Radiative Corrections

(a few typical examples at JLab and a draft plan for SoLID PVDIS)

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Radiative Corrections

• General approach at JLab

• What was done for JLab EG4

• What was done for PVDIS 6 GeV

• Note: we do not typically deal with
  • box diagrams
  • weak effects
  • QED effects (quark line)
  • so far works fine (for current precision goals)

JLab 6 GeV PVDIS long paper:

https://doi.org/10.1103/PhysRevC.91.045506
e-Print: 1411.3200 [nucl-ex]
First method

- apply correction directly to measured cross sections
- more suitable for small-acceptance spectrometers
- ("RC_external" code calculates both Born and radiated cross sections)

Radiative correction

\[
\sigma_{\text{rad}}(E_s, E_p) = \int_0^T \frac{dt}{T} \int_{E_{\text{min}}(E_p)}^{E_s} \frac{dE_p'}{E_p'} \sigma_r(E_s', E_p') I(E_p', E_s', t) \sigma_r(E_s, E_p) I(E_p, E_p, T - t)
\]

(Mo. & Tsai method, SLAC-PUB-848 (1971).)

- \(I(E, E', t)\): the probability of energy loss due to the external radiation.
- \(T\): total path length before and after scattering.
- \(\sigma_r = \sigma_r^{\text{DIS}} + \sigma_r^{\text{quasi-elastic}} + \sigma_r^{\text{elastic}} \quad \Leftarrow \text{require a cross section input}\)

\[
\begin{align*}
\text{RC} &= \sigma_{\text{model}}^{\text{born}} \sigma_{\text{model}}^{\text{rad}} \\
\sigma_{\text{data}}^{\text{born}} &= \sigma_{\text{rad}}^{\text{data}} \cdot \text{RC}
\end{align*}
\]

- For \(^3\text{H}\) and \(^3\text{He}\) born cross section model, we use \(F_2^d\) from Bodek \textit{et al.} \(^1\) and the EMC model \((F_2^{(3\text{He})}/F_2^d)\) from S. Kulagin and R. Petti (KP) \(^2\)
- RC error is the deviation caused by using different cross section models

\(^1\) Phys. Rev. D20, 1471 (1979)
\(^2\) Nucl Phys A765 (2006) 126

(from H. Liu’s talk)
Second method (fully forward simulation method)

- use a full simulation method to calculate “Born” and to simulate “measured” observables using model inputs
- if simulated “measured” observables do not agree with real data, adjustment is made to the model inputs
- more suitable for large-acceptance spectrometers
- can be added to any existing, experimental full simulation packages
- technical complications:
  - tails from elastic scattering may need to be subtracted first
  - positive and negative cross section (difference) regions need to be done separately
start from Ebeam

calculate ext Eloss (brem, ion), multiple scattering

calculate int brem.

scattering occurs (el, QE, res, DIS)
randomly pick $\theta, E'$

calculate int brem.

calculate ext Eloss (brem, ion), multiple scattering

check if the electron reaches detector

\[ E = E_{\text{beam}} - dE_{\text{ext,ion}} \]

\[ E_{\text{vtx}} = E_{\text{beam}} - dE_{\text{ext,ion}} - dE_{\text{int}} \]

\[ E' = E'_{\text{vtx}} - dE'_{\text{int}} \]

\[ E'_{\text{det}} = E'_{\text{vtx}} - dE'_{\text{int}} - dE'_{\text{ext,ion}} \]

“full simulation method”

(done for CLAS g1p, g1d measurements)

input model at

\[ \left(x_{\text{vtx}}, Q^2_{\text{vtx}}\right) \]

\[ \left(x_{\text{det}}, Q^2_{\text{det}}\right) \]

use the difference between observed and simulated spectra to apply corrections

INT Workshop “PVDIS at JLab 12 GeV and Beyond”
Radiative Corrections for CLAS EG4

\[ \Delta \sigma_{||} = \frac{d^2 \sigma_{\uparrow\uparrow}}{d\Omega dE'} - \frac{d^2 \sigma_{\uparrow\downarrow}}{d\Omega dE'} = \left[ \frac{N^+}{N^e_+} - \frac{N^-}{N^e_-} \right] \frac{1}{N_{\text{targ}} P_b P_t \Delta \Omega} \frac{1}{\eta_{\text{detector}}} \]
Simulation of EG4 Proton Elastic Peak

- simulation reproduces measured double-polarized yield (N/Ne) difference
- cross-checking PbPt measurement, tuning detector smearing, material thickness, etc.
- Radiative tail from elastic peak can be determined and subtracted from inelastic data
Simulation of EG4 Proton Resonance Region

1.1 GeV nh3b runs inclusive N/fcup differ

Comparison of polarized yield difference $N^+ - N^-$

red: data
blue: simulation with “best” A1 model
green: simulation with “best” A1 model shifted by +0.1

\[
g_1^{\text{data}} = g_1^{\text{sim0}} + (g_1^{\text{sim1}} - g_1^{\text{sim0}}) \frac{\Delta n^{\text{data}} - \Delta n^{\text{sim0}}}{\Delta n^{\text{sim1}} - \Delta n^{\text{sim0}}} \]

Extracted $g_1$ structure function:
INT Workshop “PVDIS at JLab 12 GeV and Beyond”

“full simulation method”
(done for 6 GeV PVDIS using modified HAMC)

E_{beam}

E = E_{beam} - dE_{ext,ion}

E_{vtx} = E_{beam} - dE_{ext,ion} - dE_{int}

E'_{vtx} = E_{vtx} - dE'_{int}

E'_{det} = E'_{vtx} - dE'_{int} - dE'_{ext,ion}

start from E_{beam}

calculate ext Eloss (brem, ion), multiple scattering

calculate int brem.

scattering occurs (el, QE, res, DIS) randomly pick \( \theta, E' \)

calculate int brem.

calculate ext Eloss (brem, ion), multiple scattering

check if the electron reaches detector

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calculate
kinematics and
observables at

\( (x_{vtx}, Q_{vtx}^2) \)

and

\( (x_{det}, Q_{det}^2) \)

(HEP/Djangoh)

use the difference between the two to apply corrections

\[ 1 + \bar{f}_{rc} = \frac{A(Q_{det}^2, x_{det})}{A(Q_{vtx}^2, x_{vtx})} \]
Radiative Correction for 6 GeV PVDIS

Q2_vertex vs. W_vertex for 6 GeV that includes both internal and external radiations

internal use Mo&Tsai’s effective radiator formula: 
\[ t_{\text{equiv}} = \frac{3 \alpha}{4 \pi} \ln \left( \frac{Q^2}{m^2} \right) - 1 \] (see HAMC manual)

radiative correction for 6 GeV PVDIS

spectrometer sits here (and where we think DIS events occurs)

where scattering actually occurs

resonance Apv: used model, checked with data (next slide)
Radiative Correction for 6 GeV PVDIS

Q2_vertex vs. W_vertex for 6 GeV that includes both internal and external radiations

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[Graph showing scattering and resonances]

taken as a control point, but did not see any HT

spectrometer sits here (and where we think DIS events occurs)

where scattering actually occurs

resonance Apv: used model, checked with data (next slide)
Resonance data taken during 6 GeV

"DIS#1"

"DIS#2"

Elastic and QE calculated separately;

Uncertainty in resonance $A_{pv}$ as input to radiative corrections:

- $W<1.4$ GeV: 25%
- $1.4<W<1.7$ GeV: 10%
- $1.7<W<2.0$ GeV: 7.7%
Resonance data taken during 6 GeV

Caveat:
– We didn’t really have so many (small) W bins and these points overlap (are correlated)
– HAMC implementation of $t_{\text{equiv}}$ was “off”
Q2_vertex vs. W_vertex for 6 GeV that includes only internal radiations (Djangoh simulation)

Simulation from 6 GeV (both int and ext radiation), barely any seen for final state radiation (note that this is a linear z plot), or could it be that initial state radiation dominates for fixed-target experiments (due to extended target material)?
6 GeV PVDIS long paper: \[ A^{\text{rad-corrected}} = A^{\text{meas}}(1 + f^{\text{rc}}) \]

**DIS Kine #1:**
- \( E_{\text{beam}} = 6.067 \text{ GeV} \)
- \( \theta = 12.9^\circ, E' = 3.66 \text{ GeV} \)
- \( \langle x \rangle_{\text{data}} = 0.241, \langle Q^2 \rangle_{\text{data}} = 1.085 \text{ GeV}^2 \)
- \( 1 + f^{\text{rc}} = 1.015 \pm 0.02 \)
- \( f_{yy} = -0.002 \pm 0.002 \)
- \( A_{\text{phys}} = -91.10 \pm 4.30 \text{ ppm (4.7\%)} \)

**DIS Kine #2:**
- \( E_{\text{beam}} = 6.067 \text{ GeV} \)
- \( \theta = 20^\circ, E' = 2.63 \text{ GeV} \)
- \( \langle x \rangle_{\text{data}} = 0.295, \langle Q^2 \rangle_{\text{data}} = 1.901 \text{ GeV}^2 \)
- \( 1 + f^{\text{rc}} = 1.019 \pm 0.004 \)
- \( f_{yy} = -0.003 \pm 0.003 \)
- \( A_{\text{phys}} = -160.80 \pm 7.12 \text{ ppm (4.4\%)} \)
\( \gamma-Z \) box

Electroweak radiative corrections were applied to all couplings used in the calculation of the asymmetry. The electromagnetic fine structure constant \( \alpha \) was evolved to the measured \( Q^2 \)-values from \( \alpha_{EM|Q^2=0} = 1/137.036 \) [52]. The evaluation takes into account purely electromagnetic vacuum polarization. The Fermi constant is \( G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} \) [52]. The \( C_{1q,2q} \) were evaluated using Table 7 and Eq. (114-115) of Ref. [91] at our measured \( Q^2 \)-values in the modified minimal subtraction (\( \overline{\text{MS}} \)) scheme using a fixed Higgs mass \( M_H = 125.5 \text{ GeV} \):

\[
C_{1u}^{SM} = -0.1887 - 0.0011 \times \frac{2}{3} \ln(\langle Q^2 \rangle/0.14\text{GeV}^2)
\]

(86)

\[
C_{1d}^{SM} = 0.3419 - 0.0011 \times \frac{1}{3} \ln(\langle Q^2 \rangle/0.14\text{GeV}^2)
\]

(87)

\[
C_{2u}^{SM} = -0.0351 - 0.0009 \ln(\langle Q^2 \rangle/0.078 \text{ GeV}^2)
\]

(88)

\[
C_{2d}^{SM} = 0.0248 + 0.0007 \ln(\langle Q^2 \rangle/0.021 \text{ GeV}^2)
\]

(89)

and it is expected that the uncertainty is negligible. Equations (86-89) include the “charge radius effect” and an estimate of the interference between \( \gamma \)-exchange and the \( \gamma Z \) box, but not the effect from the \( \gamma\gamma \) box. The effect from the \( \gamma\gamma \) box was applied as a correction to the measured asymmetry as described in previous sections.
**6 GeV PVDIS long paper:**

\[ A_{\text{rad-corrected}} = A_{\text{meas}} \left( 1 + f_{rc} \right) \]

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**Recent calculation using stand-alone Mo&Tsai equivalent radiator:**

**internal:** -0.7\% (original HAMC -0.33\%) -1.2\% (original HAMC -0.7\%)

**Djangoh:**

internal with lepton radiation: -0.75\% -1.23\%

internal with both lepton and quark radiation: -0.3\% -0.7\%

\[ f_{\gamma\gamma} \text{ and } f_{\gamma Z} \text{ boxes:} \quad 0.026\% \quad 0.03\% \]

pure weak: +1.14\% +1.4\%

**DIS Kine #2:**

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DIS Kine #1:

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Recent calculation using stand-alone Mo&Tsai equivalent radiator:

- internal: -0.7% (original HAMC -0.33%)
- Djangoh: -1.2% (original HAMC -0.7%)

Djangoh:

- internal with lepton radiation: -0.75%
- internal with both lepton and quark radiation: -0.3%
- gg and gZ boxes: 0.026%
- pure weak: +1.14%

\[ E_{\text{beam}} = 6.067 \text{ GeV} \\
\theta = 20^\circ, E' = 2.63 \text{ GeV} \\
\langle x \rangle_{\text{data}} = 0.295, \langle Q^2 \rangle_{\text{data}} = 1.901 \text{ GeV}^2 \]

\[ 1 + f_{rc} = \frac{A_{\det}(Q^2_{\det}, x_{\det})}{A_{\text{vtx}}(Q^2_{\text{vtx}}, x_{\text{vtx}})} \]

2012 vs. now:

- size of internal Bremsstrahlung seems to be consistent/comparable;
- slight difference between SM prediction quoted in 2014 paper and Djangoh output, could be due to RC of C_{1,2};
- no correction for pure weak (WW and ZZ boxes) in 2012 – note from HS: weak was in the equations for C1,2 two slides up (which are themselves approximations)
- all are “small” compared with precision of 6 GeV measurement, but non-trivial now for SoLID.
Radiative Correction for SoLID PVDIS – some ideas

Internal: Mo&Tsai does not deal with weak, box, etc → switch to Djangoh or another modern tool

Djangoh generator:
- specify Ebeam, specify \((x,Q^2)\) range
- parton-model based physics
- custom input \(F_{1,2}\) possible
- can run in 3 modes:
  - generate full events (lepton, hadron)
  - generate just final-state lepton
  - do not generate events, calculate cross section only:
    - unpolarized (also for event-gen mode)
    - R-L (PV) or LC difference
- can turn on/off leptonic radiation, quark (QED) radiation, and interference
- can turn on/off pure-weak box diagrams

External: using GEANT-based SoLID simulation

technicality:
- beam energy loss in target cannot be implemented easily
- what about low \(W\), low \(Q^2\)?
- custom-input of \(F^{gZ}\) would be helpful, for R-L (PV) cross section calculation
- could be useful for background study (?)
- can combine with SoLID sim for external energy loss correction in the final state
- can these corrections be separated from int/ext radiative corrections?
Calculation of $A_{pv}$
Pure-weak (WW, ZZ):

Apv (with pure-weak box correction)

Apv (without...)

Ratio: Asym_eleDpv_rad21211121111_jam22/rad21211121110_jam22 -1 (%)

Q^2

0.2 0.3 0.4 0.5 0.6 0.7

x

0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8
Also a convenient tool to calculate PDF uncertainties.
Low $W$, low $Q^2$ and high $x$?

Unpolarized cross section:

F1F2_21 vs. JAM22 PDF input

→ R?
→ TMC?

Compare apples with oranges, not sure if this indicates a real problem.
General Ideas for SoLID PVDIS Radiative Corrections

- Generate energy spectrum for electron beam in 40-cm LD2 target
- Choose 100(?) different $E_{ELE}$, sampled from the spectrum above
General Ideas for SoLID PVDIS Radiative Corrections

- Generate energy spectrum for electron beam in 40-cm LD2 target
- Choose 100(?) different $EELE$, sampled from the spectrum above
- Use Django to generate events for the full allowed region for each $EELE$ value, generate 1M (?) events (lepton only)
  - with internal Bremsstrahlung turned on
General Ideas for SoLID PVDIS Radiative Corrections

- Generate energy spectrum for electron beam in 40-cm LD2 target
- Choose 100(?) different \( EELE \), sampled from the spectrum above
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\[
Q^2
x
\]

for \( EELE < 11 \text{ GeV} \)
where the event is actually coming from due to internal Brems \( (x,Q^2)_H \)
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  - output final-states in LUND format that can be passed onto SoLID sim
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- Use cross-section only mode of Djangoh, calculate $\sigma_0$ and $Apv$ for 100x(~9000) bins
  - If there is an alternative method, calculate/generate the same “grid”

where the event is detected $(x,Q^2)_{DET}$

where the event is actually coming from due to internal Brems $(x,Q^2)_{H}$
General Ideas for SoLID PVDIS Radiative Corrections

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- Choose 100(?) different EELE, sampled from the spectrum above
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- Use cross-section only mode of Djangoh, calculate $\sigma_0$ and Apv for 100x(~9000) bins
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- Generate 1000(?) MC events along target length
- look for closest(?) EELE Djangoh simulation and input all 1M events
- pass 1G final-state electrons to SoLID simulation for evaluating final-state electrons
- for each detected events, look for Apv at the interaction vertex $(x,Q^2)_H$
- apply proper normalization (?)
- evaluate Apv_detected vs. Apv_true(H), the difference would be the RC factor
General Ideas for SoLID PVDIS Radiative Corrections

- Generate energy spectrum for electron beam in 40-cm LD2 target
- Choose 100(?) different $\EELE$, sampled from the spectrum above
- Use Djangoh to generate events for the full allowed region for each $\EELE$ value, generate 1M (?) events (lepton only)
  - with internal Bremsstrahlung turned on
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- Generate 1000(?) MC events along target length
- look for closest(?) $\EELE$ Djangoh simulation and input all 1M events
- pass 1G final-state electrons to SoLID simulation for evaluating final-state electrons
- for each detected events, look for $Apv$ at the interaction vertex $(x,Q^2)_H$
- apply proper normalization (???)
- evaluate $Apv_{\text{detected}}$ vs. $Apv_{\text{true}}(H)$, the difference would be the RC factor
- Can test a small-scale simulation to use for the on-going beam test in Hall C, precision? computing power?
Summary

For SoLID 11 GeV PVDIS (note: statistical goal 0.4% on Apv, ideally, need RC uncertainty at 0.2% or smaller)

- Can external, internal EM effects be determined to <0.1% precision?
  - Can we do a data-driven approach (like 6 GeV) for low W, low $Q^2$?
  - Three methods now exist for internal: 6 GeV approach, JLab’s factorization approach, and Djangoh/SoLID MC. Is any of these tools working for the precision needed? What is the difference among three and what if there is a large difference?

- Can ext/int EM effects be separated from all box diagram corrections (as in 6 GeV)?
- What is pure-weak box diagram? Do we need 2-loop corrections? Can we have two parallel methods for these higher-order corrections and constrain them to <<(?)0.1% precision?
- What about QCD, HT? → factorization approach (global constraint provide consistency in HT fitting, one single experiment cannot be used to determine both HT and EW parameters)
- When is a good time to put in (non-negligible) resources in this work?