

Emergent phenomena in nonperturbative QCD

The holographic light-front perspective

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Outline

Emergent QCD properties

Semiclassical approximation to light-front QCD

QCD(1 + 1)

QCD(1 + 3)

Higher-spin wave equations in AdS

Integer spin

Half-integer spin

Superconformal QM: Emergence of a mass scale and confinement

Light-front mapping and baryons

Superconformal meson-baryon-tetraquark symmetry

Veneziano amplitudes and holographic QCD

Hadron form factors and quark distribution functions

Gravitational form factors and gluon distribution functions



Emergent QCD properties

A basic understanding of fundamental features of hadron physics from first principles QCD has remained elusive

Other important aspects of the strong interactions which were manifest in dual models, before QCD, are also not explicit properties of the QCD Lagrangian

Emergent properties of QCD:

- 1 Mechanism of color confinement
- 2 The origin of the hadron mass scale
- 3 Chiral symmetry breaking (CSB)
- 4 A massless pion vs. a massive proton in the chiral limit
- 5 The pattern of hadronic excitations
- 6 How does Regge theory emerge from QCD at large distances?

...

How emerging QCD properties would appear in an effective computational framework of hadron structure?



Critical role of the number of space-time dimensions

Start with the $SU(N)_C$ Lagrangian of QCD

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu}$$

where $D_\mu = \partial_\mu - igT^a A_\mu^a$ and $G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f_{abc} A_\mu^b A_\nu^c$

Dimensional analysis from the QCD action

$$S = \int d^d x \mathcal{L}$$

in d -dimensional space-time gives

$$[\psi] \sim M^{(d-1)/2}$$

$$[A] \sim M^{(d-2)/2}$$

$$[g] \sim M^{(4-d)/2}$$

QCD(1 + 1) $[g] \sim M$: Can be solved for any number of constituents and colors (DLCQ), but no emerging phenomena

QCD(3 + 1) $[g] \sim 1$: Extremely complex theory in the nonperturbative domain which should, in principle, explain all emerging phenomena in hadronic physics



Holographic light-front QCD (HLFQCD)

Present analytic approach follows from a semiclassical approximation to light-front QCD and its holographic embedding in AdS space: It leads to relativistic wave equations similar to the Schrödinger equation in atomic physics

Further constraints from a superconformal algebraic structure introduce a mass scale and fix the effective confinement potential

The zero mass eigenmode is identified with the pion and it is thus massless in the chiral limit

The new framework leads to relations between the Regge trajectories of mesons, baryons, and tetraquarks

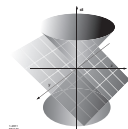
Holographic QCD also incorporates features of the Veneziano model as emerging properties

Further extensions incorporate the exclusive-inclusive connection in QCD and provide nontrivial relations between hadron form factors and quark and gluon distributions



Semiclassical approximation to light-front QCD

Light-front (LF) quantization uses the null plane $x^+ = x^0 + x^3 = 0$ tangent to the light cone as the initial surface (Dirac 1949), thus without reference to a specific Lorentz frame



Evolution in LF time x^+ is given by the Hamiltonian equation

$$i \frac{\partial}{\partial x^+} |\psi\rangle = P^- |\psi\rangle, \quad P^- |\psi\rangle = \frac{\mathbf{P}_\perp^2 + M^2}{P^+} |\psi\rangle,$$

for a hadron with 4-momentum $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, where P^- is a dynamical generator and P^+ and \mathbf{P}_\perp kinematical

Hadron mass spectra from LF invariant Hamiltonian $P^2 = P_\mu P^\mu = P^+ P^- - \mathbf{P}_\perp^2$

$$P^2 |\psi(P)\rangle = M^2 |\psi(P)\rangle, \quad |\psi\rangle = \sum_n \psi_n |n\rangle$$

Simple structure of the LF vacuum allows for a quantum-mechanical probabilistic interpretation of hadronic states in terms of invariant LF wave functions, $\psi_n = \langle n | \psi \rangle$, written in terms of the quark and gluon degrees of freedom



QCD(1+1)

A semiclassical LF Schrödinger equation derived from first principles QCD

G. 't Hooft (1974), K. Hornbostel, S. J. Brodsky and H. C. Pauli, PRD (1990)

Properties of QCD(1 + 1)

- Gluons are not dynamical, no self-couplings of gluons
- quarks have chirality but no spin
- Coupling g has dimensions of mass
- Confining field theory for any coupling

Express the hadron 2-momentum generator $P = (P^+, P^-)$, $P^\pm = P^0 \pm P^3$, in the $A^+ = 0$ gauge in terms of the fields $\psi_{L,R}$ and A^-

Interaction component of P^- is given in terms of ψ_R , only independent degree of freedom, from inverting A^- from the LF constraint equations

$$V = -\frac{1}{4}g^2 \int dx^- dy^- j^{+a}(x^-) |x^- - y^-| j^{+a}(y^-)$$

where $\frac{1}{2}j^{+a} = \psi_R^\dagger T^a \psi_R$



Mass spectrum computed from LF eigenvalue equation

$$P^+ P^- |\chi(P^+)\rangle = M_\pi^2 |\chi(P^+)\rangle$$

For the $q\bar{q}$ valence state (or large N_C) it leads to the 't Hooft equation (1974)

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right)\chi(x) + \frac{g^2 N_C}{\pi} P \int_0^1 dx' \frac{\chi(x) - \chi(x')}{(x-x')^2} = M_\pi^2 \chi(x)$$

Cancellation of singularities at $x = \epsilon$ and $x = 1 - \epsilon$ for the approximate solution

$$\chi(x) \sim x^{\beta_q} (1-x)^{\beta_{\bar{q}}}$$

leads for $m_q^2/\pi g^2 N_C \ll 1$ to $\beta_q = (3m_q^2/\pi g^2 N_C)^{1/2}$ and

$$M_\pi^2 = g \sqrt{\frac{\pi N_C}{3}} (m_q + m_{\bar{q}}) + \mathcal{O}((m_q + m_{\bar{q}})^2)$$

QCD(1+1): Both, the value of the CSB “condensate”

$$\langle \psi\psi \rangle = g f_\pi^2 \sqrt{\frac{\pi N_C}{3}}$$

and the strength of linear confinement depend on the value of the coupling g in the QCD Lagrangian, and are not emerging properties



QCD(1+ 3)

GdT and S. J. Brodsky (2009)

Mass spectrum is computed from the hadron matrix element

$$\langle \psi(P') | P_\mu P^\mu | \psi(P) \rangle = M^2 \langle \psi(P') | \psi(P) \rangle$$

We factor out the longitudinal $X(x)$ and orbital $e^{iL\varphi}$ dependence from the LFWF ψ

$$\psi(x, \zeta, \varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$



where $\zeta^2 = x(1-x)b_\perp^2$ is the invariant transverse separation between two quarks

Ultra relativistic limit $m_q \rightarrow 0$ longitudinal modes $X(x)$ decouple and the LF invariant Hamiltonian equation $P_\mu P^\mu | \psi \rangle = M^2 | \psi \rangle$ becomes a LF wave equation for ϕ

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$

- The effective potential U includes all interactions, including those from higher Fock states
- Critical value $L = 0$ corresponds to the lowest possible stable solution
- Relativistic and frame-independent semiclassical WE
- It has identical structure of AdS WE provided that $z = \zeta$

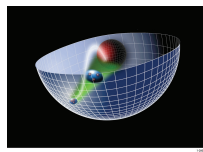


Higher-spin wave equations in AdS

GdT, H. G. Dosch and S. J. Brodsky (2013)

Integer spin

Start with AdS_{d+1} action for a tensor- J field $\Phi_{N_1 \dots N_J}$ with a dilaton φ to modify the IR region of AdS space



$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \frac{(\mu R)^2}{z^2} \right] \Phi_J(z) = M^2 \Phi_J(z)$$

Upon the substitution $\Phi_J(z) = z^{(d-1)/2-J} e^{-\varphi(z)/2} \phi_J(z)$ we find for $d = 4$ the semiclassical QCD light-front wave equation (z is the fifth dimension of AdS space)

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$

where $\zeta^2 = z^2 = x(1-x)b_{\perp}^2$ and

$$U(\zeta, J) = \frac{1}{2} \varphi''(\zeta) + \frac{1}{4} \varphi'(\zeta)^2 + \frac{2J-3}{2\zeta} \varphi'(\zeta)$$

the effective LF confinement potential with AdS mass-radius $(\mu R)^2 = -(2-J)^2 + L^2$



Half-integer spin

Start with Rarita-Schwinger action in AdS for spinor- J field $\Psi_{N_1 \dots N_{J-1/2}}$ with potential V (No dynamical dilaton for fermions)

Upon the substitution $\Psi_J^\pm(z) = z^{(d-1)/2-J} \psi_J^\pm(z) u^\pm$ we find for the chiral components ψ^\pm

$$-\frac{d}{dz} \psi_- - \frac{\nu + \frac{1}{2}}{z} \psi_- - V(z) \psi_- = M \psi_+$$
$$\frac{d}{dz} \psi_+ - \frac{\nu + \frac{1}{2}}{z} \psi_+ - V(z) \psi_+ = M \psi_-$$

with $|\mu R| = \nu + 1/2$ and equal probability $\int dz \psi_+(z)^2 = \int dz \psi_-^2(z)$

System of linear Eqs. equivalent to second order Eqs. ($z \rightarrow \zeta$):

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U^+(\zeta) \right) \psi_+ = M^2 \psi_+$$
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4(L+1)^2}{4\zeta^2} + U^-(\zeta) \right) \psi_- = M^2 \psi_-$$

the semiclassical LF WE with ψ_+ and ψ_- corresponding to LF orbital L and $L+1$ with

$$U^\pm(\zeta) = V^2(\zeta) \pm V'(\zeta) + \frac{1+2L}{\zeta} V(\zeta), \quad L = \nu,$$

a J -independent potential in agreement with the observed degeneracy in the baryon spectrum



Superconformal QM: Emergence of a mass scale and confinement

de Alfaro, Fubini and Furlan, (1976), Witten (1981) and Fubini and Rabinovici (1984)

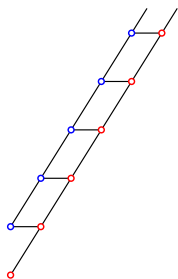
Embedding LF physics in AdS leads to important insights on the nonperturbative structure of bound state equations for any spin, but the effective confinement potential is not determined

The potential $V(\zeta)$ in the baryon equations plays the role of the superpotential in SUSY QM: Its form is fixed by superconformal QM:

$$W(x) = f/x + \lambda x$$

It corresponds to a scale deformed algebra with special conformal transformations: It generates a scale-dependent Hamiltonian that closes under the graded algebra

It leads to harmonic confinement in the LF equations, thus to Regge trajectories, and a massless pion !



$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f - \lambda + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \right) \phi_1 = E \phi_1$$
$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f + \lambda + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \right) \phi_2 = E \phi_2$$

$$H = \frac{1}{2} \{Q, Q^\dagger\}$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[Q, H] = [Q^\dagger, H] = 0$$
$$H|E\rangle = E|E\rangle$$

Note: Since $[Q^\dagger, H] = 0$, the states $|E\rangle$ and $Q^\dagger|E\rangle$ are degenerate for $E \neq 0$, but for $E = 0$ we have $Q^\dagger|E = 0\rangle = 0$. The pion has no SUSY baryon partner



Light-front mapping and baryons

GdT, Dosch and Brodsky (2015)

Upon the substitution in the superconformal equations

$$x \mapsto \zeta, \quad E \mapsto M^2, \quad f \mapsto L + \frac{1}{2}$$

$$\phi_1 \mapsto \psi_-, \quad \phi_2 \mapsto \psi_+$$

we recover the LF/AdS nucleon bound-state equations

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(L+1) \right) \psi_+ = M^2\psi_+$$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4(L+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda L \right) \psi_- = M^2\psi_-$$

Eigenvalues

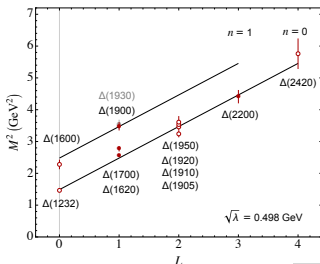
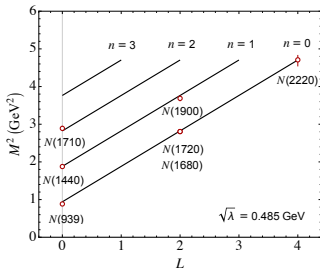
$$M^2 = 4\lambda(n + L + 1)$$

Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+L} e^{-\lambda\zeta^2/2} L_n^L(\lambda\zeta^2)$$

$$\psi_-(\zeta) \sim \zeta^{\frac{3}{2}+L} e^{-\lambda\zeta^2/2} L_n^{L+1}(\lambda\zeta^2)$$

See also: Abidin and Carlson (2009) and Gutsche, Lyubovitskij, Schmidt and Vega (2012)



Superconformal meson-baryon-tetraquark symmetry

Dosch, GdT, and Brodsky (2015)

Upon the substitution in the superconformal equations

$$x \mapsto \zeta, \quad E \mapsto M^2,$$

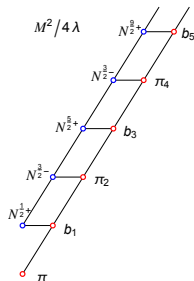
$$\lambda \mapsto \lambda_B = \lambda_M, \quad f \mapsto L_M - \frac{1}{2} = L_B + \frac{1}{2}$$

$$\phi_1 \mapsto \phi_M, \quad \phi_2 \mapsto \phi_B$$

we find the LF meson/baryon bound-state equations

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \lambda_M^2 \zeta^2 + 2\lambda_M(L_M - 1) \right) \phi_M = M^2 \phi_M$$

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + \lambda_B^2 \zeta^2 + 2\lambda_B(L_B + 1) \right) \phi_B = M^2 \phi_B$$



$$\Phi = \begin{pmatrix} \phi_M & \phi_B^- \\ \phi_B^+ & \phi_T^- \end{pmatrix}$$

Superconformal QM imposes the condition $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes)

and the remarkable relation $L_M = L_B + 1$

L_M is the LF angular momentum between the quark and antiquark in the meson and L_B between the active quark and spectator diquark cluster in the baryon

Full hadron 4-plet: meson-baryon-tetraquark

Brodsky, GdT, Dosch, GdT and Lorcé (2016)



Special role of the pion as a unique state of zero energy $Q^\dagger|M, L\rangle = |B, L-1\rangle$, $Q^\dagger|M, L=0\rangle = 0$

Zero mode invariant under deformations of scale λ

Spin-dependent Hamiltonian

$$H = \{Q_\lambda^\dagger, Q_\lambda\} + 2\lambda s$$

s internal spin of the meson, or the spin of the diquark cluster of the baryon partner

Supersymmetric 4-plet

$$M_M^2 = 4\lambda(n + L_M) + 2\lambda S$$

$$M_B^2 = 4\lambda(n + L_B + 1) + 2\lambda S$$

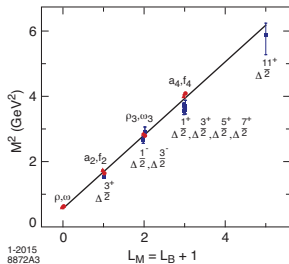
$$M_T^2 = 4\lambda(n + L_T + 1) + 2\lambda S$$

Quark masses and CSB from longitudinal dynamics

$$\left(-\sigma^2 \partial_x (x(1-x) \partial_x) + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right) \chi(x) = \Delta M^2 \chi(x)$$

$$M_\pi^2 = \Delta M^2 = \sigma(m_q + m_{\bar{q}}) + \mathcal{O}((m_q + m_{\bar{q}})^2)$$

Recent work: Li and Vary (2021), GdT and Brodsky (2021), Ahmady *et al.* (2021), Shuryak and Zahed (2021), Weller and Miller (2021), Lyubovitskij and Schmidt (2022), Rinaldi *et al.* (2022)



Veneziano amplitudes and holographic QCD

Veneziano 4-point amplitude (1968)

$$A(s, t) \sim B(1 - \alpha(s), 1 - \alpha(t))$$

where $\alpha(t) = \alpha_0 + \alpha' t$ is the linear Regge trajectory

Sum of poles in the direct s or crossed t -channels: Accounts for duality in strong interactions (Dolen-Horn-Schmid (1967))

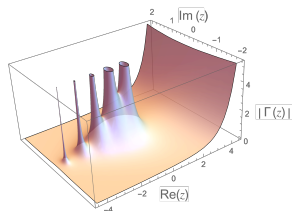
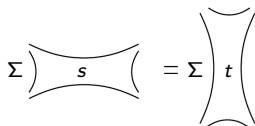
For fixed t and large s the result $A(s, t) \sim s^{\alpha(t)-1}$ is found

Scattering is exponentially suppressed with increasing t : It cannot produce collisions at large angles (soft scattering)

Veneziano model can be extended to N-particle amplitudes and/or external currents

Pole structure and high-energy Regge behavior at tree level (nonperturbative mathematical structure)

Features of Regge theory and the Veneziano model as emerging properties of holographic QCD

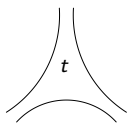


Hadron form factors

Form Factor as a 3-point Veneziano amplitude

Ademollo and Del Giudice (1969), Landshoff and Polkinghorne (1970)

GdT, Liu, Sufian, Dosch, Brodsky and Deur (HLFHS 2018)



s -channel dependence is replaced by a fixed pole allowed by unitarity

$$F(t) \sim B(\gamma, 1 - \alpha(t)) \rightarrow Q^{-2\gamma} \quad \text{for large } Q^2 = -t > 0$$

Compare with Brodsky-Farrar (B-F) hard counting rules (1973) for large Q^2

$$F_\tau(Q^2) \sim \left(\frac{1}{Q^2}\right)^{\tau-1}$$

where the twist τ is the number of constituents in a given Fock component: $\gamma = \tau - 1$

The B-F asymptotic counting rules allow us to incorporate the underlying (hard) constituent pointlike structure of hadrons into the (soft) Veneziano amplitudes with external currents

$$F_\tau(t) = \frac{1}{N_\tau} B(\tau - 1, 1 - \alpha(t))$$

$\alpha(t)$ is the Regge trajectory of the VM which couples to the quarks in the hadron

Note: Mapping to the LF one finds effective LFWFs $\psi_{eff}(x, k_\perp)$ and $\psi_{eff}(x, b_\perp)$



For $\tau = N$, the number of constituents, the FF is an $N - 1$ product of poles

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_{n=0}^2}\right) \left(1 + \frac{Q^2}{M_{n=1}^2}\right) \cdots \left(1 + \frac{Q^2}{M_{n=\tau-2}^2}\right)}$$

located at

$$-Q^2 = M_n^2 = \frac{1}{\alpha'} (n + 1 - \alpha(0))$$

It generates the radial spectrum of the exchanged t -channel vector mesons

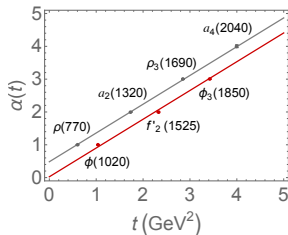
For the ρ trajectory $\alpha_0 = 1/2$ and $\alpha' = 1/4\lambda$, thus

$$M_n^2 = 4\lambda \left(n + \frac{1}{2}\right)$$

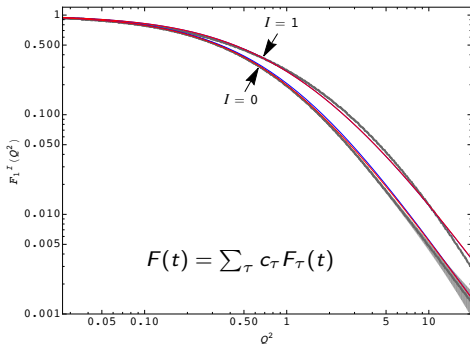
corresponding to the HLFQCD result for the ρ VM and its radial excitations for $n = 0, 1, 2, \dots, \tau - 2$

Note: The AdS Polchinski & Strassler FF for the soft-wall model leads to $M_n^2 = 4\lambda(n + 1)$, thus to the wrong intercept $\alpha_0 = 0$ Karch, Katz, Son, and Stephanov (2006)

Note: HLFQCD originates from the precise mapping of the AdS and LF expressions of FFs for an arbitrary number of partons Brodsky and GdT (2006)



Nucleon isospin form factors $F^{I=0,1}(t) = F_p(t) \pm F_n(t)$



HLFHS (2020): — Valence contribution only

HLFHS (2020): — Including $u\bar{u}$ and $d\bar{d}$

Ye et al. (2018): — z-expansion data analysis



Quark distribution functions

Using the integral representation of the Beta function the FF is expressed in a reparametrization invariant form

$$F(t)_\tau = \frac{1}{N_\tau} \int_0^1 dx w'(x) w(x)^{-\alpha(t)} [1 - w(x)]^{\tau-2}$$

with $w(0) = 0$, $w(1) = 1$, $w'(x) \geq 0$

Flavor FF is given in terms of the valence GPD $H_\tau^q(x, \xi = 0, t)$ at zero skewness

$$F_\tau^q(t) = \int_0^1 dx H_\tau^q(x, t) = \int_0^1 dx q_\tau(x) \exp[tf(x)]$$

with the profile function $f(x)$ and PDF $q(x)$ determined by $w(x)$

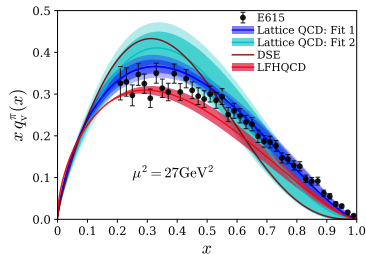
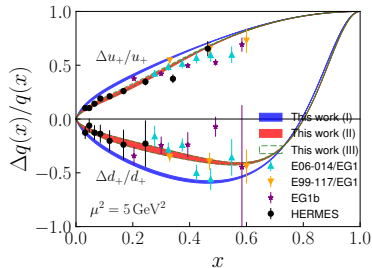
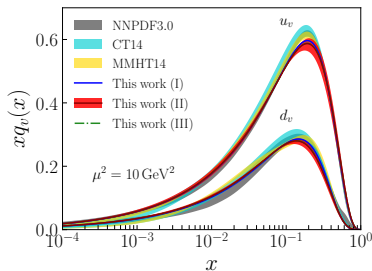
$$f(x) = \frac{1}{4\lambda} \log\left(\frac{1}{w(x)}\right)$$

$$q_\tau(x) = \frac{1}{N_\tau} [1 - w(x)]^{\tau-2} w(x)^{-\alpha(0)} w'(x)$$

At $x \sim 0$, $w(x) \sim x$ from Regge behavior, and $w'(1) = 0$ to recover the counting rules at $x \rightarrow 1$, $q_\tau(x) \sim (1-x)^{2\tau-3}$ (inclusive-exclusive connection)

If $w(x)$ fixed by nucleon PDFs then pion PDF is a prediction.





Separation of chiralities from the axial current
Coefficients c_τ are fixed from the vector current

Regge trajectory from HLFQCD

$$\alpha_A(t) = \frac{t}{4\lambda}$$

$$\lim_{x \rightarrow 1} \frac{\Delta q(x)}{q(x)} = 1, \quad \lim_{x \rightarrow 0} \frac{\Delta q(x)}{q(x)} = 0$$

Note: DGLAP NNLO evolution from initial scale $\mu \simeq 1 \text{ GeV}$ from soft-hard matching in α_s
Deur, Brodsky and GdT (2010, 2016)



Intrinsic charm-anticharm asymmetry in the proton

Sufian, T. Liu, Alexandru, Brodsky, GdT, Dosch, Draper, K. F. Liu and Y. B. Yang (2020)

Intrinsic charm in the proton introduced by Brodsky, Hoyer, Peterson and Sakai (1980)

Charm FF normalization computed with with three gauge ensembles in LGTH
(one at the physical pion mass)

Intrinsic charm asymmetry $c(x) - \bar{c}(x)$,

$$c(x) - \bar{c}(x) = \sum_{\tau} c_{\tau} (q_{\tau}(x) - q_{\tau+1}(x))$$

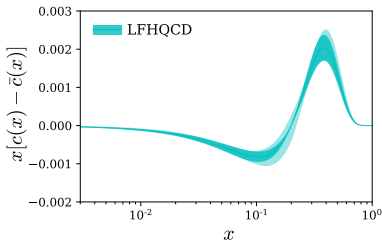
with $\int_0^1 dx [c(x) - \bar{c}(x)] = 0$, from HLFQCD

J/ψ Regge trajectory

$$\alpha(t)_{J/\psi} = -2.066 + \frac{t}{4\lambda_c}, \quad \lambda_c = 0.874 \text{ GeV}^2$$

from HLFQCD and HQET

Nielsen, Brodsky, GdT, Dosch, Navarra and Zou (2018)



Gravitational form factors and gluon distribution functions

(HLFHS 2021)

Spin-2 gluon gravitational FF $A(t)$ from the coupling of the metric fluctuations induced by the spin-two Pomeron with the energy momentum tensor in AdS

$$A_{\tau}^g(t) \sim B(\tau - 1, 2 - \alpha_P(t))$$

with Pomeron Regge trajectory

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t$$

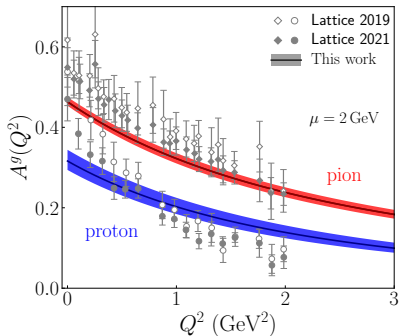
where $\alpha_P(0) \simeq 1.08$ and $\alpha'_P = 0.25 \text{ GeV}^{-2}$

Radial spectrum from t -channel poles in the 2^{++} trajectory

$$-Q^2 = M_n^2 = \frac{1}{\alpha'_P} (n + 2 - \alpha(0))$$

with $M_0 \simeq 1.92 \text{ GeV}$

GdT, Dosch, Liu, Sufian, Brodsky, Deur (2021)



Lattice data from Shanahan *et al.* (2018) and Pefkou *et al.* (2021)



Intrinsic gluon distribution in the proton and pion

Gluon GPD $H_{\tau}^g(x, t) = g_{\tau}(x)e^{tf(x)}$

$$f(x) = \alpha'_P \log\left(\frac{1}{w(x)}\right),$$

$$g_{\tau}(x) = \frac{1}{N_{\tau}} \frac{w'(x)}{x} [1 - w(x)]^{\tau-2} w(x)^{1-\alpha_P(0)}$$

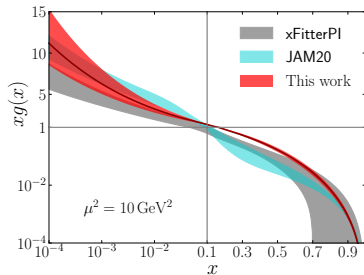
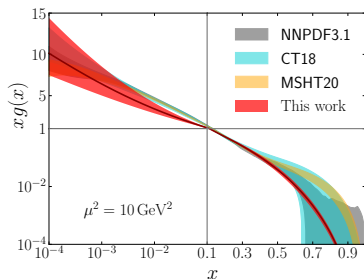
Normalization of $A^g(0)$ determined from the sum rule:

$$\sum_q \langle x \rangle_q + \sum_{\bar{q}} \langle x \rangle_{\bar{q}} + \langle x \rangle_g = 1$$

Basic parameters fixed in quark sector:
No adjustable parameters

Single Pomeron (HLFHS 2022))

Hard Pomeron from the evolution of the nonperturbative gluon distribution function



Proton (upper) and pion (bottom)