Nuclear matter in mergers: the quest for equilibrium

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Alford, Bovard, Hanauske, Rezzolla, Schwenzer
arXiv:1707.09475

Alford and Harris, arXiv:1907.03795

Alford, Harutyunyan, Sedrakian, 2209.04717, 2306.13591

Alford, Haber, Zhang, arXiv:2306.06180

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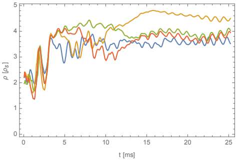
Outline

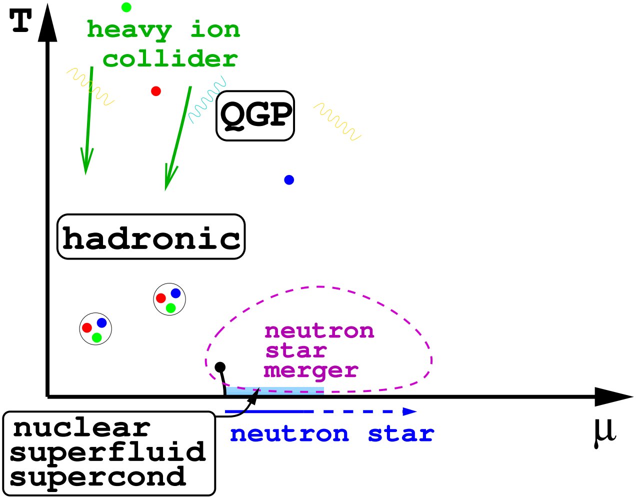
- Neutron star mergers are like experiments that probe the properties of dense matter. People mostly talk about the *Equation of State*.
- Also potentially important: **Out-of-equilibrium phenomena**
  - Flavor equilibration — bulk viscosity
  - Thermal equilibration — thermal conductivity
  - Shear flow equilibration — shear viscosity

  etc

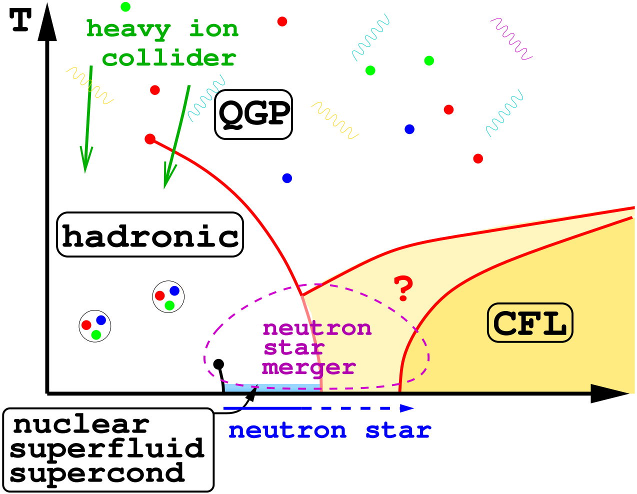
  Better than the equation of state for probing phase structure!
- Flavor equilibration: is it important in mergers?
  - relaxation time for the proton fraction
  - *Critical equilibration*: when relaxation should be included in the dynamics
  - physical manifestations: bulk viscosity and sound attenuation



QCD Phase diagram



Conjectured QCD Phase diagram

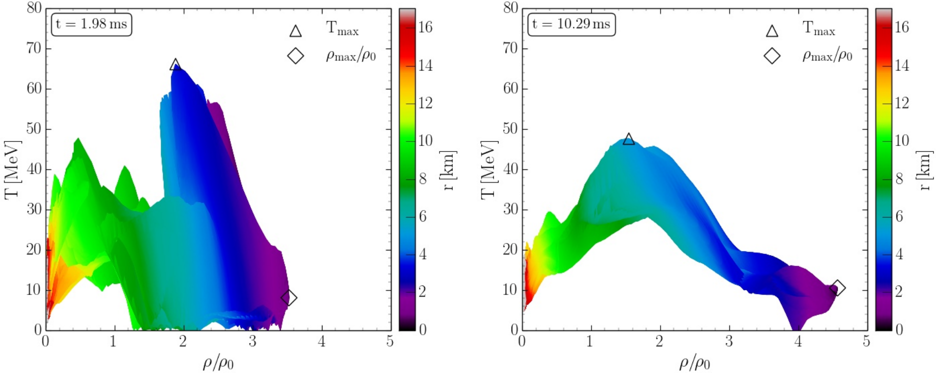


heavy ion collisions: deconfinement crossover and chiral critical point

neutron stars: quark matter core?

neutron star mergers: dynamics of warm and dense matter

Nuclear material in a neutron star merger



M. Hanauske, Rezzolla group, Frankfurt

Significant spatial/temporal variation in:

- temperature
- fluid flow velocity
- density ⇒ flavor content

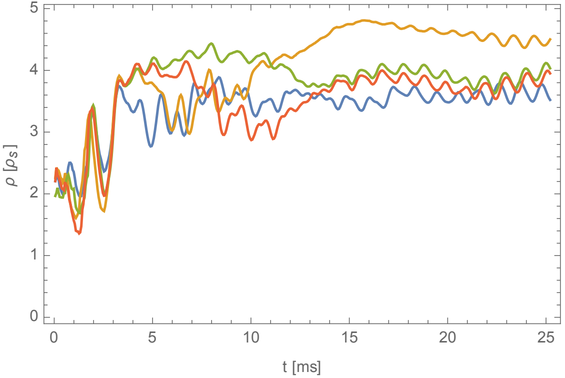
so we need to allow for

- thermal conductivity
- shear viscosity
- bulk viscosity

Density oscillations in mergers

Density vs time for tracers in merger

Flavor equilibration neglected



Tracers (co-moving fluid elements) show dramatic density oscillations, especially in the first 5 ms.

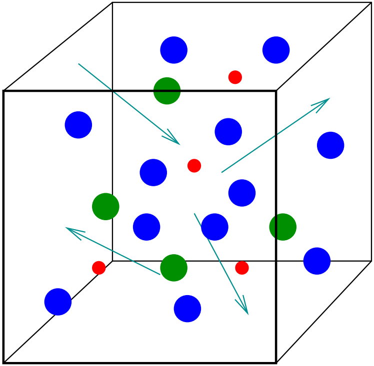
Amplitude: up to 50%
Period: 1–2 ms
Freq: ~ 1 kHz

Do density oscillations drive the system out of flavor equilibrium?

Does flavor equilibration affect the oscillations?

The nuclear matter fluid

Generic fluid element



neutrons: dominant constituent
protons: small fraction
electrons: maintaining local neutrality
neutrinos: *thermally equilibrated?*

Fluid is described by 3-4 parameters:

$\boxed{n_B} = n_\mathsf{n} + n_\mathsf{p}$	baryon density
$\boxed{T}$	temperature
$\boxed{x_p} = n_\mathsf{p}/n_B$	proton fraction
($\boxed{x_L} = n_L/n_B$ [if neutrinos are trapped]	lepton fraction)

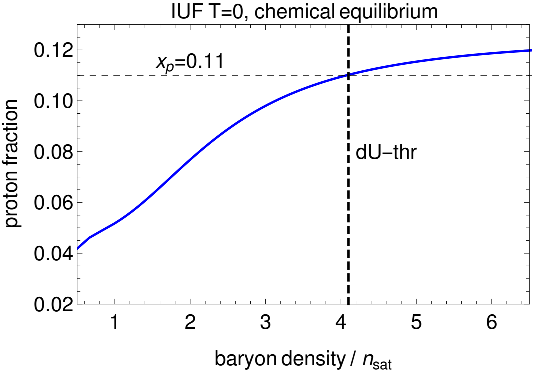
Equation of state relates these to relevant quantities: pressure, energy density etc,

$$p(n_B, T, x_p, x_L)$$
$$\varepsilon(n_B, T, x_p, x_L)$$
$$\ldots$$

Density oscillations and beta equilibration

Each fluid element relaxes to the *equilibrium proton fraction* $x_p^{\mathrm{eq}}(n_B,T)$ via weak interactions.

$x_p^{\mathrm{eq}}(n_B,T)$ is determined by properties of the strong interaction (nuclear symmetry energy) and the requirement of electrical neutrality.



So when you compress nuclear matter, the proton fraction wants to change.

But this doesn't happen instantaneously!

Density oscillations can drive the system away from flavor ("beta", "chemical", "isospin") equilibrium.

Fast and slow equilibration

- Fluid element undergoes density oscillation of angular frequency ω

$$n_B(t) = \bar{n} + \delta n \cos(\omega t)$$

- Proton fraction relaxes to equilibrium at *relaxation rate* $\gamma(n_B,T)$

$$\partial_t x_p = -\gamma\big(x_p - x_p^{\mathsf{eq}}(n_B,T)\big)$$

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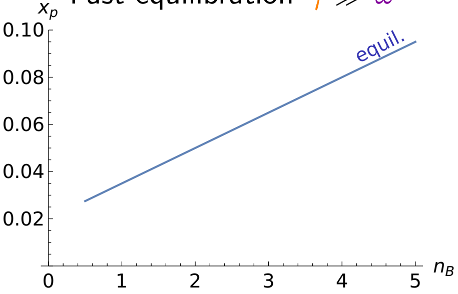
Fast equilibration $\gamma \gg \omega$



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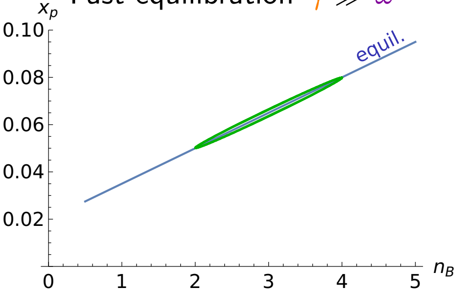

Proton fraction stays equilibrated

No need to solve relaxation equation

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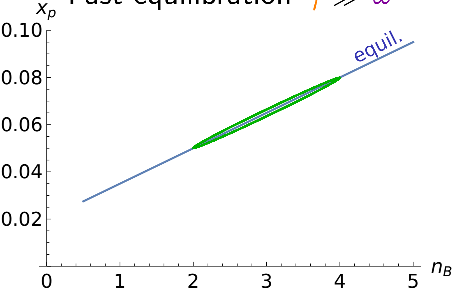

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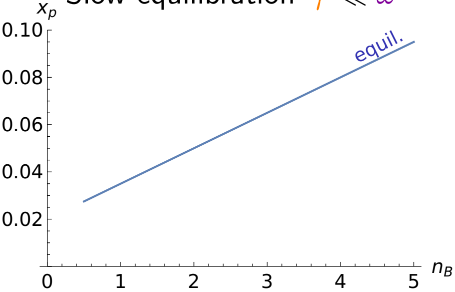

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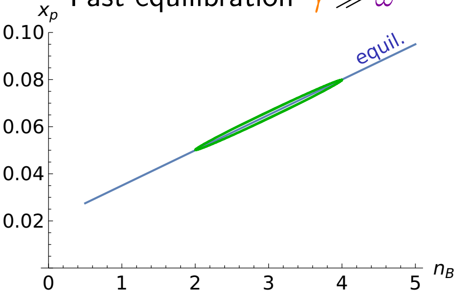

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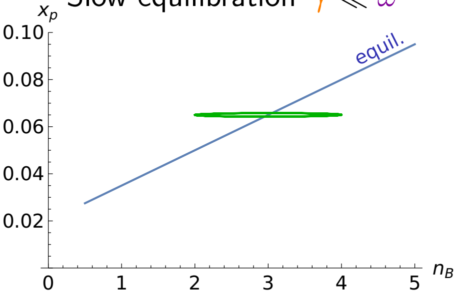

Proton fraction hardly changes

No need to solve relaxation equation

Fast and slow equilibration

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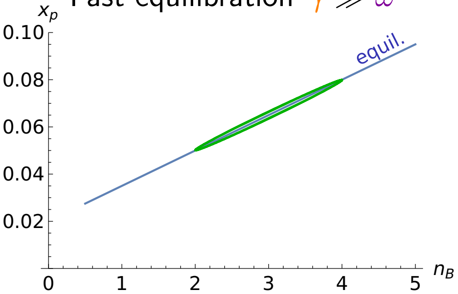

Proton fraction stays equilibrated

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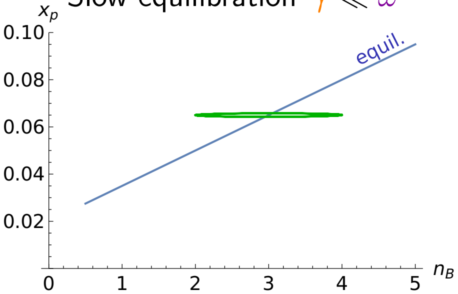
Slow equilibration $\gamma \ll \omega$



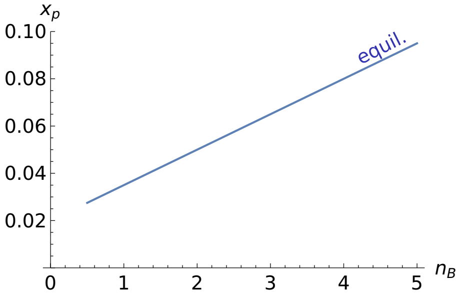
Proton fraction hardly changes

No need to solve relaxation equation

What happens if $\gamma \sim \omega$?

Critical equilibration

Critical equilibration $\gamma = \omega$



Critical equilibration

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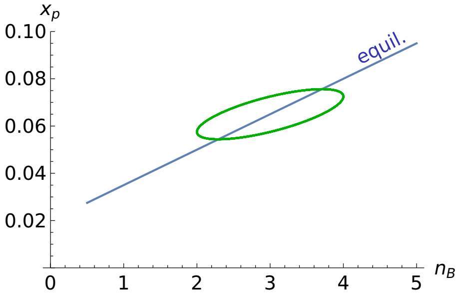

Critical equilibration

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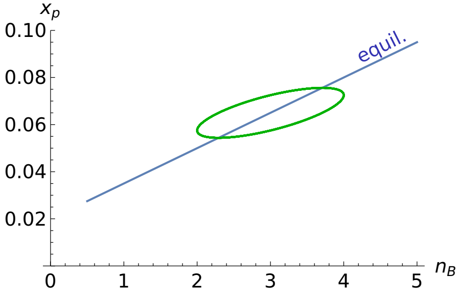
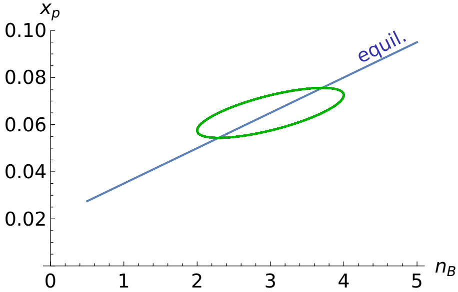


- The value of $x_p(t)$ depends on its *recent history*, not just $n_B(t)$.
- Should include the relaxation equation in the fluid dynamics

Critical equilibration

Critical equilibration $\gamma = \omega$



- The value of $x_p(t)$ depends on its *recent history*, not just $n_B(t)$.
- Should include the relaxation equation in the fluid dynamics

Other features of critical equilibration:

- Maximal phase lag between density and proton fraction
- Maximal bulk viscosity $\Rightarrow$ Maximal damping of density oscillations

Is there critical equilibration in mergers?

Critical equilibration ($\gamma = \omega$) in mergers?

Frequency for typical density oscillations in a merger: $\boxed{\omega \approx 2\pi \times 1\,\text{kHz}}$

Relaxation rate $\gamma(n_B,T)$ for proton fraction: determined by weak interaction "Urca processes" in which neutrinos play an essential role.

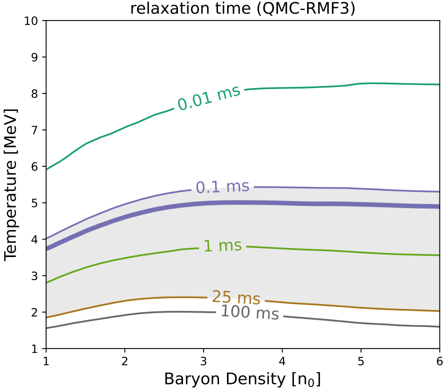
We can calculate the relaxation rate in two limiting cases:

Urca process	neutrino-transparent	neutrino-trapped
neutron decay	$\text{n} \to \text{p} + \text{e}^- + \bar{\nu}_e$	$\nu_e + \text{n} \to \text{p} + \text{e}^-$
electron capture	$\text{p} + \text{e}^- \to \text{n} + \nu_e$	$\text{p} + \text{e}^- \to \text{n} + \nu_e$

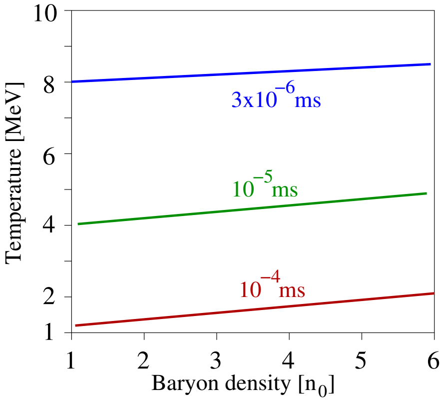
At what density and temperature is
$\gamma(n_B,T)$ comparable to the $1\,\text{kHz}$ timescale?

Proton fraction relaxation time $\tau = 1/\gamma$,

neutrino-transparent



neutrino-trapped



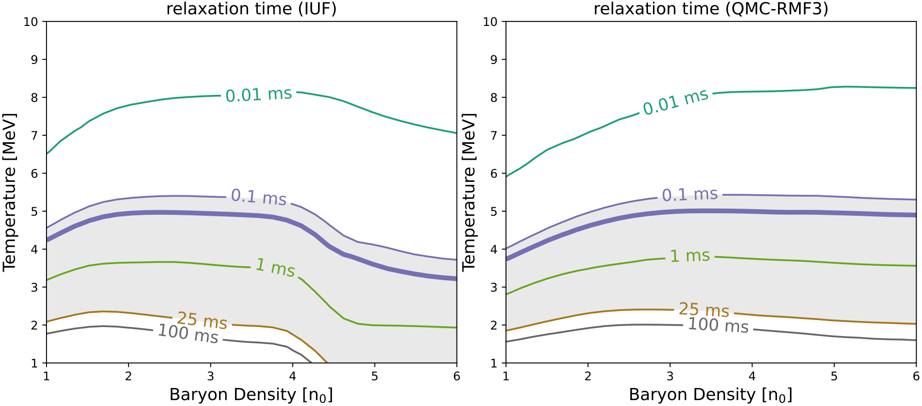
Alford, Haber, Zhang `arXiv:2306.06180`

Alford, Harutyunyan, Sedrakian `arXiv:2209.04717`

- Relaxation is faster at higher temperatures, insensitive to density
- neutrino-trapped matter: relaxation is very fast
- neutrino-transparent matter: relaxation on merger timescales!
- Thick contour shows critical equilibration, where $\tau = 1\,\text{ms}/2\pi$

Dependence on Equation of State

Relaxation time $\tau = 1/\gamma$, for two representative equations of state.



- Relaxation is slow at low temperatures, fast at high temperatures
- Relaxation is not sensitive to density (except at dUrca threshold)
- Thick contour shows critical equilibration, where $\tau = 1\,\text{ms}/2\pi$

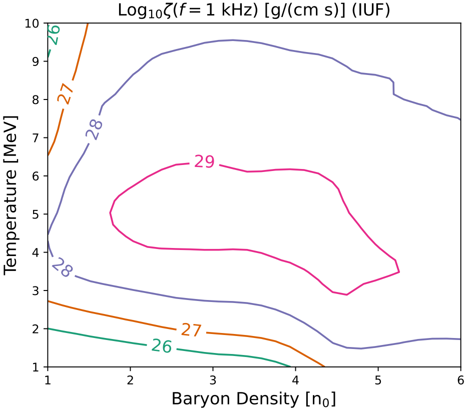
Critical equilibration occurs at $T \sim 5\,\text{MeV}$

Conclusions so far

- Neutrino-trapped matter:
  proton fraction relaxes *quickly*, $\tau \lesssim 10^{-3}$ ms at $T \geqslant 1$ MeV.
  Merger simulations with very short timesteps will need to include this process.
- Neutrino-transparent matter:
  at $T \sim 2$ to 5 MeV, proton fraction relaxes on the same timescale as the merger dynamics.
  Proton fraction equilibration is part of the dynamics.
- In reality, neutrinos in mergers have some non-thermal distribution with an energy-dependent mean free path.
  Need to develop tools to deal with this.

If critical equilibration (relaxation time $\approx$ oscillation period) occurs in mergers, are there physical consequences?

Bulk viscosity for 1 kHz oscillations
(neutrino-transparent)



- *Non-monotonic T-dependence:* bulk viscosity reaches a maximum at $T \sim 5\,\mathrm{MeV}$
- Not very sensitive to density

Does this sound familiar?

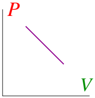
Bulk viscosity: phase lag in system response

Some property of the material (proton fraction) takes time to equilibrate.

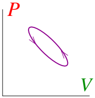
Baryon density n and hence fluid element volume V go out of phase with applied pressure P:

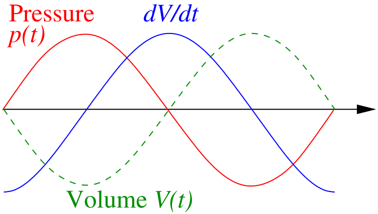
$$\text{Dissipation} = -\int P\, dV = -\int P\, \frac{dV}{dt} dt$$

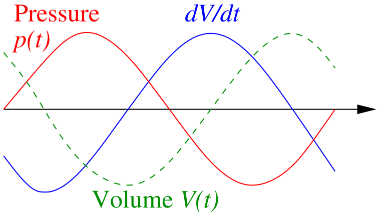
No phase lag.
Dissipation $= 0$



Some phase lag.
Dissipation > 0







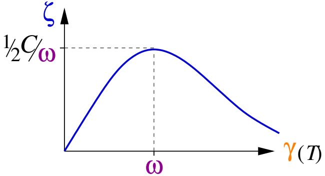
Bulk viscosity: a resonant phenomenon

Bulk viscosity is **maximum** at critical equilibration, when

(flavor relaxation rate) = (freq of density oscillation)

γ ω

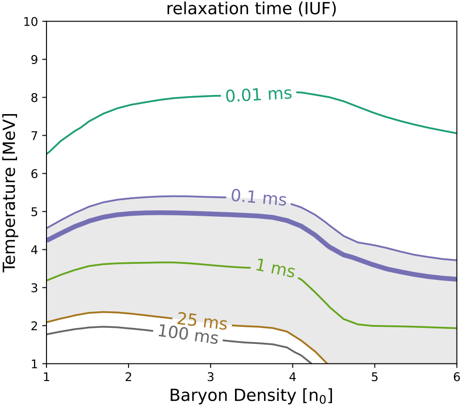
$$\zeta = C \frac{\gamma}{\gamma^2 + \omega^2}$$



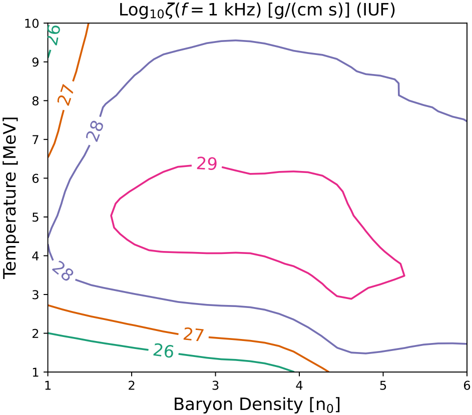
- Fast equilibration: $\gamma \to \infty \;\Rightarrow\; \zeta \to 0$
  System is always in equilibrium. No pressure-density phase lag.
- Slow equilibration: $\gamma \to 0 \;\Rightarrow\; \zeta \to 0$.
  System does not try to equilibrate: Proton fraction fixed.
  No pressure-density phase lag.
- Critical equilibration: $\omega = \gamma \;\Rightarrow$ maximum phase lag between pressure and density $\Rightarrow$ maximum dissipation

Critical equilibration and bulk viscosity
(neutrino-transparent)

Relaxation time



Bulk viscosity for $1\,$kHz oscillation



- Relaxation time $(1/\gamma)$ drops rapidly as temperature rises
- Bulk viscosity for an oscillation of frequency ω has a resonant maximum at critical equilibration, i.e. when $\gamma = \omega$

Density oscillation damping time τ_{damp}

Different from proton fraction relaxation time τ

Density oscillation of amplitude Δn at angular freq ω:

$$n(t) = \bar{n} + \Delta n \cos(\omega t)\ \mathrm{e}^{-t/2\tau_{\mathsf{damp}}}$$

Energy of density oscillation:
(K = nuclear incompressibility)

$$E_{\mathrm{comp}} = \frac{K}{18}\bar{n}\left(\frac{\Delta n}{\bar{n}}\right)^2$$

Compression dissipation rate:
(ζ = bulk viscosity)

$$W_{\mathrm{comp}} = \zeta\frac{\omega^2}{2}\left(\frac{\Delta n}{\bar{n}}\right)^2$$

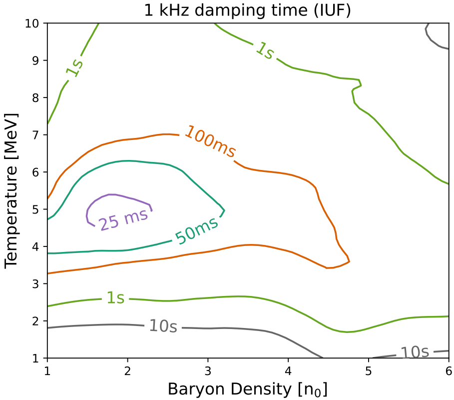
Damping Time: $\tau_{\mathsf{damp}} = \dfrac{E_{\mathrm{comp}}}{W_{\mathrm{comp}}} = \dfrac{K\bar{n}}{9\omega^2\,\zeta}$

Damping (sound attenuation) due to flavor equilibration
is important in mergers if $\tau_{\mathsf{damp}} \lesssim 20\,\mathrm{ms}$

Damping time (ν-transparent)

$$\tau_{\text{damp}} = \frac{K\,\bar{n}}{9\omega^2\,\zeta}$$

Damping of a $1\,\text{kHz}$ density oscillation can occur on merger timescale

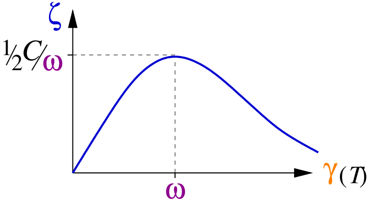


- Damping gets *slower at higher density*.
  Baryon density $\bar{n}$ and incompressibility K are both increasing.
  Oscillations carry more energy $\Rightarrow$ slower to damp
- *Non-monotonic T-dependence:* damping is fastest at $T \sim 5\,\text{MeV}$ because bulk viscosity peaks there.

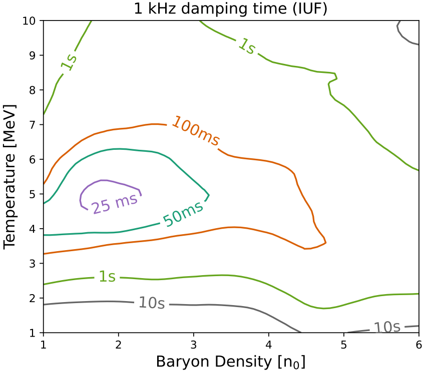
Resonant peak in bulk viscosity

Critical equilibration ($\gamma = \omega$) means

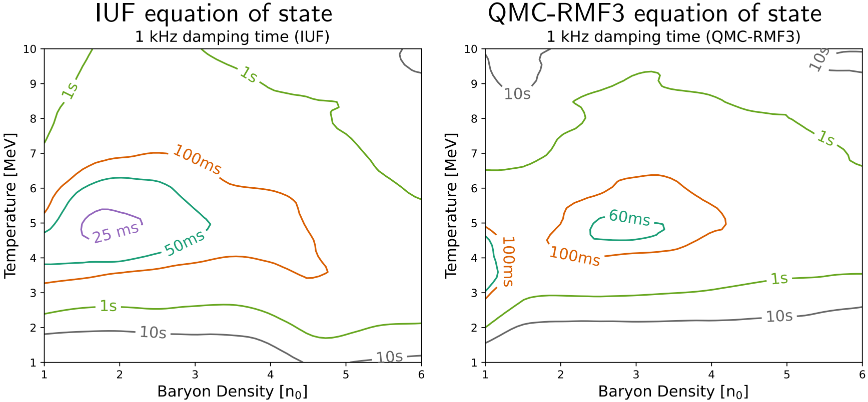
- Maximum bulk viscosity
- Fastest damping of density oscillations
- In ν-transparent matter this occurs at $T \sim 5\,\text{MeV}$



$$\zeta(T) = C\,\frac{\gamma(T)}{\gamma(T)^2 + \omega^2}$$



Two different EoSes



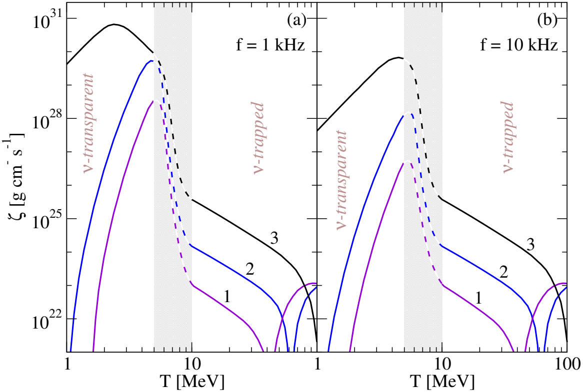
The damping time for density oscillations is shortest around $T \sim 5,$ MeV, independent of the EoS.

In neutrino-transparent matter, damping time is short enough to be relevant for mergers, especially at low density.

Bulk viscosity in neutrino-trapped regime

$$\zeta = C \frac{\gamma}{\gamma^2 + \omega^2}$$

Relaxation is much faster
Susceptibility C is smaller



Plot shows bulk viscosity,

$T < 5\,$MeV:
neutrino-transparent
$n \to p\, e^- \,\bar{\nu}_e$
$p\, e^- \to n\, \nu_e$

$T > 10\,$MeV:
neutrino-trapped
$\nu_e\, n \leftrightarrow p\, e^-$

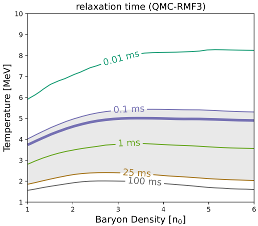
Bulk viscosity is *lower* in hot matter ($T \gtrsim 5\,$MeV)
$\Rightarrow$ damping time is much longer.

Summary

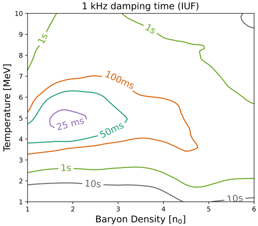
- Neutron star mergers probe the dynamical response of high-density matter on the millisecond timescale.
- In neutrino-transparent nuclear matter at $T \sim 2$ to $5\,\mathrm{MeV}$: *critical equilibration*.

  Proton fraction relaxes in milliseconds.

  Include relaxation in simulations?



- Resultant bulk viscosity damps density oscillations in 20 to 100 ms

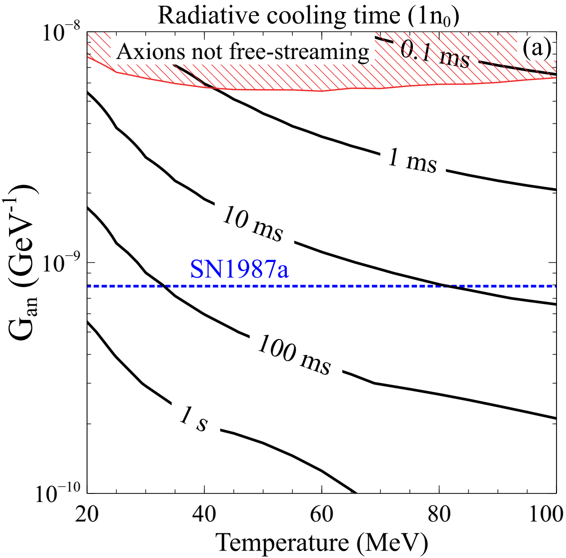


Next steps

- Beyond neutrino transparent/trapped:
  Flavor equilibration rates for arbitrary neutrino distributions
- Beyond npe:
  Flavor equilibration rates for other forms of matter.
  - Hyperonic: fast relaxation (Alford and Haber, arXiv:2009.05181)
  - Pion condensed, nuclear pasta, quark matter, etc
- Beyond bulk viscous damping:
  Other manifestations of flavor equilibration:
  - Heating
  - neutrino emission
- Beyond flavor equilibration:
  Thermal conductivity and shear viscosity may become significant in the neutrino-trapped regime if there are gradients of scale $\lesssim 100\,\text{m}$.
- Beyond Standard Model physics?

Cooling by axion emission

Time for a hot region to cool to half its original temperature:



Harris, Fortin, Sinha, Alford
arXiv:2003.09768