Probing the behavior of strongly interacting matter at the highest densities

S@INT, Institute for Nuclear Theory
01 April 2024

Based on
2312.14127, 2403.03246

In collaboration with
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Tyler Gorda
Goethe University

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Outline

- Introduction
- Constraining the NS EoS at n_{TOV} and beyond (2312.14127)
- Listening to the "long ringdown" in binary NS mergers (2403.03246)

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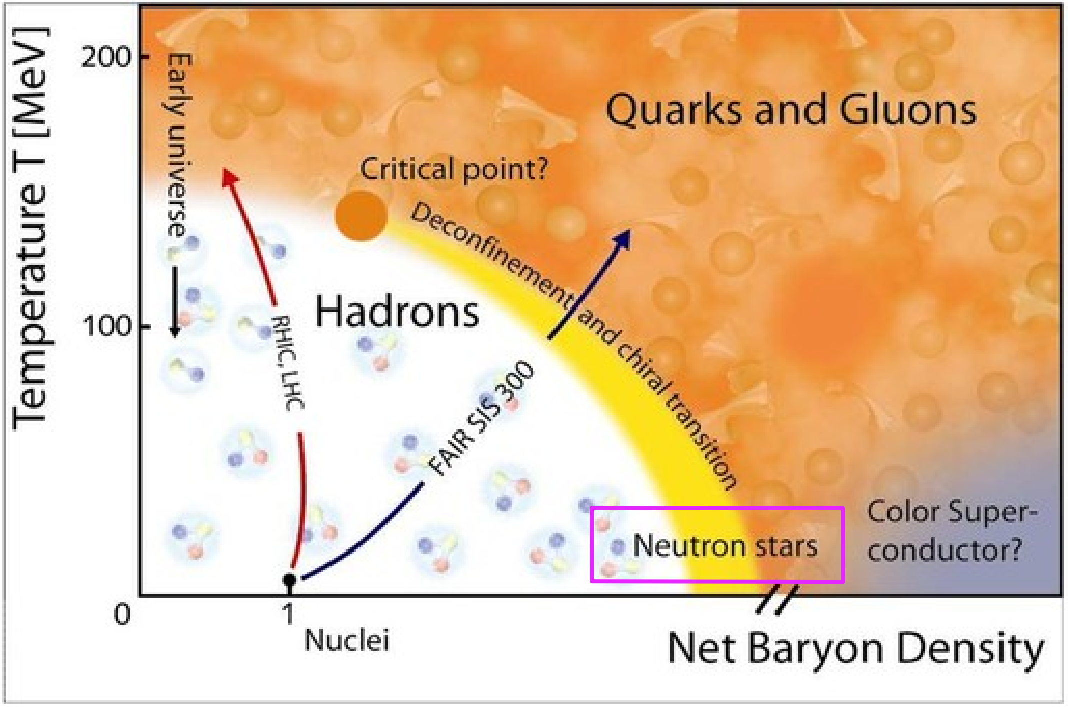
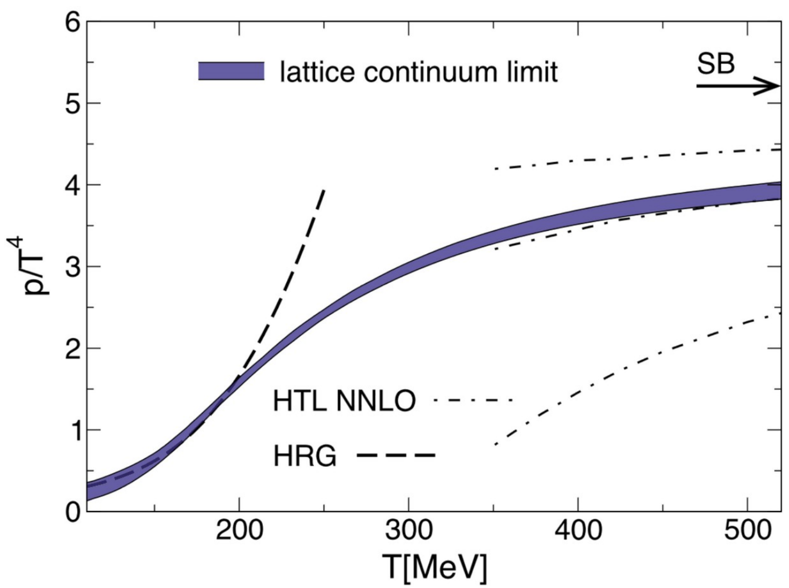
Understanding the phase diagram of QCD

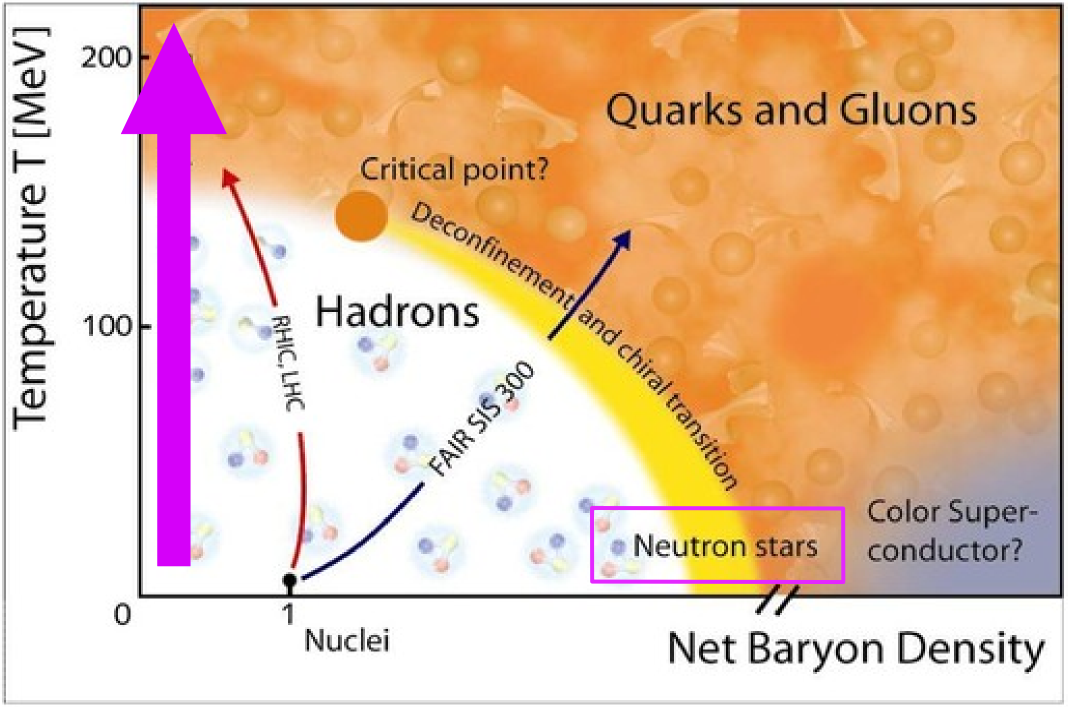


Compressed Baryonic Matter (CBM) experiment

Understanding the phase diagram of QCD



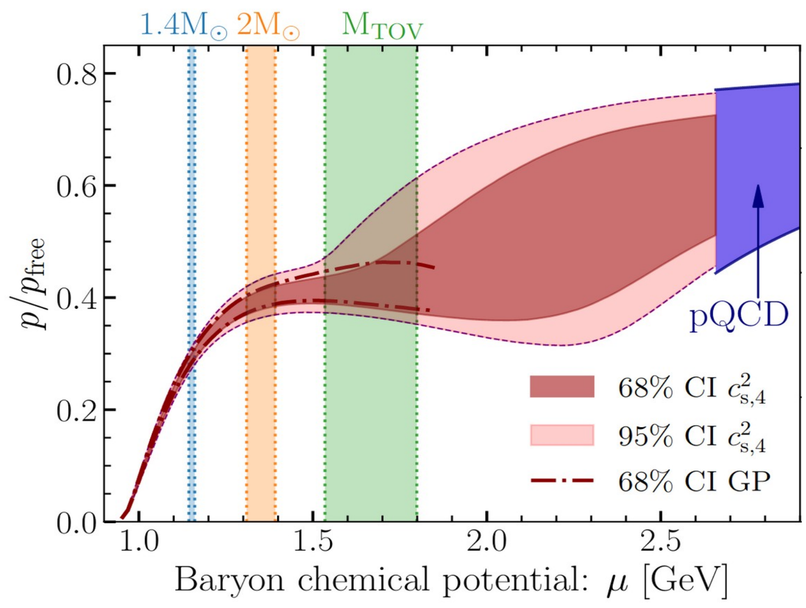
Borsanyi, Fodor, Hoelbling, Katz, Krieg, Szabo
Phys. Lett. B 370 (2014)



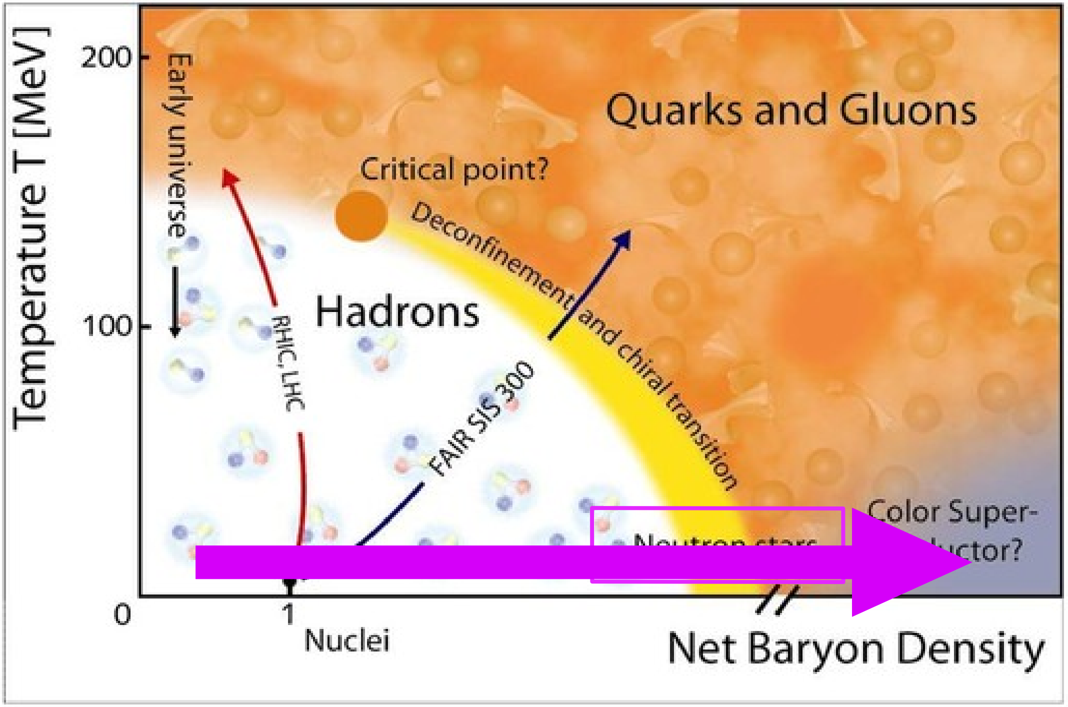
Compressed Baryonic Matter (CBM) experiment

Lattice calculations at high-T
directly tell us the EoS

Understanding the phase diagram of QCD



Annala, Gorda, Hirvonen, Komoltsev, Kurkela, Nättilä, Vuorinen Nat. Comm. 14 (2023)

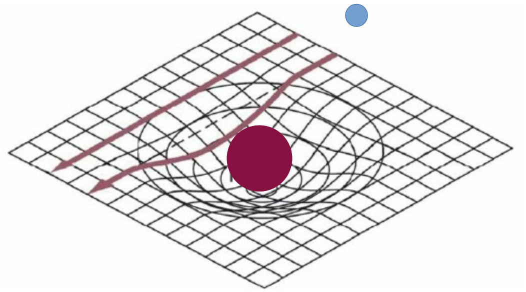


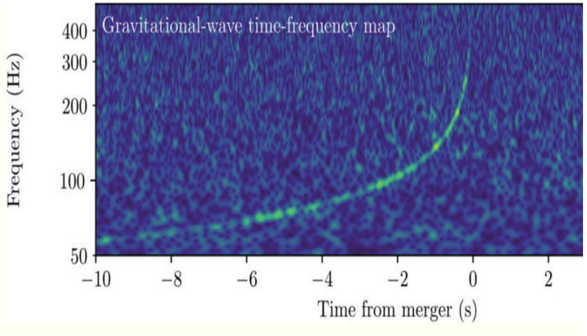
Compressed Baryonic Matter (CBM) experiment

At T = 0, no lattice, but we have **astrophysics**; and calculations in **nuclear and particle theory**

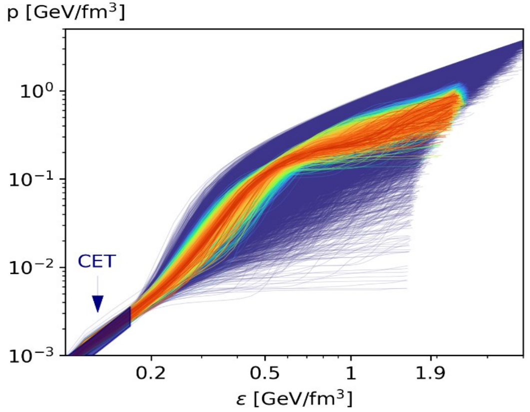
T = 0 is a synthesis of theory and experiment

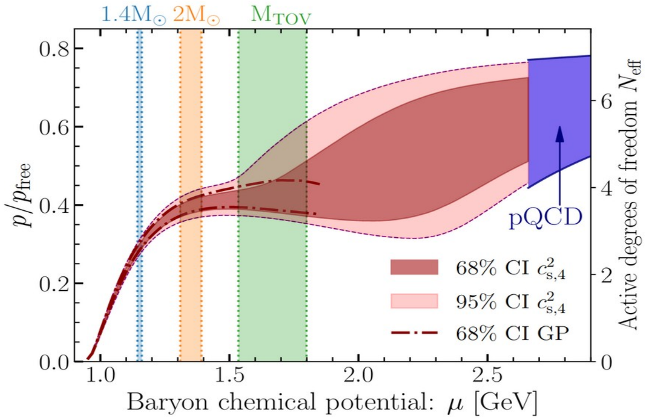
Astronomy +Astrophysics



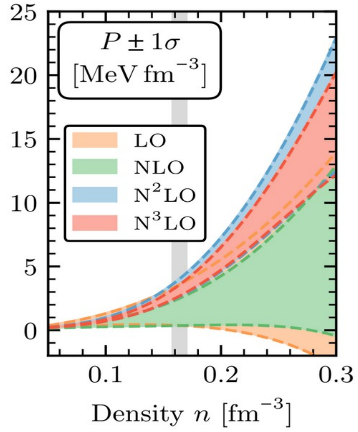


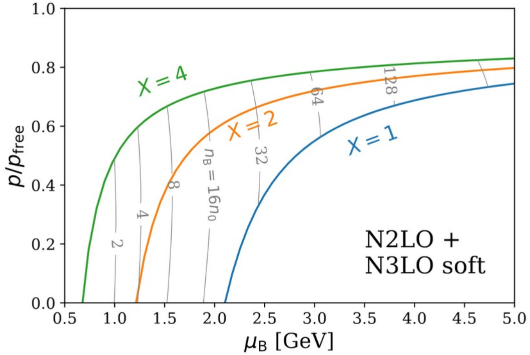
EoS Inference





Nuclear + Particle Theory

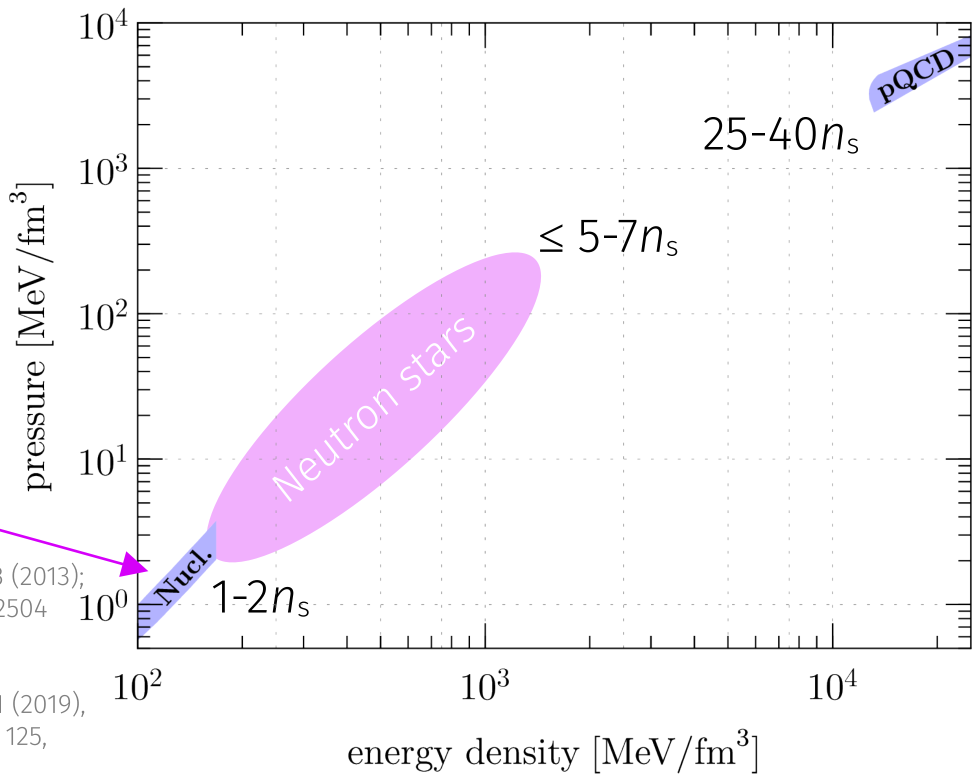




Different inputs constrain different parts of the EOS

Perturbative QCD

Kurkela, Romatschke, Vuorinen, PRD 81, 105021 (2010),
Gorda, Kurkela, Romatschke, Säppi, Vuorinen, PRL 121, 202701 (2018),
Gorda, Kurkela, Paatelainen, Säppi, Vuorinen, PRL 127, 162003 (2021), PRD 104, 074015 (2021),
Gorda, Paatelainen, Säppi, Seppänen PRL 131 (2023)



Chiral EFT

Hebeler, Lattimer, Pethick, Schwenk. ApJ. 773 (2013);
Tews, Krüger, Hebeler, Schwenk, PRL 110, 032504 (2013)
Lynn, Tews, Carlson+ PRL 116, 062501 (2016),
Drischler, Hebeler, Schwenk, PRL 122, 042501 (2019),
Drischler, Furnstahl, Melendez, Phillips, PRL 125, 202702 (2020),
Keller, Hebeler, and Schwenk, PRL 130, 072701 (2023).

Different inputs constrain different parts of the EOS

Thermodynamics

Komoltsev, Kurkela PRL 128, 202701 (2022)

Chiral EFT

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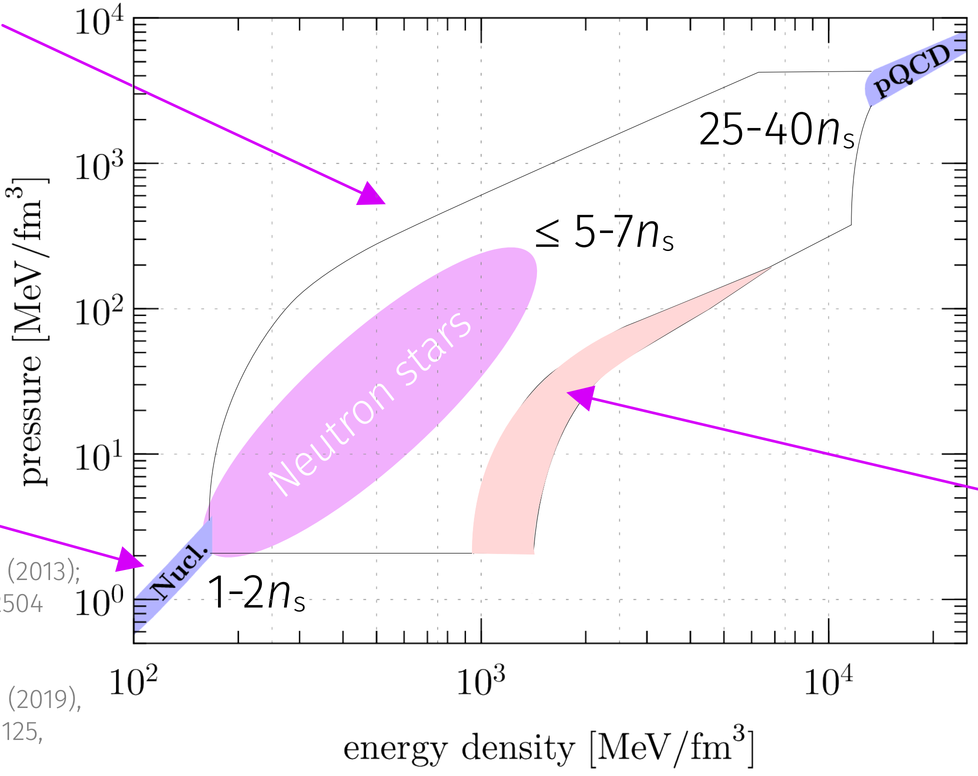

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Gorda, Kurkela, Paatelainen, Säppi, Vuorinen, PRL 127, 162003 (2021), PRD 104, 074015 (2021),
Gorda, Paatelainen, Säppi, Seppänen PRL 131 (2023)

Lattice (Isospin QCD)

Abbot, Detmold, Romero-López+ 2307.15014,
Brandt, Cuterib, Endrődi JHEP 07 (2023) 055

Fujimoto, Reddy, PRD 109, 014020 (2024),

(see also Moore, Gorda, JHEP 12 (2023) 133,
Navarrete, Paatelainen, Seppänen 2403.02180)

Chiral EFT + pQCD + Thermodynamics constrain extreme EoSs

1. Stability

$$\partial_\mu^2 \Omega(\mu) \leq 0 \implies \partial_\mu n(\mu) \geq 0$$

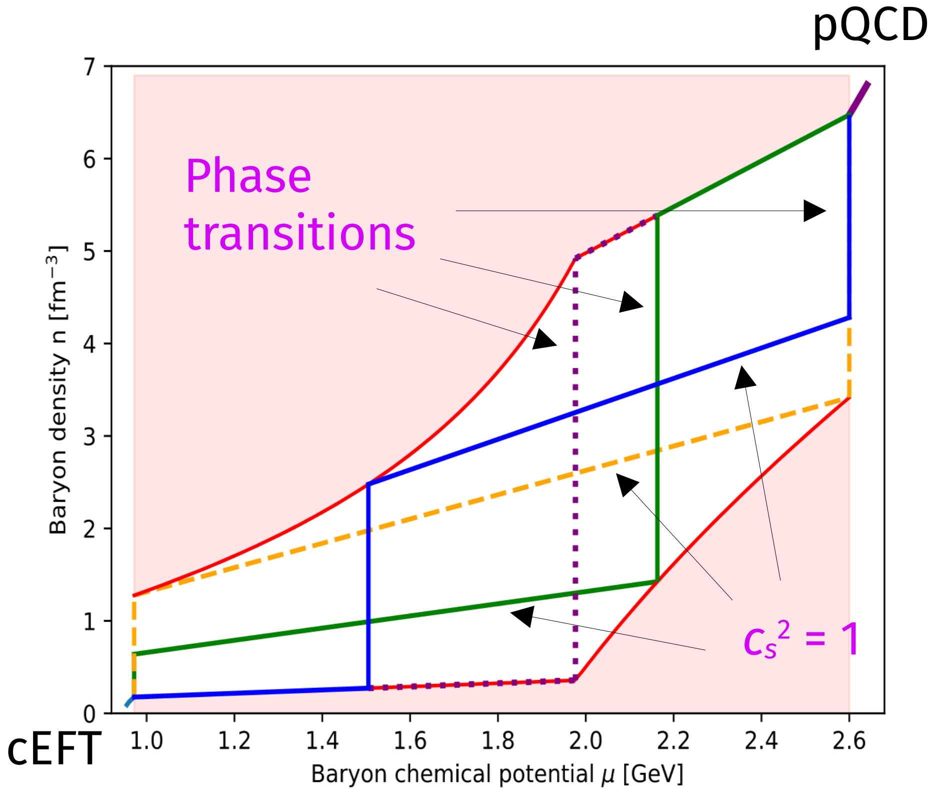
2. Causality

$$c_s^{-2} = \frac{\mu}{n}\frac{\partial n}{\partial \mu} \geq 1 \implies \partial_\mu n(\mu) \geq \frac{n}{\mu}$$

3. Consistency

$$\int_{\mu_{\mathrm{CET}}}^{\mu_{\mathrm{QCD}}} \mathrm{d}\mu\, n(\mu) = p_{\mathrm{QCD}} - p_{\mathrm{CET}}$$ Fixed!

"integral constraints"



Komoltsev and Kurkela, PRL 128 (2022)

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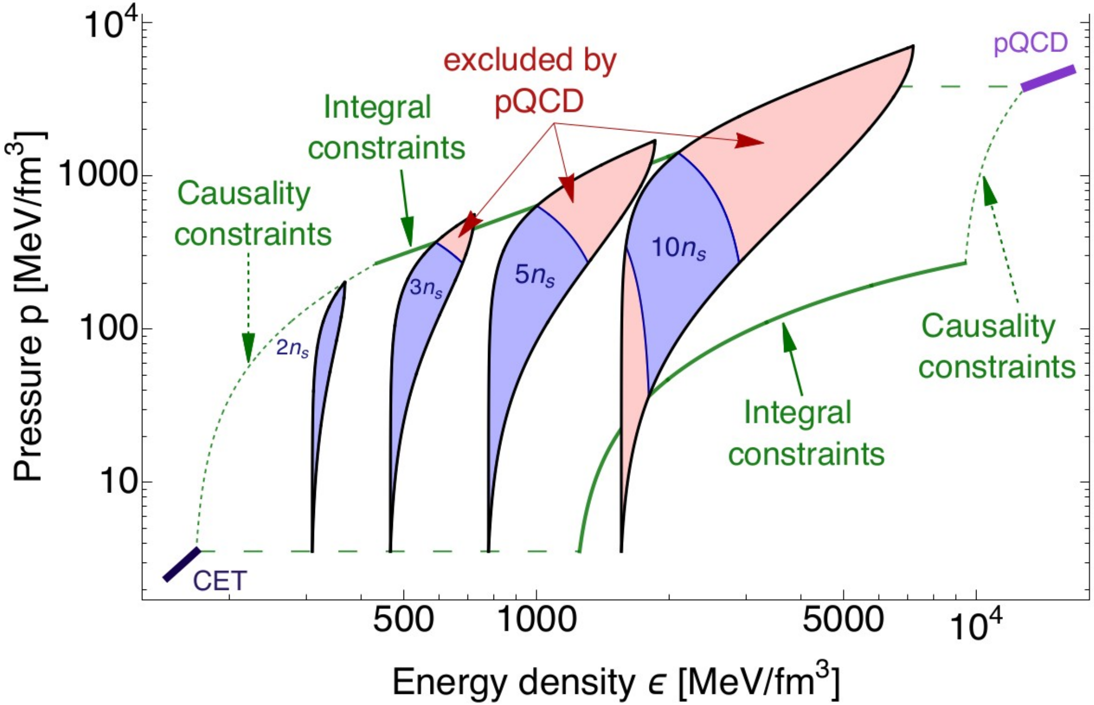


Region of (ε, p) at fixed n constrained by general principles

Komoltsev and Kurkela, PRL 128 (2022)

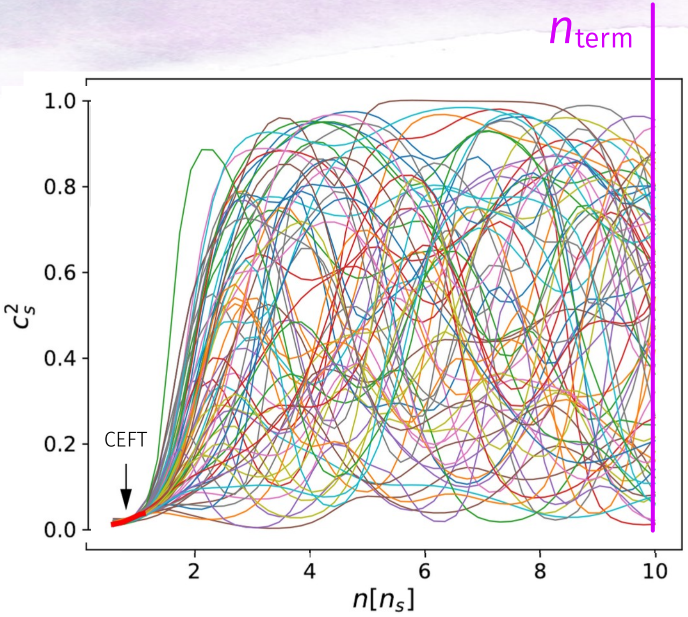
Bayesian EoS inference setup

Likelihood of EoS
given data + theory

EoS Prior

$$P(\mathrm{EoS}\,|\,\mathrm{data}) = \frac{P(\mathrm{EoS})P(\mathrm{data}\,|\,\mathrm{EoS})}{P(\mathrm{data})}$$

Bayesian EoS inference setup



Gorda, Komoltsev, Kurkela, ApJ 950 (2023)

e.g.

Likelihood of EoS given data + theory

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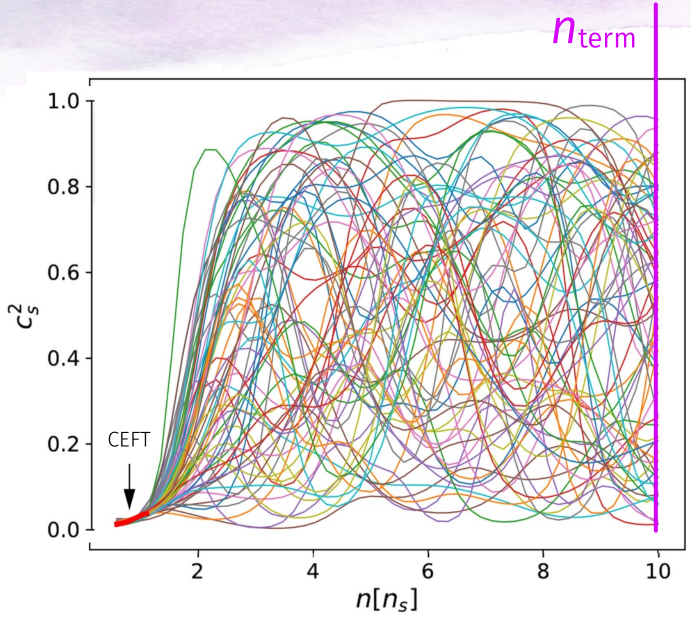

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Gorda, Komoltsev, Kurkela, ApJ 950 (2023)

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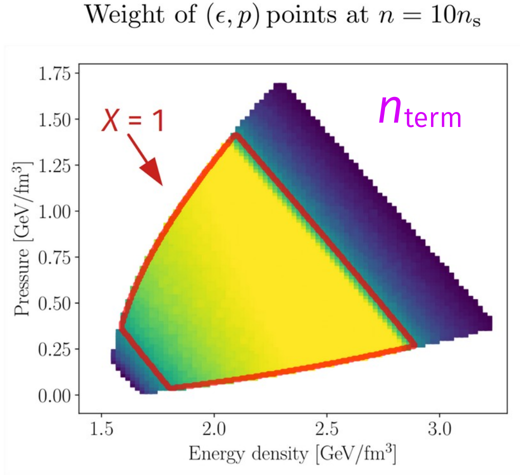
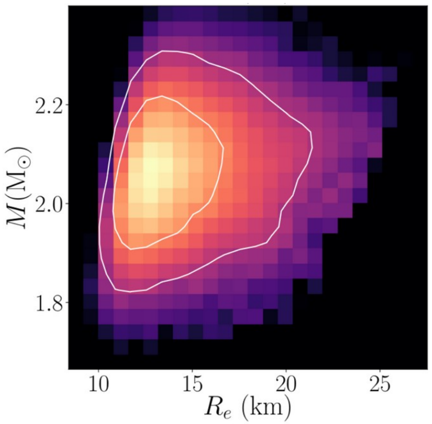
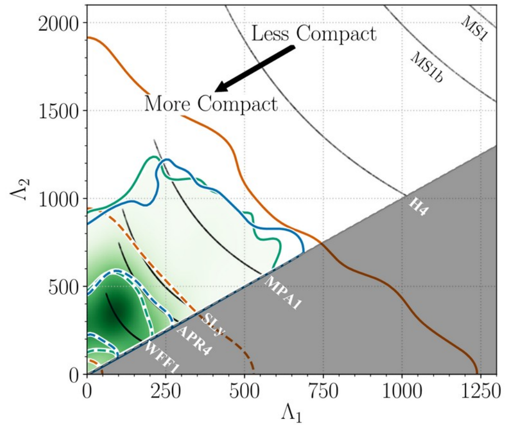
e.g.







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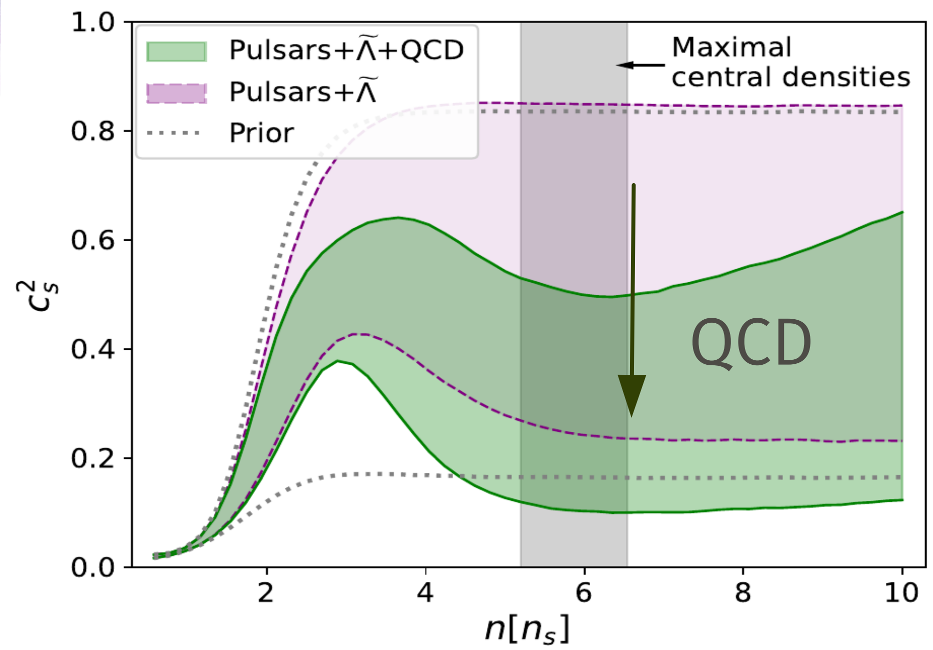
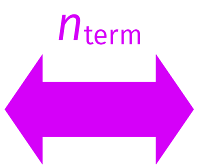
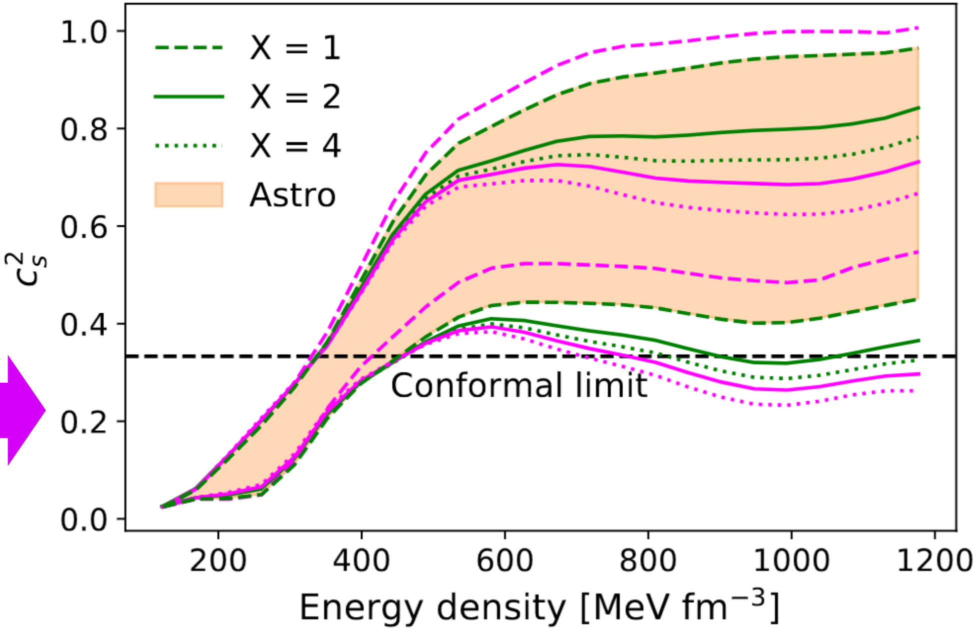
What does pQCD add to astrophysical data?







Softening seen in works applying pQCD at $\geq 10 n_{\text{sat}}$

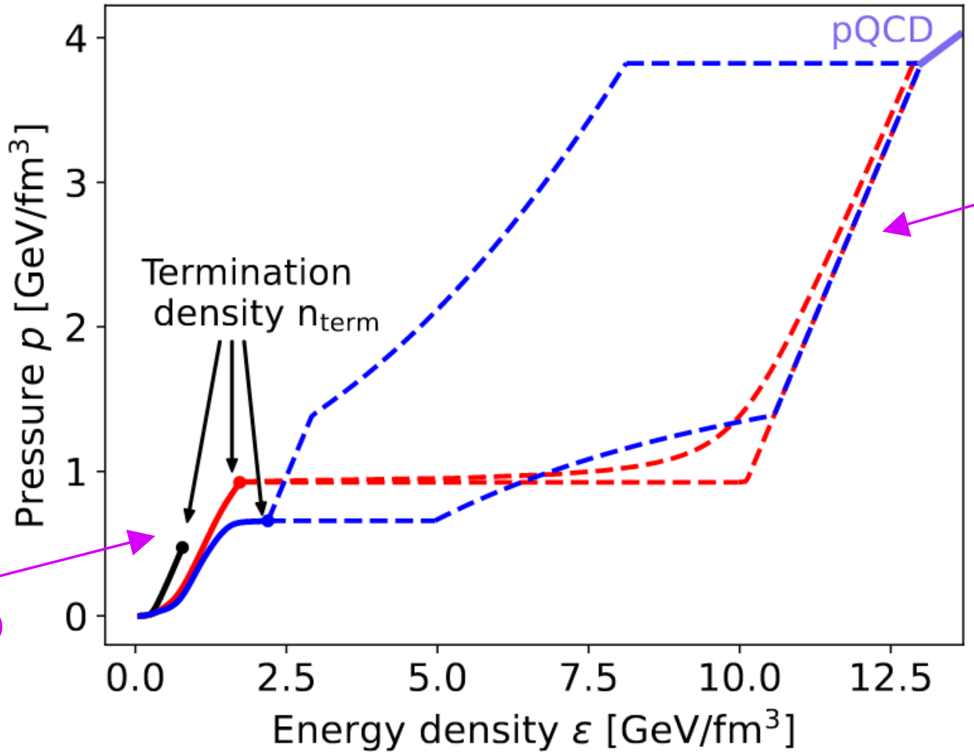
Annala, Gorda, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020), Altiparmak, Ecker, Rezzolla, ApJ.Lett. 939 (2022); Ecker & Rezzolla, ApJ.Lett. 939 (2022); Fujimoto, Fukushima, McLerran, Praszalowicz, PRL 129 (2022); Marczenko, McLerran, Redlich, Sasaki, PRC 107 (2023);

No softening seen in works applying pQCD at n_{TOV}

Somasundaram, Tews, Margueron PRC 107 (2023); Brandes, Weise, Kaiser PRD 108 (2023); Essick, Legred, Chatziioannou, PRD 108 (2023) Mroczek, Miller, Noronha-Hostler, Yunes 2309.02345

Quantifying the *tension* with perturbative QCD

Komoltsev, Somasundaram, Gorda, Kurkela, Margueron, Tews 2312.14127



Thermodynamics restricts EoS behavior beyond n_{term}

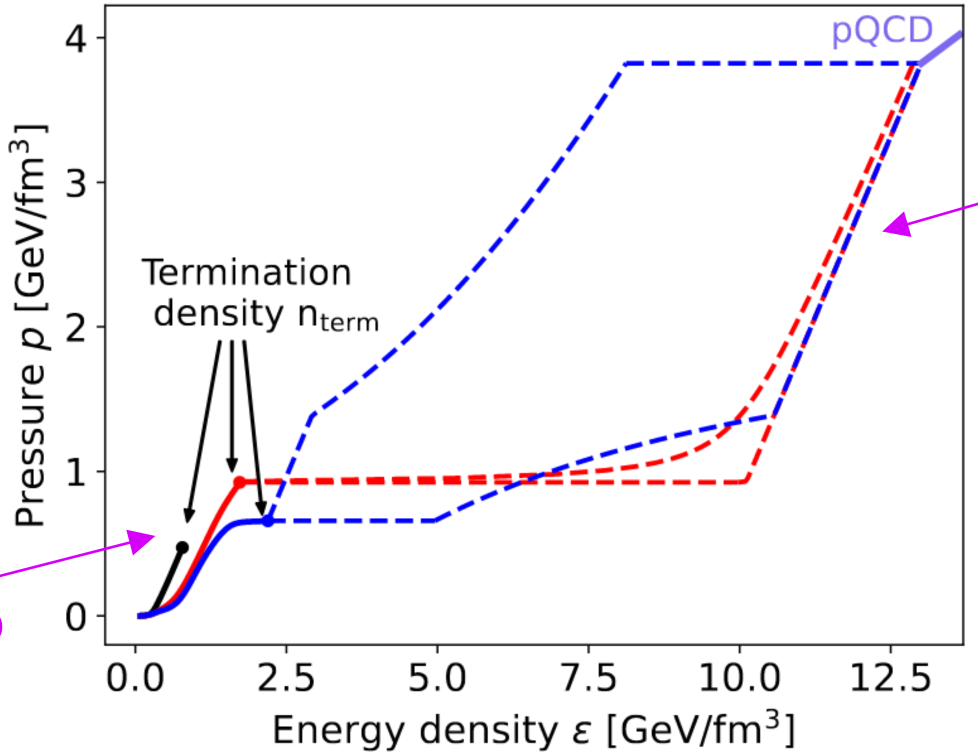
EoS modeled up to n_{term} using:

1. Gaussian process for c_s^2
2. Piecewise linear c_s^2

Gorda, Komoltsev, Kurkela, ApJ 950 (2023),
Somasundaram, Tews, Margueron PRC 107, L052801 (2023).

Quantifying the *tension* with perturbative QCD

Komoltsev, Somasundaram, Gorda, Kurkela, Margueron, Tews 2312.14127



Thermodynamics restricts EoS behavior beyond n_{term}

Introduce pQCD tension index I_{pQCD} to quantify how close to excluded a point is:

$$0 \leq I_{pQCD} \equiv \frac{\Delta p - \Delta p_{min}}{\Delta p_{max} - \Delta p_{min}} \leq 1$$

(stiffer EOSs have larger I_{pQCD})

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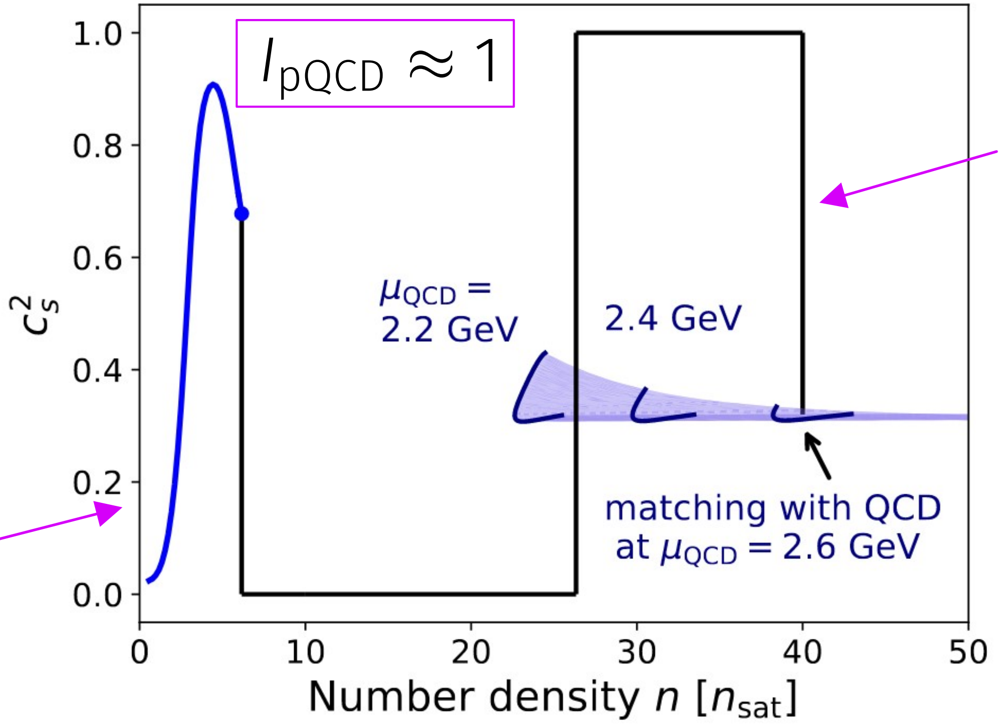
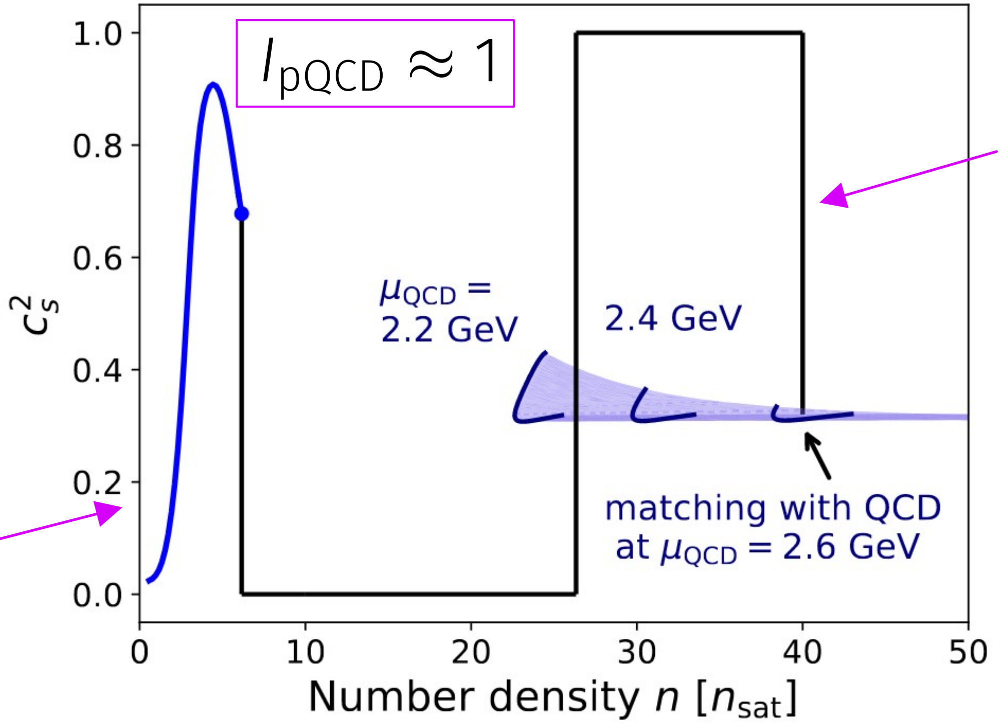


Thermodynamics restricts EoS behavior beyond $n_{\rm term}$

EoS modeled up to $n_{\rm term}$

Quantifying the *tension* with perturbative QCD

Komoltsev, Somasundaram, Gorda, Kurkela, Margueron, Tews 2312.14127



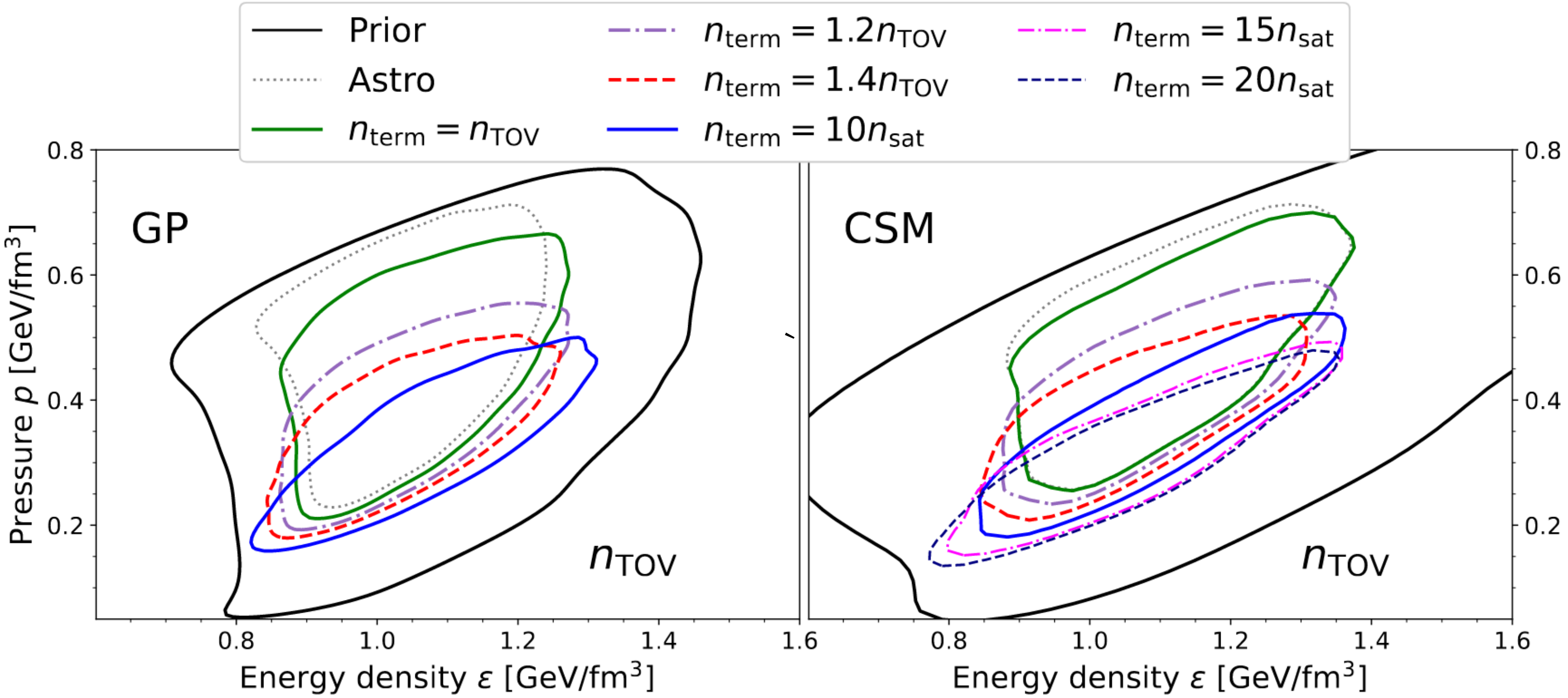
EoS modeled up to $n_{\rm term}$

Thermodynamics restricts EoS behavior beyond $n_{\rm term}$

Observation: High pQCD tension index also inconsistent with well-converged pQCD c_s^2

Softening of the EOS at n_{TOV} depends on n_{term}

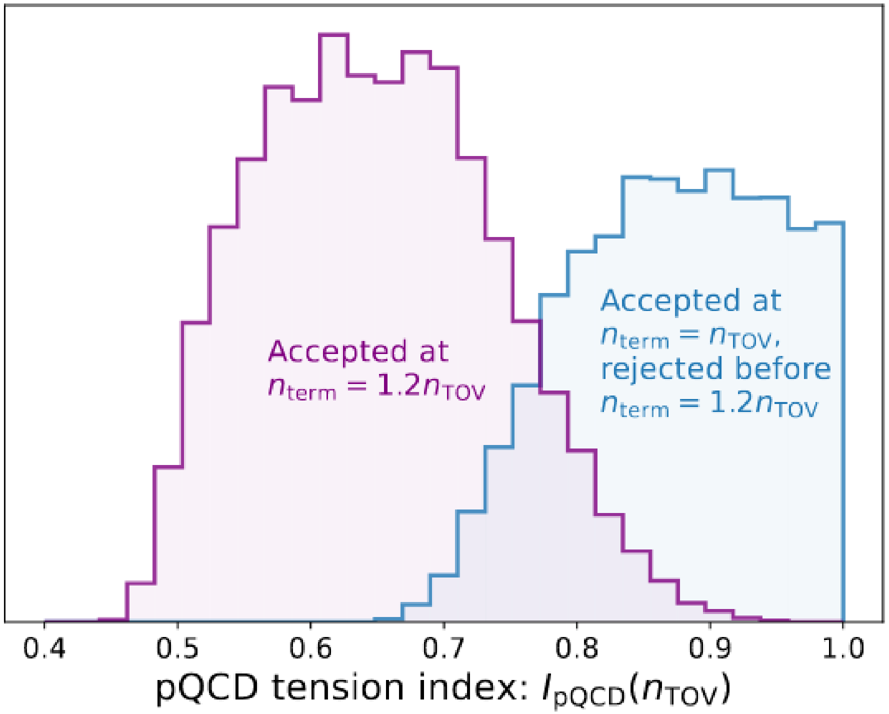
Komoltsev, Somasundaram, Gorda, Kurkela, Margueron, Tews 2312.14127

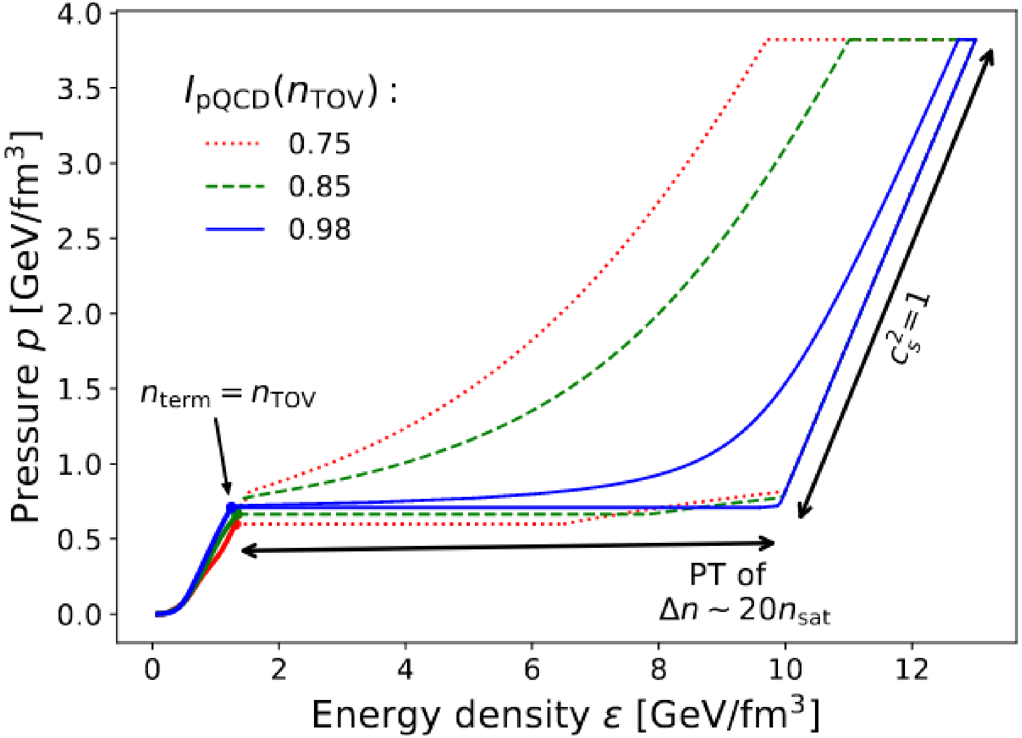


Noticeable softening for $n_{term} > n_{TOV}$, but $n_{term}=n_{TOV}$ unclear

Examining $n_{\text{term}} = n_{\text{TOV}}$: Many EOSs have high I_{pQCD}

Komoltsev, Somasundaram, Gorda, Kurkela, Margueron, Tews 2312.14127

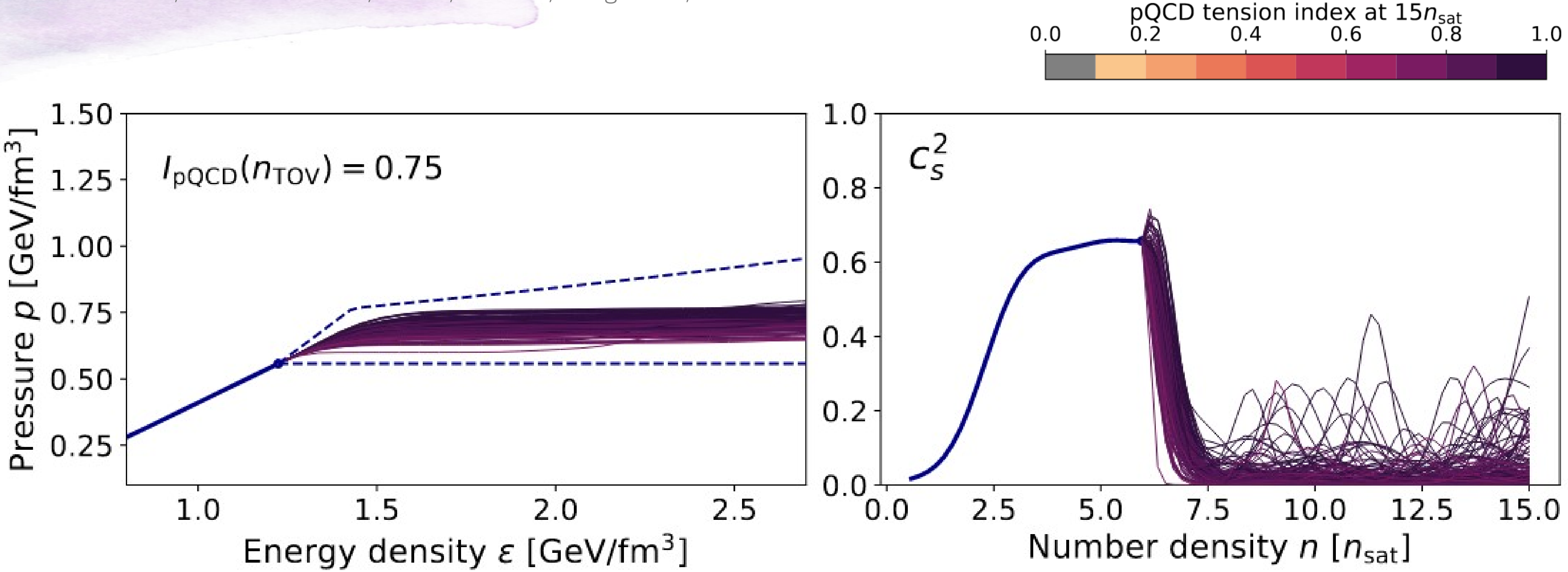




Allowed extensions for stiff EoSs have tightly constrained region beyond n_{TOV}

Extending stiff EoSs indeed show strong, prolonged softening

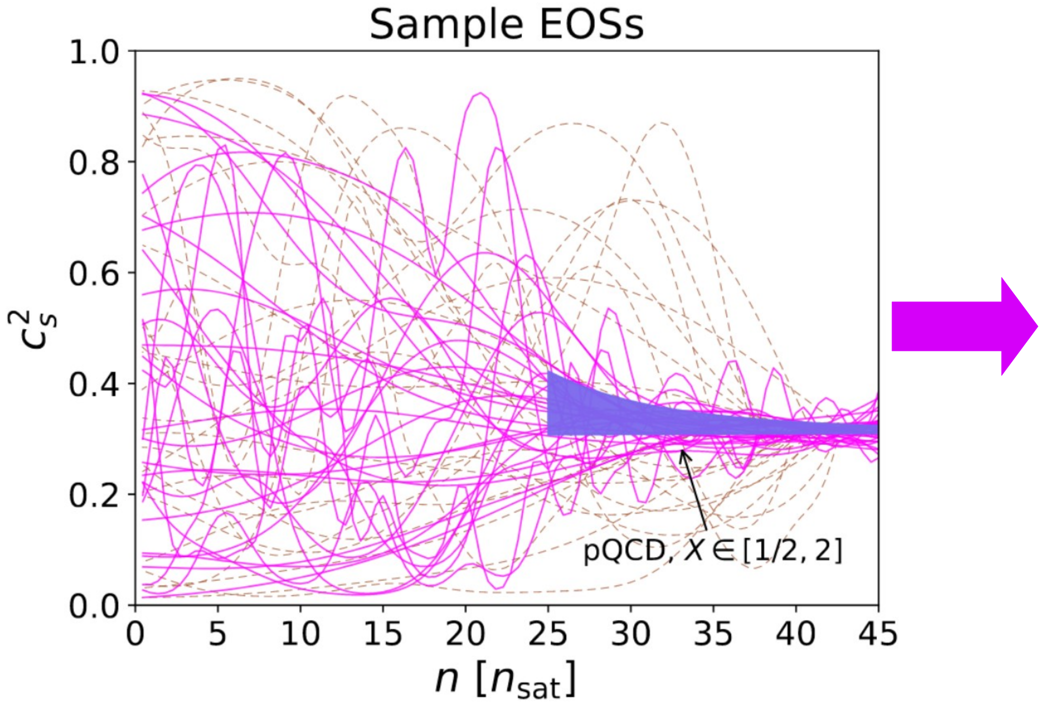
Komoltsev, Somasundaram, Gorda, Kurkela, Margueron, Tews 2312.14127



Takeaway: Softening before n_{TOV} **OR** Strong, prolonged softening just after, **followed by $c_s^2 \approx 1$**

Conditioning with pQCD c_s^2 leads to robust softening

Komoltsev, Somasundaram, Gorda, Kurkela, Margueron, Tews 2312.14127



Can use 2nd GP to marginalize over EoS extensions and condition with pQCD c_s^2

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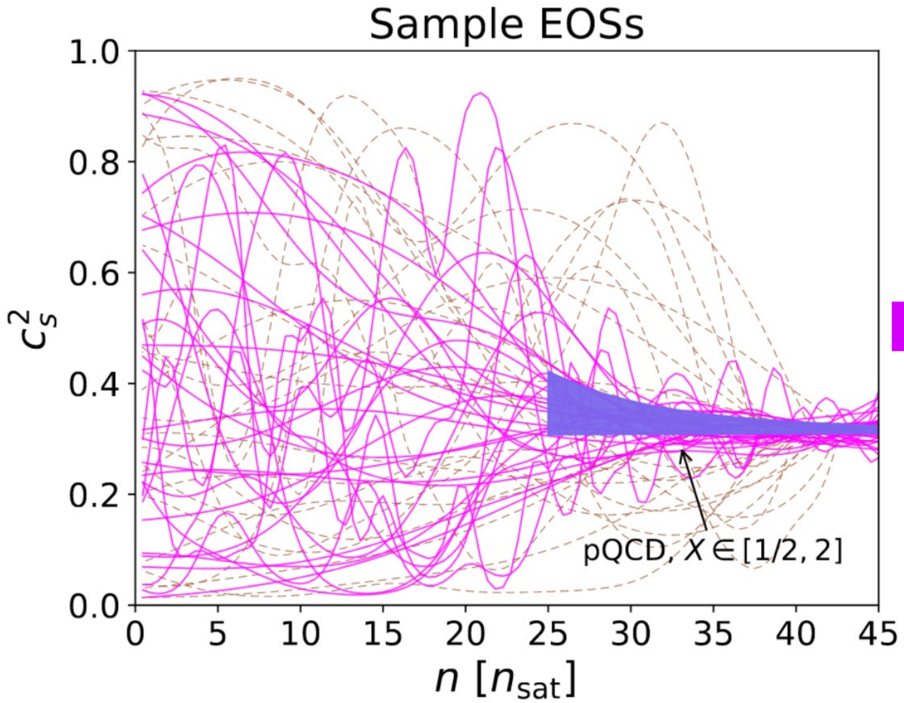

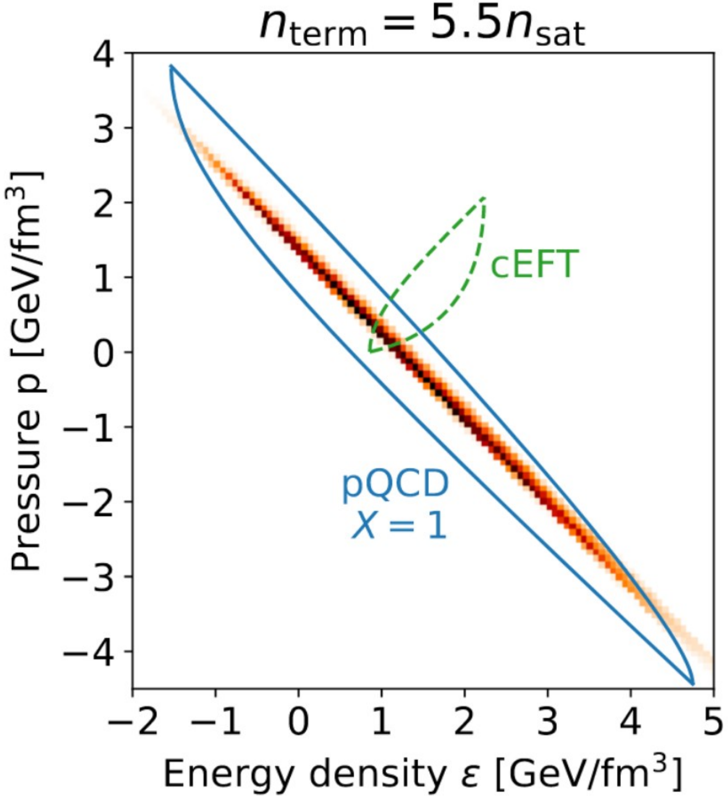

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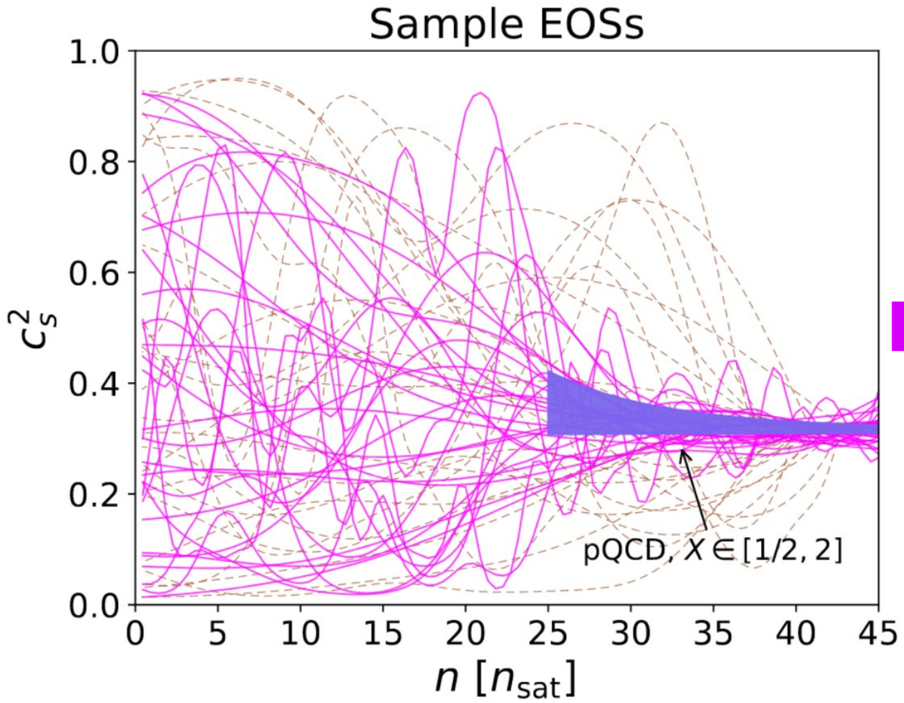

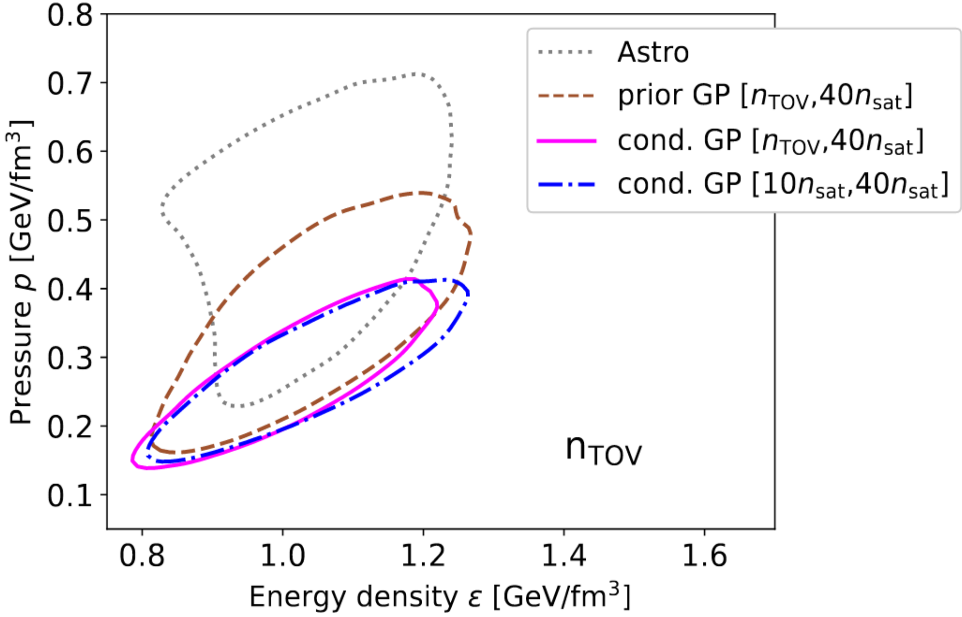

Takeaway: marginalized QCD input yields results independent of n_{term}

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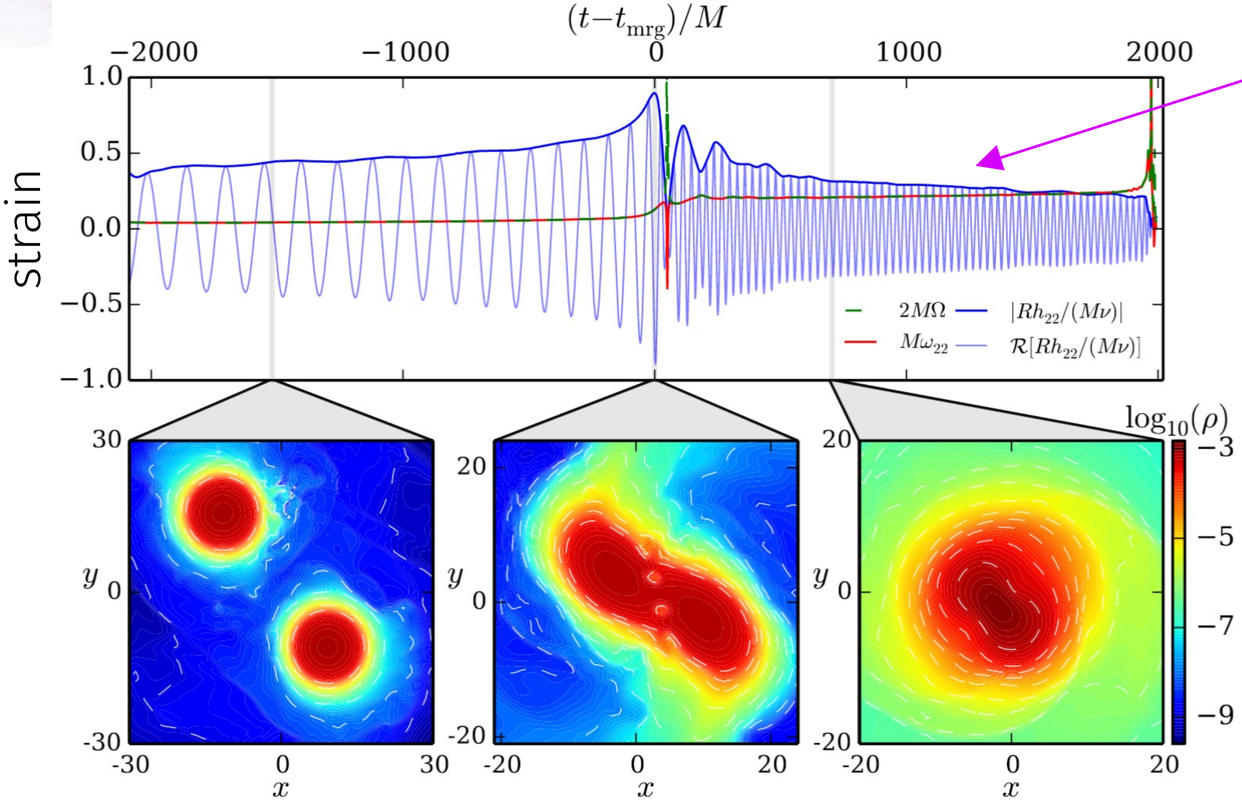
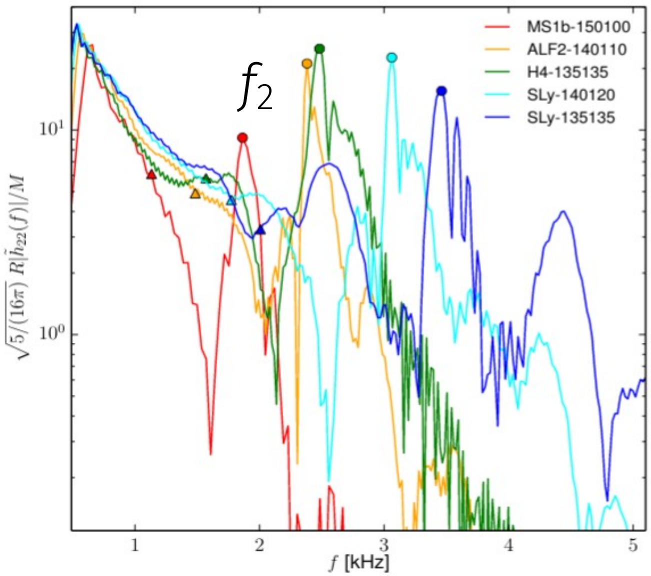
Evolution of a BNS merger – focus on post-merger phase



Power Spectral Density



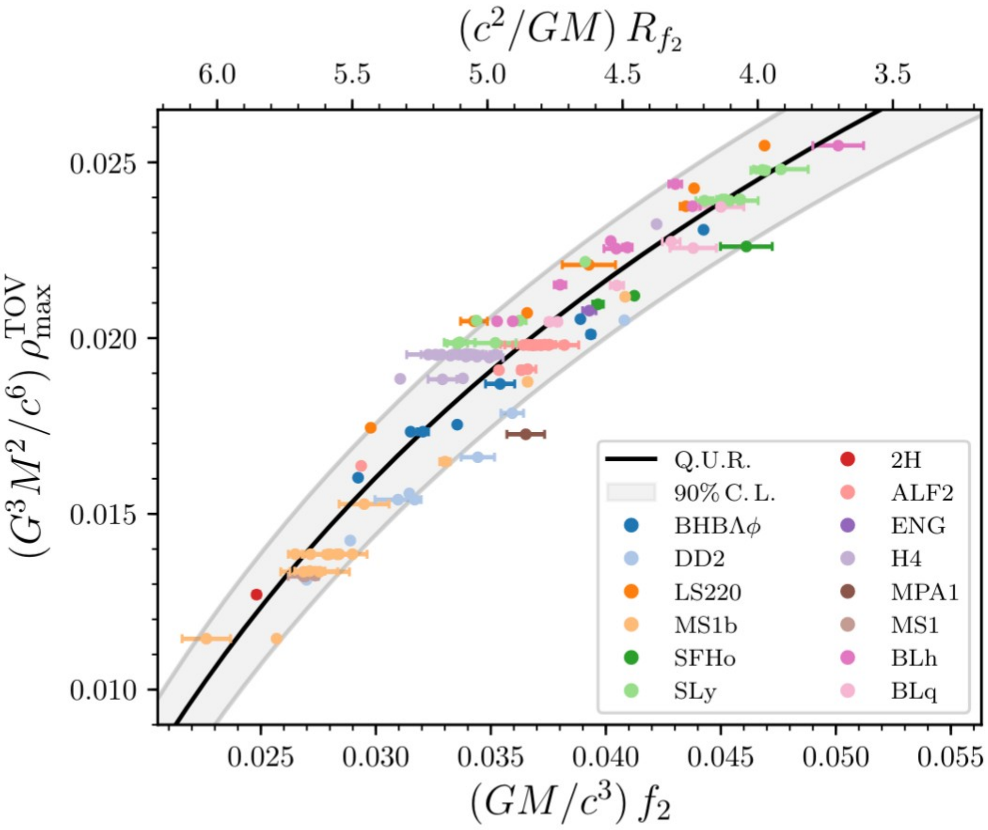
Peaks (*“f_2 frequency”*) associated with post-merger (quasistationary evolution)

Bernuzzi, Dietrich, Nagar, PRL 115, 091101 (2015);
see also Takami, Rezzolla, Baiotti PRD 91, 064001 (2015), many others

Known correlations between f_2 and the EOS

Past works using individual models have seen correlation between f_2 and the underlying EOS

Bauswein, Stergioulas, PRD 91, 124056 (2015);
Takami, Rezzolla, Baiotti PRD 91, 064001 (2015);
Rezzolla, Takami, PRD 93, 124051 (2016);
Bauswein, Nikolaos Stergioulas, J. Phys. G: Nucl. Part. Phys. 46 113002 (2019)
...



Breschi, Bernuzzi, Godzieba+ PRL 128 (2022)

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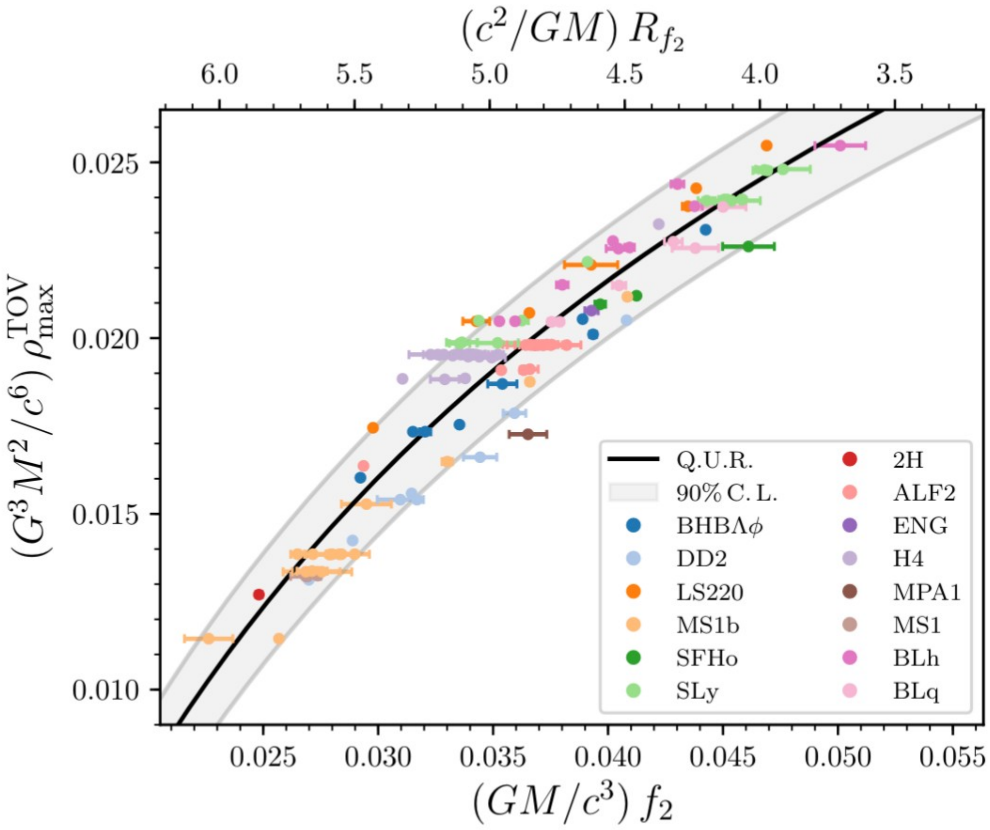
Rezzolla, Takami, PRD 93, 124051 (2016);
Bauswein, Nikolaos Stergioulas, J. Phys. G: Nucl. Part. Phys. 46 113002 (2019)
...

Goal:

Can we perform a model-agnostic analysis of the post-merger evolution, using the same GP ensemble as before?

Can we find other correlations in the post-merger phase?



Breschi, Bernuzzi, Godzieba+ PRL 128 (2022)

Selecting a small, smart sample of model-agnostic EOSs (1)

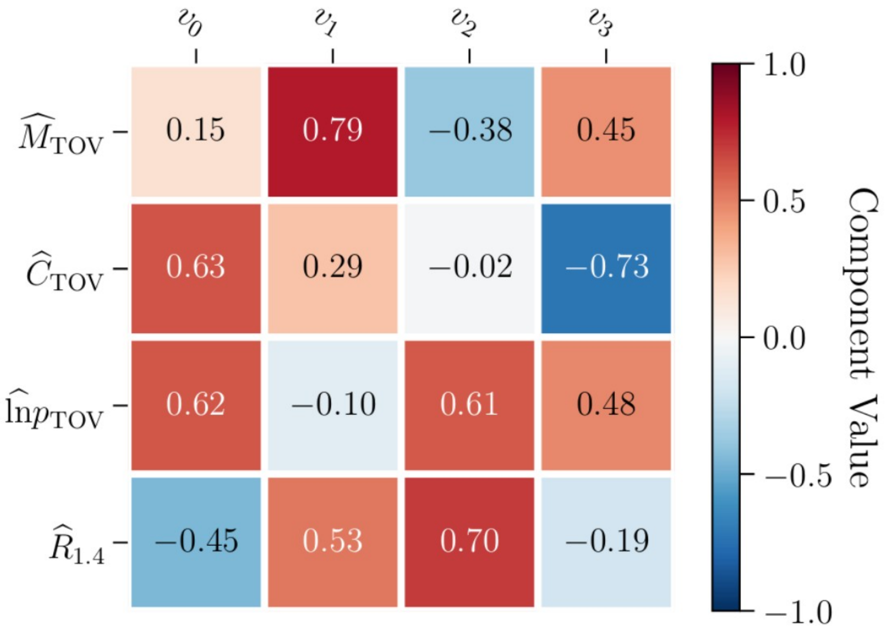
- >10^5 BNS simulations are **too expensive to simulate; need to select a small, smart sample**

- Focus on a few variables that characterize the high-density part of the EOS, and one to break degeneracy at lower densities:

$$(M_{\mathrm{TOV}}, R_{\mathrm{TOV}}, \ln p_{\mathrm{TOV}}, R(1.4M_{\odot}))$$

- Consider the 4d distribution of these variables, **normalized** as $\hat{x} \equiv (x - \bar{x})/\sigma_x$ to have mean 0 and variance 1

- Find the **principal components** in this 4D space that **capture the majority of the variance** $(v_0,\ldots,v_3)$ – eigenvectors & eigenvalues of covariance matrix

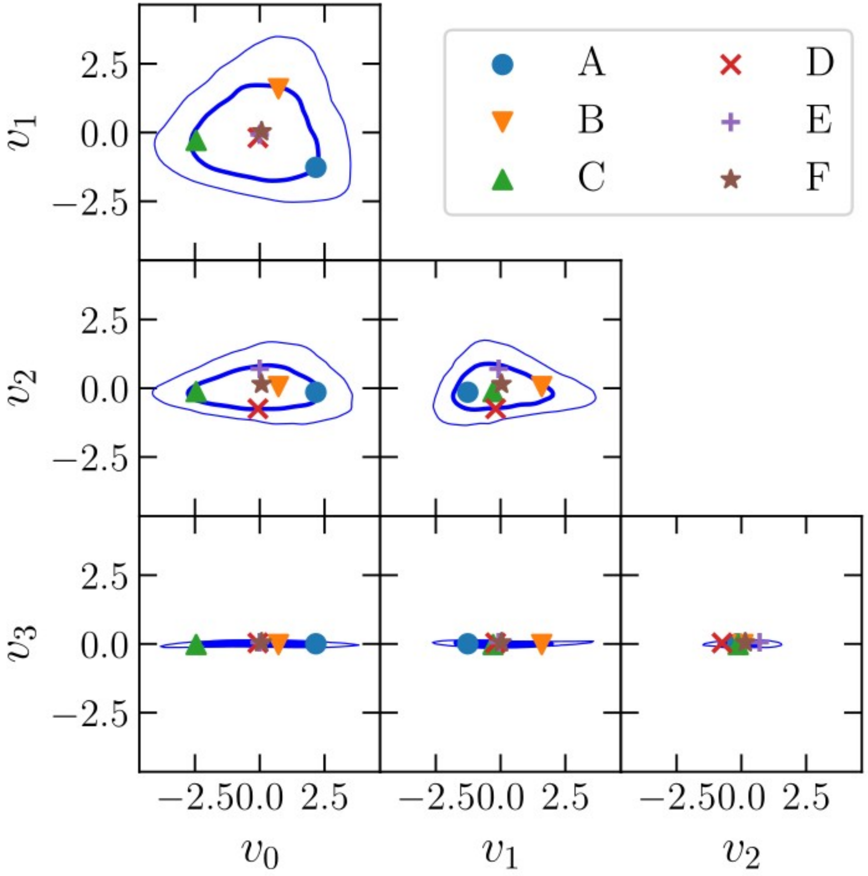


Selecting a small, smart sample of model-agnostic EOSs (2)

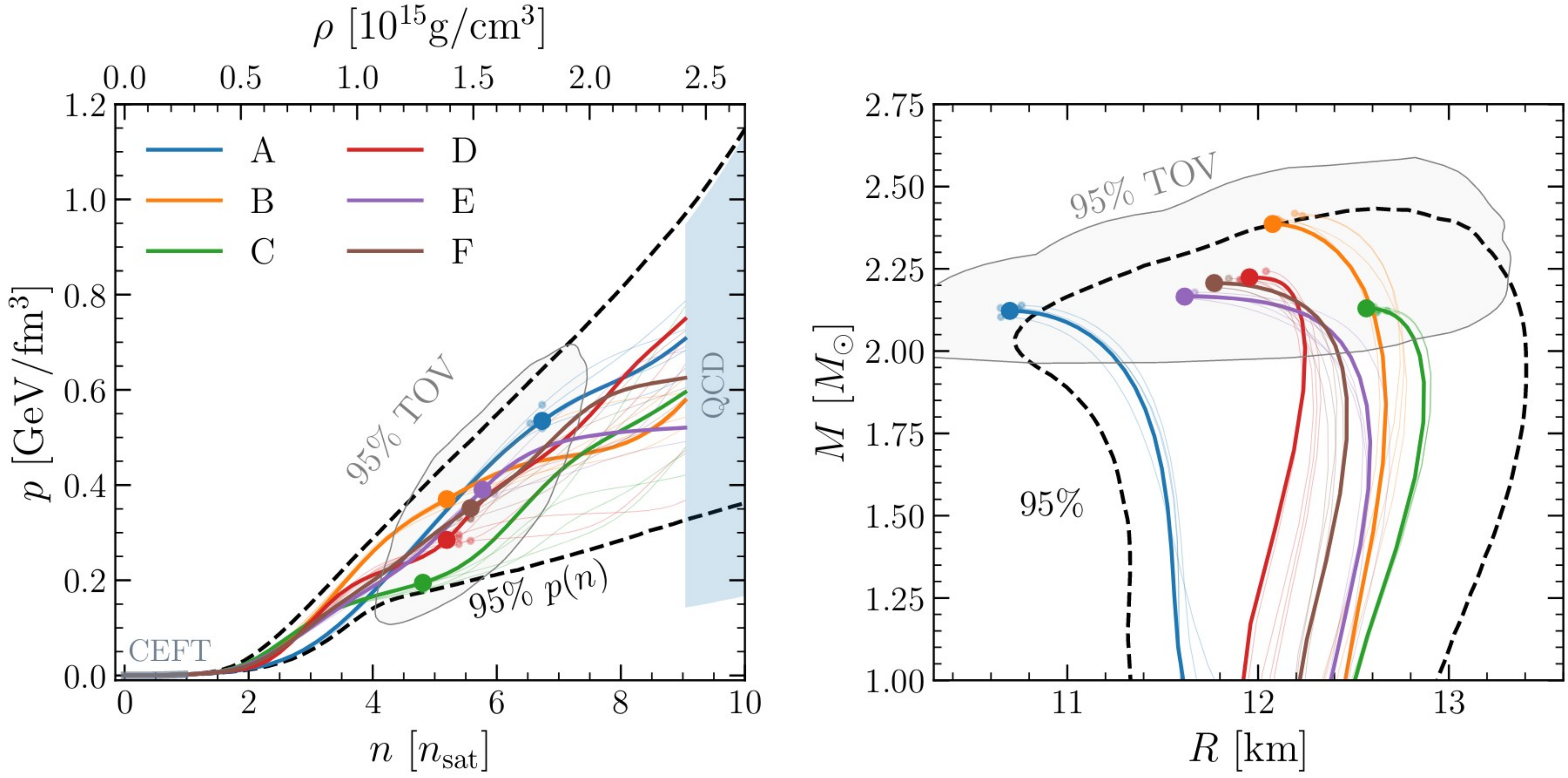
- We find the 4D distribution in the PCA coordinates is primarily 3D, with prominent triangular shape in (v_0, v_1) plane

- Identify 6 "golden EOSs" near the 68% credible region of the 3D part (+1 in center) to simulate – we select the highest-likelihood EoSs out of the 30 closest EoSs

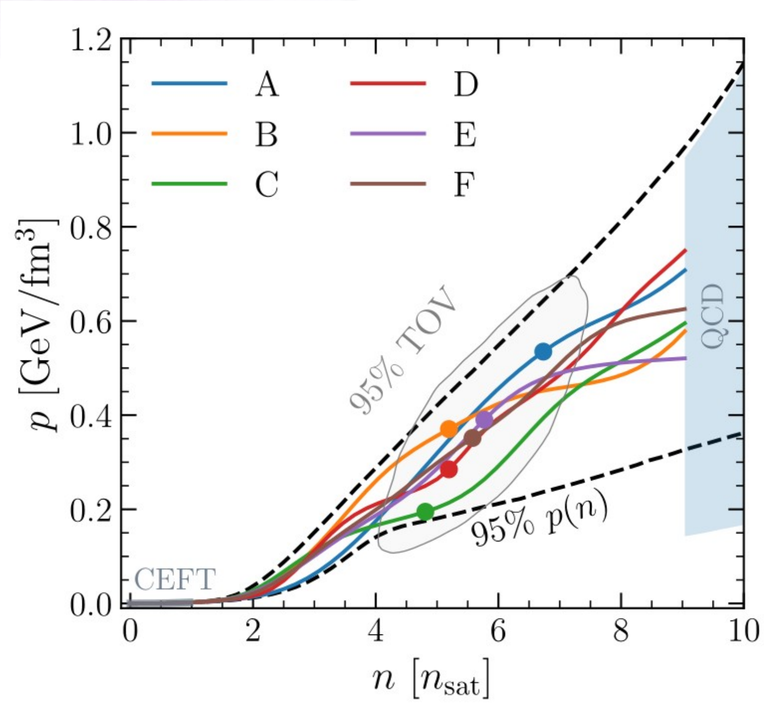
- Simulate $q = M_2/M_1$ = 1.0, 0.85, 0.7 for GW170817 chirp mass + diagnostics (use simple hybrid EoS construction with fixed Γ_{th}=1.75)



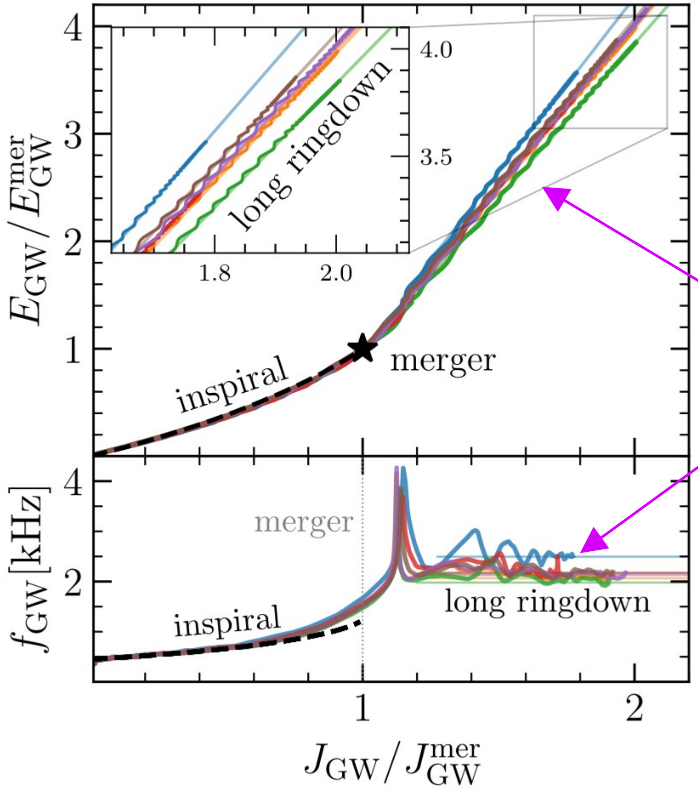
The six “golden EoSs”



Correlation observed in the "Long ringdown" of the HMNS



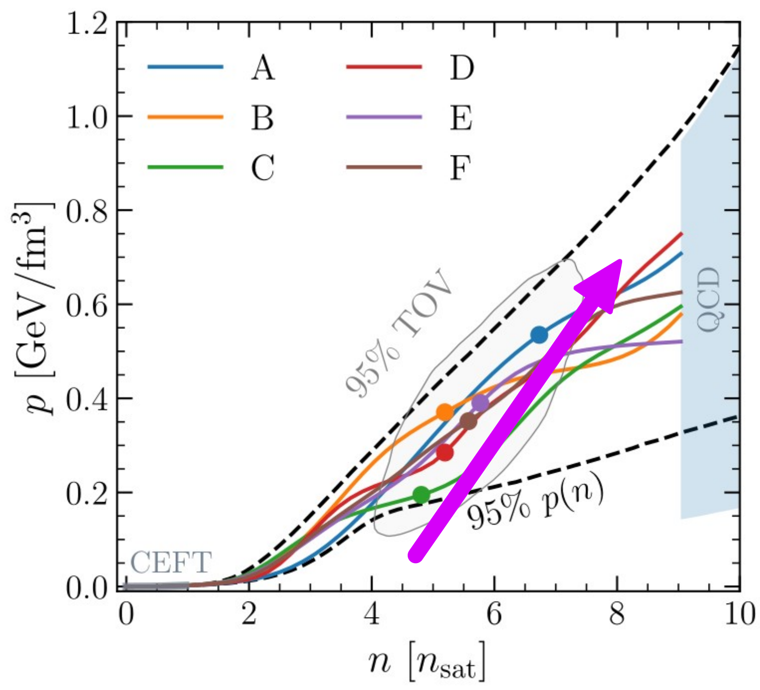
q = 1 results:



Slope given by f_{GW} if pure quadrupole (good approximation)

Striking linear relation between emitted GW energy E and angular momentum J

Correlation observed in the "Long ringdown" of the HMNS



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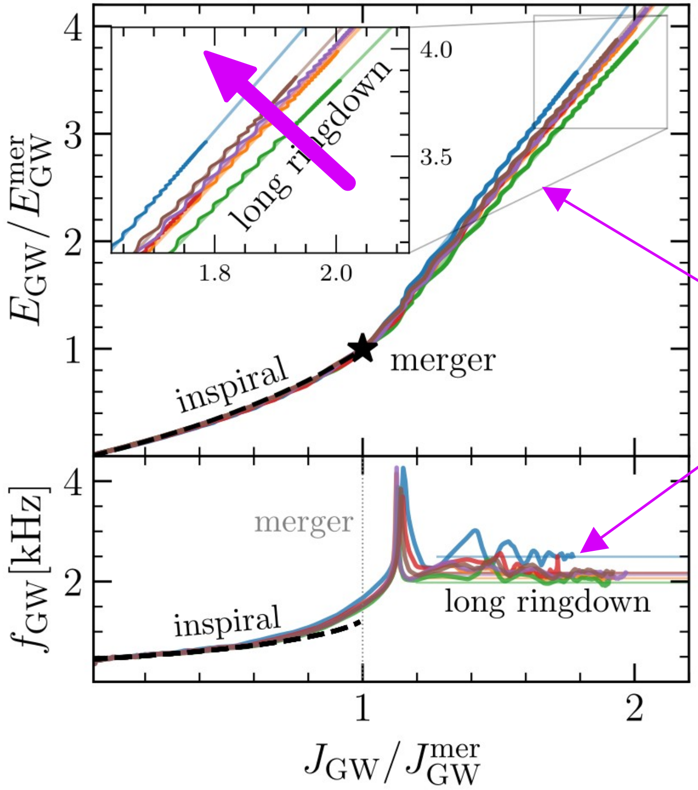
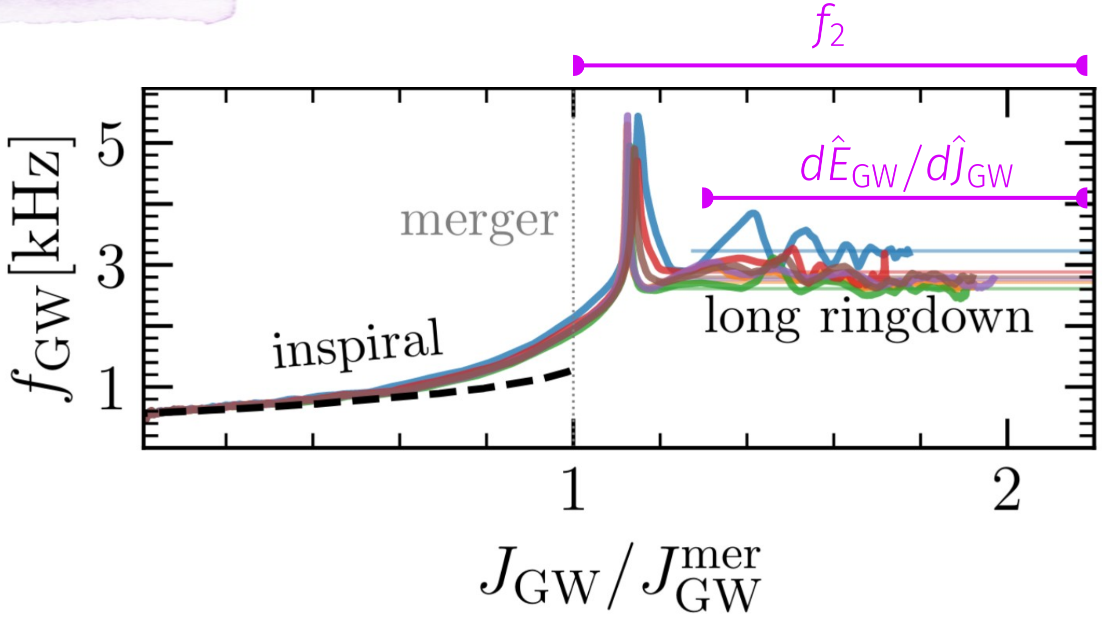


Slope given by f_{GW} if pure quadrupole (good approximation)

Find that TOV properties correlated with this linear slope

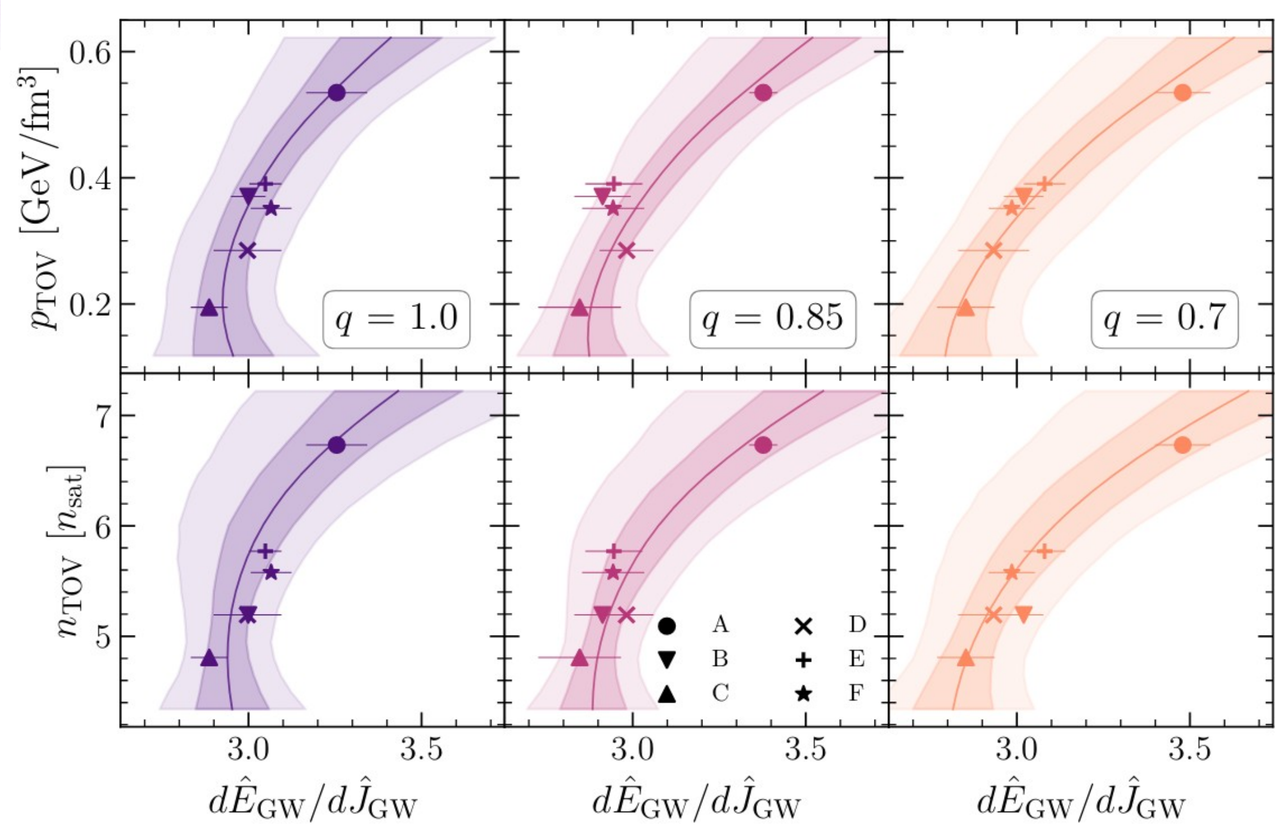
Long ring-down slope and f_2 are not the same



f_2 picks up power even during the transient first few ms, and it is measured from Fourier transform, instead of linear fit to emitted E_{GW}, J_{GW}

We find $d\hat{E}_{GW}/d\hat{J}_{GW}$ better correlated with the TOV point (though both are well correlated)

Slope is correlated with the TOV pressure and density



Use bilinear model : $\frac{d\hat{E}_{\rm GW}}{d\hat{J}_{\rm GW}} = \beta_0 + \beta_1 n_{\rm TOV} + \beta_2 p_{\rm TOV} + \beta_3 q + \beta_4 n_{\rm TOV} \cdot q + \beta_5 p_{\rm TOV} \cdot q + \beta_6 n_{\rm TOV} p_{\rm TOV}$

Use correlation in a simple mock measurement

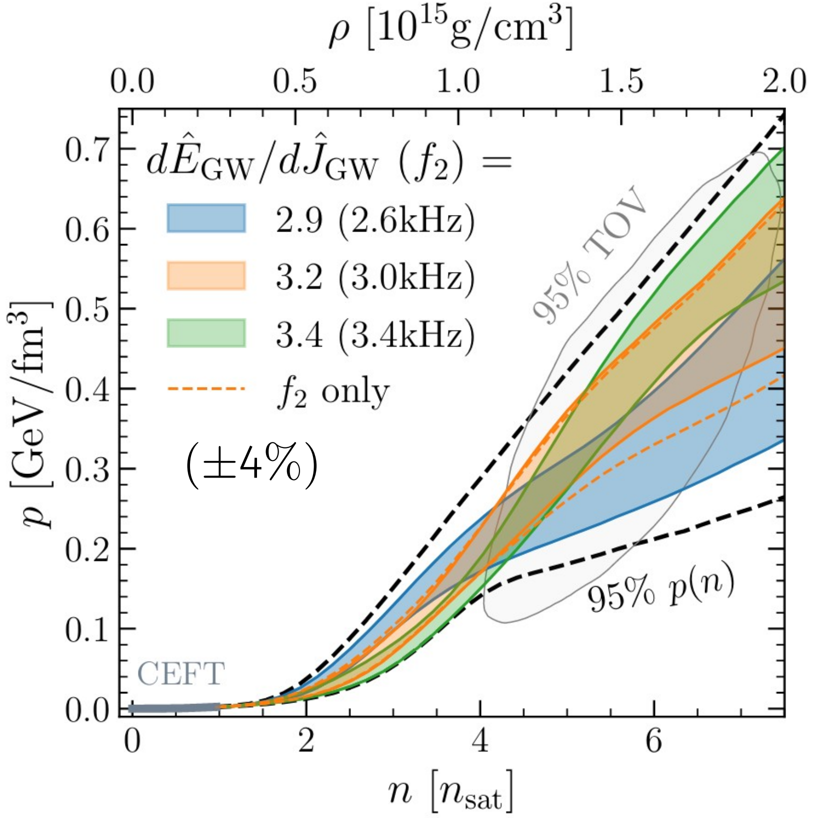
New component of likelihood in Bayes's Theorem

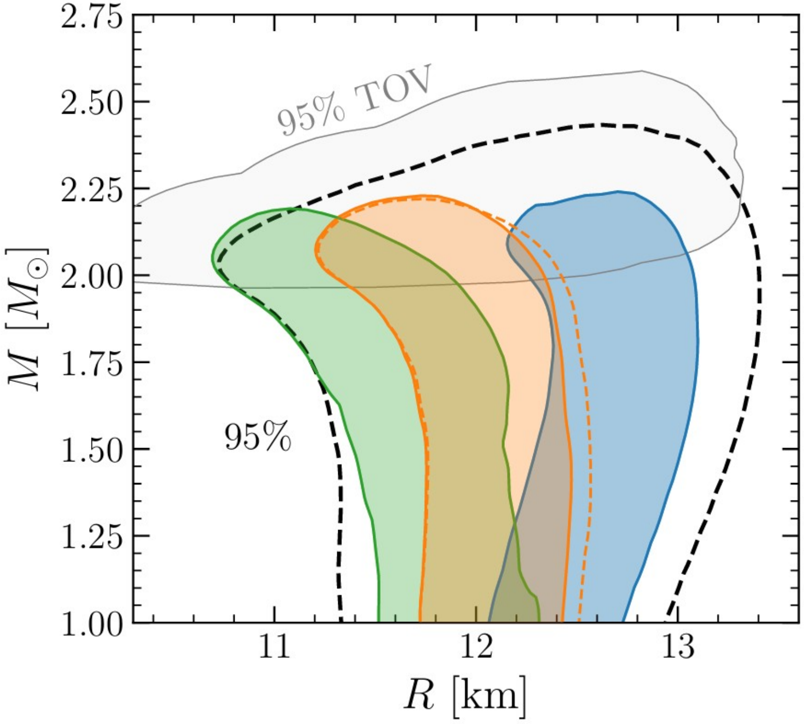
Joint likelihood from measurement (assumed multivariate Gaussian in f_2 and slope)

$$P(\mathrm{data}|\mathrm{EOS}, q) = \int df_2\, d\Big(d\widehat{E}_{\mathrm{GW}}/d\widehat{J}_{\mathrm{GW}}\Big) \times P_{\mathrm{meas}}(\mathrm{data}|f_2, d\widehat{E}_{\mathrm{GW}}/d\widehat{J}_{\mathrm{GW}}, q) \times P_{\mathrm{mod}}(d\widehat{E}_{\mathrm{GW}}/d\widehat{J}_{\mathrm{GW}}, f_2|\mathrm{EOS}, q)\,,$$

Bilinear model(s)

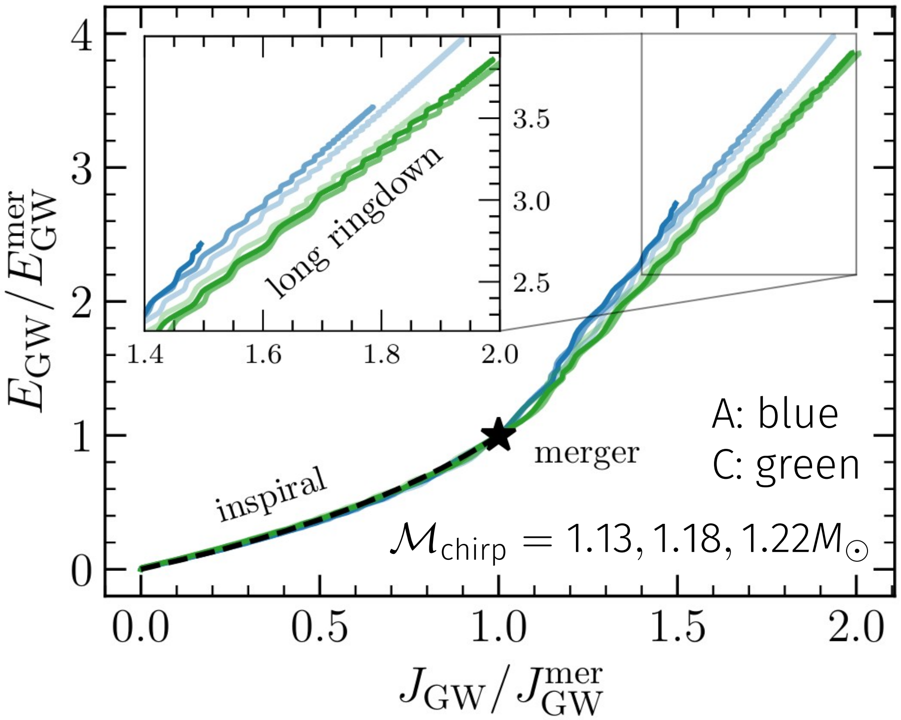
Long ringdown measurement constrains the whole EOS



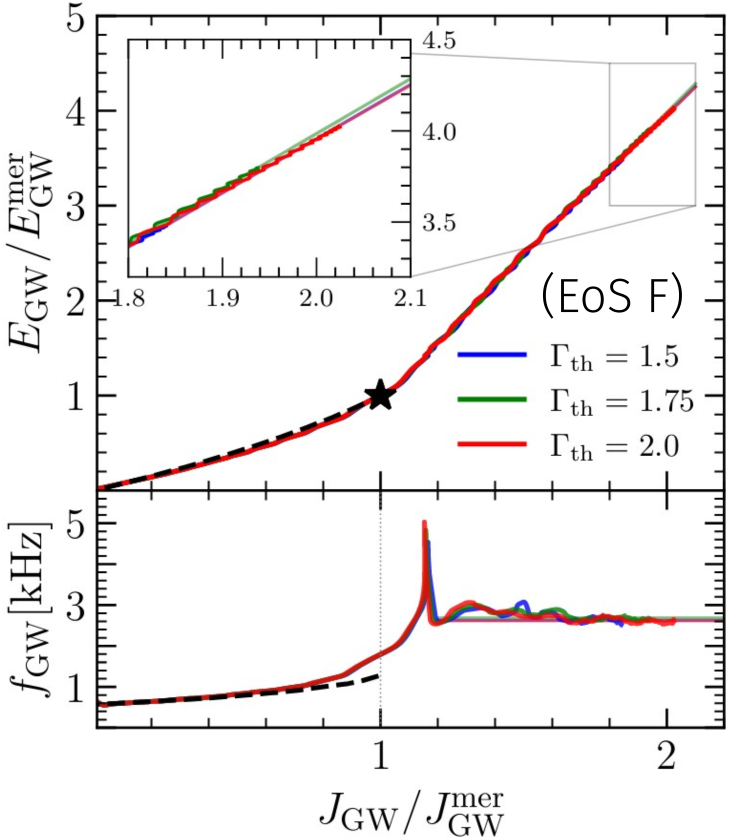


Measurement improves upon constraints from f_2 alone due to better correlation (here assuming flat prior for $q \in [0.7, 1]$)

Slope insensitive chirp mass and thermal effects



Thermal effects ($\Gamma_{\rm th}$)

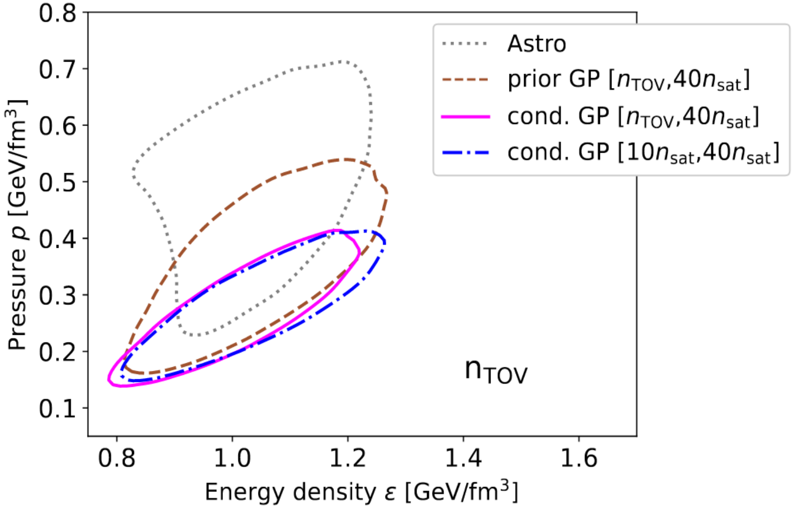


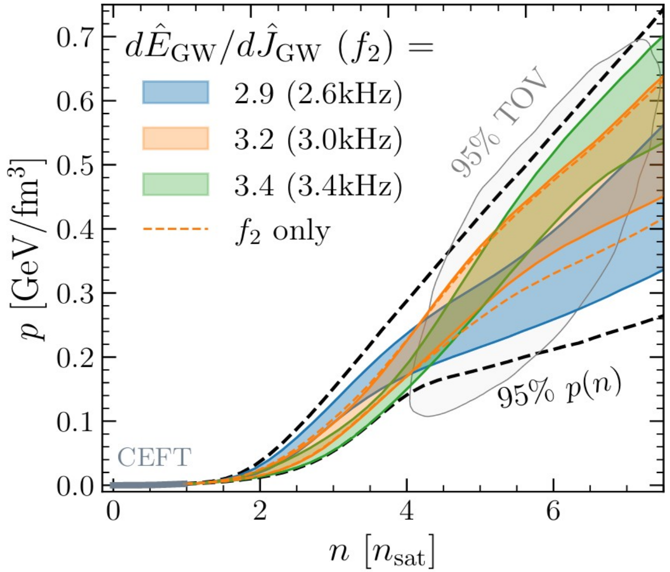
Conclusions

- ✔ pQCD +thermodynamics implies **softening before n_{TOV}** or **Strong, prolonged softening just after**, **followed by $c_s^2 \approx 1$**

  Stiff EoS at n_{TOV} incompatible with pQCD c_s^2 at 25-40n_{sat}

- ✔ Marginalized pQCD input yields results independent of n_{term}





- ✔ "Long ringdown" in BNS mergers will constrain EoS
  - PCA analysis allows us to **capture ensemble behavior**
  - **Linear postmerger relation** between GW energy and angular momentum **correlated with TOV properties**
  - Yields **improved constraints** beyond of f_2

Conclusions

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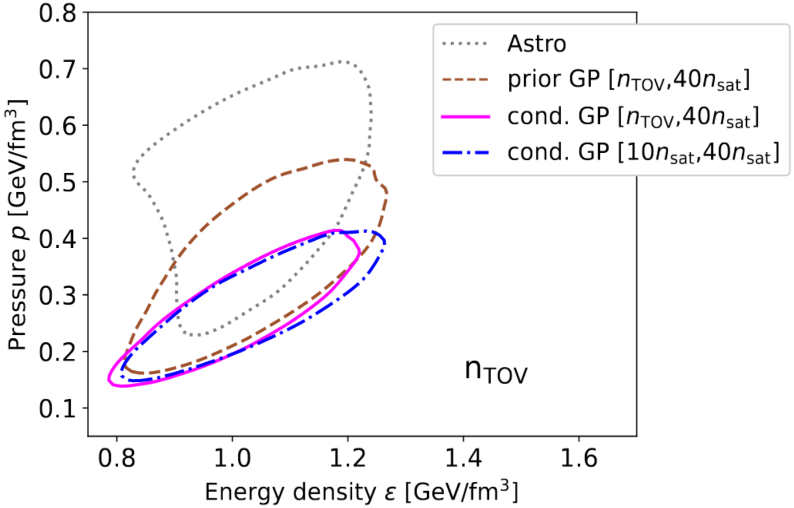

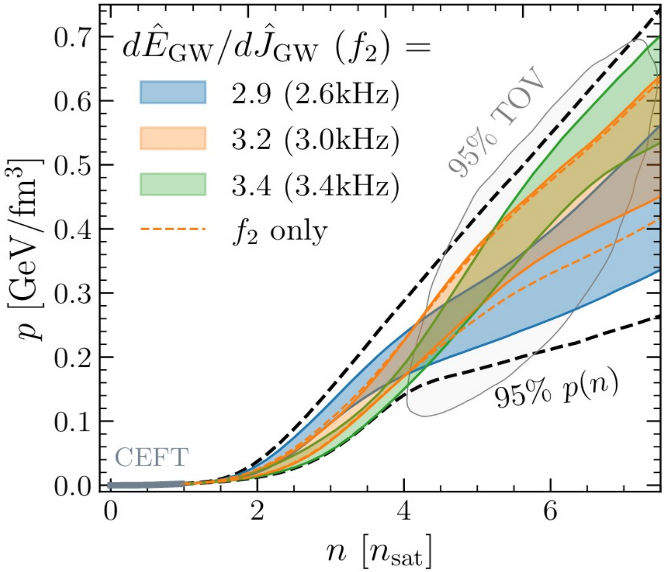

✔ "Long ringdown" in BNS mergers will constrain EoS

- PCA analysis allows us to capture ensemble behavior
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Thanks for your attention!

Here there be details…

Gaussian Process regression priors

- Follow Landry & Essick Phys. Rev. D 99 (2019) and implement a Gaussian Process Regression in an auxiliary variable $\varphi(n) = -\ln(c_s^{-2}(n) - 1)$, but as function of n
- Use hierarchical model, for wide range of behavior

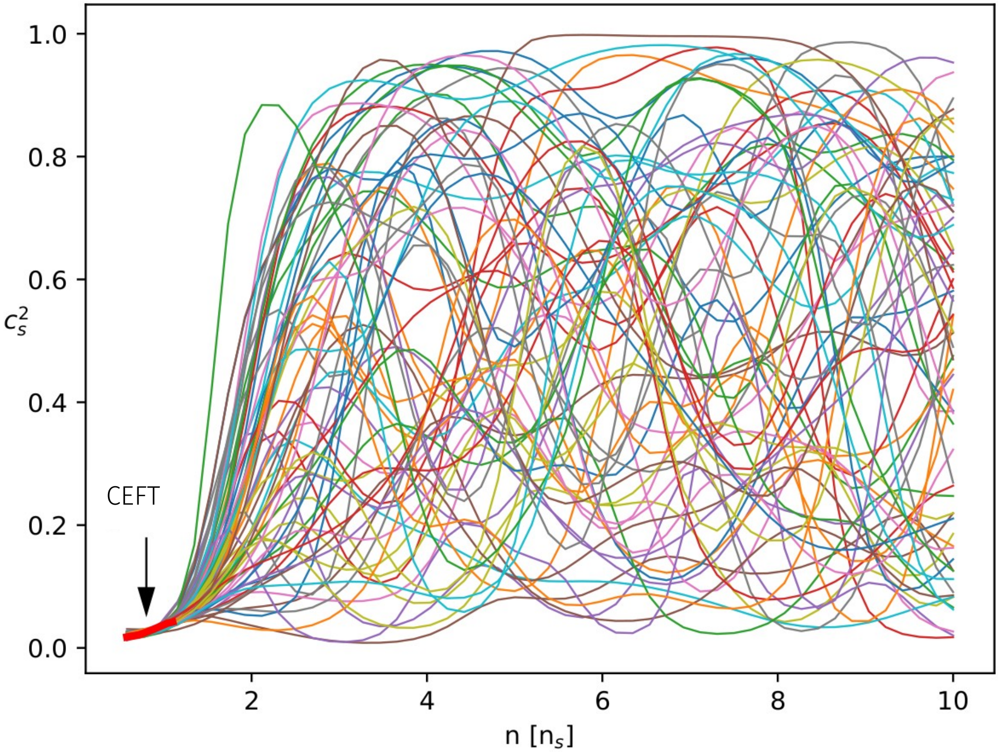
$$\varphi(n) \sim \mathcal{N}\Big(-\ln(\bar{c}_s^{-2} - 1), K(n, n')\Big),$$

$$K(n, n') = \eta e^{-(n-n')^2/2\ell^2}$$

$$\bar{c}_s^2 \sim \mathcal{N}(0.5, 0.25^2),$$

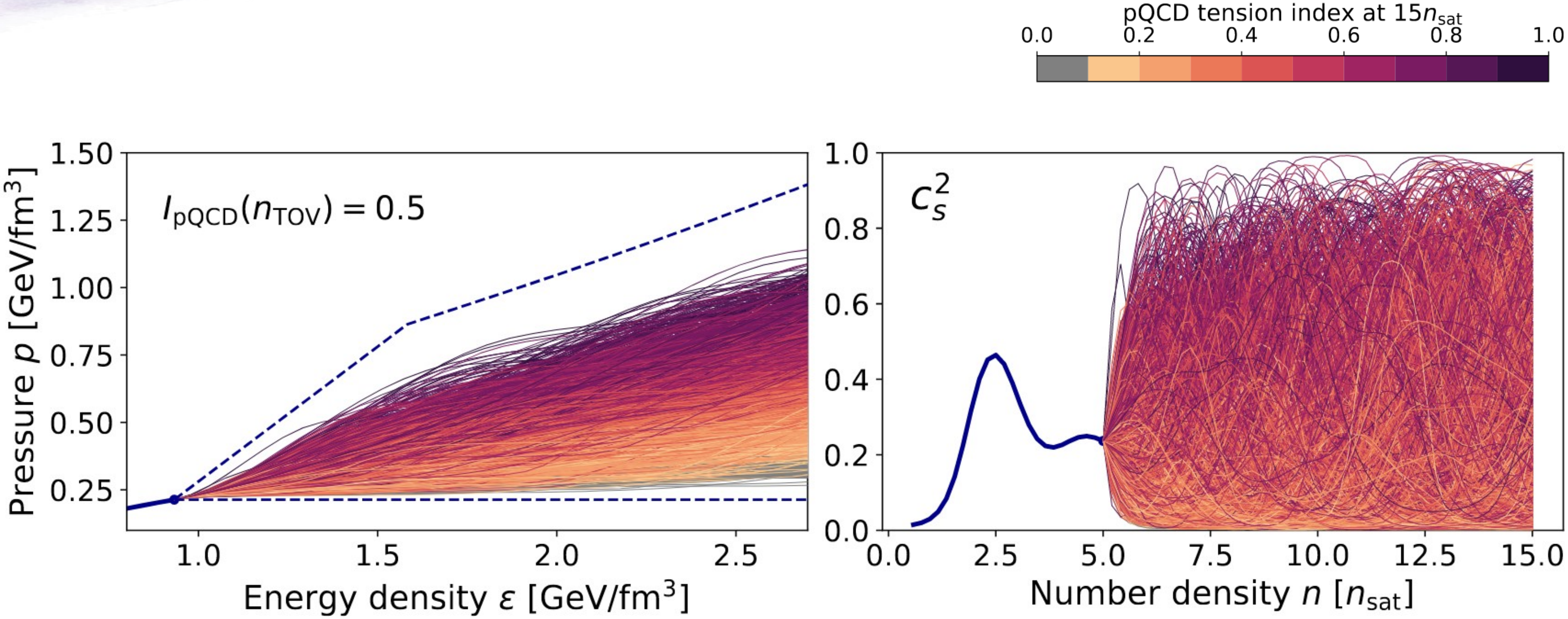
$$\ell \sim \mathcal{N}\big(1.0 n_s, (0.2 n_s)^2\big),$$

$$\eta \sim \mathcal{N}(1.25, 0.25^2).$$



Gorda, Komoltsev, Kurkela, ApJ 950 (2023)
https://github.com/OKomoltsev/QCD-likelihood-function

EoS extensions with least extreme I_{pQCD}



Why is a 2nd GP marginalization?

(Low and high parts of EoS)

1. Expectation value of Q: $$P(Q) = \int d\eta_l d\eta_h P(Q|\eta_l, \eta_h) P(\eta_l, \eta_h)$$

2. Assume l and h are un- (or weakly) correlated: $$P(\eta_l, \eta_h) = P(\eta_l) P(\eta_h)$$

3. Q only depends on h through ε_L, p_L at n_{term}: $$P(Q|\eta_l, \eta_h) = P(Q|\eta_l; p_L, \varepsilon_L),$$

4. Then can marginalize over ε_L, p_L as

$$P(Q) = \int d\eta_l dp_L d\varepsilon_L P(Q|\eta_l; p_L, \varepsilon_L) P(\eta_l) w(p_L, \varepsilon_L),$$

$$w(p_L, \varepsilon_L) \equiv \int d\eta_h P(p_L, \varepsilon_L|\eta_h) P(\eta_h)$$

Quantity that we plotted before

Simulation details:

- FUKA code initial data solver; initial separation ≈ 45 km

  Papenfort, Tootle, Grandclément, Most, Rezzolla, PRD 104, 024057 (2021)

- Evolution with Einstein-Toolkit, including the fixed-mesh box-in-box refinement framework Carpet

  Haas+ The Einstein Toolkit. Zenodo. (http://einsteintoolkit.org) (2020).
  Schnetter, Hawley, Hawke, Class. Quantum Grav. 21, 1465–1488 (2004)

  - six refinement levels; finest grid spacing of 295 m;
  - impose reflection symmetry across orbital plane
  - computational domain outer boundary at ±1512 km

- Hybrid EoS construction w/ fixed $\Gamma_{\rm th} \equiv (p_{\rm th}/\varepsilon_{\rm th}) + 1$
- Extract $\psi_4 \equiv \ddot{h}_+ + i\ddot{h}_\times$ with sampling rate ≈ 634 kHz from a spherical surface with radius ≈ 574 km (spin-weighted spherical-harmonic modes with $l \leq 4$)

Sample waveforms