

Neutron Star g-mode Oscillations at Finite Temperature

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INSTITUTE for
NUCLEAR THEORY



A brief history

- In the 1910s, Eddington proposed Cepheid variables as pulsating stars.
- In 1940s-60s, Cox and others complete non-adiabatic pulsation theory.
- Early 1960s, Chandrasekhar extended it to compact objects.
- In 1967, Thorne linked oscillations to gravitational waves.
- In 1980s, Numerical relativistic EOS-dependent mode studies.
- In the 1990s, various g-modes and their detectability: discontinuity, entropy, composition, ...
- Up until today, No definitive observational confirmation of NS modes.

ODEs of Non-radial Oscillation

Eigen value problem of even quasi-normal modes

- Solve Einstein's equations:

$$\delta\pi\delta \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T \right) = \delta R_{\mu\nu}$$

with fluid perturbations, and metric perturbations:

$$\xi_{\text{even}}^{r, \theta, \phi} = \nabla (R_n(r) Y_m^l(\theta, \phi) e^{i\omega t})$$

$$\delta T_{\mu\nu} \text{ from } \xi^{r, \theta, \phi} \text{ and } C_{dy}^2 \quad \delta g_{\mu\nu}^{\text{even}} = \begin{pmatrix} H_0 e^\nu & H_1 & 0 & 0 \\ H_1 & H_2 e^\lambda & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta K \end{pmatrix} Y_{lm}(\theta, \phi) e^{i\omega t}$$

- To get eigenvalue $\omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$,

and eigenfunction $\xi_{\text{even}}^{r, \theta, \phi}$ and H_0, H_1, H_2, K

NS Asteroseismology

- Regge-Wheeler metric,

$$\begin{aligned}
 ds^2 = & - e^{\nu(r)}(1 + H_0(r)e^{i\omega t}Y_{lm}(\phi, \theta))c^2 dt^2 \\
 & + e^{\lambda(r)}(1 - H_0(r)e^{i\omega t}Y_{lm}(\phi, \theta))dr^2 \\
 & - 2i\omega r^{l+1}H_1(r)e^{i\omega t} dt dr \\
 & + (1 - K(r)e^{i\omega t}Y_{lm}(\phi, \theta))r^2 d\Omega^2
 \end{aligned}$$

Static metric:

$$\nu, \lambda$$

Metric perturbation:

$$H_0, H_1, K$$

- Eigen value problem set by ODEs derived from Einstein's Equation:

- Perturbations of the fluid (Lagrangian displacement),

$$\xi^r = r^{l-1} e^{-\frac{\lambda}{2}} W Y_m^l e^{i\omega t}$$

$$\xi^\theta = - r^{l-2} V \partial_\theta Y_m^l e^{i\omega t}$$

$$\xi^\phi = - \frac{r^{l-2}}{\sin^2 \theta} V \partial_\phi Y_m^l e^{i\omega t}$$

Static fluid:

$$\varepsilon, P$$

Fluid perturbation:

$$W, V$$

Eigen value:

$$\omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$$

Static ODEs (TOV equations)

Static metric:

ν, λ

Static fluid:

ε, P

Schwarzschild metric:

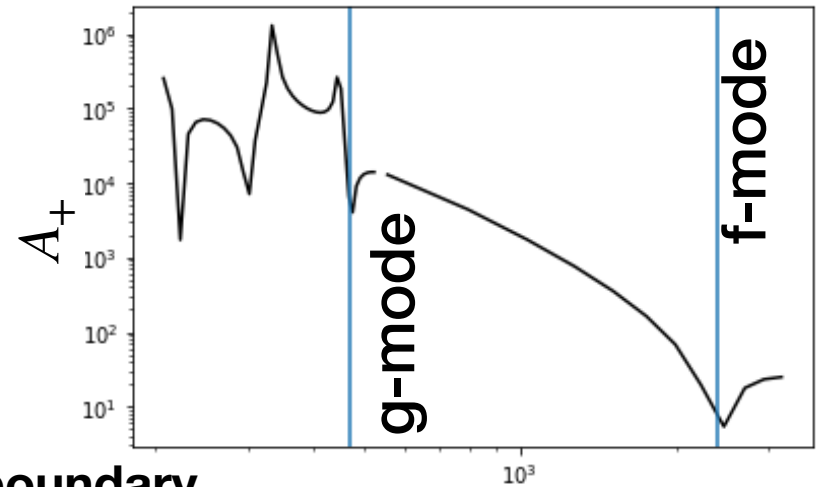
$$\nu = -\lambda = \log\left(1 - \frac{2GM}{rc^2}\right)$$

- $$ds^2 = -e^{\nu(r)}(1 + H_0(r)e^{i\omega t}Y_{lm}(\phi, \theta))c^2 dt^2 + e^{\lambda(r)}(1 - H_0(r)e^{i\omega t}Y_{lm}(\phi, \theta))dr^2 - 2i\omega r^{l+1}H_1(r)e^{i\omega t}dtdr + (1 - K(r)e^{i\omega t}Y_{lm}(\phi, \theta))r^2 d\Omega^2$$
- TOV Equations: $H_0 = H_1 = K = \omega = 0$

$$\frac{dp}{dr} = -\frac{\varepsilon + p}{2} \frac{d\nu}{dr}, \quad \frac{dm}{dr} = 4\pi r^2 \frac{\varepsilon}{c^2},$$

$$\frac{d\nu}{dr} = \frac{2G(mc^2 + 4\pi r^3 p)}{r^2 c^4 \left(1 - \frac{2Gm}{rc^2}\right)}, \quad e^\lambda = \frac{1}{1 - \frac{2Gm}{rc^2}}$$

Perturbation ODEs



Surface boundary

Asymptotic boundary

Metric perturbation:

$$H_0, H_1, K$$

Fluid perturbation:

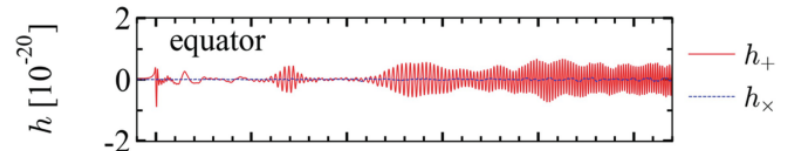
$$W, V$$

$$Z_{\pm} \propto \frac{A_{\pm}}{r} e^{\pm i\omega r/c}$$

$\omega/(2\pi)$ kHz

Observer:
LIGO...

GW strain h

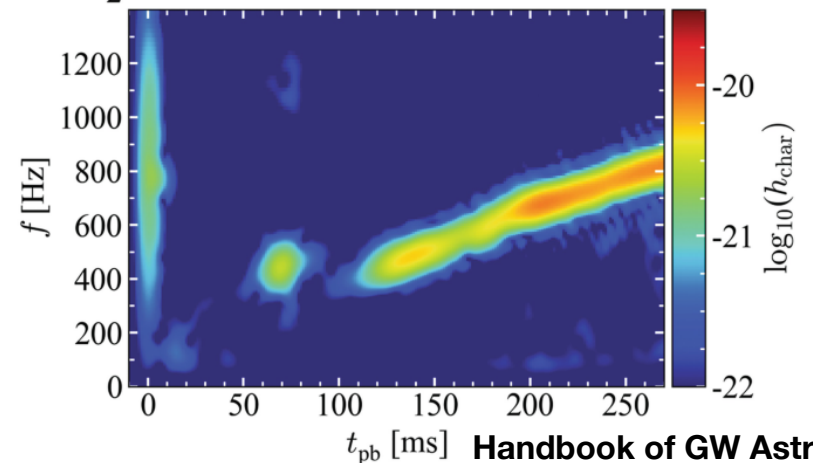


- Thorne, Kip S. 1967:
four 1st ODEs

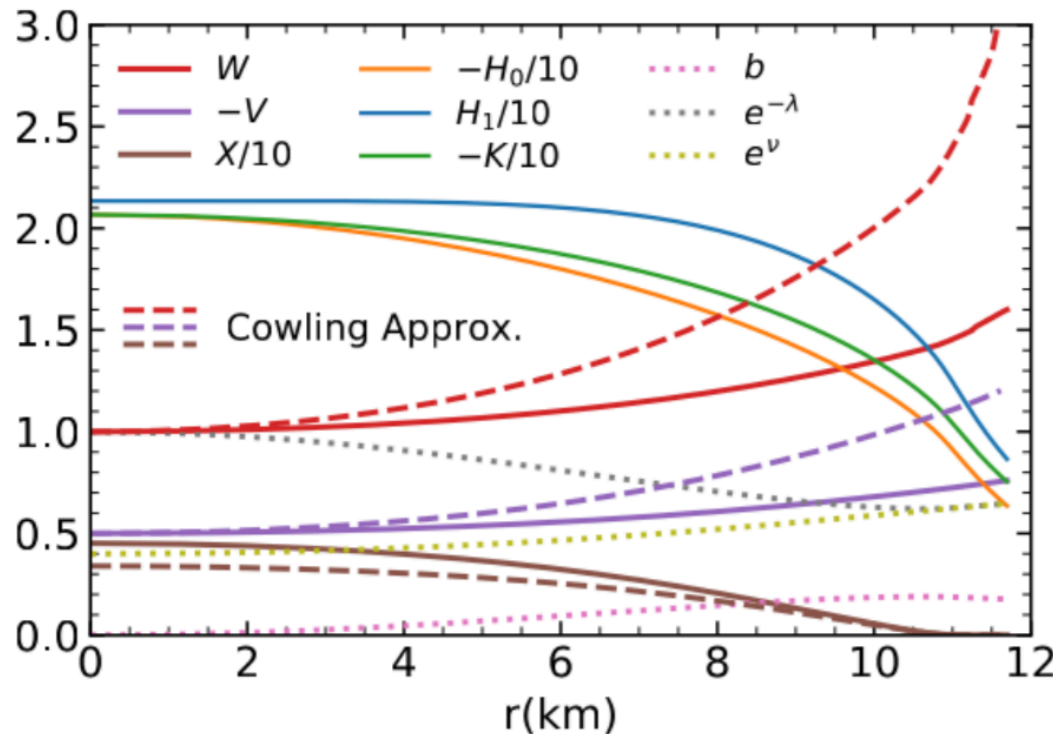
Metric perturbation:

$$\underbrace{H_0, H_1, K}_Z$$

- Zerilli's Equation:
one 2nd ODEs



Typical f-mode oscillation

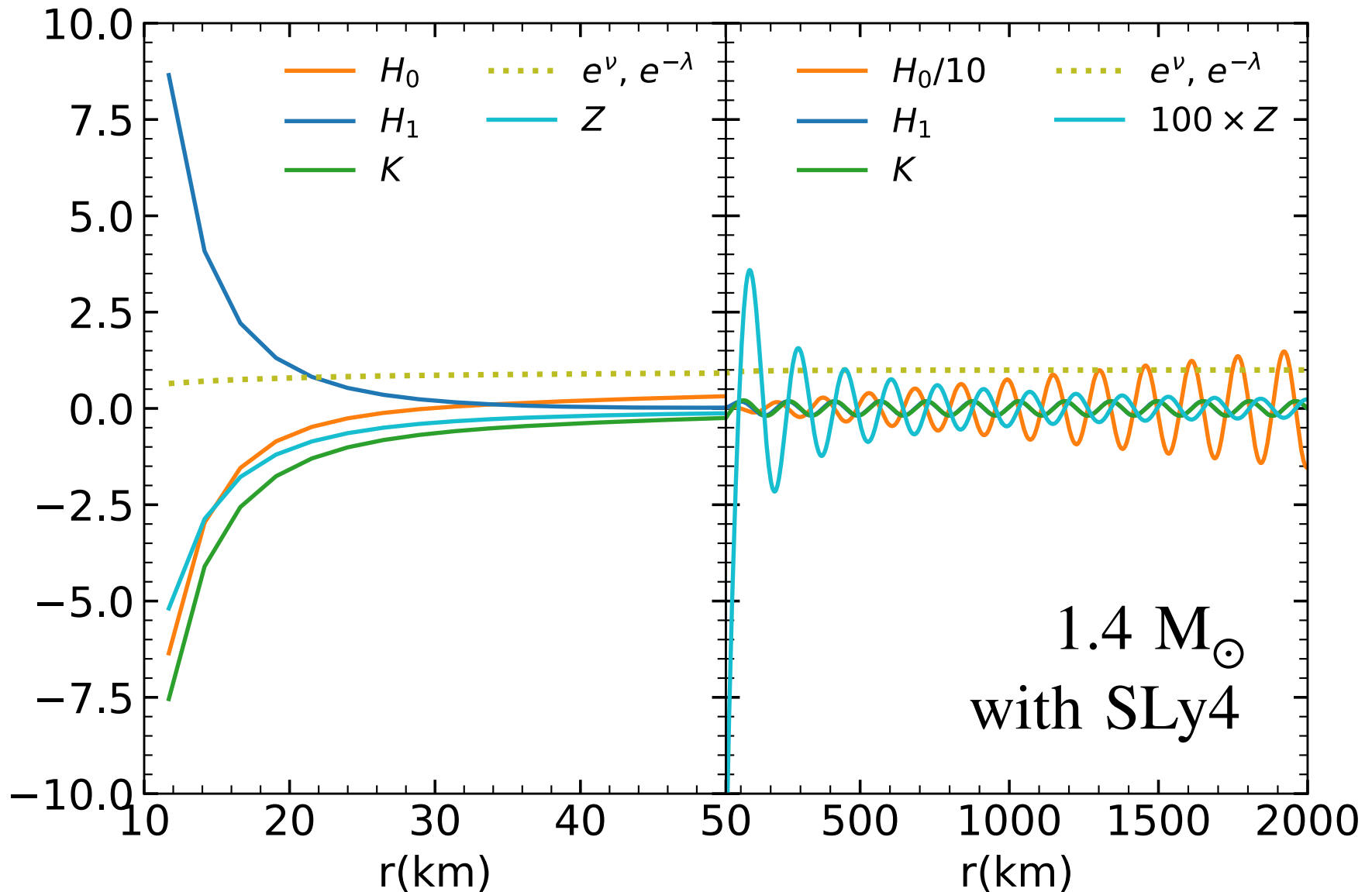


Zhao & Lattimer 2022

<https://arxiv.org/abs/2204.03037>

FIG. 14. Metric perturbation amplitudes, fluid perturbation amplitudes for non-radial oscillations with $\ell = 2$ with (dashed curves) and without (solid curves) the Cowling approximation, and static metric functions (dotted curves) inside a $1.4M_{\odot}$ NS computed with the Sly4 EOS [85]. H_0 , H_1 and K are in units of $\varepsilon_s = 152.26 \text{ MeV fm}^{-3}$, X is in units of ε_s^2 , and W , V , ν and λ are dimensionless. Only real parts of the perturbation amplitudes are plotted.

Outside metric perturbation (f-mode as example)



ODEs of Non-radial Adiabatic Oscillation

Eigen value problem of even quasi-normal modes

- Linearized Full GR:
4 1st ODEs (inside) Thorne, Kip S. 1967
1 2nd ODEs (outside)
Zerilli's Eq Fackerell, Edward D. 1971
Lee Lindblom and Steven L. Detweiler 1983

Take Newtonian limit for static gravity and perturbation

- Newtonian: Cox, John P. 1980
2 1st ODEs + 1 2nd ODE
Analytical for some modes,
e.g. f-mode and interface g-mode
in stellar asteroseismology.

(fluid)
Drop spacetime perturbation

Zhao, Constantinou,
Jaikumar, Prakash 2022
<https://arxiv.org/abs/2202.01403>

Drop gravity perturbation

- Relativistic Cowling approximation:
2 1st-order ODEs
or 1 2nd-order ODE
P. N. McDermott et. al. 1983

Take Newtonian limit for static gravity

- Newtonian Cowling approximation:
2 ODEs
Cowling, Thomas G 1941

Oscillations of NS

Fluid perturbations

- Radial oscillation ($l=0$): $\epsilon^r = R_n^r(r)e^{i\omega t}$
don't couple to gravitational waves

$$\omega = 2\pi\nu + \frac{i}{\tau}$$

- Non-radial oscillation ($l \geq 2$):

$$\xi_{\text{Seven}}^{r, \theta, \phi} = \nabla (R_n(r) Y_m^l(\theta, \phi) e^{i\omega t})$$

$$\xi_{\text{odd}}^{\theta, \phi} = \hat{r} \times \nabla (R_n(r) Y_m^l(\theta, \phi) e^{i\omega t})$$

- f-mode (fundamental $n=0$) (even),
- p-modes (pressure $n=1, 2, \dots$) (even)
- g-modes (gravity $n=1, 2, \dots$) (even)
- r-modes (rotation $m=\pm 1, \pm 2, \dots$) (odd)

	ν (kHz)	τ (s)
f-mode	1.3-2.8	0.1-1
g-mode	<0.8	>100
p-mode	>2.7	1-1000
r-mode	~ spin	<0
w-mode	~10	~1E-5

- Spacetime perturbations:**
 - Family I w-modes (even)
 - Family II w-modes (odd)
 - important for BBH ring-down

even-parity
(polar mode)

odd-parity
(axial modes)

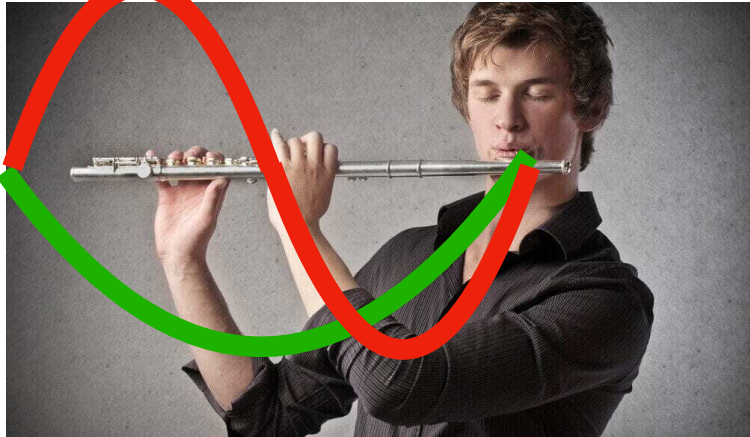
$$h_{\mu\nu}^{\text{even}} = \begin{pmatrix} H_0 & H_1 & 0 & 0 \\ H_1 & H_2 & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta K \end{pmatrix} Y_{lm}(\theta, \phi)$$

$$h_{\mu\nu}^{\text{odd}} = \begin{pmatrix} 0 & 0 & \frac{h_0 \partial_\phi}{\sin \theta} & -h_0 \sin \theta \partial_\theta \\ 0 & 0 & \frac{h_1 \partial_\phi}{\sin \theta} & -h_1 \sin \theta \partial_\theta \\ \dots & \dots & 0 & 0 \\ \dots & \dots & 0 & 0 \end{pmatrix} Y_{lm}(\theta, \phi)$$

Oscillation modes

$$A(r)e^{i\omega t} \quad \omega = 2\pi\nu + \frac{i}{\tau}$$

Pressure supported



Standing sound wave of order n:

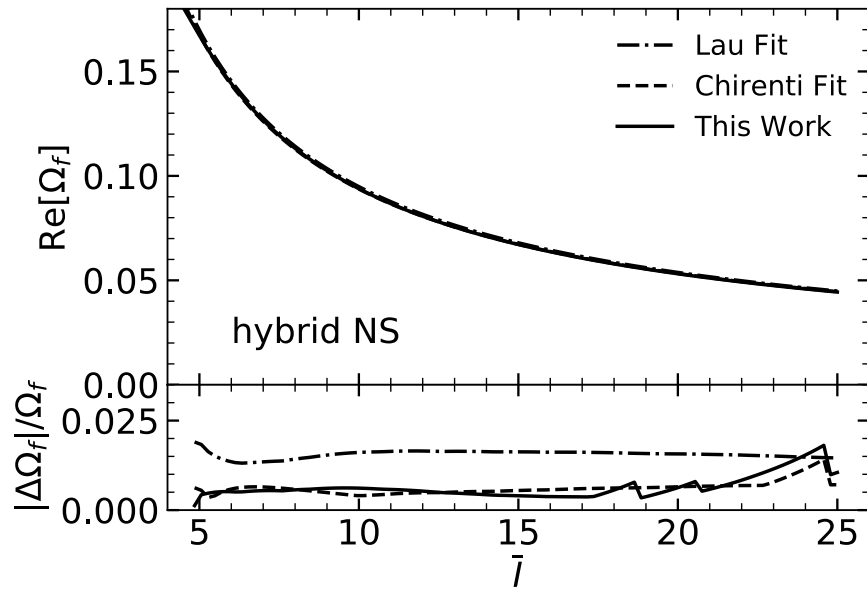
$$\omega^2 \approx \frac{dp}{d\varepsilon} k^2 \quad k = \frac{\sqrt{l(l+1)}n}{2\pi R}$$

n=0 f-mode (fundamental)

n=1 p-mode (pressure)

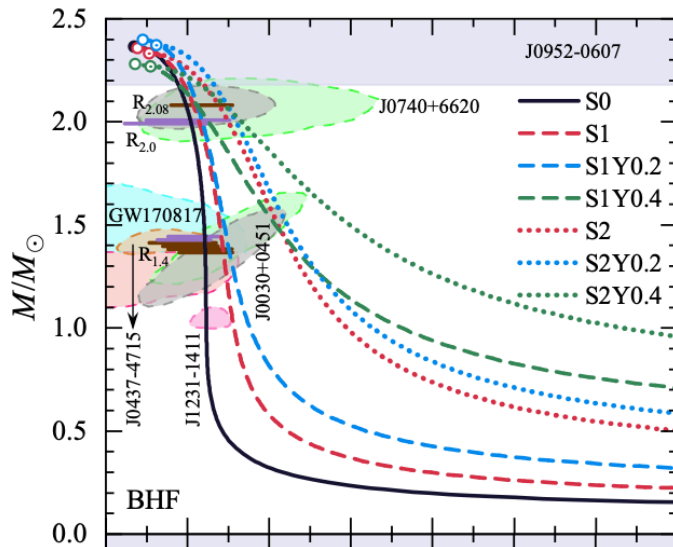
f-mode + I-love-Q universal relations

First proposed
Lau+2010

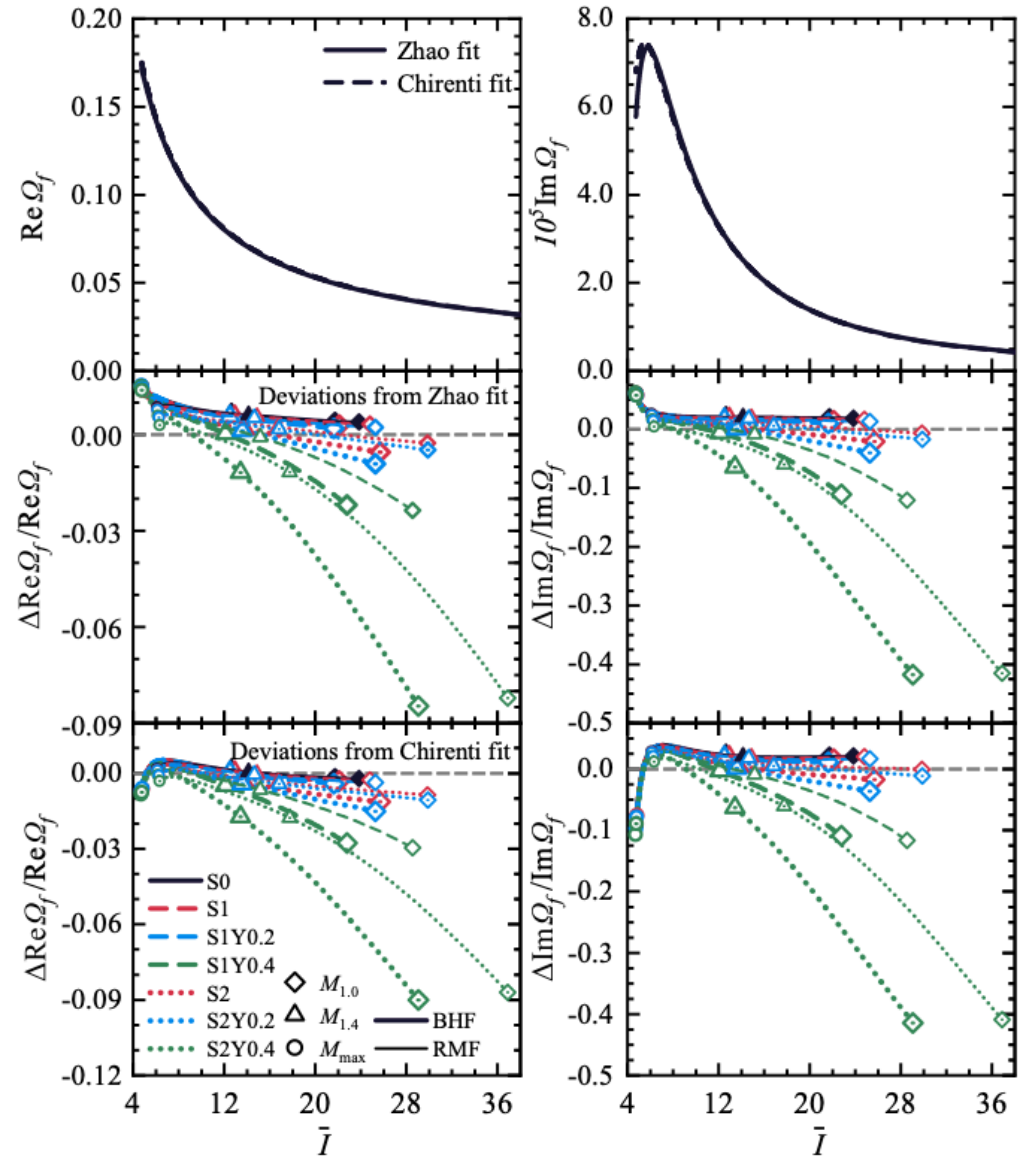


0.1% deviation except for one-node branches (strong 1st PT)

Zhao & Lattimer 2022



$$S = \frac{\pi^2 k_B^2 T m_L}{k_F^2}$$



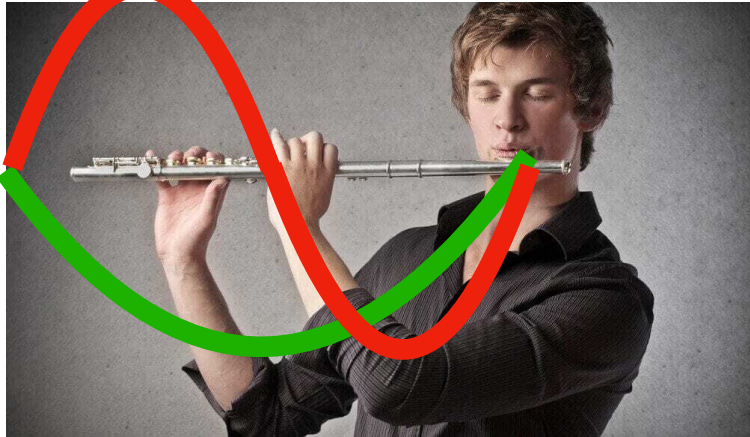
Universal relations preserved with finite-T + neutrino trapping

Zi-Yue Zheng et al. 2025

Oscillation modes

$$A(r)e^{i\omega t} \quad \omega = 2\pi\nu + \frac{i}{\tau}$$

Pressure supported



Standing sound wave of order n:

$$\omega^2 \approx \frac{dp}{d\varepsilon} k^2 \quad k = \frac{\sqrt{l(l+1)}n}{2\pi R}$$

n=0 f-mode (fundamental)

n=1 p-mode (pressure)

Gravity & interface



Stratified fluid in uniform gravity g:

$$\omega^2 = \frac{(\varepsilon_+ - \varepsilon_-)gk}{\varepsilon_+/\tanh(kd_+) + \varepsilon_-/\tanh(kd_-)}$$

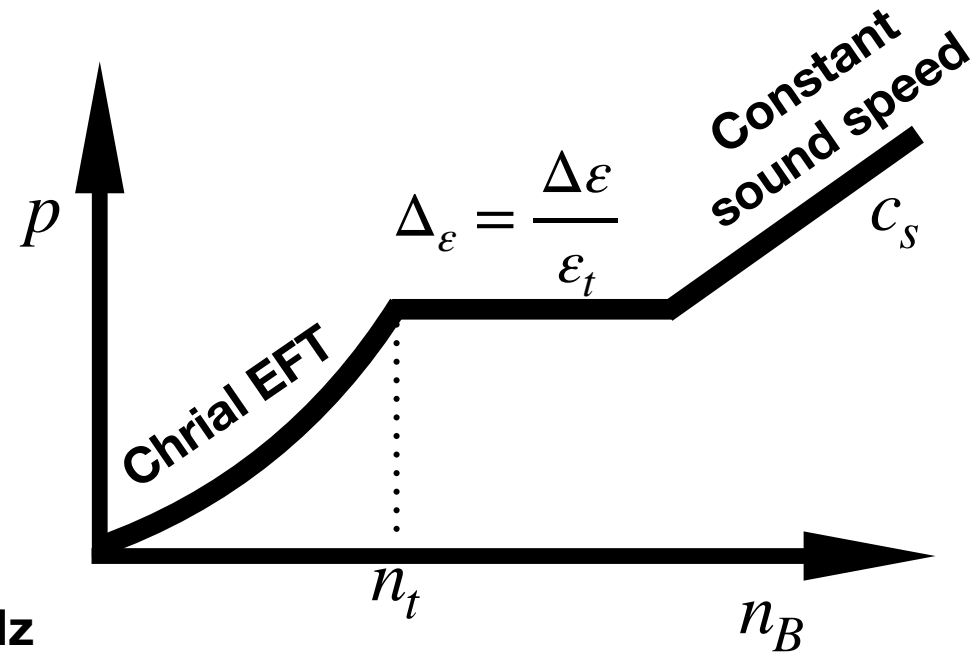
Discontinuity g-mode (interface gravity mode)

Discontinuity g-mode

First order transition

$$\Omega_g^2 \approx \frac{\beta^3 (M_t/M) (R/R_t)^3 (\Delta\varepsilon/\varepsilon_t) D \tanh[D]}{1 + \Delta\varepsilon/\varepsilon_t + \tanh[D]/\tanh[D(R/R_t - 1)]}$$

Ω_g is sensitive to structure factors, $\frac{R_t}{R}$, $\frac{M_t}{M}$, $\frac{\Delta\varepsilon}{\varepsilon}$

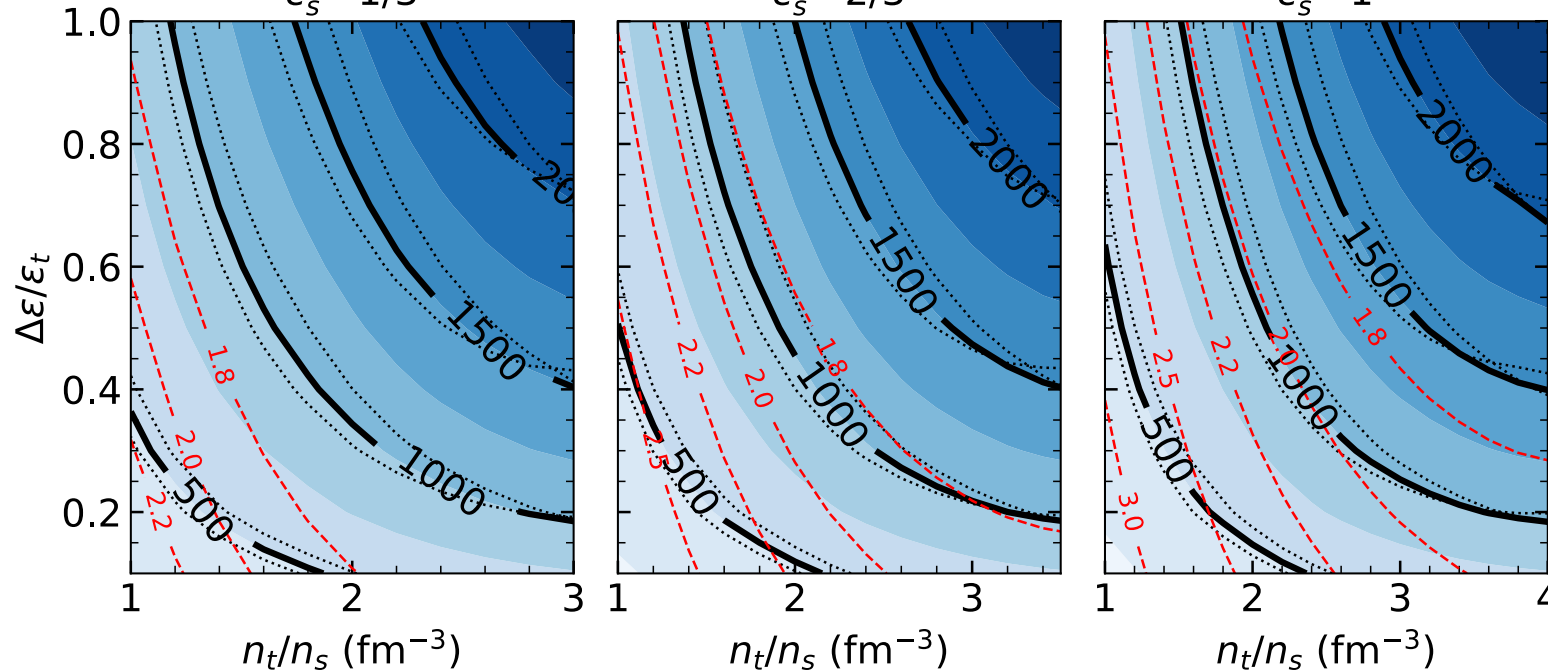


Contour of $\nu_g(n_t, \Delta\varepsilon, c_s^2) |_{M=M_{max}}$ Hz

$c_s^2 = 1/3$

$c_s^2 = 2/3$

$c_s^2 = 1$



Dotted: Chiral EFT
Uncertainty 5%

Dashed:
maximum mass

Frequency: $\nu_g < 0.8$ (1.5) kHz for $c_s^2 = c^2/3$ (c^2)

Zhao & Lattimer 2022

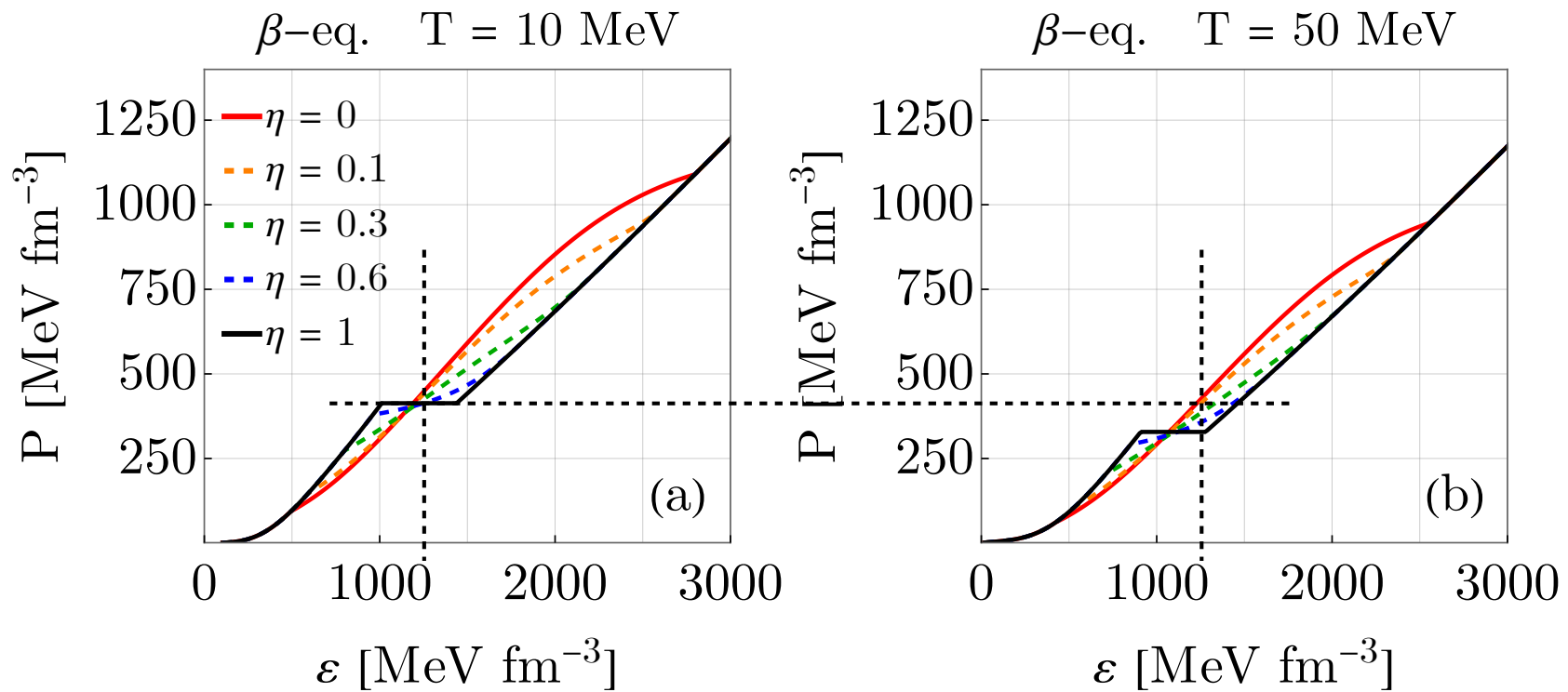
Damping time: $\tau > 100$ (10000) s

BACK

Discontinuity g-mode

First order transition

- Transition pressure (chemical potential) decrease with temperature.



$$S = \frac{\pi^2 k_B^2 T m_L}{k_F^2}$$

Constantinou, Guerrini, Zhao, Han, Prakash 2026

Oscillation modes

$$A(r)e^{i\omega t} \quad \omega = 2\pi\nu + \frac{i}{\tau}$$

Pressure supported



Standing sound wave of order n:

$$\omega^2 \approx \frac{dp}{d\varepsilon} k^2 \quad k = \frac{\sqrt{l(l+1)n}}{2\pi R}$$

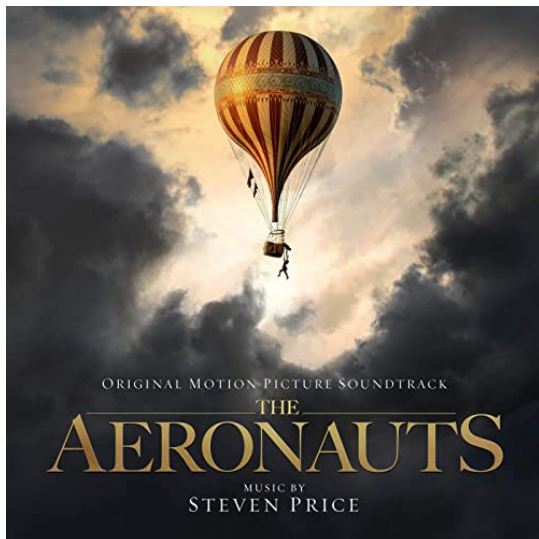
n=0 f-mode (fundamental)

n=1 p-mode (pressure)

Gravity & interface



Gravity & x gradient



Stratified fluid in uniform gravity g:

$$\omega^2 = \frac{(\varepsilon_+ - \varepsilon_-)gk}{\varepsilon_+/\tanh(kd_+) + \varepsilon_-/\tanh(kd_-)}$$

Discontinuity g-mode (interface gravity mode)

Buoyancy oscillation in uniform gravity g: $\{x\} = \left\{ \frac{n_p}{n_B}, \frac{n_n}{n_B}, T, \dots \right\}$

$$\omega^2 \approx \mathcal{N}_{BV}^2 = -\frac{g}{\varepsilon} \left(\frac{\partial \varepsilon}{\partial x} \right)_p \frac{dx}{dr} = g^2 \left(\frac{1}{c_{eq}^2} - \frac{1}{c_{ad}^2} \right)$$

Chemical g-mode (gravity with composition gradient)

gravity mode (g-mode)

Adiabatic (de)compression:

$$\varepsilon - \delta\varepsilon, p - \delta p$$

$$\varepsilon, p$$

Ambient environment:

$$\varepsilon - d\varepsilon, p - dp$$

$$\varepsilon, p$$

Gravity field $\vec{g}(r)$

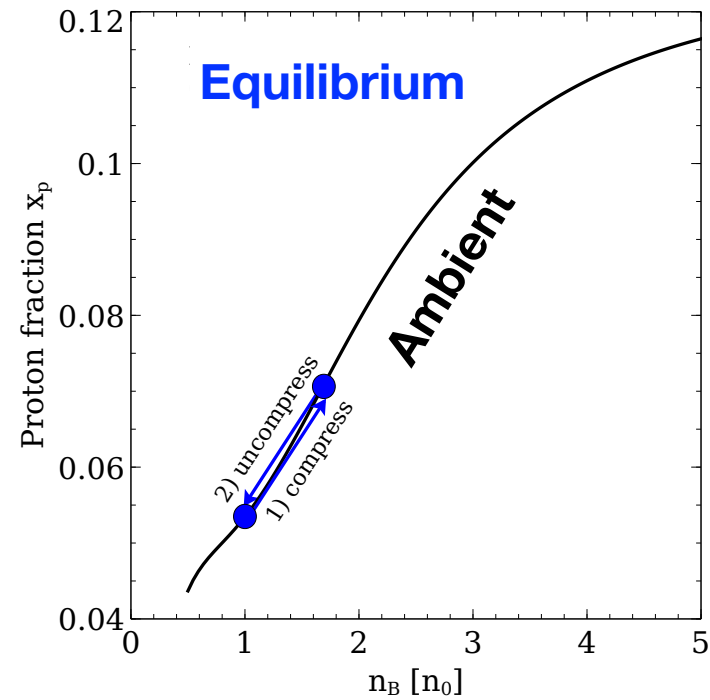
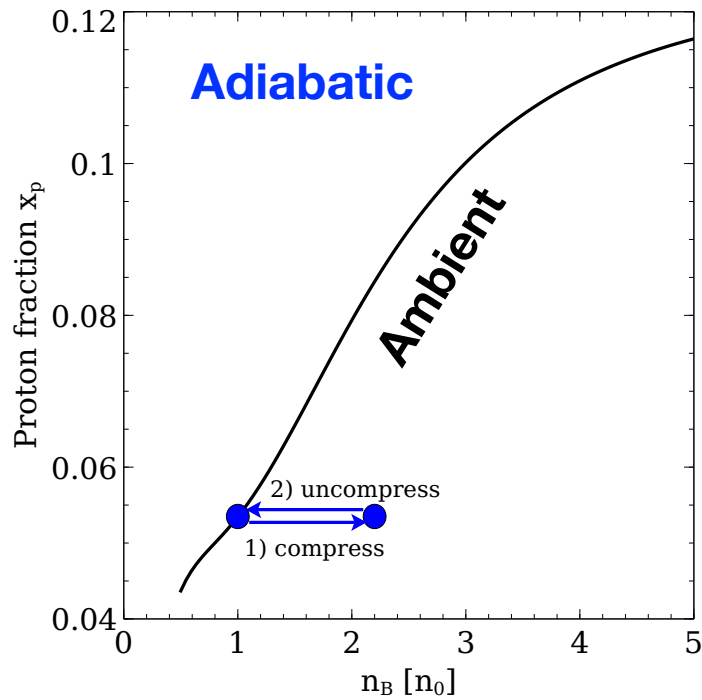


- Mechanical equilibrium: $p - \delta p = p - dp$
- Gravity as recovering: $\varepsilon - \delta\varepsilon > \varepsilon - d\varepsilon$
- Non-convecting(stable): $c_{ad}^2 = \frac{\delta p}{\delta\varepsilon} > \frac{dp}{d\varepsilon} = c_{eq}^2$

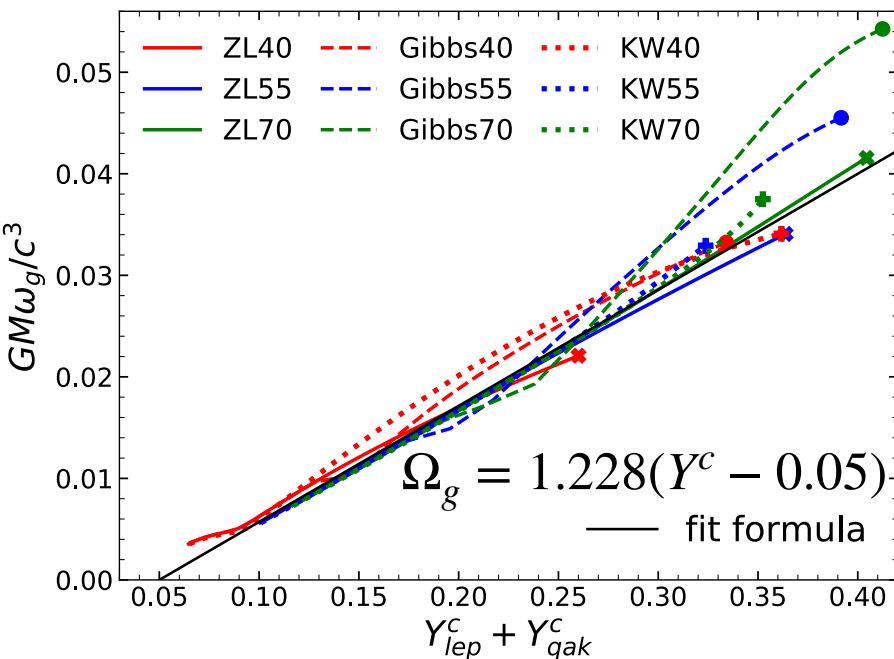
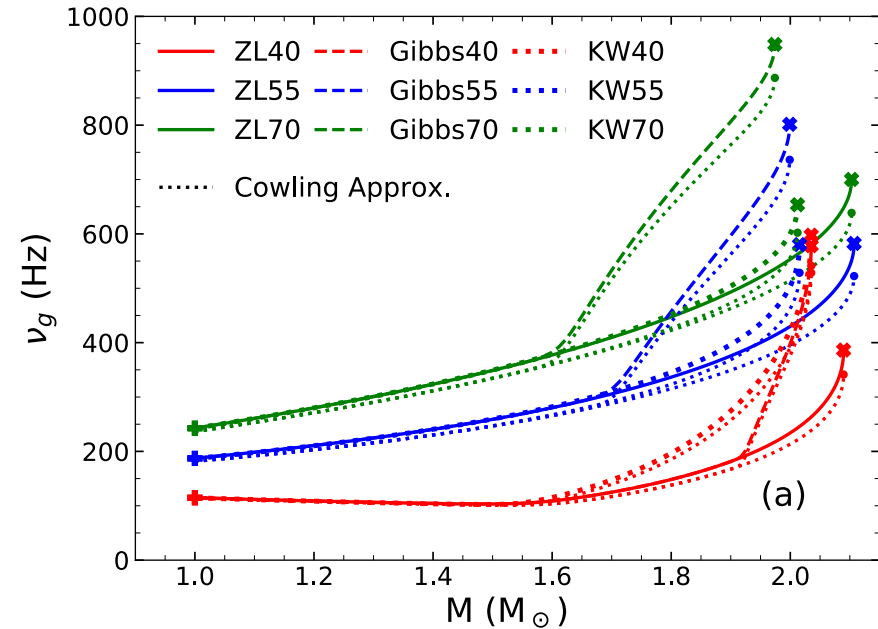
$$c_{ad}^2 = \left[\frac{\partial p}{\partial n_B} \right]_{\{x\}} \left[\frac{\partial \varepsilon}{\partial n_B} \right]_{\{x\}}^{-1} \quad c_{eq}^2 = \frac{dp}{dr} \left[\frac{d\varepsilon}{dr} \right]^{-1}$$

fix composition & entropy, e. g. $\{x\} = \left\{ \frac{n_p}{n_B}, \frac{n_n}{n_B}, S, \dots \right\}$

Adiabatic vs Equilibrium



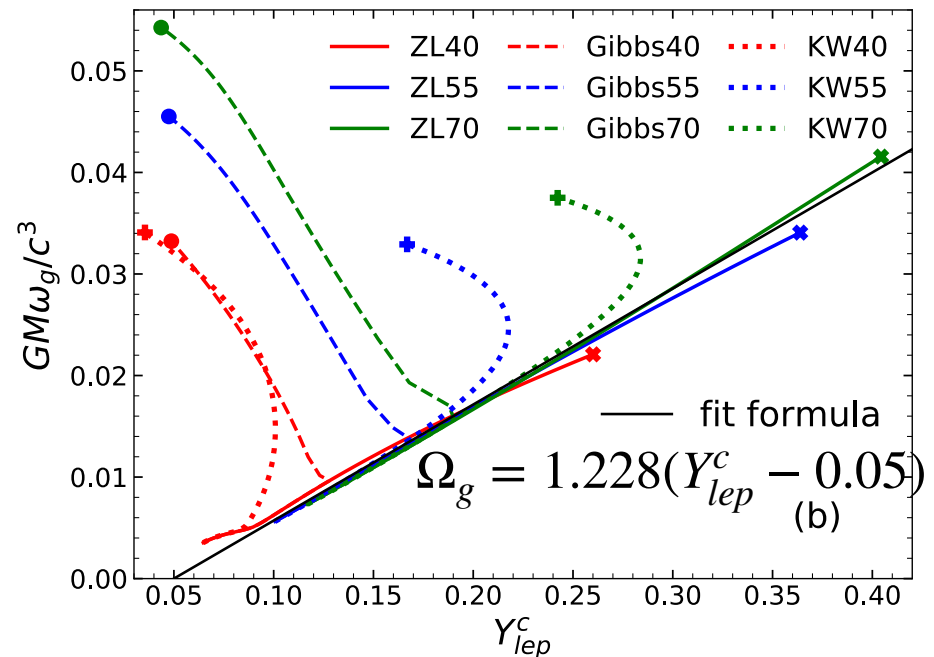
Adiabatic compositional g-mode universal relation



$$\mathcal{N}_{BV}^2 = g^2 e^{\nu-\lambda} \Delta(c^{-2}) \quad \Delta(c^{-2}) = \frac{1}{c_{ad}^2} - \frac{1}{c_{eq}^2}$$

is sensitive to:

1. Symmetry energy $S(n)$
2. Particle ratio gradient
3. Number of particle species

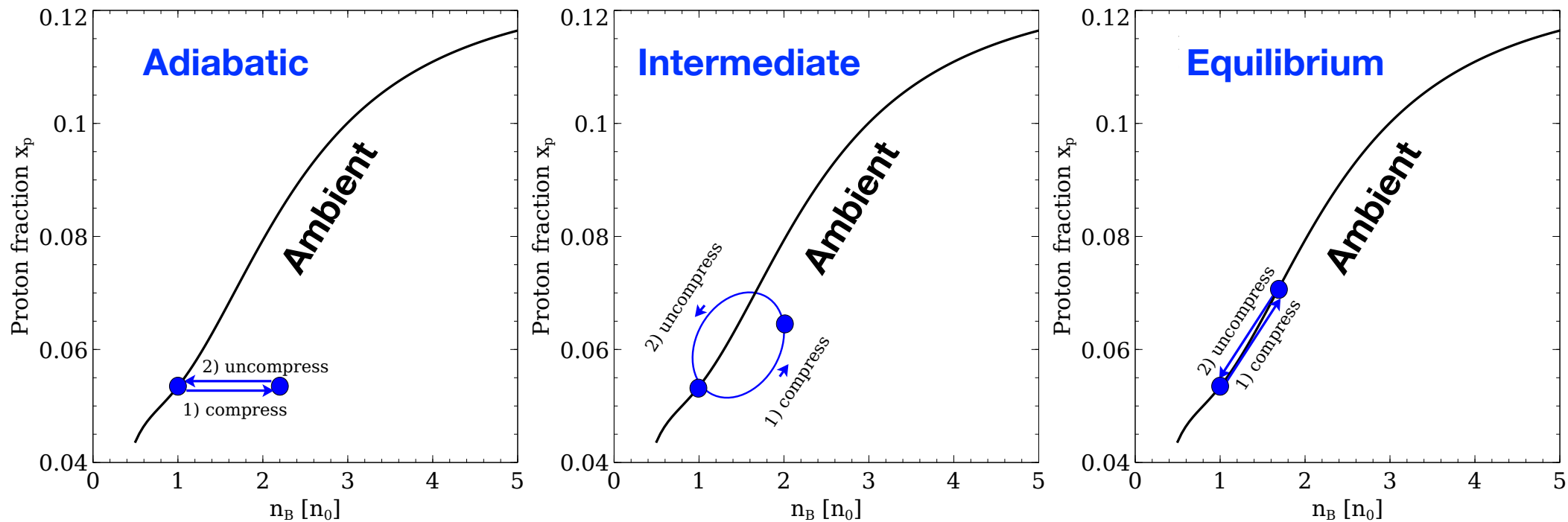


Zhao, Constantinou,
Jaikumar, Prakash 2022

BACK

Adiabatic vs Non-adiabatic

- Path of a fluid element as it is compressed and uncompressed.

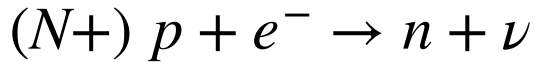
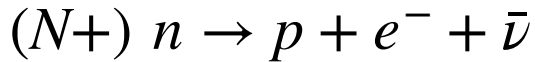


- Path in $\{x_p, n_B\}$ plane maps to path in $\{P, V\}$ plane.

Traversing the path leads to dissipative work, $\oint P \cdot dV$

Dissipation from β -reactions

- Nucleon weak β -equilibrium at low neutrino opacity,



leads to $\delta_\mu = \mu_n - \mu_p - \mu_e \rightarrow 0$, and $x = \frac{n_p}{n_B} \rightarrow x_{eq}$.

- off β -equilibrium, when $\delta_x = x - x_{eq}$,

$$\partial_t \delta_x = \frac{1}{n_B} (\Gamma_{n \rightarrow p} - \Gamma_{p \rightarrow n})$$

$$\Gamma_{n \rightarrow p}^{\text{dUrca}} \sim \int \left(\prod_{n,p,e,\bar{\nu}} \frac{d^3 p_i}{2E_i} \right) \delta^4 \left(\sum_{n,p,e,\bar{\nu}} p_i \right) \sum_{\text{spins}} |\mathcal{M}|^2 f_n (1 - f_p) (1 - f_e) \propto T^5$$

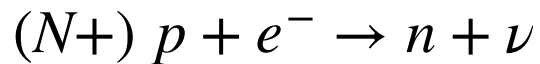
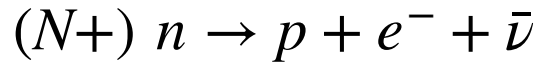
$$\Gamma_{p \rightarrow n}^{\text{dUrca}} \sim \int \left(\prod_{n,p,e,\nu} \frac{d^3 p_i}{2E_i} \right) \delta^4 \left(\sum_{n,p,e,\nu} p_i \right) \sum_{\text{spins}} |\mathcal{M}|^2 (1 - f_n) f_p f_e \propto T^5$$

$\Gamma_{n \rightarrow p}^{\text{mUrca}}$ and $\Gamma_{p \rightarrow n}^{\text{mUrca}}$ have additional factors of $d^3 p_N^{\text{in}} d^3 p_N^{\text{out}} f_N (1 - f_N) \propto T^2$

- Treat dUrca+mUrca jointly: Nucleon Width Appro. (Alford, Haber, & Zhang 2406.13717)

Dissipation from β -reactions

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leads to $\delta_\mu = \mu_n - \mu_p - \mu_e \rightarrow 0$, and $x = \frac{n_p}{n_B} \rightarrow x_{eq}$.

- off β -equilibrium, when $\delta_x = x - x_{eq}$,

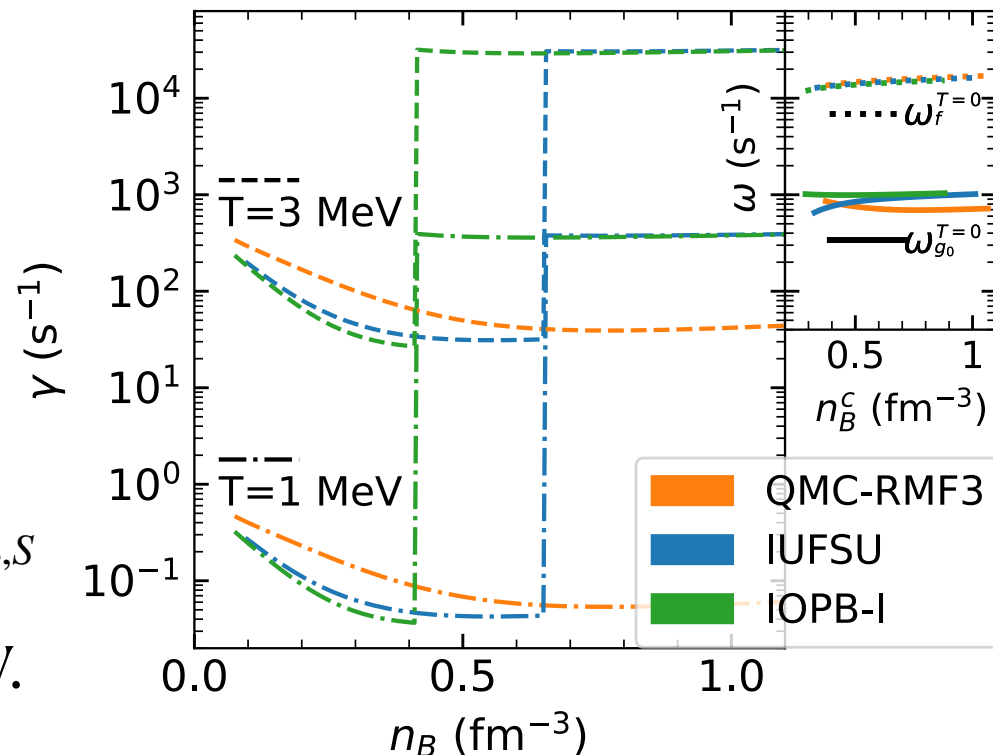
$$\partial_t \delta_x = \frac{1}{n_B} (\Gamma_{n \rightarrow p} - \Gamma_{p \rightarrow n}) \approx -\gamma \delta_x,$$

(linear order)

where γ is the weak relaxation rate,

$$\gamma = - \frac{1}{n_B} \frac{\partial(\Gamma_{n \rightarrow p} - \Gamma_{p \rightarrow n})}{\partial \delta_\mu} \bigg|_{n_B, S} \frac{\partial \delta_\mu}{\partial \delta_x} \bigg|_{n_B, S}$$

- γ reaches kHz frequency at $T \sim 3$ MeV.



Dynamic sound speed

$$c_{dy}^2 = c_{eq}^2 + \frac{c_{ad}^2 - c_{eq}^2}{1 - i\frac{\gamma}{\omega}}$$

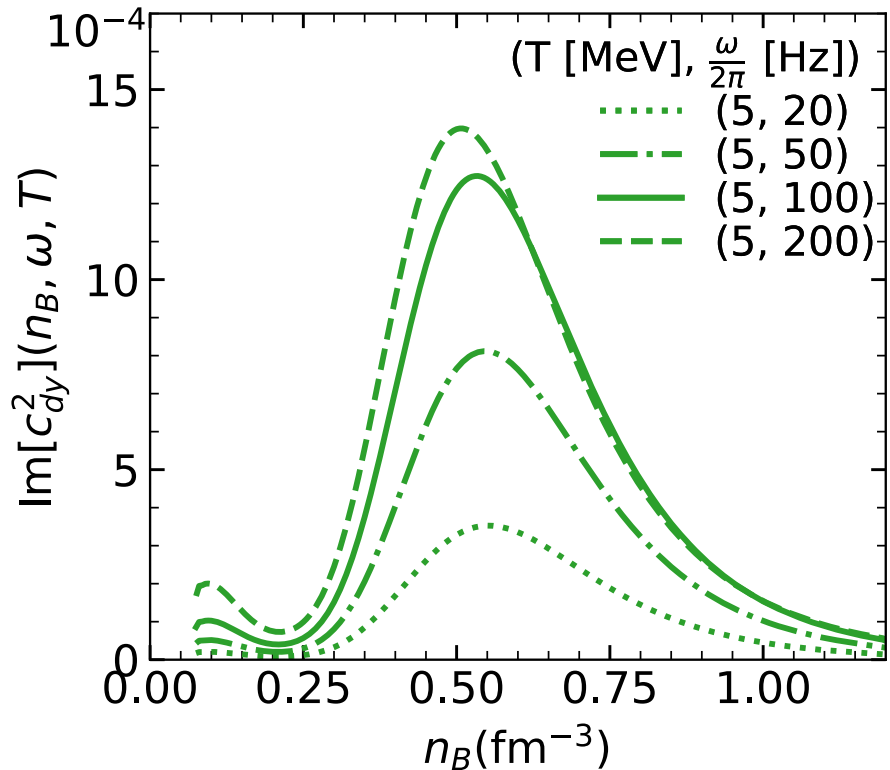
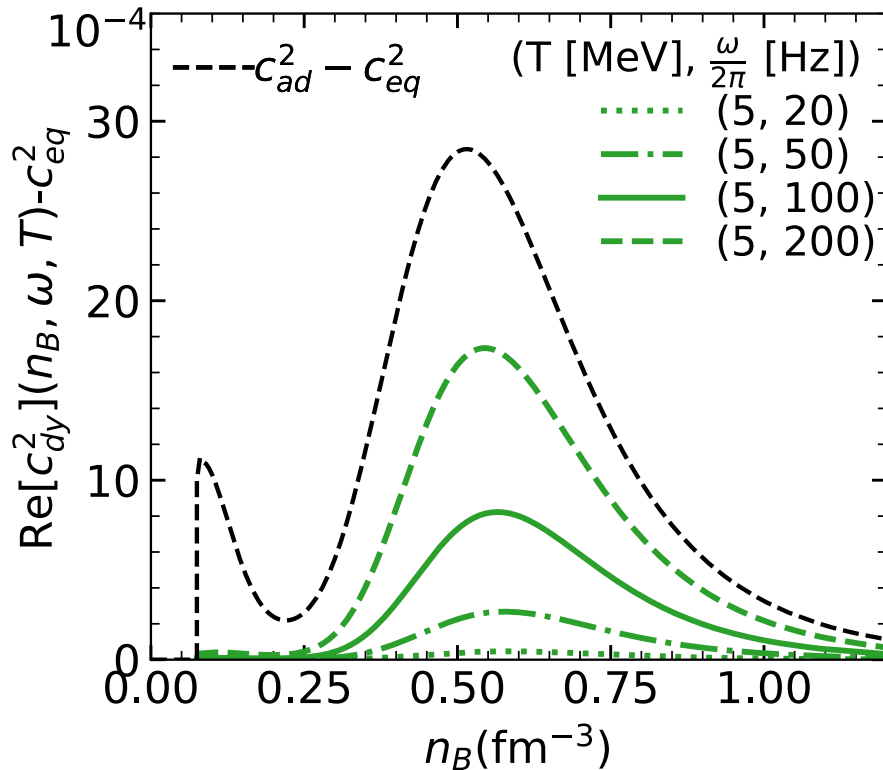
- Complex, frequency-dependent quantity derived from the **period ansatz** that captures both **restoring force** and **damping**. $n_B(t) = n_B^0 + \delta n_B \cos(\omega t)$

1. Restoring force: $c_{eq}^2 < \text{Re}[c_{dy}^2] < c_{ad}^2$

$$\text{Re}[c_{dy}^2] = c_{eq}^2 + (c_{ad}^2 - c_{eq}^2) \frac{\omega^2}{\omega^2 + \gamma^2}$$

2. Dissipative damping: resonance at $\gamma \approx \omega$

$$\text{Im}[c_{dy}^2] = (c_{ad}^2 - c_{eq}^2) \frac{\omega\gamma}{\omega^2 + \gamma^2}$$



Dynamic sound speed

$$c_{dy}^2 = c_{eq}^2 + \frac{c_{ad}^2 - c_{eq}^2}{1 - i\frac{\gamma}{\omega}}$$

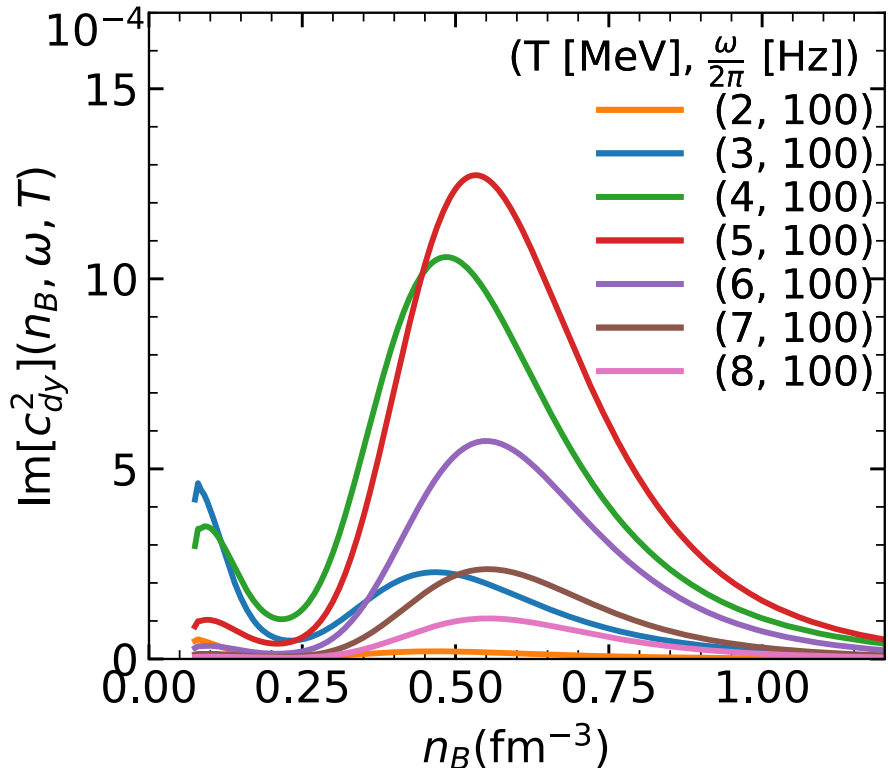
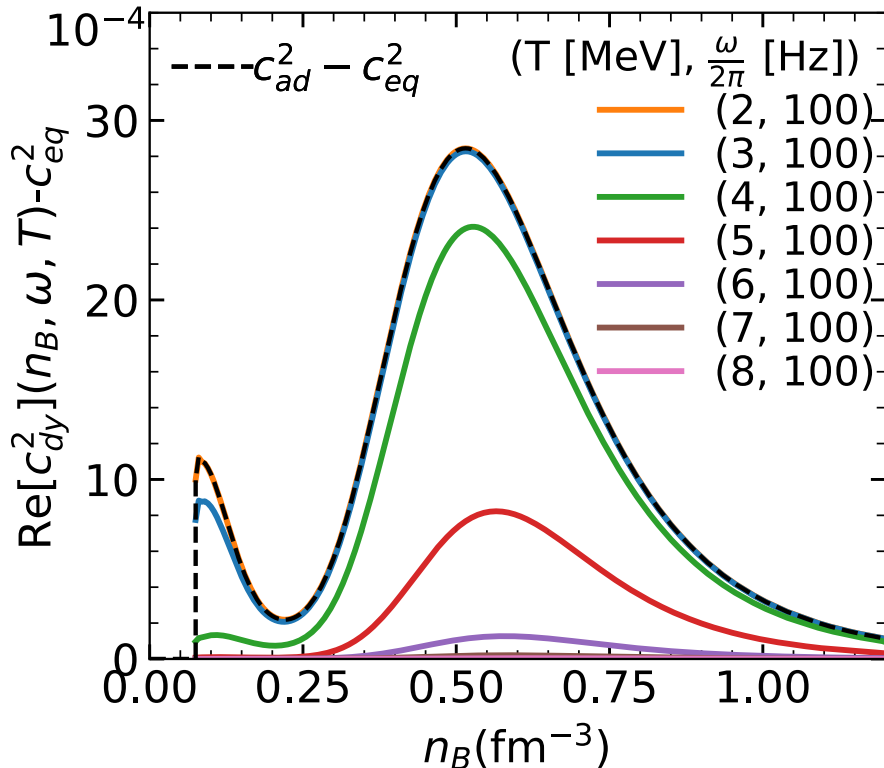
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1. Restoring force: $c_{eq}^2 < \text{Re}[c_{dy}^2] < c_{ad}^2$

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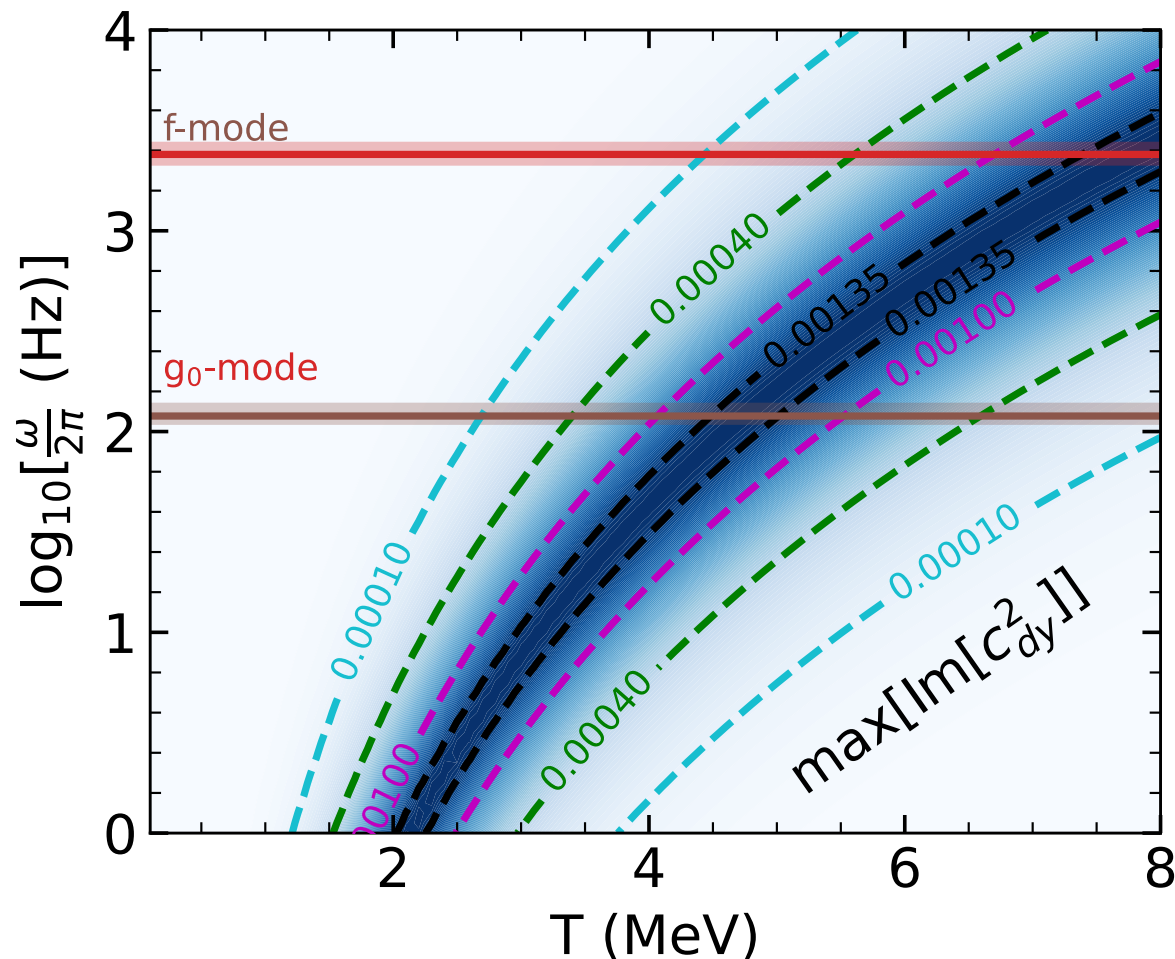
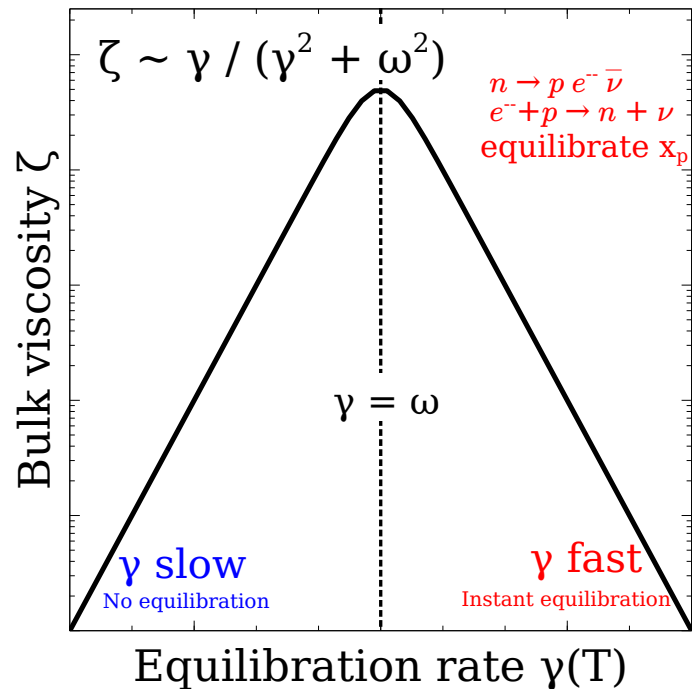


Bulk Viscosity of npe matter

- Bulk viscosity $\zeta = \frac{\varepsilon + p}{\omega} \text{Im} [c_{dy}^2] = \frac{\gamma}{\omega^2 + \gamma^2} (c_{ad}^2 - c_{eq}^2) (\varepsilon + p)$

quantifies energy dissipation from compressional flow,

$$\frac{d\varepsilon}{dt} = -\zeta (\nabla \cdot \mathbf{v})^2.$$

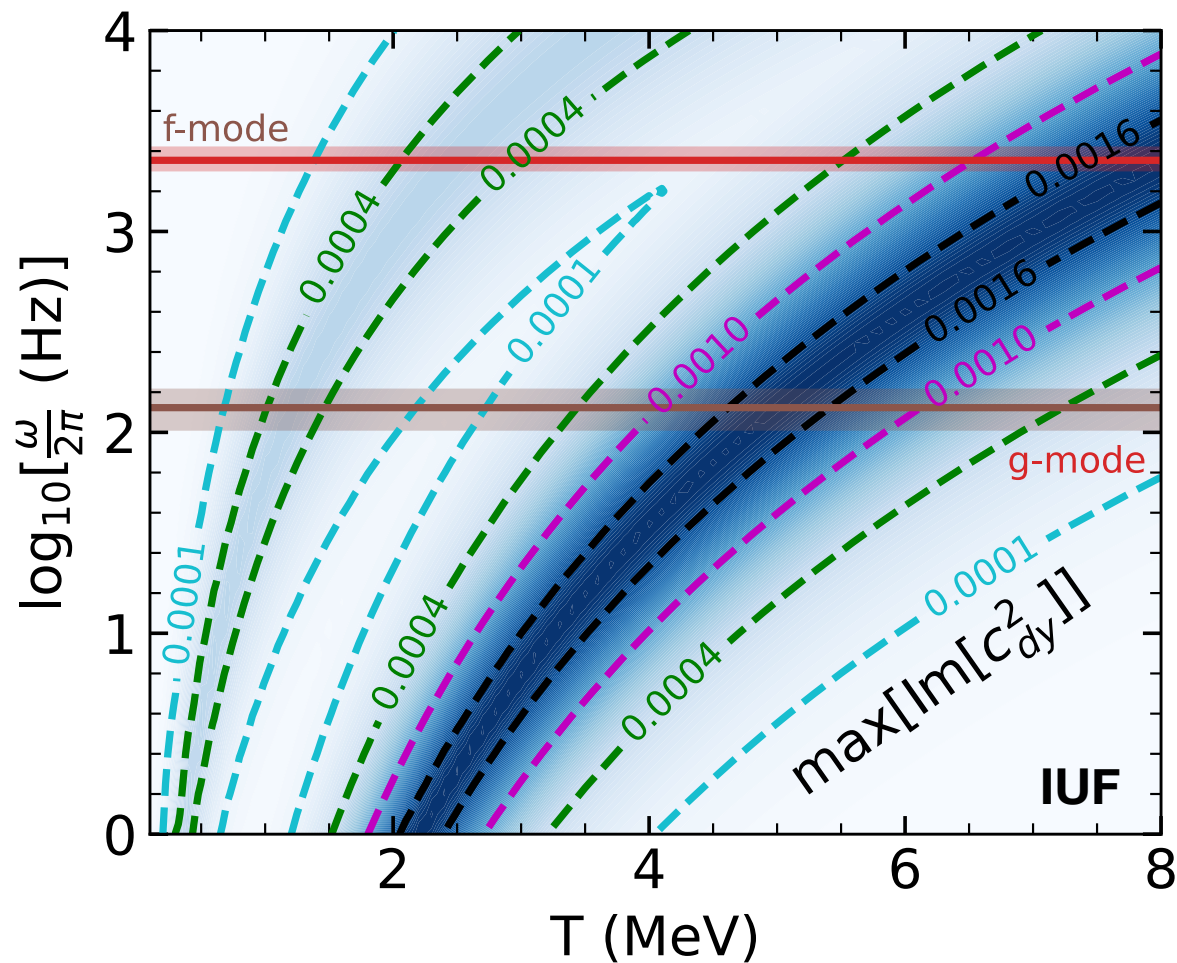
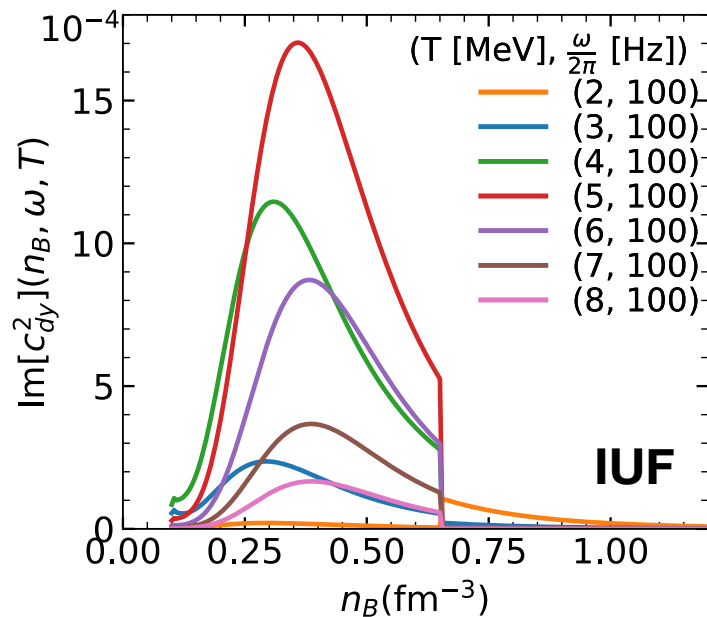
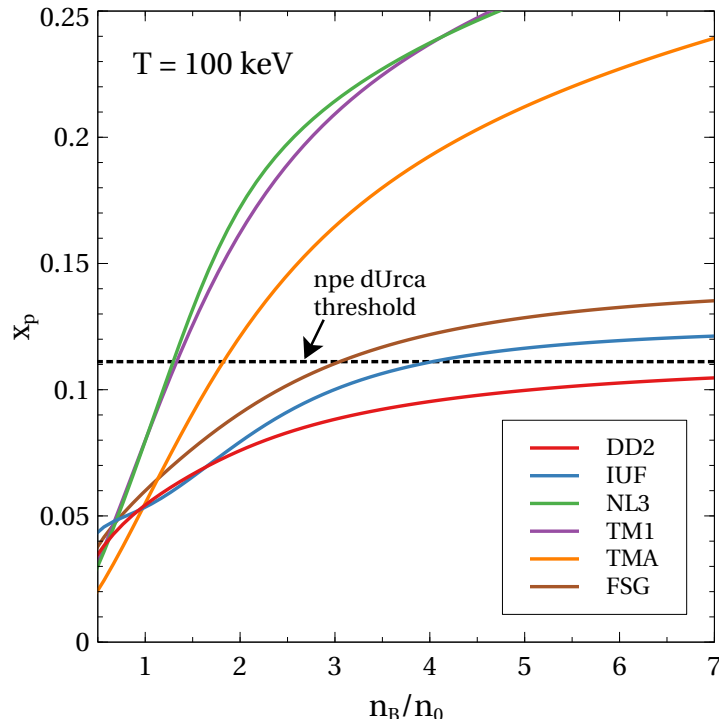


- Even modes have divergence (f-mode, g-modes, p-modes)

$$\nabla \cdot \xi_{\text{even}}^{r, \theta, \phi} = \nabla \cdot (\nabla (R_n(r) Y_m^l(\theta, \phi) e^{i\omega t}))$$

Direct Urca Process

- Threshold: $k_n < k_e + k_p = 2k_p$,
equivalent to $n_n/n_p > 8 \rightarrow x_p > 1/9$



ODEs of Non-radial Oscillation

Eigen value problem of even quasi-normal modes

- Solve Einstein's equations:

$$\delta\pi\delta \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T \right) = \delta R_{\mu\nu}$$

with fluid perturbations, and metric perturbations:

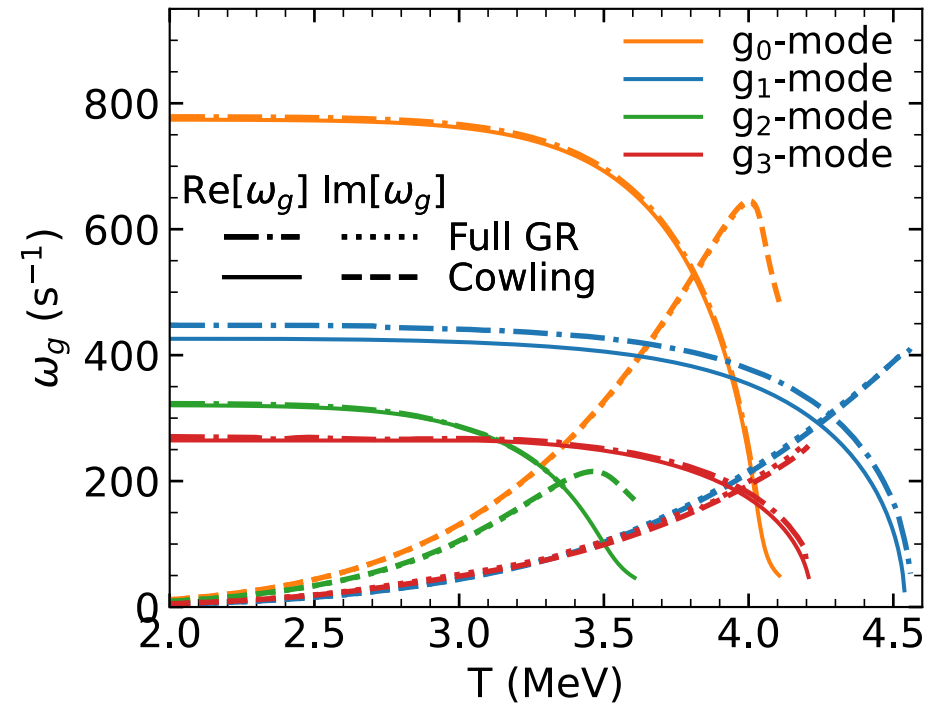
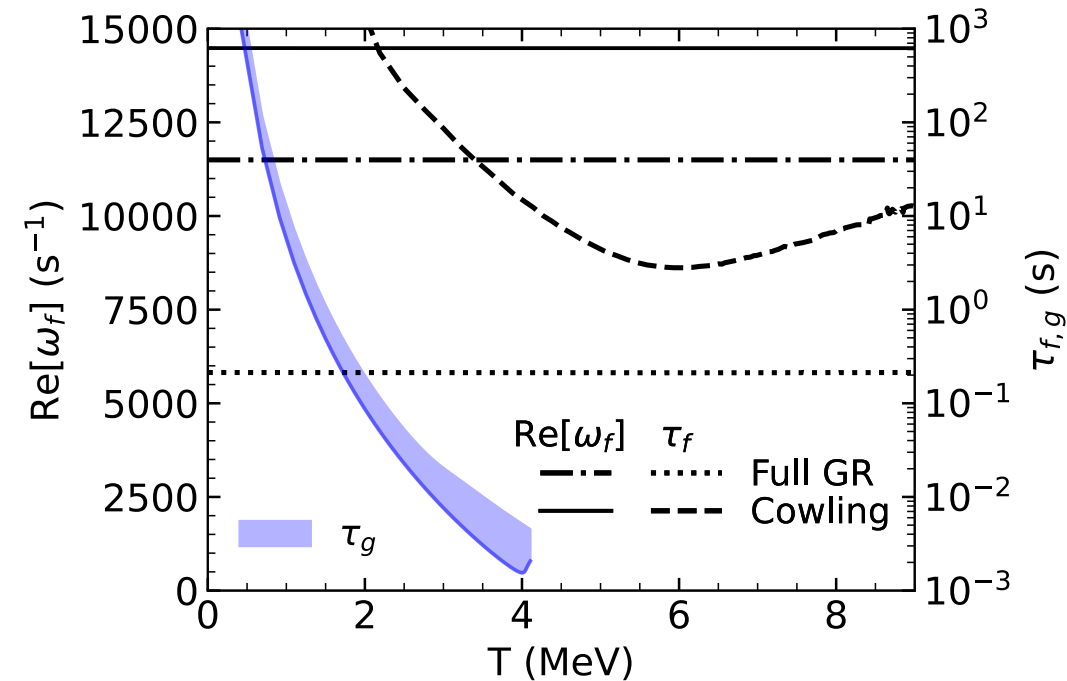
$$\xi_{\text{even}}^{r, \theta, \phi} = \nabla (R_n(r)Y_m^l(\theta, \phi)e^{i\omega t})$$

$$\delta T_{\mu\nu} \text{ from } \xi^{r, \theta, \phi} \text{ and } C_{dy}^2 \quad \delta g_{\mu\nu}^{\text{even}} = \begin{pmatrix} H_0 e^\nu & H_1 & 0 & 0 \\ H_1 & H_2 e^\lambda & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta K \end{pmatrix} Y_{lm}(\theta, \phi) e^{i\omega t}$$

- To get eigenvalue $\omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$,

and eigenfunction $\xi_{\text{even}}^{r, \theta, \phi}$ and H_0, H_1, H_2, K

Impact of bulk viscosity on $\omega = \text{Re}[\omega] + \frac{\mathbf{i}}{\tau}$



- No impact to f-mode.
f-mode is semi divergence-free.

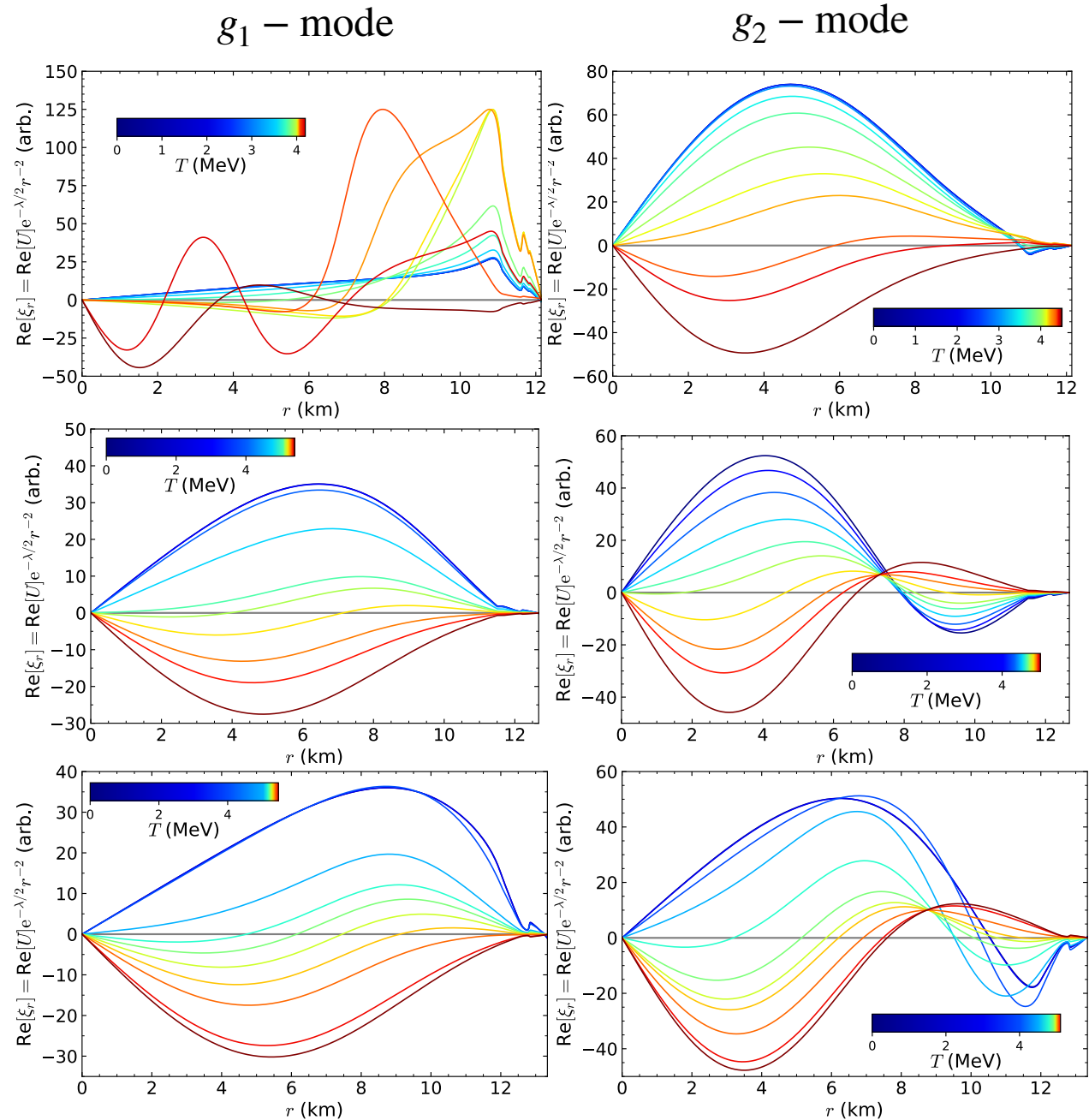
- g-mode gets suppressed at larger T .
Avoided crossing between g-modes.

Fluid perturbation

- QMC-RMF3

- IUFSU

- IOPB-I

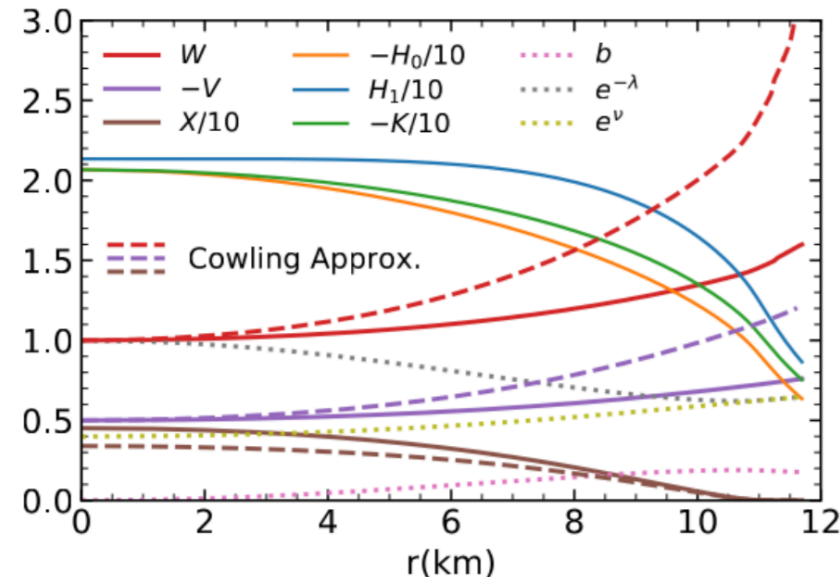
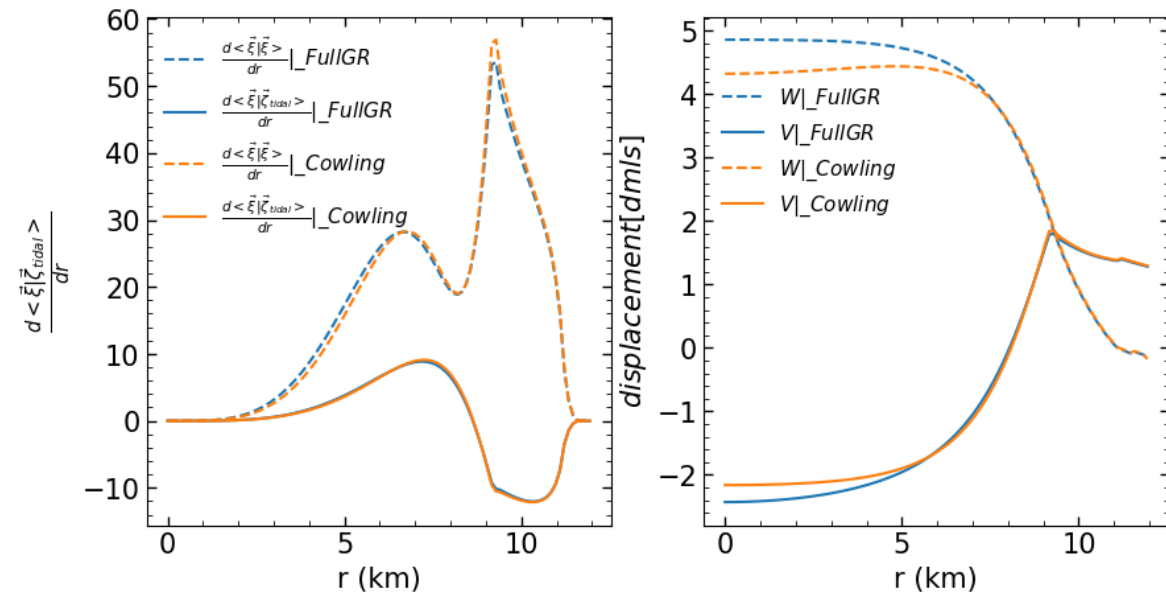


Tidal Overlapping

$$U := \frac{\langle \xi_{\text{mode}} | \xi_{\text{tidal}} \rangle}{\langle \xi_{\text{mode}} | \xi_{\text{mode}} \rangle} = \frac{\langle \xi_{\text{mode}} | \xi_{\text{tidal}} \rangle}{MR^l}$$

$$\xi_{\text{tidal}} = \nabla (r^\ell Y_{\ell m}(\theta, \phi)) = \left(\hat{\mathbf{e}}_r \frac{\partial}{\partial r} + \hat{\mathbf{e}}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\mathbf{e}}_\phi \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \right) (r^\ell Y_{\ell m}(\theta, \phi))$$

$$\xi_{\text{mode}} = r^{\ell-1} W e^{-\lambda/2} Y_{\ell m}(\theta, \phi) \hat{\mathbf{e}}_r - r^{\ell-1} V \frac{\partial Y_{\ell m}(\theta, \phi)}{\partial \theta} \hat{\mathbf{e}}_\theta - r^{\ell-1} V \frac{1}{\sin \theta} \frac{\partial Y_{\ell m}(\theta, \phi)}{\partial \phi} \hat{\mathbf{e}}_\phi$$



Dynamic Tide Effect

- Tidal fields \longrightarrow Quadrupole moments

$$-\lambda \varepsilon_{ij}(t) = Q_{ij}(t)$$

- A general deformation of NS can be decomposed into modes:

$$\xi = \sum_a A_a \xi_a \quad , \text{ where } a = \dots, g_3, g_2, g_1, f, p_1, p_2, p_3, \dots$$

- Quadrupole deformation is a special deformation close to f -mode.

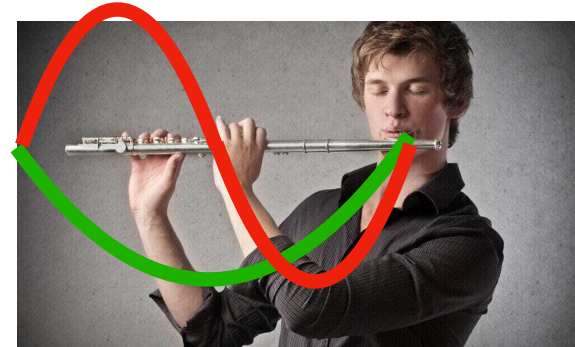
- Excitation of modes with periodic tidal external forces $\varepsilon_{ij} e^{-2i\omega t}$:

$$\ddot{A}_a + \omega_a^2 A_a = \omega_a^2 U_a \quad , \text{ where } U_a \propto \langle \xi_a | \varepsilon_{ij} \rangle e^{-2i\omega t}$$

with equilibrium solution:

$$A_a \propto \frac{\omega_a^2}{\omega_a^2 - 4\omega^2} U_a \quad , \text{ resonancing at } \omega_a = 2\omega$$

Modes Important for Dynamic Tide



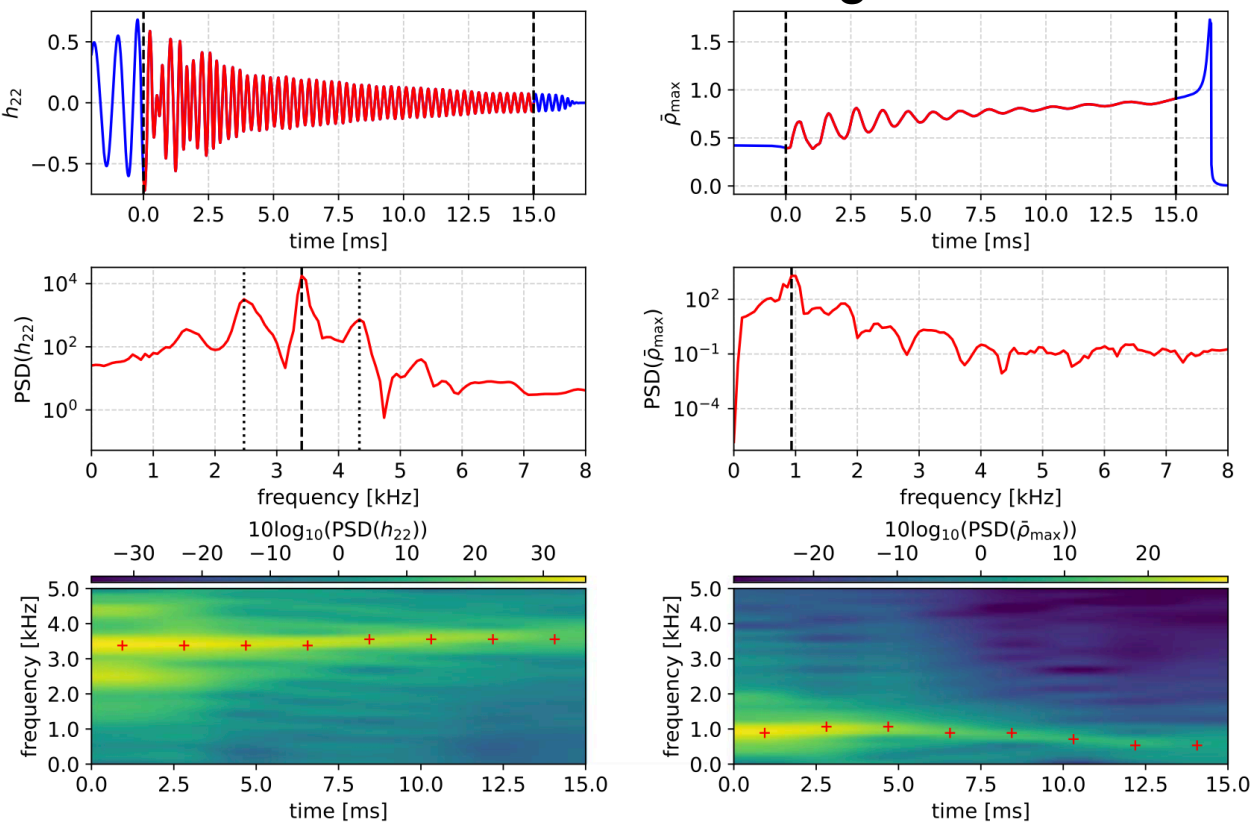
$$\frac{\omega_{orbit}}{2\pi} \lesssim 500\text{Hz}$$

- f-mode frequency is a bit too high:
no significant impact to GW170817 [G. Pratten. et. al. 2019]
- Composition and discontinuity g-modes are important for GW waveform:
Discontinuity g-modes[Z. Miao, et. al. 2024, A.R. Counsell et. al. 2025]
Composition g-modes[P. Jaikumar, et. al. 2021, T. Zhao, et. al. 2022]
- Resonant shattering flare during binary NS merger inspiraling:
Elasticity enhanced g-mode[D Neill, et. al. 2023]
- Intermode coupling might be important as well:
Resonance locking of g-modes [K. J. Kwon, et. al. 2025]

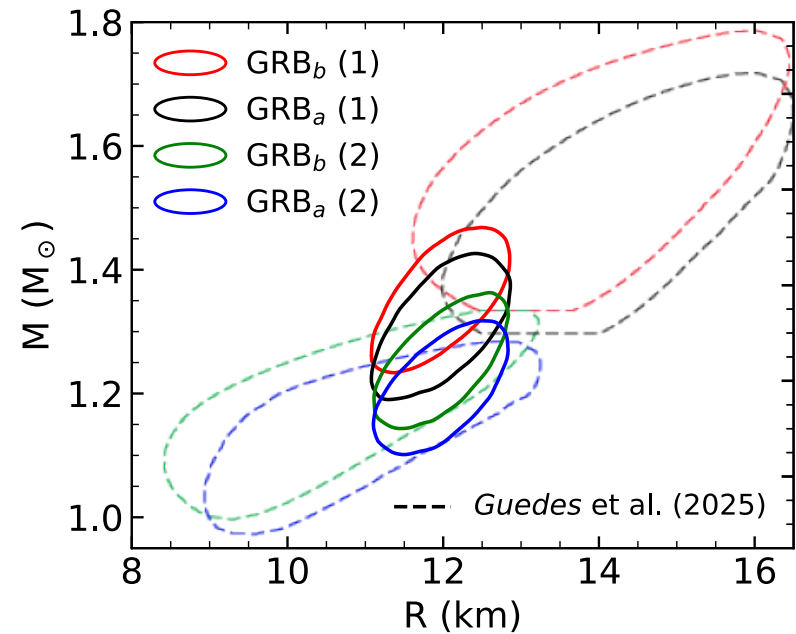
Oscillation in Gamma-ray Bursts

- Three equal spacing frequency observed GRBs 910711 and 931101B, corresponds to $f_2 - f_0$, f_2 , $f_2 + f_0$.
(for angular quantum number $l = 0, 2$)

Gravitational Wave of Post Merger Remnant



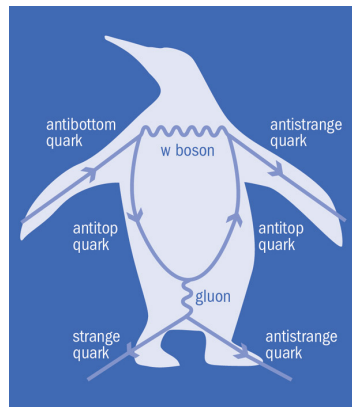
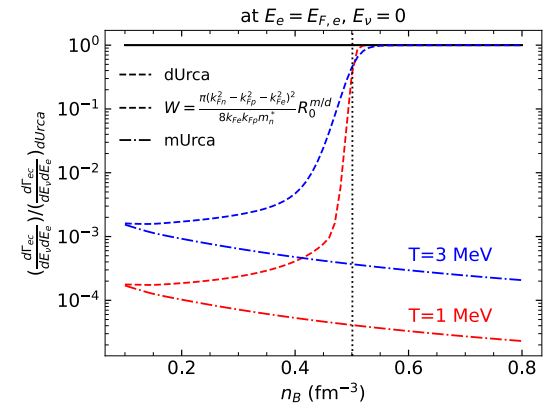
Victor Guedes *et al* 2025 *ApJ* 983 88



Zhao & Lattimer

To-do Next

- More realistic composition & entropy profile.
- Consider the Neutrino-trapped scenario.
- Improve weak rates, e.g. NWA between dUrca and mUrca.
- Viscosity due to other degrees of freedom: muons, pions, hyperons, deconfined quarks, and perhaps dark matter ...



Presenter: Tianqi Zhao

Thank you!

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