Radiative Corrections

(a few typical examples at JLab and a draft plan for SoLID PVDIS)

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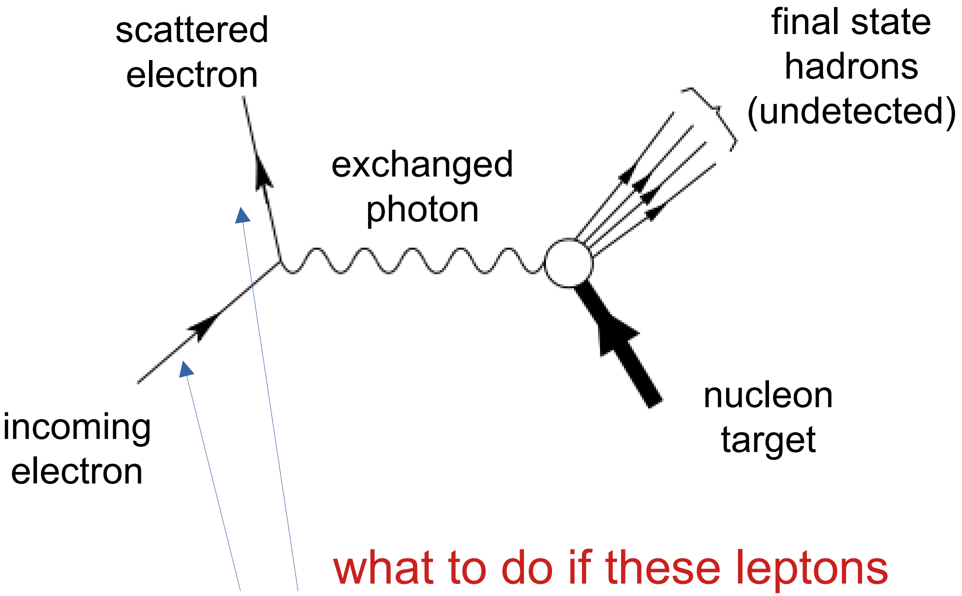
Radiative Corrections

- General approach at JLab
- What was done for JLab EG4
- What was done for PVDIS 6 GeV
- Note: we do not typically deal with
  - box diagrams
  - weak effects
  - QED effects (quark line)
  - so far works fine (for current precision goals)

JLab 6 GeV PVDIS long paper:

https://doi.org/10.1103/PhysRevC.91.045506

e-Print: 1411.3200 [nucl-ex]



what to do if these leptons
→ ionization loss
→ external bremsstrahlung
→ multiple scattering
→ internal bremsstrahlung?

First method

- apply correction directly to measured cross sections
- more suitable for small-acceptance spectrometers
- (“RC_external” code calculates both Born and radiated cross sections)

Radiative correction

$$\sigma_{rad}(E_s, E_p) = \int_0^T \frac{dt}{T} \int_{E_{s\,min}(E_p)}^{E_s} dE'_s \int_{E_p}^{E_{p\,max}(E'_s)} dE'_p I(E_s, E'_s, t)\sigma_r(E'_s, E'_p) I(E'_p, E_p, T-t)$$

(Mo. & Tsai method, SLAC-PUB-848 (1971).)

- $I(E, E', t)$: the probability of energy loss due to the external radiation.
- T: total path length before and after scattering.
- $\sigma_r = \sigma_r^{DIS} + \sigma_r^{quasi-elastic} + \sigma_r^{elastic} \Leftarrow$ require a cross section input

$$\Rightarrow \quad RC = \frac{\sigma_{born}^{model}}{\sigma_{rad}^{model}} \quad \Rightarrow \quad \sigma_{born}^{data} = \sigma_{rad}^{data} \cdot RC$$

- For ^3H and ^3He born cross section model, we use F_2^d from Bodek *et al.* [1] and the EMC model ($F_2(^3He)/F_2^d$) from S. Kulagin and R. Petti (KP) [2]
- RC error is the deviation caused by using different cross section models

[1] Phys. Rev. D20, 1471 (1979)
[2] Nucl Phys A765 (2006) 126

(from H. Liu’s talk)

Second method (fully forward simulation method)

- use a full simulation method to calculate “Born” and to simulate “measured” observables using model inputs
- if simulated “measured” observables do not agree with real data, adjustment is made to the model inputs
- more suitable for large-acceptance spectrometers
- can be added to any existing, experimental full simulation packages
- technical complications:
  - tails from elastic scattering may need to be subtracted first
  - positive and negative cross section (difference) regions need to be done separately

"full simulation method"

(done for CLAS g1p,g1d measurements)

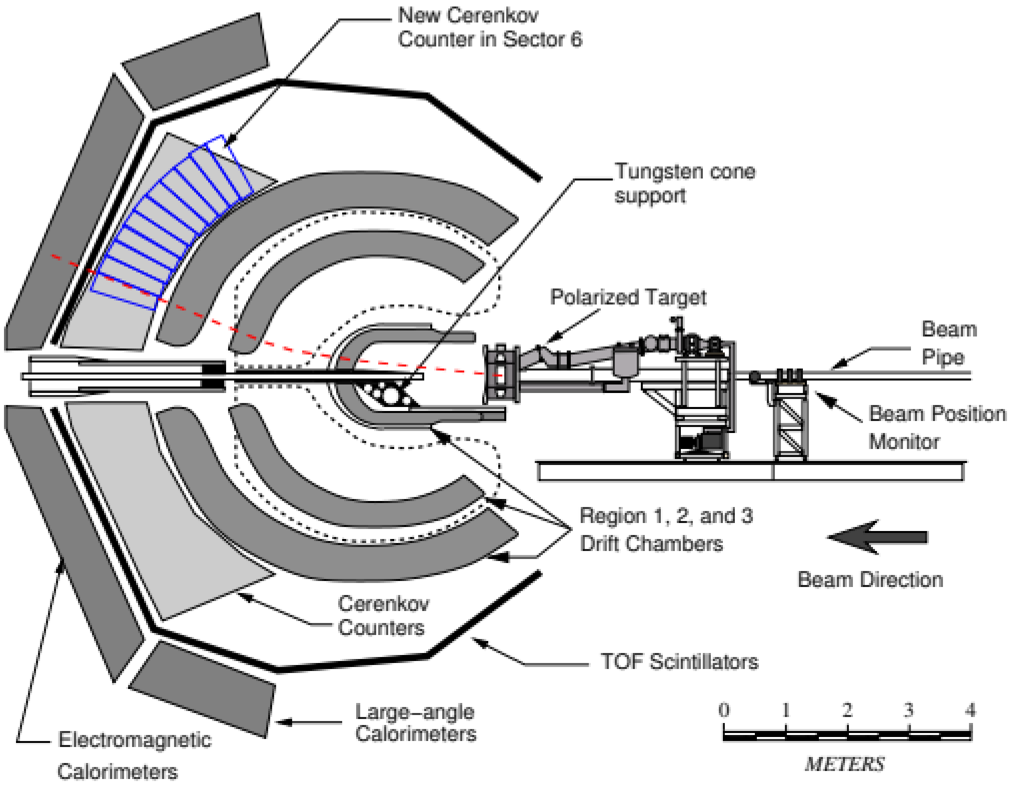
1. start from Ebeam — E_{beam}
2. calculate ext Eloss (brem, ion), multiple scattering — $E = E_{beam} - dE_{ext,ion}$
3. calculate int brem. — $E_{vtx} = E_{beam} - dE_{ext,ion} - dE_{\text{int}}$
4. scattering occurs (el, QE, res, DIS) randomly pick θ, E' — E'_{vtx}
5. calculate int brem. — $E' = E'_{\text{vtx}} - dE'_{\text{int}}$
6. calculate ext Eloss (brem, ion), multiple scattering — $E'_{\text{det}} = E'_{\text{vtx}} - dE'_{\text{int}} - dE'_{\text{ext, ion}}$
7. check if the electron reaches detector

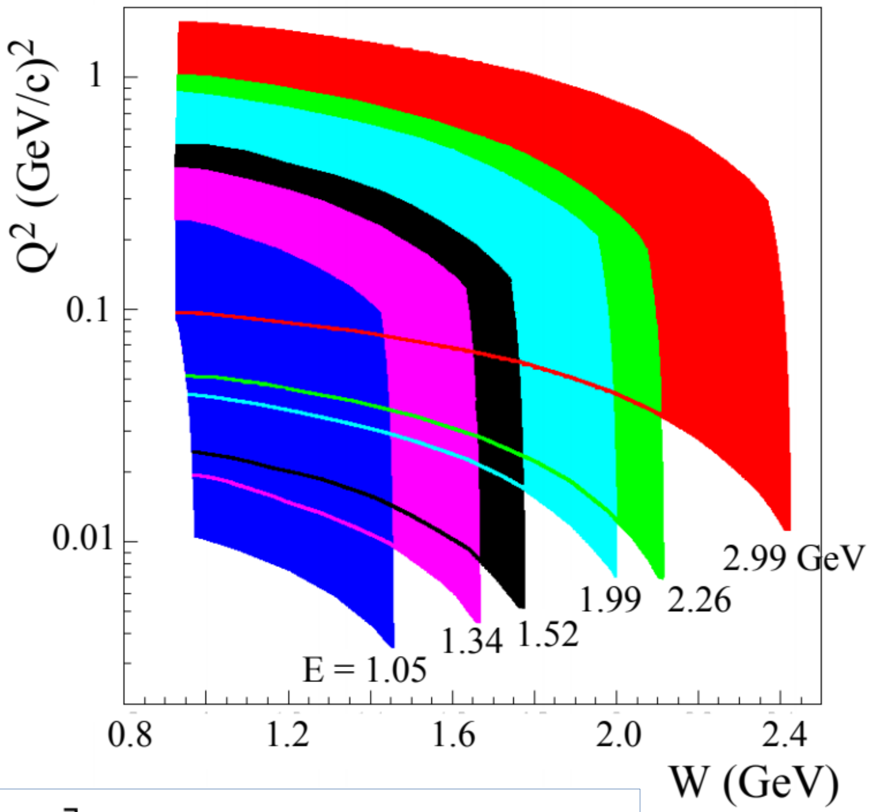
input model at $\left(x_{\text{vtx}}, Q^2_{\text{vtx}}\right)$

$\left(x_{\text{det}}, Q^2_{\text{det}}\right)$

use the difference between observed and simulated spectra to apply corrections

Radiative Corrections for CLAS EG4

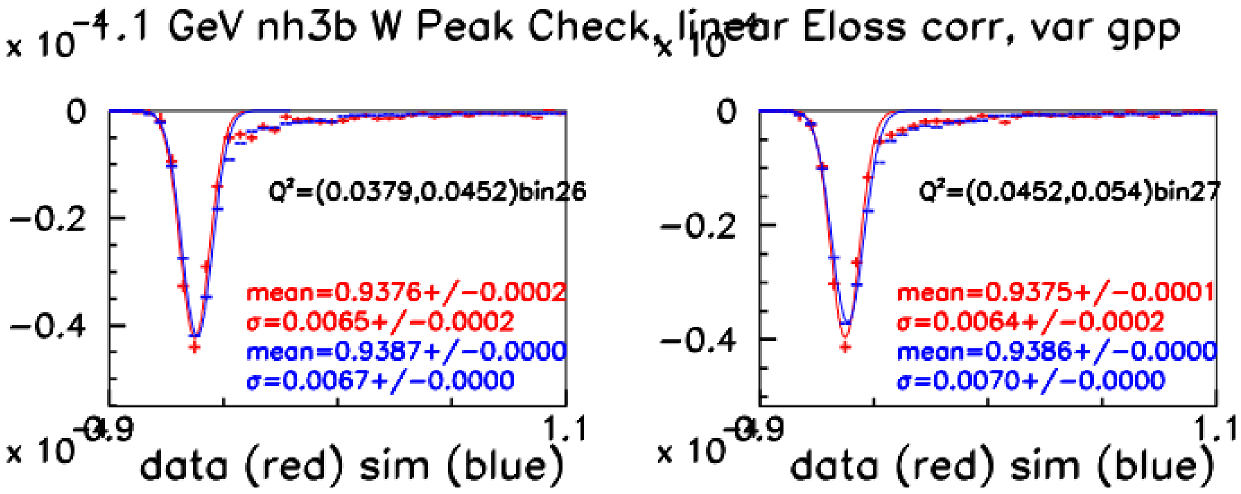




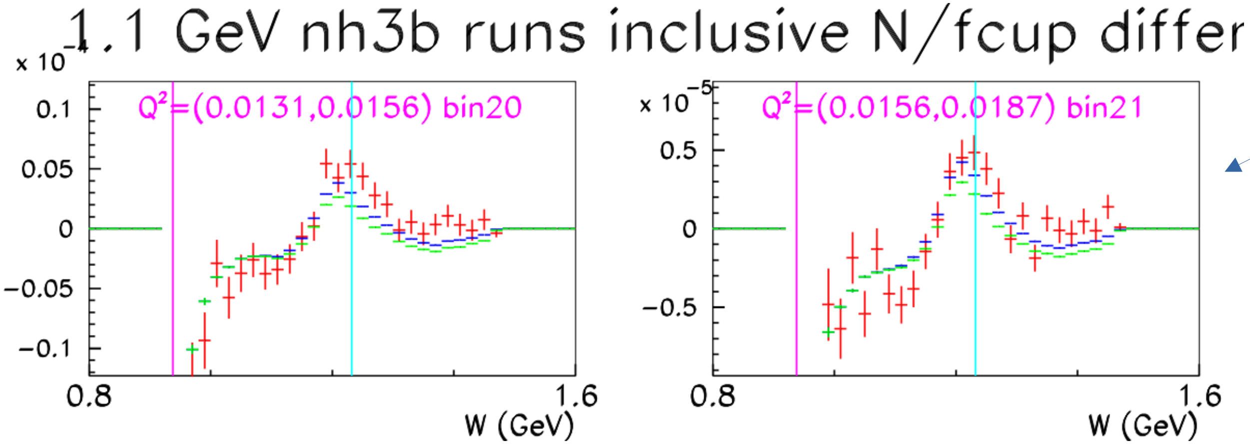
$$\Delta\sigma_{||} = \frac{d^2\sigma_{\uparrow\Uparrow}}{d\Omega dE'} - \frac{d^2\sigma_{\uparrow\Downarrow}}{d\Omega dE'} = \left[\frac{N^+}{N_e^+} - \frac{N^-}{N_e^-}\right] \frac{1}{N_{targ}} \frac{1}{P_b P_t} \frac{1}{\Delta\Omega} \frac{1}{\eta_{detector}}$$

Simulation of EG4 Proton Elastic Peak

- simulation reproduces measured double-polarized yield (N/Ne) difference
- cross-checking PbPt measurement, tuning detector smearing, material thickness, etc.
- Radiative tail from elastic peak can be determined and subtracted from inelastic data



Simulation of EG4 Proton Resonance Region

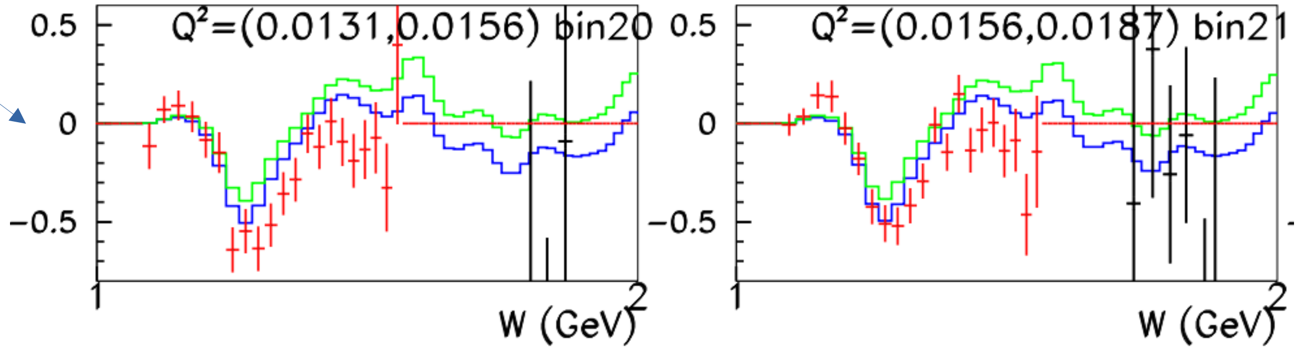


Comparison of polarized yield difference $N^+ - N^-$

red: data
blue: simulation with “best” A1 model
green: simulation with “best” A1 model shifted by +0.1

$$g_1^{\rm data} = g_1^{\rm sim0} + (g_1^{\rm sim1} - g_1^{\rm sim0}) \frac{\Delta n^{\rm data} - \Delta n^{\rm sim0}}{\Delta n^{\rm sim1} - \Delta n^{\rm sim0}}$$

Extracted g1 structure function:



"full simulation method"

(done for 6 GeV PVDIS using modified HAMC)

start from Ebeam

E_{beam}

calculate ext Eloss (brem, ion), multiple scattering

$E = E_{beam} - dE_{ext,ion}$

calculate int brem.

$E_{vtx} = E_{beam} - dE_{ext,ion} - dE_{int}$

scattering occurs (el, QE, res, DIS) randomly pick θ, E'

E'_{vtx}

calculate int brem.

$E' = E'_{vtx} - dE'_{int}$

calculate ext Eloss (brem, ion), multiple scattering

$E'_{det} = E'_{vtx} - dE'_{int} - dE'_{ext,\,ion}$

check if the electron reaches detector

calculate kinematics and observables at

$\left(x_{vtx}, Q^2_{vtx}\right)$ and $\left(x_{det}, Q^2_{det}\right)$

$\left(x_{hadron}, Q^2_{hadron}\right)$ (HEP/ Djangoh)

use the difference between the two to apply corrections

$$1 + \bar{f}_{rc} = \frac{A\left(\langle Q^2_{det}\rangle, \langle x_{det}\rangle\right)}{A\left(\langle Q^2_{vtx}\rangle, \langle x_{vtx}\rangle\right)}$$

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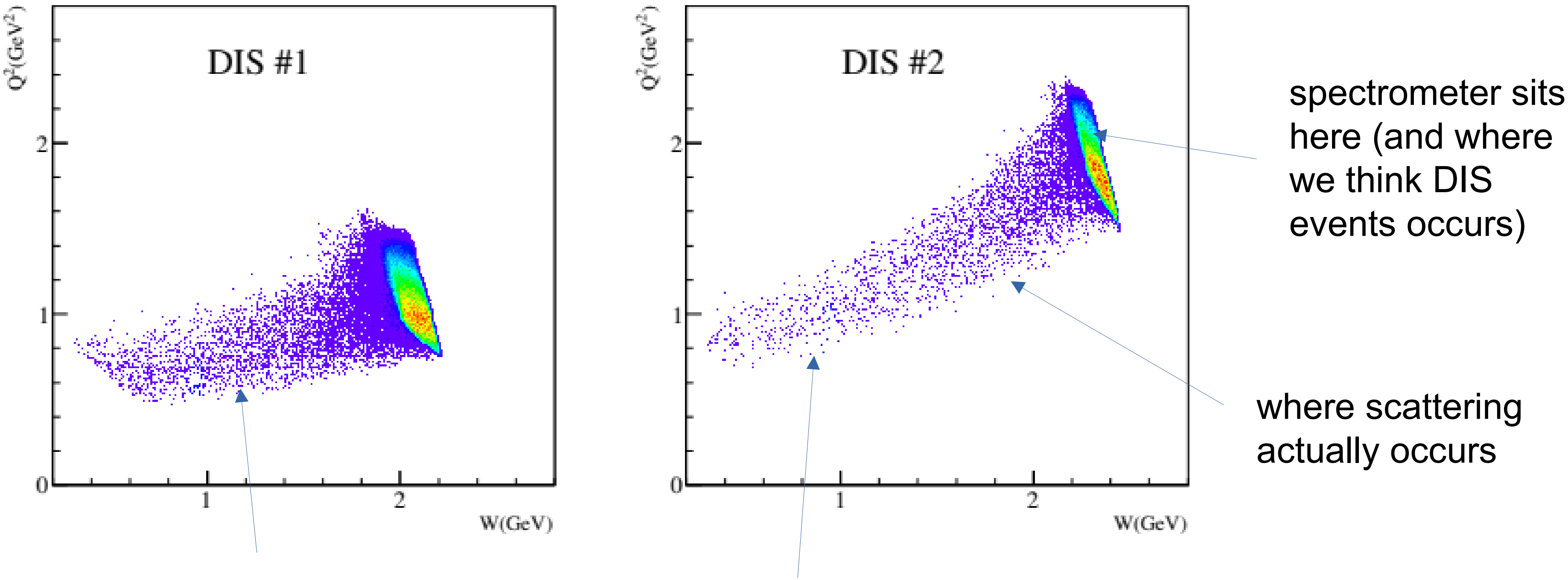
Radiative Correction for 6 GeV PVDIS

Q2_vertex vs. W_vertex for 6 GeV that includes both internal and external radiations

internal use Mo&Tsai's effective radiator formula: $t_{\text{equiv}} = \frac{3\alpha}{4\pi}\left[\ln\left(\frac{Q^2}{m^2}\right) - 1\right]$ (see HAMC manual)



resonance Apv: used model, checked with data (next slide)

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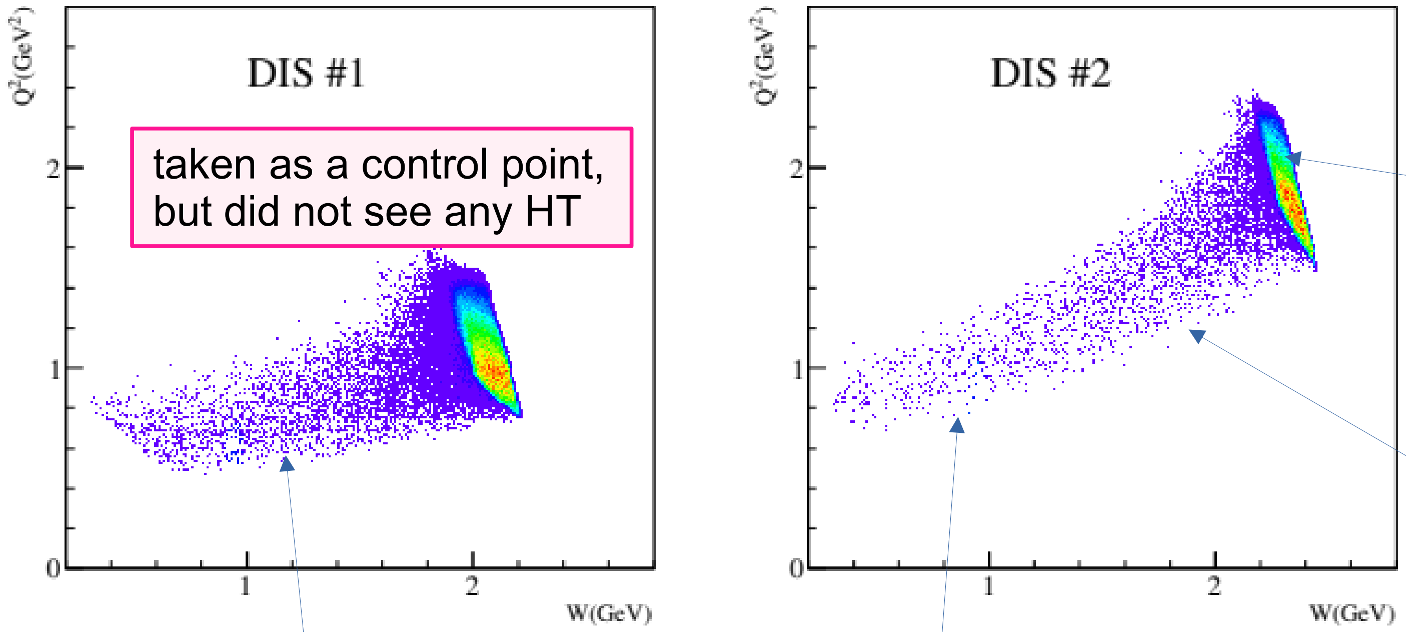
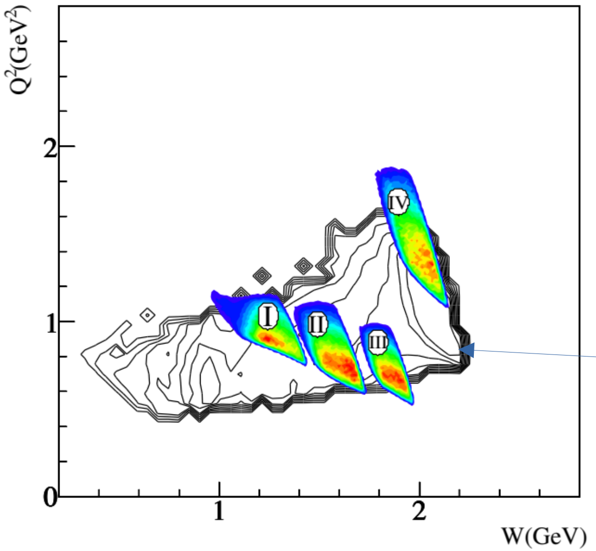


spectrometer sits here (and where we think DIS events occurs)

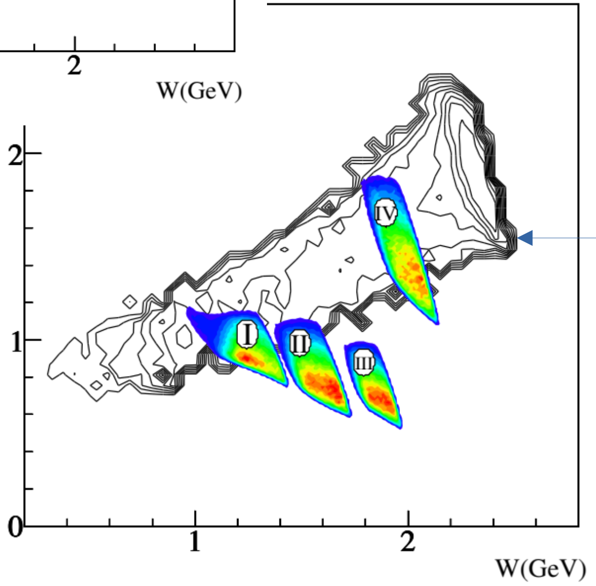
where scattering actually occurs

resonance Apv: used model, checked with data (next slide)

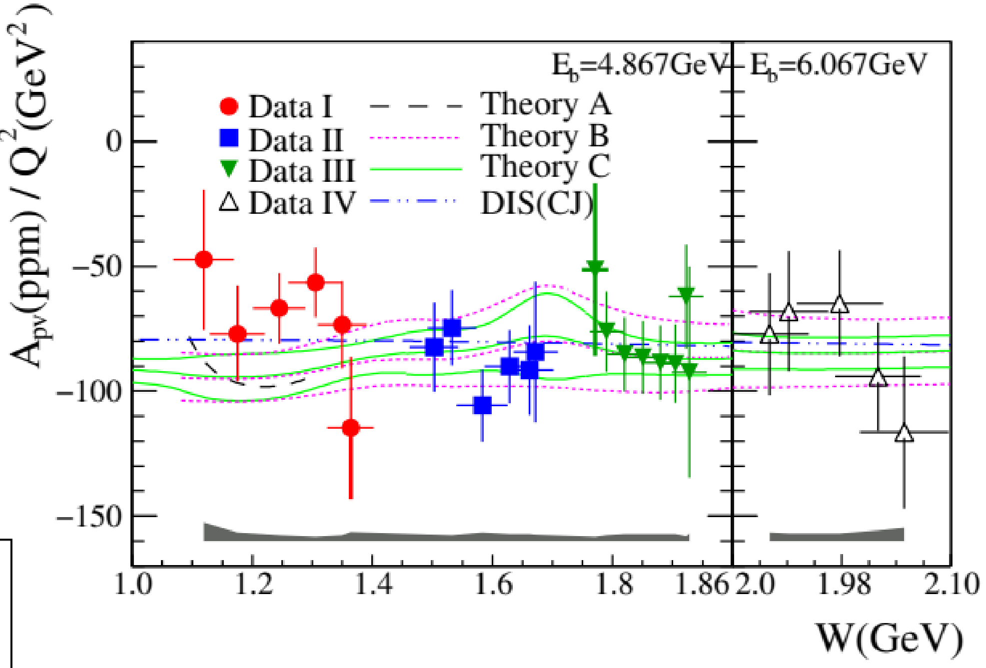
Resonance data taken during 6 GeV



"DIS#1"



"DIS#2"

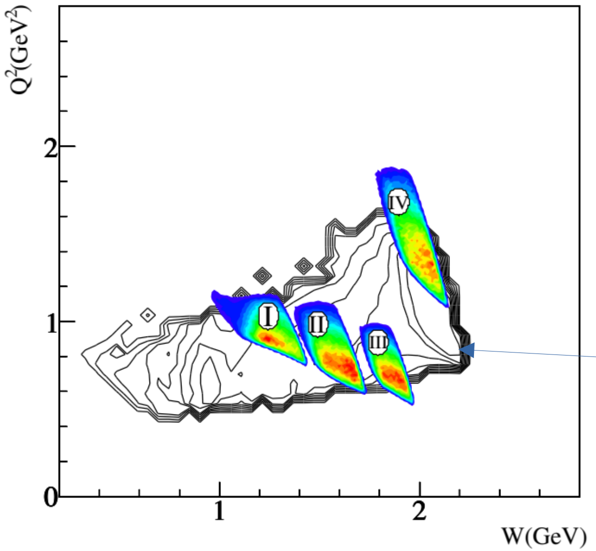


Elastic and QE calculated separately;

Uncertainty in resonance Apv as input to radiative corrections:

– W<1.4 GeV: 25%
– 1.4<W<1.7 GeV: 10%
– 1.7<W<2.0 GeV: 7.7%

Resonance data taken during 6 GeV



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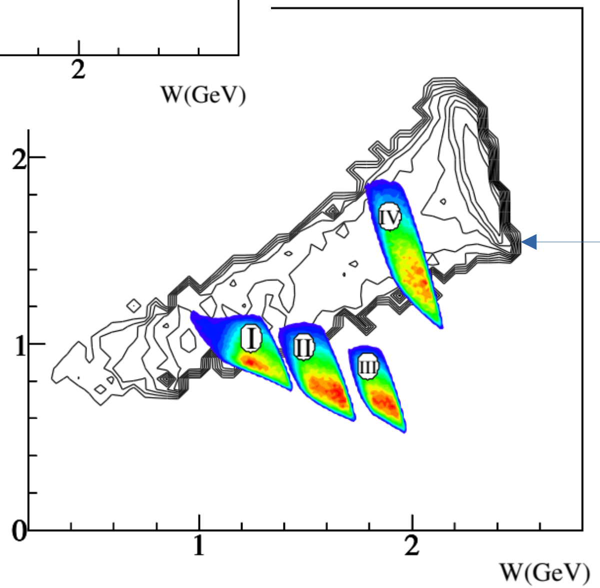
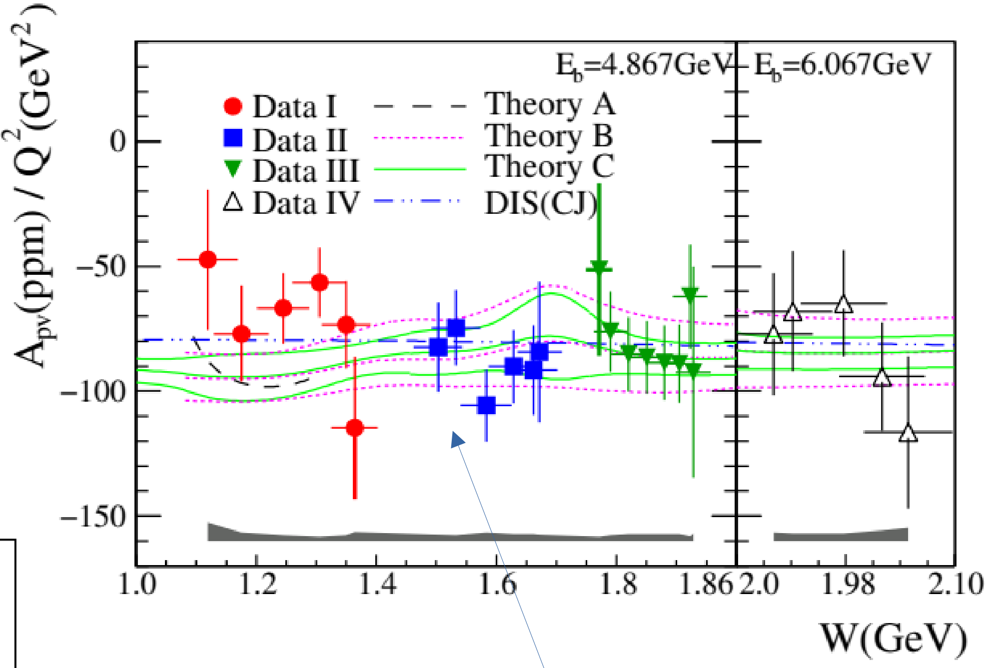


“DIS#2”

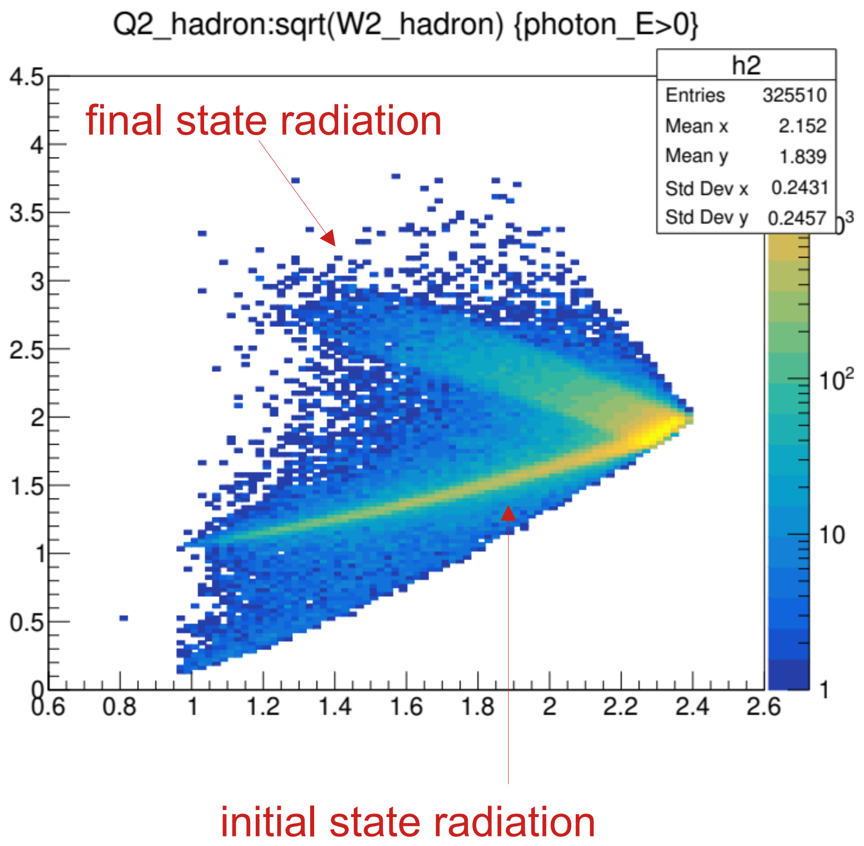


Caveat:

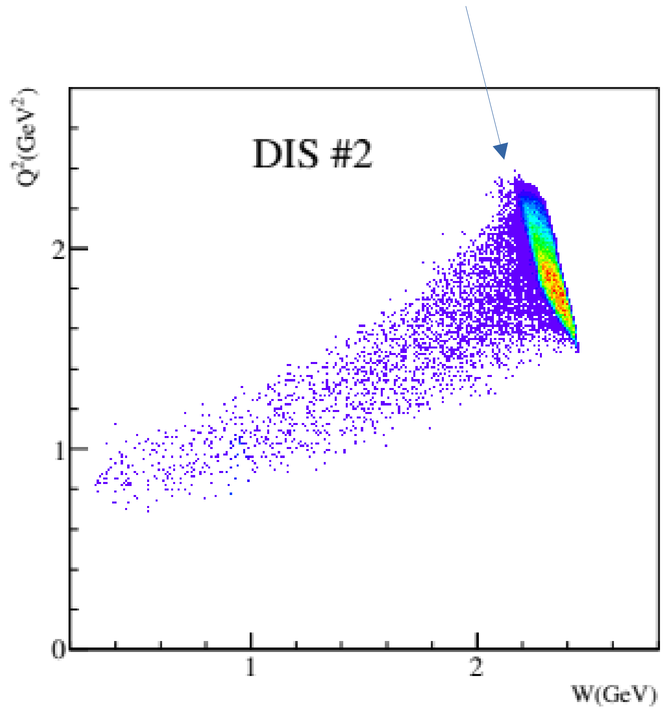
– We didn’t really have so many (small) W bins and these points overlap (are correlated)

– HAMC implementation of t_{equiv} was “off”

Q2_vertex vs. W_vertex for 6 GeV that includes only internal radiations (Djangoh simulation)



simulation from 6 GeV (both int and ext radiation), barely any seen for final state radiation (note that this is a linear z plot), or could it be that initial state radiation dominates for fixed-target experiments (due to extended target material)?



6 GeV PVDIS long paper: $A^{\text{rad-corrected}} = A^{\text{meas}}\left(1+\bar{f}_{rc}\right)$

$$1+\bar{f}_{rc} = \frac{A\left(\left\langle Q^2_{\text{det}}\right\rangle, \left\langle x_{\text{det}}\right\rangle\right)}{A\left(\left\langle Q^2_{\text{vtx}}\right\rangle, \left\langle x_{\text{vtx}}\right\rangle\right)}$$

DIS Kine #1:

$$E_{\text{beam}} = 6.067\,\text{GeV}$$

$$\theta = 12.9^o, E' = 3.66\,\text{GeV}$$

$$\left\langle x\right\rangle_{\text{data}} = 0.241, \left\langle Q^2\right\rangle_{\text{data}} = 1.085\,\text{GeV}^2$$

$$1+f_{\text{rc}} = 1.015 \pm 0.02$$

$$f_{\gamma\gamma} = -0.002 \pm 0.002$$

$$A_{\text{phys}} = -91.10 \pm 4.30\,\text{ppm}\,(4.7\,\%)$$

DIS Kine #2:

$$E_{\text{beam}} = 6.067\,\text{GeV}$$

$$\theta = 20^o, E' = 2.63\,\text{GeV}$$

$$\left\langle x\right\rangle_{\text{data}} = 0.295, \left\langle Q^2\right\rangle_{\text{data}} = 1.901\,\text{GeV}^2$$

$$1+f_{\text{rc}} = 1.019 \pm 0.004$$

$$f_{\gamma\gamma} = -0.003 \pm 0.003$$

$$A_{\text{phys}} = -160.80 \pm 7.12\,\text{ppm}\,(4.4\,\%)$$

γ–Z box

Electroweak radiative corrections were applied to all couplings used in the calculation of the asymmetry. The electromagnetic fine structure constant α was evolved to the measured Q^2-values from $\alpha_{EM}|_{Q^2=0} = 1/137.036$ [52]. The evaluation takes into account purely electromagnetic vacuum polarization. The Fermi constant is $G_F = 1.1663787(6) \times 10^{-5}$ GeV^{-2} [52]. The $C_{1q,2q}$ were evaluated using Table 7 and Eq. (114-115) of Ref. [91] at our measured Q^2-values in the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme using a fixed Higgs mass $M_H = 125.5$ GeV:

$$C_{1u}^{\mathrm{SM}} = -0.1887 - 0.0011 \times \frac{2}{3} \ln(\langle Q^2 \rangle / 0.14\mathrm{GeV}^2) \quad (86)$$

$$C_{1d}^{\mathrm{SM}} = 0.3419 - 0.0011 \times \frac{-1}{3} \ln(\langle Q^2 \rangle / 0.14\mathrm{GeV}^2) \quad (87)$$

$$C_{2u}^{\mathrm{SM}} = -0.0351 - 0.0009 \ln(\langle Q^2 \rangle / 0.078\ \mathrm{GeV}^2) \quad (88)$$

$$C_{2d}^{\mathrm{SM}} = 0.0248 + 0.0007 \ln(\langle Q^2 \rangle / 0.021\ \mathrm{GeV}^2) \quad (89)$$

and it is expected that the uncertainty is negligible. Equations (86-89) include the "charge radius effect" and an estimate of the interference between γ-exchange and the γZ box, but not the effect from the $\gamma\gamma$ box. The effect from the $\gamma\gamma$ box was applied as a correction to the measured asymmetry as described in previous sections.

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Recent calculation using stand-alone Mo&Tsai equivalent radiator:

	Kine #1	Kine #2
internal:	-0.7% (original HAMC -0.33%)	-1.2% (original HAMC -0.7%)

Djangoh:

	Kine #1	Kine #2
internal with lepton radiation:	-0.75%	-1.23%
internal with both lepton and quark radiation:	-0.3%	-0.7%
$\gamma\gamma$ and γZ boxes:	0.026%	0.03%
pure weak:	+1.14%	+1.4%

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DIS Kine #2:

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Recent calculation using …
internal: -0.7% (ori…

Djangoh:
internal with lepton radiat…
internal with both lepton a…
γγ and γZ boxes:
pure weak:

2012 vs. now:

- size of internal Bremsstrahlung seems to be consistent/comparable;
- slight difference between SM prediction quoted in 2014 paper and Djangoh output, could be due to RC of $C_{1,2}$;
- no correction for pure weak (WW and ZZ boxes) in 2012 – note from HS: weak was in the equations for C1,2 two slides up (which are themselves approximations)
- all are “small” compared with precision of 6 GeV measurement, but non-trivial now for SoLID.

Radiative Correction for SoLID PVDIS – some ideas

Internal: Mo&Tsai does not deal with weak, box, etc → switch to Djangoh or another modern tool

Django generator:

- specify Ebeam, specify (x,Q^2) range
- parton-model based physics
- custom input $F_{1,2}$ possible
- can run in 3 modes:
  - generate full events (lepton, hadron)
  - generate just final-state lepton
  - do not generate events, calculate cross section only:
    - unpolarized (also for event-gen mode)
    - R-L (PV) or LC difference
- can turn on/off leptonic radiation, quark (QED) radiation, and interference
- can turn on/off pure-weak box diagrams

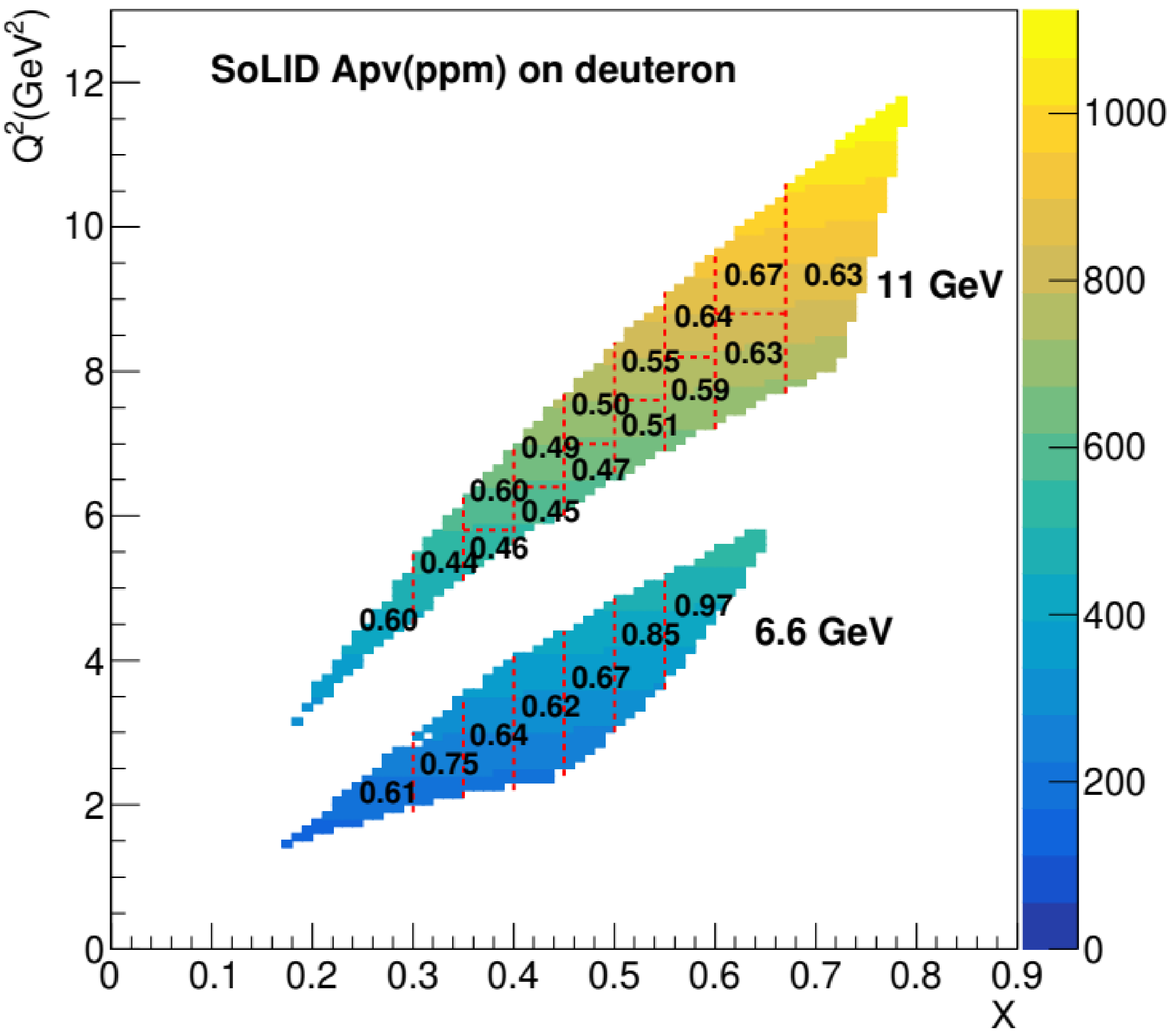
External: using GEANT-based SoLID simulation

technicality:

- beam energy loss in target cannot be implemented easily
- what about low W, low Q^2?
- custom-input of F^{gZ} would be helpful, for R-L (PV) cross section calculation
- could be useful for background study (?)
- can combine with SoLID sim for external energy loss correction in the final state

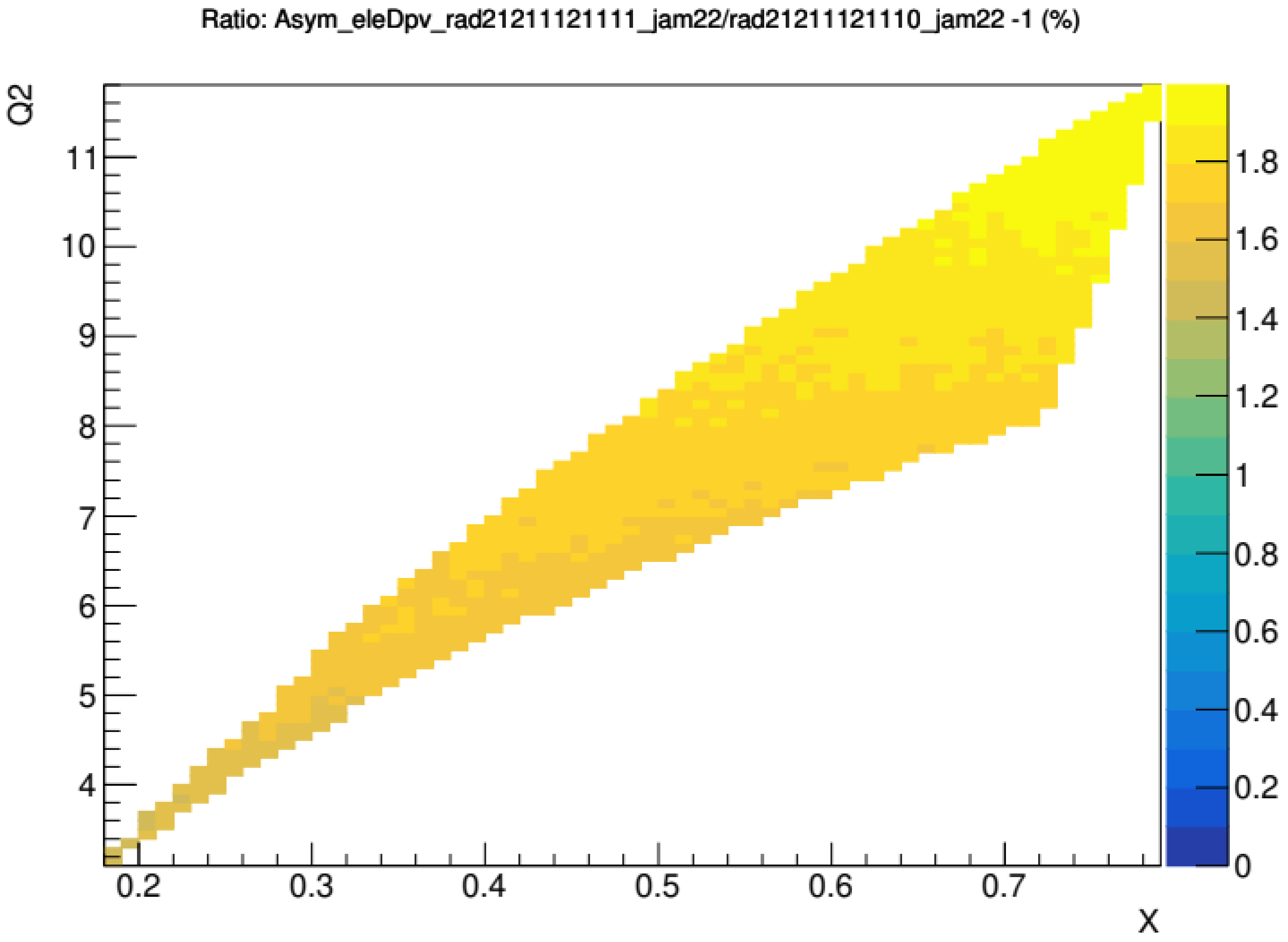
- can these corrections be separated from int/ext radiative corrections?

Calculation of Apv

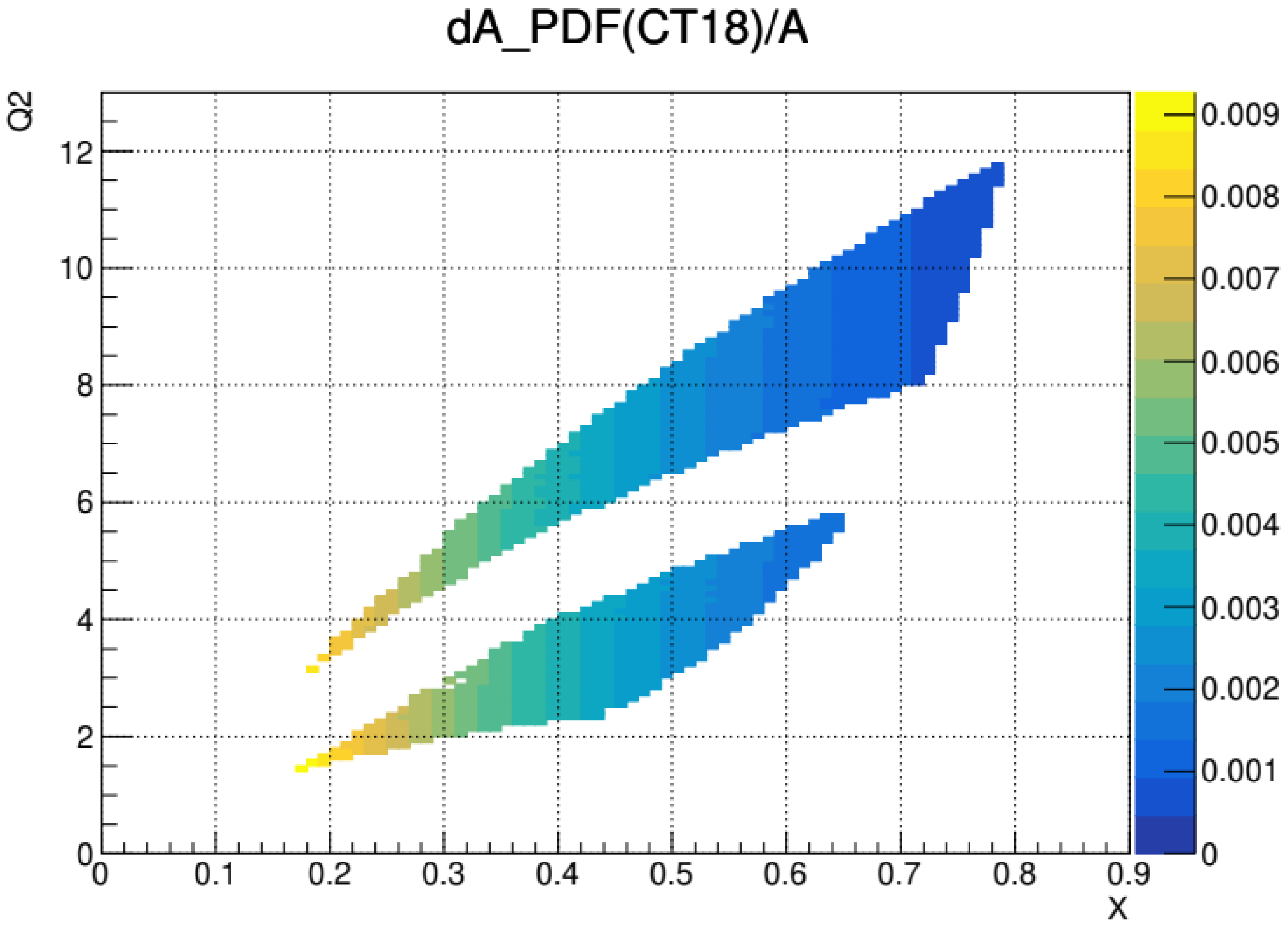


Pure-weak (WW, ZZ):

$$\frac{\text{Apv (with pure-weak box correction)}}{\text{Apv (without...)}}$$



Also a convenient tool to calculate PDF uncertainties



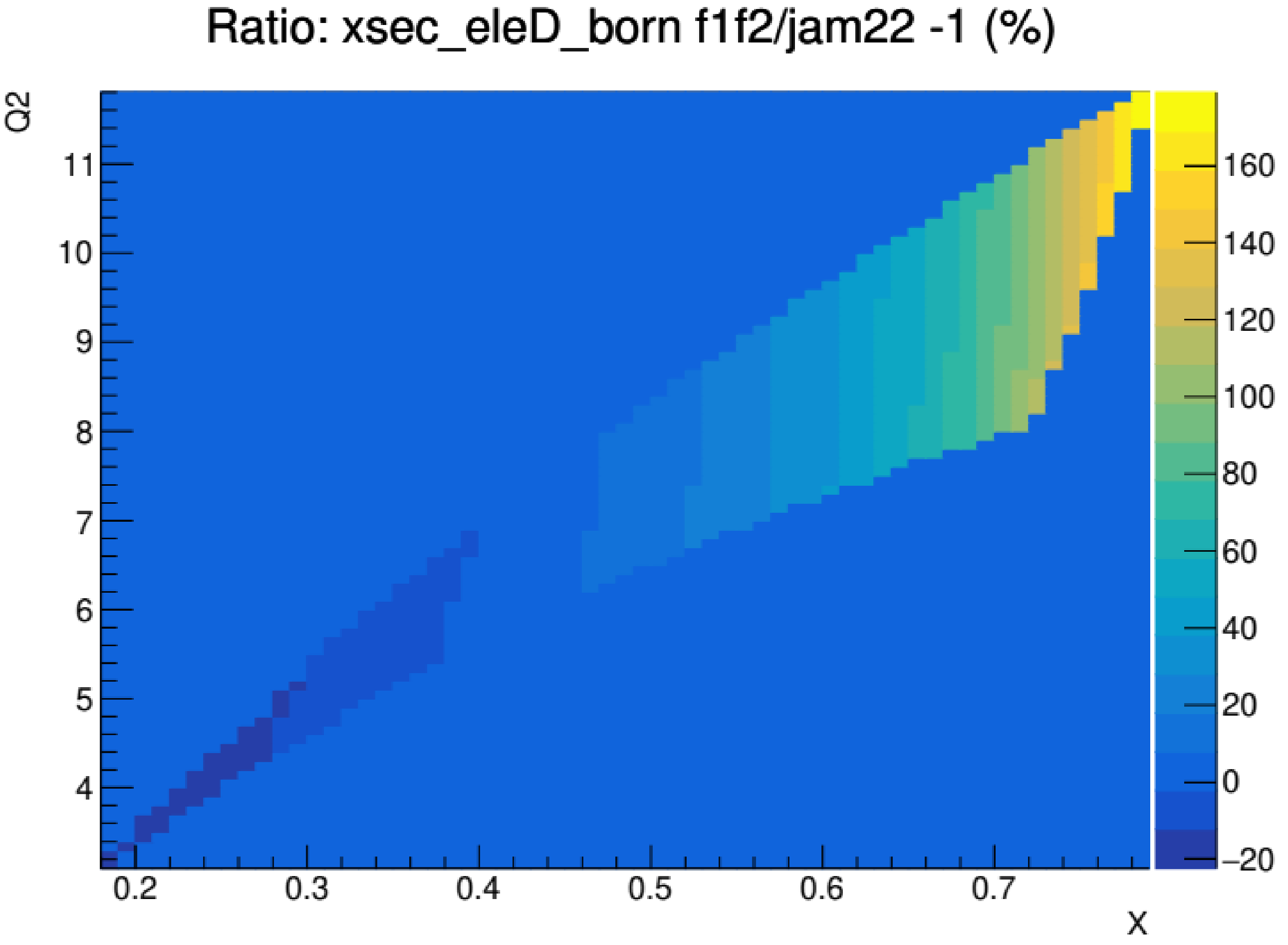
Low W, low Q^2 and high x?

Unpolarized cross section:

F1F2_21 vs. JAM22 PDF input

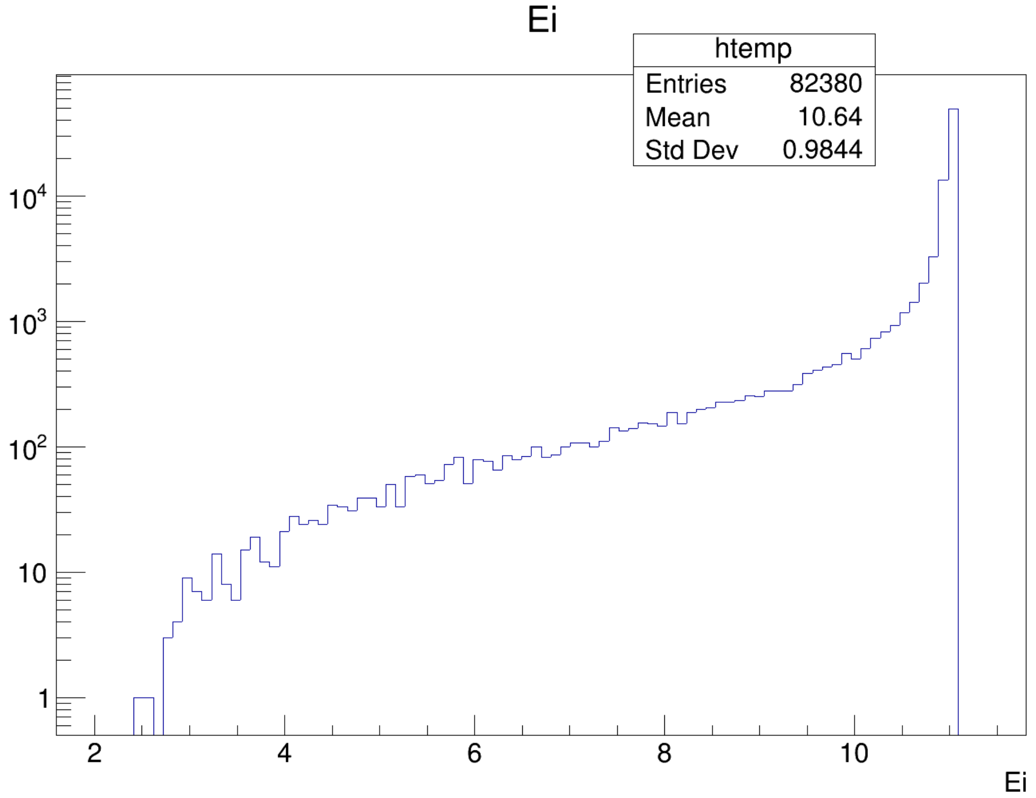
→ R?
→ TMC?

Compare apples with oranges, not sure if this indicates a real problem.



General Ideas for SoLID PVDIS Radiative Corrections

- Generate energy spectrum for electron beam in 40-cm LD2 target
- Choose 100(?) different `EELE`, sampled from the spectrum above



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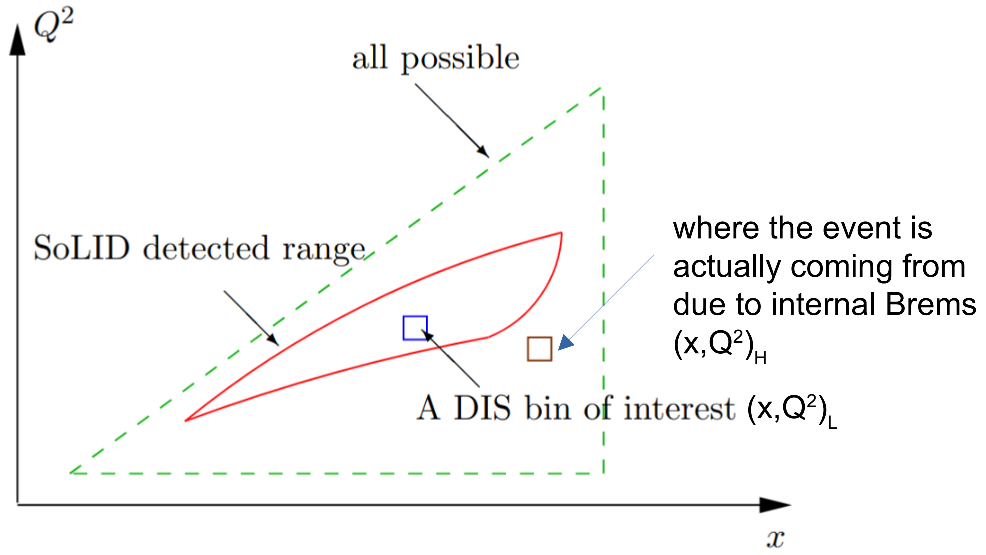
  - with internal Bremsstrahlung turned on



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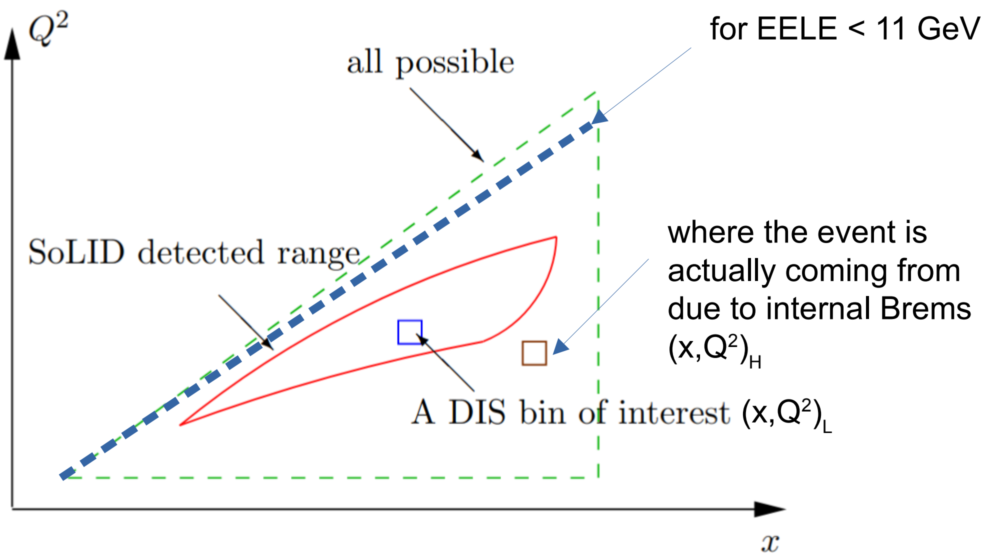

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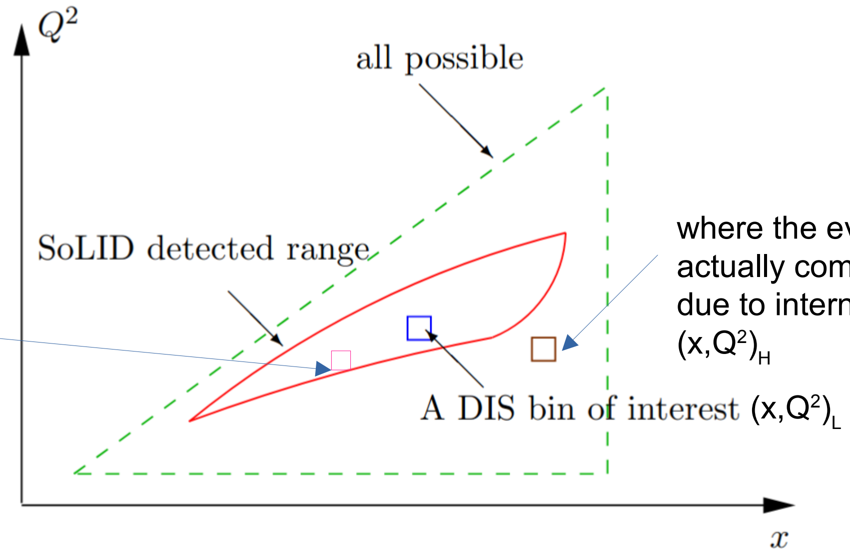

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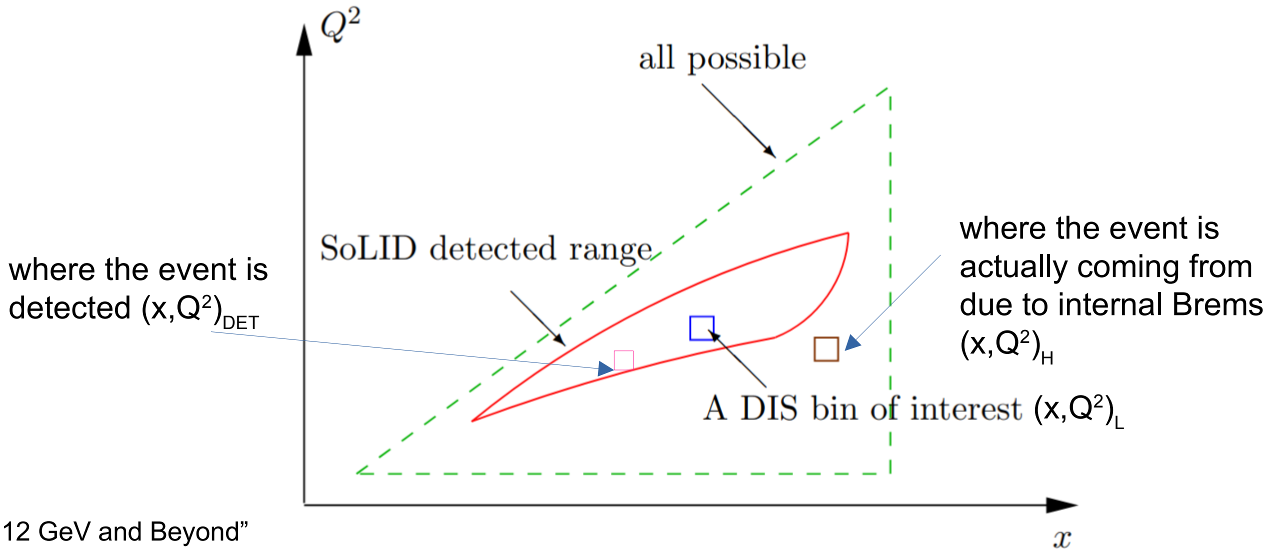
- Use cross-section only mode of Djangoh, calculate σ_0 and Apv for 100x(~9000) bins
  - If there is an alternative method, calculate/generate the same "grid"



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-
- Generate 1000(?) MC events along target length
- look for closest(?) `EELE` Djangoh simulation and input all 1M events
- pass 1G final-state electrons to SoLID simulation for evaluating final-state electrons
- for each detected events, look for Apv at the interaction vertex $(x,Q^2)_H$
- apply proper normalization (??)
- evaluate Apv_detected vs. Apv_true(H), the difference would be the RC factor

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- Generate energy spectrum for electron beam in 40-cm LD2 target
- Choose 100(?) different `EELE`, sampled from the spectrum above
- Use Djangoh to generate events for the full allowed region for each `EELE` value, generate 1M (?) events (lepton only)
  - with internal Bremsstrahlung turned on
  - output final-states in LUND format that can be passed onto SoLID sim
- Use cross-section only mode of Djangoh, calculate σ_0 and Apv for 100x(~9000) bins
  - If there is an alternative method, calculate the same “grid”
-
- Generate 1000(?) MC events along target length
- look for closest(?) `EELE` Djangoh simulation and input all 1M events
- pass 1G final-state electrons to SoLID simulation for evaluatingfinal-state electrons
- for each detected events, look for Apv at the interaction vertex $(x,Q^2)_H$
- apply proper normalization (??)
- evaluate Apv_detected vs. Apv_true(H), the difference would be the RC factor
- Can test a small-scale simulation to use for the on-going beam test in Hall C, precision? computing power?

Summary

For SoLID 11 GeV PVDIS (note: statistical goal 0.4% on Apv, ideally, need RC uncertainty at 0.2% or smaller)

- Can external, internal EM effects be determined to <0.1% precision?
  - Can we do a data-driven approach (like 6 GeV) for low W, low Q^2?
  - Three methods now exist for internal: 6 GeV approach, JLab's factorization approach, and Djangoh/SoLID MC. Is any of these tools working for the precision needed? What is the difference among three and what if there is a large difference?
- Can ext/int EM effects be separated from all box diagram corrections (as in 6 GeV)?
- What is pure-weak box diagram? Do we need 2-loop corrections? Can we have two parallel methods for these higher-order corrections and constrain them to <<(?)0.1% precision?
- What about QCD, HT? → factorization approach (global constraint provide consistency in HT fitting, one single experiment cannot be used to determine both HT and EW parameters)
- When is a good time to put in (non-negligible) resources in this work?