

Large N_c , Three Dimensional String Theory, the Thermodynamics and Phase Structure of QCD

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(A simple story that sadly could have been told years ago....)

Scientific Questions:

Is there a phase of matter between the Quark Gluon Plasma and the Hadron Gas?
Can one quantitatively describe thermodynamics below 160 MeV from first principles, no free parameters, computation? Above 160 MeV?

Work in part motivated by some observations of
Lenya Glozman, Tom Cohen, and colleagues

Motivation from lattice computations:

Chiral symmetry is restored about 160 MeV, but confinement as measured by Polyakov loop appears to disappear at about 300~Mev. This is the temperature of deconfinement in the pure glue theory

In this intermediate temperature range, entropy is smaller than expected

Old Problem:

How to characterize confinement:

When dynamical quarks are present confinement at finite temperature or density is only approximately characterized by the Polyakov line because strings always break and the free energy of an isolated quark is always finite.

If we go to large N_c , or in pure gauge theory, there is truly confinement. In pure gauge theory, the deconfinement temperature is about 300 MeV, which is much different from the chiral transition in a theory with dynamical quarks.

Perhaps the region between 160 MeV and 300 MeV is distinct from that at higher or lower T . Some evidence from lattice data showing that stringy vortices percolate at around 270 MeV. In this vortex picture, this should be the temperature of deconfinement.

How to characterize these separate regions.

Bosonic String Theory:

Originally Proposed to Describe All of Hadronic Physics

In large N_c Limit, Free String theory has a Hagedorn Spectrum and mesons are narrow width.

Theory formulated in d dimension, but only in $d = 26$ space time dimension is the theory free of tachyons or ghost in the low mass spectrum

We will assume that strings describe the spectrum of QCD mesons in the large N_c limit except for the lowest mass states, which we take to be (almost) Goldstone bosons. We treat these states distinctly.

The theory is defined with no undetermined parameters, and when the physical string tension is used, the Hagedorn, or limit temperature, is about 300 MeV

People gave up.

We did not:

We find a quantitatively precise computation of the spectrum of mesons and glueball, and the QCD thermodynamics for $T < 160$ MeV.

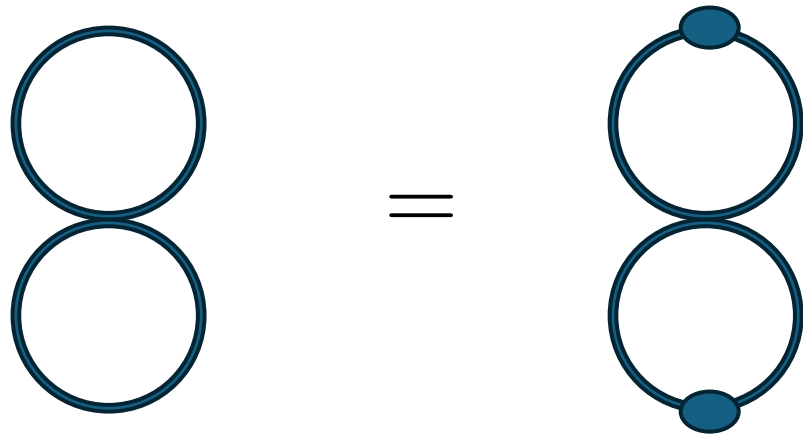
Chiral symmetry restored at 160 MeV

For $160 \text{ MeV} < T < 300 \text{ MeV}$, we argue gas of quarks/interacting-mesons, plus very low density of glueballs

Above 300 MeV, glueball melt into a quark gluon plasma

Mark Twain: It not what you don't know which gets you in trouble, it's what you think you know, but don't.

The strength of interactions:
 Contribution to the pressure are densities times an amplitude:



$$= \int dp dp' \, n(p) A(p, p') n(p')$$

Kinetic energy of hadrons ~ 1 , quarks $\sim N_c$, gluons $\sim N_c^2$

Amplitude for meson-meson $\sim 1/N_c$, glueball-glueball $\sim 1/N_c^2$

For pressure of hadrons to be of order N_c the
 hadron density should be of order N_c

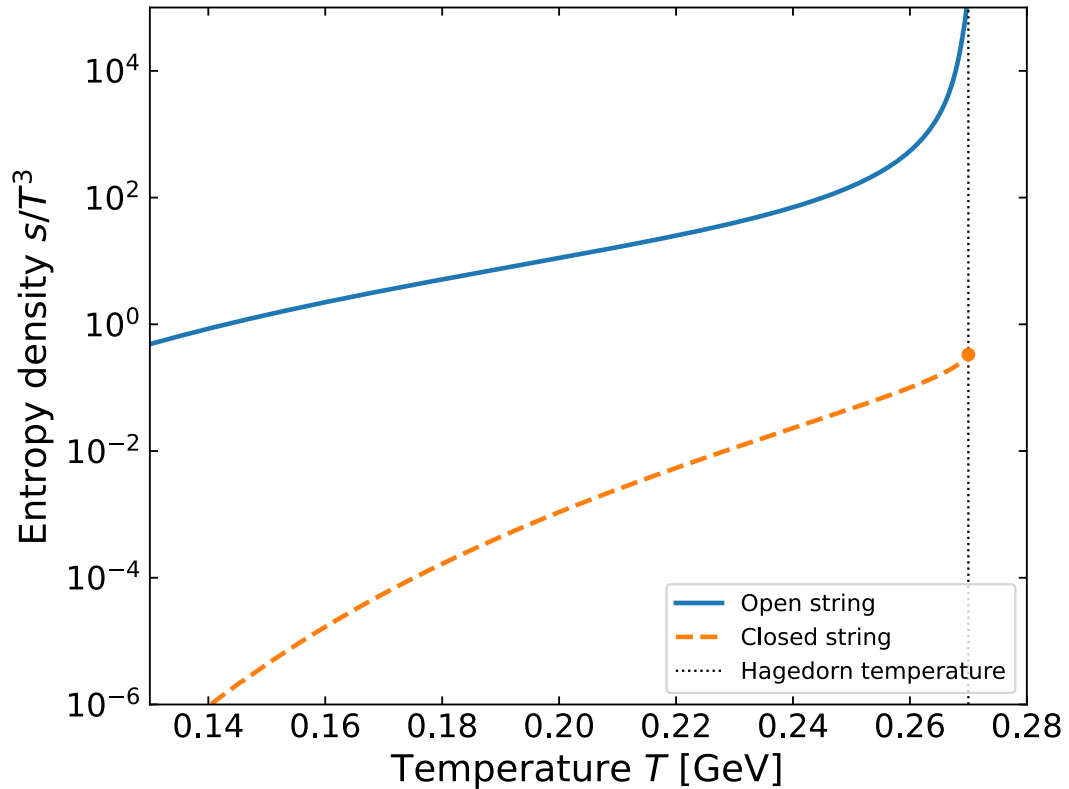
Pressure for glueballs to be of order N_c^2 the
 density of glueballs should be of order N_c^2

Mesons and glueballs, density ~ 1

Quarks and glueballs, density $\sim N_c$

Quarks and gluons, density $\sim N_c^2$

Simple model:



Contribution of glueballs is always very very small.
The closed string, glueball contribution, is non-divergent at the Hagedorn temperature

Closed strings describes well the spectrum of glueballs and thermodynamics in pure gauge theory
H. Meyer

Mesons: open strings

Glueballs: closed strings

Quarks: Free quarks with mass between constituent mass and current quarks mass

Gluons: Mass between massless and $\frac{1}{2}$ of glueball mass

Take closed and open string spectrum from string theory.

Glueball and mesons must have the same Hagedorn temperature which we take to be

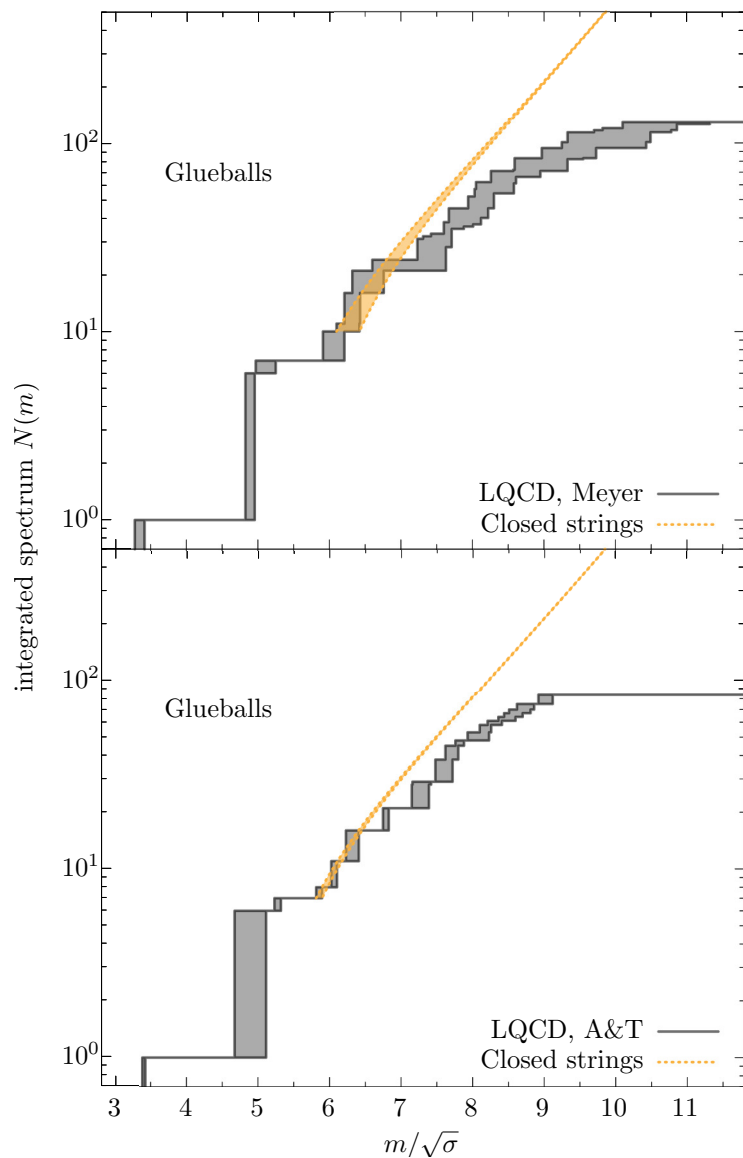
$$T_H \sim 300 \text{ MeV}$$

We handle the contribution of lowest mass glueball states, and Goldstone boson meson states separately, and then treat higher mass states by a continuum integral. The meson integration begins at .6 GeV, and the glueball at about 3~GeV

$$T_H^{string} = \sqrt{\frac{3\sigma}{2\pi}} = 304 \text{ MeV}$$

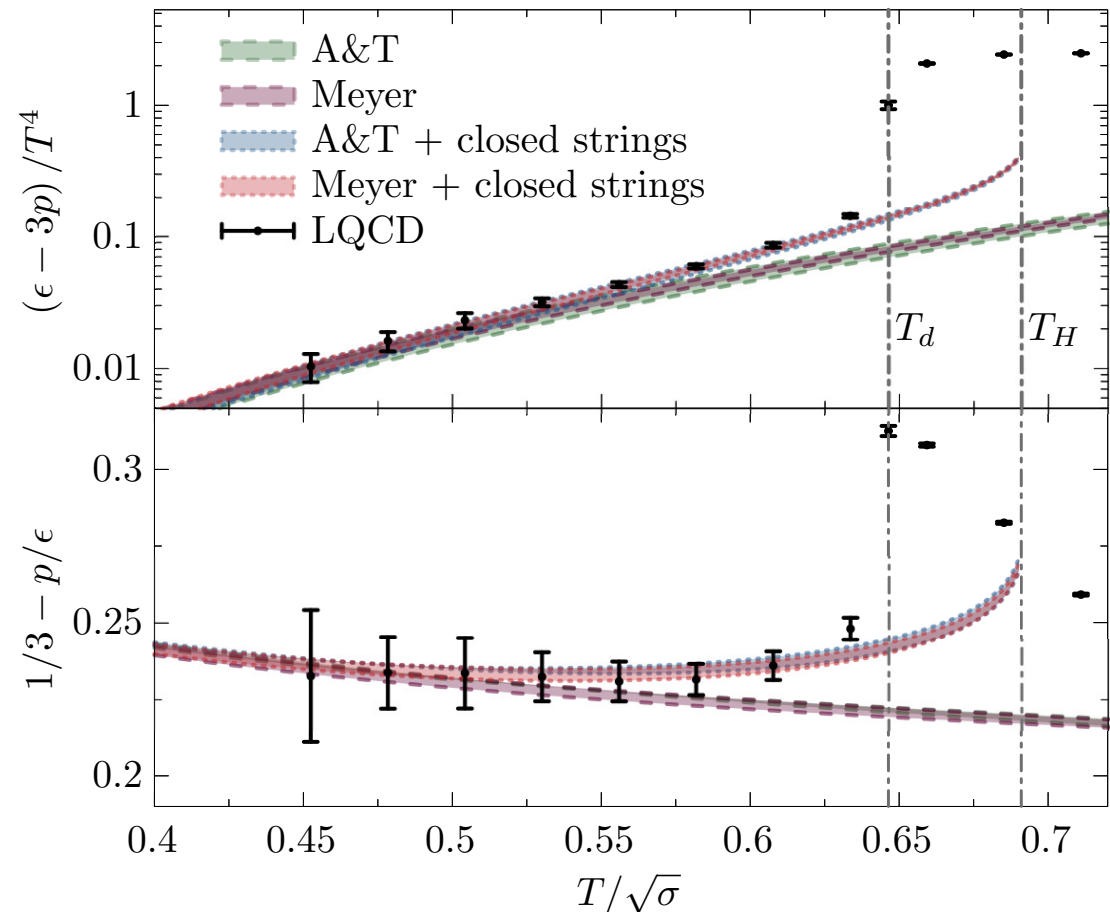
for

$$\sqrt{\sigma} = 440 \text{ MeV}$$

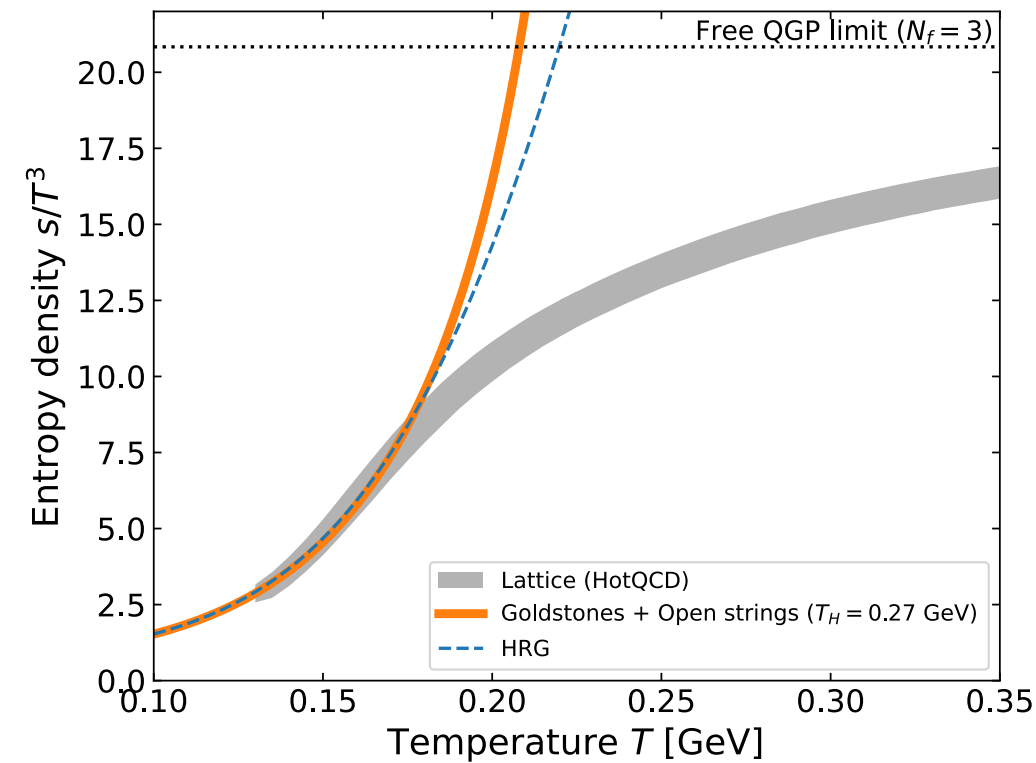


As shown by Meyer,
the string provides a
good description of
glueball spectrum
and thermodynamics
for pure glue QCD
($N_c = 3$)

Resonance spectrum
has missing states
above twice the mass
of the lowest mass
glueball, because
unstable states are
difficult to extract



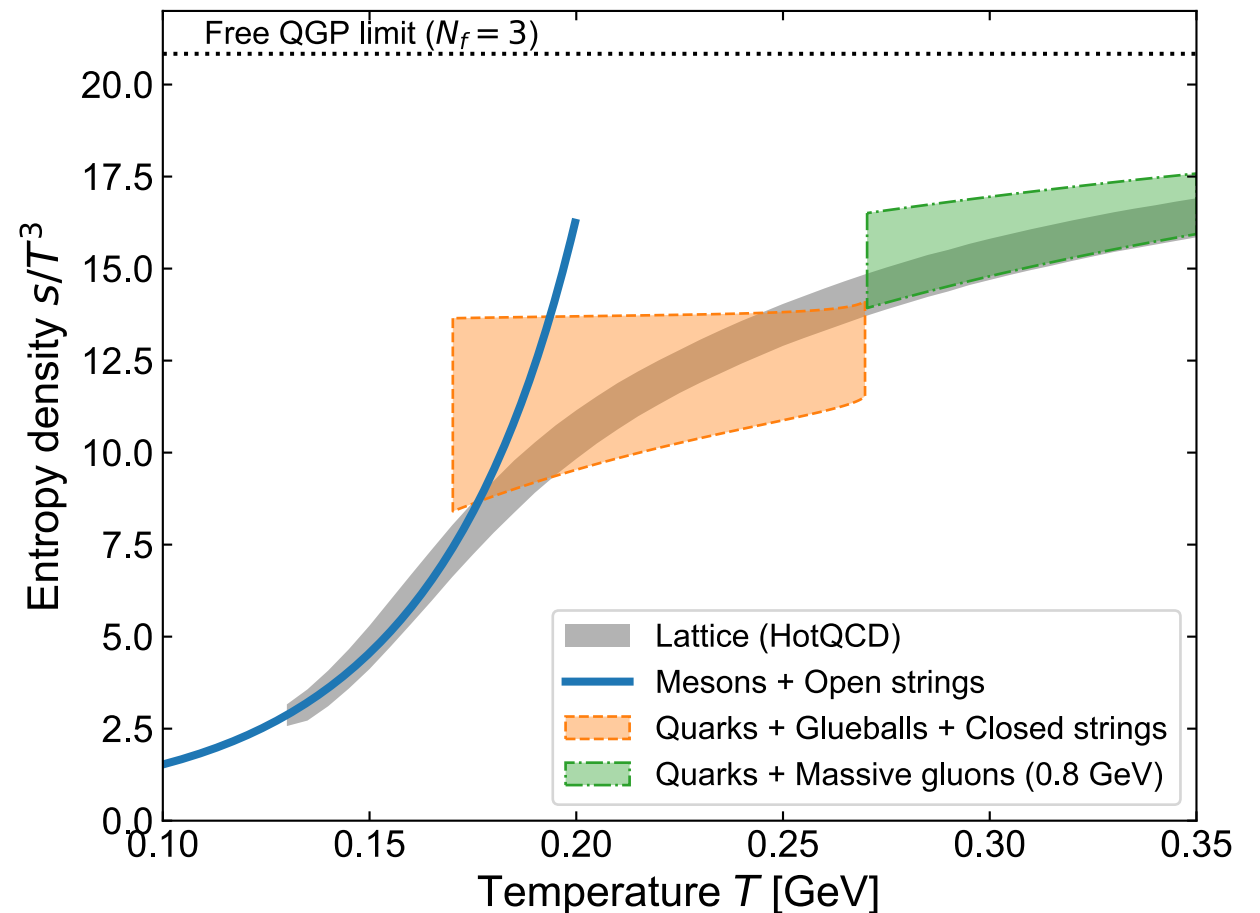
Thermodynamics for $N_c = 3$ is
determined by the trace anomaly
and is well determined below the
deconfinement temperature, which
is slightly below the Hagedorn
temperature



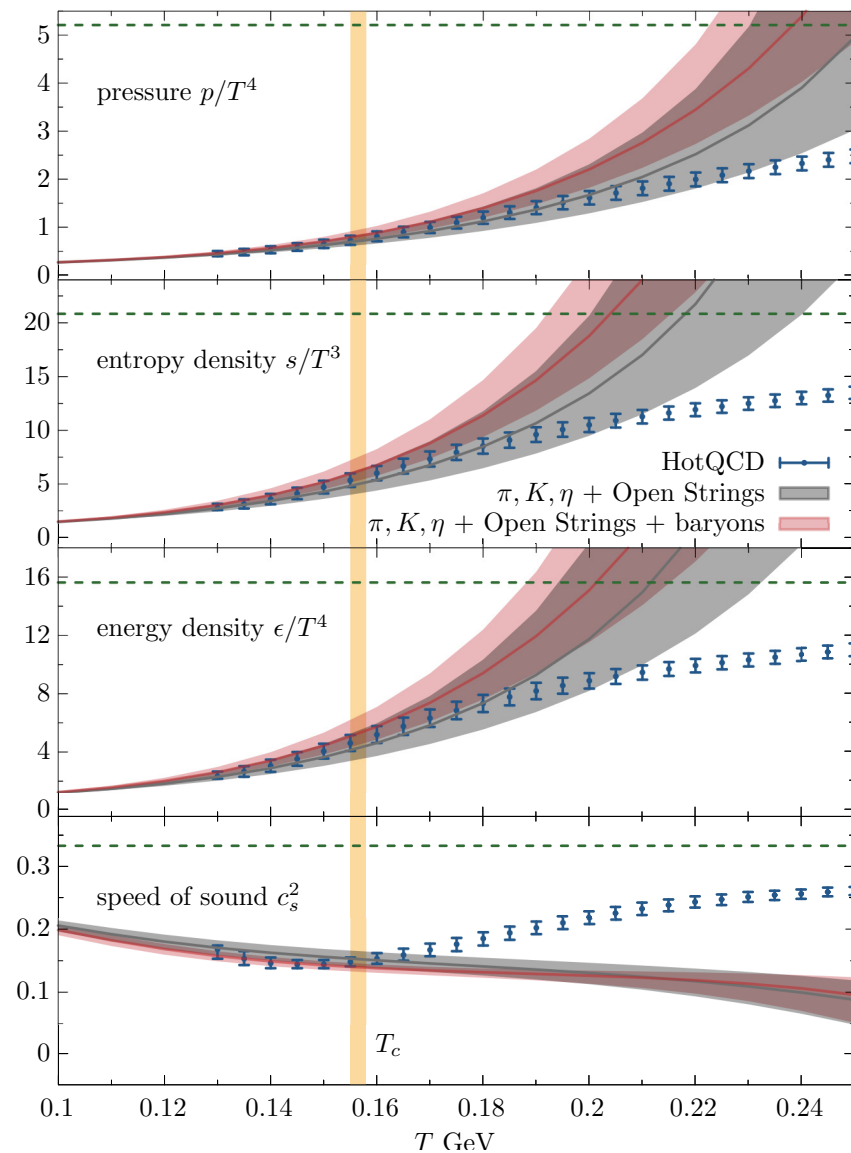
Matching on to quarks plus glueball at 160 MeV,
and to quarks plus gluons at the Hagedorn
temperature, the entropy is reasonably well
described

There is a correction to this from subtracting off
the baryon contributions from lattice data,
which makes a good fit at 310 MeV:
Redlich, Marczenko, Kovacs LM

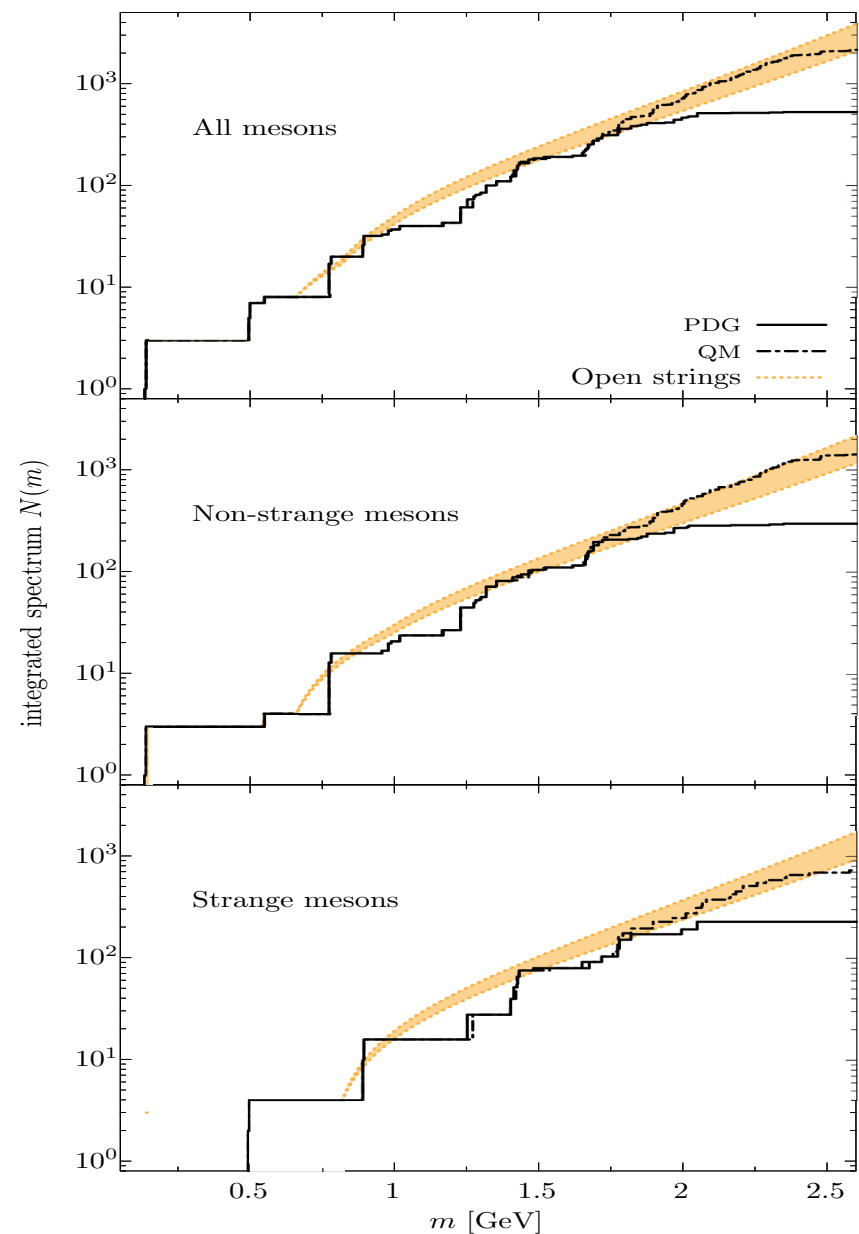
With the string theory resonance model and a
Hagedorn temperature of 285 MeV, describe the
lattice data on entropy up until the chiral
temperature, where presumably the degrees of
freedom become quarks



Thermodynamic Contributions with and without baryons included, $T_H = 275\text{--}340$ MeV



Comparison to experimental and theory extrapolated spectrum of mesons states



The quark phase with energy density of order N_c has to a good approximation, no glueballs, and hence no gluons, in it. This is because glueballs are massive. The Debye screening mass is

$$M_{Debye}^2 = g^2(N_c/3 + N_f/6)T^2$$

The first term is from gluons and the second from quarks. In the large N_c limit, one holds the 'tHooft coupling fixed,

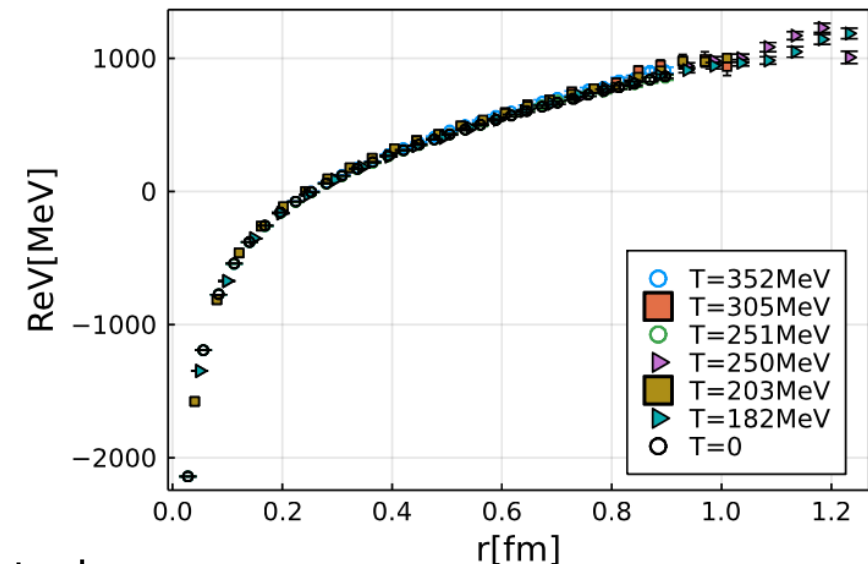
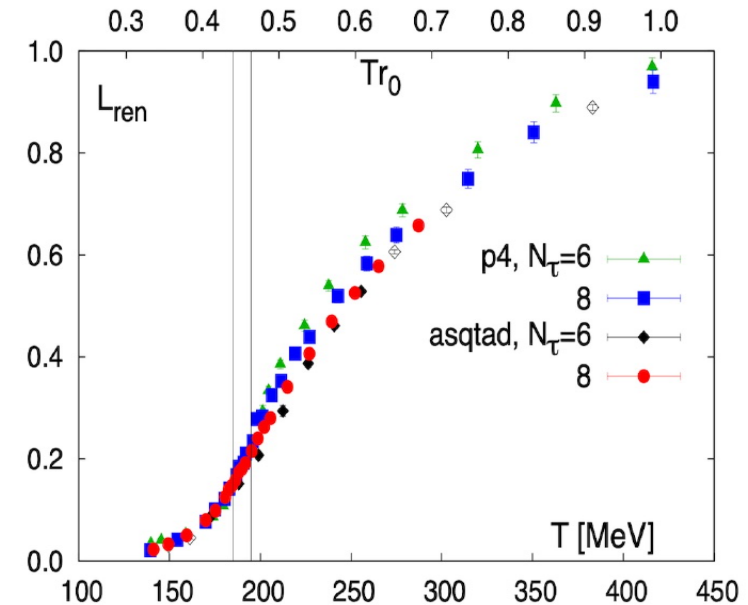
$$g_{tHooft}^2 = g^2 N_c$$

If because glueball contribution is small, we ignore the first term, the second term vanishes in the large N_c limit, so that the new phase of quarks is confined. The quarks are stringy. We call this phase,

Spaghetti of Quarks with Glueballs (SQGB)

But it is not very expensive **Quark Spaghetti with Glue Balls** since there is much spaghetti and almost no glueballs!

Karsch et. al.



Bazavov et. al

The Chiral Transition

$$\langle \bar{\psi}\psi \rangle_T = \langle \bar{\psi}\psi \rangle_0 - \frac{\partial P}{\partial m_q}$$

Take P(M) for a free resonance gas of quarks.

We need to know

$$\sigma = m_q \frac{\partial M}{\partial m_q}$$

For pions:

$$f_\pi^2 m_\pi^2 = -m_q \langle \bar{\psi}\psi \rangle_0$$

$$\sigma_\pi = \frac{m_q}{2m_\pi} \frac{dm_\pi^2}{dm_q} = -\frac{m_q}{2m_\pi f_\pi^2} \langle \bar{\psi}\psi \rangle_0 = m_\pi/2$$

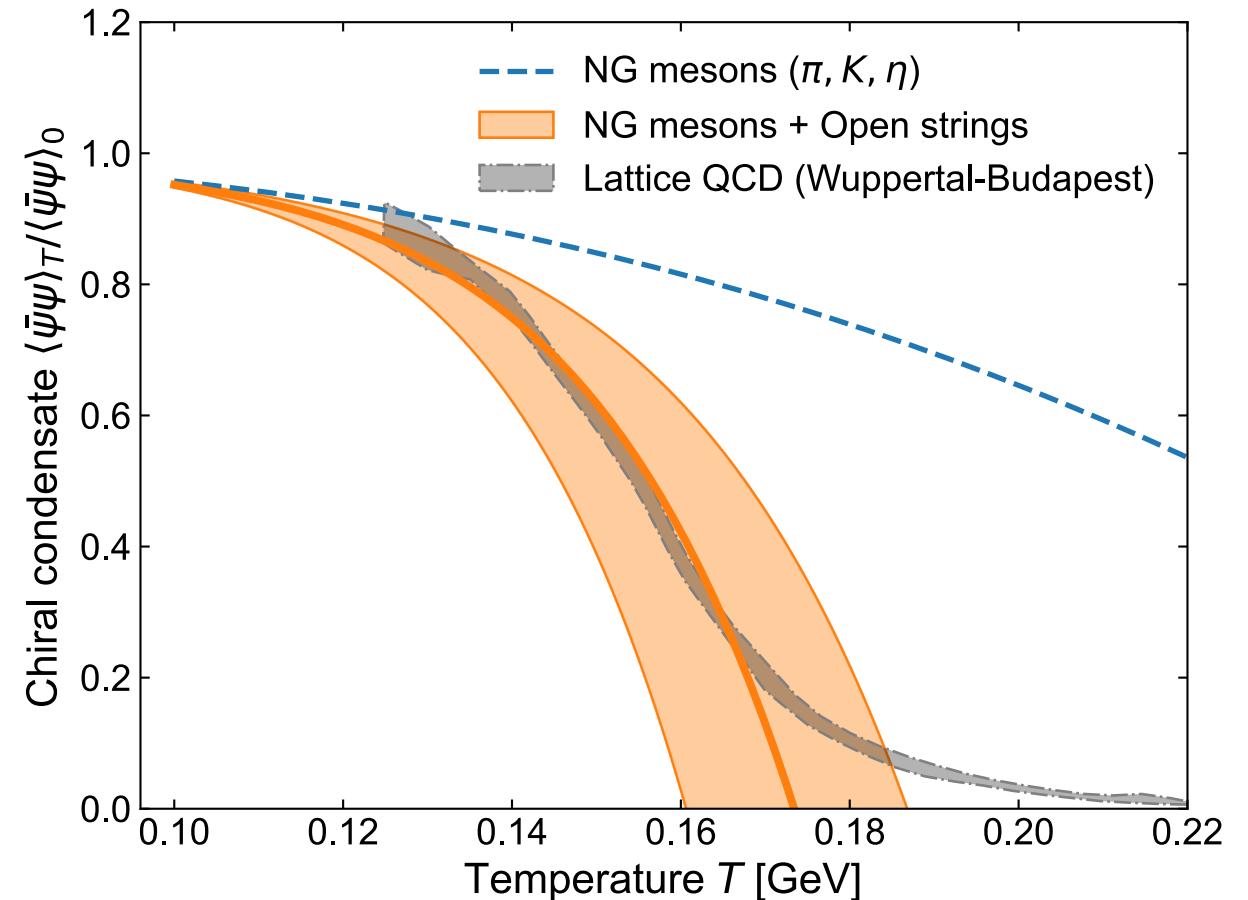
For massive mesons, we fit the sigma value to data on finite temperature values for chiral symmetry.
We assume sigma arises only from low mass quarks, so that

$$\sigma_M = n_q \bar{\sigma}$$

We find best fit is

$$\bar{\sigma} = 60 MeV$$

For Goldstone bosons, we
use current algebra value



SQGB and Quarkyonic Matter

Quarkyonic matter is for low temperatures and finite baryon number density. Because there are no gluons the matter is confined. In ordinary hadronic matter there are no baryons for temperature because

$$e^{-M_B/T} \big|_{T \sim \Lambda_{QCD}} \sim e^{-N_c}$$

Baryons exist as quark-like degrees of freedom because of a chemical potential

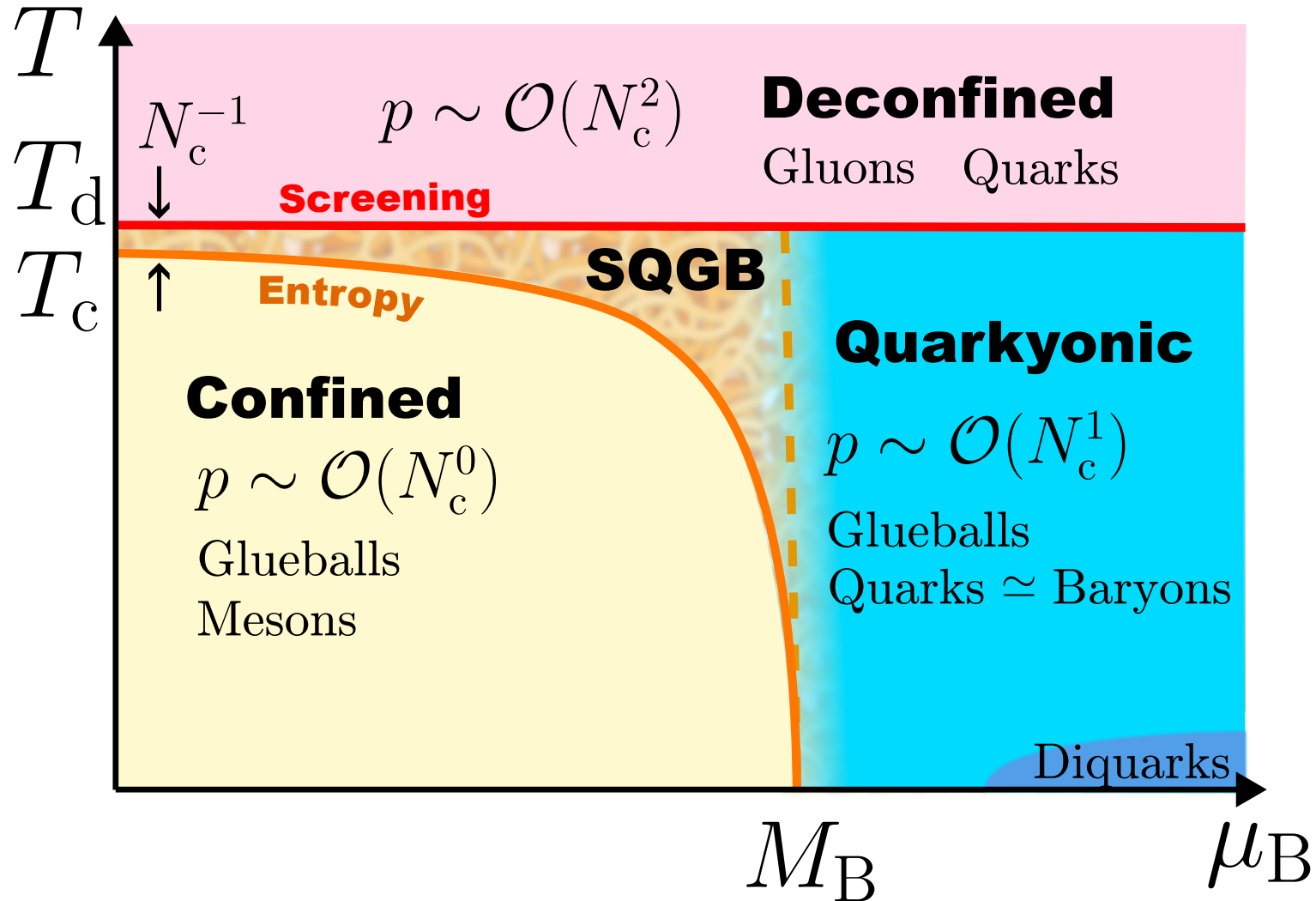
$$\mu_B - M = \kappa T, \kappa \sim 1$$

The density of quarks is of order N_c in Quarkyonic matter.

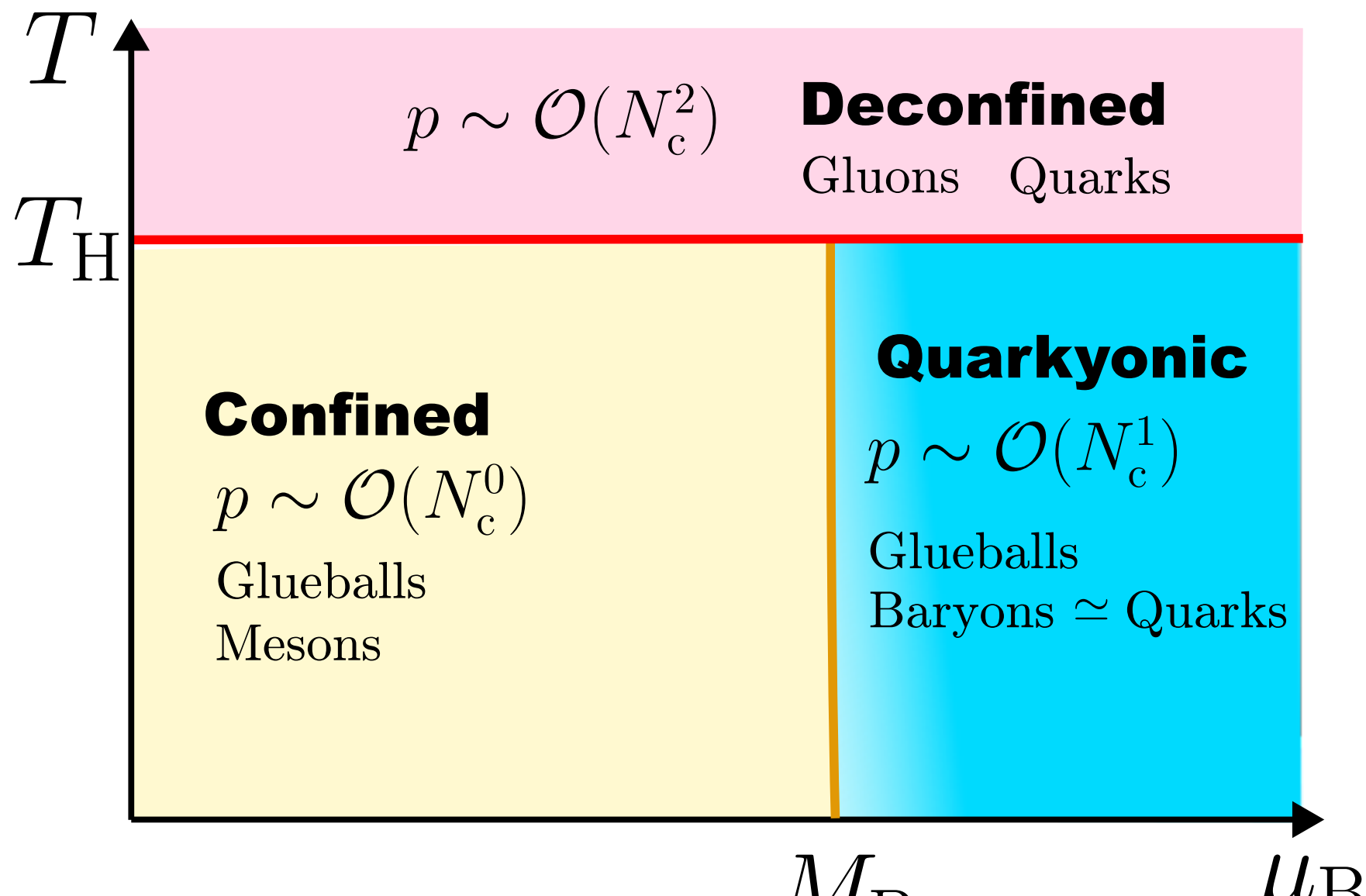
This is very similar to the SQGB, except in the SQGP there are antiquarks.

Naively, Quarkyonic Matter has restored chiral symmetry, but condensate forms at the Fermi surface that break chiral symmetry. The weak chiral symmetry breaking effects differentiate the SQGP from Quarkyonic Matter

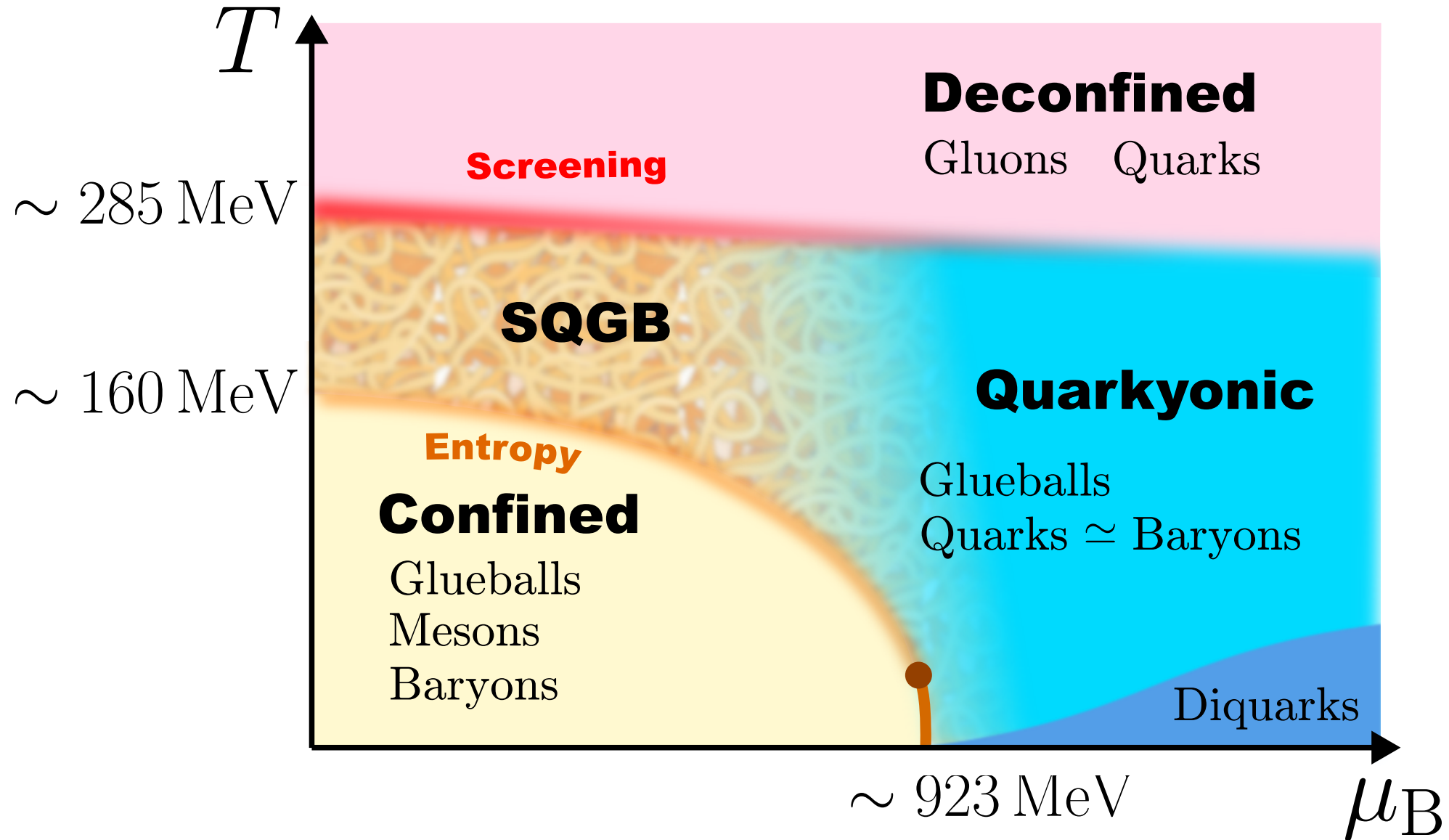
In the limit if a very large number of colors, because the density of quarks diverges, the QGP and SQGB temperatures should both become the Hagedorn temperature



For infinite N_c :

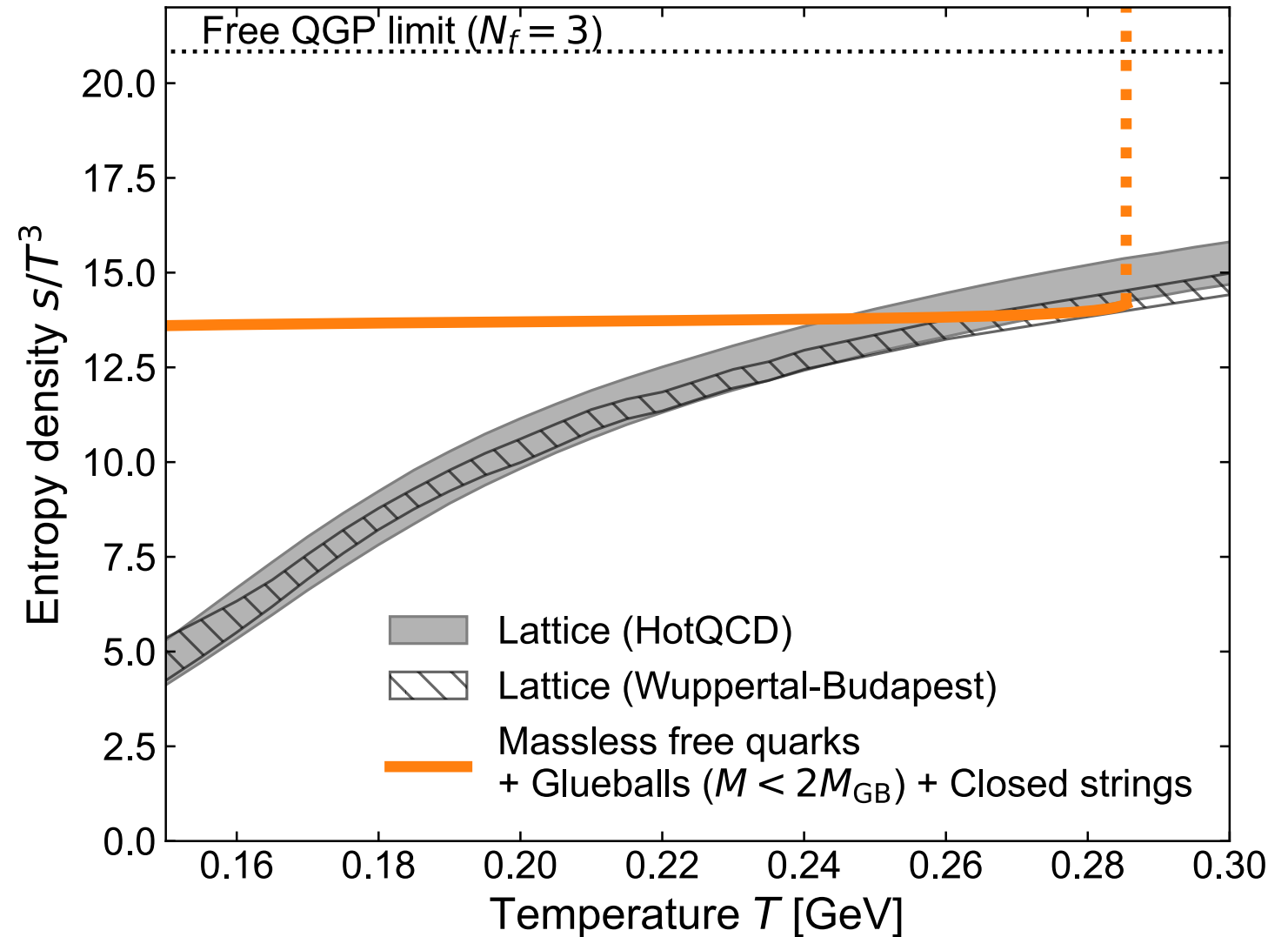


For three colors and realistic parameters



What are the degrees of freedom of this new phase? Certainly quarks. Perhaps the long distance confining nature does not much affect the description of bulk properties of matter as quarks. Interactions are summarized by the trace anomaly, which above 220 MeV is small. Resummed perturbative corrections?

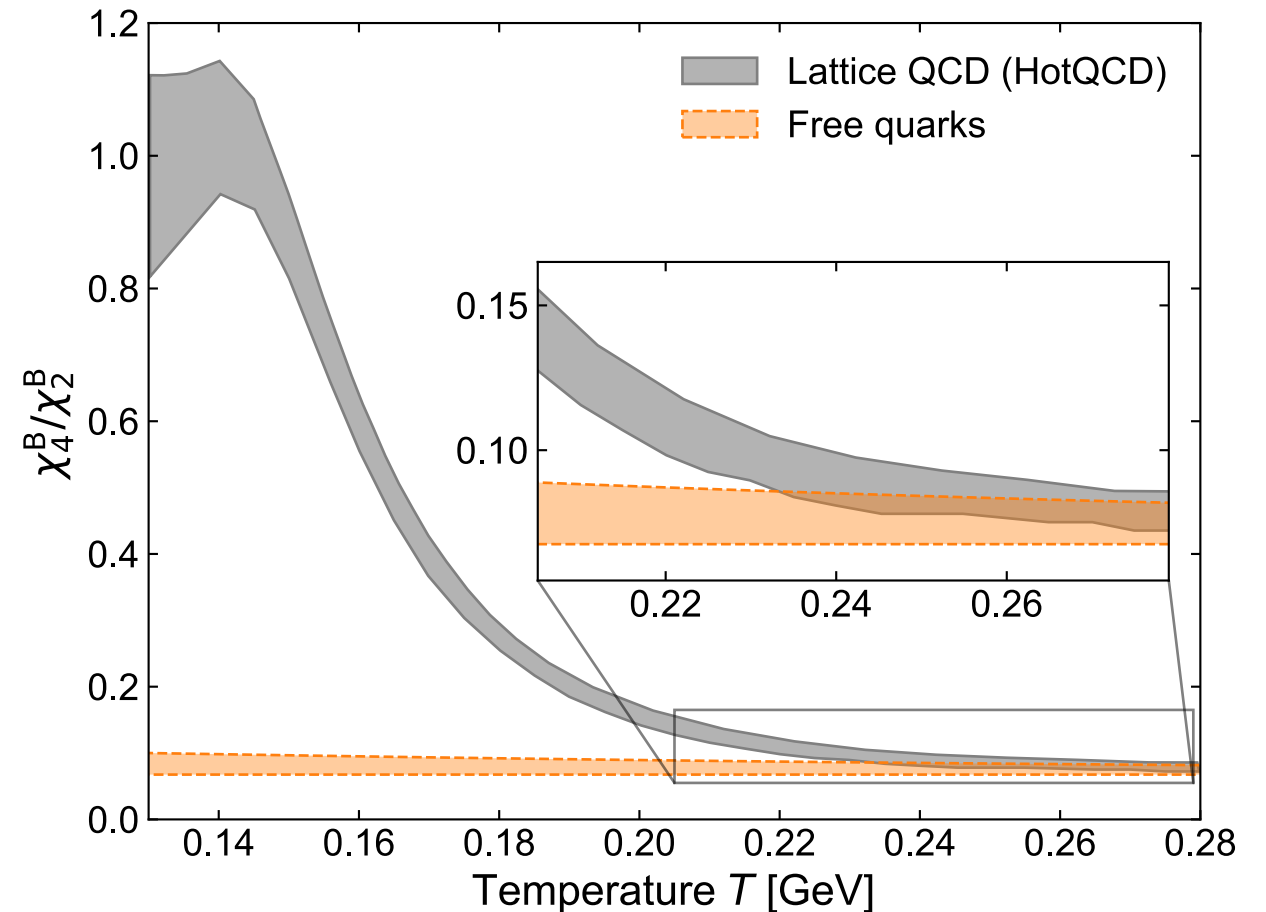
(Vovchenko and Fujimoto will argue that interacting resonance gas matched on to perturbative QCD will well described thermodynamics in intermediate region, with matching about 220 MeV. See also recent work of Pajares et. al.)



And this quasi free quark description would explain susceptibilities.

For Quarkyonic matter, there is an explicit Idylliq model that builds in this dual description either in terms of quarks or nucleons. Quarks remain confined in nucleon, the quarks have a filled Fermi sea with a small contribution from a broadened Fermi surface. At high density, therefore many quantities are correctly computed from a filled Fermi sea of non-interacting quarks.

But all of this involves understanding baryons in the string model, and this has controversy. (But can probably be resolved.)



Generic Comments:

What about transport properties that indicate system is strongly interacting?

Transport cross sections are dominated by infrared cutoff.

True even in Coulomb gas where interactions are cutoff at the Debye scale.

Here the coupling is strong.

Bulk thermodynamic processes are more sensitive to typical quark kinetic energy scales?

What about jet quenching?

No gluons present in intermediate phase, but still have strongish interactions with quarks.

Jet energy loss is very hard to compute.

Charmonium lattice computations shows that charmonium remains bound for T significantly above the chiral symmetry restoration temperature.

General arguments about $T_{\text{chiral}} > T_{\text{confinement}}$, but these arguments are largely about chiral $U(1)$ and have loopholes associated with the lack of Lorentz invariance at finite T .

Lattice computations show that string percolation occurs at T about 300 MeV.