Lattice QCD for Neutrino Oscillation

Aaron S. Meyer (asmeyer.physics@gmail.com)

UC Berkeley/LBNL

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S@INT Seminar

Outline

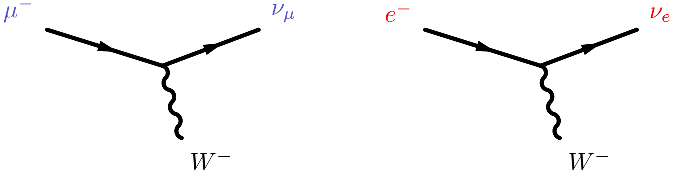
- Introduction
- Quasielastic Scattering from Experiment
  - Constraints from Deuterium Scattering
- $F_A(Q^2)$ from LQCD
  - CalLat Calculation
  - Excited States
- LQCD Survey of $F_A(Q^2)$
  - Summary of $F_A(Q^2)$ Calculations
  - T2K/DUNE Implications
- Future Directions

Note: all references in online slides are hyperlinked

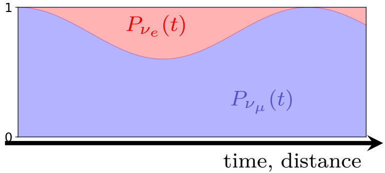
Neutrino Oscillation

Neutrino Oscillation (in a slide)

Neutrino flavor defined by charged lepton produced under weak interaction

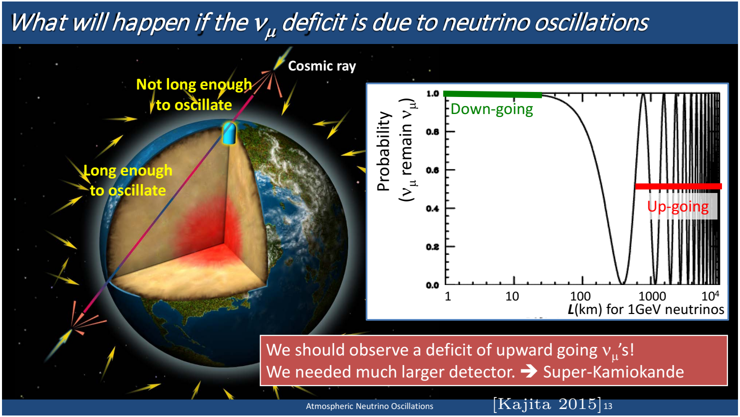


ν flavor changes spontaneously during near light-speed propagation: $\nu_\mu \rightarrow \nu_e$



Want to understand the mechanics governing this oscillation

Discovery of Neutrino Oscillation



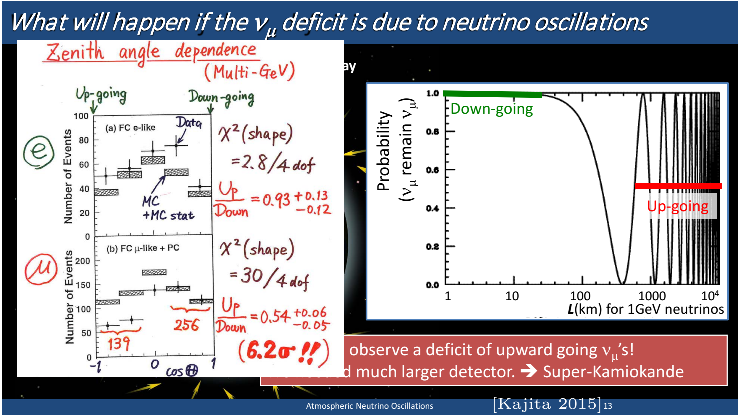
[Kajita 2015]

Difference between upward vs downward ν at Super-Kamiokande (1998)

$\implies$ Neutrinos have mass and oscillate!

2015 Nobel prize to Arthur McDonald, Takaaki Kajita

Discovery of Neutrino Oscillation



[Kajita 2015]

Difference between upward vs downward ν at Super-Kamiokande (1998)

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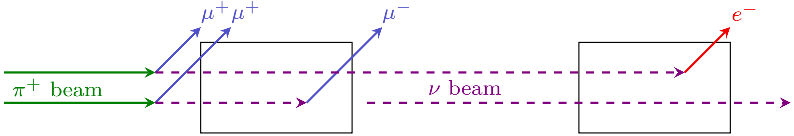
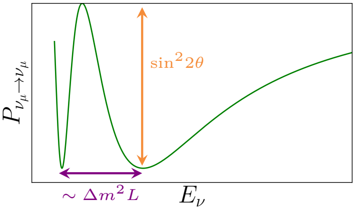
Neutrino Oscillation (with math)



$$\underbrace{|\nu_\ell\rangle}_{\text{flavor eigenstate}} = \sum_i U^*_{\ell i} \underbrace{|\nu_i\rangle}_{\text{mass eigenstate}} \qquad |\nu_i\rangle \rightarrow e^{-iE_i t}|\nu_i\rangle$$

2 flavor model:

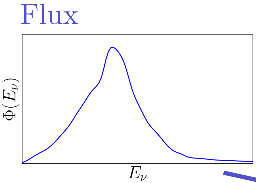
$$P_{\nu_\mu \rightarrow \nu_e} = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 [\text{eV}^2] L [\text{km}]}{E_\nu [\text{GeV}]} \right)$$

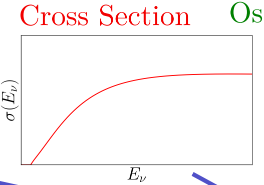


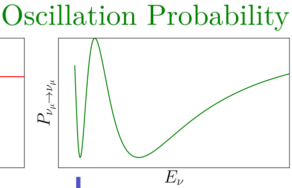
Mass eigenstates – propagation
flavor eigenstates – interaction
} Not the same

Oscillation probability is function of L/E_ν at fixed L

Measuring Oscillation Probability

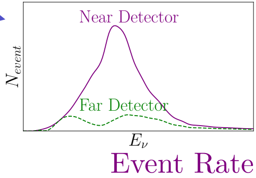






$$N_{\text{event}} = \int_{\text{bin}} dE_\nu \, \Phi(E_\nu) \sigma(E_\nu) \underbrace{\left[\times P(L/E_\nu) \right]}_{\text{far detector only}}$$

$$\left[P(L/E_\nu) \right]_{\text{bin}} \approx \left[\frac{N_{\text{event}}[\text{FAR}]}{N_{\text{event}}[\text{NEAR}]} \right]_{\text{bin}}$$



Neutrinos from tertiary beam ($p \to \pi^+ \to \nu_\mu$): broad flux

To measure ν oscillation, measure number of ν at given E_ν

Neutrino Event Topologies

Use large nucleus for more nucleons to interact with

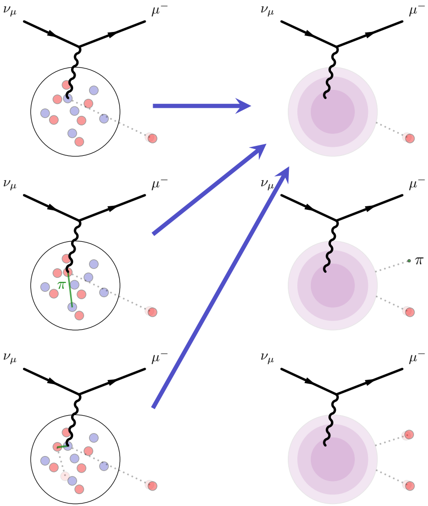
Nuclear environment complicates measurements:

- Many allowed kinematic channels
- Reinteractions within nucleus
- Only final state particles are observable

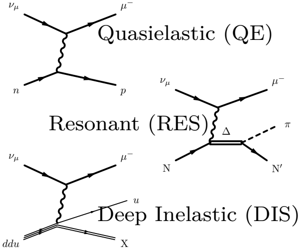
For any event, range of possible E_ν depending upon event topology

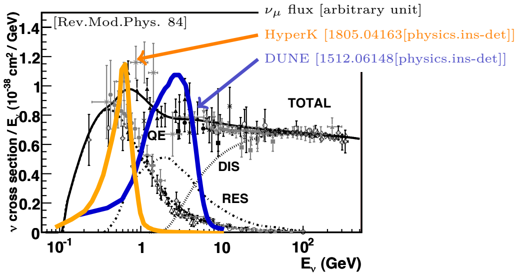
$\Longrightarrow$ Event-by-event E_ν measurements are not possible

"Reconstruct" event distributions using Monte Carlo simulations



Neutrino Cross Sections





Energy range spans several *nucleon* interaction topologies

Nucleon amplitudes used to build *nuclear* cross sections

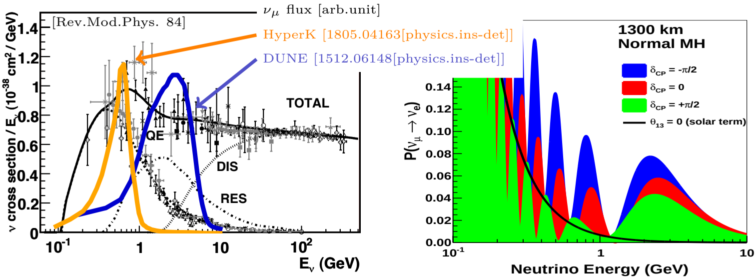
$\Longrightarrow$ inputs to Monte Carlo simulations, E_ν reconstruction

Goal: isolate, quantify, improve *nucleon* amplitudes

$\Longrightarrow$ Precise, theoretically robust *nucleon* inputs

$\rightarrow$ definitive statements about *nuclear* uncertainties

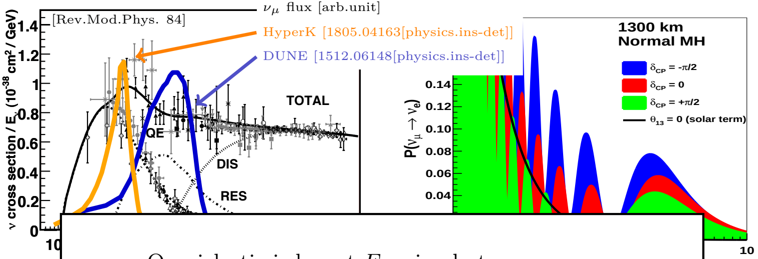
Summary of Oscillation Experiment Challenges



E_ν not known per event	$\Longrightarrow$	reconstruct E_ν from distribution
Primary interaction topologies mixed by nuclear effects	$\Longrightarrow$	Counts/kinematics/topologies change inside nucleus
ND/FD beam not same	$\Longrightarrow$	different efficiency @ near, far

High precision experiments need robust & precise cross sections
to determine topology and E_ν for interaction events

Summary of Oscillation Experiment Challenges



Quasielastic is lowest E_ν, simplest
$\Longrightarrow$ most important

Question:
How well do we know free nucleon quasielastic cross section from elementary target sources?

Two main sources:

- D_2 scattering
- Lattice QCD

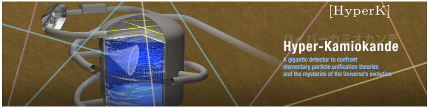
E_ν not k

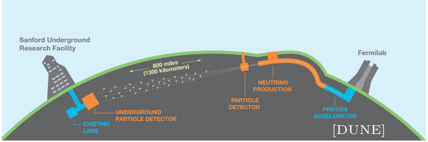
Primary
mixed

ND/FD

High prec
to deter

Neutrino Physics Goals





Flagship long baseline experiments to measure neutrino oscillation

DUNE: USA, HyperK: Japan

$O(10 - 100 \text{ kton})$ mass

$O(10^2 - 10^3 \text{ km})$ baseline

Seek to answer fundamental questions about neutrinos:

- mass ordering ($\Delta m^2_{32} > 0$?)
- octant ($\sin^2 \theta_{23} = 0.5$?)
- CP violation ($\delta_{\rm CP} =$?)
- PMNS unitarity?
- 3 ν flavors?
- precision constraints

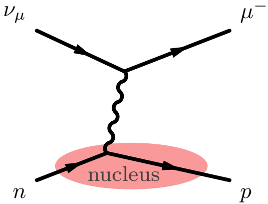
Measurements of solar, supernova ν

Data collection starts 2028-2029 $\implies$ need support from theory!

QE Experimental Constraints

Quasielastic Form Factors

Quasielastic (QE) scattering assumes quasi-free nucleon inside nucleus



$$\mathcal{M}_{\text{nucleon}} = \langle \ell | \mathcal{J}^\mu | \nu_\ell \rangle \langle N' | \mathcal{J}_\mu | N \rangle$$

$$\langle N'(p') | (V-A)_\mu(q) | N(p) \rangle$$
$$= \bar{u}(p') \Big[\gamma_\mu F_1(q^2) + \frac{i}{2M_N} \sigma_{\mu\nu} q^\nu F_2(q^2) + \gamma_\mu \gamma_5 F_A(q^2) + \frac{1}{2M_N} q_\mu \gamma_5 F_P(q^2) \Big] u(p)$$

- F_1, F_2: constrained by eN scattering
- F_P: subleading in cross section,
  $\propto F_A$ from pion pole dominance constraint

Axial form factor F_A is leading contribution to nucleon cross section uncertainty

Form Factor Parameterizations

Most common in experimental literature: dipole ansatz —

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{m_A^2}\right)^{-2}$$

- Overconstrained by both experimental and LQCD data (revisit later)
- Inconsistent with QCD, requirements from unitarity bounds
- Motivated by $Q^2 \to \infty$ limit, data restricted to low Q^2

Model independent alternative: z expansion [Phys.Rev.D 84 (2011)] —

$$F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z(Q^2; t_0, t_{\mathrm{cut}}) = \frac{\sqrt{t_{\mathrm{cut}} + Q^2} - \sqrt{t_{\mathrm{cut}} - t_0}}{\sqrt{t_{\mathrm{cut}} + Q^2} + \sqrt{t_{\mathrm{cut}} - t_0}} \qquad t_{\mathrm{cut}} \leq (3M_\pi)^2$$

- Rapidly converging expansion
- Controlled procedure for introducing new parameters

Deuterium Constraints on F_A

$\sim$ free nucleon constraints on F_A

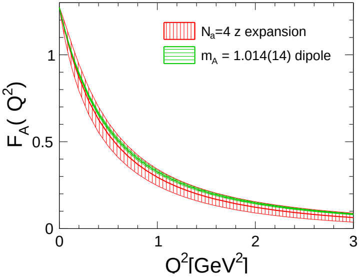
- Outdated bubble chamber experiments:
  - Total $O(10^3)$ ν_μQE events
  - Original data lost
  - Unknown corrections to data
  - Deficient deuterium correction
- Dipole overconstrained by data
  underestimated uncertainty $\times O(10)$
- Prediction discrepancies could be from
  nucleon and/or nuclear origins

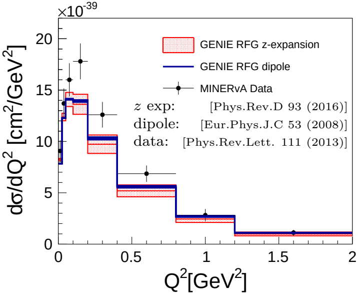
Coming soon:

MINERνA $\bar{\nu}_\mu p \to \mu^+ n$ dataset

Obtained from hydrocarbon target

See [Cai thesis (2021)]





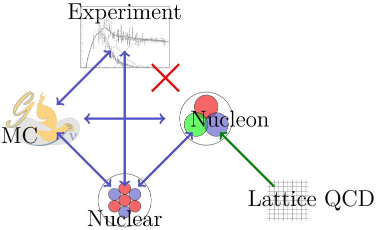
LQCD as Disruptive Technology

How can we improve precision?

Ideal: Modern high stats ν-D_2 scattering bubble chamber experiment

$\Longrightarrow$ LQCD as a alternative/complement to experiment,
especially with experimentally inaccessible quantities

- ✓ No nuclear effects
- ✓ Realistic uncertainty estimates
- ✓ Systematically improvable
- ✓ Computers are (relatively) inexpensive



Build from the ground up:

Nucleon amplitudes from first principles

Robust uncertainty quantification

Well motivated theory inputs to nuclear models/EFTs

Lattice QCD

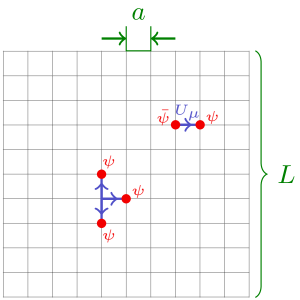
Lattice QCD Formalism

Numerical evaluation of path integral
Quark, gluon DOFs —

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, \mathcal{D}U \, \exp(-S) \, \mathcal{O}_\psi \, [U]$$

Inputs: $am_{(u,d),\mathrm{bare}}$
$am_{s,\mathrm{bare}}$
$\beta = 6/g^2_{\mathrm{bare}}$

Matching: e.g. $\frac{M_\pi}{M_\Omega}$, $\frac{M_K}{M_\Omega}$, M_Ω
Only 1 per computational input

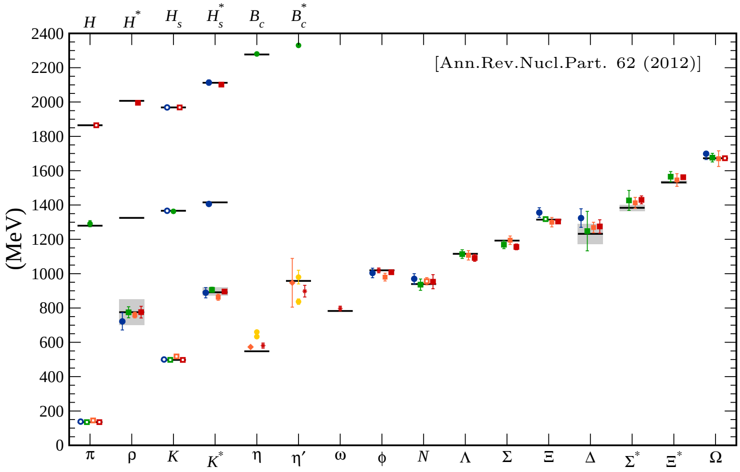


Results — first principles predictions from QCD,
gluons to all orders

"Complete" error budget $\implies$ extrapolation in a, L, M_π guided by EFT, FVχPT

- $a \to 0$ (continuum limit)
- $L \to \infty$ (infinite volume limit)
- $M_\pi \to M_\pi^{\mathrm{phys}}$ (chiral limit)

Successes of Lattice QCD



Open symbol: input   Closed symbol: (pre/post)diction   Line: expt

- Excellent agreement across board
- Widely used in flavor physics

Axial Vector Coupling: $g_A(Q^2 = 0)$

g_A is benchmark for
nucleon matrix elements in LQCD

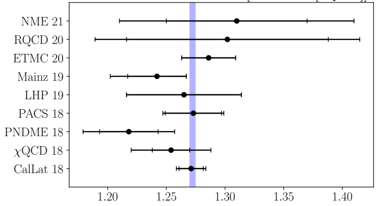
Status circa 2018 summarized by
USQCD white paper
[Eur.Phys.J.A 55 (2019)]

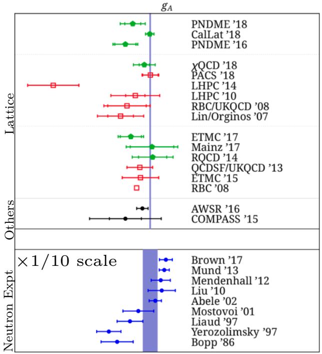
See also: FLAG review
[Eur.Phys.J.C 80 (2020)]

Historically g_A low compared to expt
excited states (+other...)

Lots of activity since 2018,
consistent agreement with PDG
full error budgets available

See refs. in: [2103.05599[hep-lat]]



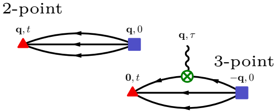


[Eur.Phys.J.A 55 (2019)]

LQCD Computation Anatomy

Correlation functions in euclidean time:

$\implies e^{-E_n t}$ decay of excited state contribs



2-point function

$$\langle \blacktriangle(t) \blacksquare(0)\rangle = \sum_n \langle 0|\blacktriangle|n\rangle\langle n|\blacksquare|0\rangle e^{-E_n t}$$

3-point function

$$\langle \blacktriangle(t) \otimes(\tau) \blacksquare(0)\rangle = \sum_{mn} \langle 0|\blacktriangle|n\rangle\langle n|\otimes|m\rangle\langle m|\blacksquare|0\rangle e^{-E_n(t-\tau)-E_m\tau}$$

Extract masses from 2-point, matrix elements from 3-point

Complications:

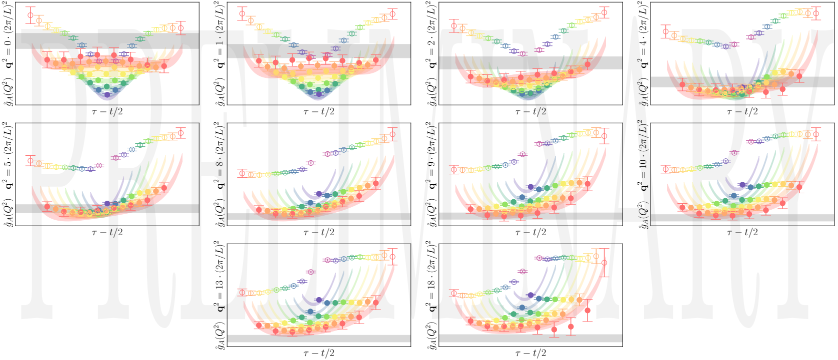
- exponentially degrading signal/noise with t
- $n > 0$ contaminations from excited states

Use many source/sink operators ($\blacksquare$, $\blacktriangle$) to suppress excited states:

$$C_{ij}(t) = \sum_n z_{i,n} z^{\dagger}_{j,n} e^{-E_n t}$$

$$\implies v^T C(t) v \approx e^{-E_0 t} \quad \text{when} \quad \sum_i v_i^T z_{i,n} \approx \delta_{0,n}$$

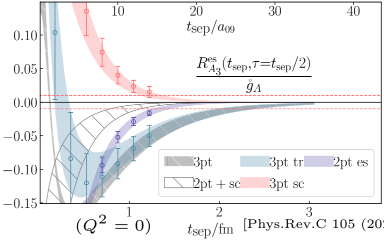
Three-Point Function Ratios



Global 3-state exponential fit $\times$ 10 momenta w/ $|q_i| \leq 3 \cdot (2\pi/L)$

- colors: $t/a \in \{3, ..., 12\}$ ▶ gray band: fit $\mathring{g}_A(Q^2)$ ▶ All $q_z = 0$
- horizontal: τ, centered about midpoint
- vertical: $\sqrt{2E_{\mathbf{q}}/(E_{\mathbf{q}} + M)}\mathcal{R}_{\mathcal{A}_z}(t_{\text{sep}}, \tau, \mathbf{q}) \to \mathring{g}_A(Q^2)$
- Minimum time separation: $\tau/a,\ (t-\tau)/a \geq 2$

Excited States in g_A



$(Q^2 = 0)$ [Phys.Rev.C 105 (2022)]

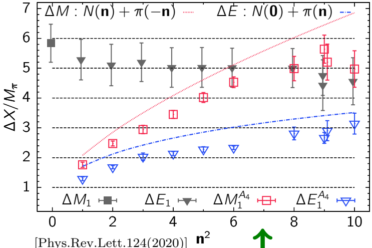
Compare fit to correlator data ratio

Remnant contamination dominated by "transition" states ($m \to n$, violet)

Statistically significant until 2 fm typical data $\lesssim$ 1 fm

Excited states still present in practically achievable large time limit

$(Q^2 \geq 0)$



[Phys.Rev.Lett.124(2020)]

NOTE: expect only approx agreement between data/curves

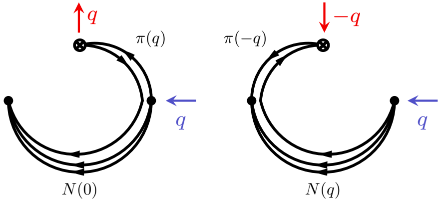
NME collab:

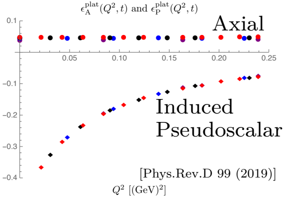
Q^2 contamination from $N \to N\pi$

Dominant contribution agrees with χPT expectation

$N\pi$ is important for $g_A(Q^2)$

Excited States - χPT and $N\pi$





Contamination primarily from enhanced $N\pi$, mostly from induced pseudoscalar

Intermediate fits not sensitive to $N\pi$

$\Longrightarrow$ need simultaneous fits including intermediate correlators

[Phys.Rev.C 105 (2022)] [Phys.Rev.D 105 (2022)]

Alternate fit strategies to remove $N\pi$ (are they comparable?):

- Kinematic constraints ($F_P = 0$)
- include $\mathcal{A}_4$ (strong $N\pi$ coupling)
- explicit $N\pi$ operators

Prediction from χPT: [Phys.Rev.D 99 (2019)]

First demonstration by NME: [Phys.Rev.Lett. 124 (2020)]

χPT-inspired fit methods for fitting form factor data
[Phys.Rev.D 105 (2022)] [JHEP 05 (2020) 126]

LQCD Survey & Implications

Now published!

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Status of Lattice QCD Determination of Nucleon Form Factors and Their Relevance for the Few-GeV Neutrino Program

Annual Review of Nuclear and Particle Science

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Aaron S. Meyer,[1,2] André Walker-Loud,[2] and Callum Wilkinson[3]

[1]Department of Physics, University of California, Berkeley, California, USA; email: asmeyer@berkeley.edu
[2]Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California, USA
[3]Physics Division, Lawrence Berkeley National Laboratory, Berkeley, California, USA

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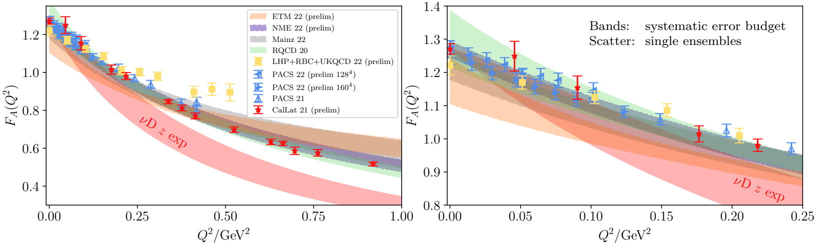
Abstract

Calculations of neutrino–nucleus cross sections begin with the neutrino–nucleon interaction, making the latter critically important to flagship neutrino oscillation experiments despite limited measurements with poor statistics. Alternatively, lattice quantum chromodynamics (LQCD) can be used to determine these interactions from the Standard Model with quantifiable theoretical uncertainties. Recent LQCD results of g_A are in excellent agreement with data, and results for the (quasi-)elastic nucleon form factors with full uncertainty budgets are expected within a few years. We review the status of the field and LQCD results for the nucleon axial form factor, $F_A(Q^2)$, a major source of uncertainty in modeling sub-GeV neutrino–nucleon interactions. Results from different LQCD calculations are consistent but collectively disagree with existing models, with potential implications for current and future neutrino oscillation experiments. We describe a road map to solidify confidence in the LQCD results and discuss future calculations of more complicated processes, which are important to few-GeV neutrino oscillation experiments.

Expected final online publication date for the *Annual Review of Nuclear and Particle Science*, Volume 72 is September 2022. Please see http://www.annualreviews.org/page/journal/pubdates for revised estimates.

Nucleon Axial Form Factor

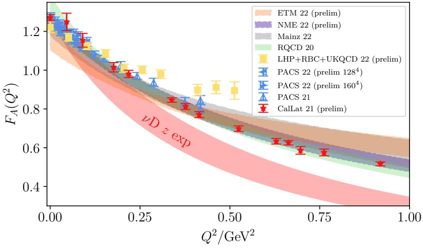


LQCD results maturing:

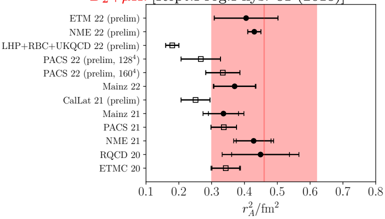
- Many results, all physical M_π:
  independent data & different methods
- Agreement w/ single ensemble
  $\implies$ Small systematic effects observed
  (expectation: largest at $Q^2 \to 0$)
- Extrapolated results (bands) satisfy nontrivial PCAC checks
  Lots of recent effort to understand

Evidence of slow Q^2 falloff, situation unlikely to change drastically

Axial Radius (r_A^2)



Filled circle: full error budget
Open square: incomplete
D$_2$+μH: [Rept.Prog.Phys. 81 (2018)]



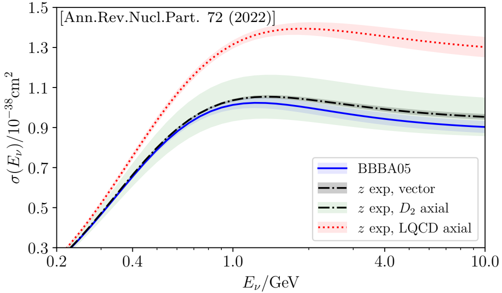
Radius related to slope: $r_A^2 = -\frac{6}{g_A}\frac{dF_A}{dQ^2}\Big|_{Q^2=0}$

Good agreement with r_A^2 from experiment, poor agreement with large Q^2

Fixing radius to agree at large Q^2 would bring radius down to $r_A^2 \sim 0.25~\mathrm{fm}^2$

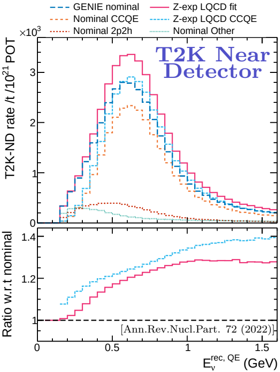
$\implies$ Incompatible with dipole ansatz

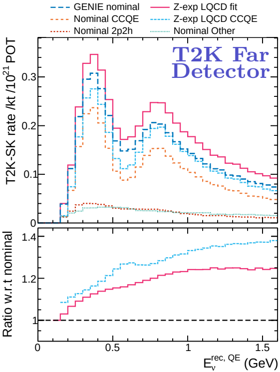
Free Nucleon Cross Section



- Representative LQCD dataset (CalLat 21)
- Integral over $Q^2 \implies$ enhancement of discrepancy
- LQCD prefers 30-40% enchancement of ν_μ CCQE cross section
- recent Monte Carlo tunes require 20% enhancement of QE
  [Phys.Rev.D 105 (2022)] [2206.11050 [hep-ph]]
  similar trend with continuum Schwinger function methods
  [Phys.Rev.D 105 (2022)] [2206.12518 [hep-ph]]
- With improved precision, sensitive to vector FF tension (black vs blue)
  [Phys.Rev.D 102 (2020)] vs [Nucl.Phys.B Proc.Suppl. 159 (2006)]

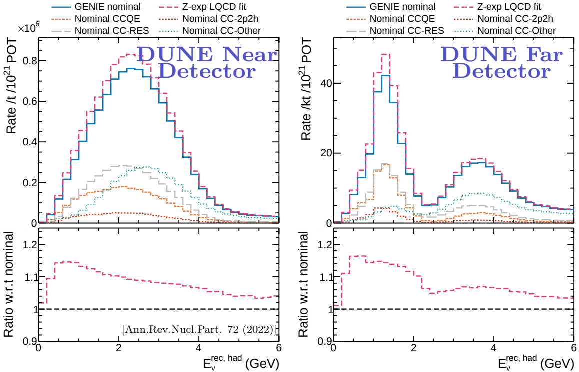
T2K Implications





- Dashed dark blue (GENIE nominal) vs solid magenta (z exp LQCD fit)
- QE enhancements produce 10-20% event rate enhancement, E_ν-dependent
- Monte Carlo tuning makes more detailed comparisons complicated
  $\implies$ All channels are adjusted to compensate for QE changes
- cross section changes at ND $\neq$ *effective* cross section changes at FD:
  insufficient CCQE model freedom $\rightarrow$ bias in FD prediction

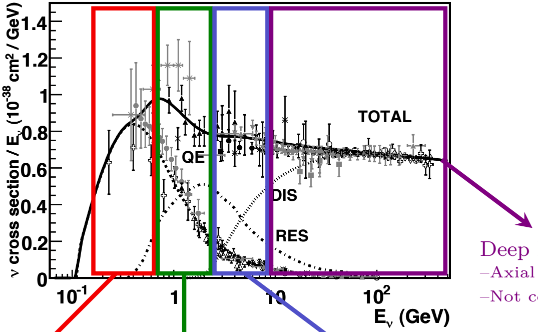
DUNE Implications



- Solid dark blue (GENIE nominal) vs dashed magenta (z exp LQCD fit)
- QE enhancements produce 10-20% event rate enhancement, E_ν-dependent
- Monte Carlo tuning makes more detailed comparisons complicated
  $\Longrightarrow$ All channels are adjusted to compensate for QE changes
- cross section changes at ND $\neq$ *effective* cross section changes at FD:
  insufficient CCQE model freedom $\rightarrow$ bias in FD prediction

Future Directions

Future Directions



Deep Inelastic Scattering

–Axial quasi/pseudo PDF

–Not covered here

Quasielastic

–Nucleon Form Factors

–Full Error Budgets

–Detailed Systematics

$N \to \Delta, N \to N^*$

–Transition Matrix Elements

–Multiparticle Operators

–Initial Calculations

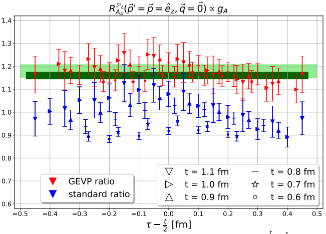
"Shallow Inelastic Scattering" (SIS)

–Hadronic Tensor

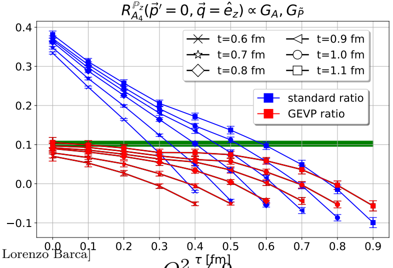
–Four Point Functions

–Exploration

Axial FF - $N\pi$ Interpolating Operators



$Q^2 = 0$



$Q^2 \neq 0$

[priv.comm. Lorenzo Barca]

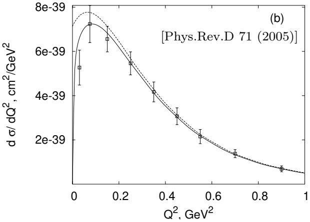
Address primary source of excited state contamination: $N\pi$

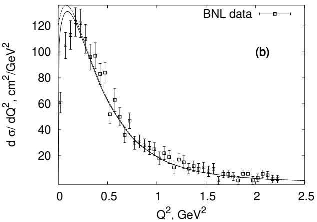
2×2 operator basis, explicit 3- and 5-quark interpolating operators

Significantly flatter ratios, simplified analysis

Will analysis with only 3-quark operators be consistent?

Resonance Production - $N \to \Delta$





$N \to \Delta$ transition form factors are poorly known, but needed

1π production cross section known to 30% [Phys.Rev.C 88 (2013)]

DUNE error budget anticipates $\lesssim$10% precision [2002.03005 [hep-ex]]

Completely unconstrained axial form factors in other $J^P = 3/2^-$ channels
$\implies$ 100% uncertainties from $V - A$, $A - A$ interference terms

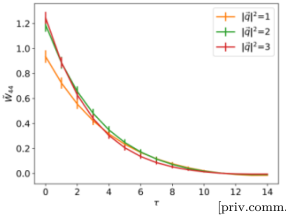
[Phys.Rev.D 74 (2006)]

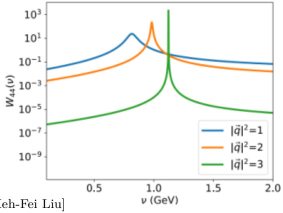
Previous work by ETM: [Phys.Rev.D 83 (2011)] [Phys.Rev.Lett. 98 (2007)]

Formal developments:

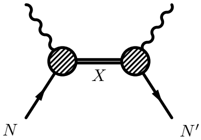
$1 + \mathcal{J} \to 2$	$(N\gamma^* \to N\pi)$	[Phys.Rev.D 103 (2021)]
$1 + \mathcal{J} \to 2 + \mathcal{J}$	$(N\gamma^* \to \Delta \to N\pi\gamma^*)$	[Phys.Rev.D 105 (2022)]

Resonance Production - $N \rightarrow N^*$





[priv.comm. Keh-Fei Liu]



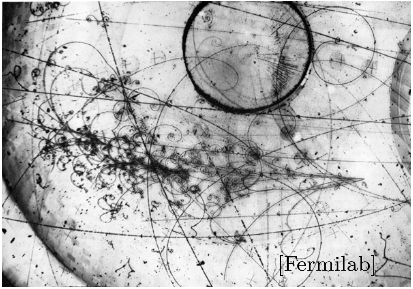
See also: [Phys.Rev.D 101 (2020)]

Four point function with $\langle \mathcal{O}(0)\mathcal{J}_4(-q)\mathcal{J}_4(q)\bar{\mathcal{O}}(0)\rangle$, $M_\pi \sim 370$ MeV
Removed elastic contribution $\implies$ resonant response (strong overlap with Roper)

Hadronic tensor methods for addressing SIS (1.4 GeV $\leq W \leq$ 2.0 GeV)
Large $N\pi$, $N\pi\pi$ contributions; strong interferences between resonant/nonresonant
Currently no practical $Q^2 \neq 0$ data in this region [S.Nakamura - NuSTEC S&DIS]

Concluding Remarks

Outlook

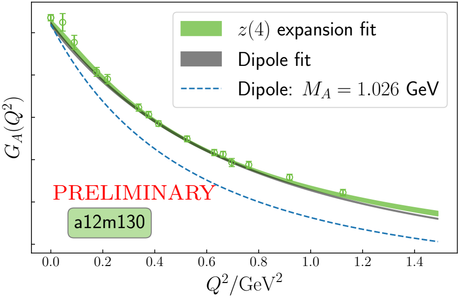


- *Nucleon* form factor uncertainty significantly underestimated
- LQCD is a proxy for missing experimental data
- $N\pi$ states are a major player in LQCD calculations
  $\implies$ Might need explicit treatment in calculations
- Mounting evidence that ν QE cross section underestimated
  $\implies$ Attention needed to avoid biased results
- Unfilled niche: need support for neutrino experimental program
  - resonant transition form factors
  - shallow inelastic scattering

Thank you for your attention!

Backup

Axial Form Factor Fit



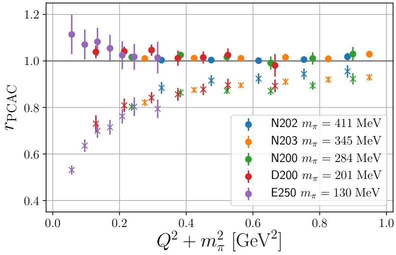
parameter source	value(err)[rel] g_A/g_V	r_A^2/fm^2	$m_{A,\text{dipole}}/\text{GeV}$
$k_{\text{max}} = 4$ z expansion	1.6%	18%	8.8%
dipole	1.6%	5.7%	2.8%
PDG	0.2%		
[Phys.Rev.D 93 (2016)]		48%	24%

Dipole underestimates r_A^2 uncertainty by factor ~ 3,
Will get worse with increased Q^2 range

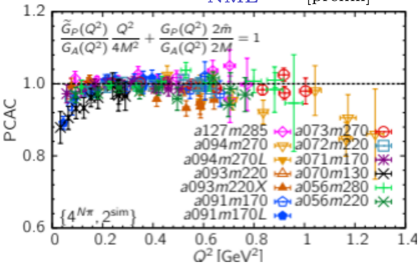
Using data up to $(qL/2\pi)^2 = 18$, data exist up to 108

PCAC Checks

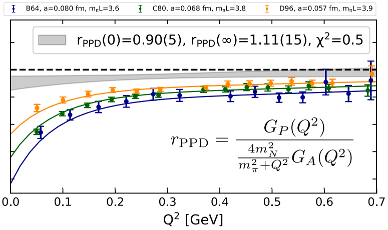
RQCD [JHEP 05 (2020)]



NME [prelim]



ETMC [prelim]

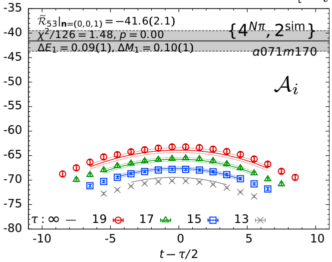


- Relation btw F_A, F_P, $\tilde{F}_P$ via PCAC
- Contamination in F_A and $\tilde{F}_P$, F_P very different [Phys.Rev.D 99 (2019)]

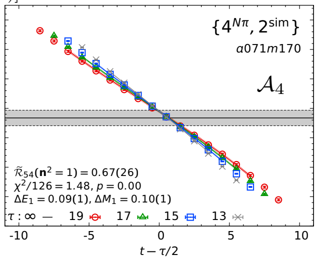
  $\Longrightarrow$ nontrivial consistency check

Excited States in Temporal Axial Current

[Phys.Rev.D 105 (2022)]



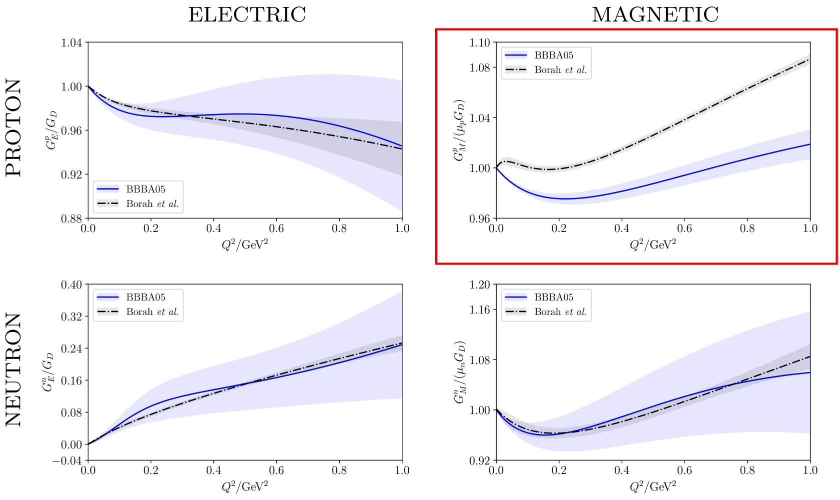
versus



Plotted: ratio combination $\sim \langle N|\mathcal{A}_\mu|N\rangle$

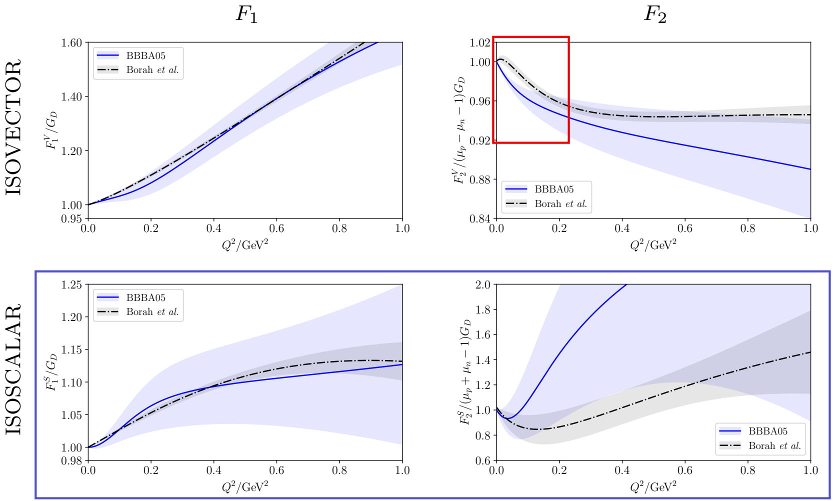
Highly enhanced excited states in $\mathcal{A}_4$, use as handle to quantify $N\pi$

Vector Form Factors - Proton/Neutron



Large tension in proton magnetic form factor

Vector Form Factors - Isospin Symmetric



Uncertain slope of F_2^V

Large uncertainty on isoscalar form factors

z Expansion in χPT

[Ann.Rev.Nucl.Part. 72 (2022)]

$$F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z(Q^2; t_0, t_{\rm cut}) = \frac{\sqrt{t_{\rm cut} + Q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} + Q^2} + \sqrt{t_{\rm cut} - t_0}} \qquad t_{\rm cut} \leq (3M_\pi)^2$$

With nonzero t_0, chiral expansion is about $Q^2 = -t_0$ rather than $Q^2 = 0$:

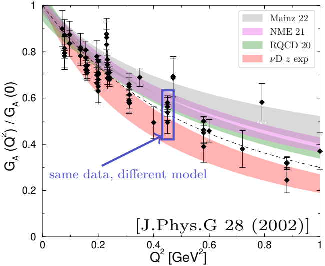
$$x \equiv \frac{Q^2 + t_0}{t_c - t_0} = 4 \sum_{k=1}^{\infty} k z^k \sim O(z)$$

$$(1+x)^{1/2} - 1 = \frac{x}{2} - 4 \sum_{k=2}^{\infty} \frac{(2k-3)!}{k!(k-2)!} \left(-\frac{x}{4} \right)^k$$

$$z = \frac{1}{x} \Big((1+x)^{1/2} - 1 \Big)^2 \sim O(x)$$

$$Q^{2m} = \Big((Q^2 + t_0) - t_0 \Big)^m = (t_c - t_0)^m \sum_{n=0}^{m} \binom{m}{n} x^n \left(\frac{-t_0}{t_c - t_0} \right)^{m-n}$$

Electro Pion Production



- Large model uncertainty, not included in world averages
- Valid only in $M_\pi \to 0$, $q \to 0$ limits
- Expansion to $O(M_\pi^2, Q^2)$:
  - restricted Q^2 validity
  - lacks shape freedom in Q^2
- Predates Heavy Baryon χPT, no systematic power counting

Modern experiments do not report $F_A(Q^2) \implies$ averages out of date

Possible argument for comparing to r_A^2 from low Q^2; high Q^2 untrustworthy

Effort needed to update prediction from photo/electro pion production

Excited States - PCAC Checks

Nontrivial checks from PCAC:

$$\langle N|\mathcal{A}_\mu|N\rangle = \bar{u}\Big[\gamma_\mu\gamma_5 F_A(q^2) + \frac{q_\mu}{2M_N}\gamma_5 F_P(q^2)\Big]u$$

$$\langle N|\mathcal{P}|N\rangle = \bar{u}\Big[\gamma_5 F_5(q^2)\Big]u$$

PCAC operator relation: $q^\mu \mathcal{A}_\mu = 2\hat{m}\mathcal{P}$

$$\implies 2M_N F_A(Q^2) - \frac{Q^2}{2M_N} F_P(Q^2) = 2\hat{m} F_5(Q^2)$$

Previously: Checks succeed for correlators, fail for form factors

Solution: better excited state quantification

$\implies$ extracted something other than $\langle N|\mathcal{A}_\mu|N\rangle$, like $\langle N\pi|\mathcal{A}_\mu|N\rangle$

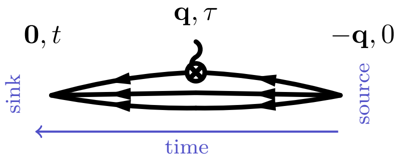
Two general strategies have been used to deal with excited states:

large t_{sep}: ground state saturation; signal-to-noise degradation
e.g. summation method [Phys.Rev.D 86 (2012)]

many t_{sep}: isolate, remove excited states; many fit parameters
[Phys.Rev.C 105 (2022)]

Can we do better?

Setup



$$\mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) = \frac{C^{3\text{pt}}_{\mathcal{A}_z}(t,\tau,\mathbf{q})}{\sqrt{C^{2\text{pt}}(t-\tau,\mathbf{0})C^{2\text{pt}}(\tau,\mathbf{q})}} \sqrt{\frac{C^{2\text{pt}}(\tau,\mathbf{0})}{C^{2\text{pt}}(t,\mathbf{0})} \frac{C^{2\text{pt}}(t-\tau,\mathbf{q})}{C^{2\text{pt}}(t,\mathbf{q})}}$$

$$\xrightarrow[t-\tau,\tau\to\infty]{} \frac{1}{\sqrt{2E_{\mathbf{q}}(E_{\mathbf{q}}+M)}} \left[-\frac{q_z^2}{2M} \mathring{\tilde{g}}_P(Q^2) + (E_{\mathbf{q}}+M)\mathring{g}_A(Q^2) \right]$$

$Q^2 = |\mathbf{q}|^2 - (E_{\mathbf{q}} - M)^2$

$\mathcal{A}_z$ w/ $q_z = 0 \implies \mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) \to \sqrt{\frac{E_{\mathbf{q}}+M}{2E_{\mathbf{q}}}} \mathring{g}_A(Q^2)$

$\implies$ No induced pseudoscalar

$\implies$ Simplified analysis of $\mathring{g}_A(Q^2)$