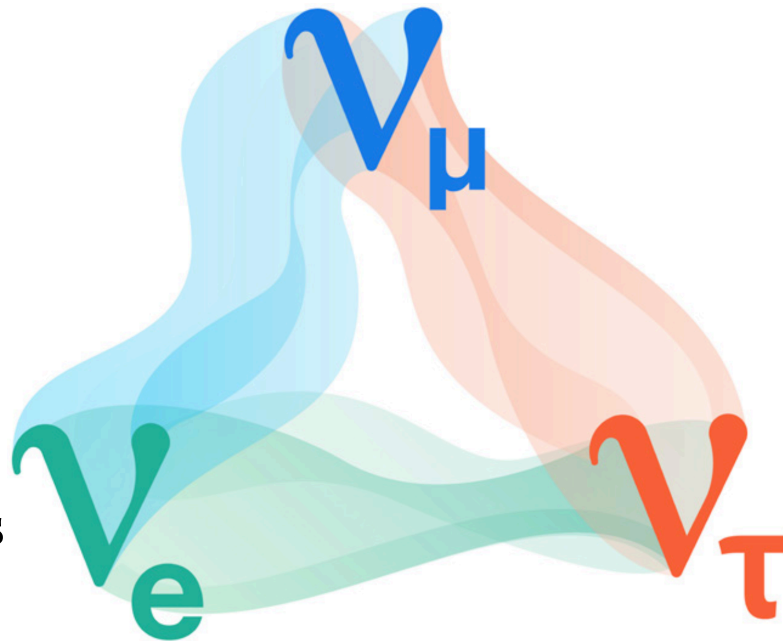
precise values of  
oscillation parameters

neutrino masses

Dirac vs Majorana

hierarchy of masses

absolute values



Astrophysics and Cosmology

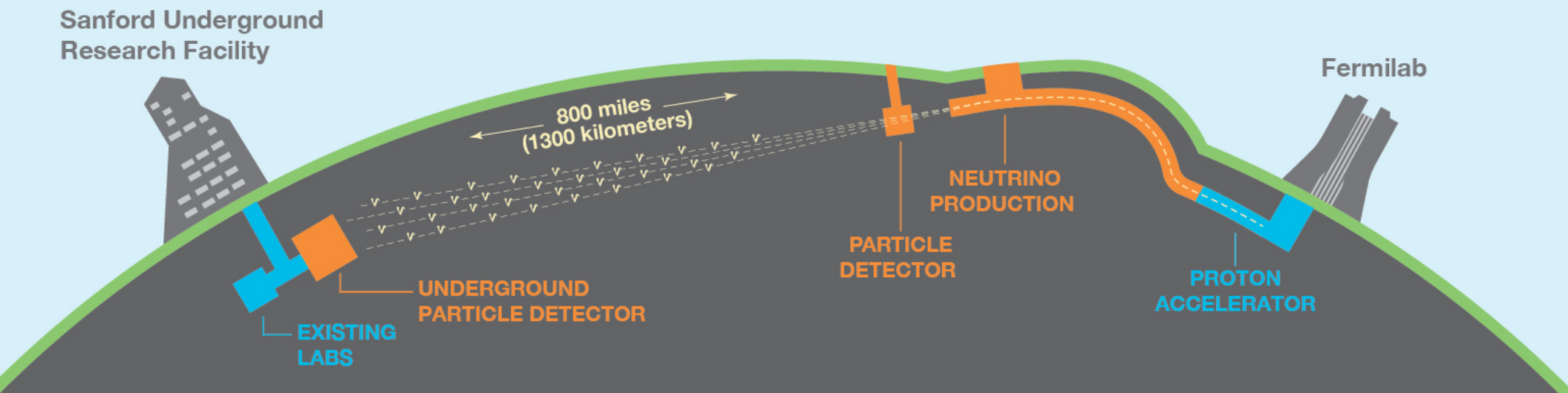
cosmic neutrinos

neutrinos as part of dark matter

detection of Big Bang relic neutrinos

# Neutrino experiments

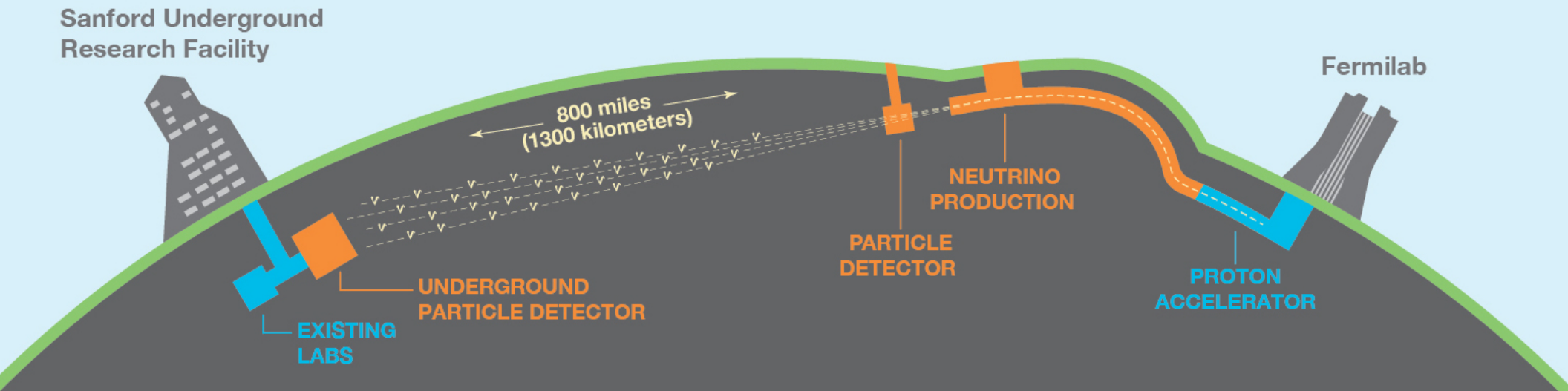
- **DUNE** and Hyper-K: leading-edge  $\nu$  science experiments



- origin of matter-antimatter asymmetry  $\delta_{CP}$
- mass hierarchy and oscillation parameters PMNS matrix,  $\Delta m_{31}^2$
- Grand Unified Theories proton decay
- dynamics of supernova explosion wait for one;)

# Neutrino experiments

- **DUNE** and Hyper-K: leading-edge  $\nu$  science experiments



- measurement of  $\nu_\mu(\bar{\nu}_\mu)$  disappearance and  $\nu_e(\bar{\nu}_e)$  appearance

$$N_\nu \sim \int dE_\nu \Phi_\nu(E_\nu) \times \sigma(E_\nu) \times R(E_\nu, E_\nu^{\text{rec}})$$

- near detector: determine flux and cross sections

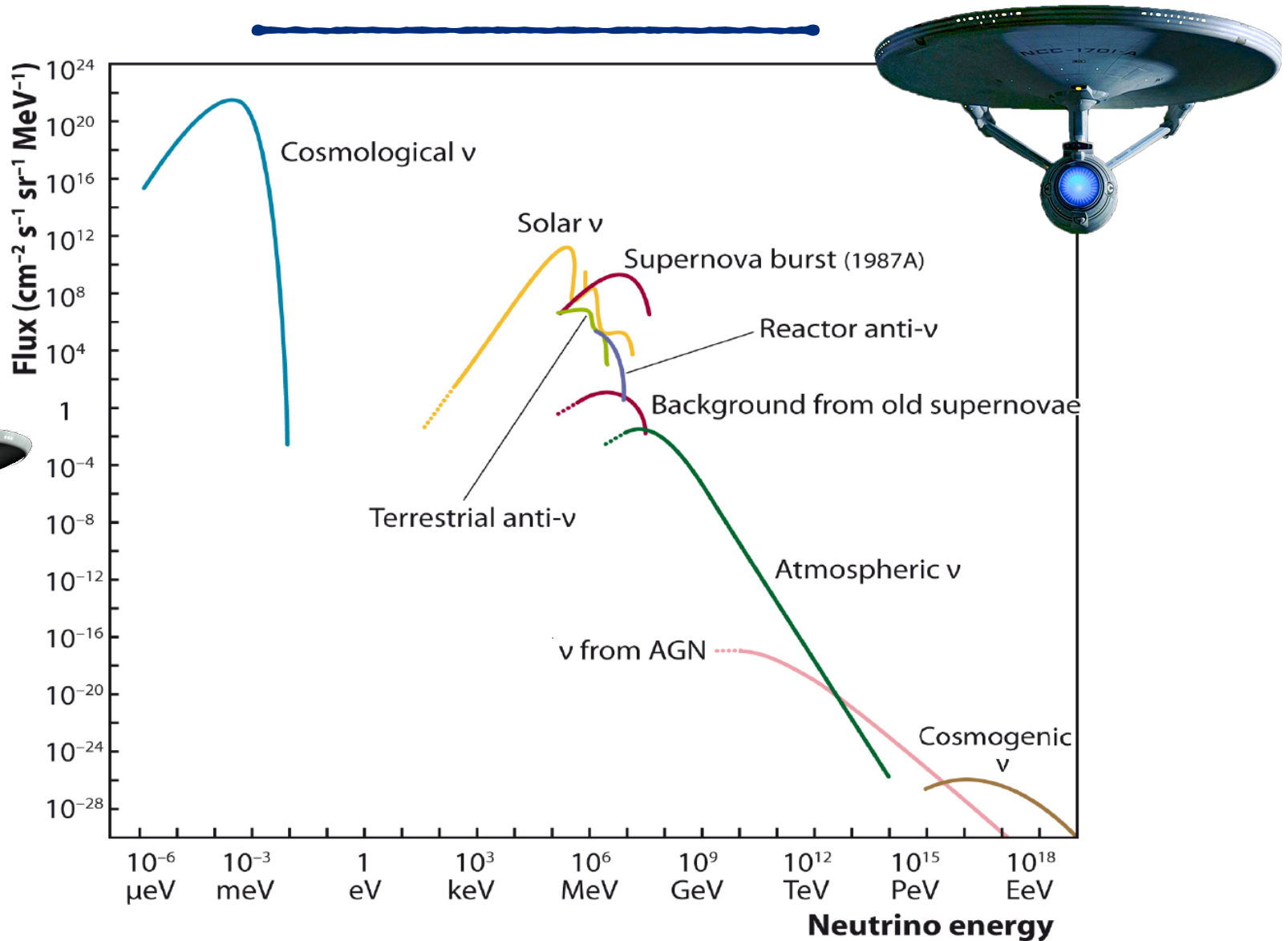
# Outline

- 1) neutrino sources across energy scales
- 2) cross sections on electrons, nucleons, and nuclei
- 3) radiative corrections in neutrino physics
- 4) charged-current scattering on **nucleons**

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# Neutrino interactions across energy scales

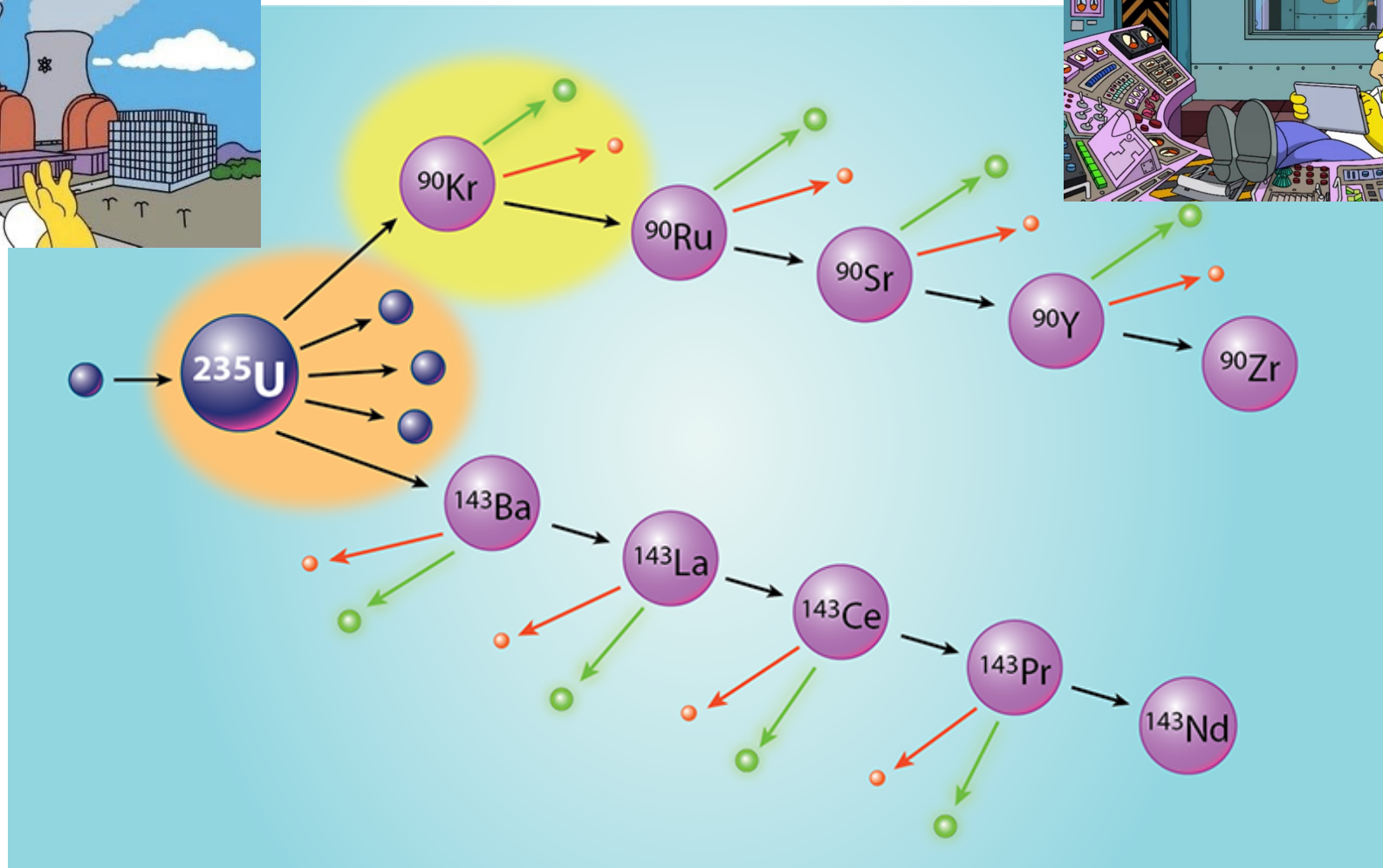
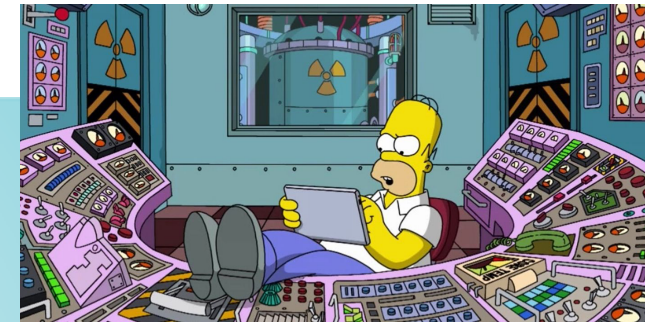
# Nature sources



U.F. Katz, Ch. Spiering, Prog. Part. Nucl. Phys. 67, 651-704 (2012)

- cosmological, cosmogenic, supernova background: to be detected

# Reactor (anti)neutrinos

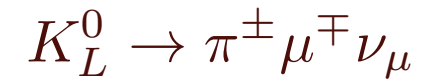
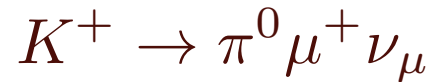
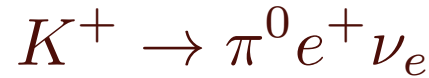
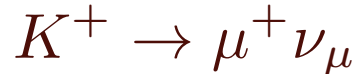


A. Bernstein, N. Bowden, B.L. Goldblum, P. Huber,  
I. Jovanovic, J. Mattingly, Rev. Mod. Phys. 92, 011003 (2020)

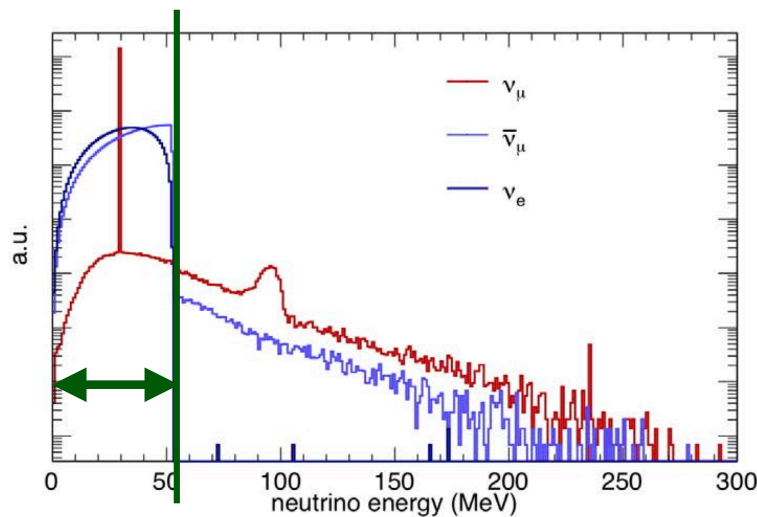
- first detected neutrinos; antineutrinos from nuclear beta decays



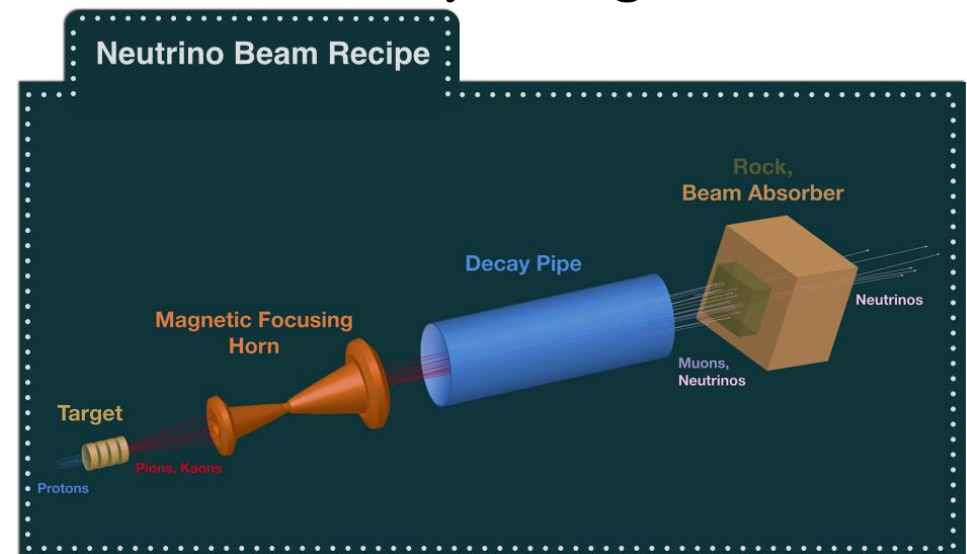
# Artificial neutrinos: accelerator



decay at rest



decay in flight



Akimov et al., Science 357 6356, 1123-1126 (2017)

Coherent and CCM

meson decay: monochromatic line

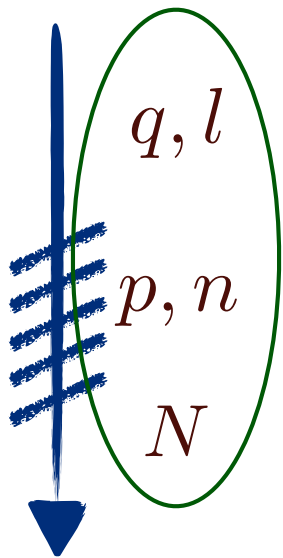
[www.fnal.gov](http://www.fnal.gov)

T2K, NOvA, MiniBooNE, MicroBooNE

MINERvA, MINOS, NuTeV

SBN, DUNE, HyperK, ESSnuSB

- precise measurements of neutrino properties, EW, and BSM search
- physics program relies on neutrino cross sections in MeV-TeV range



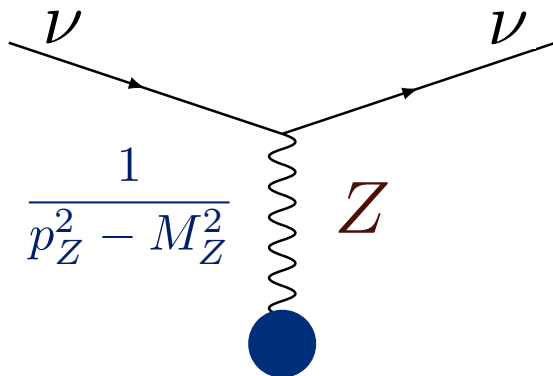
# Cross sections on electrons, nucleons, and nuclei

# Neutral- and charged-current processes

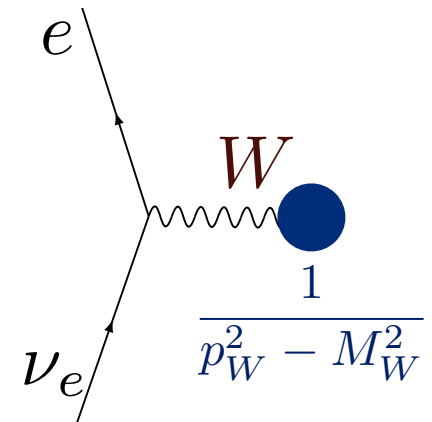
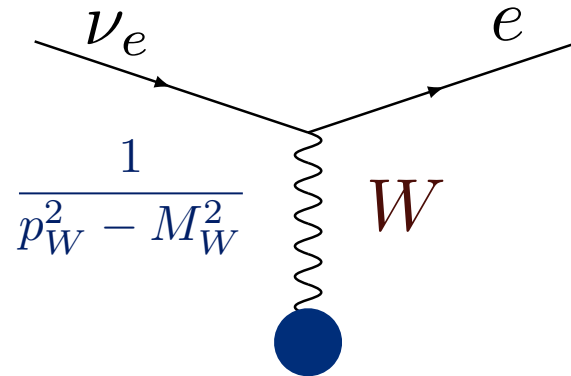
- cross sections determine neutrino-induced events

$$N_\nu \sim \int dE_\nu \Phi_\nu(E_\nu) \times \sigma(E_\nu) \times R(E_\nu, E_\nu^{\text{rec}})$$

neutral current



charged current



- contact interactions at GeV energy scale and below

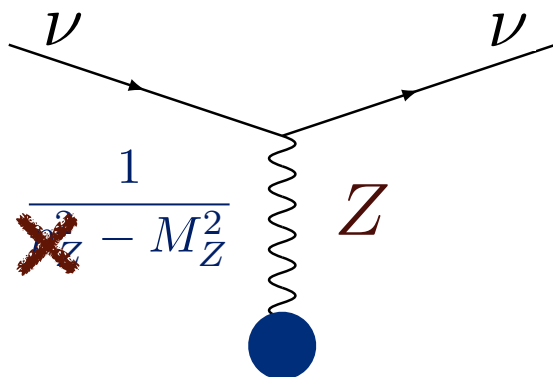
- charged current (only): **threshold** to produce lepton and recoil
- neutral current: **no thresholds**

# Neutral- and charged-current processes

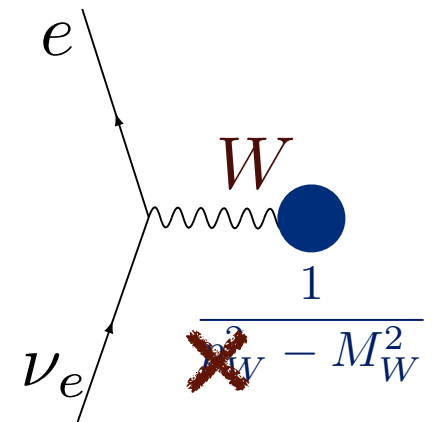
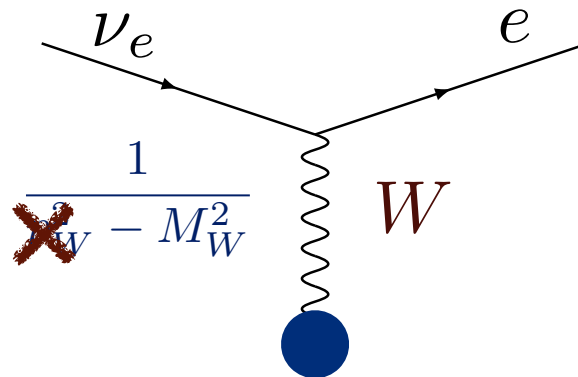
- cross sections determine neutrino-induced events

$$N_\nu \sim \int dE_\nu \Phi_\nu(E_\nu) \times \sigma(E_\nu) \times R(E_\nu, E_\nu^{\text{rec}})$$

neutral current



charged current

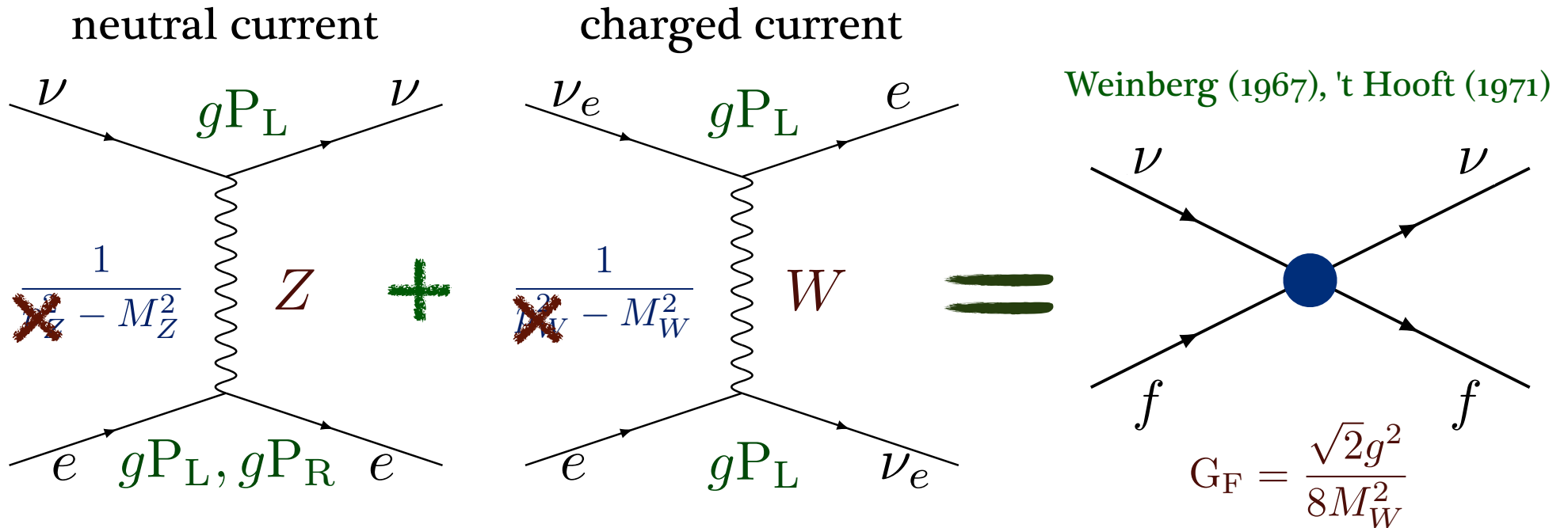


- contact interactions at GeV energy scale and below

- charged current (only): **threshold** to produce lepton and recoil
- neutral current: **no thresholds**

# Neutrino-electron scattering

- Fermi theory at GeV energies and below,  $\sigma \sim m$

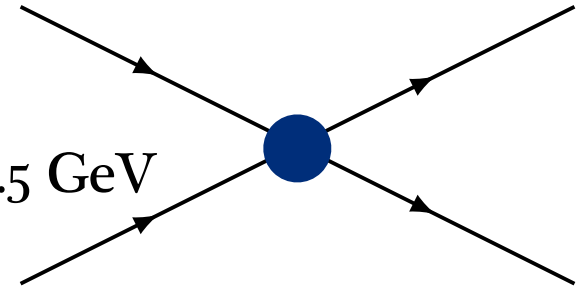


- s-channel resonant enhancement at vector-boson pole (PeV scale)

- historically: precise EW physics and BSM searches
- channel for in-situ flux constraints at accelerator experiments
- solar neutrinos@Super-K, SNO, Borexino
- recent observation of Glashow resonance by IceCube

# Neutrino-nucleon scattering (CC)

- 4-Fermi theory + ChPT @ and < pion-mass scale
- production thresholds: muon  $\sim 110$  MeV, tau  $\sim 3.5$  GeV
- only electron flavor for supernova, solar, and reactor neutrinos



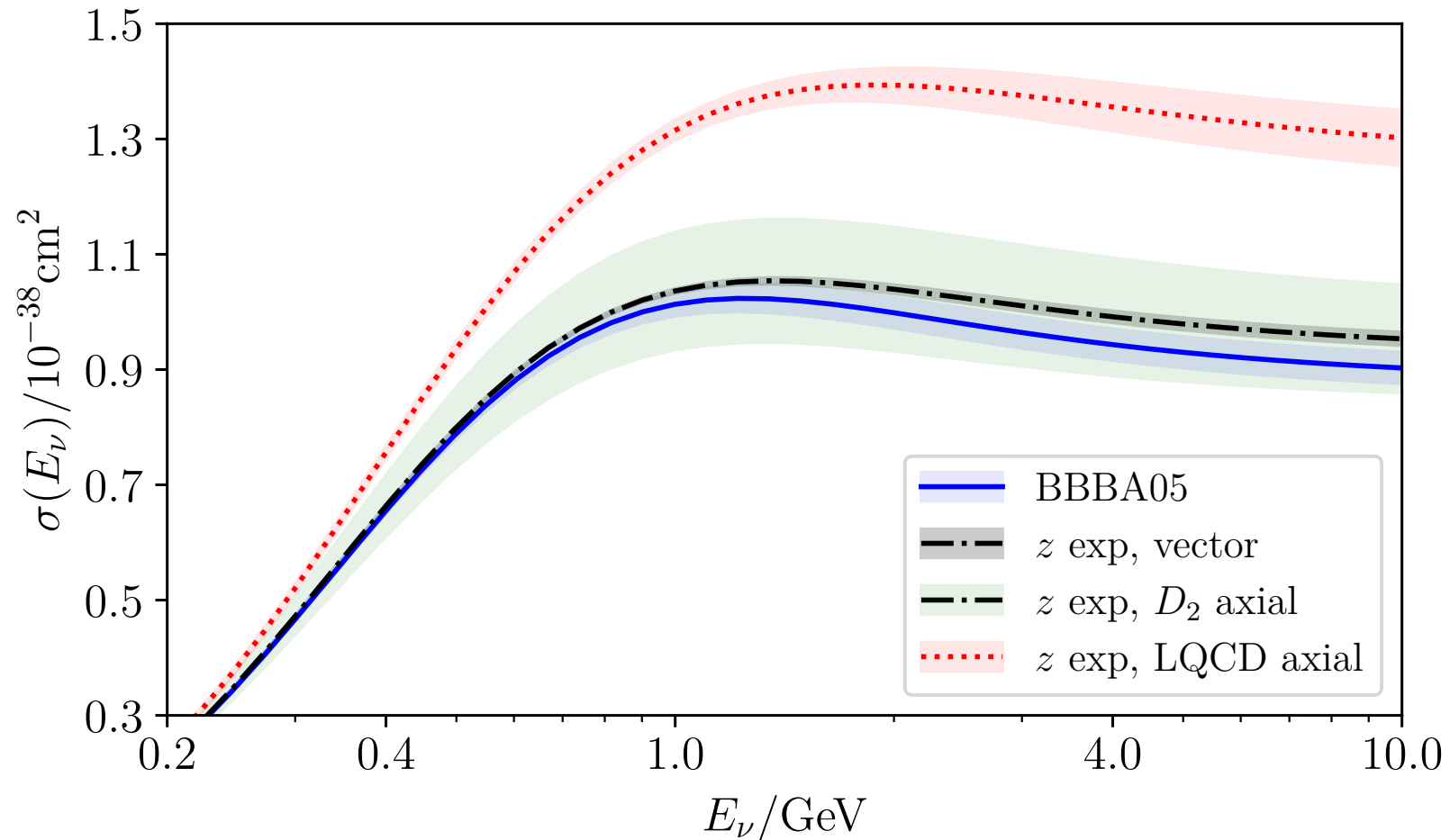
- data from deuterium bubble chambers in 1980th
- CH<sub>2</sub>-C subtraction results are anticipated
- provide nucleon axial form factor
- target for many lattice QCD groups



Fermilab bubble chamber, Richard Drew

- elastic scattering -> pion production -> deep inelastic scattering

# Neutrino-nucleon scattering (CC)



A.S. Meyer, A. Walker-Loud, C. Wilkinson, *Ann. Rev. of* 72, 010622-120608 (2022)

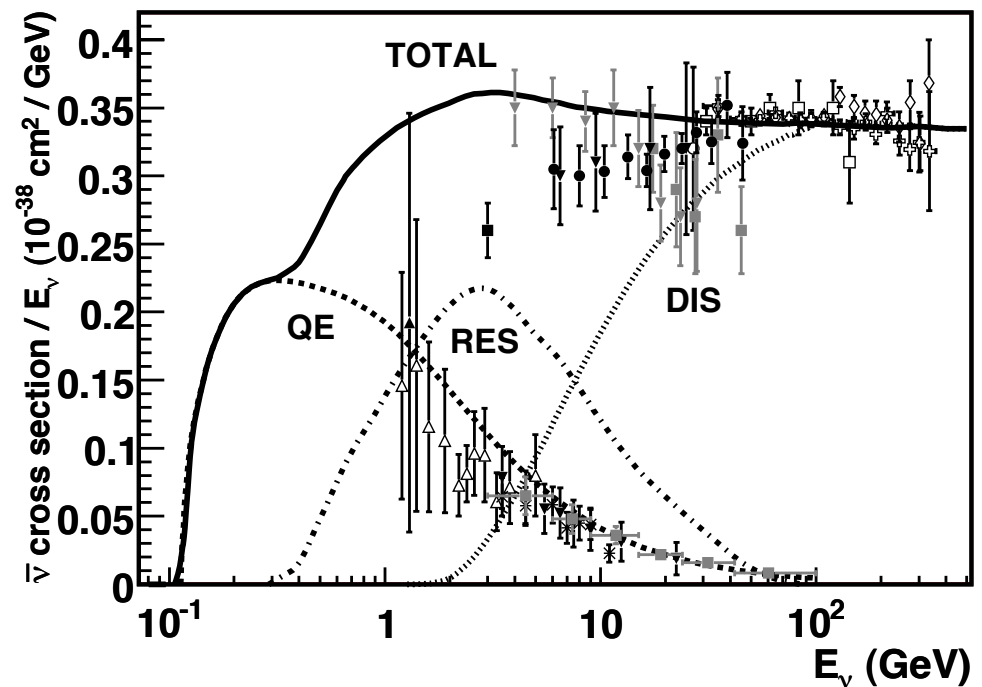
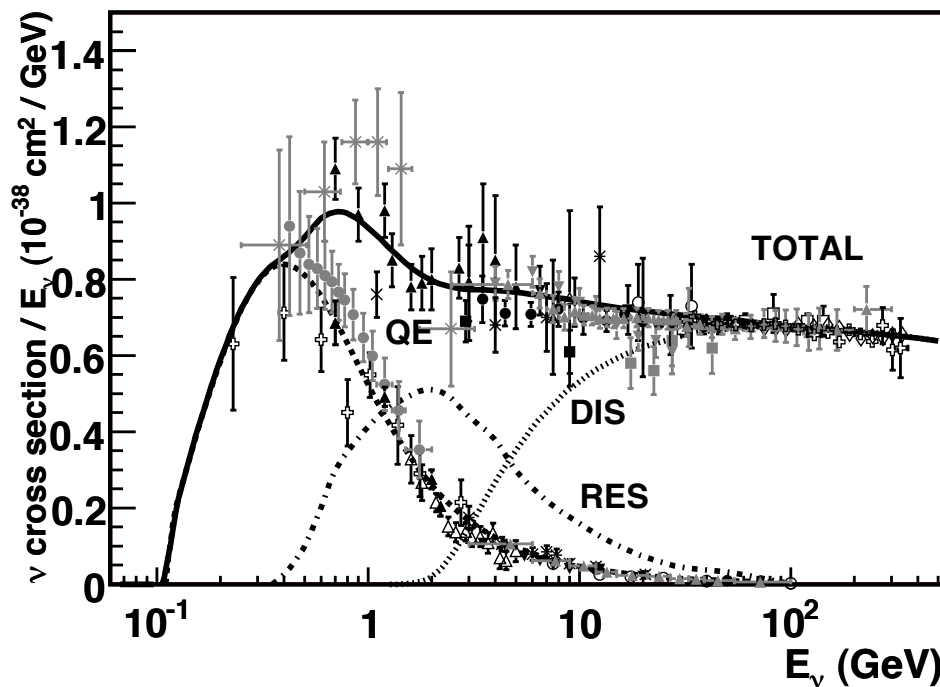
A.S. Meyer, M. Betancourt, R. Gran, and R.J. Hill, *PRD* (2016)

Kaushik Borah, Gabriel Lee, Richard J. Hill, and O. T., *PRD* (2021)

- knowledge of vector structure stops a progress in studies of axial
- acknowledged discrepancy: lattice QCD  $\leftrightarrow$  experimental data

# Neutrino-nucleus scattering

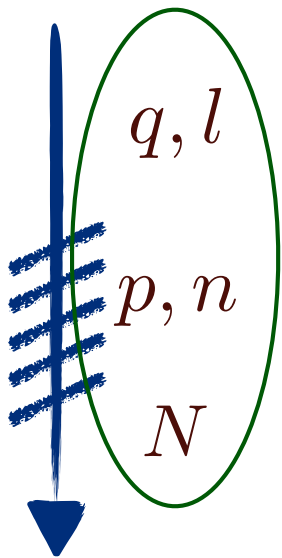
- NC scattering across all energies  $\rightarrow$  neutrino floor
- CC with electron flavor for supernova, solar, and reactor neutrinos
- same open channels as at nucleon level



Formaggio and Zeller (2013)

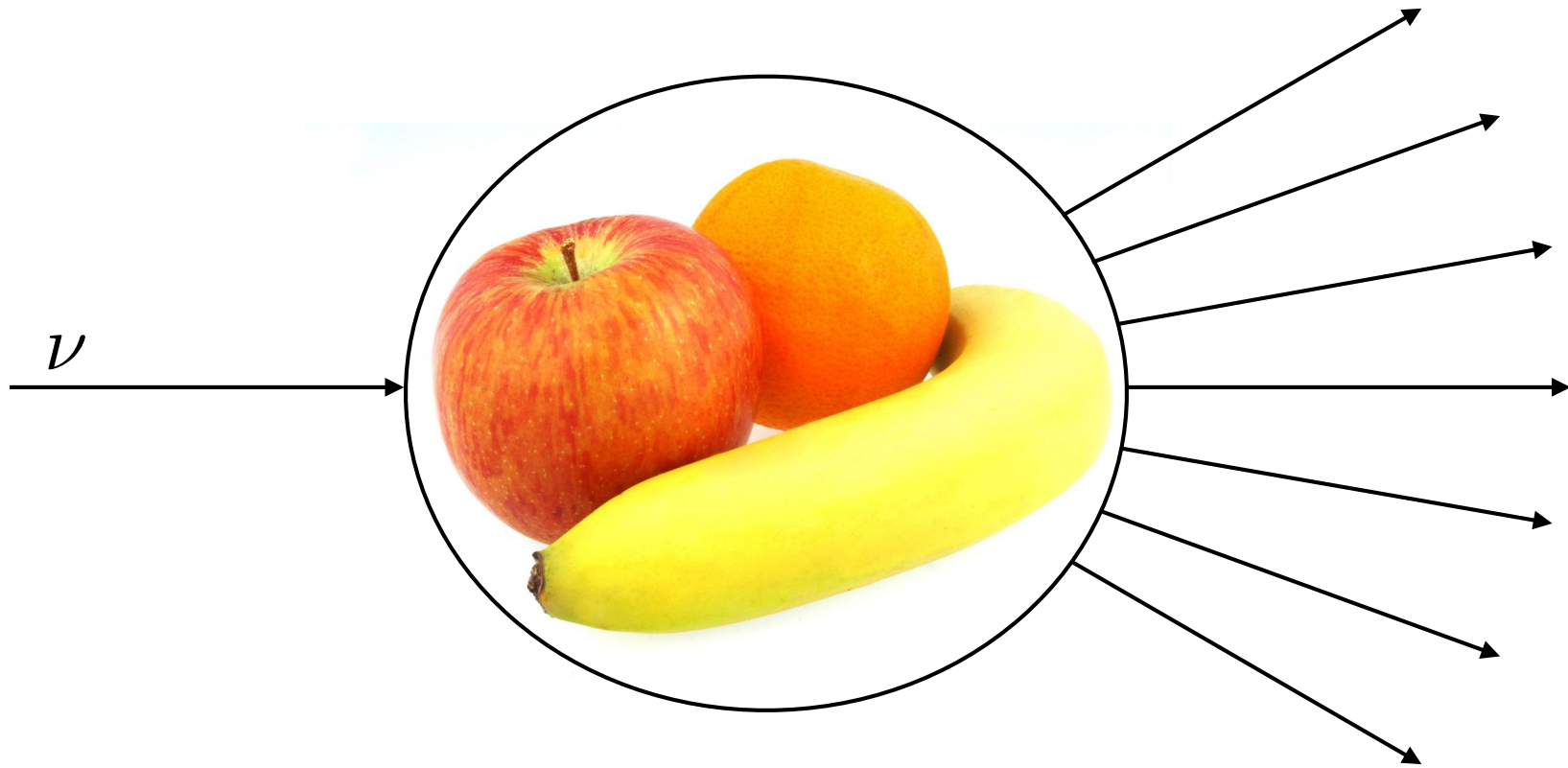
- binding effects, Fermi motion, Pauli blocking
- meson exchange, 2p-2h, final-state interaction



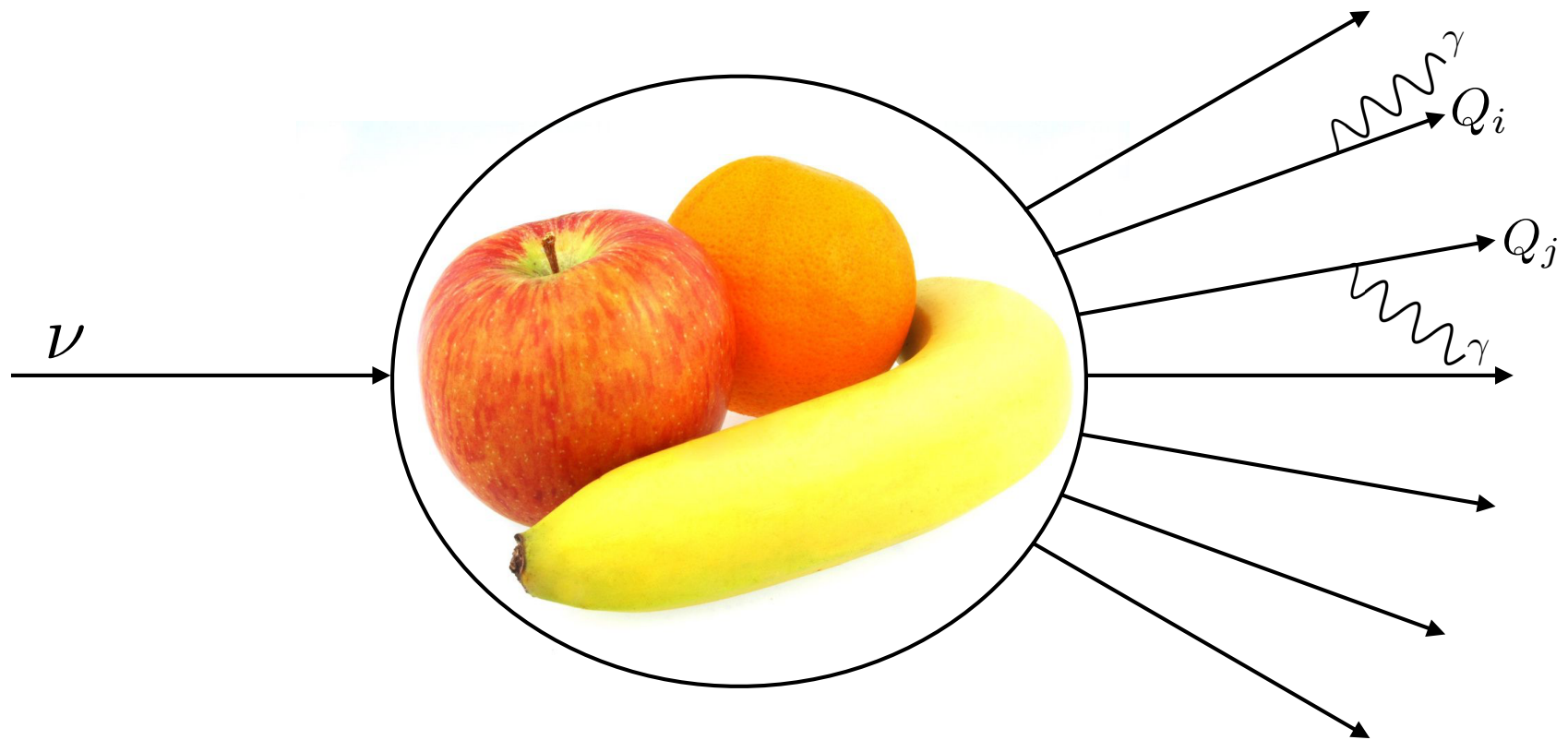


# Radiative corrections in neutrino physics (at MeV-GeV energies)

# Neutrino interactions

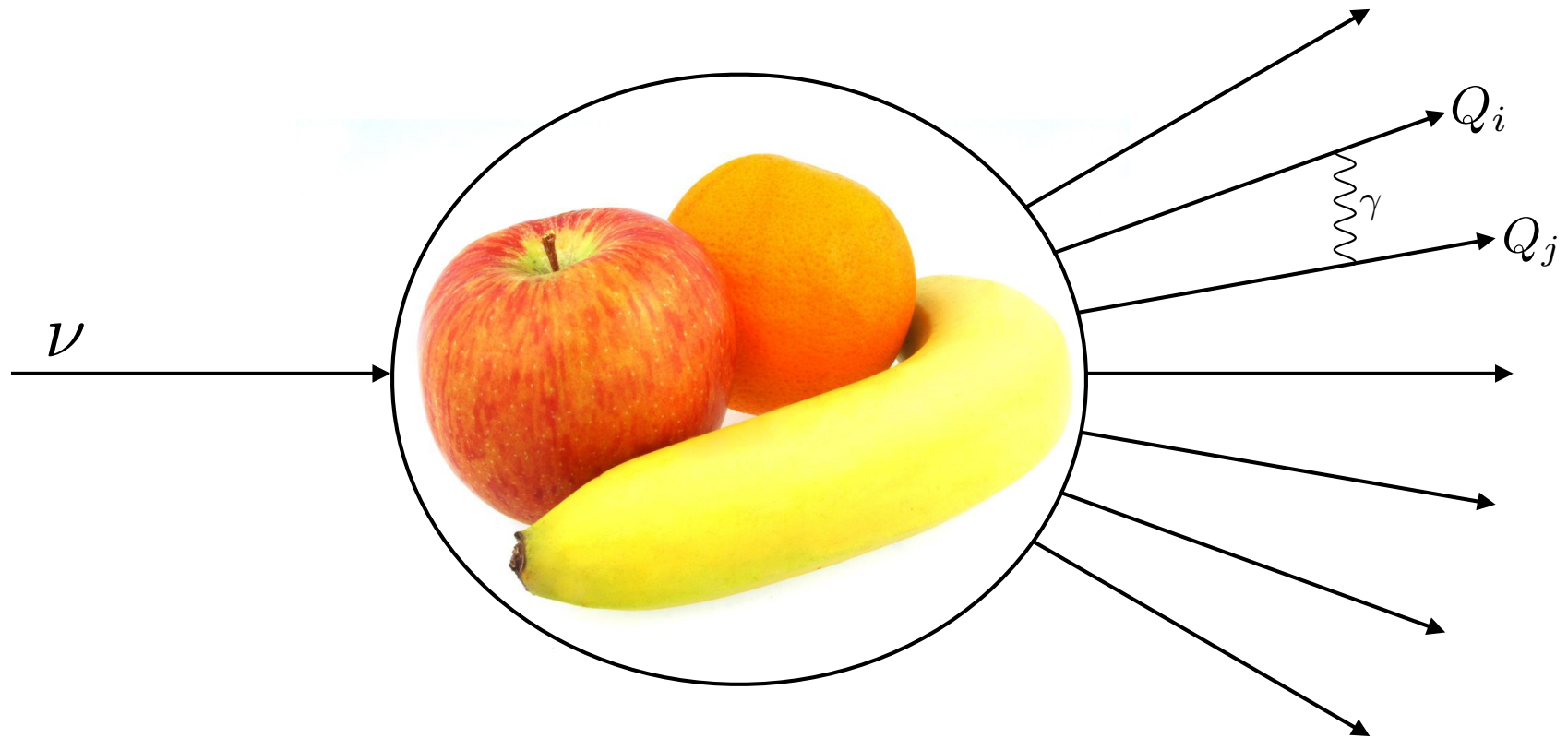


# QED corrections



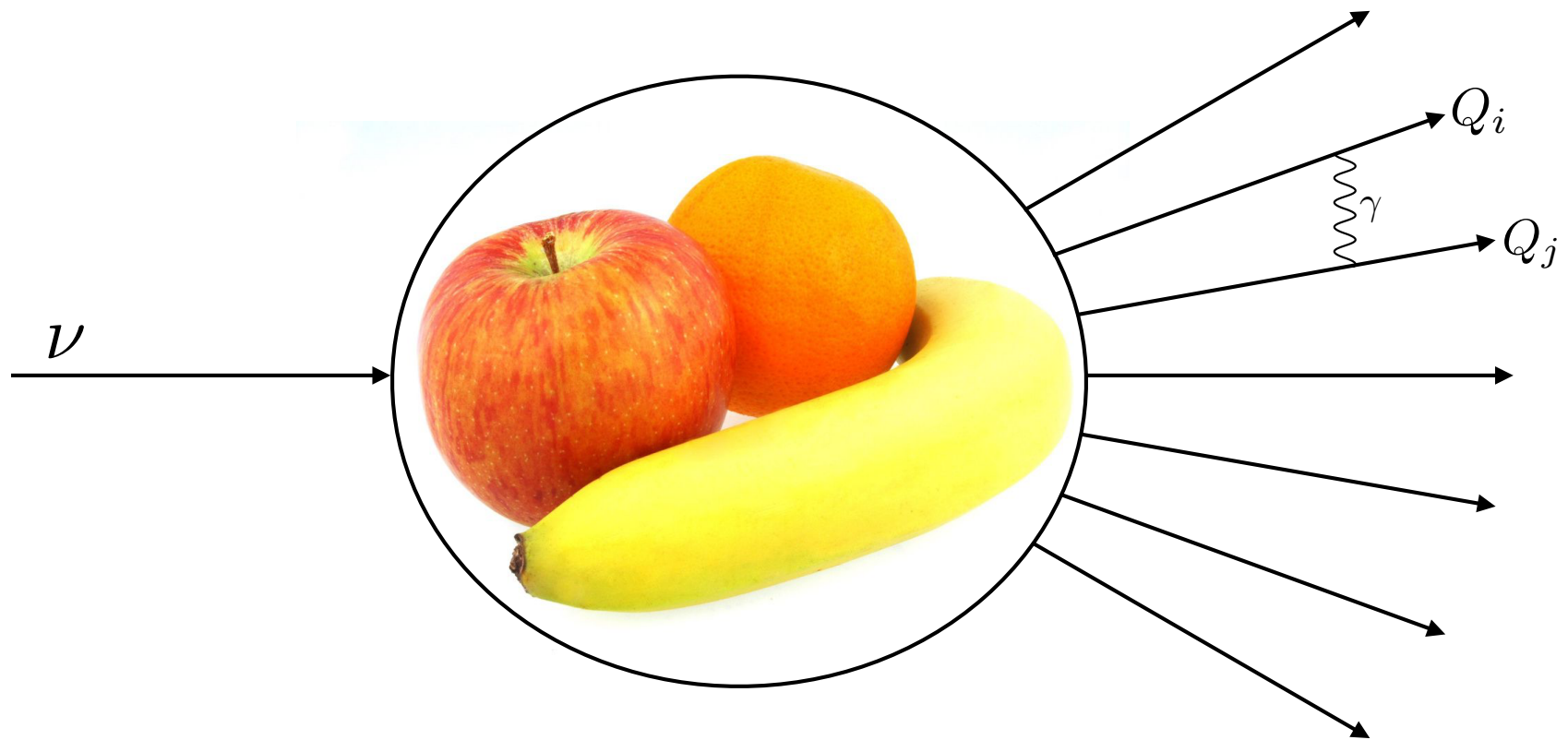
- all charged particles couple to real and virtual photons

# QED corrections



- all charged particles couple to real and virtual photons

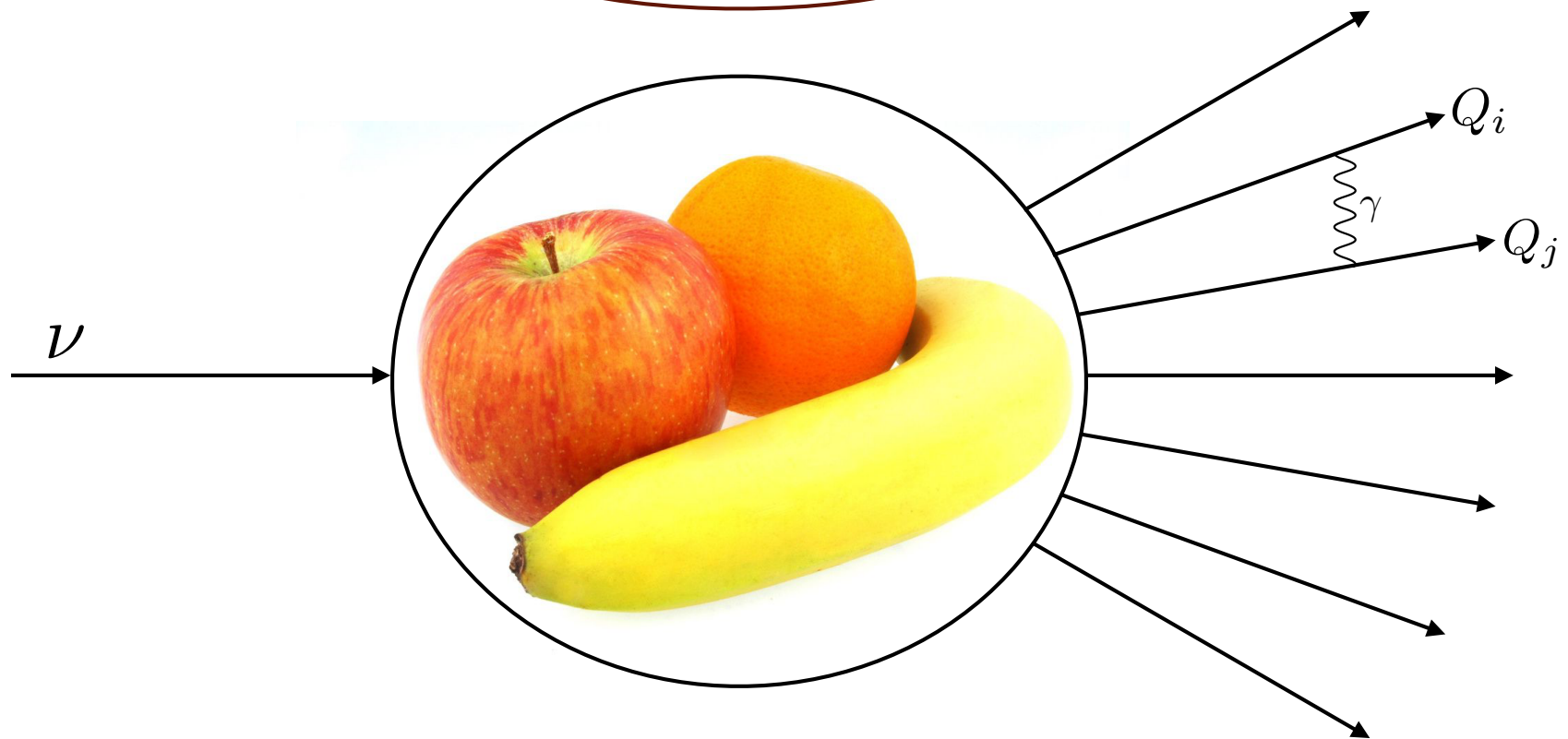
# QED corrections



-  $\frac{\alpha}{\pi} \sim 0.2\%$  suppression by electromagnetic coupling constant

# QED corrections

$$m_e \ll m_\mu \ll E_\nu$$

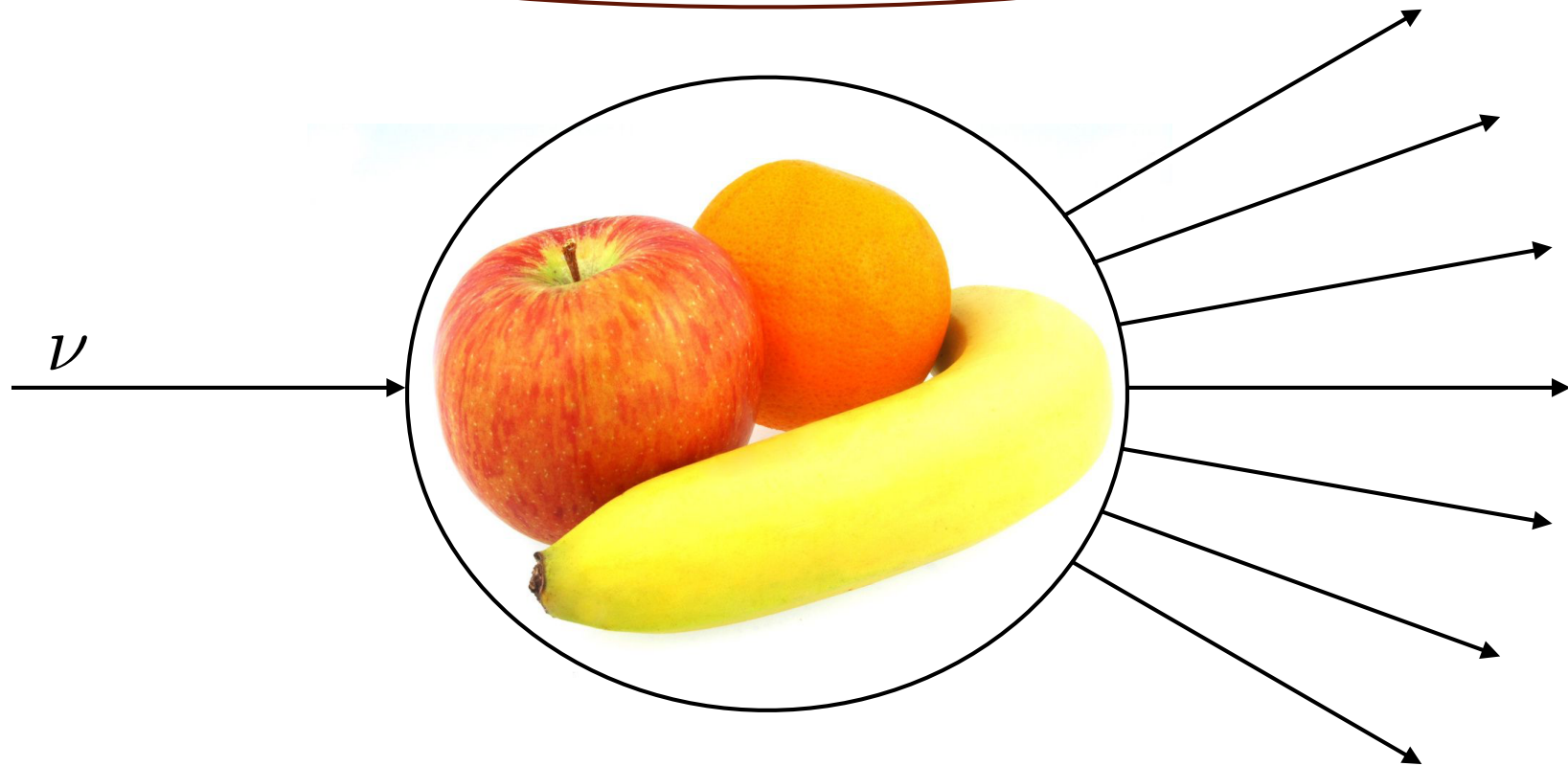


$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \ln \frac{E_\nu}{m_e} \sim 6 - 10 \text{ or } \ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$$

- scale separation introduces large flavor-dependent QED logarithms

# Electroweak corrections

$$m_e, m_\mu, M, E_\nu \ll M_W, M_Z, m_t, m_H$$



$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \frac{1}{\sin^2 \theta_W}, \ln \frac{M_Z}{M}, \ln \frac{M_t}{M}, \dots$$

- electroweak corrections can be included in low-energy interactions

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couplings of **effective Lagrangian** are precisely determined

$$\mathcal{L}_{\text{eff}}^{\text{NC}} = -\bar{\nu}_l \gamma_\mu P_L \nu_l \cdot \bar{f} \gamma^\mu (c_L^{\nu_l f} P_L + c_R^{\nu_l f} P_R) f$$

$$\mathcal{L}_{\text{eff}}^{\text{CC}} = -2\sqrt{2}G_F \sum_{l \neq l'} \bar{\nu}_{l'} \gamma^\mu P_L \nu_l \bar{\ell} \gamma_\mu P_L \ell' - c^{qq'} \sum_{q \neq q'} \bar{\ell} \gamma^\mu P_L \nu_l \bar{q} \gamma_\mu P_L q'$$

## Neutrino-lepton, neutrino-quark scattering

O.T. and Richard J Hill, Phys. Lett. B 805, 3, 135466 (2020)

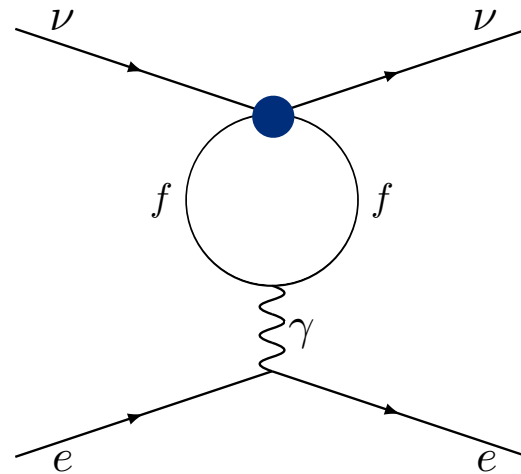
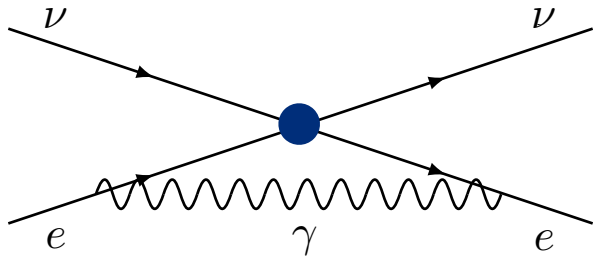
known at permille level



leading in  $G_F$  terms with loop expansion in  $\alpha$ ,  $\alpha_s$  within Standard Model

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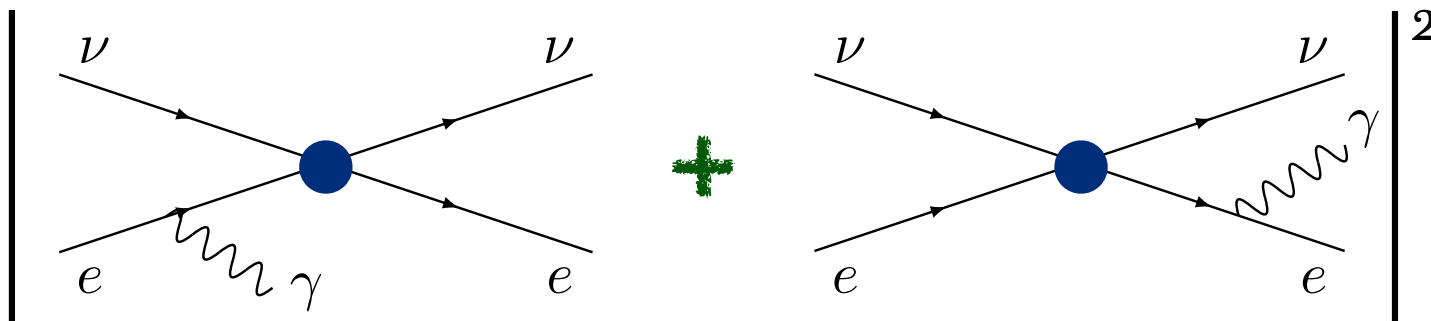


# Neutrino-electron scattering

O.T. and Richard J Hill, Phys. Rev. D 101 3, 033006 (2020)

percent-level predictions for MINERvA

known analytically at permille level for NOvA and DUNE, solar  $\nu$

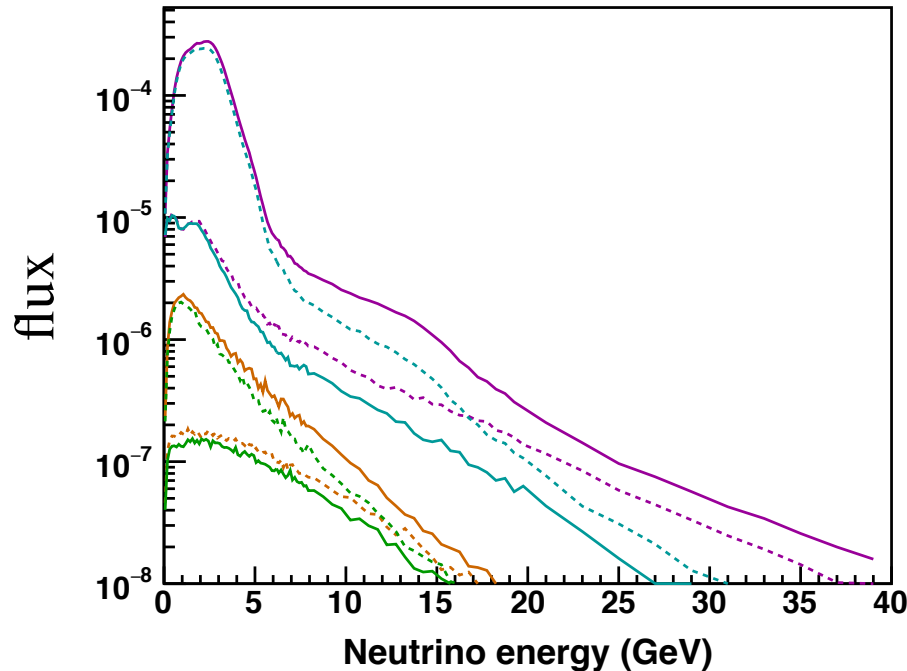


# Main theoretical uncertainty

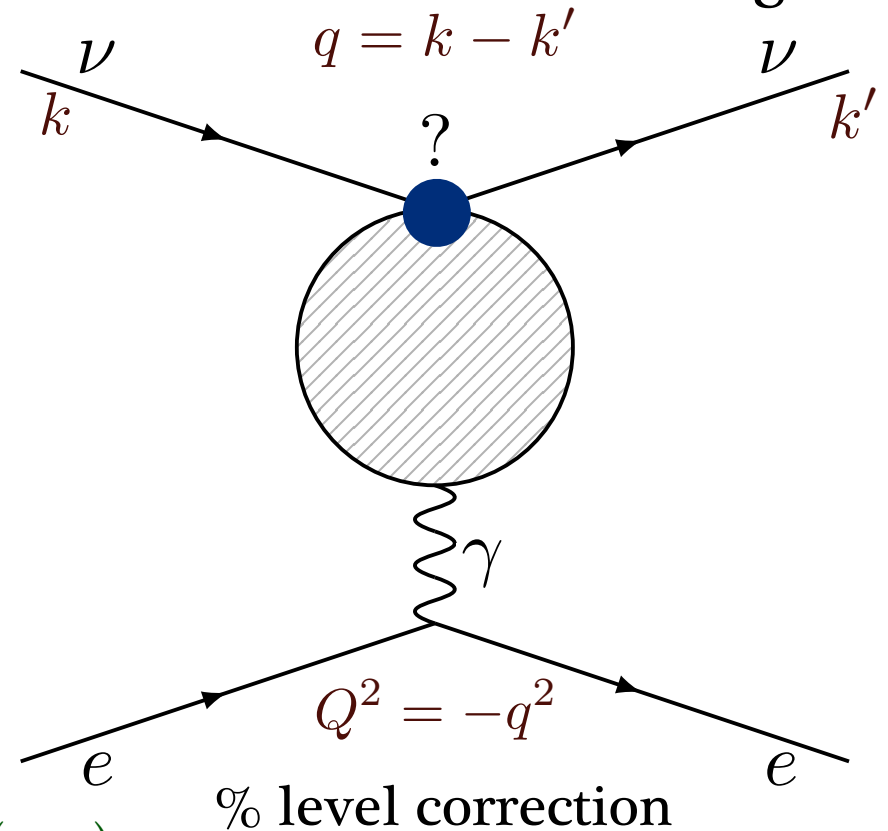
- kinematics is suppressed by electron mass:

$$s, Q^2 \lesssim 2mE_\nu \ll \Lambda_{\text{QCD}}^2$$

- description in terms of quarks is invalid for GeV neutrino energies



Ch. Marshall et al, Phys.Rev.D 101 3, 032002 (2020)

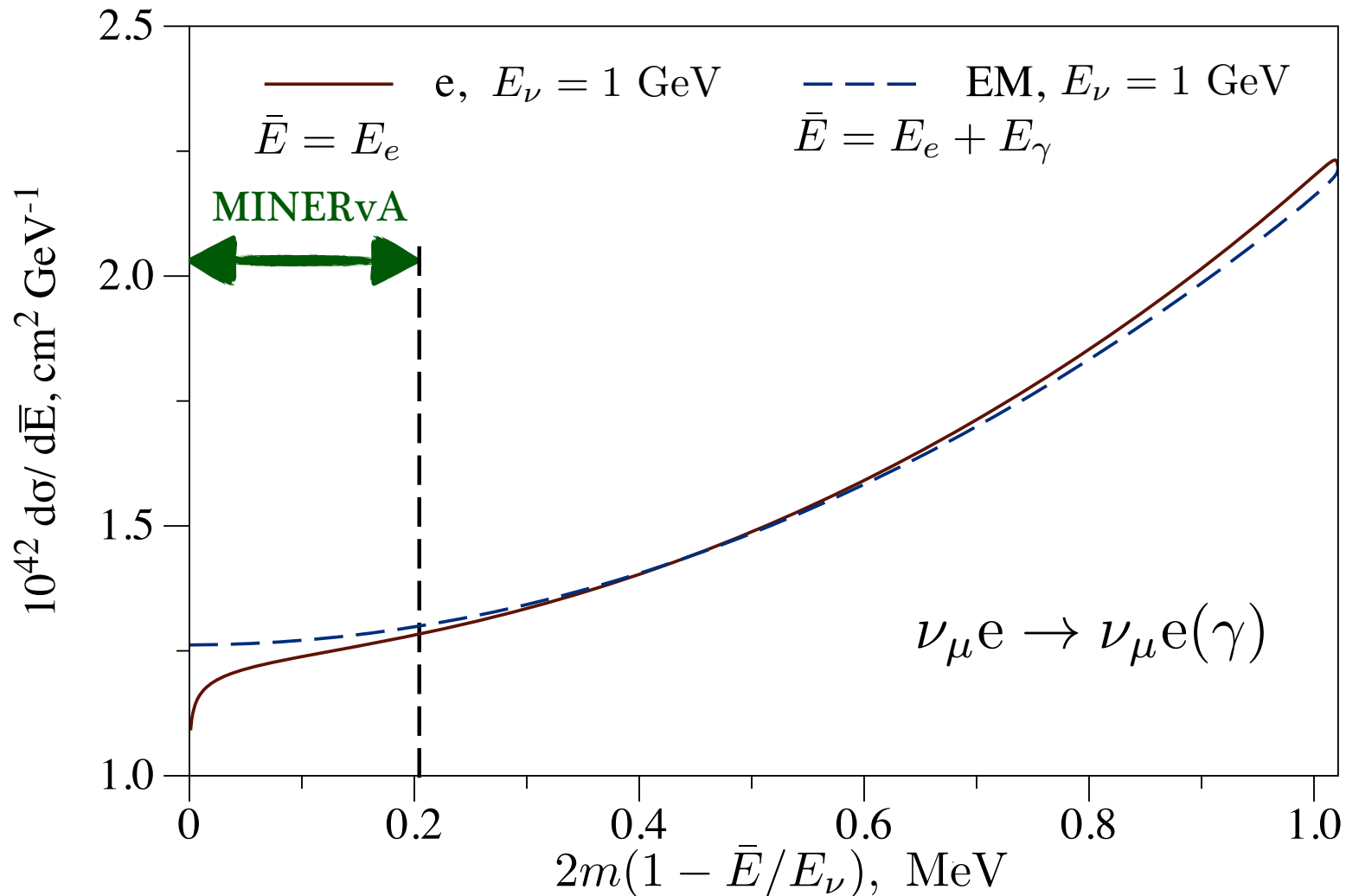


- hadronic correction is the main error in theory

# Electron vs electromagnetic (EM) spectra

- resulting spectrum:

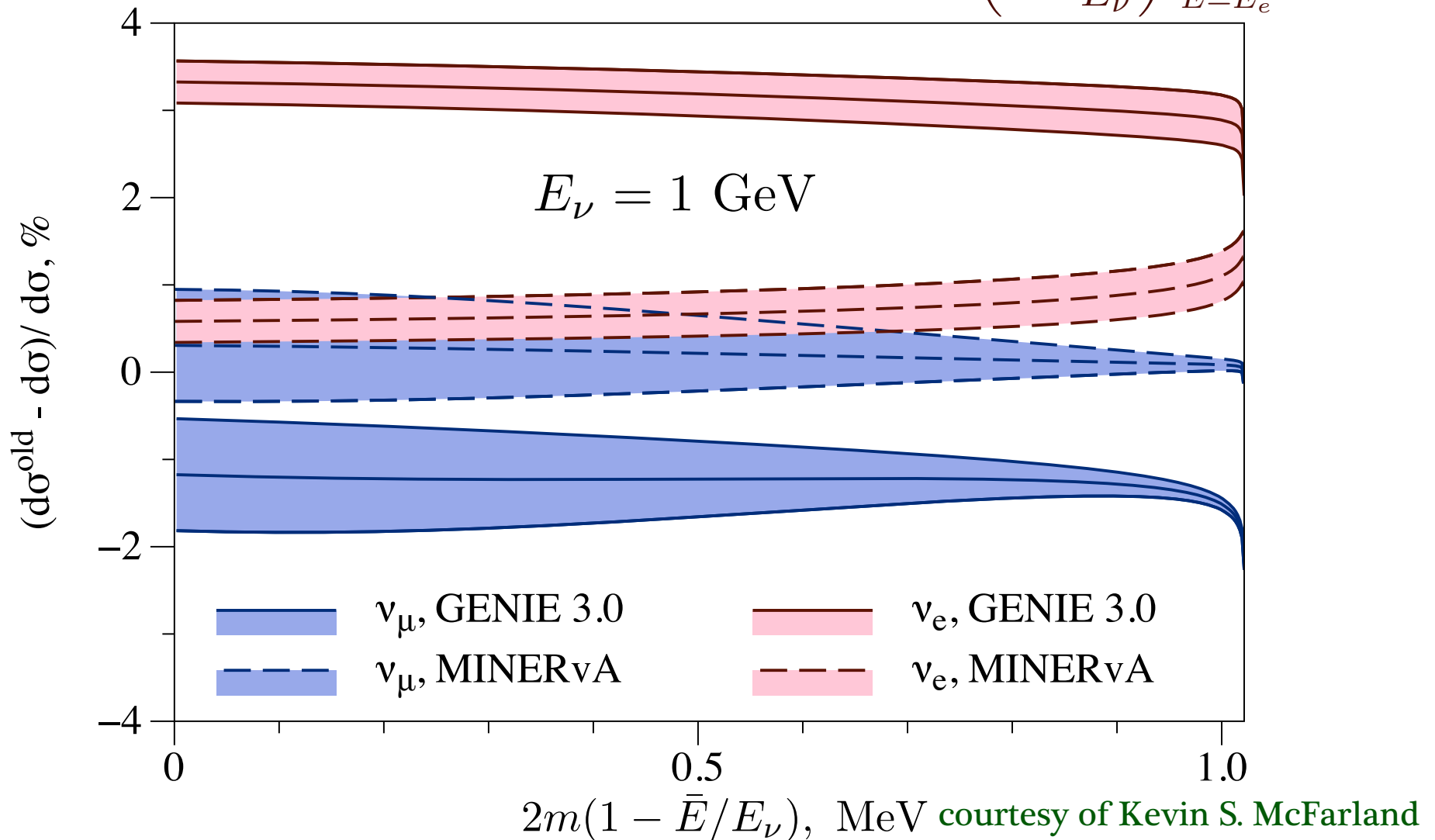
$$2m \left( 1 - \frac{\bar{E}}{E_\nu} \right) \Big|_{\bar{E}=E_e} \approx E_e \theta_e^2$$



- cut dependence after radiative corrections

# Comparison to GENIE

- electromagnetic energy spectrum:  $2m \left( 1 - \frac{\bar{E}}{E_\nu} \right) \Big|_{\bar{E}=E_e} \approx E_e \theta_e^2$

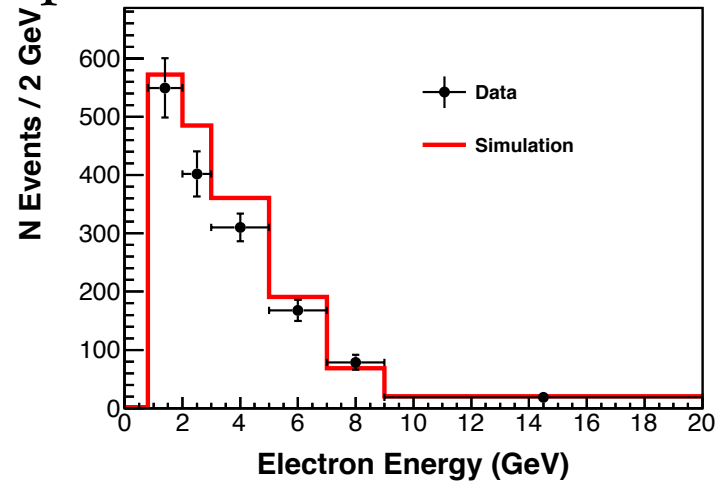


- correct description and definite improvement at GeV energies

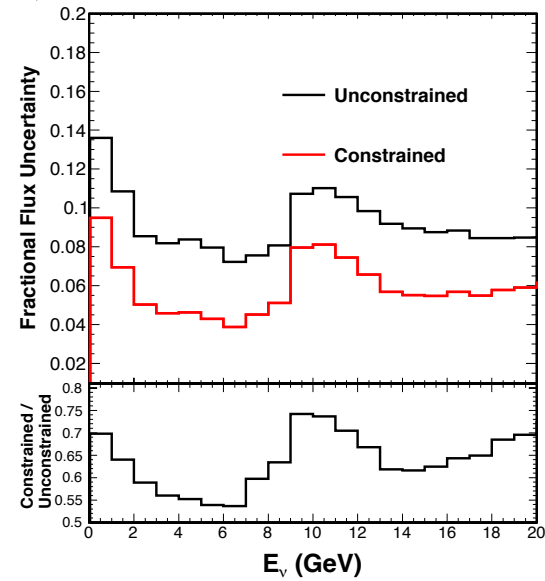
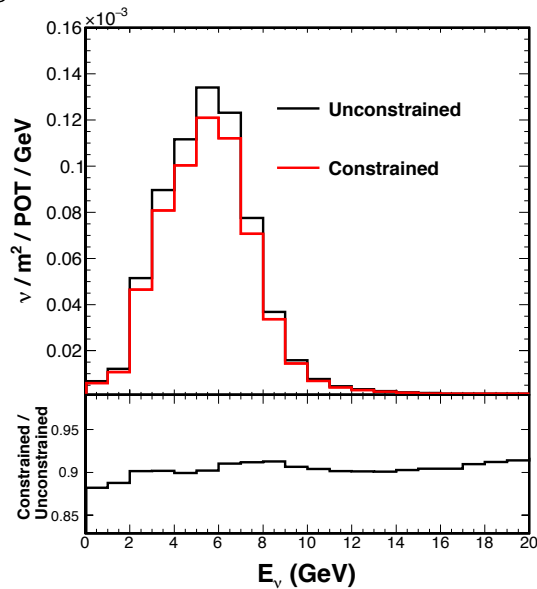
# MINERvA constraint

- electron energy spectrum:

MINERvA, Phys.Rev.D 100 9, 092001 (2019)



- 10% correction on flux normalization, reduced error



- successful implementation by MINERvA collaboration

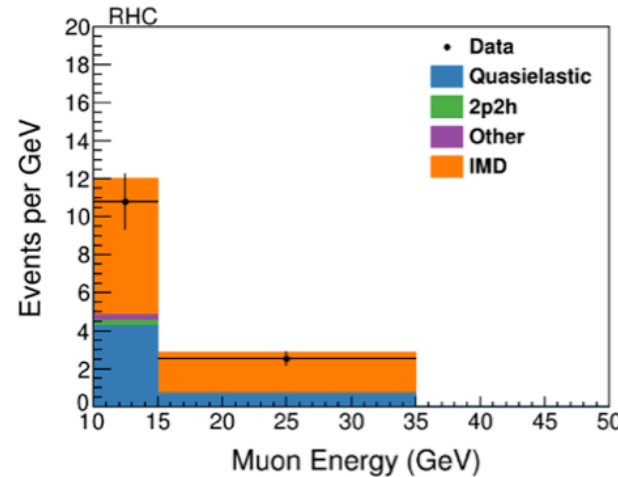
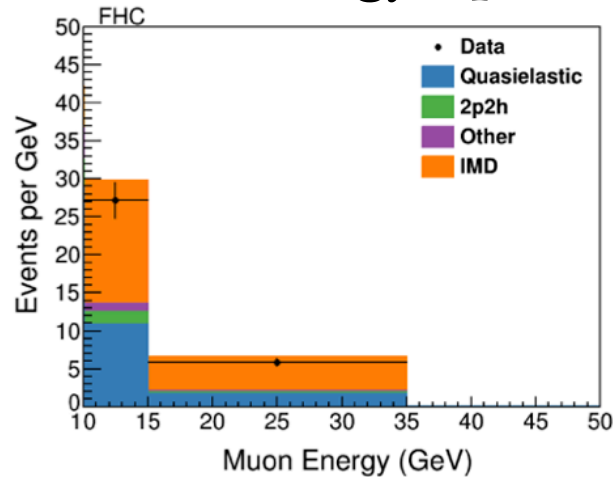
# Neutrino-electron scattering at Fermilab

- **MINERvA** experiment: measure neutrino-nucleus cross sections
  - flux constraints via scattering on atomic electrons: 7.5% to 4%  
MINERvA, Phys. Rev. D 93, 112007 (2016), Phys.Rev.D 100 9, 092001 (2019)
  - cross section scales as target mass  $m$   
 $10^{-4}$ - $10^{-3}$  of cross section on nucleons and nuclei
  - unique process **free from structure effects**
  - huge statistics of DUNE near detector vs MINERvA: 8% to 2%  
**5000-7000** events in a year vs **1100-1200** events in total  
Ch. Marshall et al, Phys.Rev.D 101 3, 032002 (2020)
- *νe* scattering: standard candle to constrain neutrino flux

# MINERvA constraint by inverse muon decay

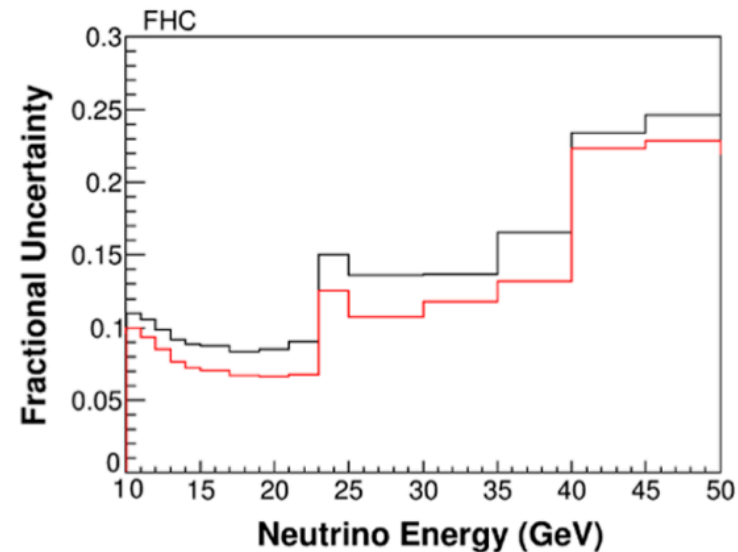
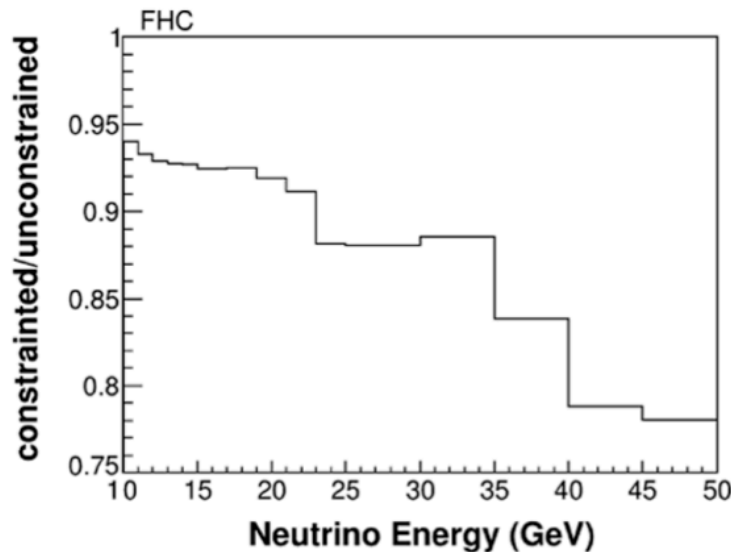
MINERvA, Phys.Rev.D 104, 092010 (2021)

- muon energy spectrum:



$$E_{\nu}^{\text{thr}} \gtrsim 10.9 \text{ GeV}$$

- 10-20% correction on flux normalization, reduced error



- successful implementation by MINERvA collaboration

# Inverse muon decay theory

- precise Lagrangian with  $G_F$  from muon decay

$$\mathcal{L}_{\text{eff}} = -2\sqrt{2}G_F\bar{\nu}_e\gamma^\lambda P_L\nu_\mu\bar{\mu}\gamma_\lambda P_L e + \text{h.c.}$$

- 2 from 3 distributions are reproduced by alternative method

Bardin and Dokuchaeva (1987)

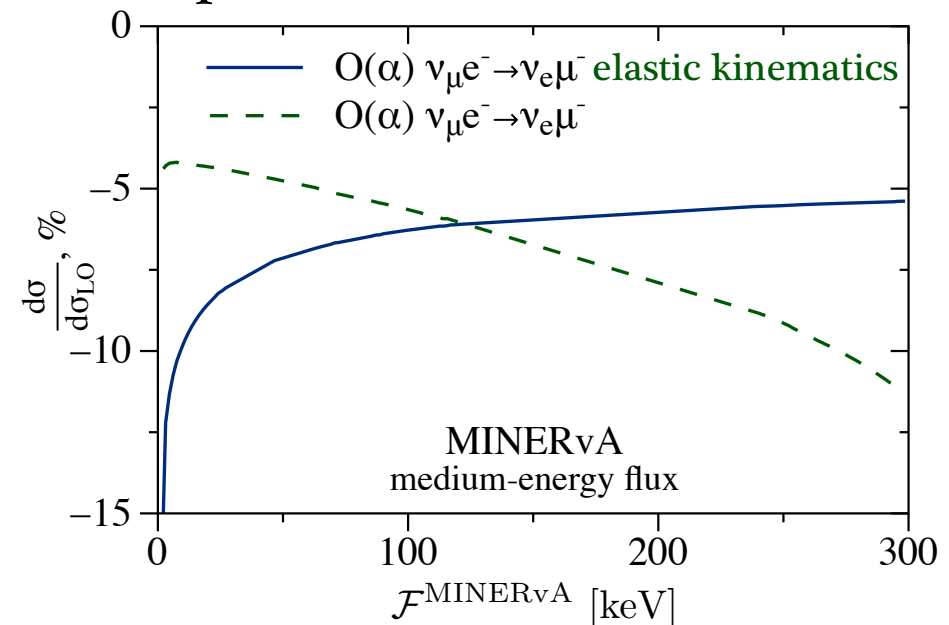
- first QED/EW form factors with different mass

- radiative corrections to distribution of experimental discriminant

$$\mathcal{F} = E_\mu\theta_\mu^2 \approx \left(1 - \frac{E_\mu}{E_\nu}\right) \left(2m_e - \frac{m_\mu^2}{E_\mu}\right)$$

$$\mathcal{F}^{\text{MINERvA}} = \frac{E_\mu\theta_\mu^2}{1 - \frac{E_\mu}{35 \text{ GeV}}}$$

O.T., Kaushik Borah, Richard J. Hill,  
Kevin S. McFarland, Daniel Ruterbories (2023)



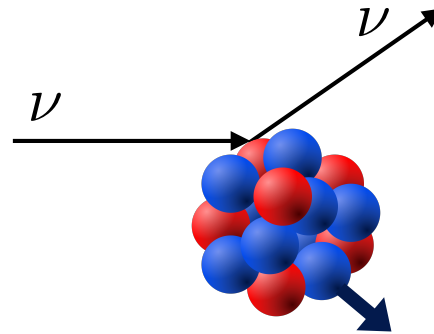
- double-differential distributions and corrections to  $\mathcal{F}$  distribution



# Coherent elastic neutrino-nucleus scattering

- at low neutrino energies (<50 MeV) nuclear state is unchanged  
nucleus recoils as a whole

Stodolsky (1966), Freedman (1974), Kopeliovich and Frankfurt (1974)



recoil nucleus energy  $T$

- large cross section scales as squared number of neutrons  $N^2$

$$\frac{d\sigma}{dT} \approx \frac{G_F^2 M_A}{4\pi} \left(1 - \frac{M_A T}{2E_\nu^2}\right) (N - (1 - 4\sin^2 \theta_W) Z)^2$$

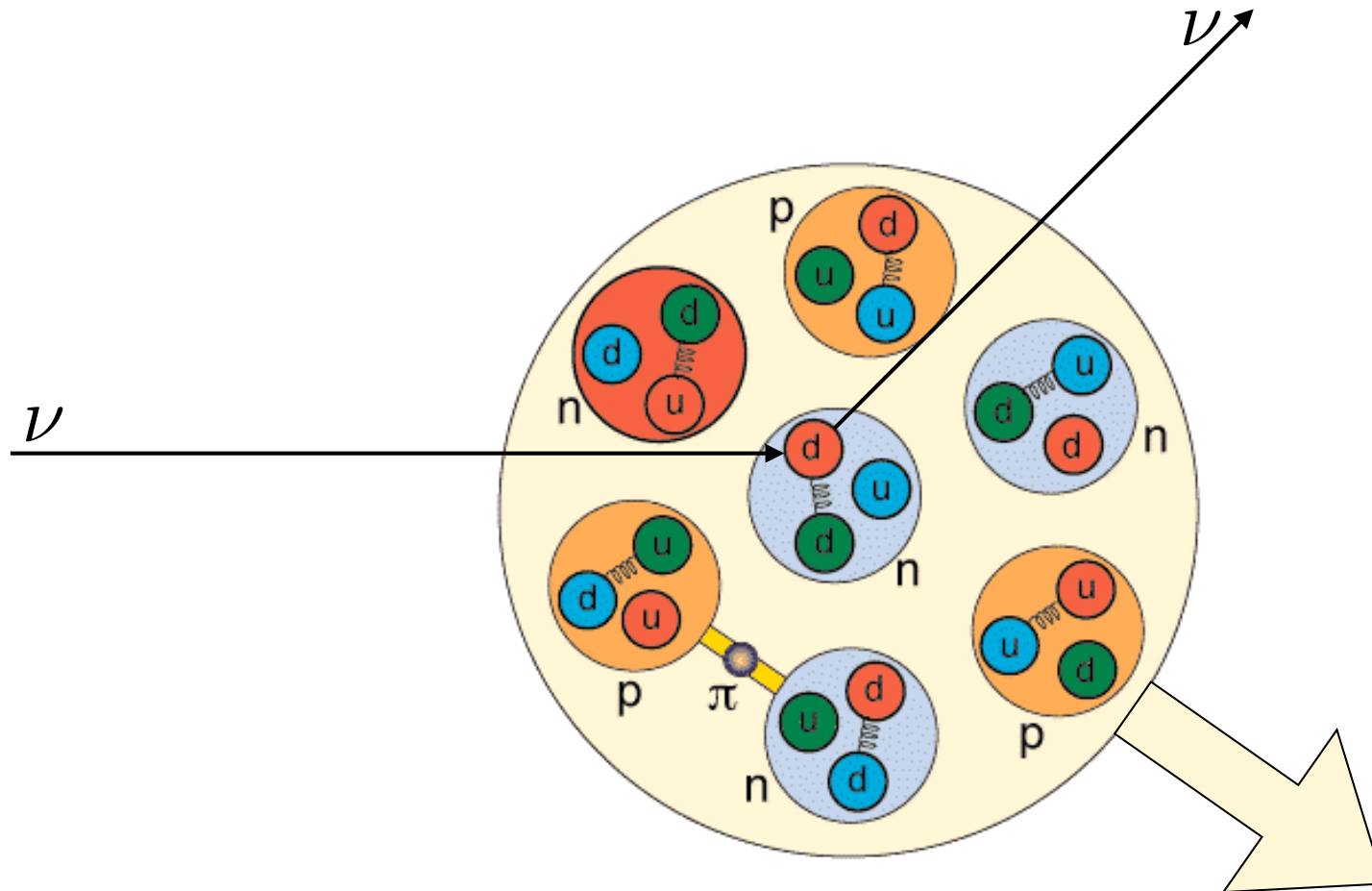
- first detection in 2017 at SNS, measured on CsI and Ar

COHERENT, Science 357 (2017) 6356, 1123-1126

- rapidly developing field nowadays

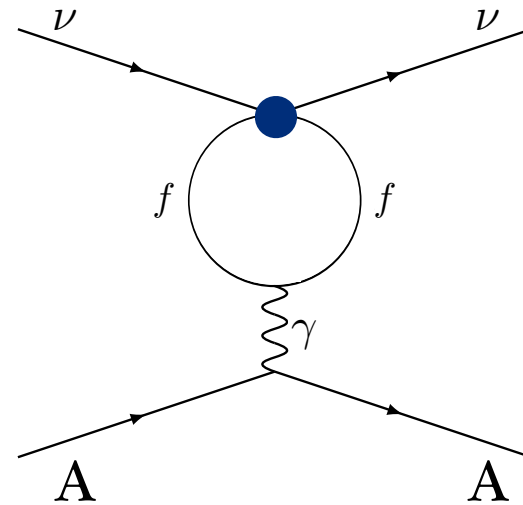
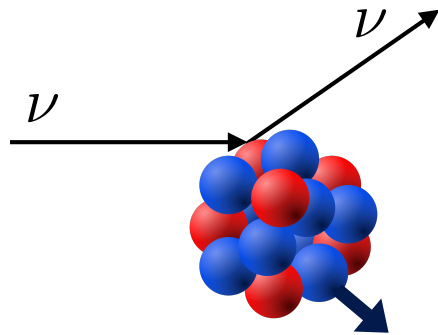
- CEvNS enters precision era with  $\pi$ DAR sources

# From quarks to nuclei



[fafnir.phyast.pitt.edu](http://fafnir.phyast.pitt.edu)

- scattering on quarks in nucleons in nucleus



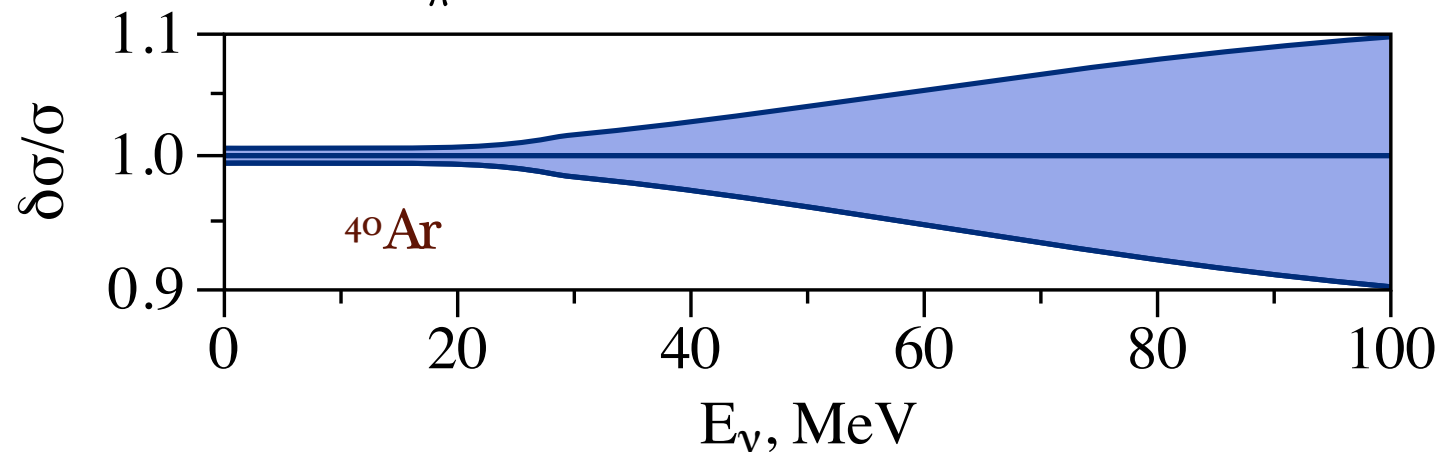
# Coherent elastic neutrino-nucleus scattering

O.T., Pedro Machado, Vishvas Pandey and Ryan Plestid, JHEP 2102, 097 (2021)

$$F_W(Q^2) \rightarrow F_W(Q^2) + \frac{\alpha}{\pi} [\delta^{\nu e} + \delta^{\text{QCD}}] F_{\text{ch}}(Q^2)$$

flavor-dependent  
at percent level

for Coherent and CCM

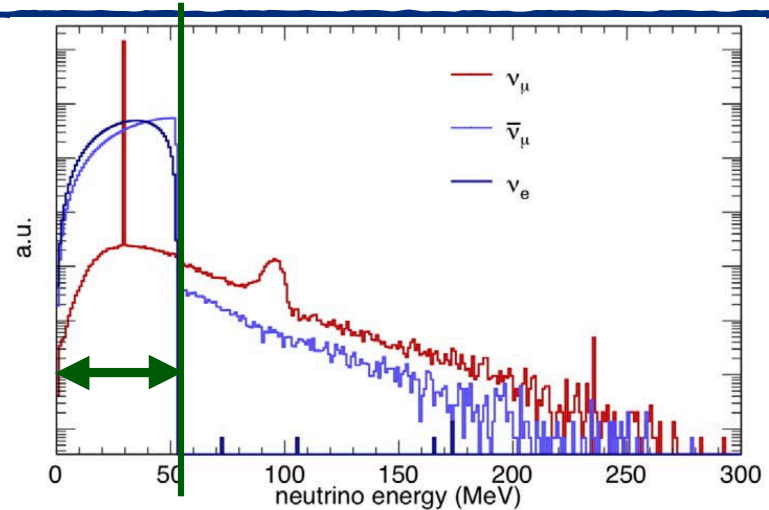


flavor-dependence at tree-level

energy spectra from  $\pi$ DAR  $\rightarrow$

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$



Akimov et al., Science 357 6356, 1123-1126 (2017)

# Neutrinos from muon, pion and kaon decays

O. T., Phys. Lett. B 829, 137108 (2022)

$$\pi^+ \rightarrow \mu^+ \nu_\mu \gamma$$

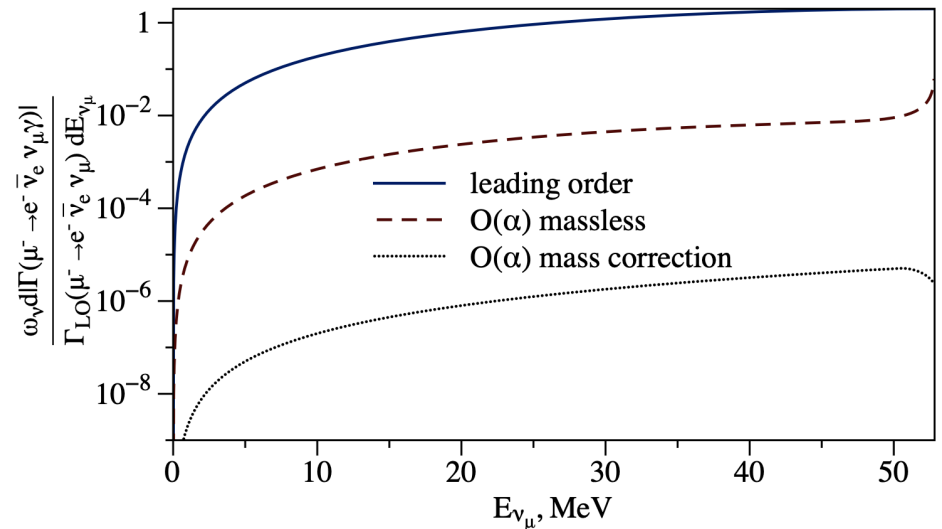
< 0.1 ‰

$$K^+ \rightarrow \mu^+ \nu_\mu \gamma$$

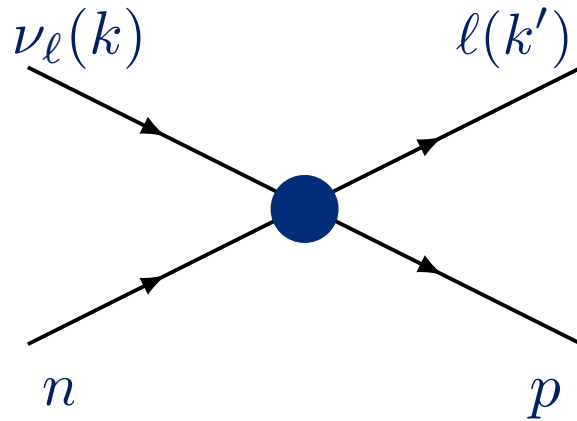
flavor-dependence is clarified  
to permille level analytically



$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma \quad 3-4 \text{ ‰}$$



first QED/EW form factors with different mass

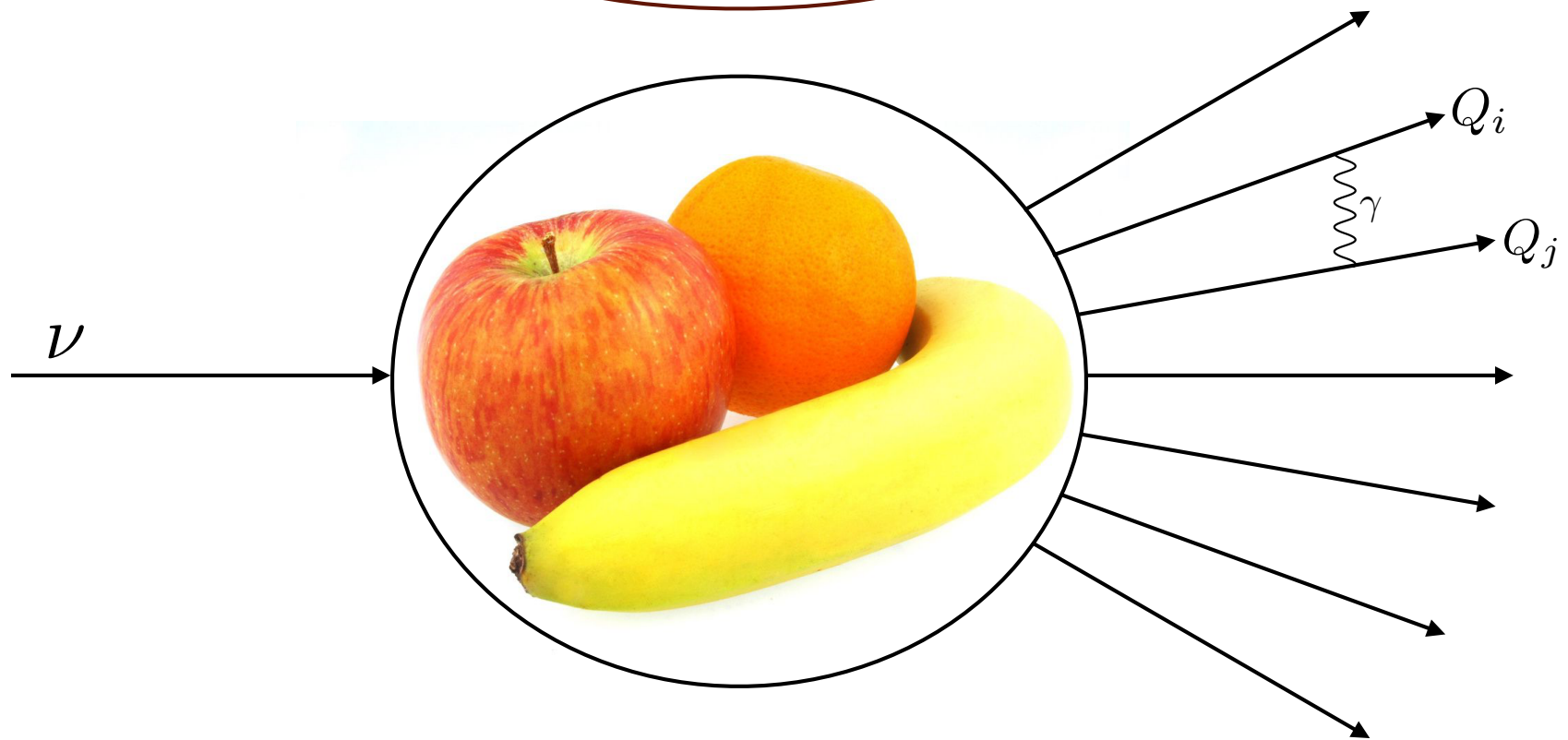


# Radiative corrections to charged-current elastic scattering on free nucleons

neutrino energy  $\sim$  GeV

# QED corrections

$$m_e \ll m_\mu \ll E_\nu$$



$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \ln \frac{E_\nu}{m_e} \sim 6 - 10 \text{ or } \ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$$

- scale separation introduces large flavor-dependent QED logarithms

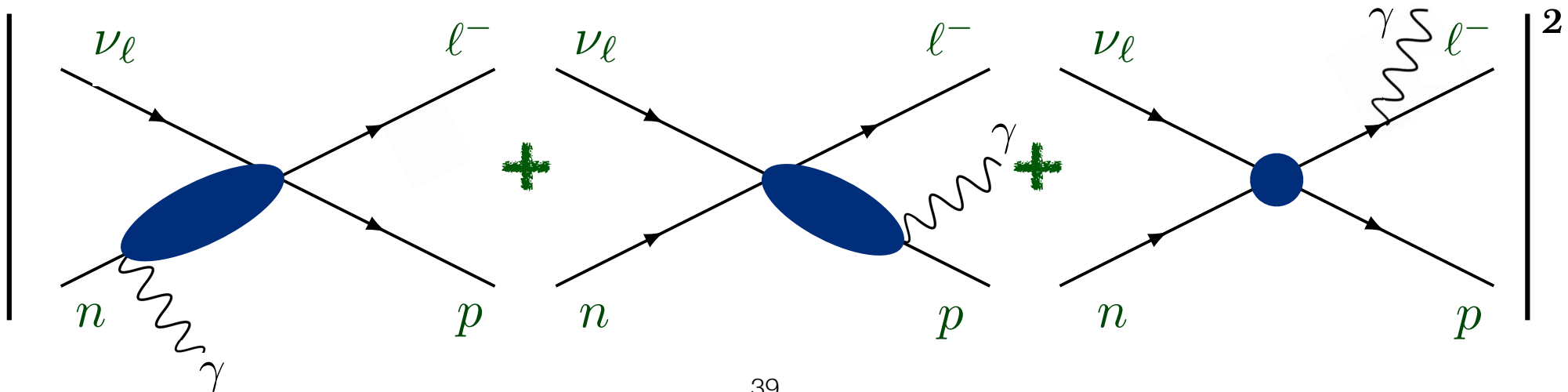
factorization for radiative corrections with model for hard function



# Charged-current elastic scattering on nucleons

precise predictions for flavor ratios and radiative corrections

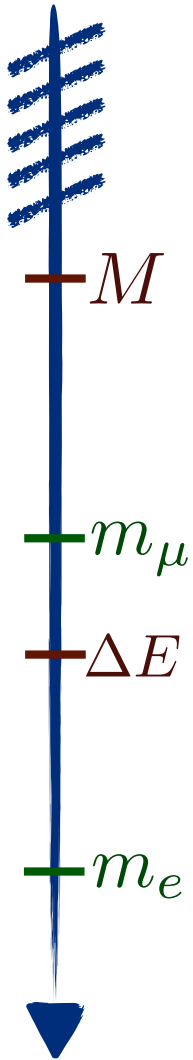
in exclusive and inclusive observables with GeV neutrino beams



# Factorization approach

- cross section is given by **factorization formula**

$$d\sigma \sim S \left( \frac{\Delta E}{\mu} \right) J \left( \frac{m_\ell}{\mu} \right) H \left( \frac{M}{\mu} \right)$$

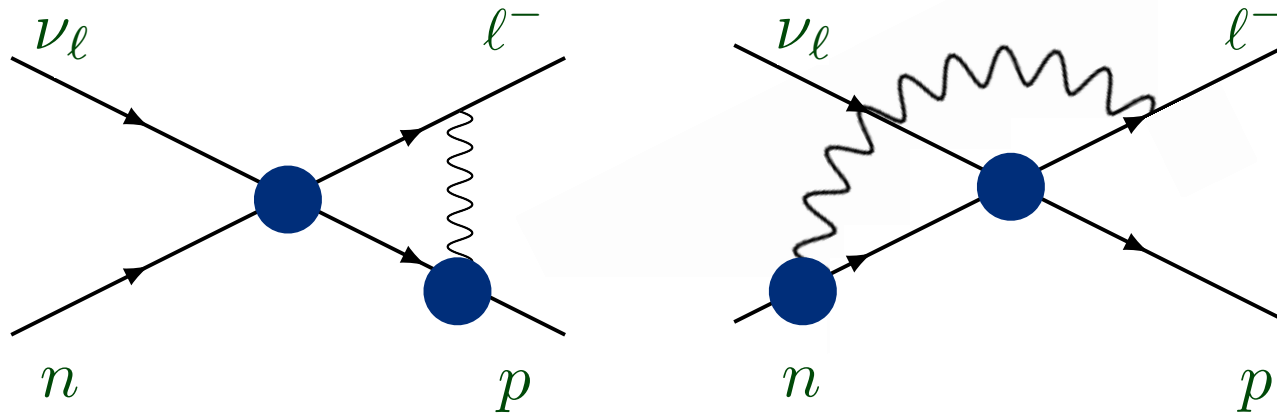


- determine **hard function** at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

- soft and collinear functions are evaluated **perturbatively**



# Hadronic model at GeV scale



- exchange of photon between the charged lepton and nucleons
- assume **onshell form** for each interaction with dipole form factors  
discussed for neutrino-nucleon scattering: Graczyk, Phys. Lett. B 732, 315-319 (2013)
- add **self energy** for charged particles
- reproduce soft and collinear regions of SCET

- best determination of hard function

# Factorization approach

- cross section is given by **factorization formula**

$$d\sigma \sim S \left( \frac{\Delta E}{\mu} \right) J \left( \frac{m_\ell}{\mu} \right) H \left( \frac{M}{\mu} \right)$$



$M$

- determine **hard function** at hard scale by matching experiment or **hadronic model** to the theory with heavy nucleon

$m_\mu$

- **RGE evolution** of the hard function to scales  $\Delta E, m_\ell$

$\Delta E$

- **soft and collinear functions** are evaluated **perturbatively**

$m_e$

- calculate cross section at low energies accounting for **all large logs** ep scattering with soft radiation only: Richard J. Hill, Phys. Rev. D 95 1, 013001 (2016)

- **soft and collinear functions** determined **analytically**
- **hard function** describes physics at GeV energies

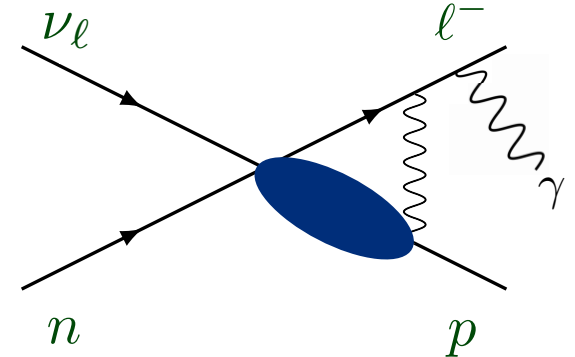
# Charged-current scattering on nucleons

- theory and 1<sup>st</sup>-ever complete calculation

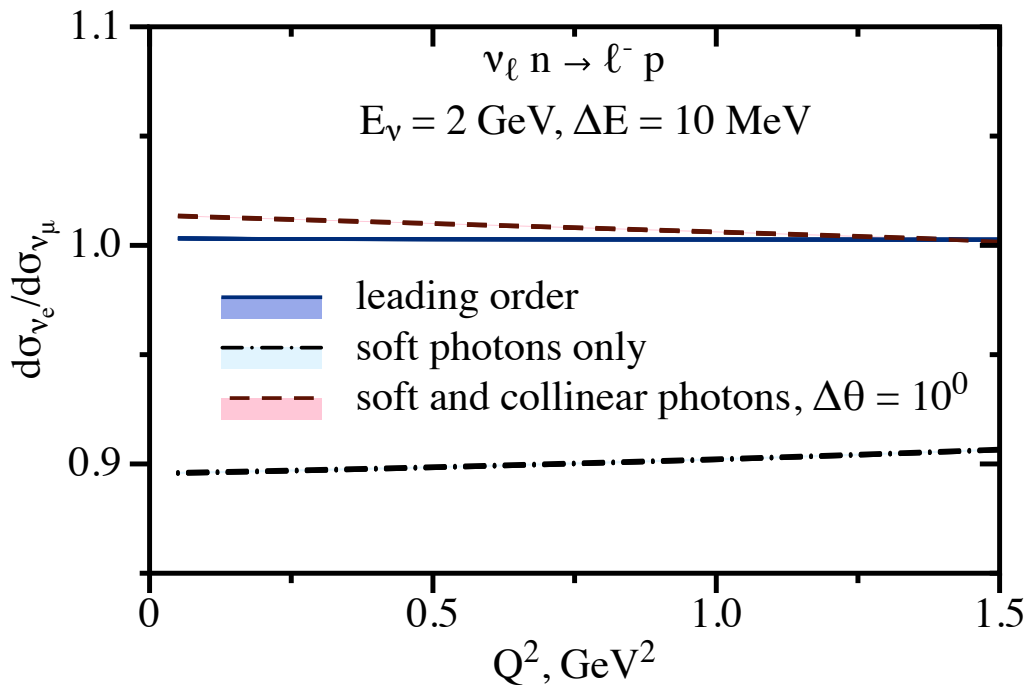
10-20% hadronic uncertainties

cancel for  $e/\mu$  ratio

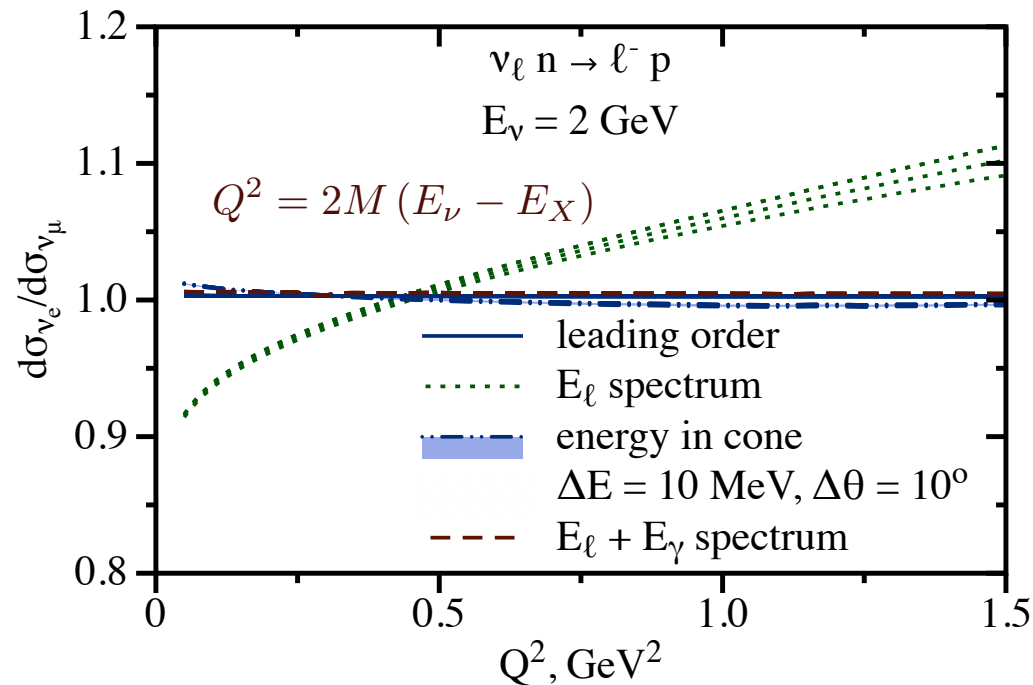
O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland,  
Nature Commun. 13 (2022), 1, 5286



exclusive



inclusive



- critical dependence on details of experimental analysis
- predict  $\sigma_{\nu_e}$  from  $\sigma_{\nu_\mu}$  measurements with neutrino beam



# Electron/muon ratio

	$E_\nu$ , GeV		$\left(\frac{\sigma_e}{\sigma_\mu} - 1\right)_{\text{LO}}$ , %	$\frac{\sigma_e}{\sigma_\mu} - 1$ , %
T2K/HyperK	0.6	$\nu$	$2.47 \pm 0.06$	$2.84 \pm 0.06 \pm 0.37$
		$\bar{\nu}$	$2.04 \pm 0.08$	$1.84 \pm 0.08 \pm 0.20$
NO $\nu$ A/DUNE	2.0	$\nu$	$0.322 \pm 0.006$	$0.54 \pm 0.01 \pm 0.22$
		$\bar{\nu}$	$0.394 \pm 0.003$	$0.20 \pm 0.01 \pm 0.19$

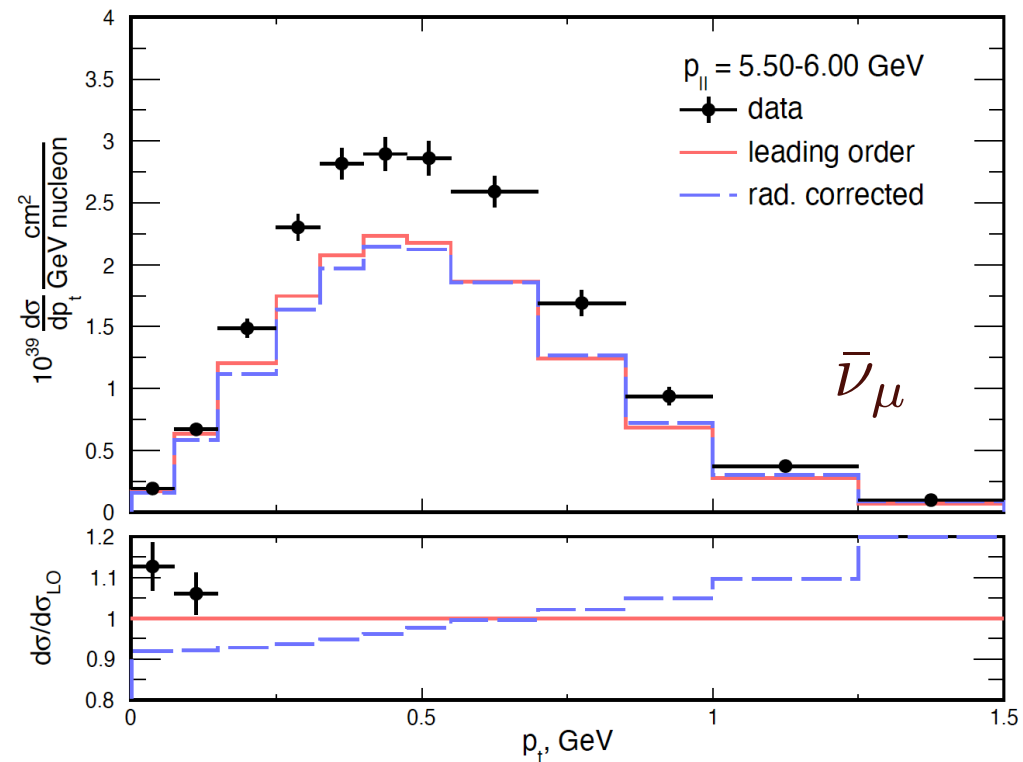
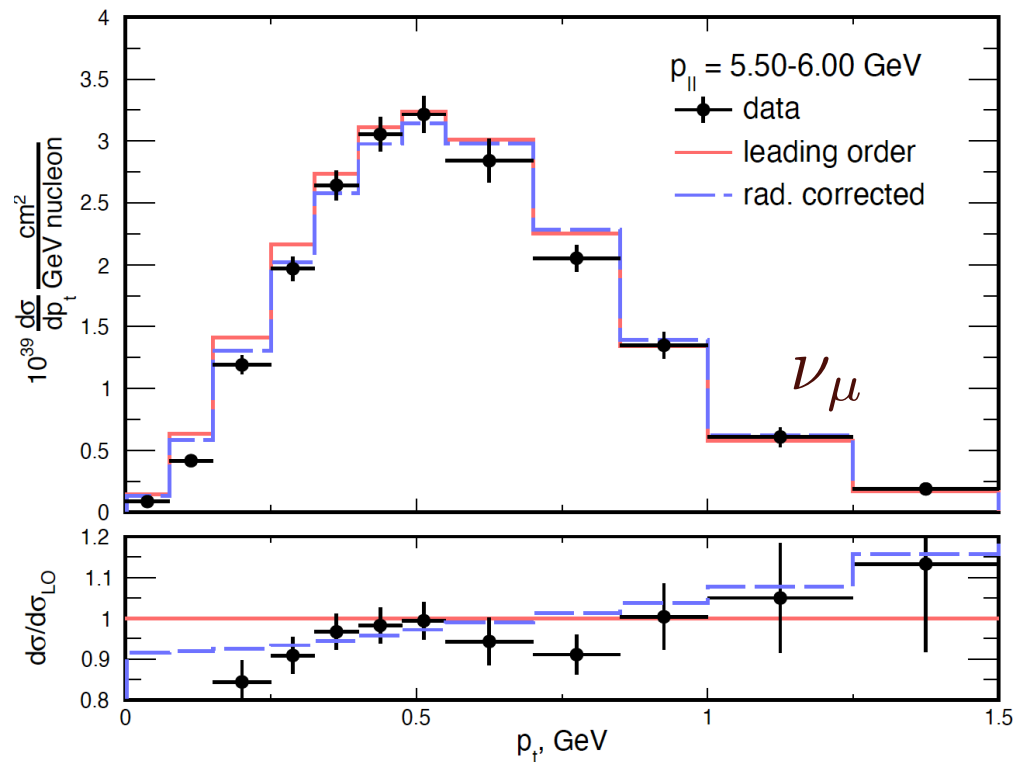
TABLE II: Inclusive electron-to-muon cross-section ratios for neutrinos and antineutrinos without kinematic cuts. Uncertainties at leading order are from vector and axial nucleon form factors. For the final result, we include an additional hadronic uncertainty from the one-loop correction to the first uncertainty, and provide a second uncertainty as the magnitude of the radiative correction.

$$\frac{\sigma(m_\ell \rightarrow 0)}{\sigma(m_\ell = 0)} \approx 1 + Am_\ell^2 + \alpha Bm_\ell^2 \ln m_\ell$$

- inclusive cross sections and flavor ratios determined by KLN
- nuclear effects: suppressed by expansion parameters squared

# Comparison to data

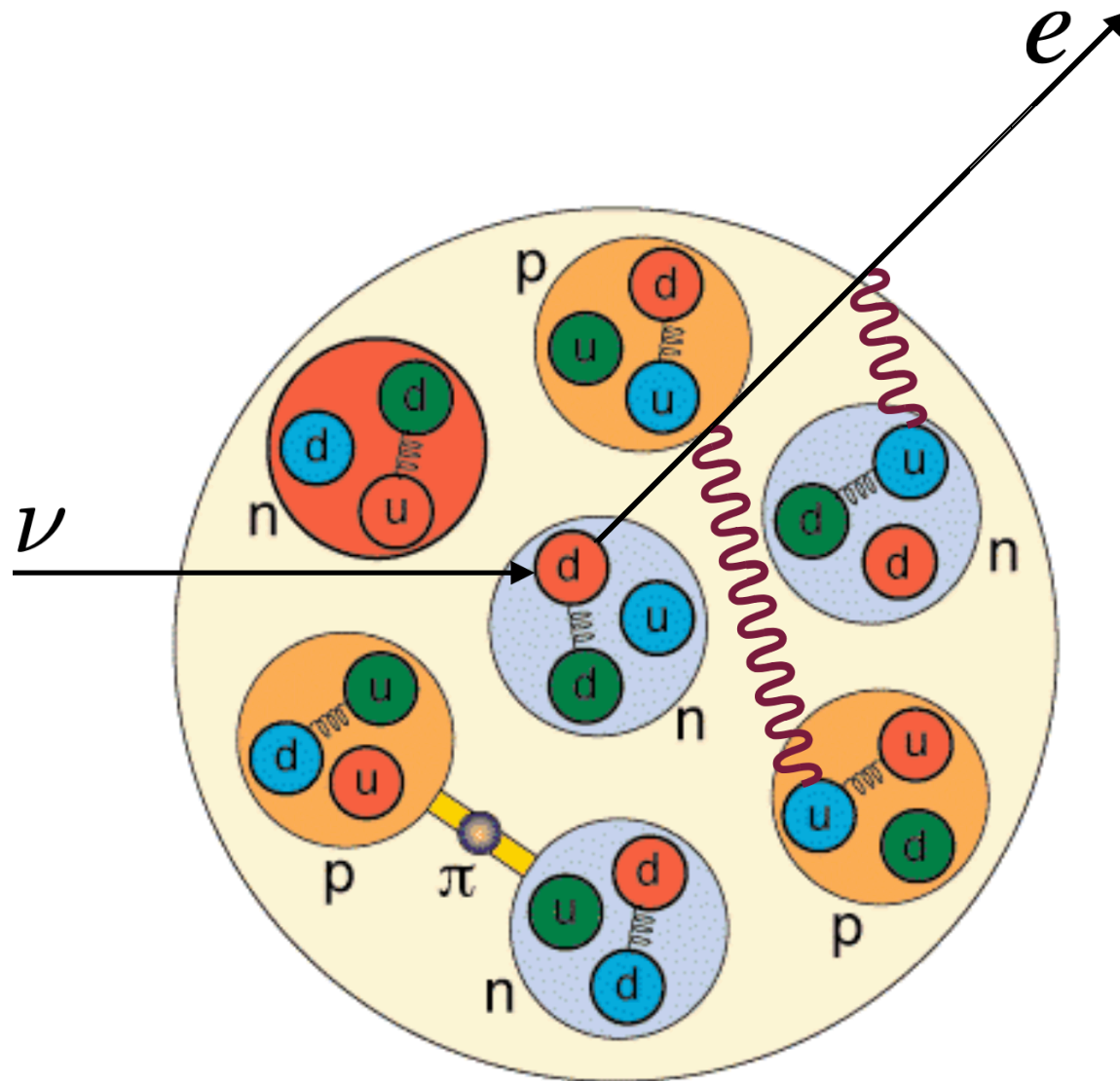
- medium-energy flux data from MINERvA@FERMILAB



O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret  
editors suggestion, Phys. Rev. D 106, 093006 (2022)

- electron flavor: measurements are uncertain
- muon flavor: comparable to experimental precision

# QED medium effects



- charged lepton exchanges photons with nuclear medium

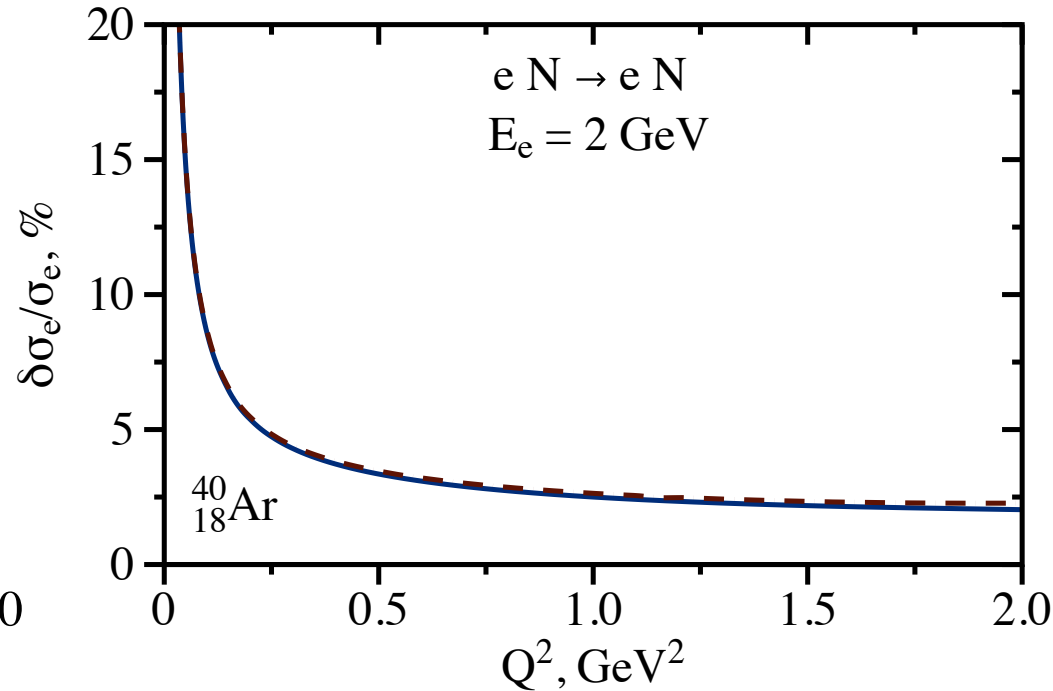
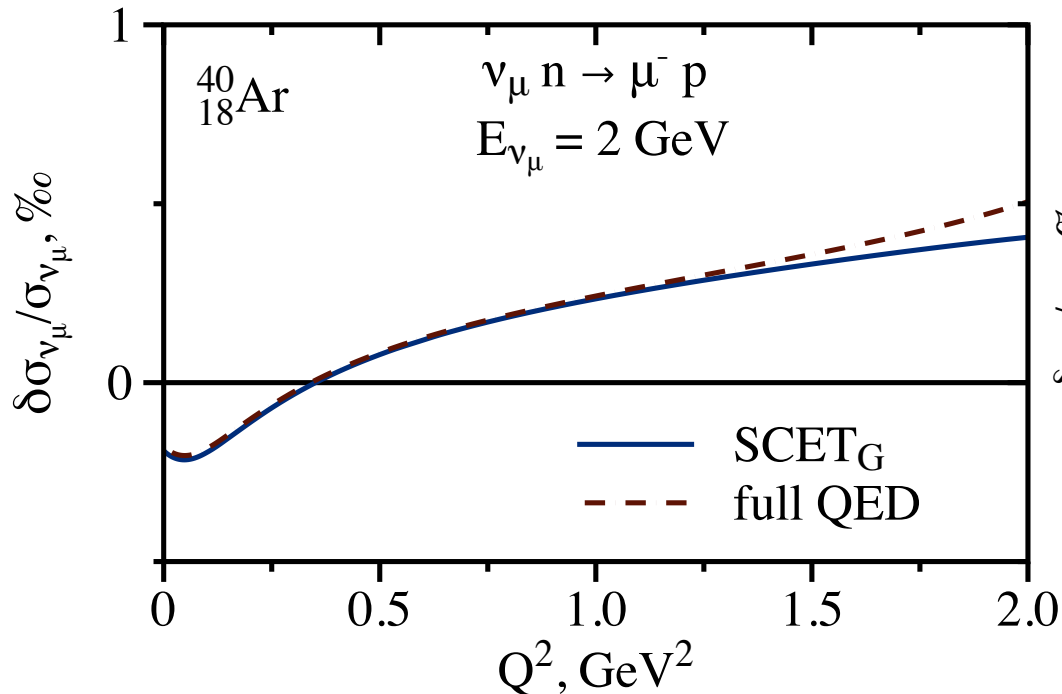
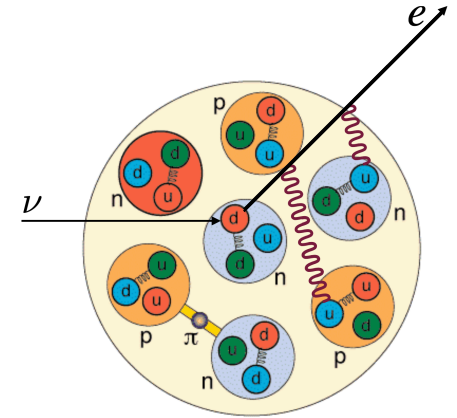
# QED nuclear medium effects

- theory and 1<sup>st</sup>-ever estimate by two methods:

SCET<sub>G</sub>: Soft-collinear effective theory (Glauber)

QED: quantum electrodynamics

O. T. and Ivan Vitev, Phys. Lett. B 805, 135466 (2022)



- permille-level for  $\nu_e A \rightarrow eA'$ , percent-level for  $eA \rightarrow eA$
- critical new effect for electron scattering experiments

# Conclusions

- neutrino cross sections is the main tool to access neutrino properties
- various production and interaction mechanisms at all energy scales
- radiative corrections (1-20%) for consistent error estimates
- radiative corrections for precise flux determinations
- QED nuclear effects in neutrino and electron scattering
- total and differential  $\nu e$ , CEvNS,  $\nu_\ell n \rightarrow \ell^- p$  and  $\bar{\nu}_\ell p \rightarrow \ell^+ n$   
flavor ratios evaluated from theory with rigorous error analysis



Thanks for your attention !!!