INSTITUTE for NUCLEAR THEORY

The S@INT seminar
The INT, University of Washington
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QCD Factorization:
Matching hadrons to quarks and gluons with controllable approximations

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50 Years of QCD: arXiv:2212.11107

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TMD Collaboration

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Outline of my talk

- **QCD at a Fermi-Scale – Nuclear Femtography**

  => Need new probes with multiple observed scales

  Need new advances in QCD factorizations

- **Why and how QCD factorization works?**

  => Necessary conditions and predictive powers

  QCD factorization with one or more identified hadrons – challenges

- **QCD factorization of exclusive processes for extracting GPDs – 3D tomography**

  => Single diffractive hard exclusive processes (SDHEP)

  QCD factorization of minimum 2=>3 SDHEP & enhanced x-dependence of GPDs

- **QCD factorization beyond the leading power and beyond QCD**

  => Necessary for understanding heavy quarkonium production from LHC to EIC

  Hybrid (collinear QED) and (TMD QCD) factorization for SIDIS

- **Summary and outlook**

Frontiers of QCD and Strong Interaction

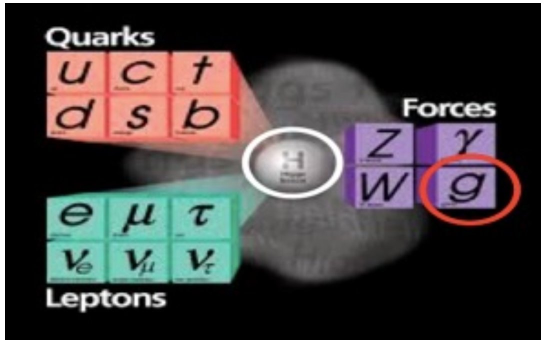
- ❑ **Understanding where did we come from?**

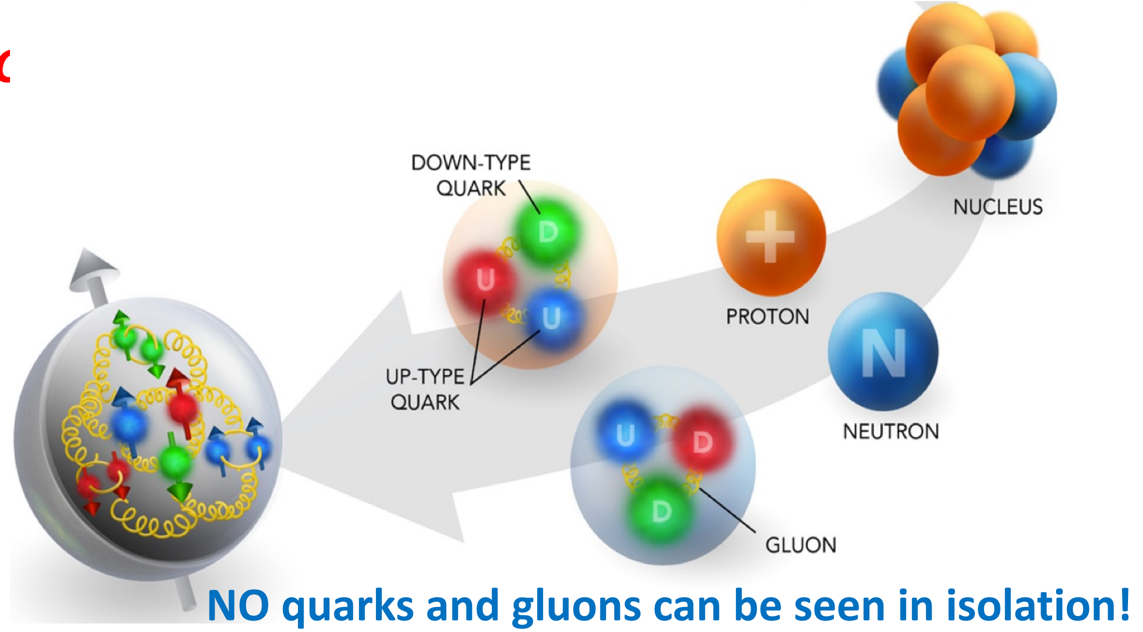
Global Time: →

QCD at high temperature, high densities, phase transition, …

Facilities – Relativistic heavy ion collisions: SPS, RHIC, the LHC

- ❑ **Understanding what are we made of?**





NO quarks and gluons can be seen in isolation!

- **How to understand the emergent properties of nucleon and nuclei (elements of the periodic table) in terms of elements of the modern periodic table?**
- **How does the glue bind us all?**

Nuclear Femtography

Search for answers to these questions at a Fermi scale!

Facilities – CEBAF, EIC, EICC, LHeC, …

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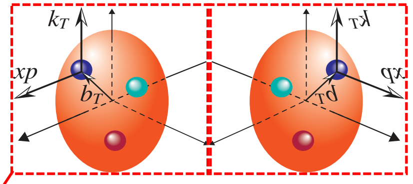
We believe we have the right Theory, …

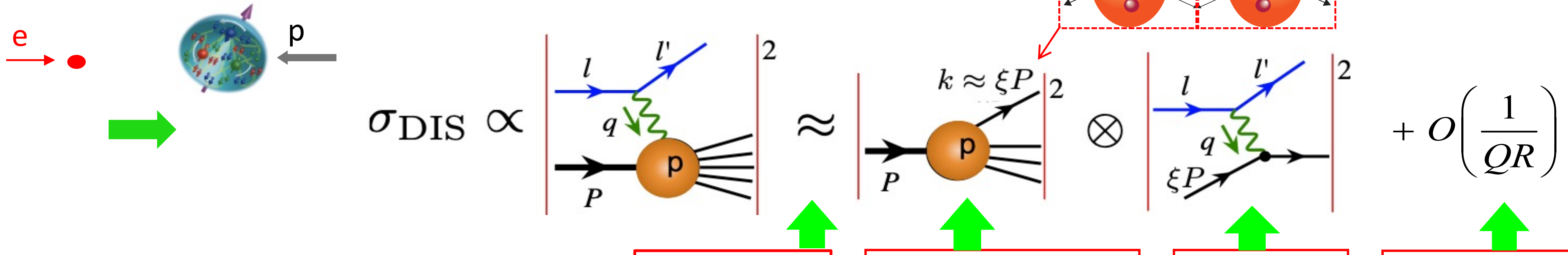
- **QCD – A theory of quarks & gluons:**

$$\mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[(i\partial_\mu \delta_{ij} - g A_{\mu,a}(t_a)_{ij})\gamma^\mu - m_f \delta_{ij} \right] \psi_j^f - \frac{1}{4}\left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - g C_{abc} A_{\mu,b} A_{\nu,c} \right]^2$$

But, we saw none of them directly !!!

- **Try to "see" quarks & gluons indirectly – QCD Factorization:**





Approximation at Feynman diagram level

Factorization Theory	Parton-distribution "Structure"	Hard-part Probe	Power corrections Approximation

- **Effective field theory (EFT) –** ***Approximation at the Lagrangian level*****:**

  Soft-collinear effective theory (SCET), Non-relativistic QCD (NRQCD), Heavy quark EFT, chiral EFT(s), …

- **Lattice QCD –** ***Approximation mainly due to computer power*****:**

  Hadron spectroscopy, phase shift, nuclear structure, ***hadron structure (with pQCD factorization)*****, …**

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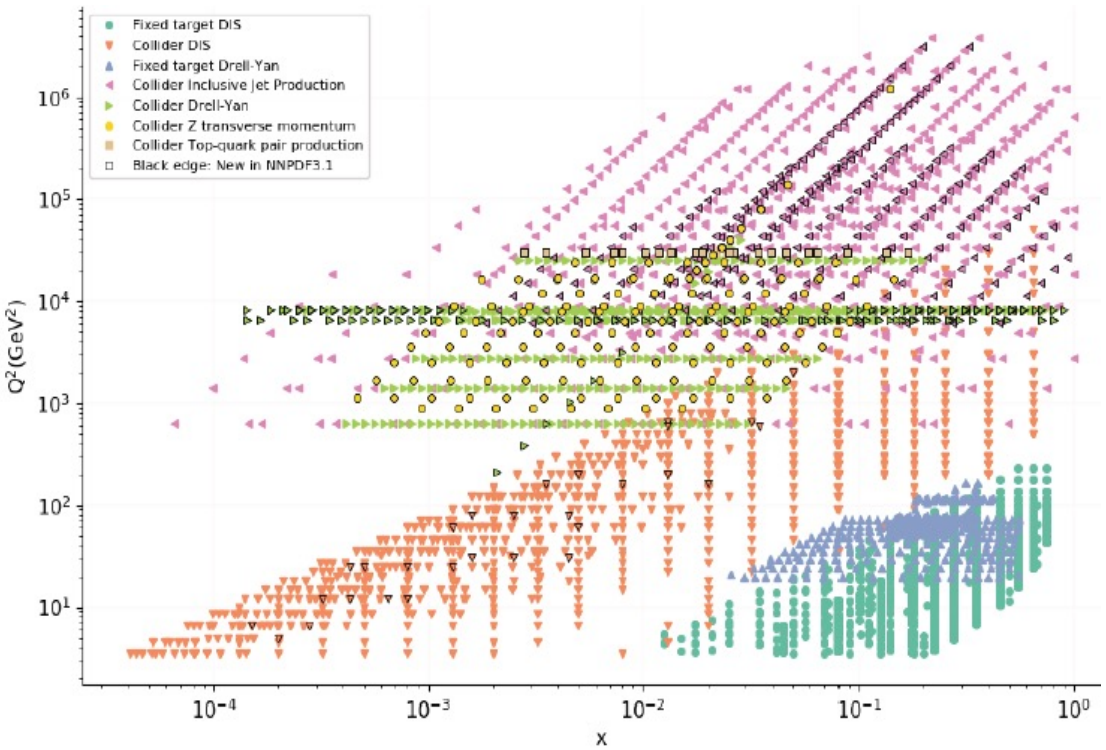
QCD Factorization Works to the Precision

❑ Data sets for Global Fits:

	Process	Subprocess	Partons	x range
Fixed Target	$\ell^\pm\{p,n\} \to \ell^\pm + X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
	$\ell^\pm n/p \to \ell^\pm + X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
	$pp \to \mu^+\mu^- + X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
	$pn/pp \to \mu^+\mu^- + X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
	$\nu(\bar{\nu}) N \to \mu^-(\mu^+) + X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
	$\nu N \to \mu^-\mu^+ + X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
	$\bar{\nu} N \to \mu^+\mu^- + X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
Collider DIS	$e^\pm p \to e^\pm + X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
	$e^+ p \to \bar{\nu} + X$	$W^+\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
	$e^\pm p \to e^\pm c\bar{c} + X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^\pm p \to e^\pm b\bar{b} + X$	$\gamma^* b \to b, \gamma^* g \to b\bar{b}$	b, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
Tevatron	$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
	$p\bar{p} \to (W^\pm \to \ell^\pm \nu) + X$	$ud \to W^+, \bar{u}\bar{d} \to W^-$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
	$p\bar{p} \to (Z \to \ell^+\ell^-) + X$	$uu, dd \to Z$	u, d	$x \gtrsim 0.05$
	$p\bar{p} \to t\bar{t} + X$	$qq \to t\bar{t}$	q	$x \gtrsim 0.1$
LHC	$pp \to \text{jet} + X$	$gg, qg, q\bar{q} \to 2j$	g, q	$0.001 \lesssim x \lesssim 0.5$
	$pp \to (W^\pm \to \ell^\pm \nu) + X$	$u\bar{d} \to W^+, d\bar{u} \to W^-$	$u, d, \bar{u}, \bar{d}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X$	$q\bar{q} \to Z$	$q, \bar{q}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X, p_\perp$	$gq(\bar{q}) \to Zq(\bar{q})$	$g, q, \bar{q}$	$x \gtrsim 0.01$
	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, Low mass	$q\bar{q} \to \gamma^*$	$q, \bar{q}, g$	$x \gtrsim 10^{-4}$
	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, High mass	$q\bar{q} \to \gamma^*$	$\bar{q}$	$x \gtrsim 0.1$
	$pp \to W^+\bar{c}, W^-c$	$sg \to W^+c, \bar{s}g \to W^-\bar{c}$	$s, \bar{s}$	$x \sim 0.01$
	$pp \to t\bar{t} + X$	$gg \to t\bar{t}$	g	$x \gtrsim 0.01$
	$pp \to D, B + X$	$gg \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to J/\psi, \Upsilon + pp$	$\gamma^*(gg) \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to \gamma + X$	$gq(\bar{q}) \to \gamma q(\bar{q})$	g	$x \gtrsim 0.005$

❑ Kinematic Coverage:

NNPDF3.1

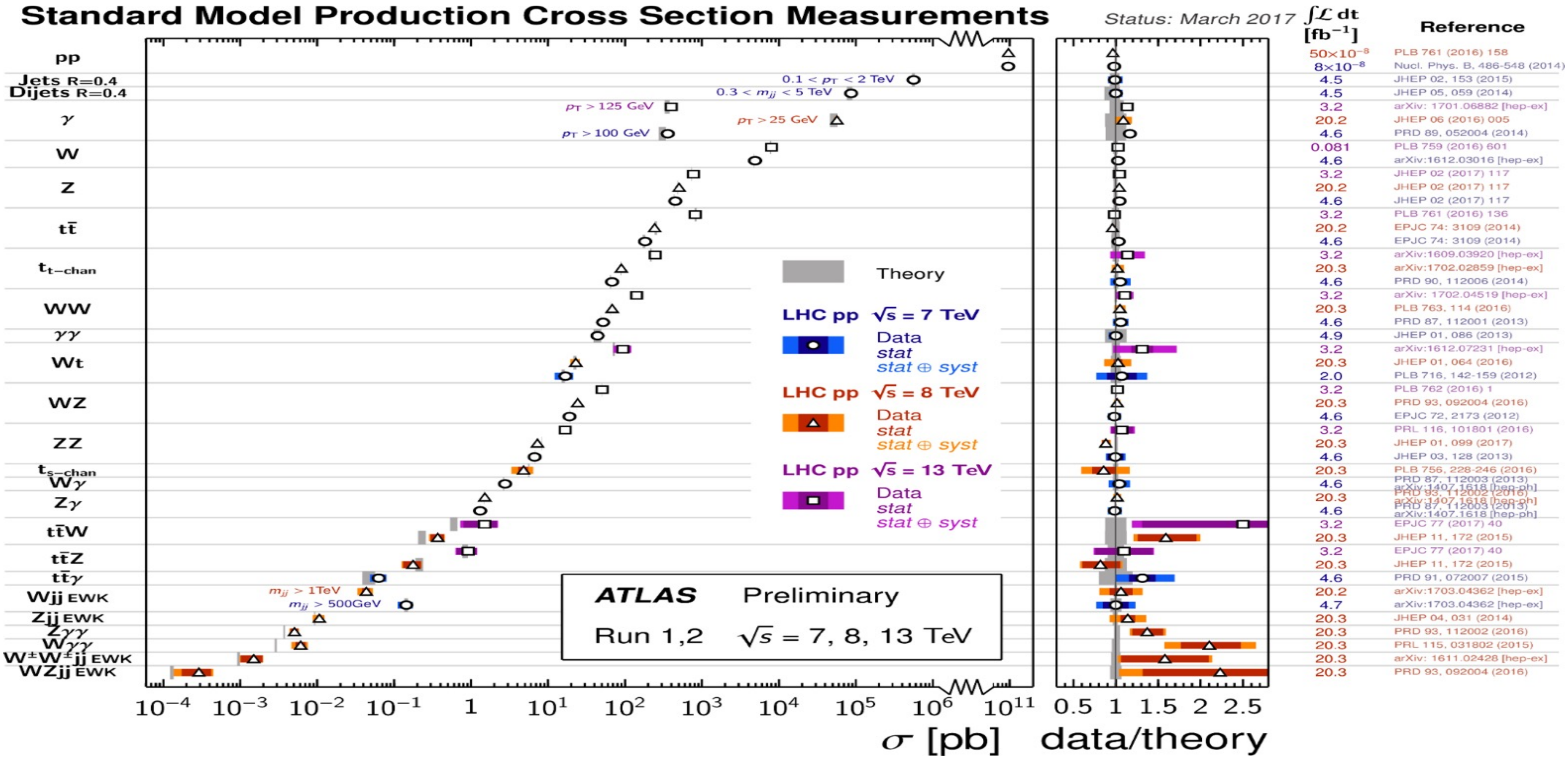


❑ Fit Quality:

$\chi^2/\text{dof} \sim 1 \;\Rightarrow$ Non-trivial check of QCD

All data sets	3706 / 2763	3267 / 2996	2717 / 2663
	LO	NLO	NNLO

Unprecedent Success of QCD and Standard Model



SM: Electroweak processes + QCD perturbation theory + PDFs works!

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QCD Landscape of Nucleons and Nuclei

Color Confinement — Asymptotic freedom

Q (GeV)

20 MeV (10 fm) — 200 MeV (1 fm) — 2 GeV (1/10 fm)

Probing scale

QCD at the Fermi Scale: Femto-science (0.1-10 fm)

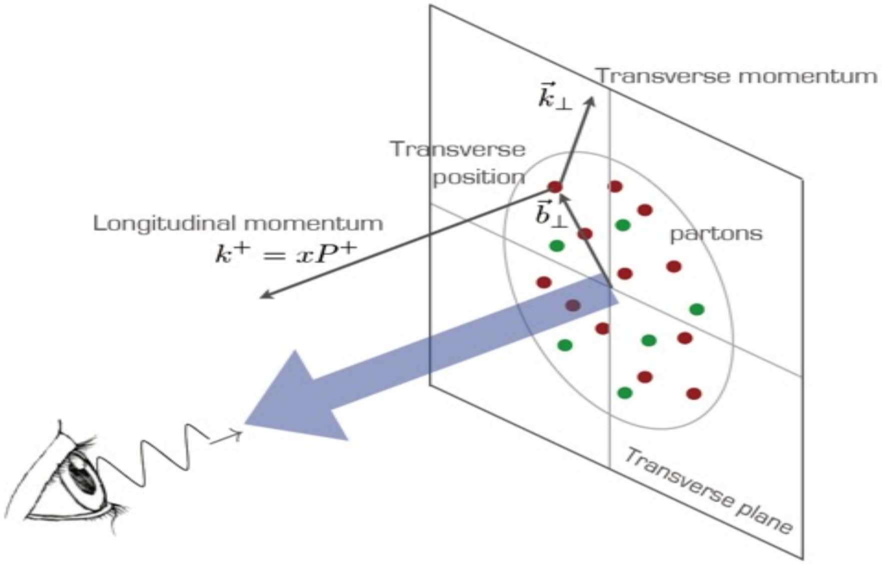
Asymptotic regime PQCD works beautifully!

- *The most interesting, rich, and complex regime of the theory!*
- *All emergent phenomena depend on the scale at which we probe them!*

❑ Need new probes/observables with two distinctive scales:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

- Hard scale: Q_1 to localize the probe to see the particle nature of quarks/gluons
- "Soft" scale: Q_2 could be more sensitive to the hadron structure ~ 1/fm



Do we have QCD factorization for two-scale observables?

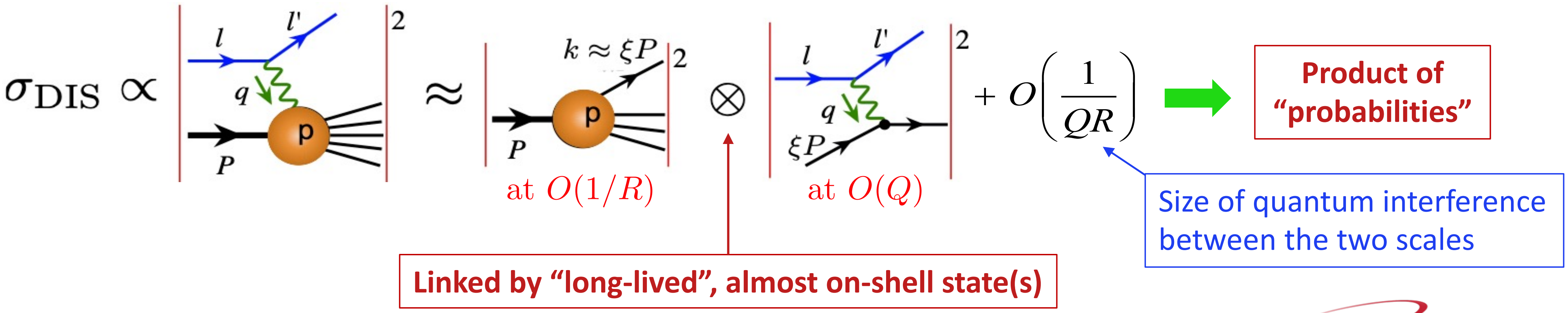
Why and How QCD factorization works?

❑ An important fact:

QCD color interaction is so strong at a typical hadronic scale O(1/R) with a hadron radius R ~ 1 fm that any scattering cross section with identified hadron(s) cannot be calculated fully in QCD perturbation theory!

❑ Why QCD factorization could work?

- The color interaction becomes weaker and calculable perturbatively at short distances – Asymptotic Freedom
- We are able to separate consistently the strong interacting dynamics at the hadronic scale (~ fm) from those taking place at short-distance (< 0.1 fm), and
- Prove that the quantum interference between the two scales are suppressed by the ratio of the two scales



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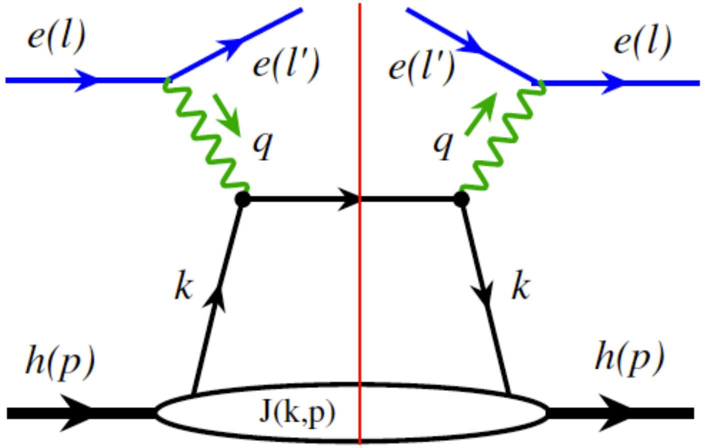

$$\sigma_{\rm DIS}^{\rm LO} \propto \int d^4k \left[\hat{\sigma}^{\rm LO}(Q,k)\frac{1}{k^2+i\varepsilon}J(k,p)\frac{1}{k^2-i\varepsilon}\right]$$

$$\approx \int \frac{dk^+}{2k^+} d^2k_T\; \hat{\sigma}^{\rm LO}(Q,\hat{k}) \sim O(Q)$$

$$\times \int dk^2 \frac{1}{k^2+i\varepsilon}J(k,p)\frac{1}{k^2-i\varepsilon} + \mathcal{O}\left[\frac{\Lambda_{\rm QCD}^2}{Q^2}\right]$$

$$\sim O(1/R)$$

$$\hat{k} = (k^+, \frac{k_T^2}{2k^+}, \vec{k}_T)$$

Perturbative pinch

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Why and How QCD factorization works?

❑ Necessary conditions for QCD factorization to work:

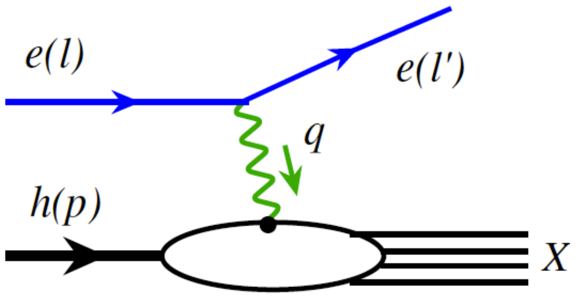
- All process-dependent nonperturbative contributions to “good” cross sections are suppressed by powers of O(1/QR), which could be neglected if the hard scale Q is sufficiently large
- All factorizable nonperturbative contributions are process independent, representing the characteristics of identified hadron(s), and
- The process dependence of factorizable contributions is perturbatively calculable from partonic scattering at the short-distance

❑ Predictive power and the value of factorization:

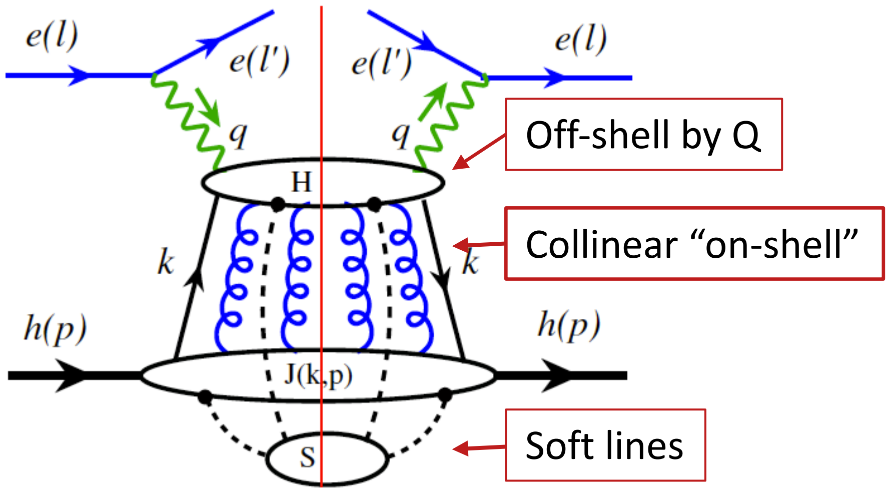
- Our ability to calculate the process-dependent short distance partonic scatterings at the hard scale Q
- Prediction follows when cross sections with different hard scatterings but the same nonperturbative long-distance effect of identified hadron are compared
- Factorization supplies physical content to these universal long-distance effects of identified hadrons by matching them to hadronic matrix elements of active quark and/or gluon operators, which could be interpreted as parton distribution or correlation functions of the identified hadrons, and allows them to be measured experimentally or by numerical simulations and model calculations

Why and How QCD factorization works?

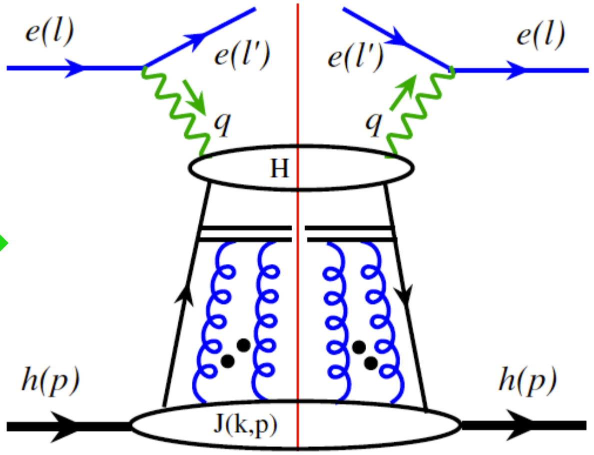
❑ QCD factorization with one identified hadron – Inclusive DIS:



$$e(l) + h(p) \to e(l') + X$$



"Leading pinch surface"
Reduced diagrams
Soft lines to "H" power suppressed



Apply Ward identity
Longitudinally polarized gluons decoupled from "H" to gauge link

- **Factorization formalism – leading power:**

$$E'\frac{d\sigma^{\rm DIS}_{eh\to eX}}{d^3l'}(l,p;l') = \sum_{f=q,\bar{q},g}\int dx\ \phi_{f/h}(x,\mu^2)\, E'\frac{d\hat{\sigma}_{ef\to eX}}{d^3l'}(l,\hat{k};l',\mu^2) + \mathcal{O}\left[\frac{\Lambda^2_{\rm QCD}}{Q^2}\right]$$

$\hat{k} \equiv xp^+$

- **Renormalization improvement:**

Hard part: $E'\frac{d\hat{\sigma}^{(n)}_{ef\to eX}}{d^3l'} = E'\frac{d\sigma^{\rm DIS(n)}_{ef\to eX}}{d^3l'}(l,p;l') - \sum_{m=0}^{n-1}\left[\sum_{f'=q,\bar{q},g} E'\frac{d\hat{\sigma}^{(m)}_{ef'\to eX}}{d^3l'} \otimes \phi^{(n-m)}_{f'/f}(x,\mu^2)\right]$

$$d\sigma_{eh\to eX}/d\log\mu^2 = 0 \quad \Rightarrow \quad \frac{d\phi_{f/h}(x,\mu^2)}{d\log\mu^2} = \sum_{f'}\int_x^1 \frac{dx'}{x'} P_{f/f'}\left(\frac{x}{x'},\alpha_s(\mu^2)\right)\phi_{f'/h}(x',\mu^2)$$

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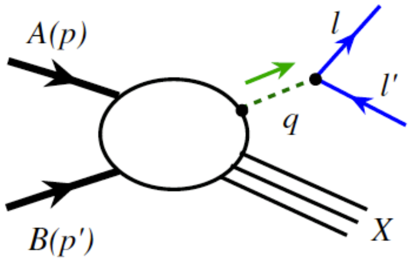
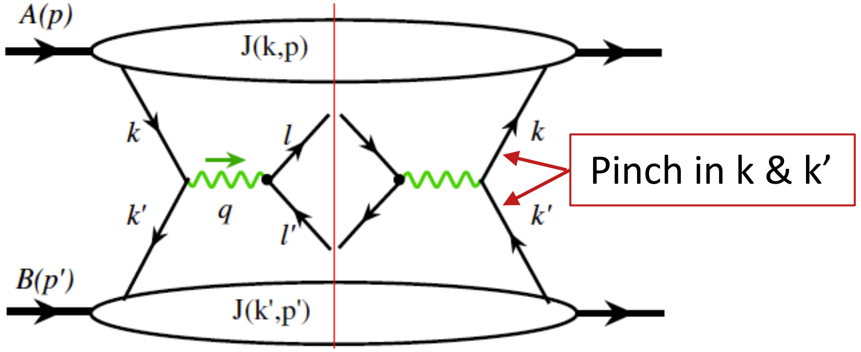
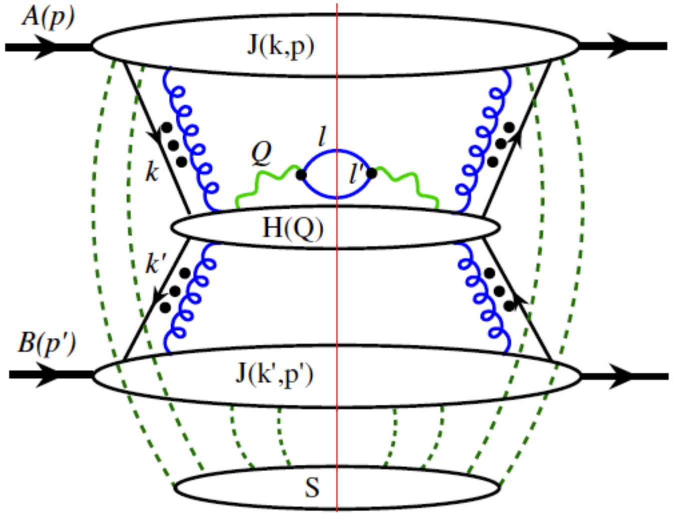
Why and How QCD factorization works?

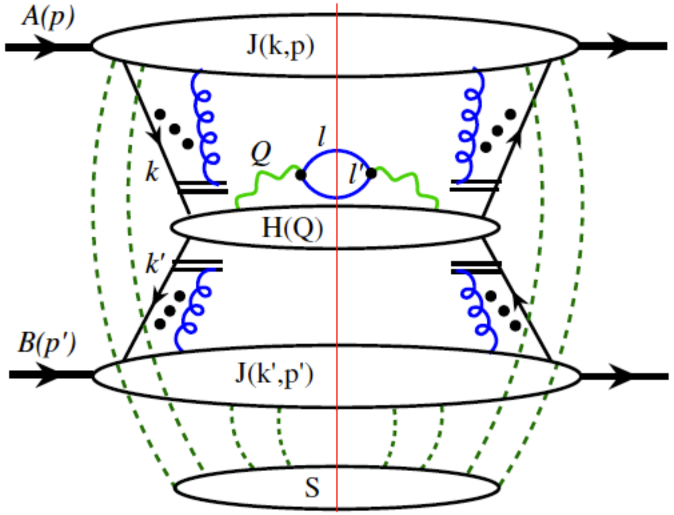
❑ QCD factorization with Two identified hadrons – Drell-Yan type:



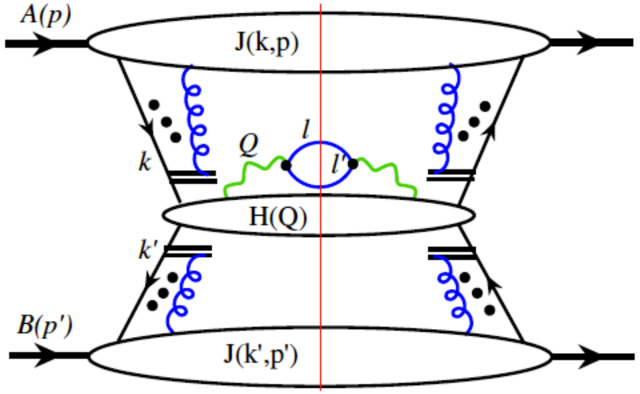




“Leading pinch surface”
n “beam jets”



Apply Ward identity
to decouple CO gluons from “H”



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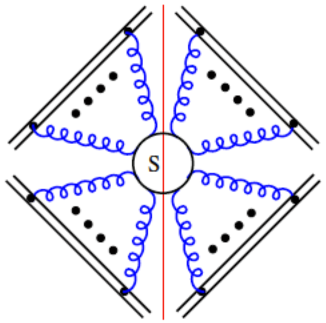

Single soft component to the beam jet
Apply Ward identity to decouple soft gluons into soft factor(s)
Soft factor = 1 for CO factorization!

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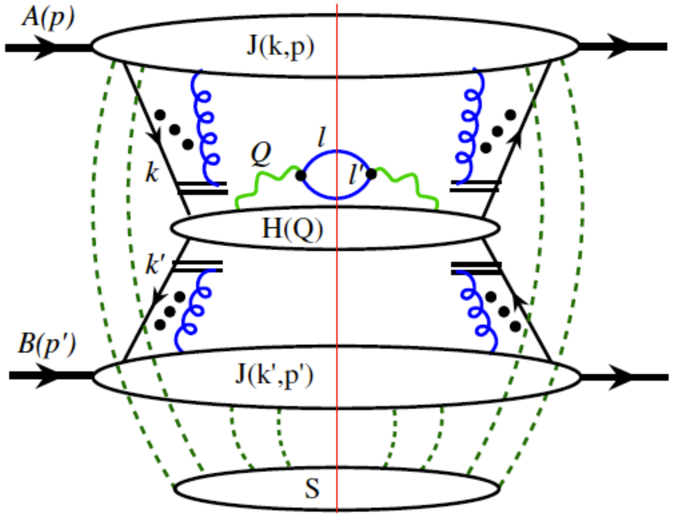
Why and How QCD factorization works?

- **QCD factorization with Two identified hadrons – Drell-Yan type:**

$$\frac{d\sigma^{(\mathrm{DY})}_{A+B\to ll'+X}}{dQ^2\,dy} = \sum_{ff'}\int dx\,dx'\,\phi_{f/A}(x,\mu)\,\phi_{f'/B}(x',\mu)\,\frac{d\hat{\sigma}_{f+f'\to ll'+X}(x,x',\mu,\alpha_s)}{dQ^2\,dy} + \mathcal{O}\left[\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right]$$

Same as that in DIS
"Universality"

But, this factorization can fail if the soft gluon momenta are trapped in the Glauber region: $k_i^{\pm} \ll k_i^{\perp}$



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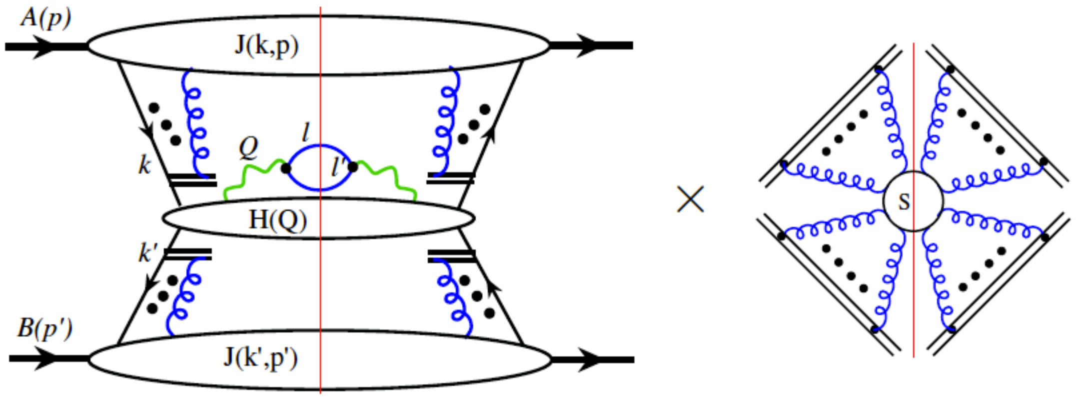

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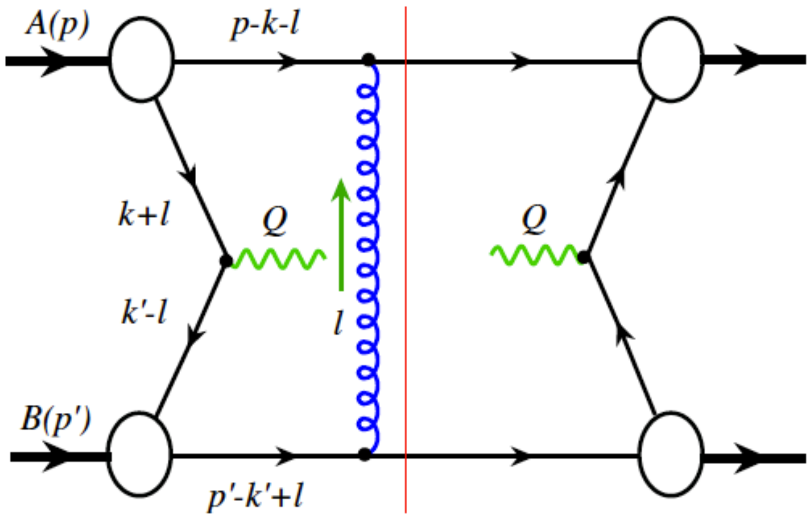
"Universality"

But, this factorization can fail if the soft gluon momenta are trapped in the Glauber region: $k_i^{\pm} \ll k_i^{\perp}$

Soft spectator interaction is responsible for this – the most challenge part of the factorization proof:



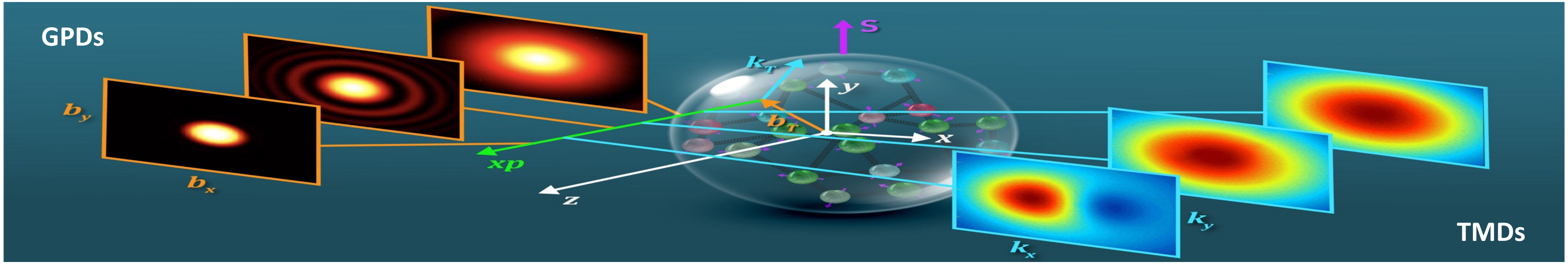
$$\frac{1}{(p-k-l)^2+i\varepsilon}\,\frac{1}{(k+l)^2++i\varepsilon} \longrightarrow \frac{1}{-l^-+i\varepsilon}\,\frac{1}{l^-+i\varepsilon}$$

Solution: (1) sum over all cuts, unitarity cancels all poles
in upper half plane for l^-, and
in lower half plane for l^+
(2) deform the other component out of Glauber region

$$\frac{1}{(p'-k'+l)^2+i\varepsilon}\,\frac{1}{(k'-l)^2++i\varepsilon} \longrightarrow \frac{1}{l^++i\varepsilon}\,\frac{1}{-l^++i\varepsilon}$$

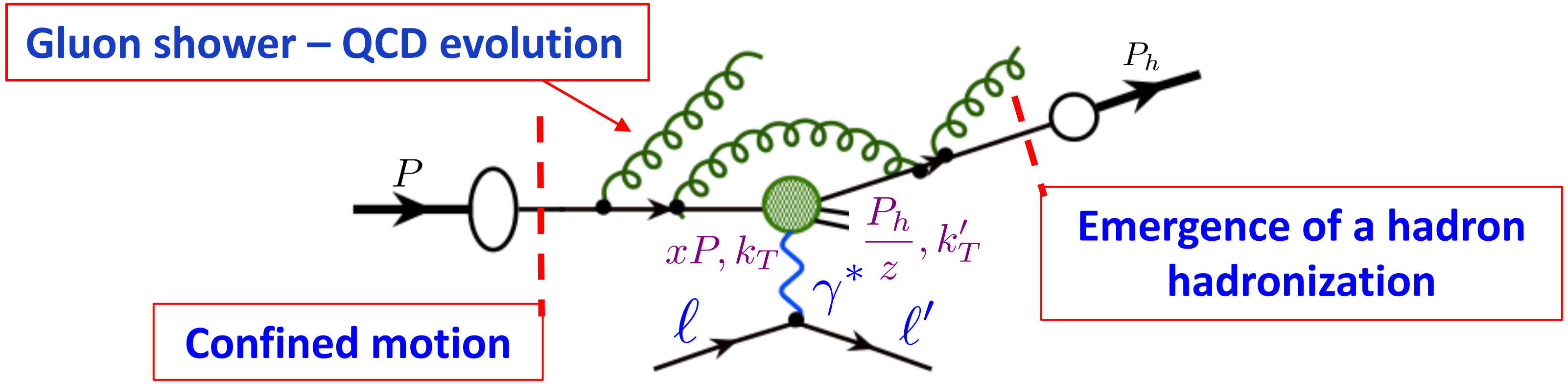
Nuclear Femtography

- 3D hadron structure extracted with two-scale probes:



NO quarks and gluons can be seen in isolation!

- If the nucleon is broken, e.g., in SIDIS, …



Transverse momentum broadening:

$$\Delta k_T^2 \propto \Lambda_{\rm QCD}^2 \times \alpha_s(C_F, C_A) \times \log(Q^2/\Lambda_{\rm QCD}^2) \times \log(s/Q^2) \quad \Big\} \gtrsim 1$$

Structure information is diluted by the collision induced shower!

- *Measured k_T is NOT the same as k_T of the confined motion!*
- *Too larger Q^2 could weaken our precision to probe the true hadron structure!*

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"See" hadron's internal structure without breaking it

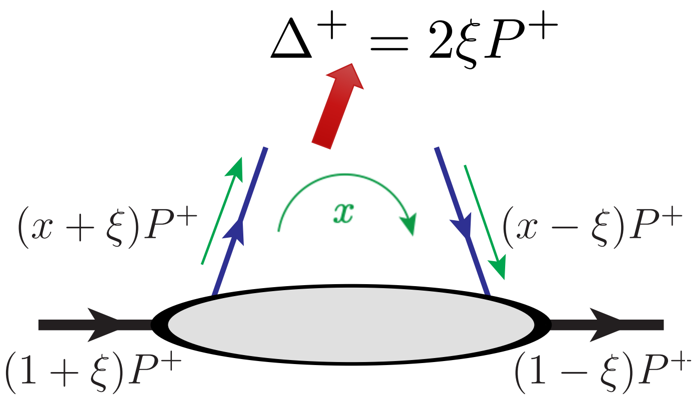
❑ Definition:

$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\frac{\gamma_5\Delta^+}{2m}u(p)\right].$$



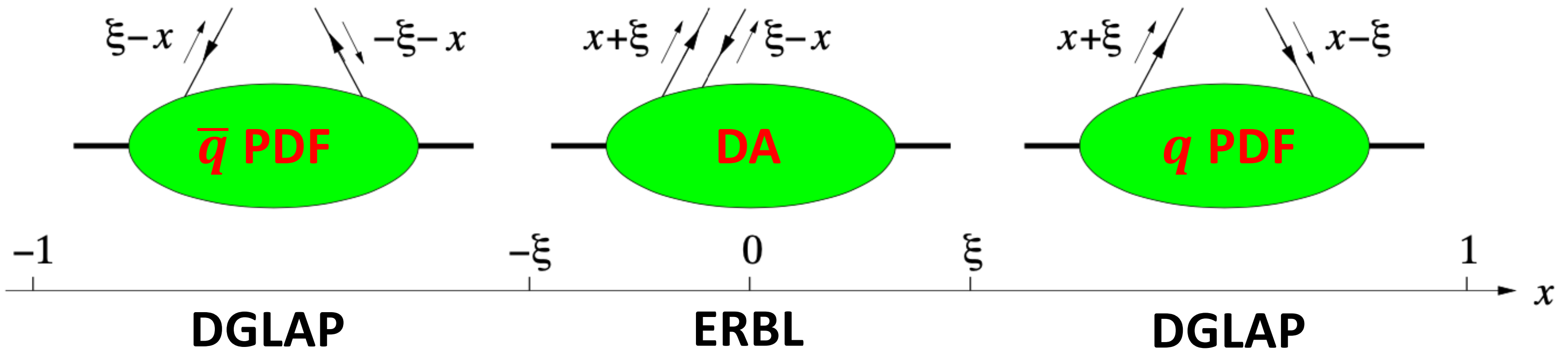
$$P^+ = \frac{p^+ + p'^+}{2}$$

$$\Delta = p - p' \qquad t = \Delta^2$$

Similar definition for gluon GPDs

❑ Combine *PDF* and *Distribution Amplitude (DA):*

Forward limit $\xi = t = 0$ **:** $\quad H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x)$



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Properties of GPDs

M. Burkdart, PRD 2000

- **Impact parameter dependent parton density distribution:**

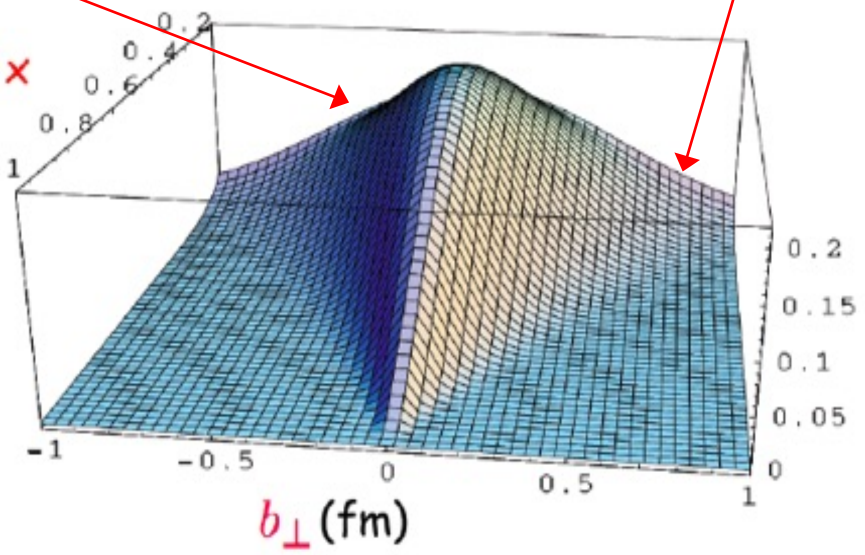
$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$

Quark density in $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$

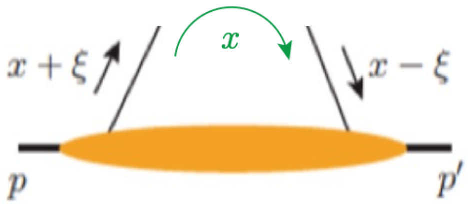
How fast does glue density fall?

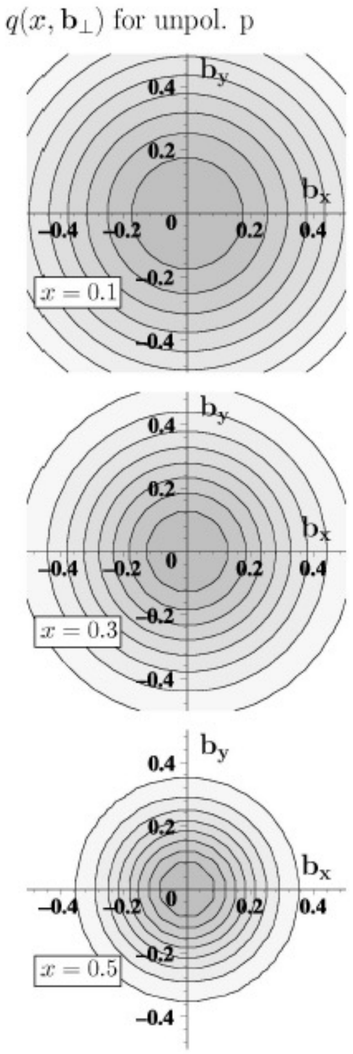
3D image

How far does glue density spread?



Unpolarized proton





- Should $r_q(x)$ > $r_g(x)$, or vice versa?
- Could $r_g(x)$ saturates as $x \to 0$
- How do they compare with known radius (EM charge radius, mass radius, …)?
- Tomographic images in slides of different x value!

➡ Proton radii of quark and gluon spatial density distribution, $r_q(x)$ & $r_g(x)$

Properties of GPDs

❑ "Mass" – QCD energy-momentum tensor:

Ji, PRL78, 1997

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \textbf{with} \quad T_q^{\mu\nu} = \bar{\psi}_q \, i\gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \, \psi_q - g^{\mu\nu} \bar{\psi}_q \left(i\gamma \cdot \overleftrightarrow{D} - m_q \right) \psi_q \quad \textbf{and} \quad T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4} g^{\mu\nu} \left(F^a_{\rho\eta} \right)^2$$

❑ Gravitational form factors:

$$\langle p' \, | T_i^{\mu\nu} | \, p \rangle = \bar{u}(p') \left[A_i(t) \frac{P^\mu P^\nu}{m} + J_i(t) \frac{i P^{(\mu} \sigma^{\nu)\Delta}}{2m} + D_i(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} + m \, \bar{c}_i(t) \, g^{\mu\nu} \right] u(p)$$

$$C_i(t) \leftrightarrow D_i(t)/4$$

Related to pressure & stress force inside h

Polyakov, schweitzer, Inntt. J. Mod. Phys. A33, 1830025 (2018)
Burkert, Elouadrhiri , Girod Nature 557, 396 (2018)

❑ Connection to GPD moments:

$$\int_{-1}^{1} dx \, x \, F_i(x,\xi,t) \propto \langle p' | T_i^{++} | p \rangle \quad \propto \quad \bar{u}(p') \left[\underbrace{(A_i + \xi^2 D_i)}_{\int_{-1}^{1} dx \, x \, H_i(x,\xi,t)} \gamma^+ + \underbrace{(B_i - \xi^2 D_i)}_{\int_{-1}^{1} dx \, x \, E_i(x,\xi,t)} \frac{i\sigma^{+\Delta}}{2m} \right] u(p)$$

❑ "Spin" – Angular momentum sum rule:

$$J_i = \lim_{t \to 0} \int_{-1}^{1} dx \, x \, [H_i(x,\xi,t) + E_i(x,\xi,t)] \qquad i = q,g$$

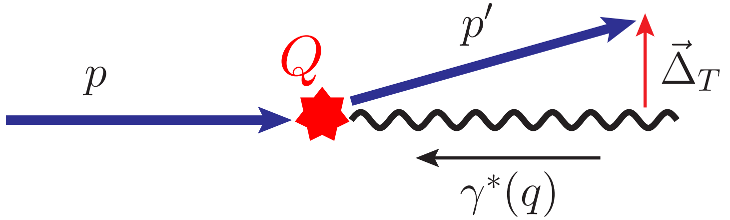
➡ **3D tomography**
Relation to GFF
Angular Momentum

➡ ***x*-dependence of GPDs!**

Need to know the x-dependence of GPDs to construct the proper moments!

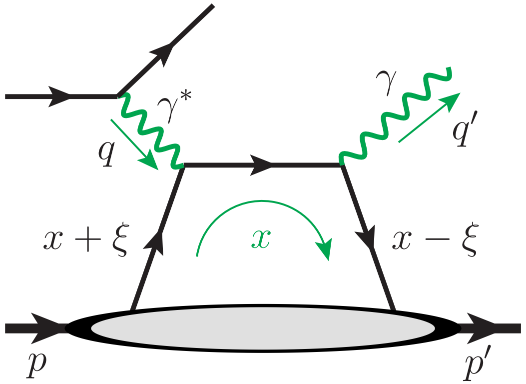
Exclusive Diffractive Process for Extracting GPDs

- ❑ **Hit the proton hard without breaking it** $\Longrightarrow$ **Diffractive scattering to keep proton intact**

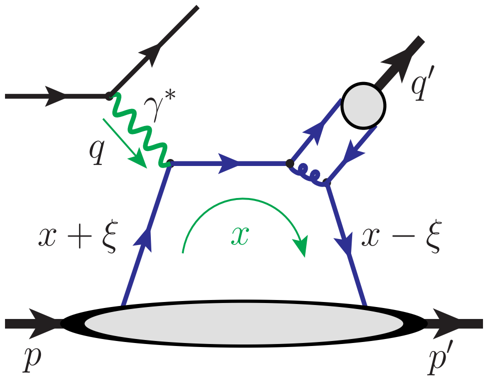


HERA discovery:
~15% of HERA events with the Proton stayed intact

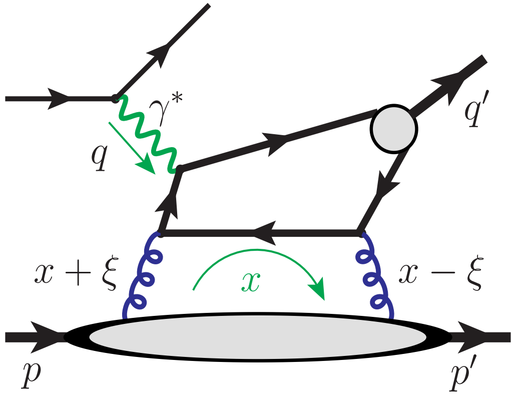
- ❑ **Known exclusive processes for extracting GPDs:**



DVCS: Q² >> |t|



DVMP



DVQP

+ DDVCS, …

Feature: Two-scale observables $Q^2 \gg |t|$ $\quad t = (p - p')^2$

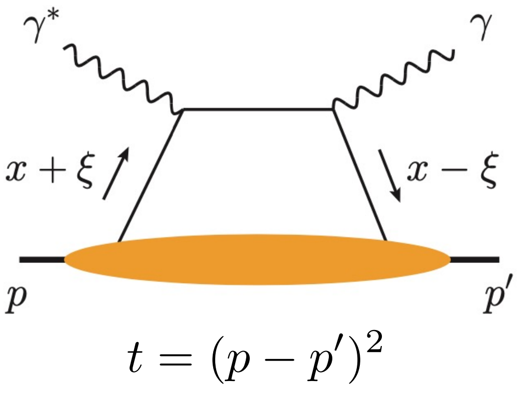
- **Hard scale Q: allows pQCD, factorization**
- **Low scale t: probes non-pert. hadron structure**

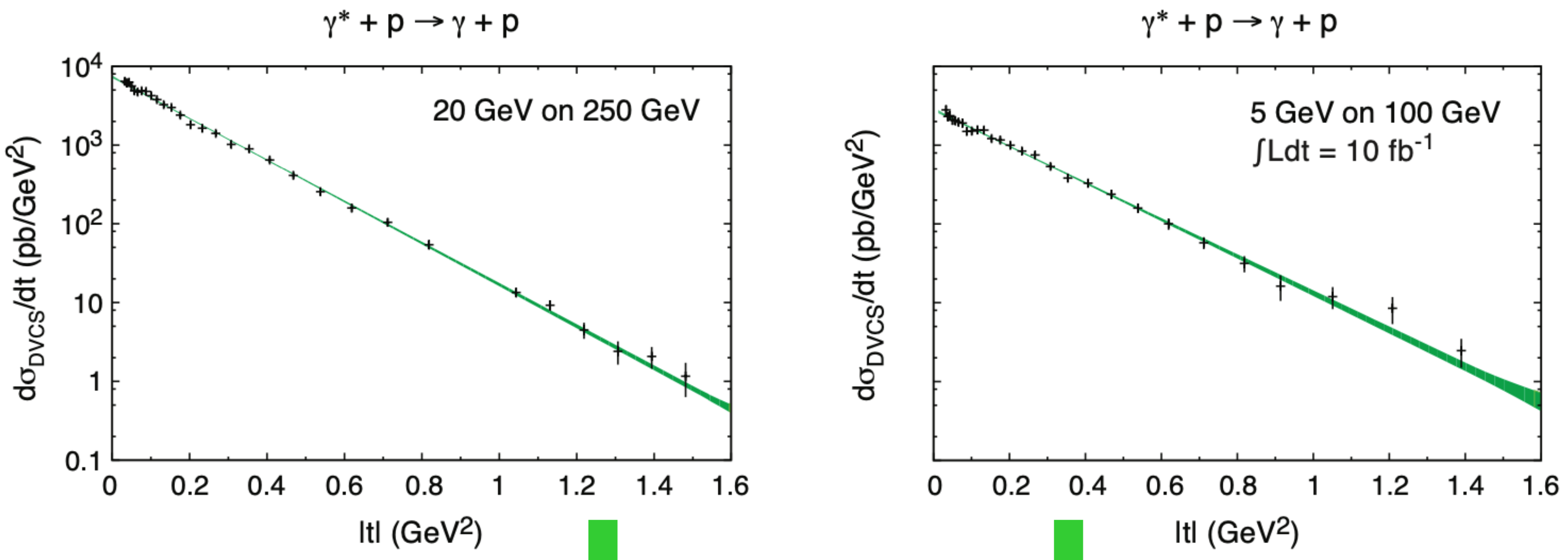
Factorization ➡ **GPDs:** $f_{i/h}(x, \xi, t; \mu)$

Jefferson Lab

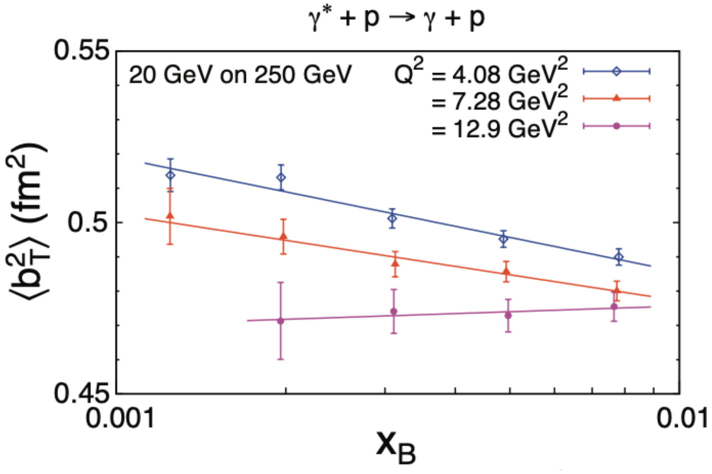
DVCS at a Future EIC (White Paper)

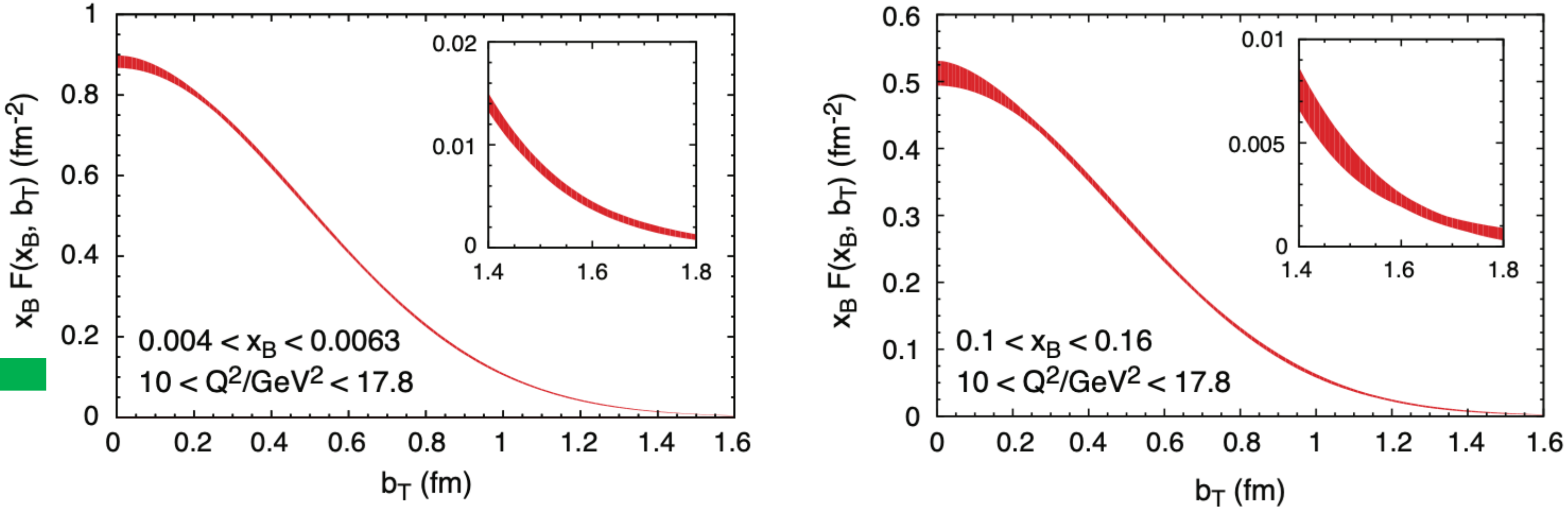
❑ Cross Sections:





❑ Spatial distributions:



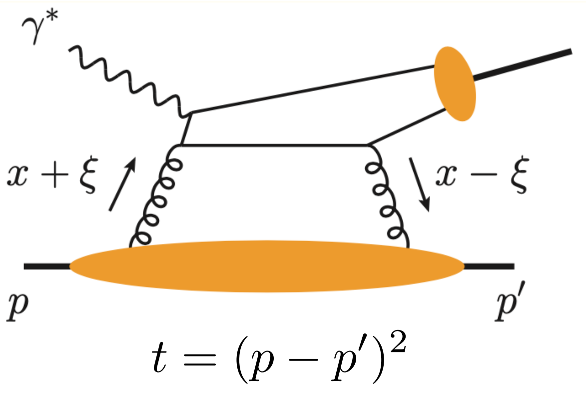


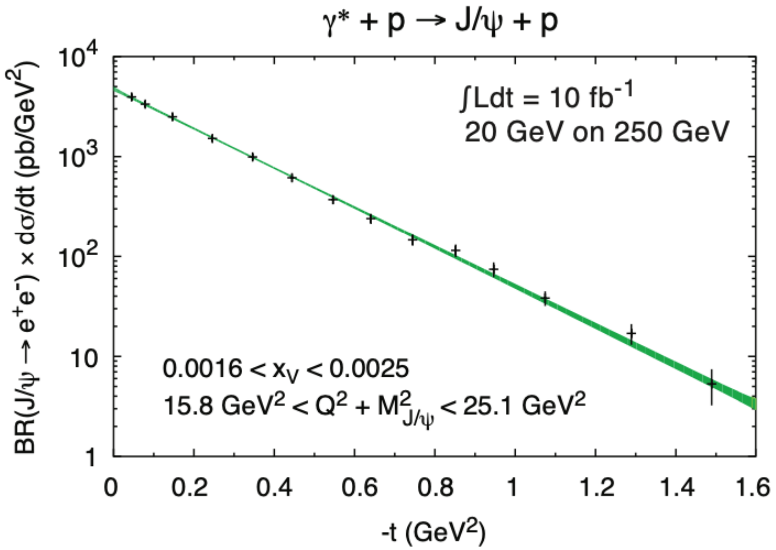
Effective "proton radius" in terms of quarks as a function of x_B

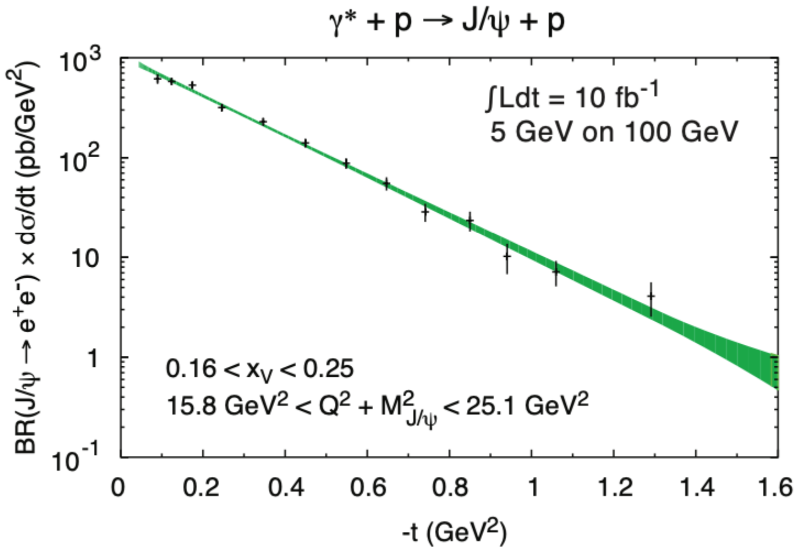
Jefferson Lab

Imaging the Gluon at the EIC (White Paper)

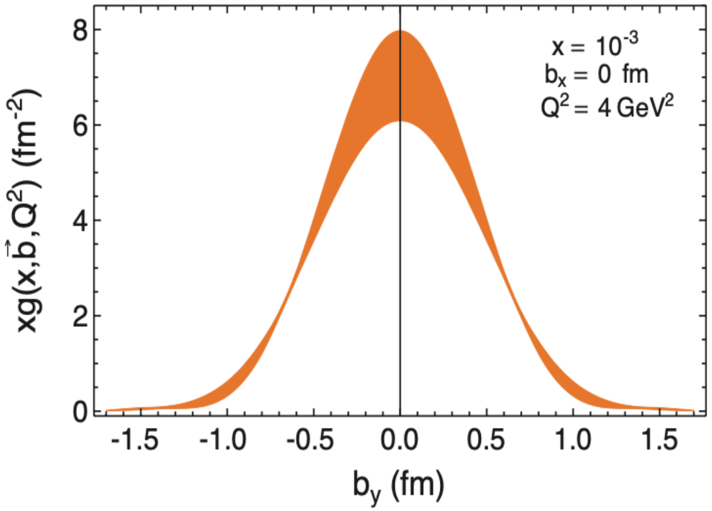
- Exclusive vector meson production:

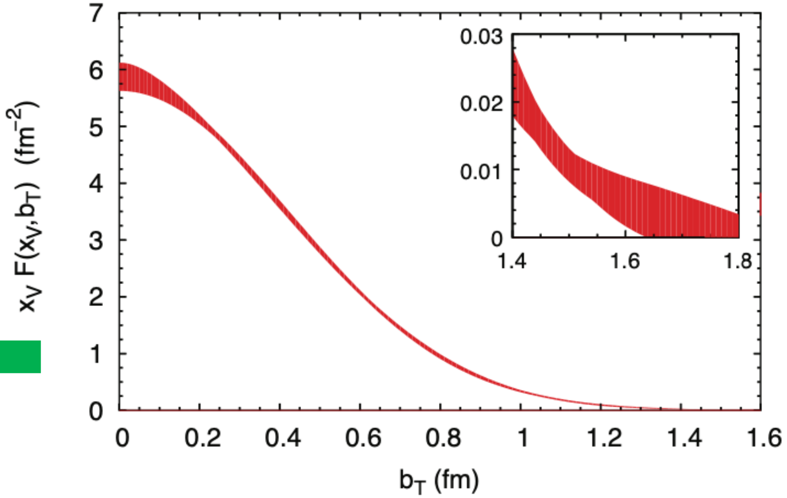


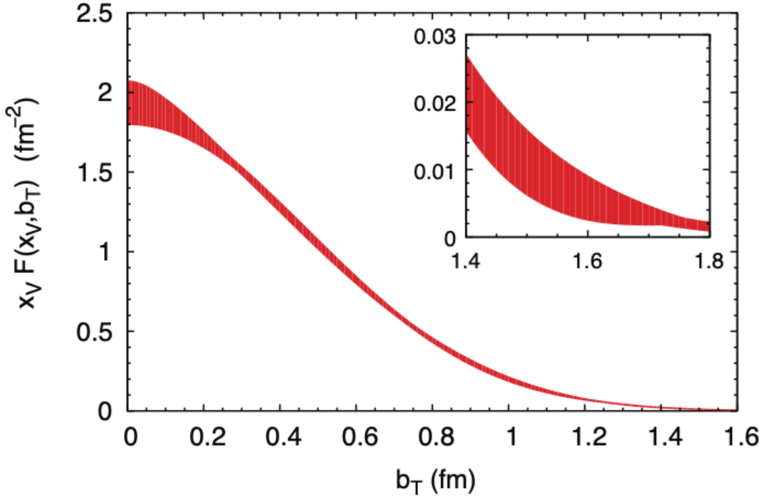




- Spatial distributions:







The b_T space density for gluons → **Effective "proton radius" in terms of gluons**

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It is difficult to extract the x-dependence of GPD – Why?

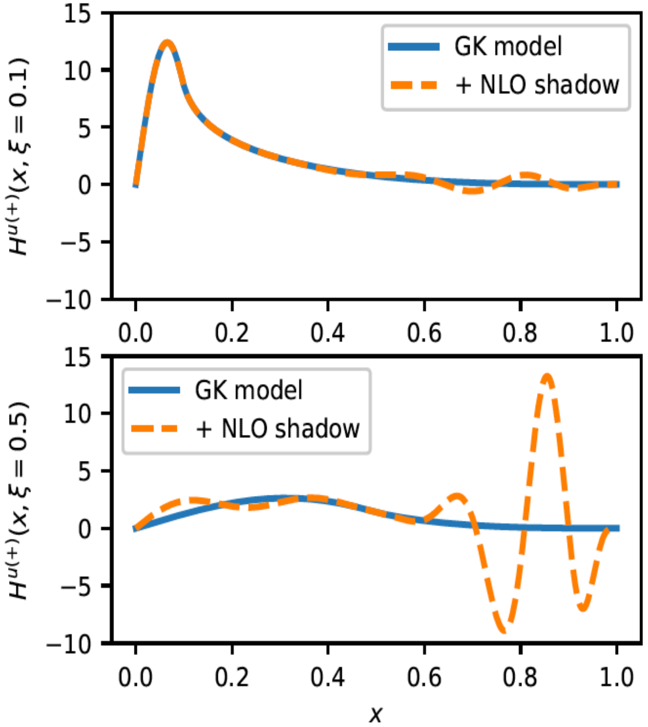
- **Amplitude nature: exclusive processes**

  $x \sim$ loop momentum

  $$\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x F(x,\xi,t) \cdot C(x,\xi;Q/\mu)$$

  never pin down to some x

- **"Shadow GPDs"** $F(x,\xi,t) \to F(x,\xi,t) + S(x,\xi,t)$



with

$$\int_{-1}^{1} \mathrm{d}x \frac{S(x,\xi,t)}{x-\xi+i\varepsilon} = 0$$

Blue and dashed Fit the same CFFs !

PRD103 (2021) 114019

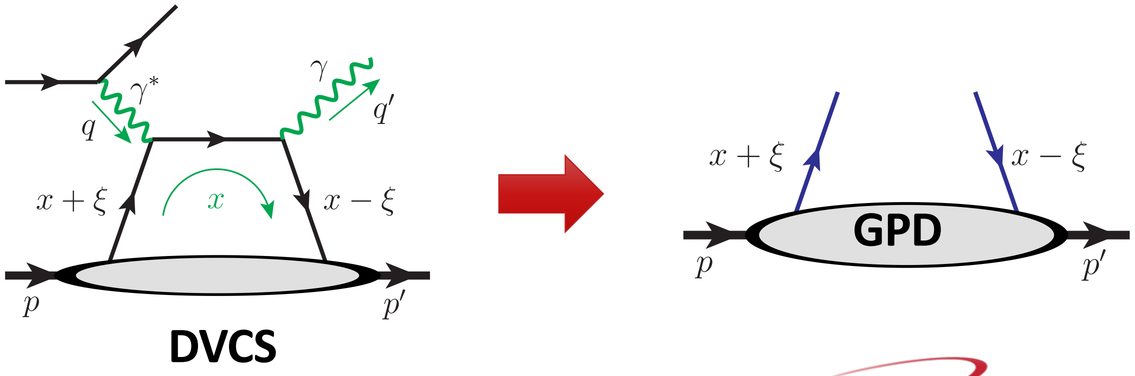
- **Sensitivity to x comes from $C(x,\xi;Q/\mu)$**

  At LO, DVCS hard coefficient factorizes

  $$C(x,\xi;Q/\mu) = C_Q(Q/\mu) \cdot C_x(x,\xi) \propto \frac{1}{x-\xi+i\varepsilon} \cdots$$

  $$\Rightarrow \quad i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \frac{F(x,\xi,t)}{x-\xi+i\varepsilon} \equiv \text{“}F_0(\xi,t)\text{”}$$

  - also true for most other processes
  - x-dependence is only constrained by a "moment"
  - easy to fit to the data



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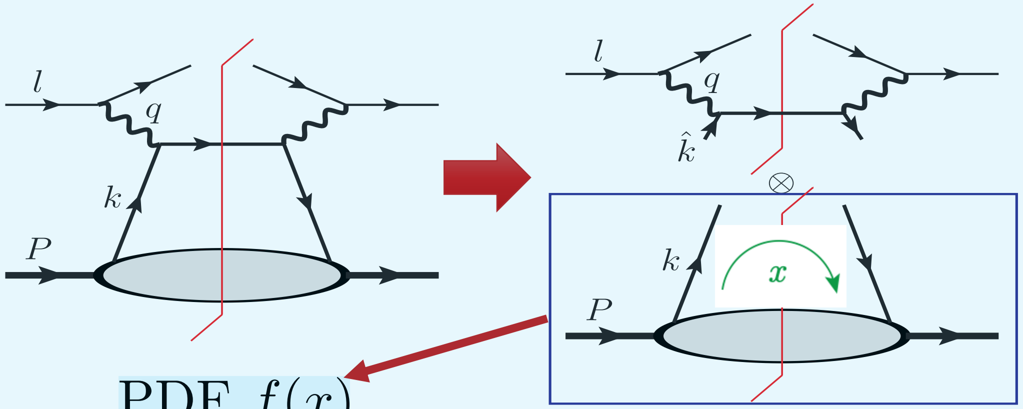
Inclusive Process vs. Exclusive Process

❑ Deeply Inelastic Scattering (DIS):



PDF $f(x)$

$x = k^+/P^+$

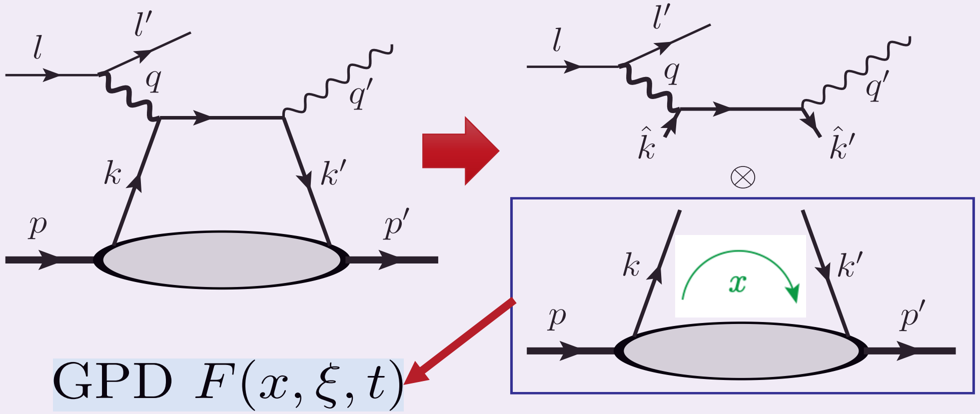
Cross section: **Cut diagrams**

$$\sigma_{\rm DIS} \simeq \int_{x_B}^{1} {\rm d}x\, f(x)\, \hat{\sigma}(x/x_B)$$

- **PDF ~ probability**
- **At LO: $x = x_B$**
- **Beyond LO: $x \in [x_B, 1]$**

x-dependence: **Part of measurement**

❑ Deeply Virtual Compton Scattering (DVCS)



GPD $F(x, \xi, t)$

$$x = \frac{(k+k')^+}{(p+p')^+}$$

$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

$$t = (p-p')^2$$

Amplitude: **Uncut diagrams**

$$\mathcal{M}_{\rm DVCS}(\xi, t) \simeq \int_{-1}^{1} {\rm d}x\, F(x, \xi, t)\, \hat{\mathcal{M}}(x, \xi)$$

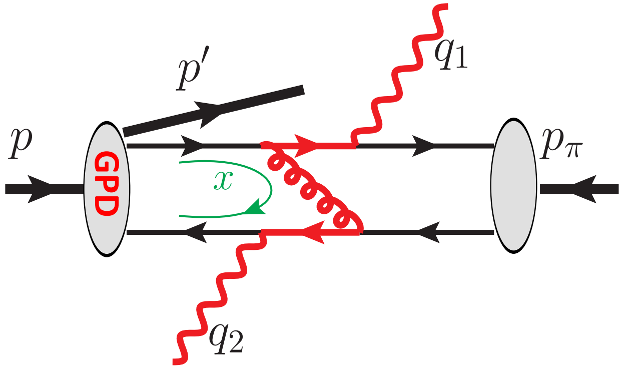
- **GPD ~ amplitude**
- **$k^+ = (x + \xi)\, P^+$ is loop momentum**
- **At any order: $x \in [-1, 1]$**

x-dependence: **Hard to measure**

What kind of process/observable could be sensitive to the x-dependence?

- Create an entanglement between the internal x and an externally measured variable?
  - **Production of two back-to-back high pT particles (say, two photons):**

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$



Qiu & Yu, JHEP 08 (2022) 103

  - **Kinematical observables:**
    - $t = (p - p')^2$
    - $\xi = (p^+ - p'^+)/(p^+ + p'^+)$

Hard scale: $q_T \gg \Lambda_{\rm QCD}$

Soft scale: $t \sim \Lambda^2_{\rm QCD}$

  - **Factorization:**

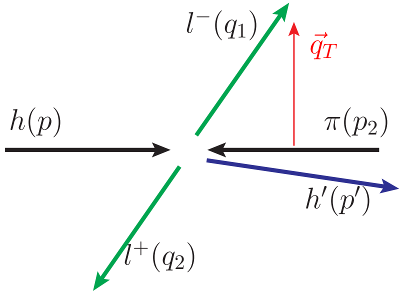
[suppressing pion DA factor]

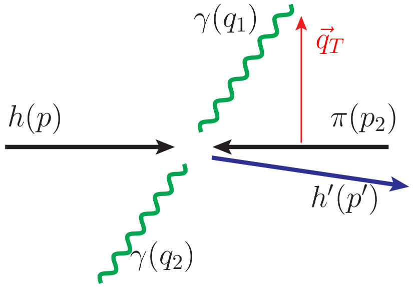
$$\mathcal{M}(t, \xi, q_T) = \int_{-1}^{1} dx\, F(x, \xi, t; \mu) \cdot C(x, \xi; q_T/\mu) + \mathcal{O}(\Lambda_{\rm QCD}/q_T)$$

$$\Rightarrow \quad \frac{d\sigma}{dt\, d\xi\, dq_T} \sim |\mathcal{M}(t, \xi, q_T)|^2$$

$x \leftrightarrow q_T$

q_T distribution is "conjugate" to x distribution





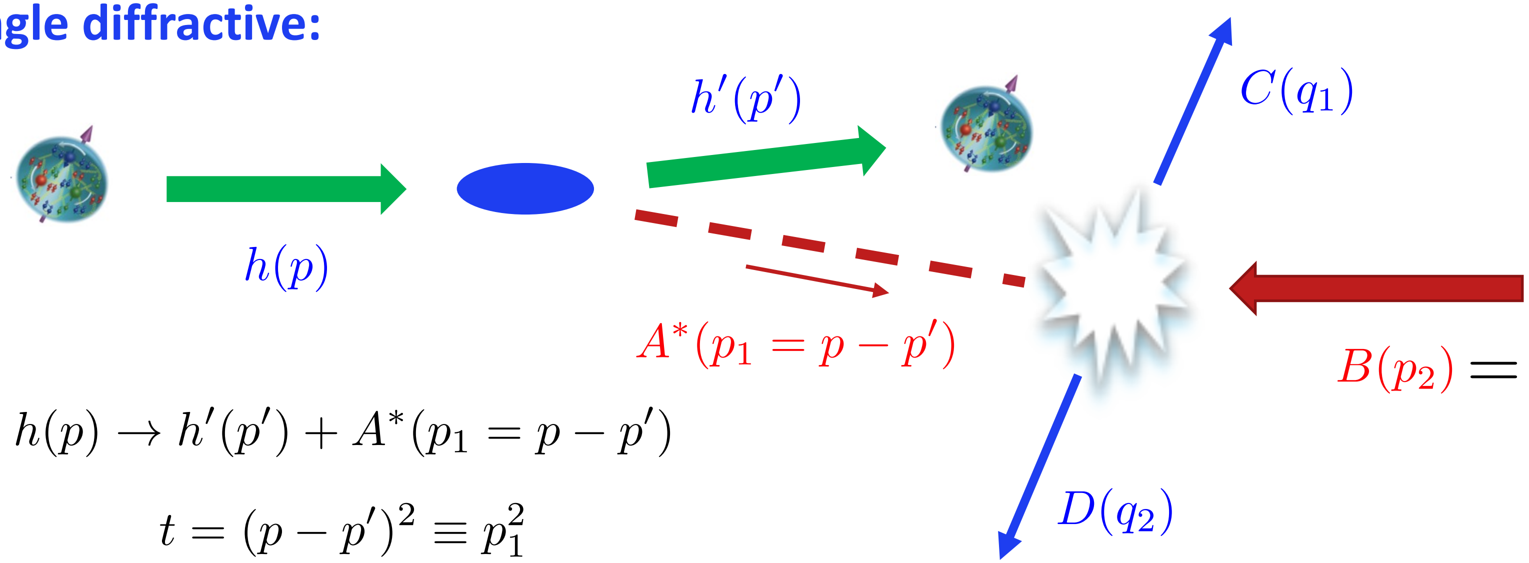
\+ photon-meson pair, meson-meson pair

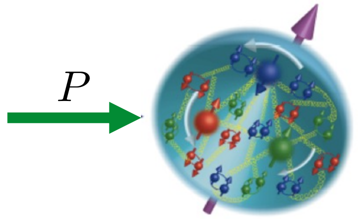
Single-Diffractive Hard Exclusive Processes (SDHEP)

- **Two-stage diffractive $2 \to 3$ hard exclusive processes:**

Qiu & Yu, JHEP 08 (2022) 103, PRD 107 (2023) 1, in preparation

  - **Single diffractive:**





Probing its structure without breaking it!

$$h(p) \to h'(p') + A^*(p_1 = p - p')$$

$$t = (p - p')^2 \equiv p_1^2$$

  - **Hard probe:** $2 \to 2$ **high** q_T **exclusive process**

$$A^*(p_1) + B(p_2) \to C(q_1) + D(q_2)$$

$$(p - p') \cdot n \gg \sqrt{|t|} \quad \Longleftrightarrow \quad |q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$$

  - **The single diffractive $2 \to 3$ exclusive hard processes:**

$$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$$

- **Necessary condition for QCD factorization:**

$$|q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$$

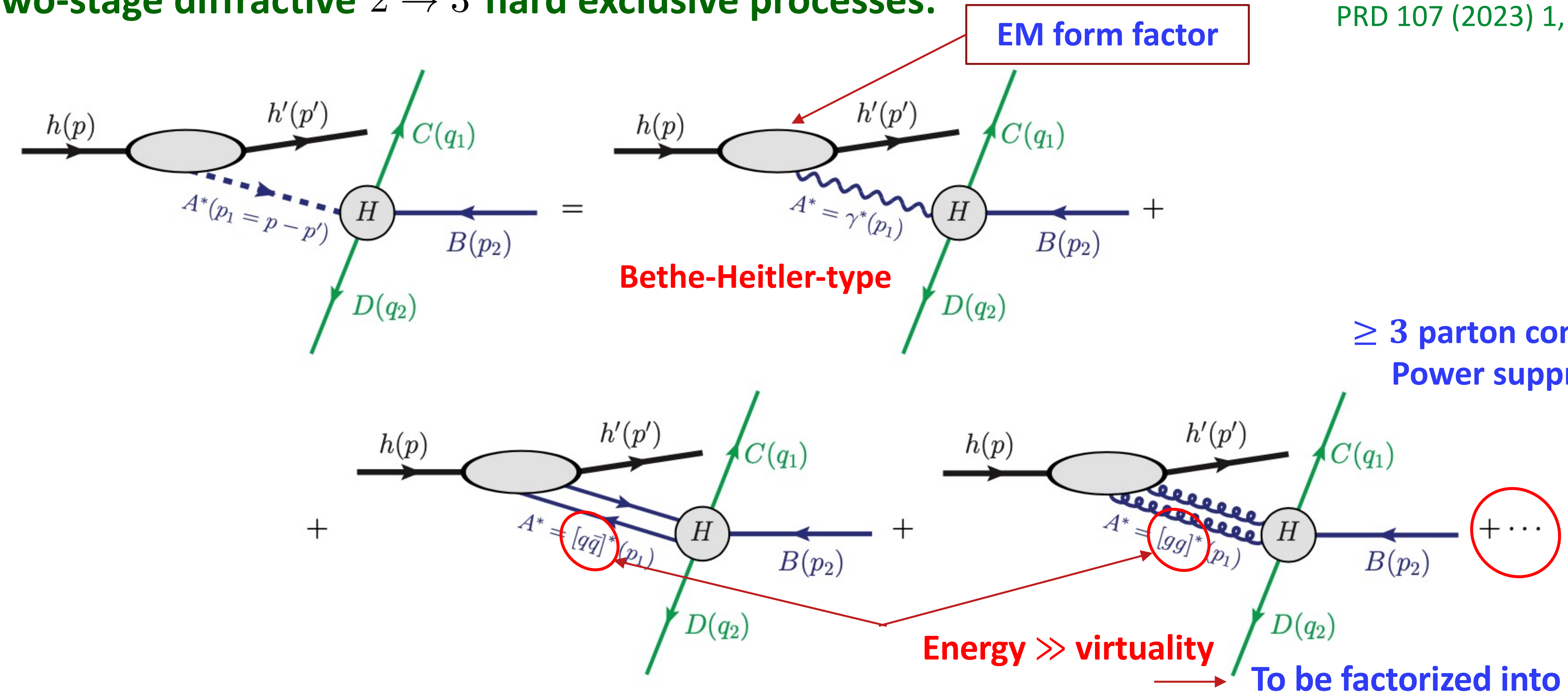
The state $A^*(p_1)$ lives much longer than $2 \to 2$ hard exclusive collision!

Not necessarily sufficient!

Single-Diffractive Hard Exclusive Processes (SDHEP)

Qiu & Yu, JHEP 08 (2022) 103, PRD 107 (2023) 1, in preparation

- **Two-stage diffractive $2 \to 3$ hard exclusive processes:**



The exchanged state *A*(p-p')* is a sum of all possible partonic states, $\sum_{n=1,2,..}$, allowed by

- **Quantum numbers of *h(p) – h'(p')***
- **Symmetry of producing non-vanishing *H***

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General Discussion on n=1 state: γ^*

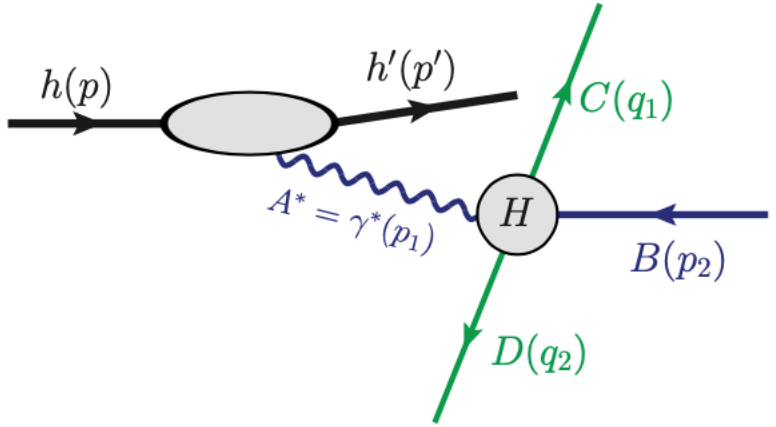
Qiu & Yu, PRD 107 (2023) 1

❑ Exchange of a virtual photon:

$$\mathcal{M}^{(1)} = \frac{ie^2}{t}\langle h'(p')|J^\mu(0)|h(p)\rangle\,\langle C(q_1)D(q_2)|J_\mu(0)|B(p_2)\rangle$$

$$\equiv \frac{ie^2}{t}F^\mu(p,p')\,\mathcal{H}_\mu(p_1,p_2,q_1,q_2)$$

$$J^\mu = \sum_{i\in q} Q_i\bar{\psi}_i\gamma^\mu\psi_i$$



$$F^\mu(p,p') = \langle h'(p')|J^\mu(0)|h(p)\rangle$$

$$= F_1^h(t)\,\bar{u}(p')\gamma^\mu u(p) + F_2^h(t)\,\bar{u}(p')\frac{i\sigma^{\mu\nu}p_{1\nu}}{2m_h}u(p)$$

Forbidden for $p\to n$ (or $n\to p$) transition GPDs
Or not allowed by H

Has a leading component , $F^+\propto\mathcal{O}(Q)$, as h-h' fast along "+"

$$F^+\mathcal{H}^- = \frac{1}{p_1^+}F^+\left(p_1^+\mathcal{H}^-\right) = \frac{1}{p_1^+}F^+\left(p_1\cdot\mathcal{H} + \boldsymbol{p}_{1\perp}\cdot\boldsymbol{\mathcal{H}}_\perp - p_1^-\mathcal{H}^+\right) \sim \mathcal{O}(\sqrt{|t|})$$

Leading power of $F\cdot\mathcal{H}$

➡ $\mathcal{M}^{(1)} \sim \mathcal{O}(1/\sqrt{|t|})$

Higher power than n=2 contribution, but, higher power in power of α_{EM}

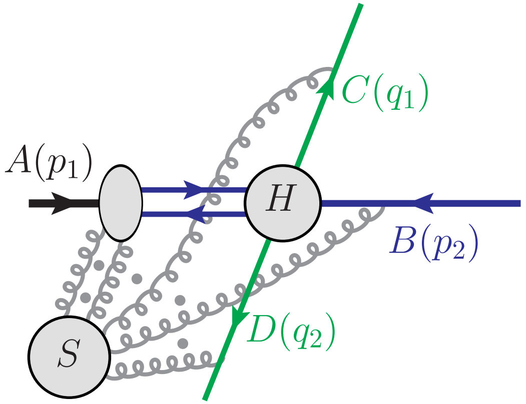
$\mathcal{M}^{(2)} \sim \mathcal{O}(1/Q)$ ➡ $\mathcal{M}^{(1)}/\mathcal{M}^{(2)} \sim \mathcal{O}(Q/\sqrt{|t|})$

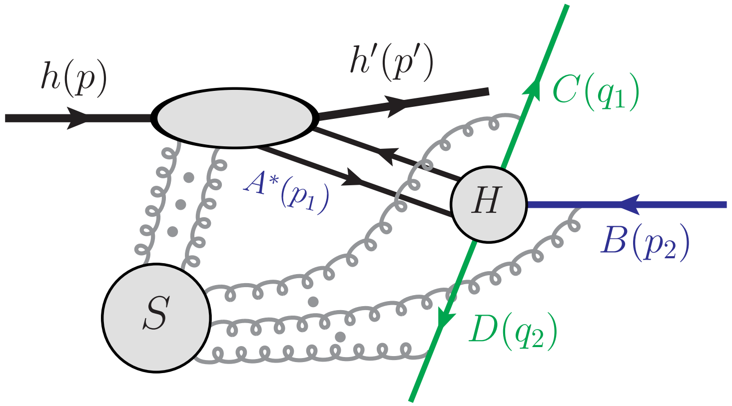
If we neglect contribution from $n\geq 3$, $\mathcal{M}^{(1+2)}_{\rm SDHEP} \sim$ is up to corrections at $\mathcal{O}(\sqrt{|t|}/Q^2)$

Factorization for SDHEP in the Two-stage Paradigm

Qiu & Yu, JHEP 08 (2022) 103,
PRD 107 (2023) 1

❑ Factorization for 2-parton channel factorization:





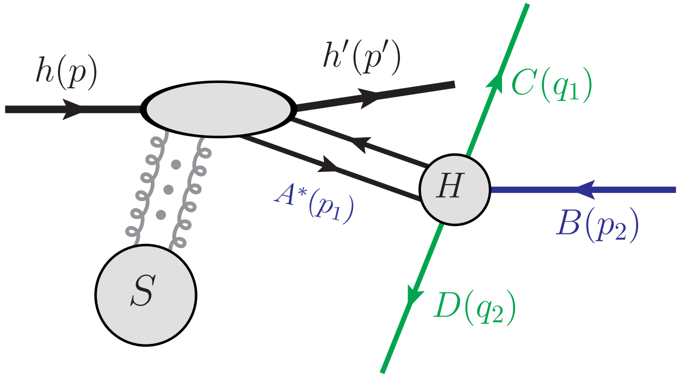
Only complication:

k_s^- is **pinched** in Glauber region for DGLAP region.

$$k_s^+ \mapsto k_s^+ \pm i\mathcal{O}(Q)$$

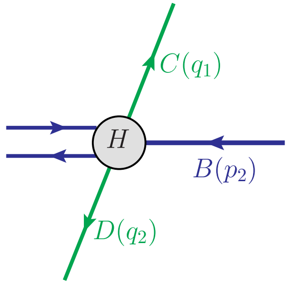
Glauber → ***h*-collinear region**

❑ Soft gluons cancel for the meson-initialized process if *C* and *D* are mesons:

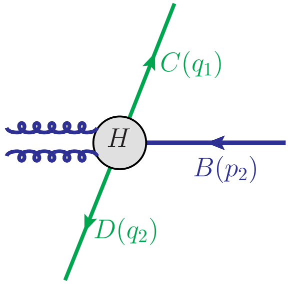


Soft gluons are no longer pinched and can be deformed into *h*-collinear region

GPD $\otimes$



or



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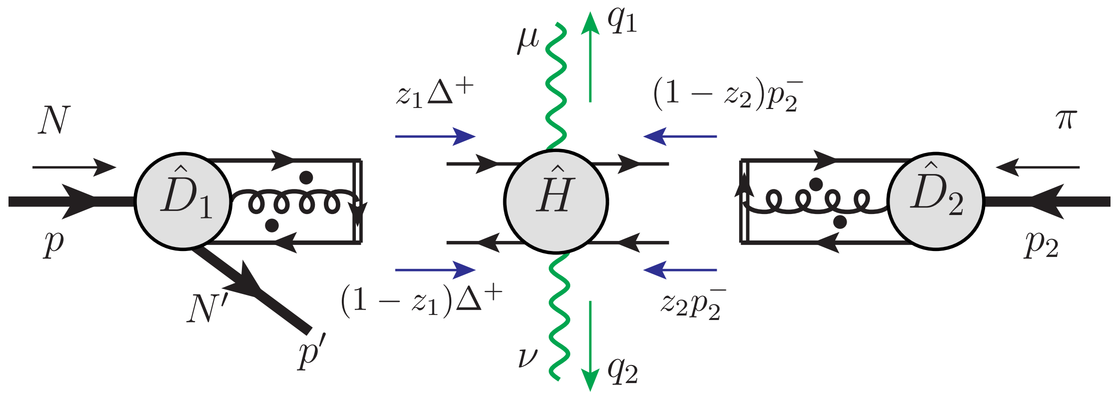
Exclusive massive photon-pair production in meson-hadron collision

- **Factorization formula:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$

Qiu & Yu, JHEP 08 (2022) 103

$$\mathcal{M}^{\mu\nu} = \int \mathrm{d}z_1 \mathrm{d}z_2 \left[\widetilde{\mathcal{F}}^{ud}_{NN'}(z_1,\xi,t) D(z_2) C^{\mu\nu}(z_1,z_2) + \mathcal{F}^{ud}_{NN'}(z_1,\xi,t) D(z_2) \widetilde{C}^{\mu\nu}(z_1,z_2) \right] + \mathcal{O}(\Lambda_{\mathrm{QCD}}/q_T)$$



Similar factorized form for SDHEP with lepton, photon beam

PRD 107 (2023) 1

$$\mathcal{F}^{ud}_{NN'}(z_1,\xi,t) = \int \frac{\mathrm{d}y^-}{4\pi} e^{iz_1\Delta^+ y^-} \langle N'(p')|\bar{d}(0)\gamma^+ \Phi(0,y^-;w_2)\, u(y^-)|N(p)\rangle$$

$$= \frac{1}{2P^+}\left[H^{ud}_{NN'}(z_1,\xi,t)\, \bar{u}(p')\gamma^+ u(p) - E^{ud}_{NN'}(z_1,\xi,t)\, \bar{u}(p') \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_p} u(p) \right],$$

$$\widetilde{\mathcal{F}}^{ud}_{NN'}(z_1,\xi,t) = \int \frac{\mathrm{d}y^-}{4\pi} e^{iz_1\Delta^+ y^-} \langle N'(p')|\bar{d}(y^-)\gamma^+\gamma_5 \Phi(0,y^-;w_2)\, u(0)|N(p)\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^{ud}_{NN'}(z_1,\xi,t)\, \bar{u}(p')\gamma^+\gamma_5 u(p) - \widetilde{E}^{ud}_{NN'}(z_1,\xi,t)\, \bar{u}(p') \frac{i\gamma_5\sigma^{+\alpha}\Delta_\alpha}{2m_p} u(p) \right]$$

Jefferson Lab

Exclusive massive mhoton-mair production in meson-hadron collision

❑ **Challenge for QCD factorization:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$ $\quad \lambda \sim m_\pi/Q, \quad Q \sim q_T$

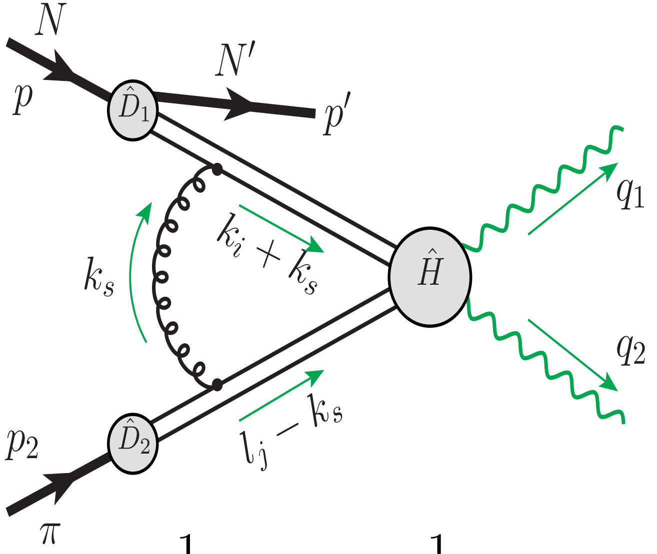
Gluons in the Glauber region: $k_s = (\lambda^2, \lambda^2, \lambda)\, Q$ *Transverse component contribute to the leading region!*

$$k_i = (1, \lambda^2, \lambda)\, Q$$

$$l_j = (\lambda^2, 1, \lambda)\, Q$$

ERBL region

(Efremov, Radyushkin, Brodsky, Lepage)



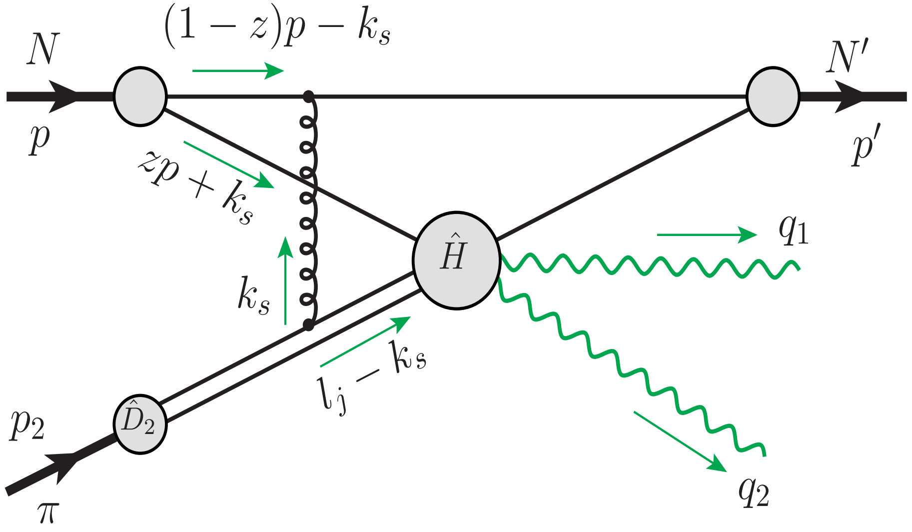
$$\frac{1}{k_s^2 + i\varepsilon} \to \frac{1}{-\mathbf{k}_s^2 + i\varepsilon}$$

$$\frac{1}{(k_i + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

$$\frac{1}{(l_j - k_s)^2 + i\varepsilon} \to \frac{1}{-k_s^+ + i\varepsilon}$$

No pinch!

DGLAP region



$$\frac{1}{((1-z)p - k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- - i\varepsilon}$$

$$\frac{1}{(zp + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

Pinched!

Same conclusion if k_s flows Through N'!

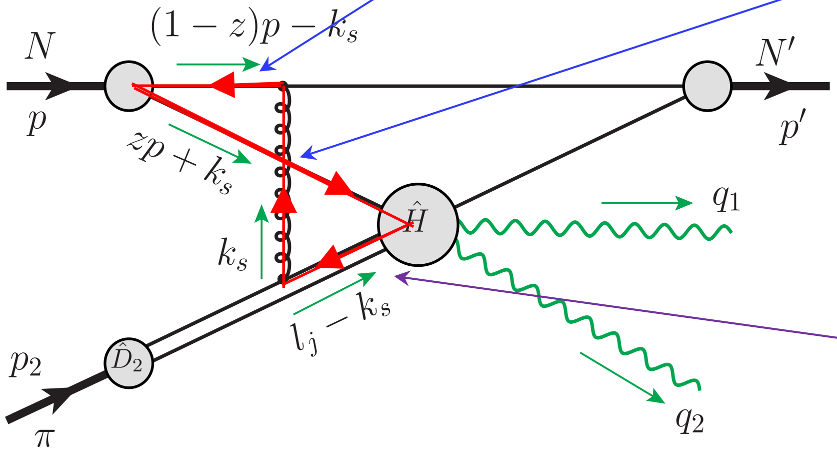
Exclusive Massive Photon-Pair Production in Meson-Hadron Collision

❑ Factorization: $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$

Qiu & Yu, JHEP 08 (2022) 103

$$\frac{1}{-k_s^- + i\varepsilon}$$

$$\frac{1}{+k_s^- + i\varepsilon}$$



k_s^- is pinched around 0

$$\frac{1}{-k_s^+ + i\varepsilon}$$

Only pole for k_s^+

DGLAP region: pinched

Deformation out of the Glauber region:

$k_s^+ \to k_s^+ - i\mathcal{O}(Q)$ ➡ $k_s \sim (1, \lambda^2, \lambda)Q$ **Collinear region**

Works for both ERBL and DGLAP regions!

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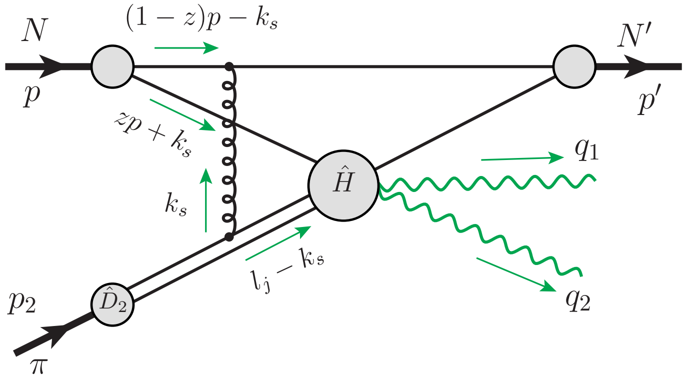
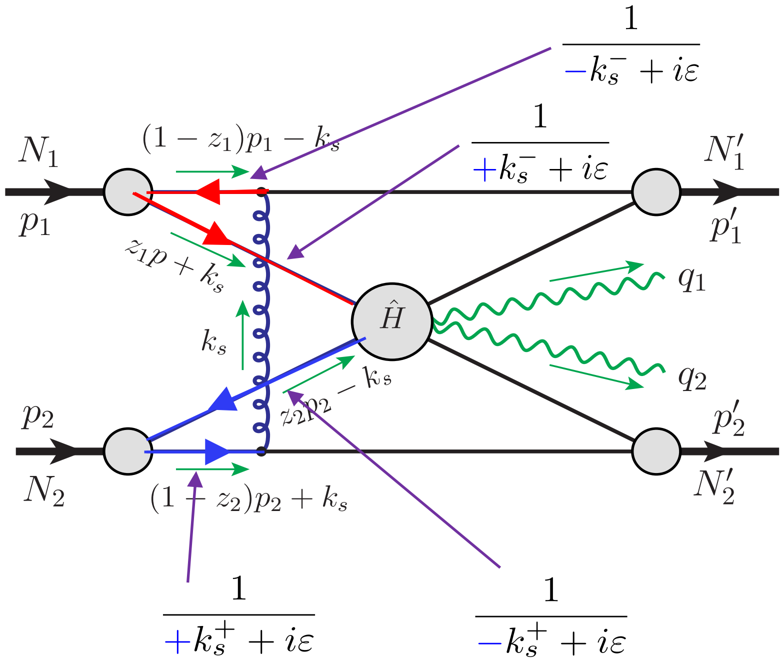
Why *single* diffractive?

- **Double diffractive process**

Glauber pinch for diffractive scattering



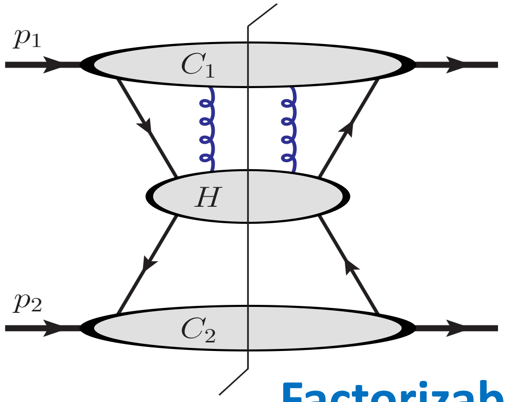


Both k_s^+ and k_s^- are pinched in Glauber region!

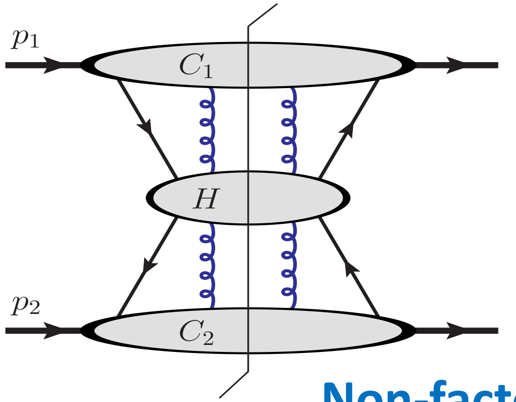
Break of factorization

Factorizable if all pion momentum flows into hard part

- **Compare: Drell-Yan process at high twist:**



Factorizable



Non-factorizable

Only the 1st sub-leading twist is factorizable!

Qiu & Sterman, NPB, 1991

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Numerical results

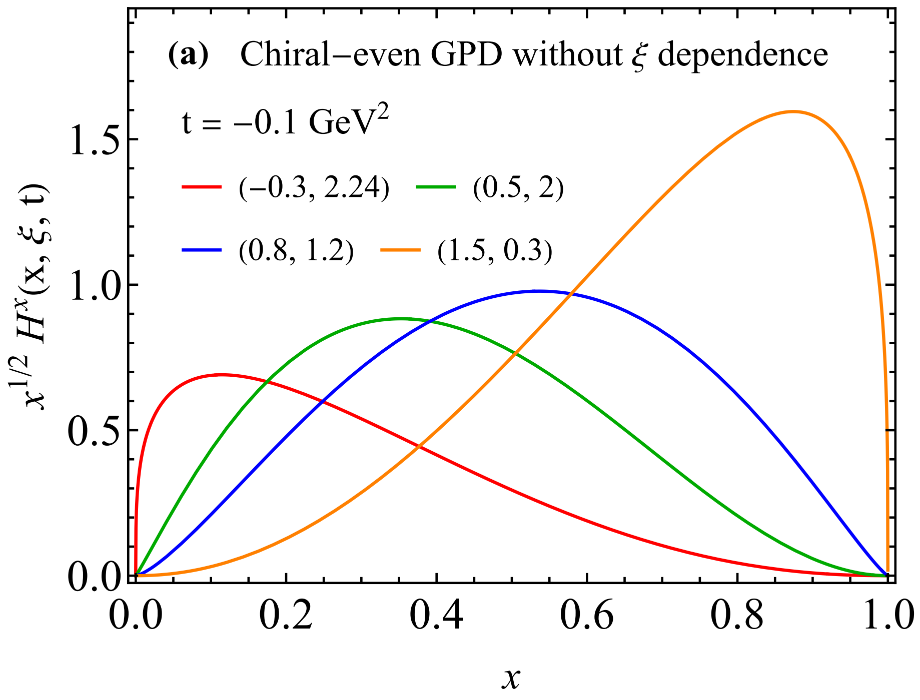
❑ GPD models – simplified GK model:

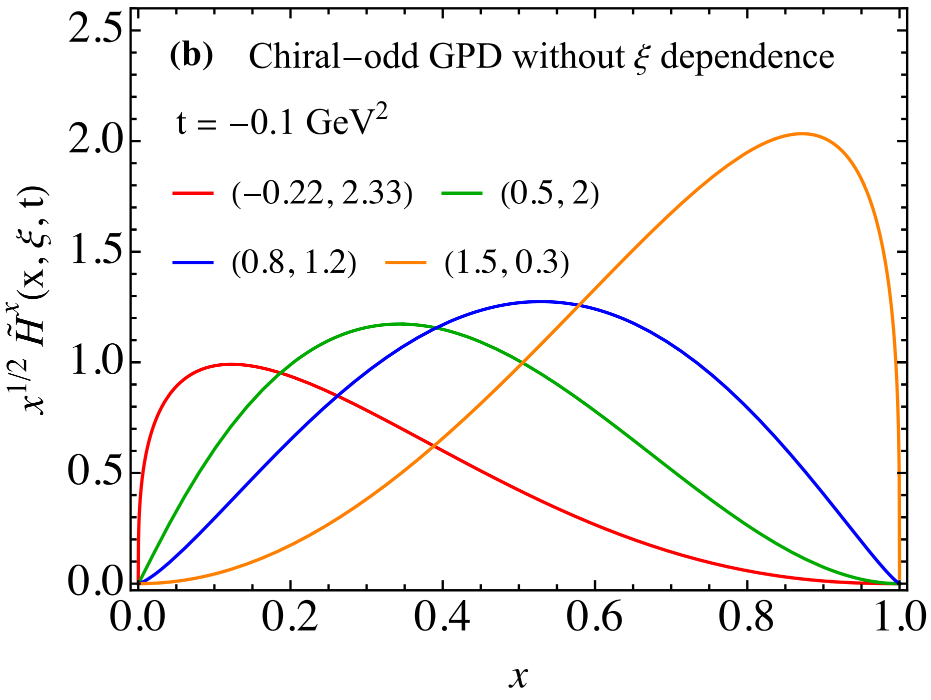
Goloskokov, Kroll
hep-ph/0501242
arXiv: 0708.3569
arXiv: 0906.0460

$$H_{pn}(x,\xi,t) = \theta(x)\, x^{-0.9\,(t/\mathrm{GeV}^2)}\, \frac{x^{\rho}(1-x)^{\tau}}{B(1+\rho,1+\tau)}$$

$$\widetilde{H}_{pn}(x,\xi,t) = \theta(x)\, x^{-0.45\,(t/\mathrm{GeV}^2)}\, \frac{1.267\, x^{\rho}(1-x)^{\tau}}{B(1+\rho,1+\tau)}$$

- Neglect $E, \widetilde{E}$. Neglect evolution effect.
- Tune (ρ, τ) to control x shape.
- Fix DA: $D(z) = N\, z^{0.63}(1-z)^{0.63}$





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Numerical results

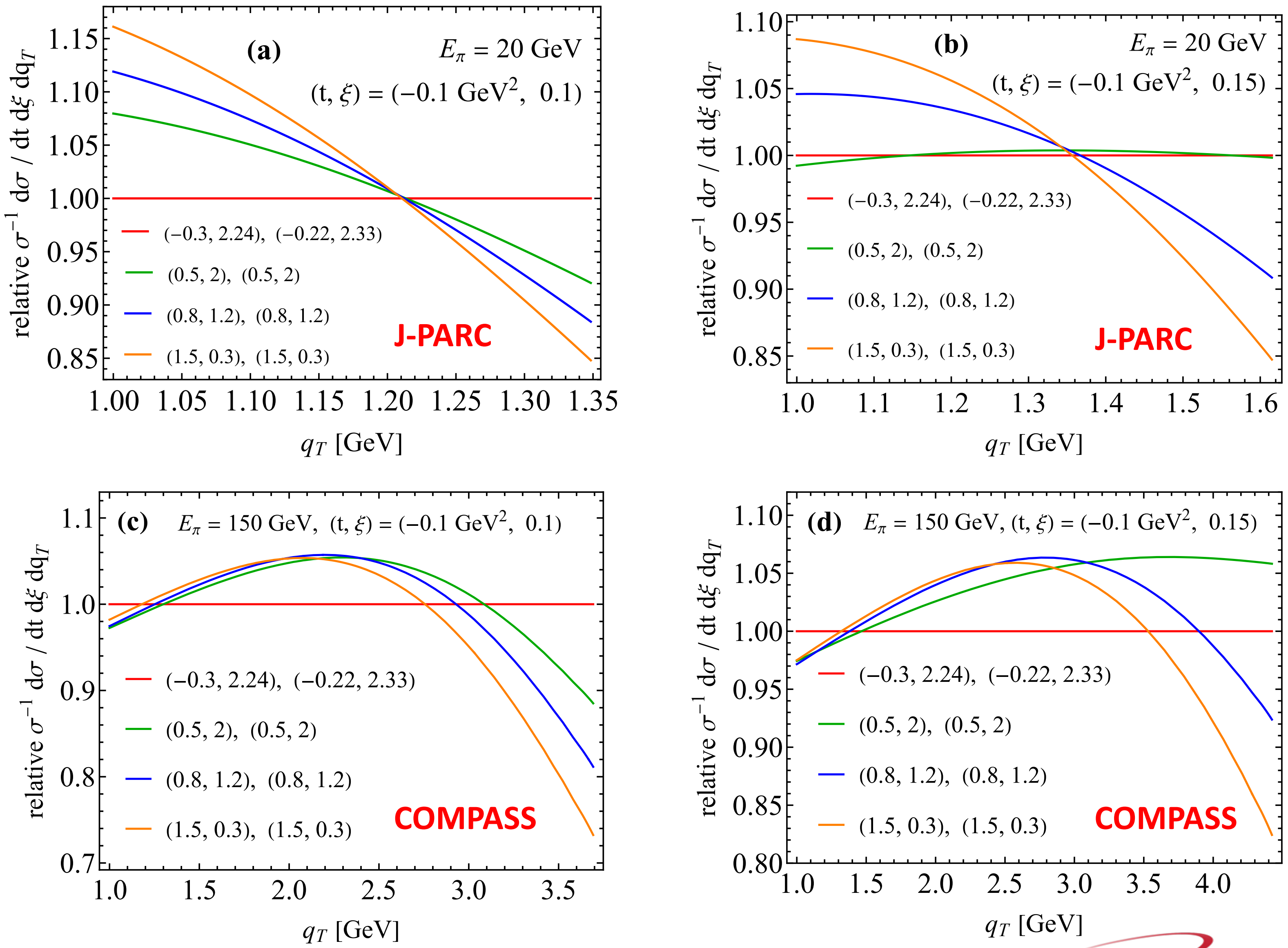
$$\frac{d\sigma}{dt\, d\xi\ dq_T} \sim |H(x, \xi, t)|^2$$

↓

Relative q_T shape

$$\frac{\sigma_{\rm tot}^{-1} d\sigma/dq_T}{\text{some shape func}}$$

$$\sigma_{\rm tot} = \int_{1\ \rm GeV}^{\sqrt{\hat{s}}/2} dq_T \frac{d\sigma}{dt d\xi dq_T}$$



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Exclusive Photo-Production of a π γ Pair – Hall D at JLab

- **Process:** $\gamma(p_\gamma) + h(p) \to \pi^\pm(q_1) + \gamma(q_2) + h'(p')$

  First introduced by G. Duplancic et al. [JHEP 11 (2018) 179],
  No contribution from gluon GPDs

- **Factorization:**

  Proved to be valid when $q_T \gg \sqrt{|t|} \gtrsim \Lambda_{\rm QCD}$

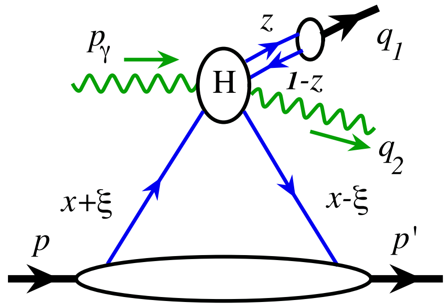
- **Polarization of photon and hadron:**

$$\frac{d\sigma}{d|t|\, d\xi\, d\cos\theta\, d\phi} = \frac{1}{2\pi} \frac{d\sigma}{d|t|\, d\xi\, d\cos\theta} \cdot \left[1 + \lambda_N \lambda_\gamma A_{LL} + \zeta A_{UT} \cos 2(\phi - \phi_S) + \lambda_N \zeta A_{LT} \sin 2(\phi - \phi_S)\right]$$

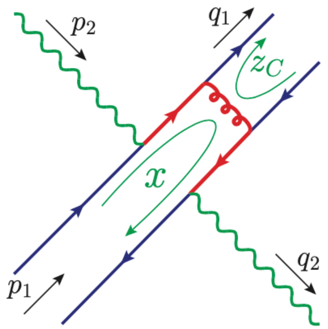
Unpolarized cross section:

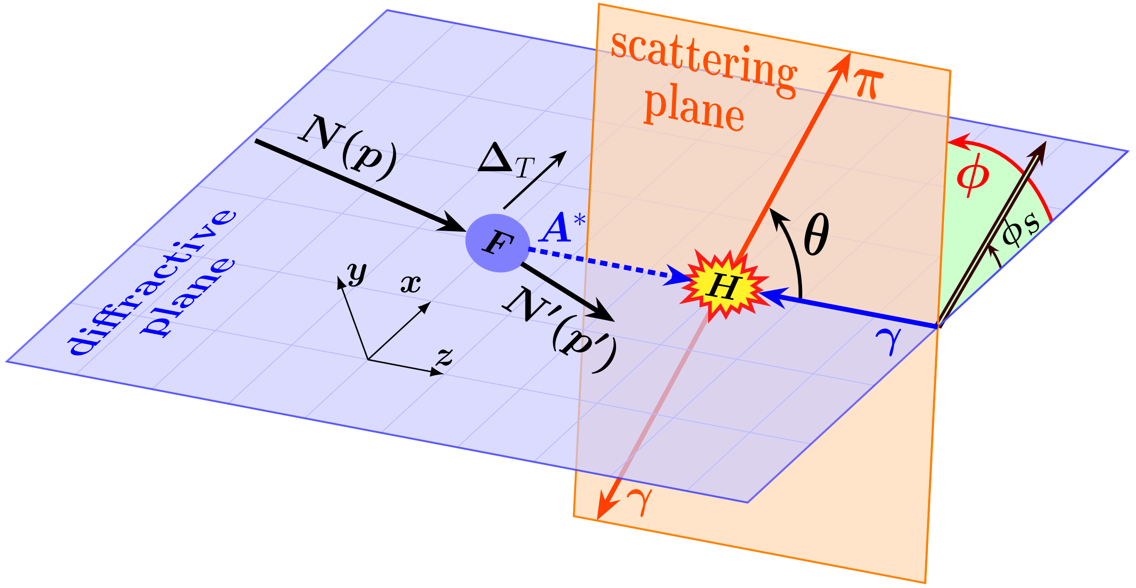
$$\frac{d\sigma}{d|t|\, d\xi\, d\cos\theta} = \frac{N^2 (1-\xi^2)}{32\, s\, (2\pi)^3 (1+\xi)^2} \Sigma_{UU}$$

$$\Sigma_{UU} = |\widetilde{C}_+^{[H]}|^2 + |\widetilde{C}_-^{[H]}|^2 + |C_+^{[\widetilde{H}]}|^2 + |C_-^{[\widetilde{H}]}|^2$$



$$\frac{d\sigma}{dt\,d\xi\,dq_T^2} \quad \text{or} \quad \frac{d\sigma}{dt\,d\xi\,d\cos\theta}$$





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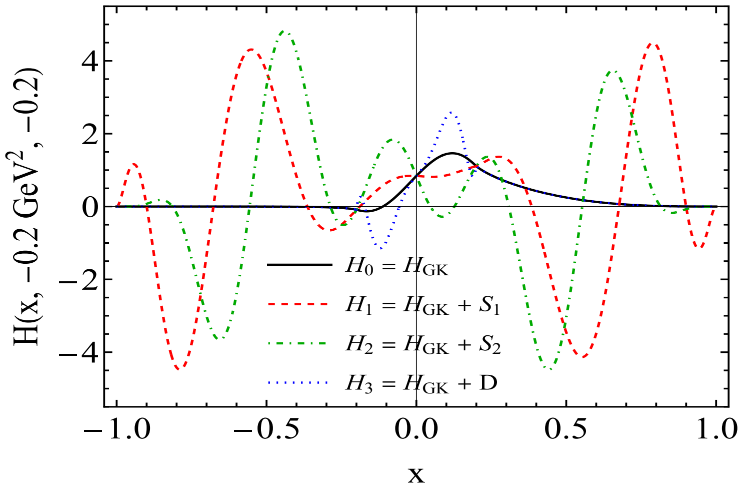
Exclusive $\pi^0\gamma$ Pair Production – Phenomenology

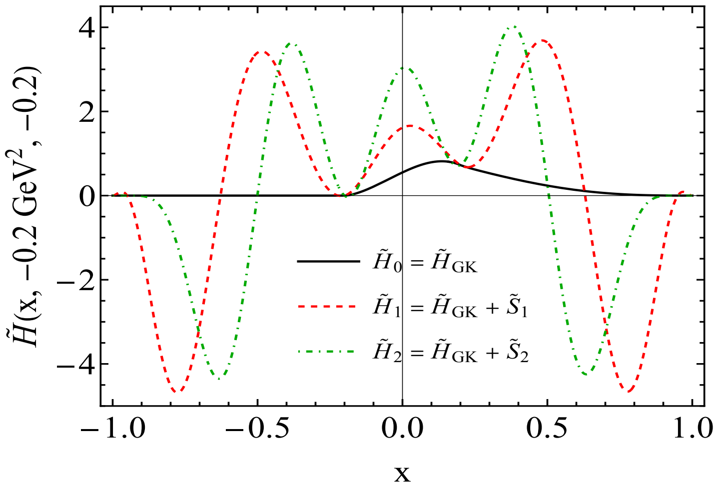
❑ Impact of shadow GPDs:

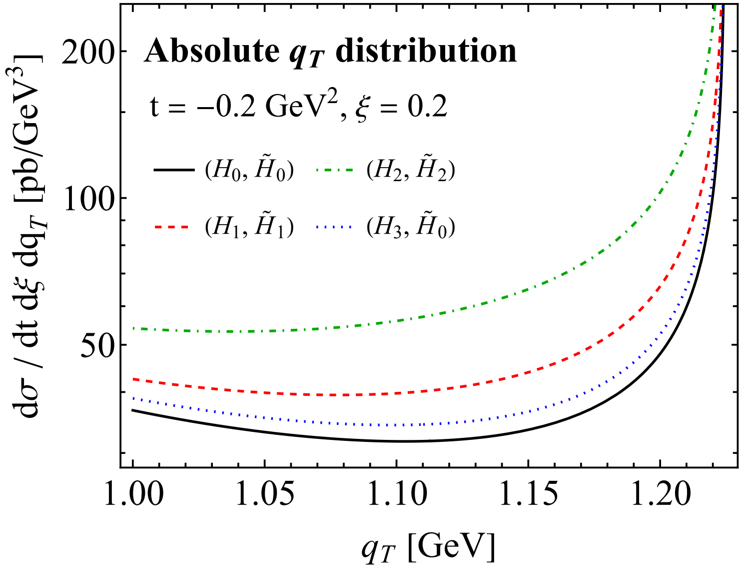
$$F(x,\xi,t) \to F(x,\xi,t) + S(x,\xi,t)$$

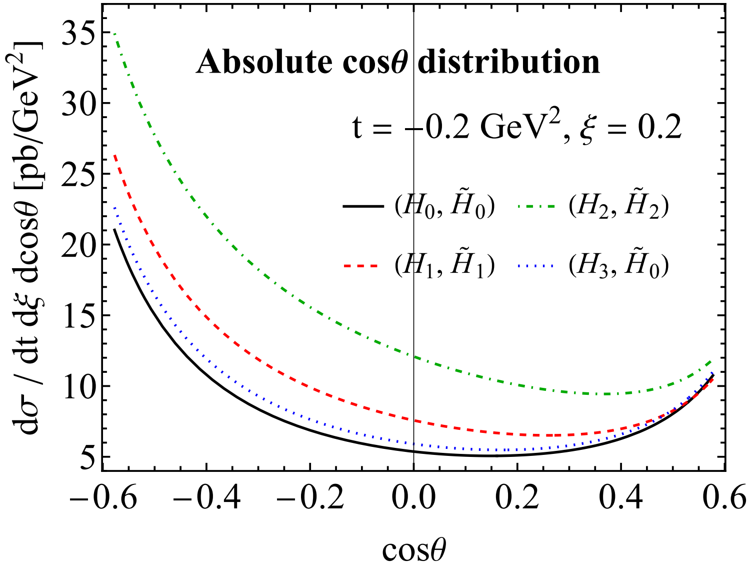
with

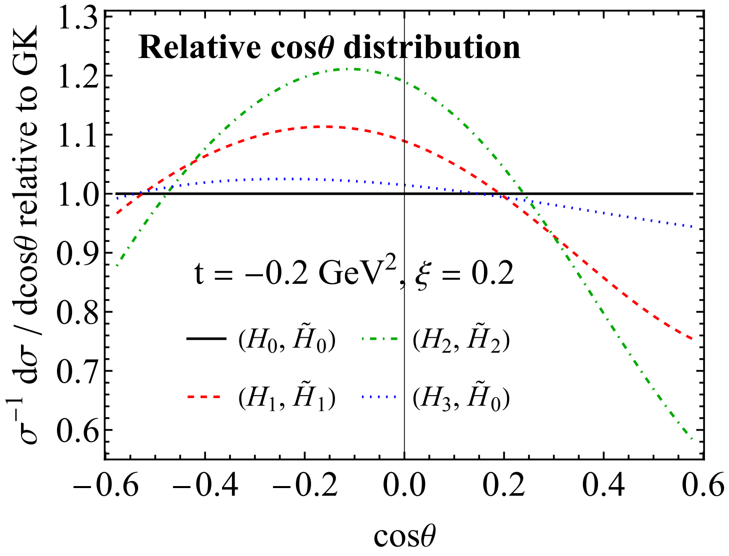
$$\int_{-1}^{1} \mathrm{d}x \, \frac{S(x,\xi,t)}{x-\xi+i\varepsilon} = 0$$





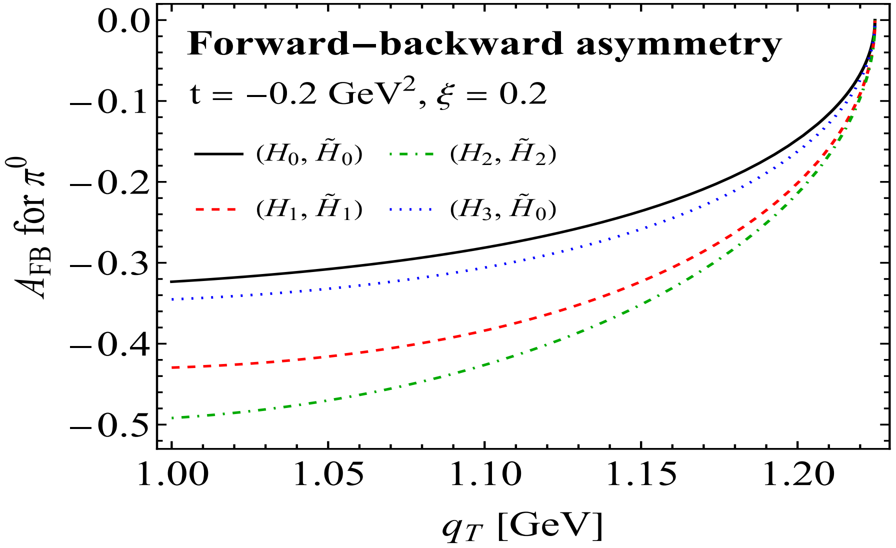


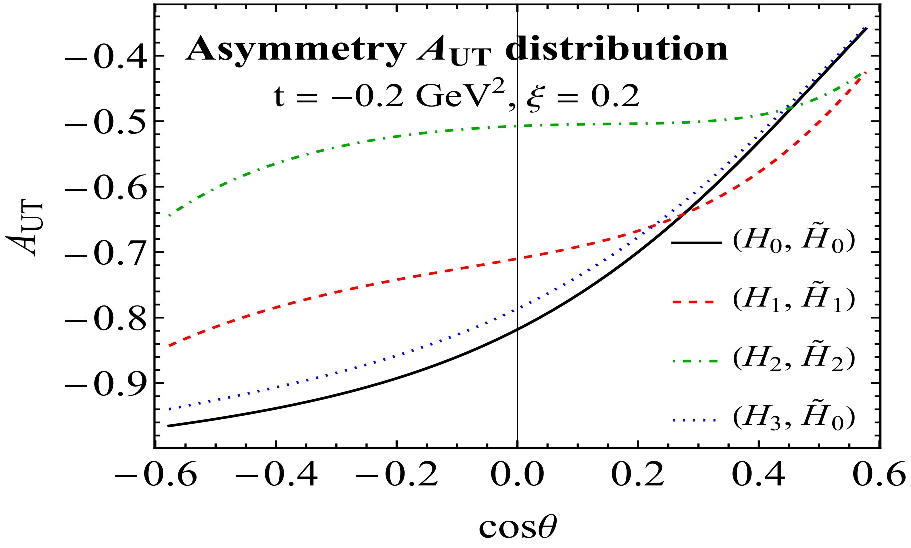


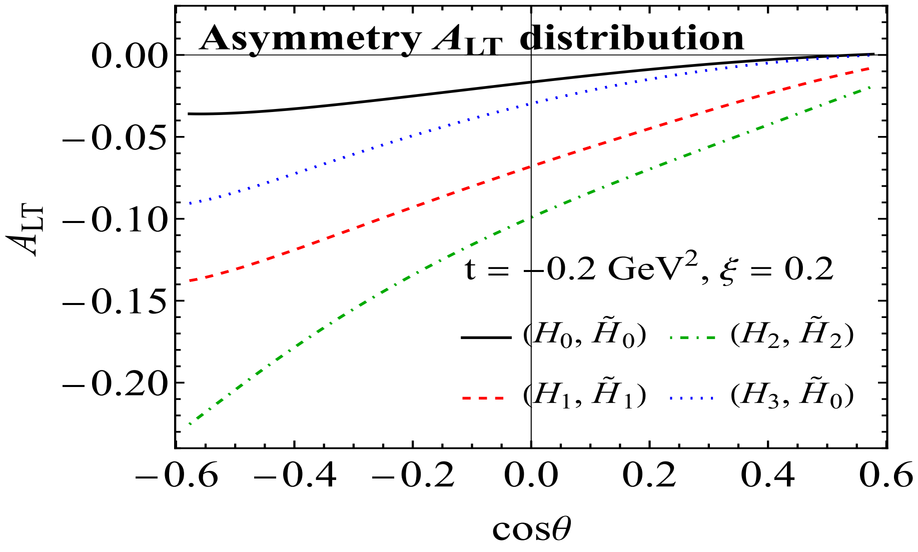


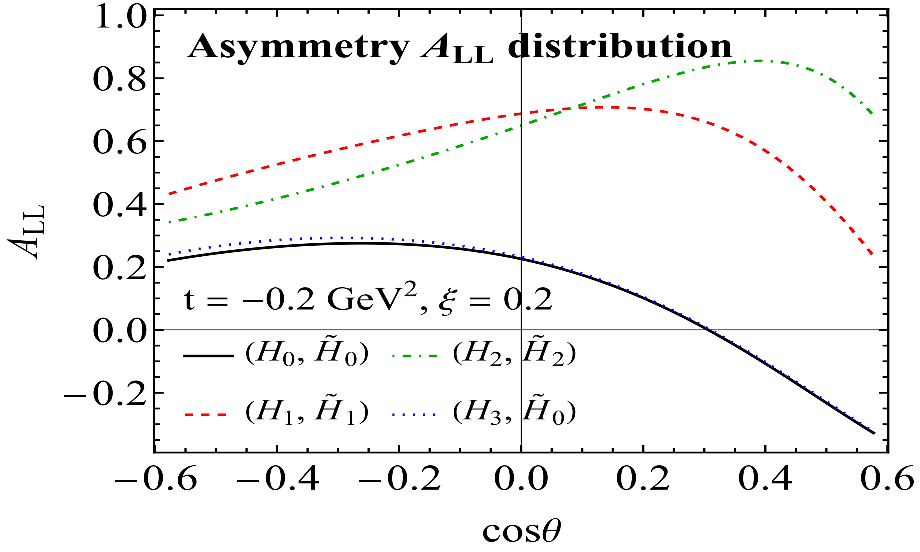
Exclusive $\pi^0\gamma$ Pair Production – Phenomenology

❑ Asymmetries:









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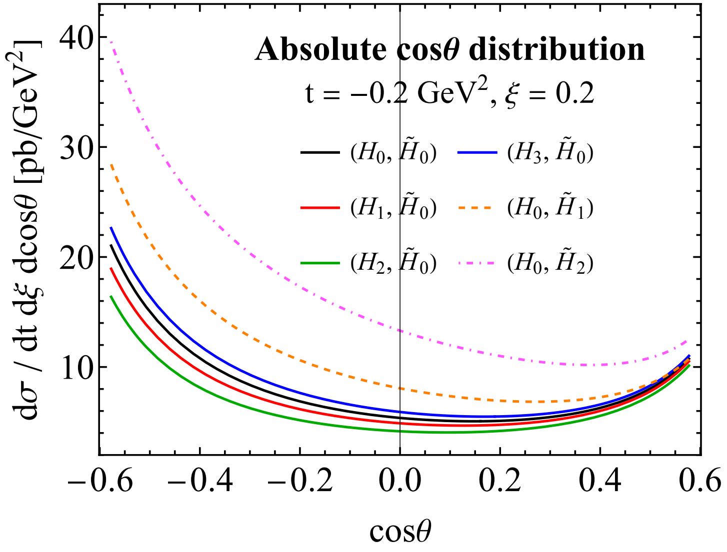
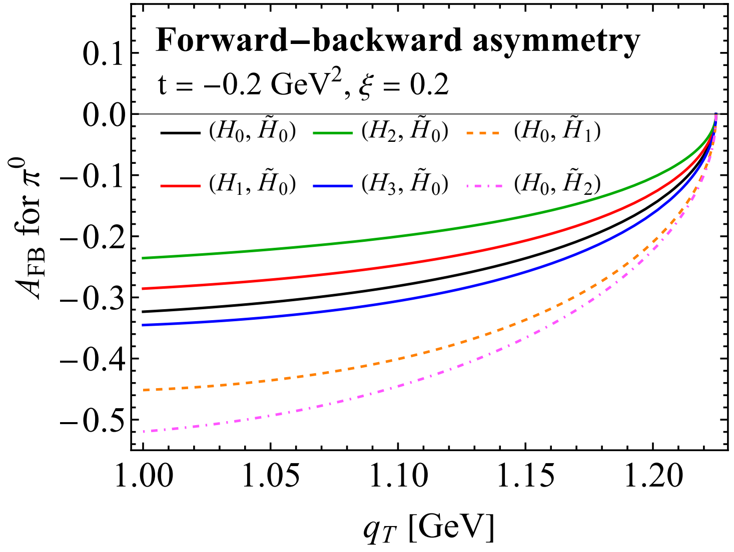
Exclusive $\pi^0\gamma$ Pair Production – Phenomenology

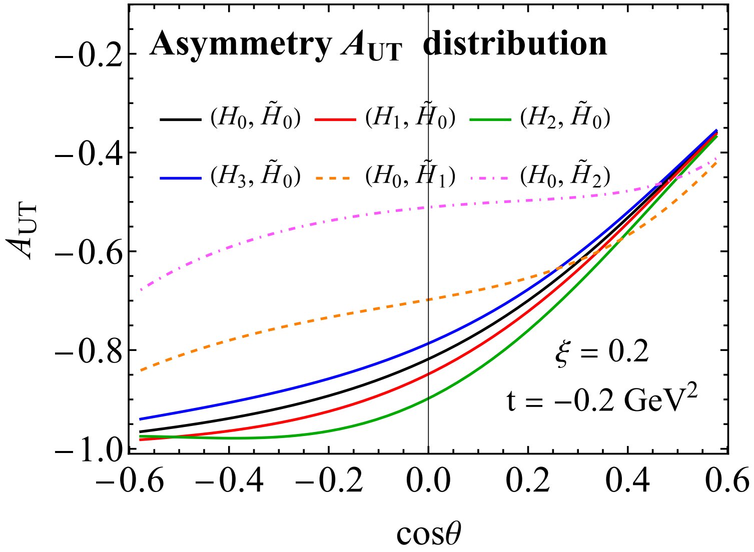
❑ Sensitivities on GPDs:

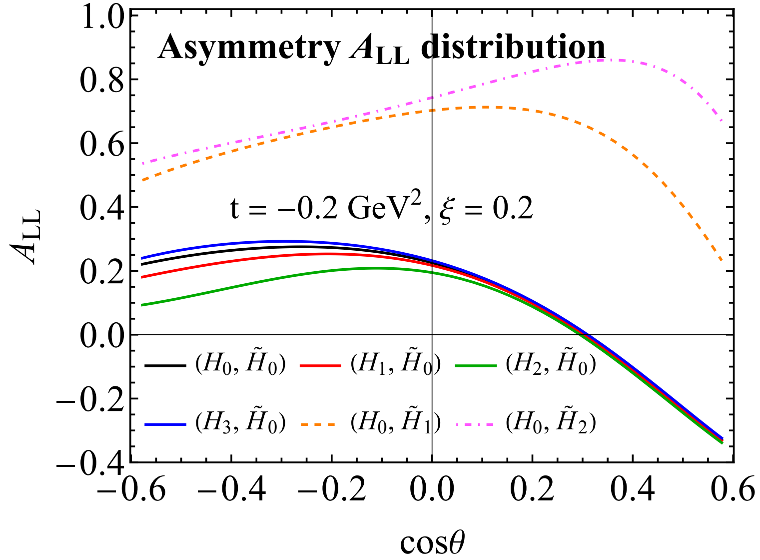
GPD models
= simplified GK model

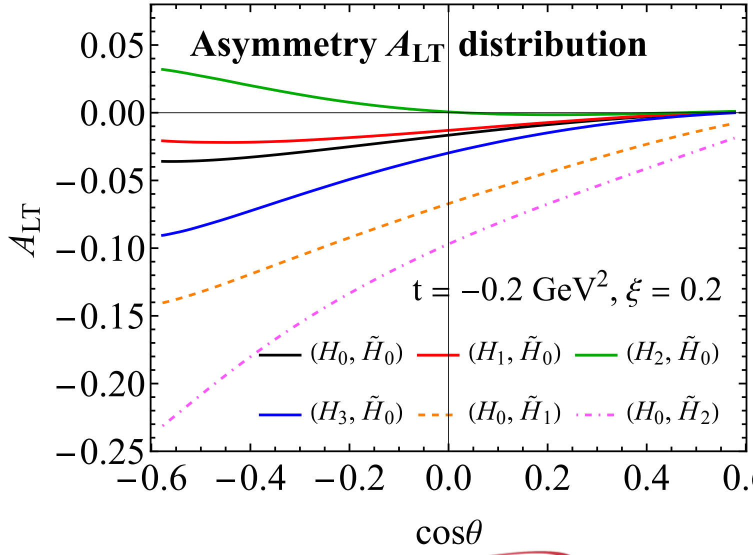
JLab Hall D is exploring these opportunities











Jefferson Lab

QCD factorization beyond the leading power

Lee, Qiu, Sterman, Watanabe, 2022

❑ Heavy quarkonium production at high P_T:

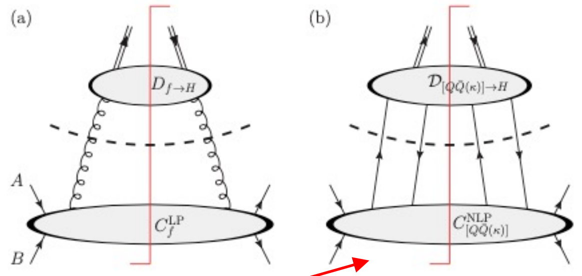
$$E\frac{d\sigma_{hh'\to J/\psi(P)X}}{d^3P} = \sum_{c\bar{c}[n]} F_{c\bar{c}[n]\to J/\psi} \otimes \sum_{a,b}\int dx_a\, f_{a/h}(x_a,\mu_f^2)\int dx_b\, f_{b/h'}(x_b,\mu_f^2)$$
$$\times\left[E\frac{d\tilde{\sigma}^{\rm Resum}_{ab\to c\bar{c}[n](P)X}}{d^3P} + E\frac{d\tilde{\sigma}^{\rm NRQCD}_{ab\to c\bar{c}[n](P)X}}{d^3P} - E\frac{d\tilde{\sigma}^{\rm Asym}_{ab\to c\bar{c}[n](P)X}}{d^3P}\right]$$

NRQCD:

$$F_{c\bar{c}[n]\to J/\psi} = \langle O^{J/\psi}_{c\bar{c}[n]}(0)\rangle$$
$$c\bar{c}[n] = c\bar{c}[^{2S+1}L_J^{[1,8]}]$$

- **PQCD factorization + FFs:** $\kappa = (v,a,t)^{[1,8]} = (\gamma^+, \gamma^+\gamma_5, \gamma^+\gamma_\perp^i)^{[1,8]}$

$$E\frac{d\tilde{\sigma}^{\rm Resum}_{ab\to c\bar{c}[n](P)X}}{d^3P} \approx \sum_f \int\frac{dz}{z^2} D_{f\to c\bar{c}[n]}(z,\mu_f^2) E_f \frac{d\hat{\sigma}_{ab\to f(p_f)X}}{d^3p_f}(z, p_f = P/z, \mu_f^2)$$
$$+ \sum_{[c\bar{c}(\kappa)]}\int\frac{dz}{z^2} D_{[c\bar{c}(\kappa)]\to c\bar{c}[n]}(z,\mu_f^2) E_c \frac{d\hat{\sigma}_{ab\to [c\bar{c}(\kappa)](p_c)X}}{d^3p_c}(z, p_c = P/z, \mu_f^2)$$



Kang, Ma, Qiu, Sterman, 2014

- **PQCD fixed-order:**

$$E\frac{d\tilde{\sigma}^{\rm NRQCD}_{ab\to c\bar{c}[n](P)X}}{d^3P}$$ **Known to NLO**

- **PQCD Asymptotic contribution:**

$$E\frac{d\tilde{\sigma}^{\rm Asym}_{ab\to c\bar{c}[n](P)X}}{d^3P} = E\frac{d\tilde{\sigma}^{\rm Resum}_{ab\to c\bar{c}[n](P)X}}{d^3P}\Bigg|_{\rm fixed\ order}$$

When $P_T \gg m_c$, $E\frac{d\tilde{\sigma}^{\rm Asym}_{ab\to c\bar{c}[n](P)X}}{d^3P}$ **cancels** $E\frac{d\tilde{\sigma}^{\rm NRQCD}_{ab\to c\bar{c}[n](P)X}}{d^3P}$

When $P_T \gtrsim m_c$, $E\frac{d\tilde{\sigma}^{\rm Asym}_{ab\to c\bar{c}[n](P)X}}{d^3P}$ **cancels** $E\frac{d\tilde{\sigma}^{\rm Resum}_{ab\to c\bar{c}[n](P)X}}{d^3P}$

Renormalization group improvement

Kang, Ma, Qiu, Sterman, PRD 90, 034006 (2014)

❑ Renormalization group:

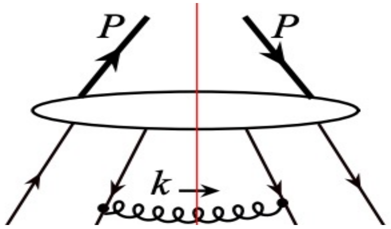
$$\frac{d}{d\ln\mu_f^2}\left[E\frac{d\tilde{\sigma}^{\rm Resum}_{ab\to c\bar{c}[n](P)X}}{d^3P}\right]=0$$

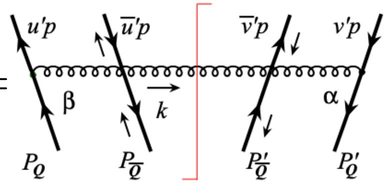
To be accurate up to the 1st power correction

❑ Modified evolution equations:

NRQCD: $H=c\bar{c}[^{2S+1}L_J^{[1,8]}]$

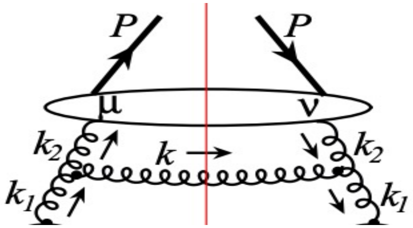
$$\frac{\partial\mathcal{D}_{[Q\bar{Q}(n)]\to H}}{\partial\ln\mu_f^2}=\Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}\otimes\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}$$



$\Gamma =$ 

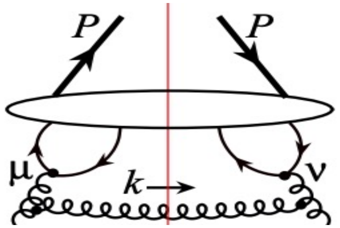
DGLAP-type: Heavy quark pair produced at the hard scale

$$\frac{\partial D_{[f]\to H}}{\partial\ln\mu_f^2}=\gamma_{[f]\to[f']}\otimes D_{[f']\to H}$$



Heavy quark pair produced at the input scale

$$+\frac{1}{\mu_f^2}\,\overline{\gamma}_{[f]\to[Q\bar{Q}(\kappa)]}\otimes\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}$$



$\longleftarrow\quad \overline{\gamma}_{g\to[Q\bar{Q}]}$

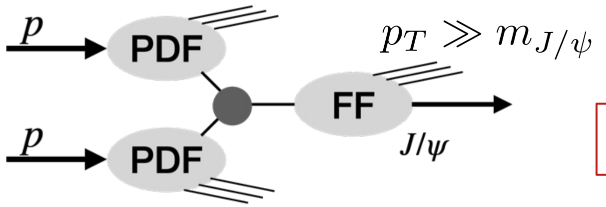
Heavy quark pair produced between the hard scale and the input scale

Modified DGLAP – inhomogeneous evolution

Single inclusive high P_T J/ψ-production in hadronic collisions

❑ Test the consistency:

$p + p \to J/\psi + X$



NLO

LO x K_{NLP}

$$\frac{d\sigma_{p+p\to J/\psi+X}}{dp_T} \approx f_{i/p} \otimes f_{j/p} \otimes \left[D_k^{J/\psi} \otimes C_{ij\to k} + D_{c\bar{c}}^{J/\psi} \otimes C_{ij\to c\bar{c}} \right]$$

❑ Input FFs from NRQCD:

Ma, Qiu, Zhang, PRD89 (2014) 094029; ibid. 94030

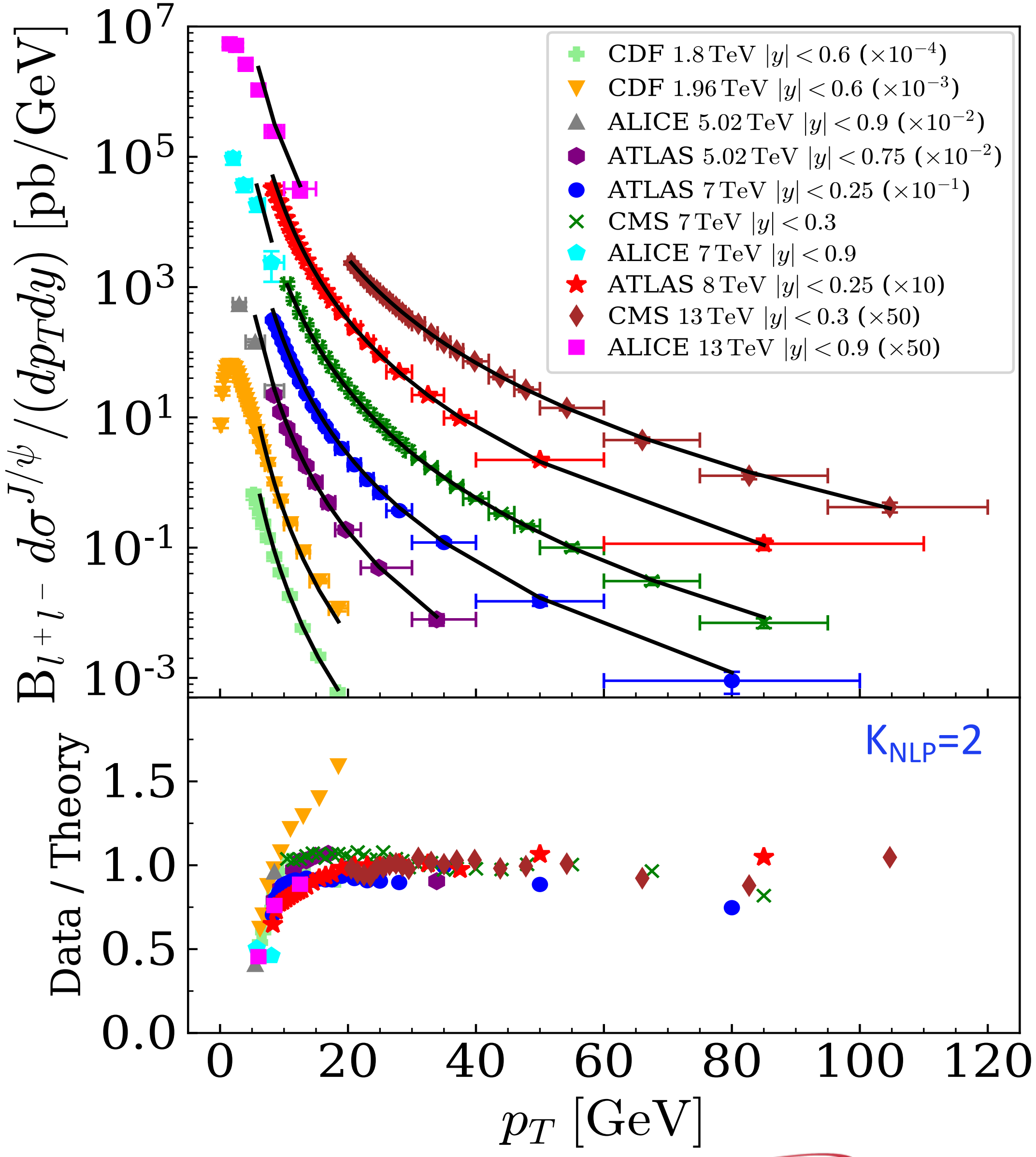
$$D_{f\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \pi\alpha_s \left\{ \hat{d}^{(1)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(2)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+3}}$$

$\kappa = v^{[c]}, a^{[c]}, t^{[c]}, \quad n = {}^{2S+1}L_J^{[c]}$

$$D_{[Q\bar{Q}(\kappa)]\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+1}}$$

$\mu_0 = \mathcal{O}(2m)$: input scale, $\mu_\Lambda = \mathcal{O}(m)$: NRQCD factorization scale

$$\Rightarrow \quad D_{f\to H}(z) = N_f \frac{z^{\alpha_f}(1-z)^{\beta_f}}{B(1+\alpha_f, 1+\beta_f)}$$



Lee, Qiu, Sterman, Watanabe, 2108.00305, 2211.12648

Matching to fixed-order PQCD calculation

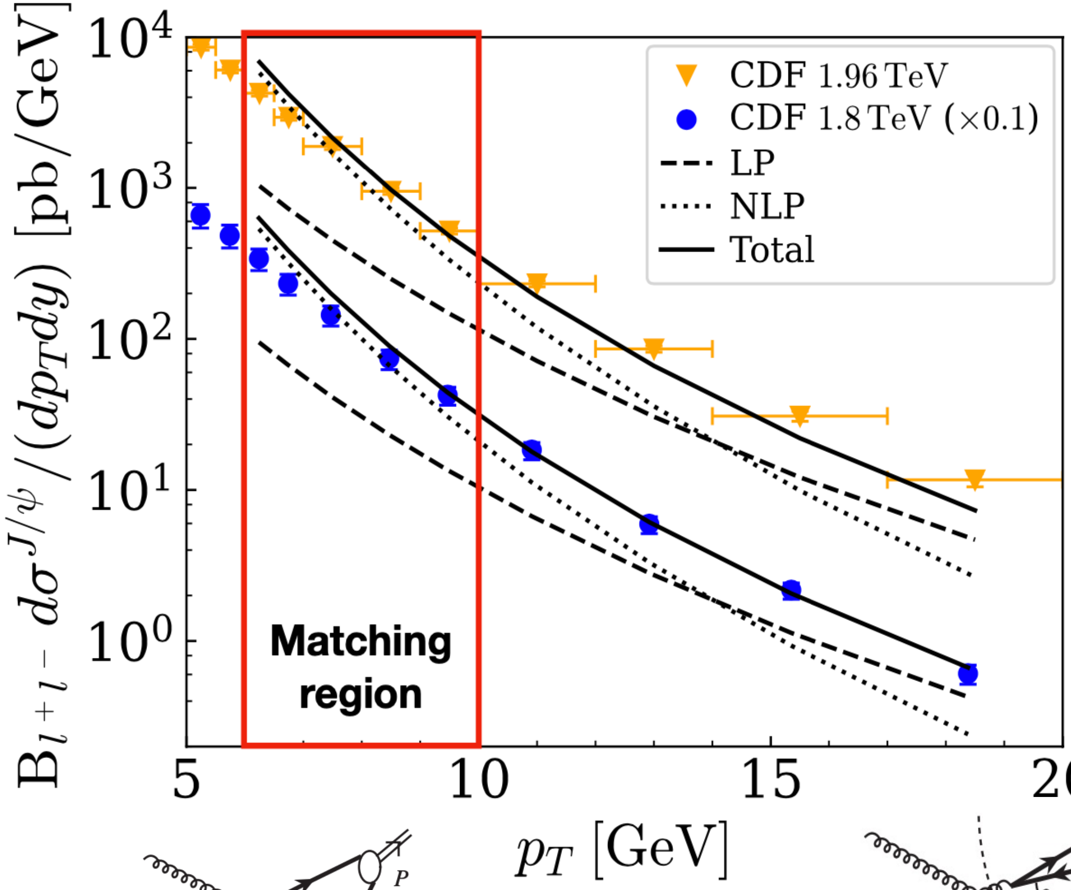
- ❑ **Leading power logarithmically enhanced contributions start to dominate when**
  $P_T \gtrsim 5(2m_c) \sim 15$ GeV

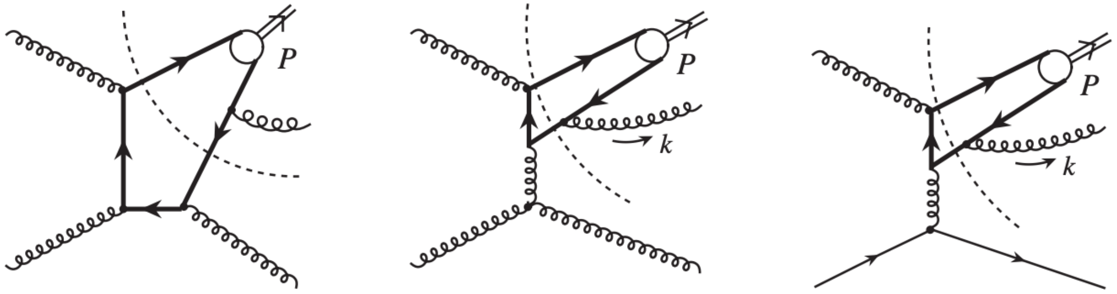
- ❑ **Next-to-leading power is important for**
  $5(2m_c) \gtrsim P_T \gtrsim (2m_c)$

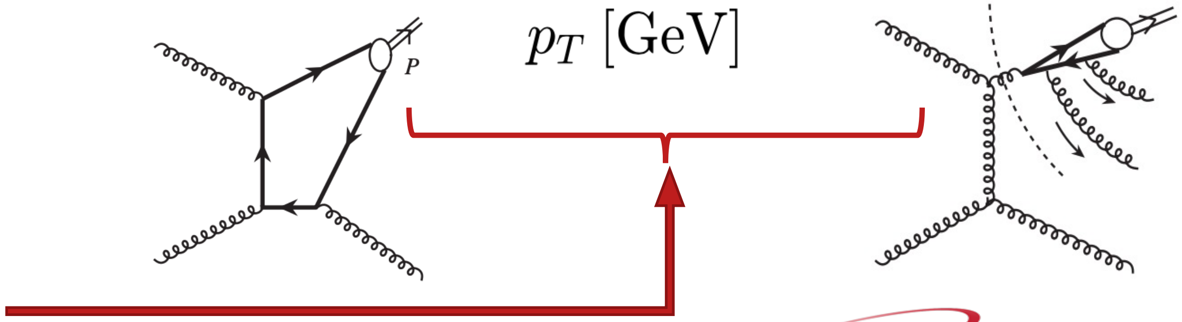
- ❑ **Matching to fixed-order NRQCD calculation**
  $P_T \sim (2m_c)$
  NLP term is necessary for the matching

- ❑ **Further improvement by exploring the FFs**
  Use the medium as a filter?

Lee, Qiu, Sterman, Watanabe, 2022





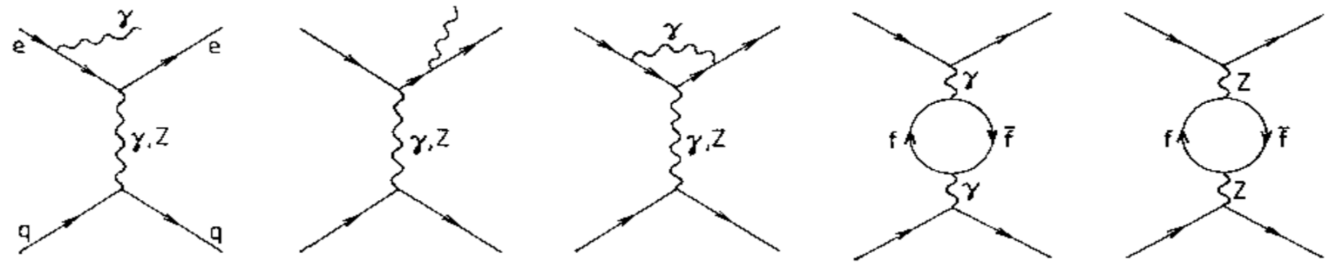


Jefferson Lab

Joint factorization beyond QCD – important for observables at EIC

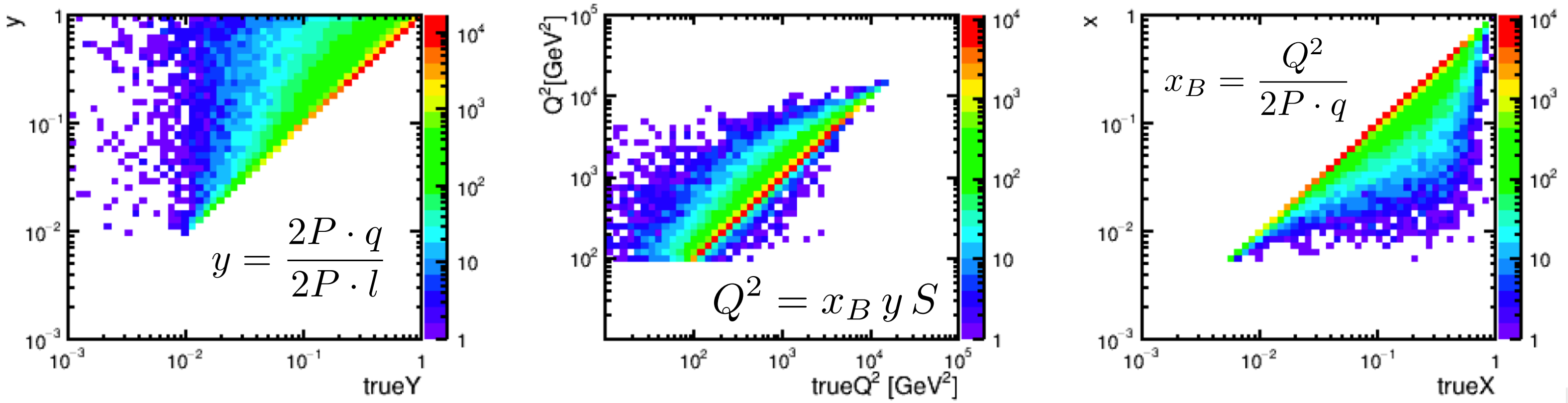
- **"Probe" for the hadron is smeared by the induced QED radiation:**

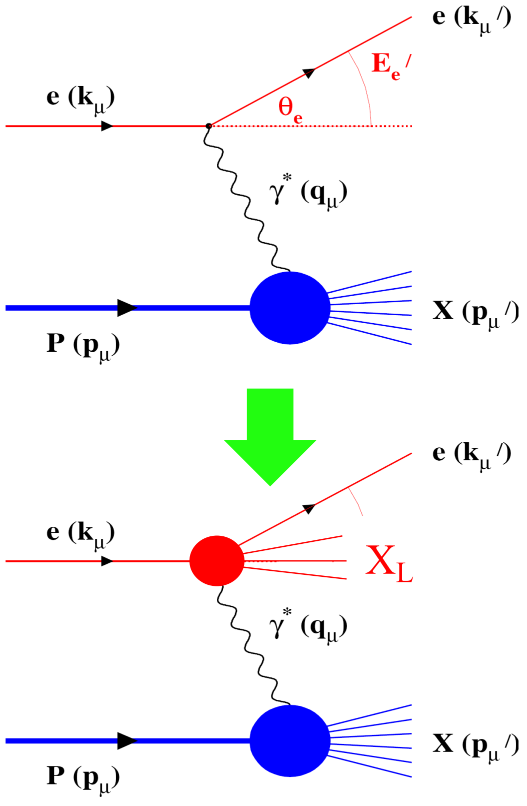
Data sample : Int L = 10 fb^{-1}, Kinematics settings: 0.01<y<0.95, 10^2 GeV^2 <Q^2<10^5 GeV^2



initial **final** **vacuum** **loops**

See Xiaoxuan Chu @2nd EIC YR workshop





$$q_\mu \to \hat{q}_\mu$$

$$Q^2 = -q^2 \to \widehat{Q}^2 = -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \to \hat{x}_B = \frac{\widehat{Q}^2}{2P\cdot \hat{q}}$$

Instead of a straight line – linear correlation,

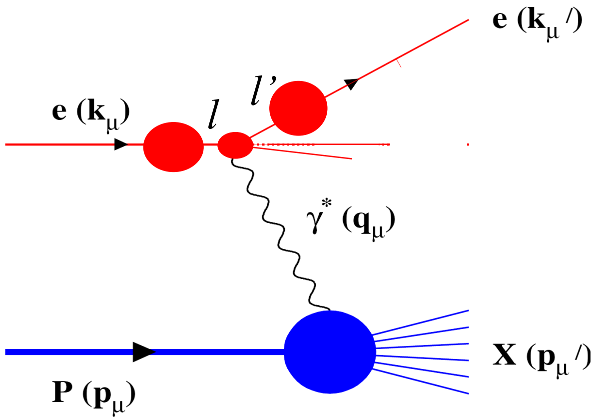
the kinematic variables, y, Q^2 , x_B, from the leptons are smeared so much to make them different from what the scattered "quark" experienced!

Ill-defined "photon-hadron" frame?!

Inclusive lepton-hadron deep inelastic scattering (DIS)

- **Collinear factorization with the "one-photon" approximation:**

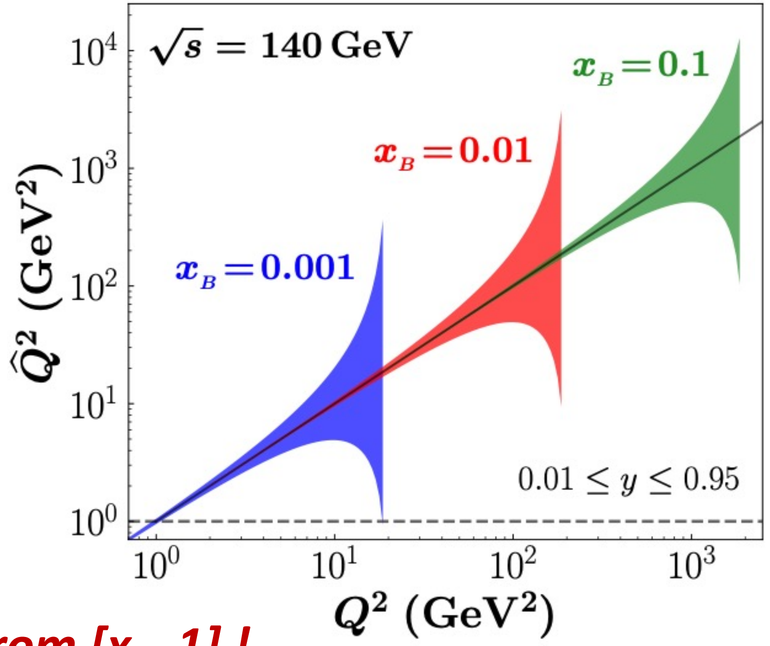
Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B\,dy} \approx \int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1} d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2)\left[\frac{Q^2}{x_B}\,\frac{\hat{x}_B}{\widehat{Q}^2}\right]$$
$$\times\frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2F_1(\hat{x}_B,\widehat{Q}^2) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

- QED radiation prevents a well-defined "photon-hadron" frame
- Radiation is CO sensitive as $m_e/Q \to 0$, factorized into LDFs & LFFs
- Hadron is probed by $(x_B, Q^2) \to (\hat{x}_B, \widehat{Q}^2)$

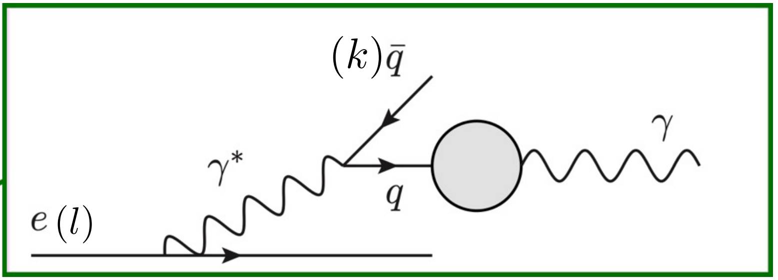
$$x_B \to \hat{x}_B \in [x_B, 1] \qquad \widehat{Q}^2_{\min} = Q^2\,\frac{(1-y)}{(1-x_B\,y)} \qquad \widehat{Q}^2_{\max} = Q^2\,\frac{1}{(1-y+x_B\,y)}$$



A simple RC factor at x_B is necessarily sensitive to hadronic information from [x_B, 1] !

Quantum evolution of LDFs

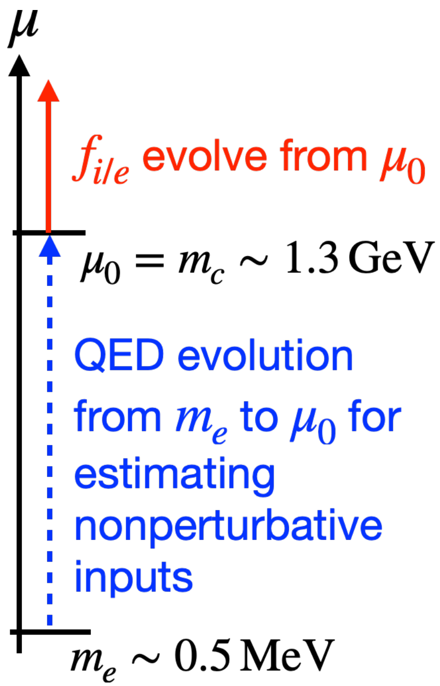
- **DGLAP evolution:** $\xi = \dfrac{k^+_{\text{activelepton(quark)}}}{l^+_{\text{lepton}}}$



QED part — Mixing part

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix} = \begin{pmatrix} P^{(1,0)}_{ee} & P^{(2,0)}_{e\bar{e}} & P^{(1,0)}_{e\gamma} & P^{(2,0)}_{eq} & P^{(2,0)}_{e\bar{q}} & P^{(2,1)}_{eg} \\ P^{(2,0)}_{\bar{e}e} & P^{(1,0)}_{\bar{e}\bar{e}} & P^{(1,0)}_{\bar{e}\gamma} & P^{(2,0)}_{\bar{e}q} & P^{(2,0)}_{\bar{e}\bar{q}} & P^{(2,1)}_{\bar{e}g} \\ P^{(1,0)}_{\gamma e} & P^{(1,0)}_{\gamma\bar{e}} & P^{(1,0)}_{\gamma\gamma} & P^{(1,0)}_{\gamma q} & P^{(1,0)}_{\gamma\bar{q}} & P^{(1,1)}_{\gamma g} \\ P^{(2,0)}_{qe} & P^{(2,0)}_{q\bar{e}} & P^{(1,0)}_{q\gamma} & P^{(0,1)}_{qq} & P^{(0,2)}_{q\bar{q}} & P^{(0,1)}_{qg} \\ P^{(2,0)}_{\bar{q}e} & P^{(2,0)}_{\bar{q}\bar{e}} & P^{(1,0)}_{\bar{q}\gamma} & P^{(0,2)}_{\bar{q}q} & P^{(0,1)}_{\bar{q}\bar{q}} & P^{(0,1)}_{\bar{q}g} \\ P^{(2,1)}_{ge} & P^{(2,1)}_{g\bar{e}} & P^{(1,1)}_{g\gamma} & P^{(0,1)}_{gq} & P^{(0,1)}_{g\bar{q}} & P^{(0,1)}_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix}$$

Mixing part — QCD part



Splitting functions in QED+QCD:

$$P_{ij}(\xi,\mu^2) = \sum_{n,m=0}^{\infty}\left(\frac{\alpha_{em}(\mu^2)}{2\pi}\right)^n \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^m \hat{P}^{(n,m)}_{ij}(\xi) \equiv \sum_{n,m=0}^{\infty} P^{(n,m)}_{ij}(\xi,\mu^2)$$

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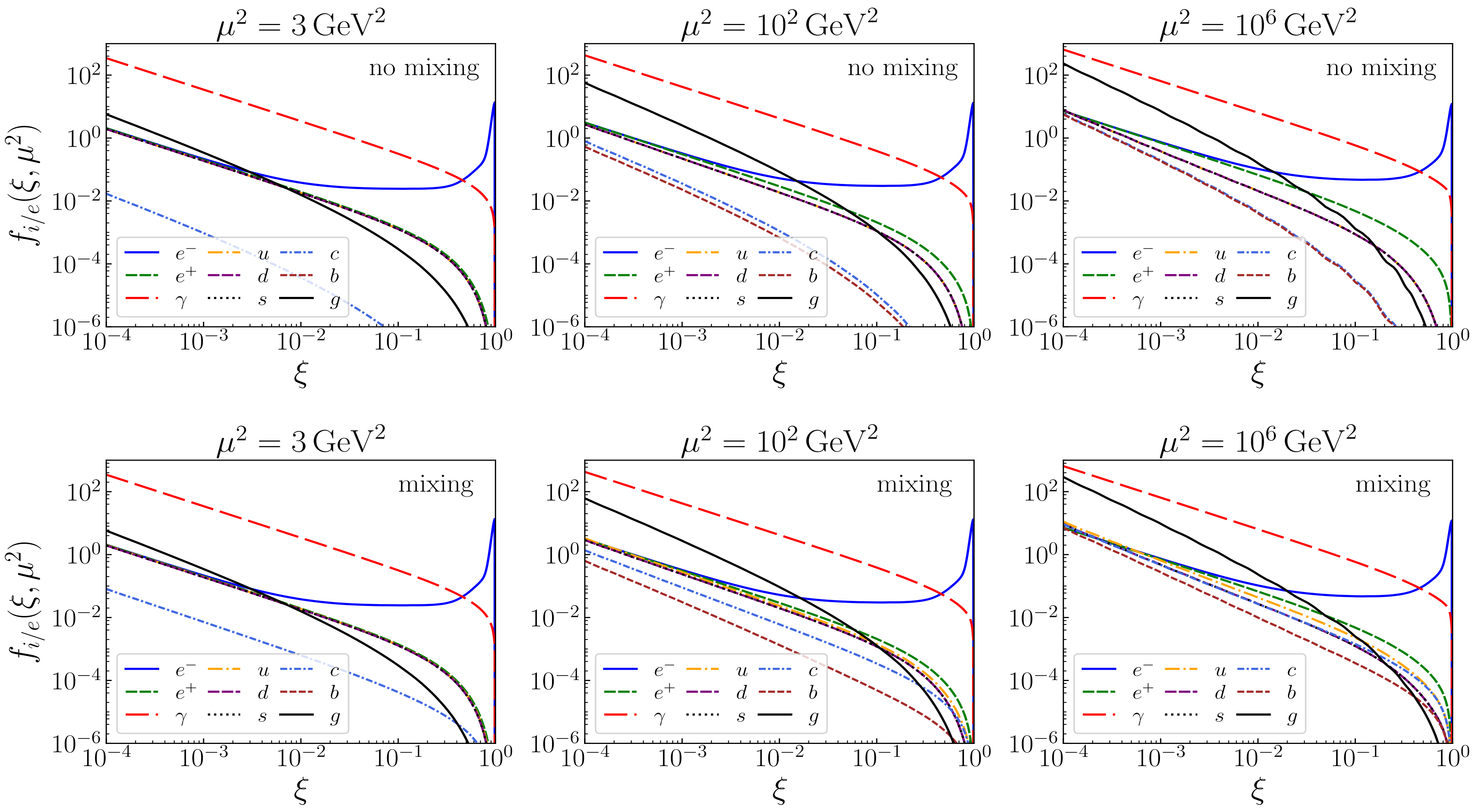
Lepton distribution functions (LDFs) after evolution

Qiu, Watanabe
in preparation



QED (QCD) evolution is slow (fast) due to the weak (strong) μ-dependence of $\alpha_{em}(\alpha_s)$

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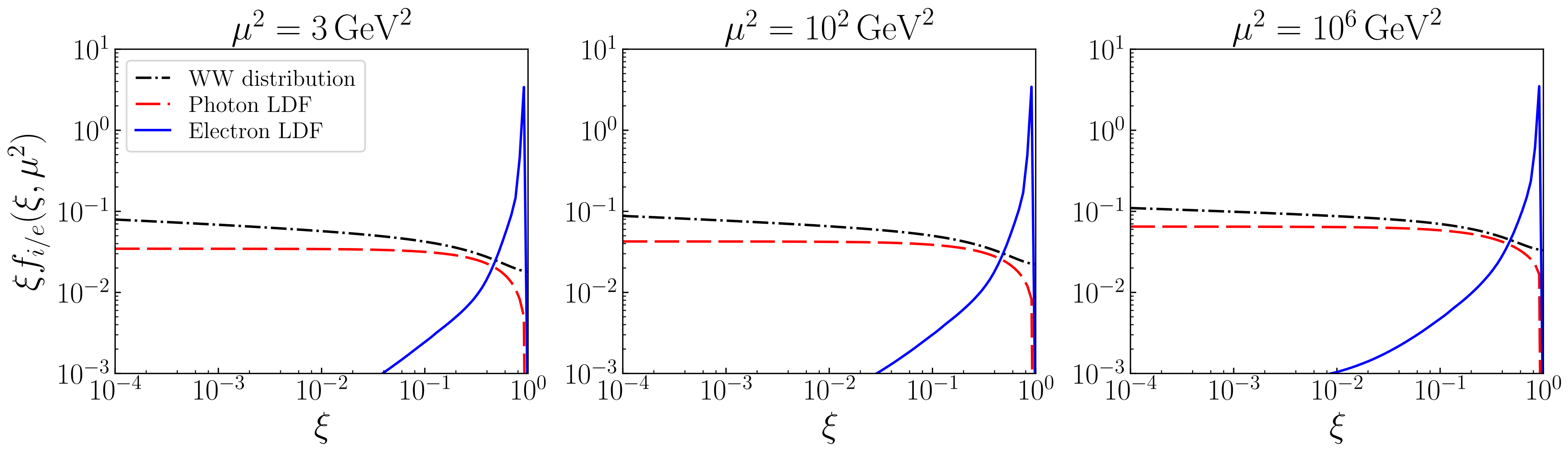
Photon LDF vs. Weizsäcker-Williams distribution

Qiu, Watanabe
in preparation



Weizäcker-Williams (WW) distribution at LO with $\overline{\mathrm{MS}}$-scheme: Hinderer, Schlegel, Vogelsang, PRD92, no.1, 014001 (2015)

$$f_{\gamma/l}^{WW}(\xi,\mu^2) = \frac{\alpha_{\rm em}}{2\pi} P_{\gamma l}(\xi) \left[\ln\left(\frac{\mu^2}{\xi^2 m_l^2}\right) - 1 \right] + \mathcal{O}(\alpha_{\rm em}^2)$$

- Photon LDF is smaller to WW distribution, but different because of the resummation of large logs, and higher-order corrections, such as $\gamma \to e^+e^-, q^+\bar{q}, \ldots$.
- Photon LDF depends on our purely QED evolution from m_e to μ_0; a global fitting could systematically improve the "red" dashed line.

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Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

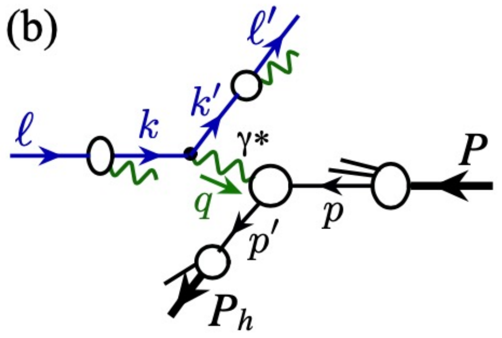
- **QED factorization of collision-induced radiation – collinear:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$E_{\ell'} E_{P_h} \frac{d^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{d^3\ell'\, d^3P_h} \approx \sum_{ij\lambda_k} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} D_{e/j}(\zeta) \int_{\xi_{\min}}^1 d\xi\, f_{i(\lambda_k)/e(\lambda_\ell)}(\xi) \left[E_{k'} E_{P_h} \frac{d^6\hat{\sigma}_{k(\lambda_k)P(S)\to k' P_h X}}{d^3k'\, d^3P_h} \right]_{k=\xi\ell, k'=\ell'/\zeta} + \mathcal{O}\left(\frac{m_e^n}{Q^n}\right)$$

- Leading power IR sensitive contribution is universal, as $m_e/Q \to 0$, factorized into LDFs and LFFs
- IR safe contributions are calculated order-by-order in powers of α
- Neglect m_e/Q power suppressed contributions
- Collinear QED factorization for both inclusive DIS and SIDIS, or e^+e^-, … [global fits of LDFs, LFFs]

- **"One photon"-approximation** ⟹ Hybrid factorization: CO for QED and TMD for QCD!



$$\frac{d^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{dx_B dy\, d\psi\, dz_h\, d\phi_h dP_{hT}^2} = \sum_{ij\lambda_k} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^1 \frac{d\xi}{\xi} f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\, D_{e/j}(\zeta)$$
$$\times \frac{\hat{x}_B}{x_B \xi\zeta} \left[\frac{\alpha^2}{\hat{x}_B\, \hat{y}\, \widehat{Q}^2} \frac{\hat{y}^2}{2(1-\hat{\varepsilon})} \left(1 + \frac{\hat{\gamma}^2}{2\hat{x}_B}\right) \sum_n \hat{w}_n F_n^h(\hat{x}_B, \widehat{Q}^2, \hat{z}_h, \widehat{P}_{hT}^2) \right]$$

Evaluated in a "virtual photon-hadron" frame

Apply a (ξ, ζ)-dependent Lorentz transformation:

$$\{\hat{q}, P, \widehat{P}_h\} \xrightarrow{(\xi,\zeta)} \{q, P, P_h\}$$

In a frame to compare with exp. measurements

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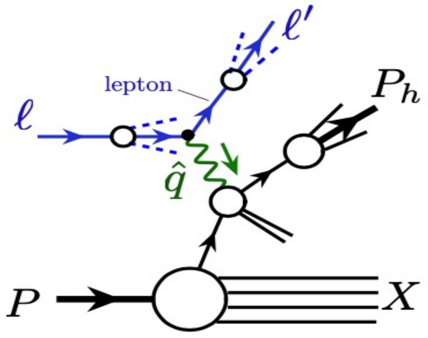
Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

❑ Two-step approach to SIDIS:



One-photon approximation

1) In "virtual-photon" frame, defined by $\hat{q}(\xi, \zeta) - p$

- TMD factorization when $\widehat{P}_T^2 \ll \widehat{Q}^2$
- CO factorization when $\widehat{P}_T^2 \sim \widehat{Q}^2$
- Matching to get the $\widehat{P}_T$-distribution

2) Lorentz transformation from the "virtual-photon" frame to any experimentally defined frame
– lepton-hadron Lab frame, Breit frame (x_B,Q^2), …

QED contribution (not correction) can be systematically improved order-by-order in power α!

❑ Case study F_{UU}:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = {} & \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{ \boxed{F_{UU,T}} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} \\
& + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, F_{LU}^{\sin\phi_h} \\
& + S_\parallel\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h}\Big] \\
& + S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\, F_{LL}^{\cos\phi_h}\Big] \\
& + |\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) \\
& \quad + \varepsilon\,\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\, F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
& + |\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\, F_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

- Case study F_{UU}:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$\frac{d\sigma^h_{\rm SIDIS}}{dx_B dy\, dz\, dP^2_{hT}} = \int_{\zeta_{\rm min}}^1 d\zeta \int_{\xi_{\rm min(\zeta)}}^1 d\xi\, D_{e/e}(\zeta)\, f_{e/e}(\xi) \times \left[\frac{\hat{x}_B}{x_B\, \xi\zeta}\right] \left[\frac{(2\pi)^2\, \alpha}{\hat{x}_B \hat{y}\, \widehat{Q}^2} \frac{\hat{y}^2}{2(1-\hat{\varepsilon})} F^h_{UU}(\hat{x}_B, \widehat{Q}^2, \hat{z}, \widehat{P}_{hT})\right]$$

Evaluated in a “virtual photon-hadron” frame

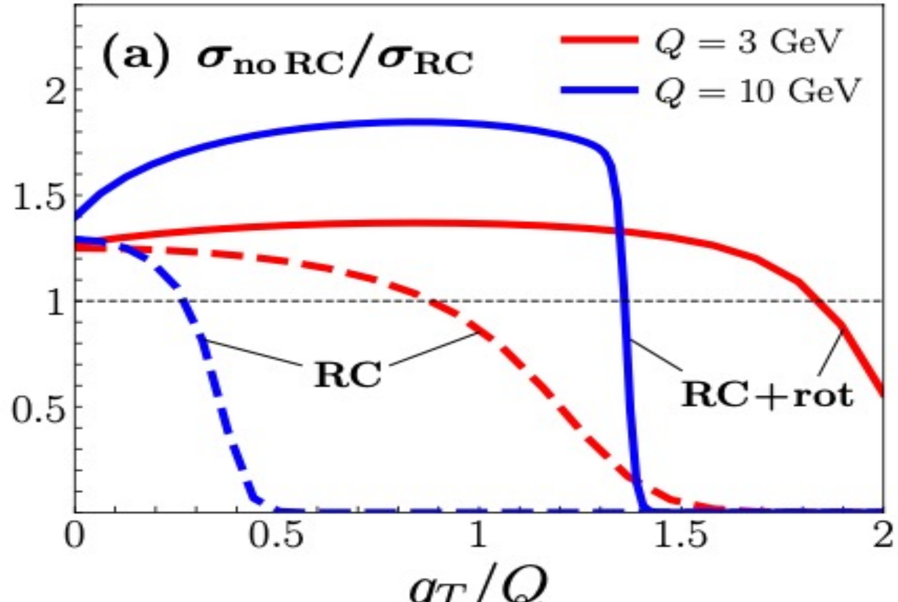
Unpolarized structure function:

$$F^h_{UU} = x_B \sum_q e_q^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{q}_T) \times f_{q/N}(x_B, \boldsymbol{p}_T^2)\, D_{h/q}(z, \boldsymbol{k}_T^2) \qquad \boldsymbol{q}_T = \boldsymbol{P}_{hT}/z$$

(ξ, ζ) - Dependent Lorentz transformation

Effectively, a rotation in hadron-rest frame

- Solid – with Lorentz transformation
- Dashed – without Lorentz transformation

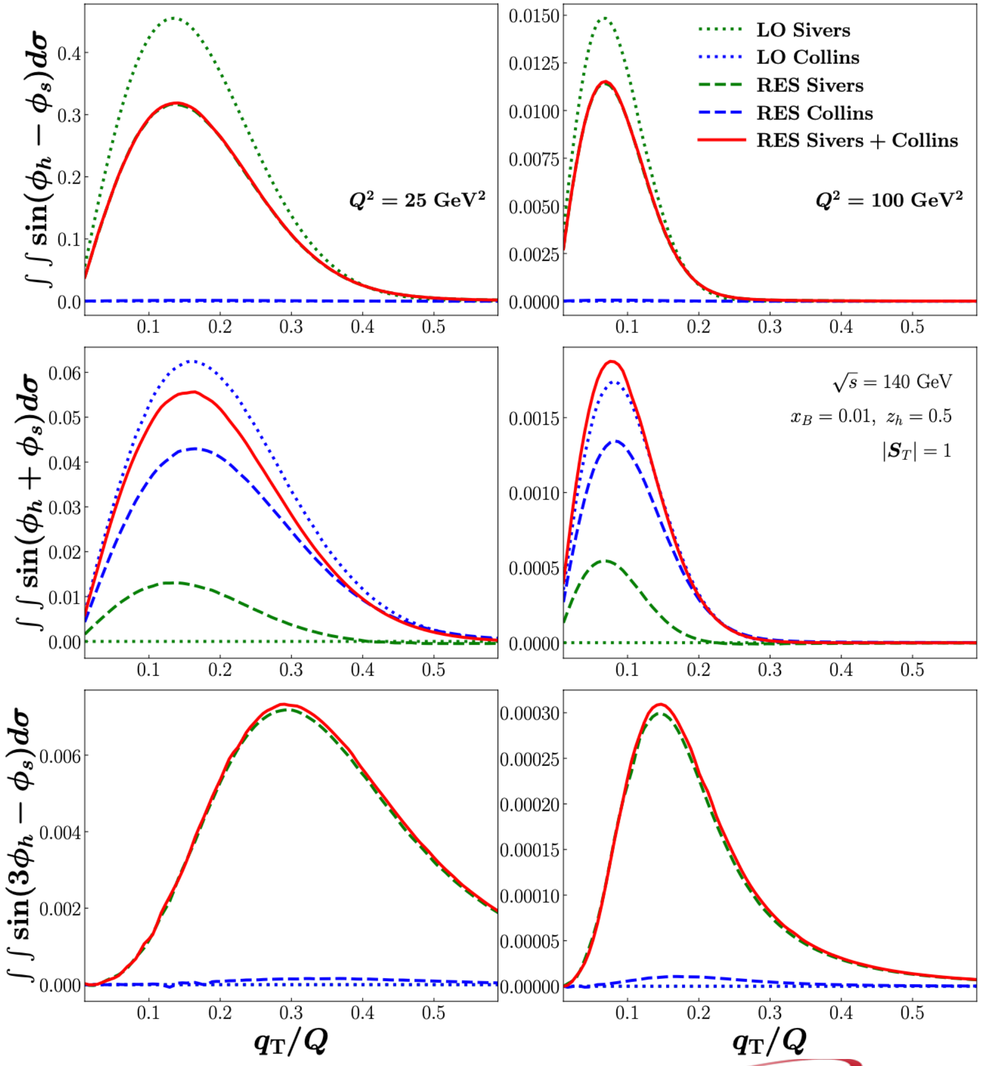


Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

- **Case study – single transverse spin asymmetry:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&\quad+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\quad+S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&\quad+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\qquad+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$



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Summary and Outlook

- Reliable factorization is necessary for probing QCD dynamics with identified hadrons(s)
  - Need for exploring QCD dynamics
  - Need for probing hadron's internal structure
- QCD factorization beyond the leading power is important and necessary
  - It is necessary for heavy quarkonium production where a heavy quark-pair is required
  - It is also necessary for better understanding of QCD multiple scattering (not discussed in this talk)
  - New form of evolution equations and modified scale dependence
- Joint factorization between QCD and QED is critical for the EIC and high energy lepton-hadron facilities
  - QED radiation is a part of production cross sections, treated in the same way as QCD radiation from quarks and gluons
  - No artificial and/or process dependent scale(s) introduced for treating QED radiation, other than the standard factorization scale, universal lepton distribution and fragmentation functions
  - All perturbatively calculable hard parts are IR safe for both QCD and QED
  - All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)

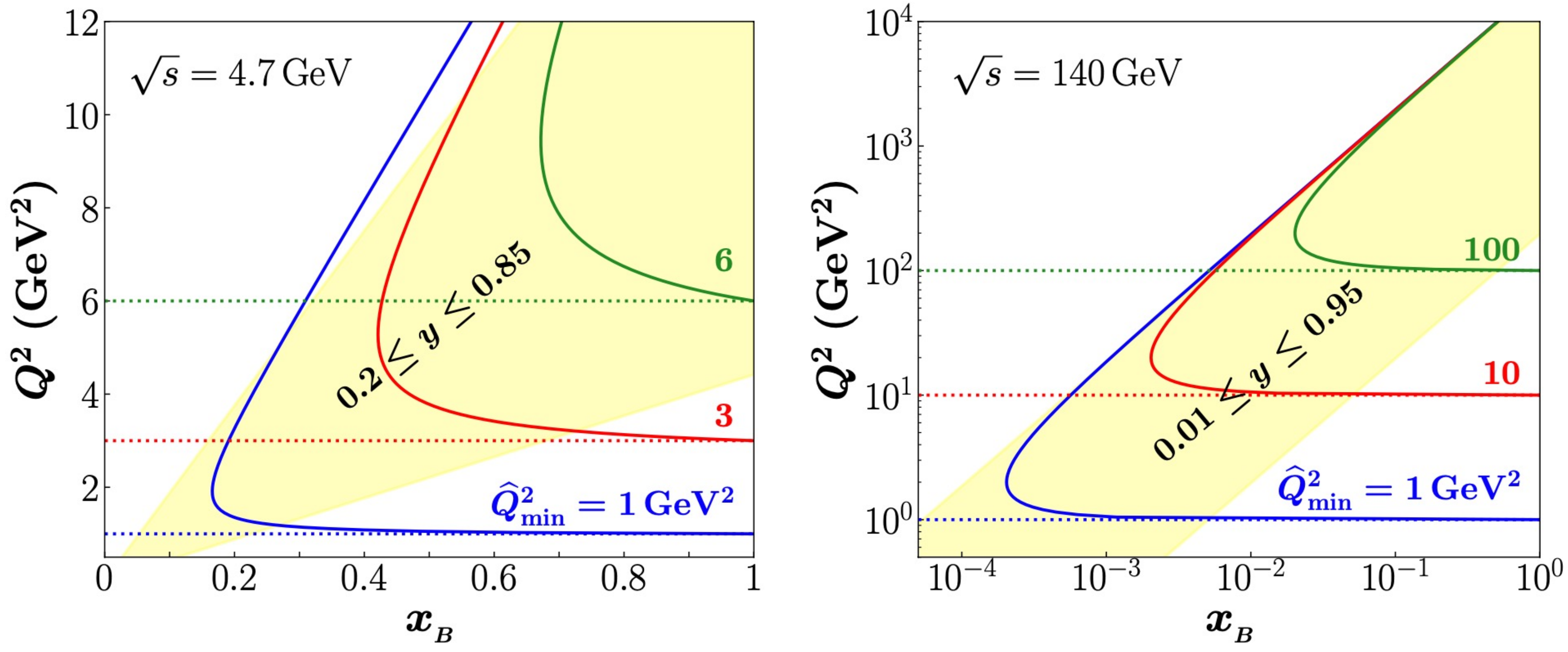
Thank you!

Inclusive lepton-hadron deep inelastic scattering (DIS)

- QED radiation effectively reduces the reach of the "hard" probe:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$x_B \to \hat{x}_B \in [x_B, 1] \qquad \widehat{Q}^2_{\min} = Q^2 \frac{(1-y)}{(1-x_B\, y)} \qquad \widehat{Q}^2_{\max} = Q^2 \frac{1}{(1-y+x_B\, y)}$$



For example, for $Q^2 > 3$ GeV^2, amount of the reach to the small-x regime is significant (red curves)!

Smaller x, more phase space for radiation, both QCD and QED!