^{48}Ca Radius EXperiment – CREX
^{208}Pb Radius EXperiment – PREX

Learning from PREX-II and CREX: What Neutron Skins Tell Us

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- arXiv.2406.05267

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries
PHYSICS FRONTIER CENTER

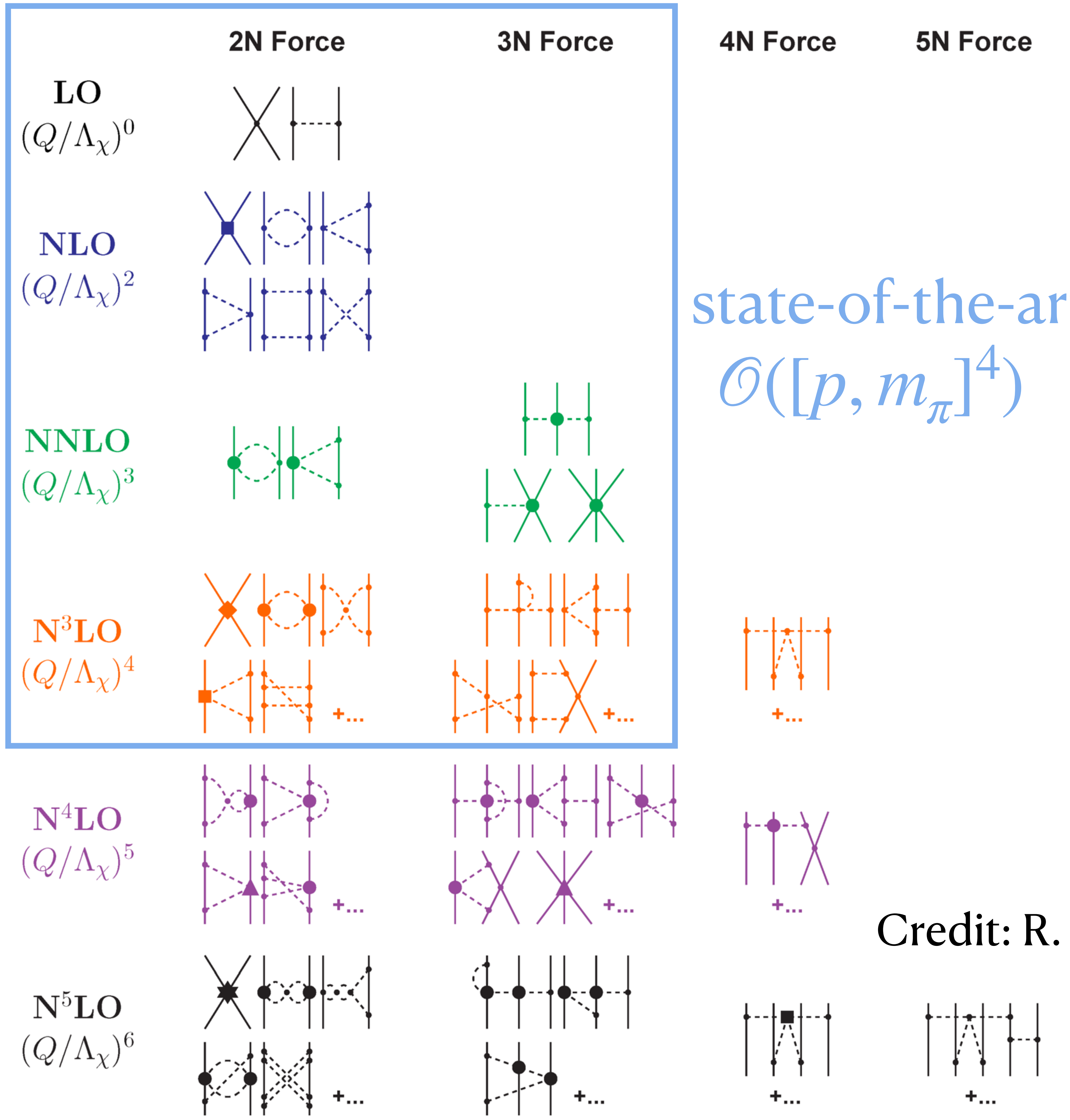
INSTITUTE for NUCLEAR THEORY

OUTLINE

- Nuclear models
- Symmetry energy & Neutron skin experiments (PREX & CREX)
- Tensions between PREX+CREX and mean field models
- Constraints on bulk properties (symmetry energy)
- Constraints on surface properties (spin-orbit coupling)
- Takeaways

Chiral Effective Field Theory (χEFT)

- Low-energy-constant uncertainty: nucleon contact vortex is fitted to light-bound states, e.g. Deuteron.
  Piarulli and Tews 2021
- Regulator uncertainty for EFT: Cut-off $\Lambda = 450\ \text{MeV},\ 500\ \text{MeV}$ tested.
  Entem and Machleidt 2003
- Manybody uncertainty: tested to be subdominant, controlled by model mixing.
  Hu et al. 2022
- Truncation uncertainty for χEFT: modeled with Gaussian Process.
  Drischler et al.

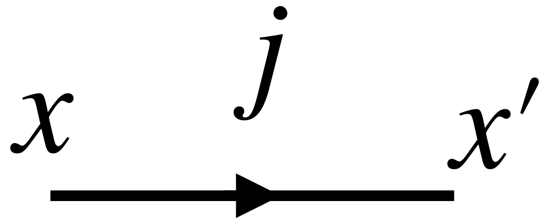


state-of-the-art
$\mathcal{O}([p, m_\pi]^4)$

Credit: R. Machleid

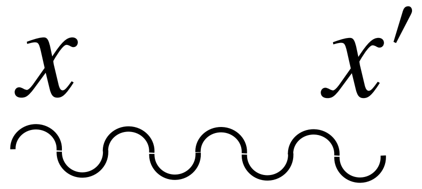
Hartree-Fork Approximation

- Nucleon Green's function:



  $G_j(x, x') = < \psi_j(x)\psi_j^\dagger(x') >$

- Two body interactions:



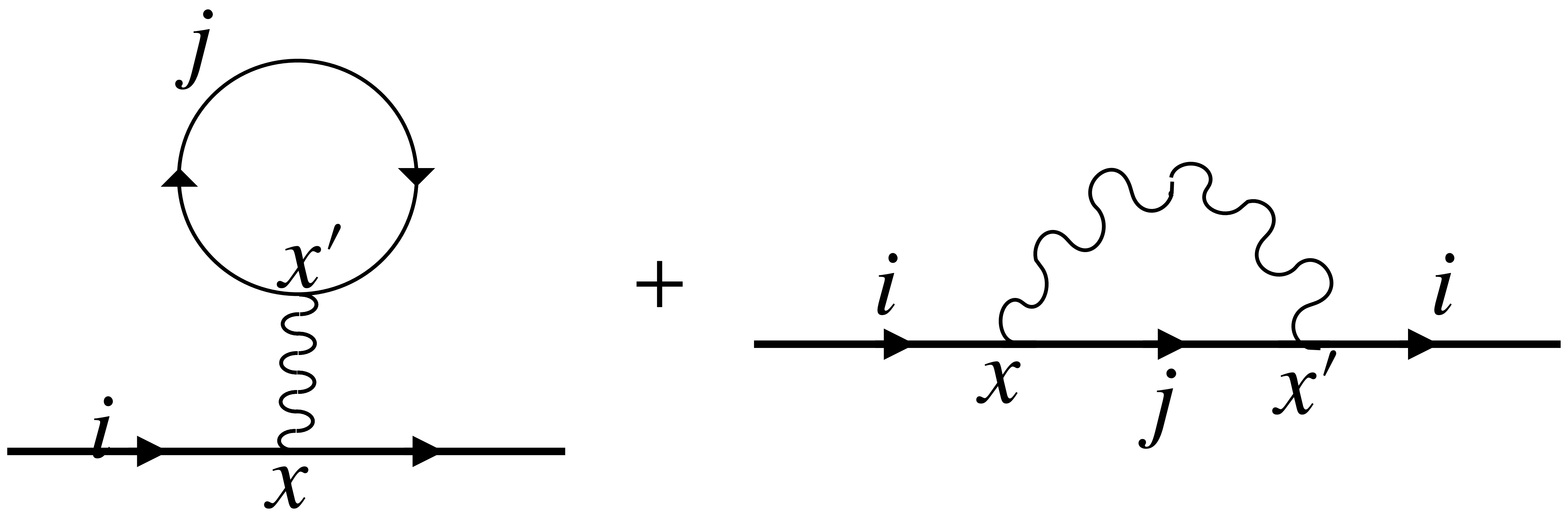
  $V(x, x')$

- Hartree potential:

$$V_{Hartree}(x) = -\sum_j \int V(x, x')G_j(x', x')dx'$$

- Fork potential:

$$V_{Fock}(x)\psi_i(x) = \sum_j \int V(x, x')G_j(x, x')\psi_i(x')dx'$$



- Schrödinger equation:

  $(H_{kinetic} + V_{Hartree} + V_{Fork})\psi_i = \epsilon_i\psi_i$

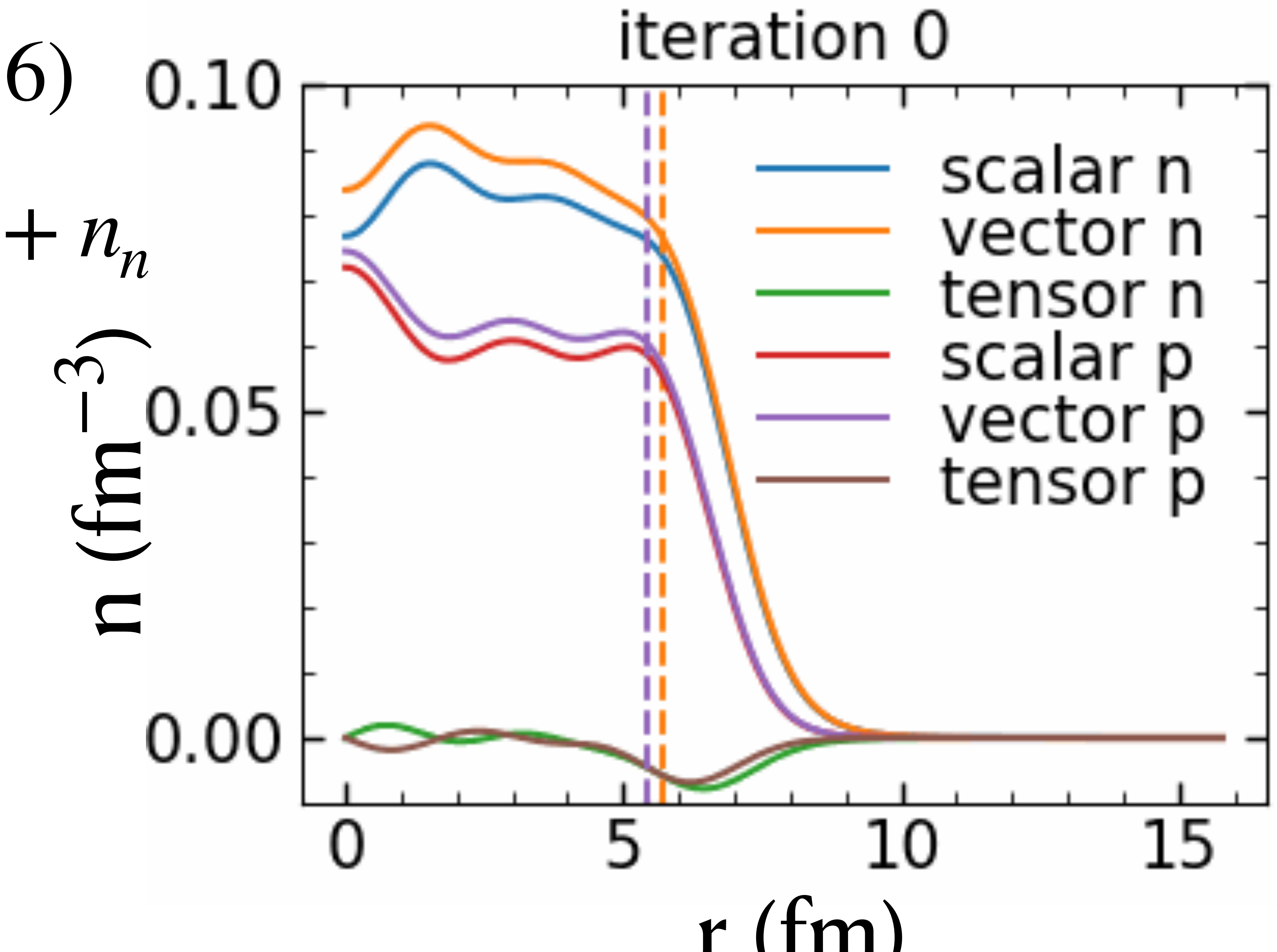
- Skyrme model:

  $V(x, x') \propto \delta(r - r')\times$ (spin, momentum)

Relativistic mean-field model (RMF)

Relativistic Hartree Approximation

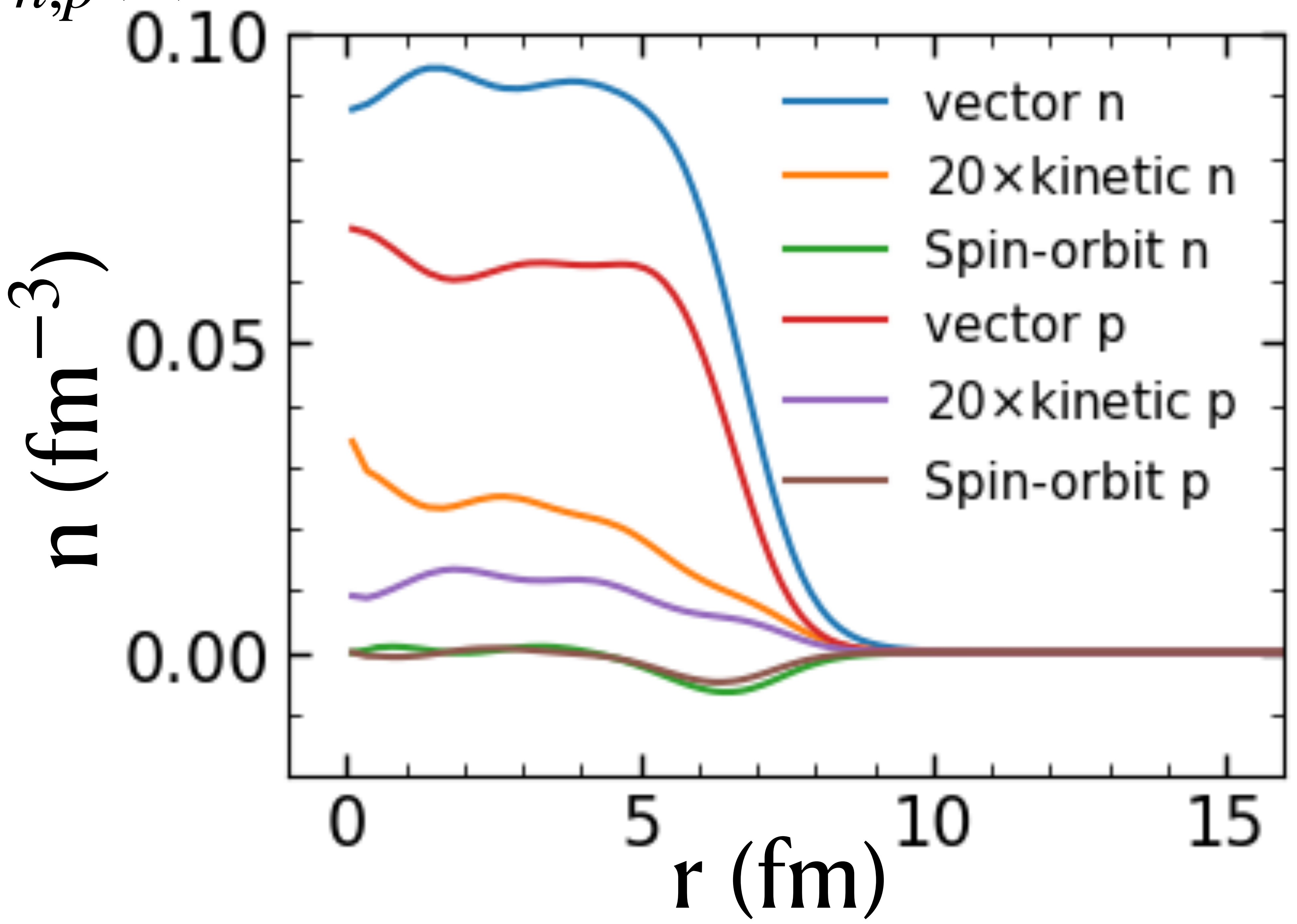
- 1. Nucleon interactions: e.g. $g_\omega \psi^\dagger \gamma_\mu \omega^\mu \psi$ for vector isoscalar sector
  Yukawa interactions mediated by scalar(vector)-isoscalar(isovector) mesons
- 2. Relativistic Hartree potential $V_{Hartree}(x)$:
  from classical meson fields $\sigma(500), \delta(980), \omega(783), \rho(776)$
- 3. Klein–Gordon equation: e. g. $(\Box + m^2 + V_{\omega\rho})\omega = n_p + n_n$
  nucleons source meson fields
- 4. Dirac equation: $(i\gamma^\mu \partial_\mu - m + V_{Hartree})\psi = 0$
  eigenvalue problem determines nucleon levels.
  spin is included automatically in the spinor.



Non-relativistic mean-field model (Skyrme)

Non-relativistic Hartree-fork Approximation

- 1. Zero range nucleon interactions: $V(\overrightarrow{r_1}, \overrightarrow{r_2}, \overrightarrow{p_1}, \overrightarrow{p_2}, P_{\text{spin}}) = \delta(\overrightarrow{r_1} - \overrightarrow{r_2}) \otimes (\overrightarrow{p_1}, \overrightarrow{p_2}, P_{\text{spin}})$
- 2. As in local density approximation, Hamiltonian $\mathscr{H}(\rho_{n,p}, \tau_{n,p}, \overrightarrow{J}_{n,p})$ contains: spin-orbit current $\overrightarrow{J}_{n,p}(\vec{r}) = -i\sum \psi^{\dagger}_{n,p}(\vec{r})\left(\overrightarrow{\nabla} \times \vec{s}\right)\psi_{n,p}(\vec{r})$ baryon density $\rho_{n,p}(\vec{r}) = \sum |\psi_{n,p}(\vec{r})|^2$, and kinetic density $\tau_{n,p}(\vec{r}) = \sum |\overrightarrow{\nabla}\psi_{n,p}(\vec{r})|^2$, arrived from summing up single particle levels.
- 4. Shrodinger equation: $\mathscr{H}\psi = E\psi$ eigenvalue problem determines nucleon levels. Spin appears through densities, not operators



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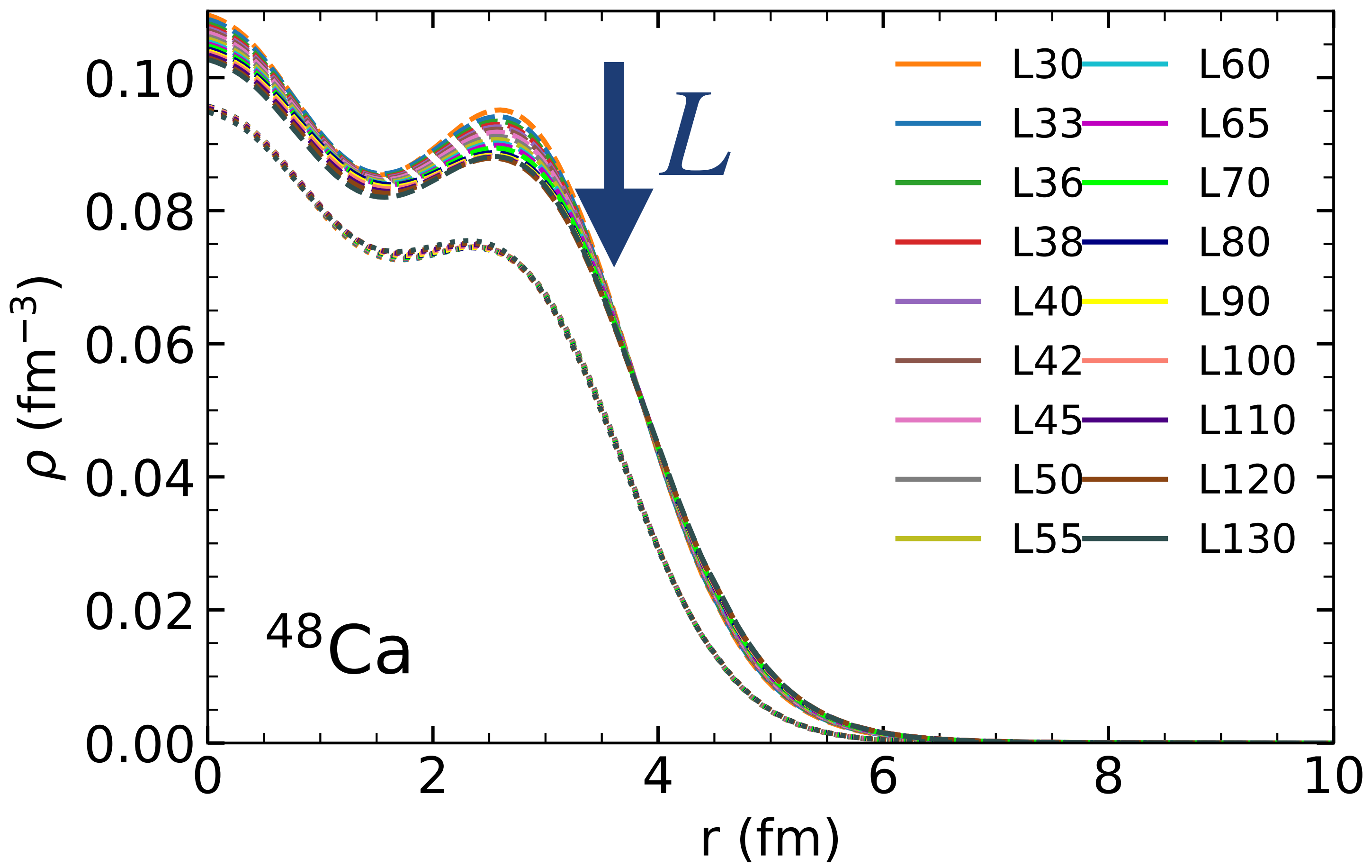
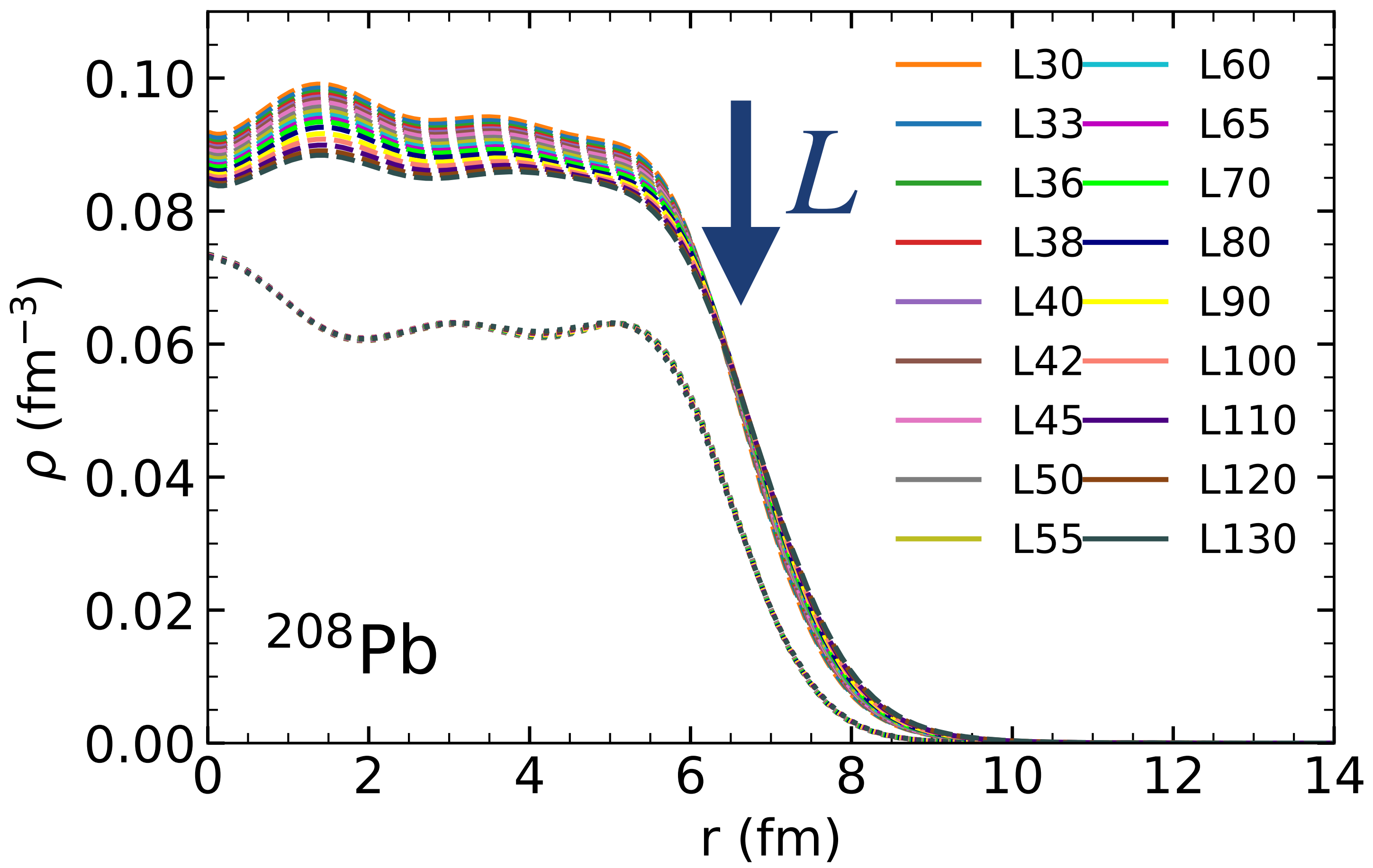
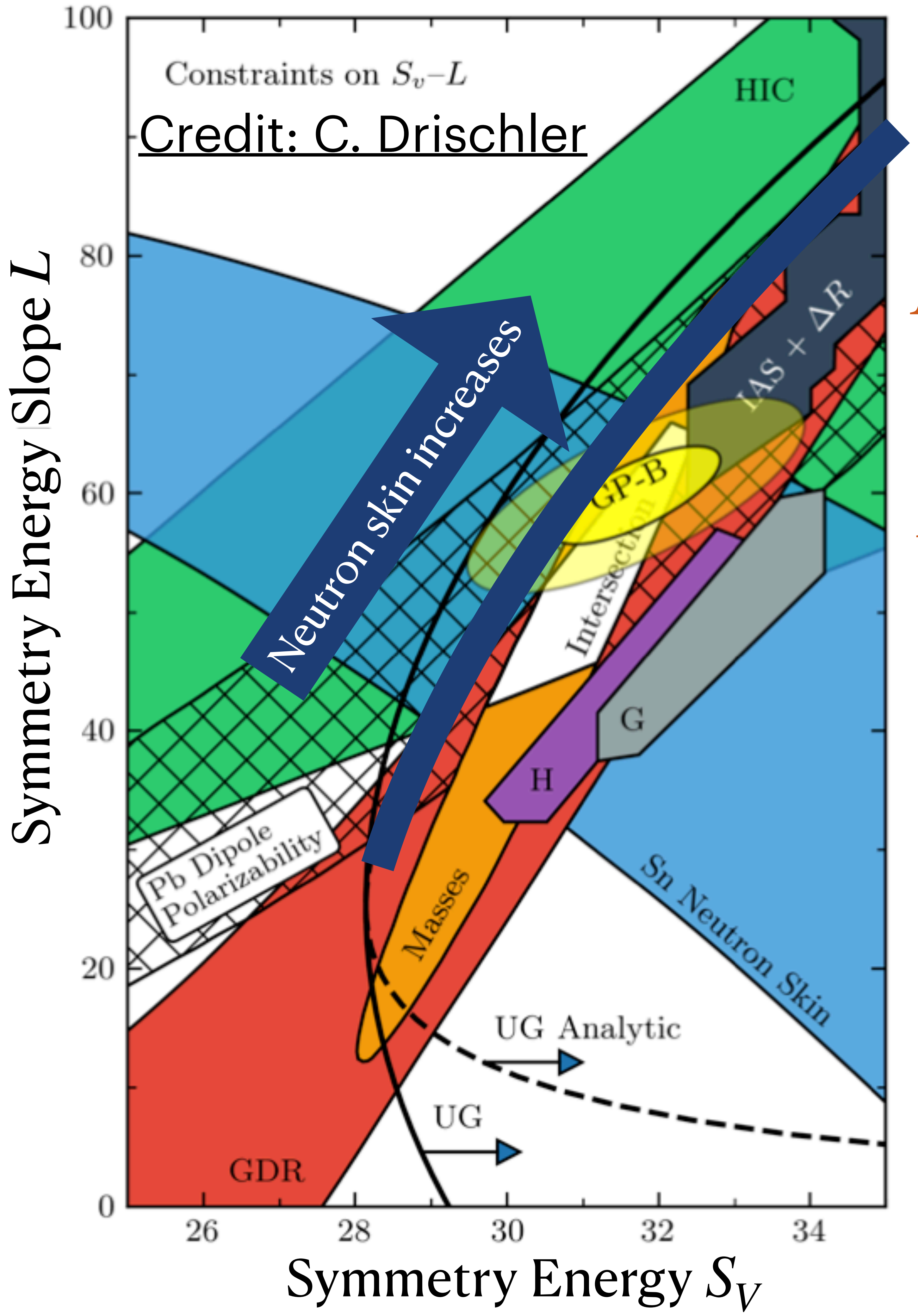
Infinite Nuclear Matter $E(u = n_B/n_s, x = n_p/n_B)$

Neutron star matter ≈ Pure neutron matter = Symmetric nuclear matter + Symmetry energy






Credit: C. Drischler

$$E(n_B, x) \approx$$

$$E_{SNM}(u) + E_{SYM}(u)\,(1-2x)^2 + \dots$$

$$BE + \frac{K}{18}(u-1)^2 + \dots$$

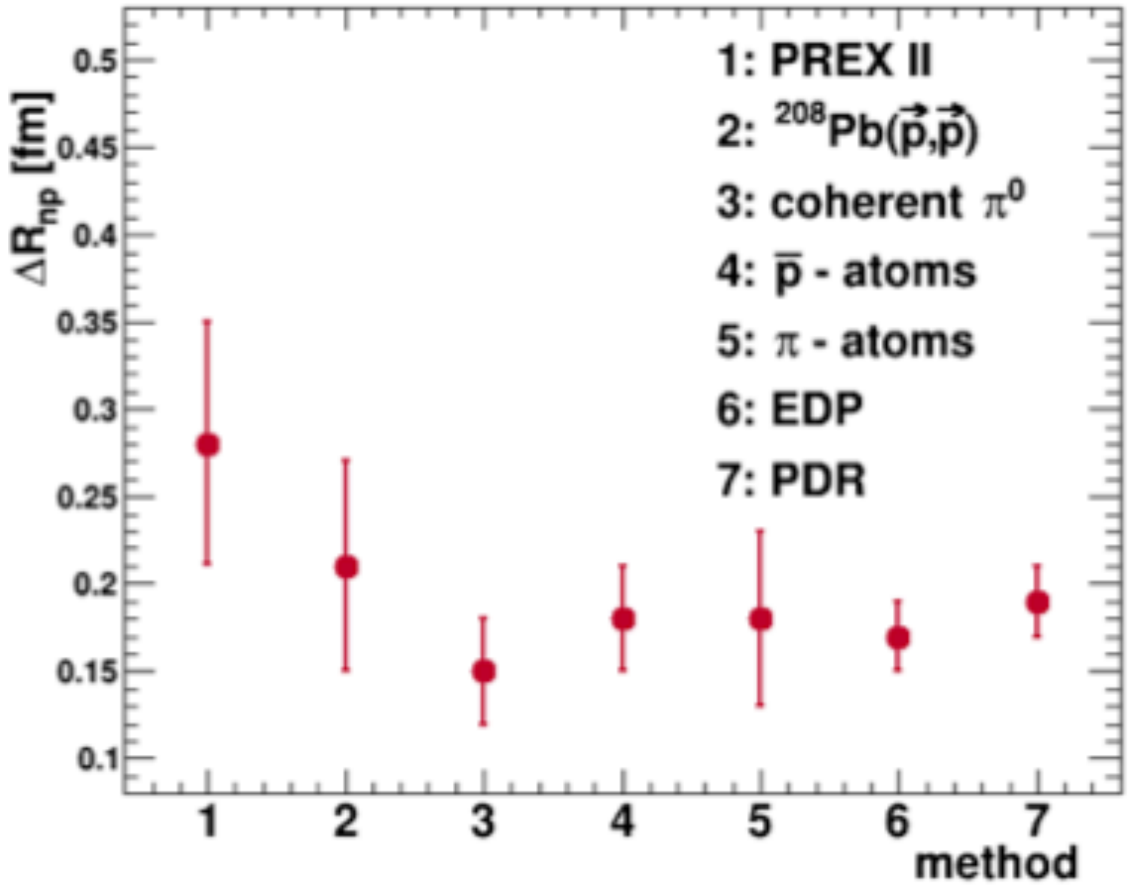
$$S_v + \frac{L}{3}(u-1) + \frac{K_{SYM}}{18}(u-1)^2 + \dots$$

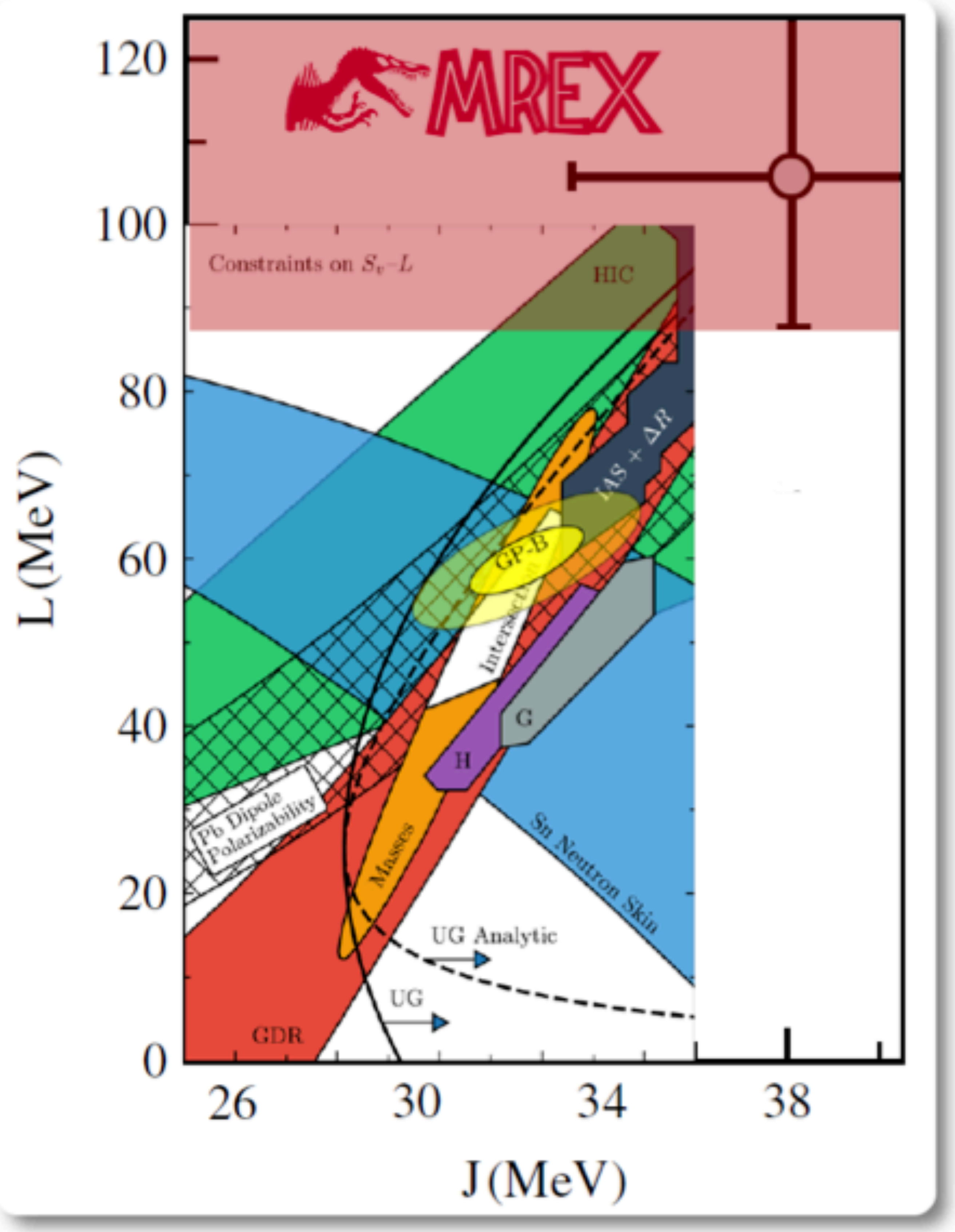
Neutron Skin $\Delta R = R_n - R_p$
is "**perpendicular**" to others

$$L = 30 - 90 \text{ MeV}$$
$$\Delta R_{^{208}Pb} = 0.11 - 0.25 \text{ fm}$$

Neutron skin experiments

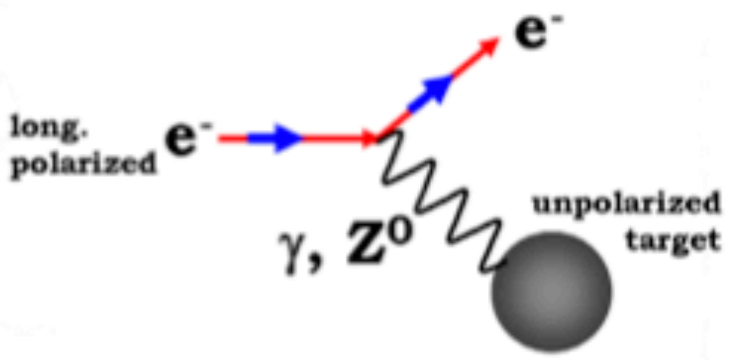
Credit: Michaela Thiel

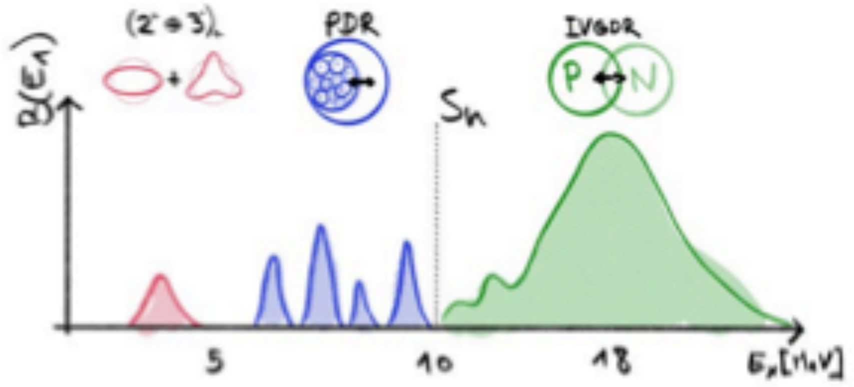


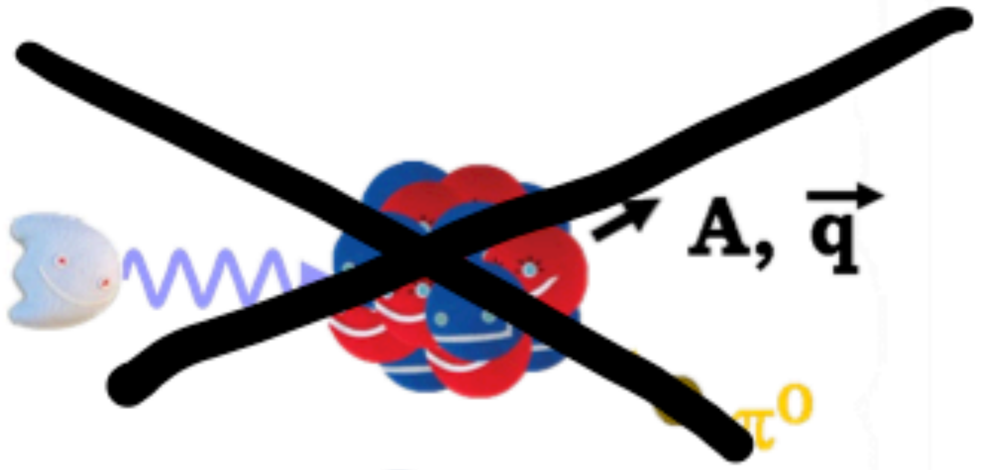


B. T. Reed et al., PRL 126 (2021) 172503

Experimental Challenges (in unit of frustration)







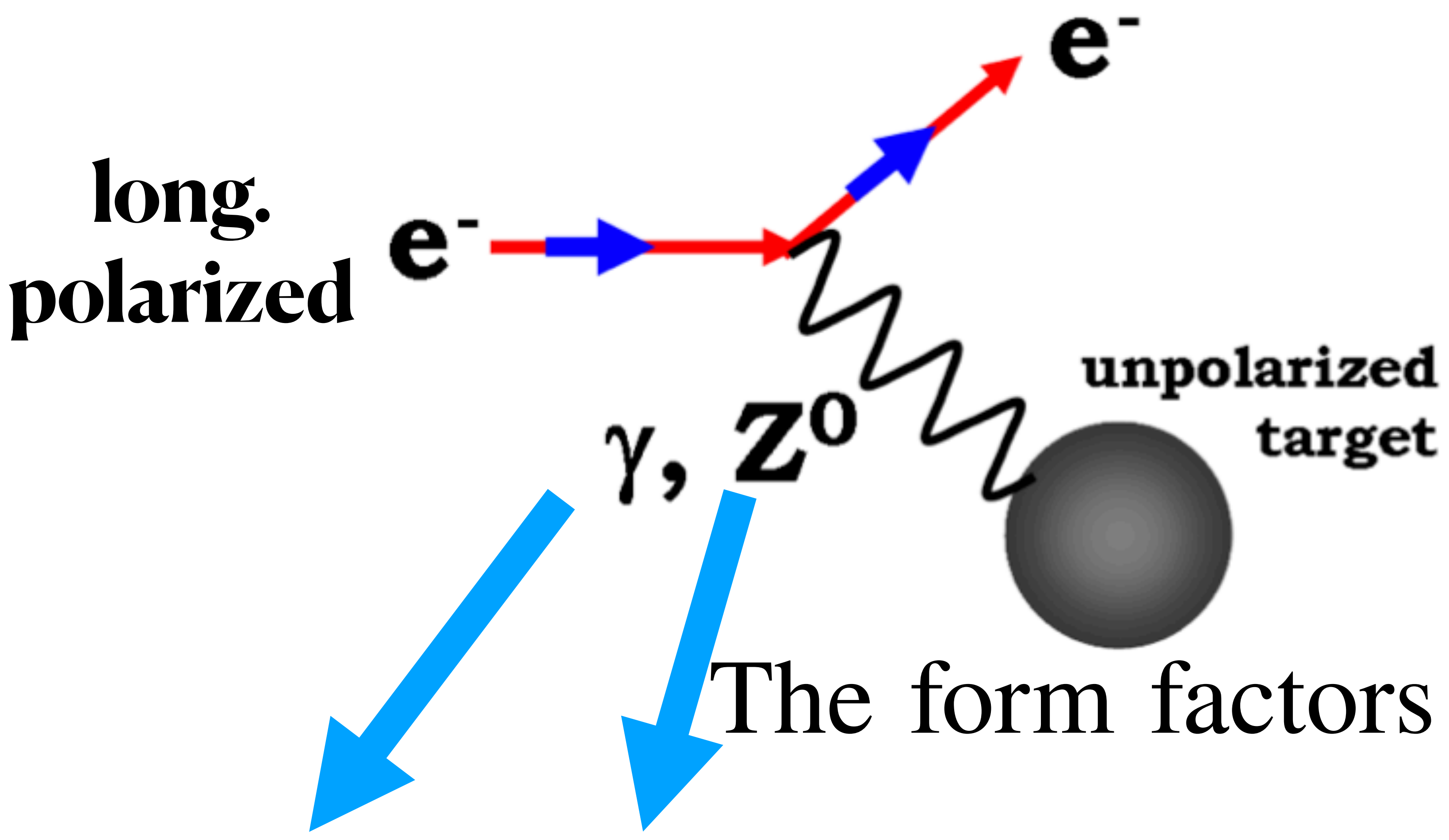
Theo. uncertainties (a.u)

Parity violating electron scattering

	CREX	PREX
(N,Z)	(28,20) Ca	(126,82) Pb
q (fm-1)	0.8733	0.3977
Fch, Rch(fm)	0.1581, 3.481	0.409, 5.503
Apv	2668±106(stat) ±40(syst)	550±16(stat) ±8(syst)
Fw	0.1304±0.0052(stat)±0.002(syst)	0.368±0.013(exp) ±0.001(theo)
Fch-Fw	0.0277±0.0052(stat)±0.002(syst)	0.041±0.013(exp) ±0.001(theo)
Rw	3.64±0.026(exp) ±0.023(theo)	5.8±0.075(tot)
Rw-Rch	0.159±0.026(exp) ±0.023(theo)	0.297±0.075(tot)
Rn-Rp	0.121±0.026(exp) ±0.024(theo)	0.283±0.071(tot)

CREX 2022 PREX I 2012 PREX II 2021

MREX: 208Pb at different momentum q (expected 2030)



The form factors

$$F_{ch} - F_W = \Delta F$$

Bohn approx.
“back-up slide”

The parity violating asymmetry :

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

The weak interaction violates parity :

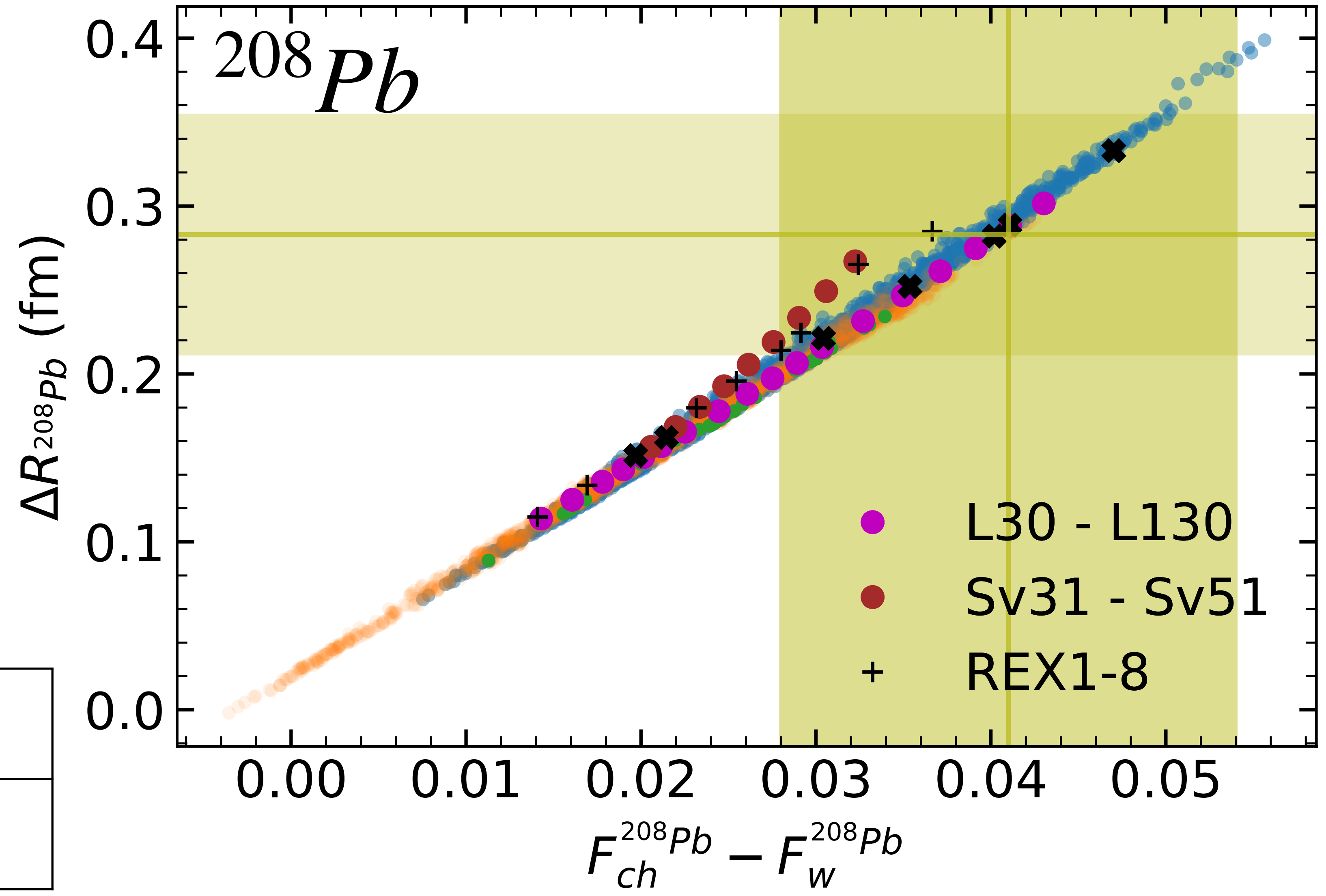
$$J^\mu_Z = -\frac{1}{2}\bar{\psi}_L\gamma^\mu\psi_L - \sin^2(\Theta)\bar{\psi}\gamma^\mu\psi$$

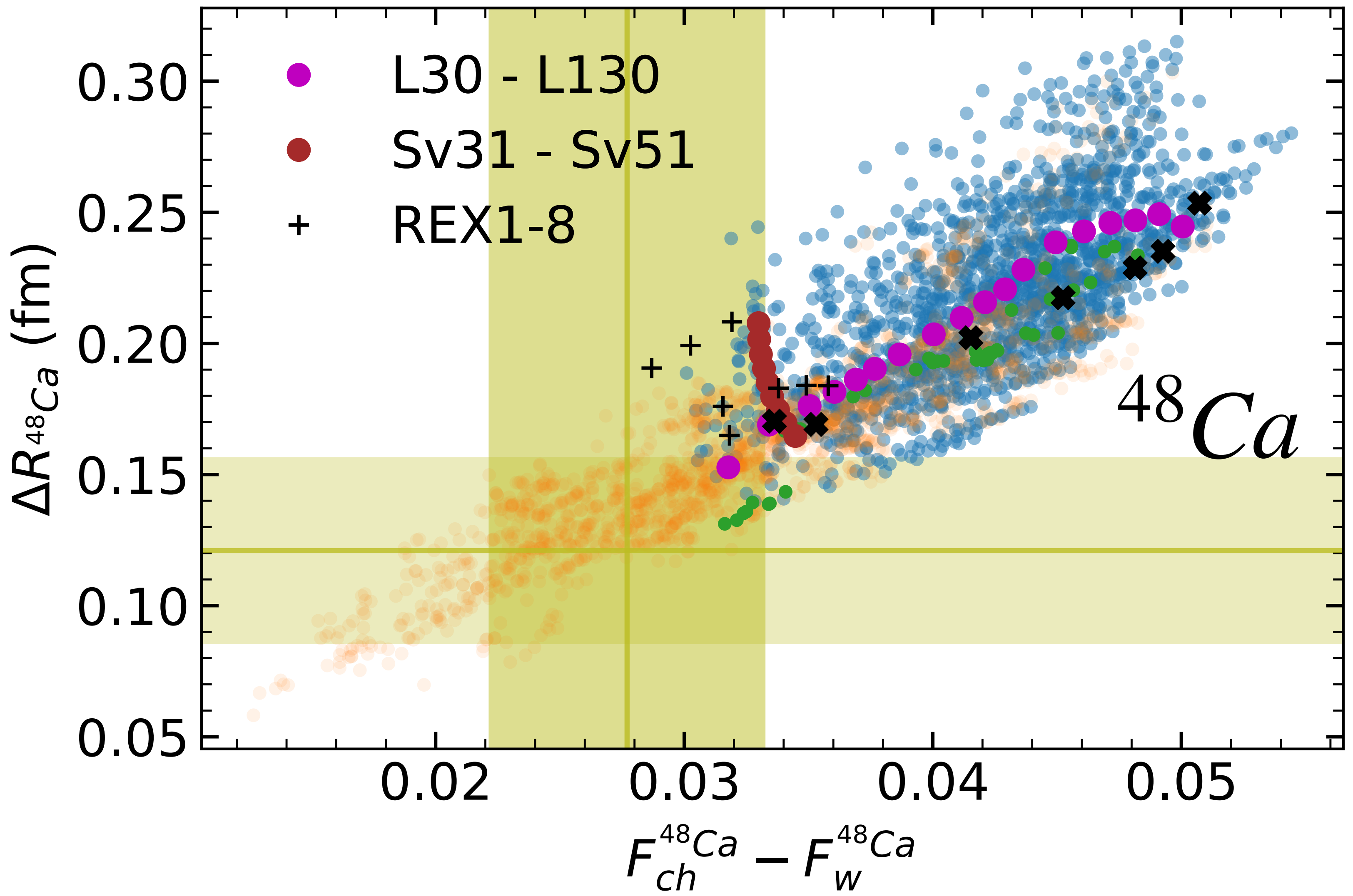
Can CREX Measure Neutron skin?

- RMS radius: $< r^2 > = \frac{1}{Q}\int r^2\rho(\mathbf{r})d^3r$
- Form Factor: $F(\mathbf{q}) = \frac{1}{Q}\int e^{i\mathbf{q}\cdot\mathbf{r}}\rho(\mathbf{r})d^3r = 1 - \frac{1}{6}q^2 < r^2 > + \ldots$
- $\lim_{q<<1/\sqrt{< r^2 >}} < r^2 > = \frac{6[1 - F(q)]}{q^2}$

208Pb	1	-0.8	0.19
48Ca	1	-1.56	0.73

- Form factor to radius mapping is much less accurate for CREX:
  1. Momentum transfer q is a bit too large for CREX.
  2. MFT uncertainty increase for lighter nuclei.
- Better use form factor to constrain nuclear model.





Pearson Correlation

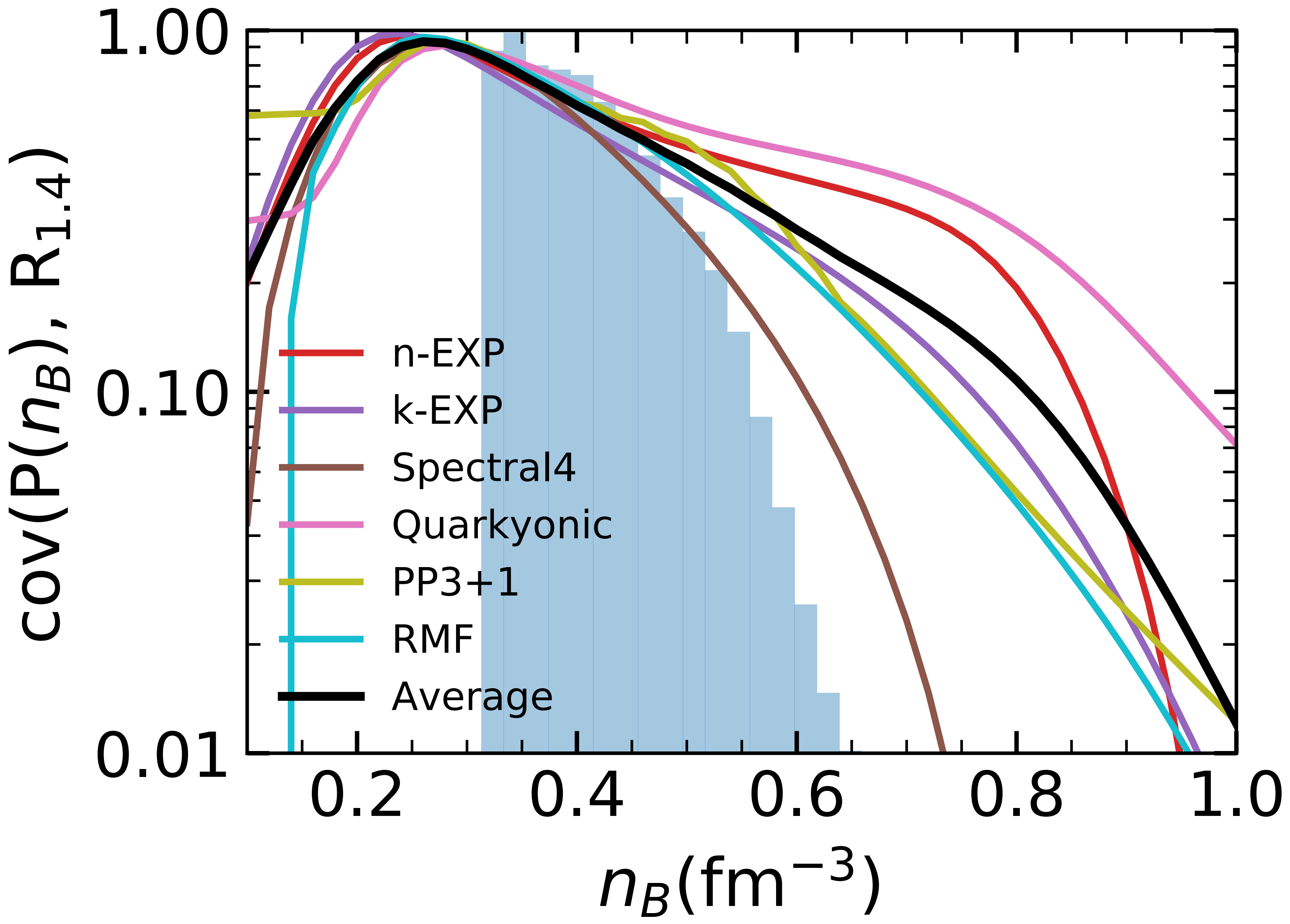
arXiv:2009.06441

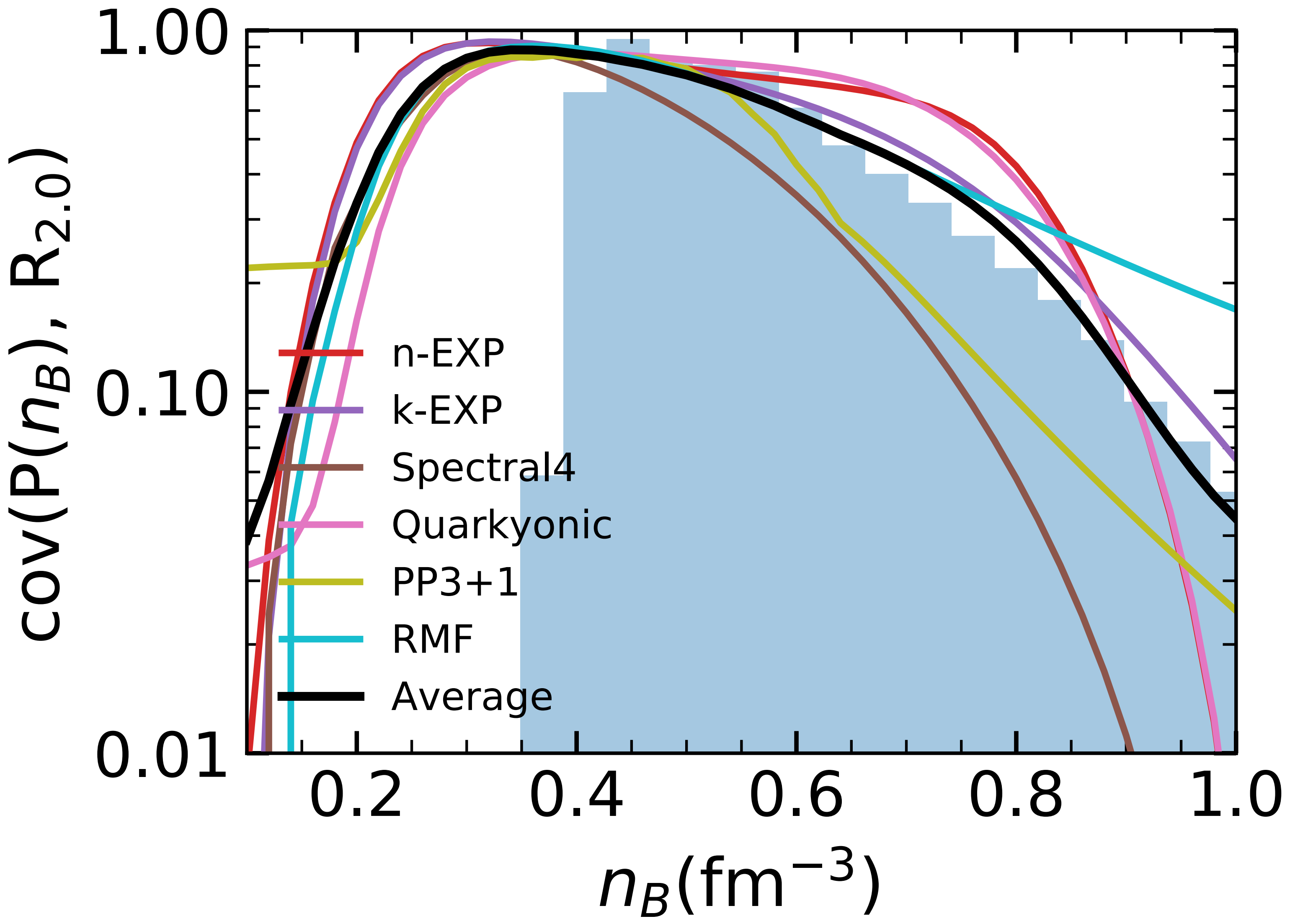
$$\mathrm{cov}[X, p(n_B), P] = \sum_i P_i \frac{(X_i - \bar{X})(p_i - \bar{p})}{\sigma_X \sigma_p}$$

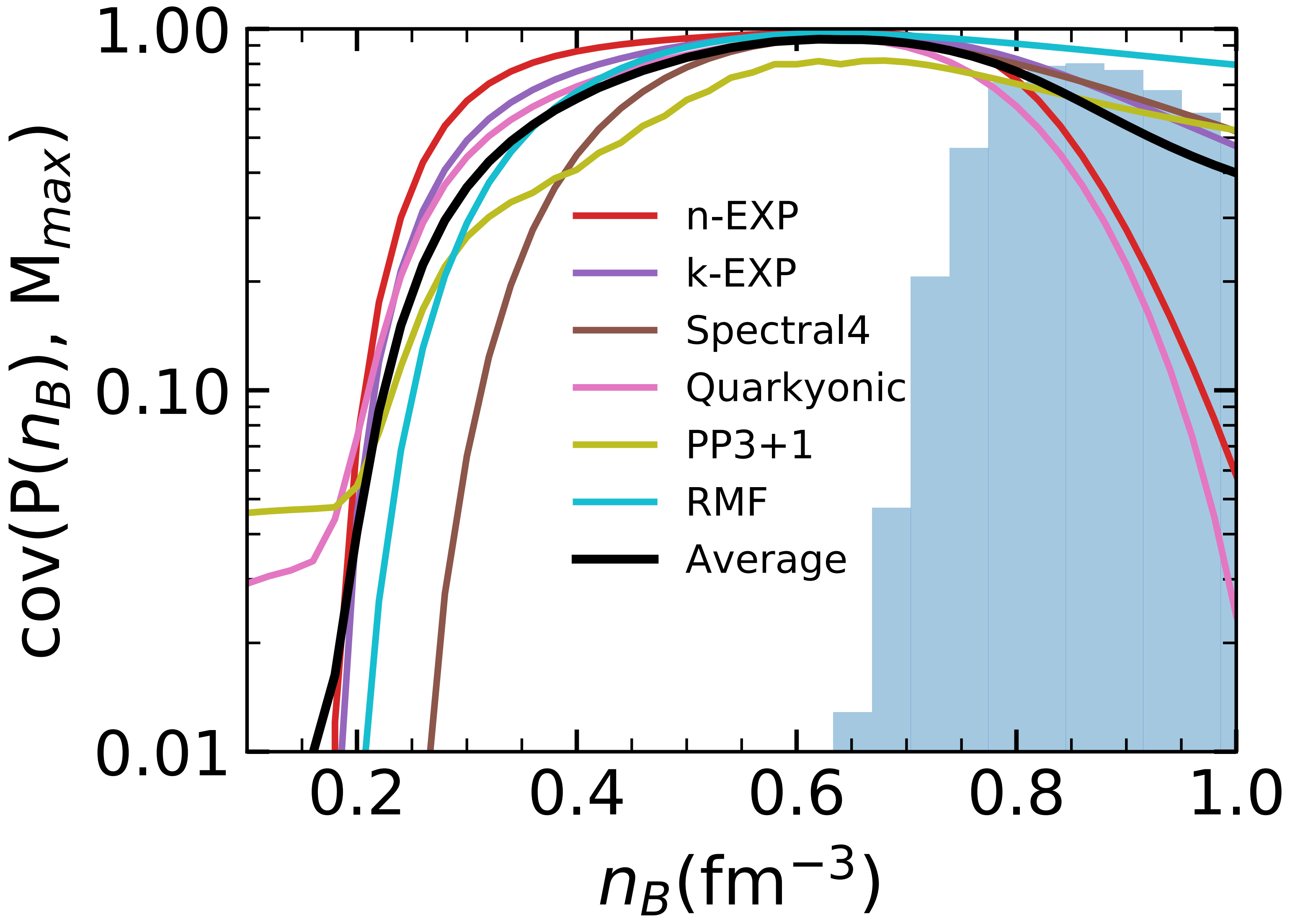
Observable

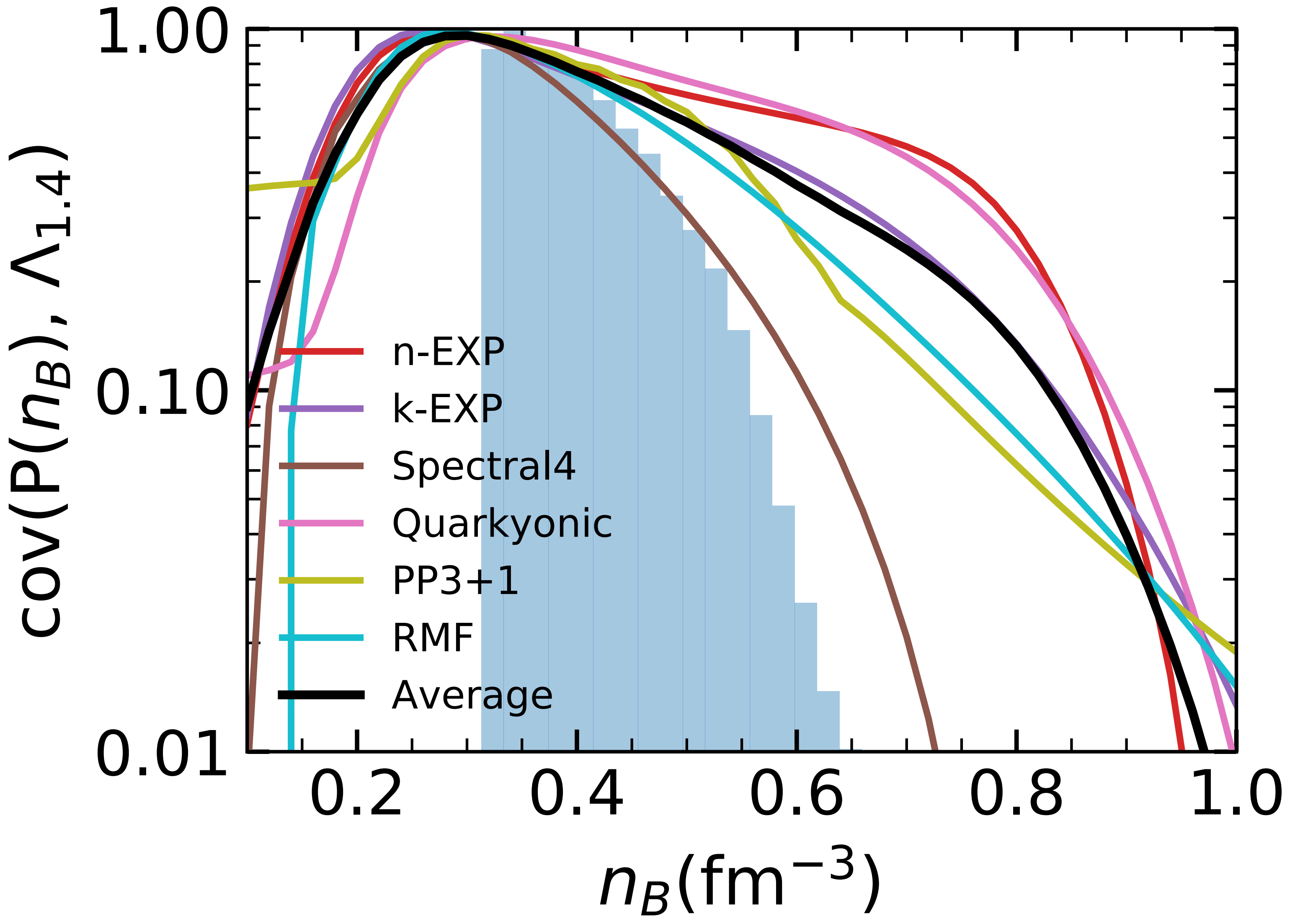
Likelihood

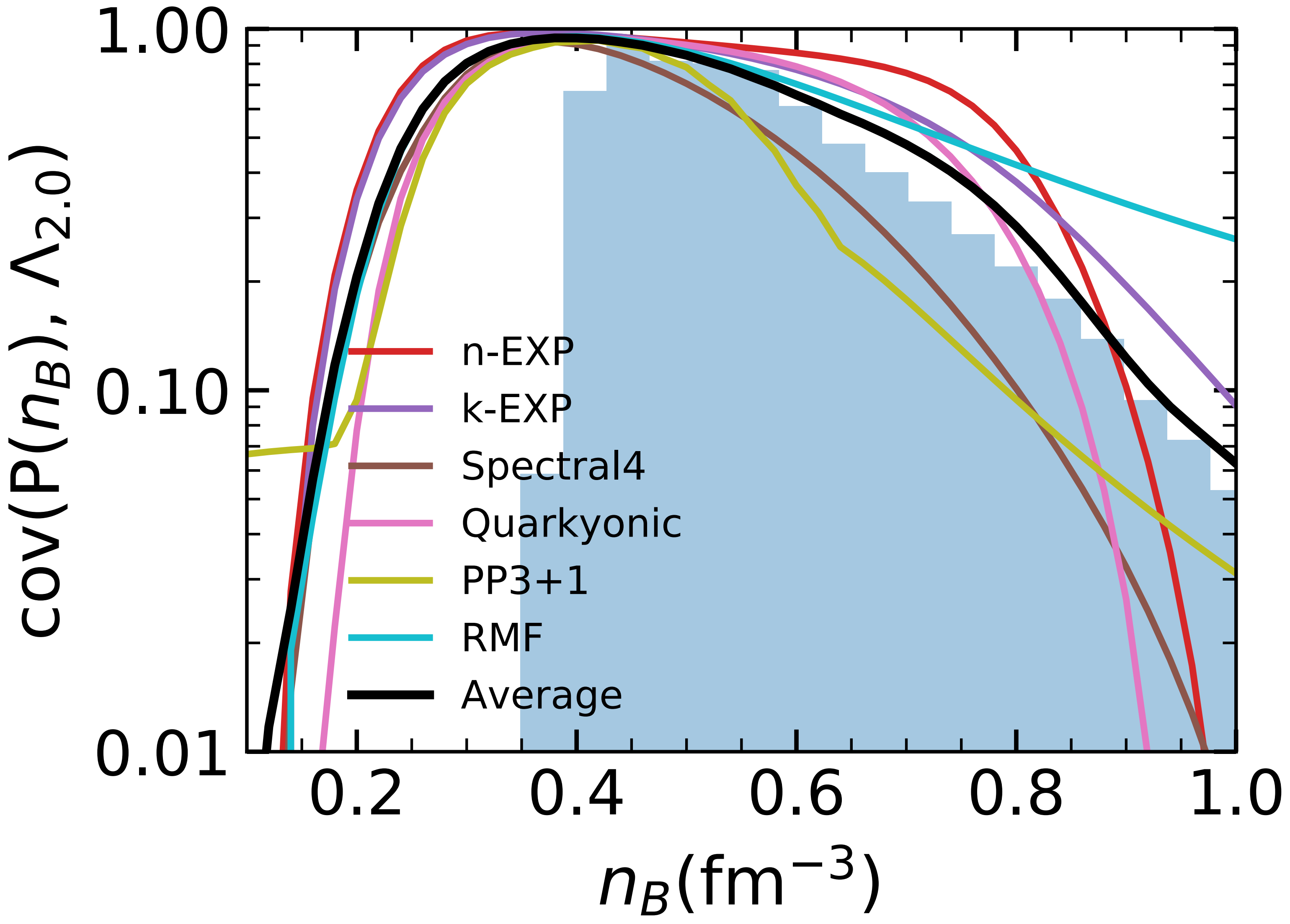
Pressure at given density











Pearson Correlation

$$\mathrm{cov}[\mathrm{X}, \mathrm{p}(\mathrm{n_B}), \mathrm{P}] = \sum_{\mathrm{i}} \mathrm{P_i} \frac{(\mathrm{X_i} - \bar{\mathrm{X}})(\mathrm{p_i} - \bar{\mathrm{p}})}{\sigma_{\mathrm{X}}\sigma_{\mathrm{p}}}$$

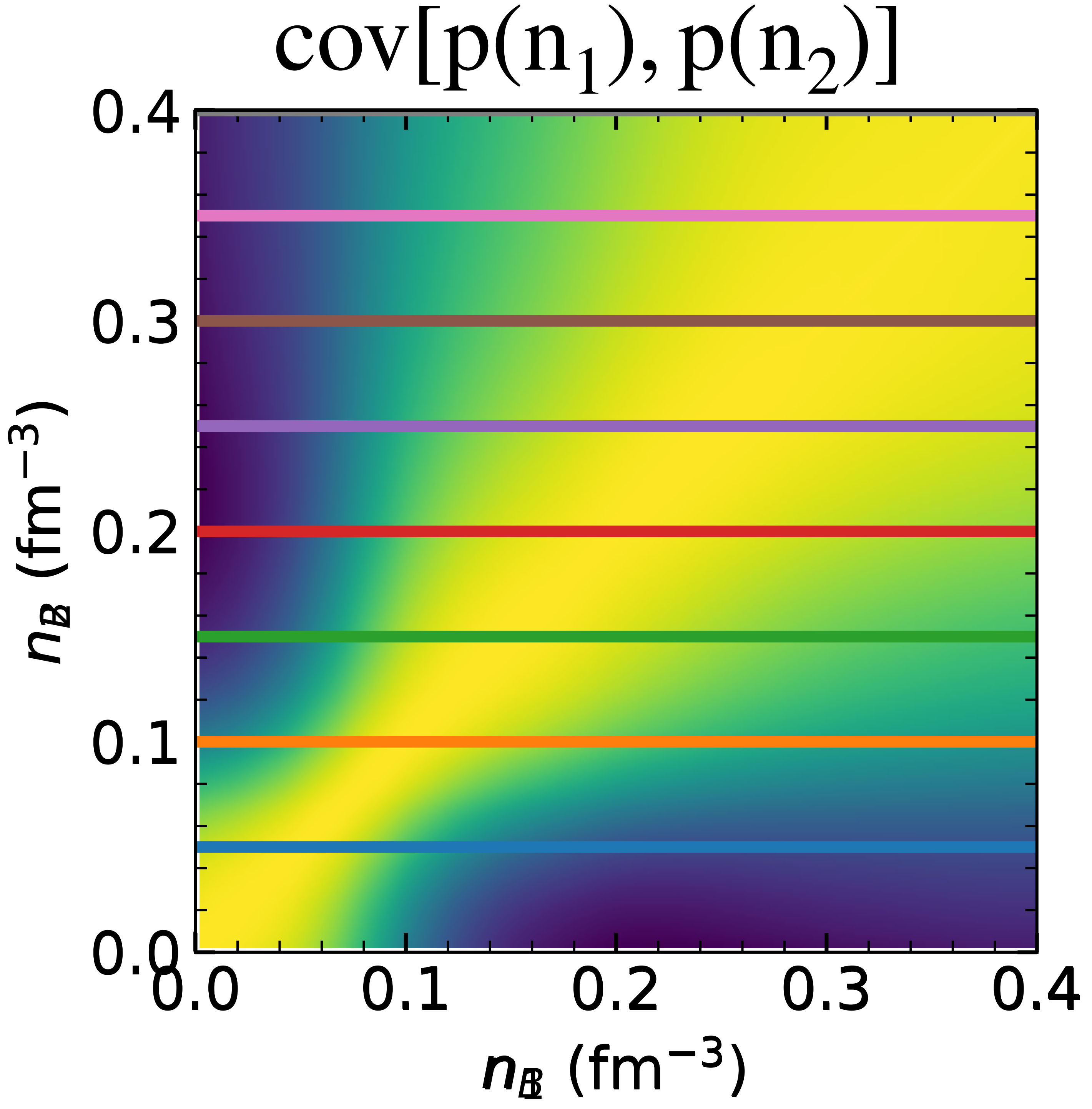
Observable

Pressure at given density

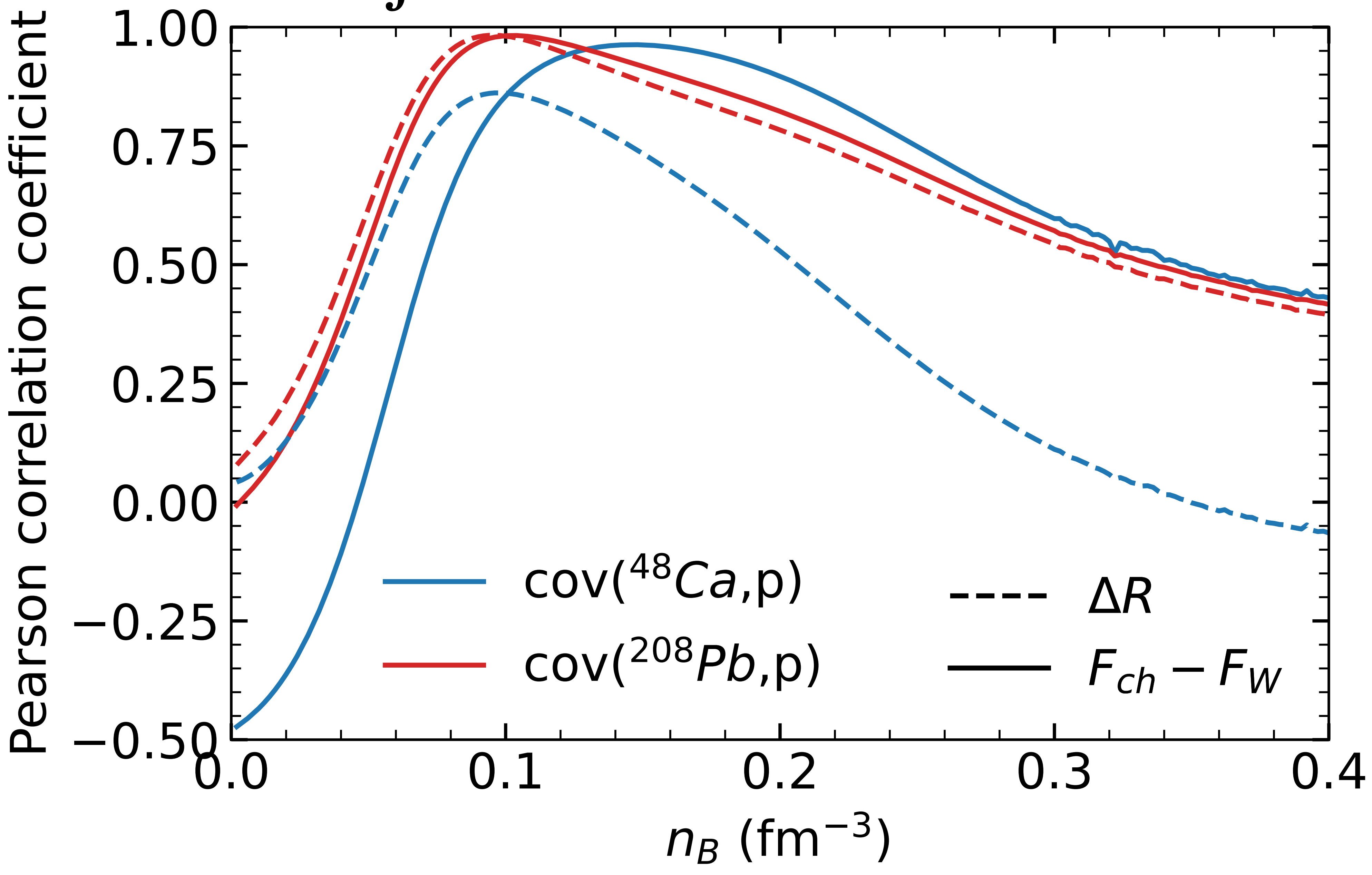
Likelihood

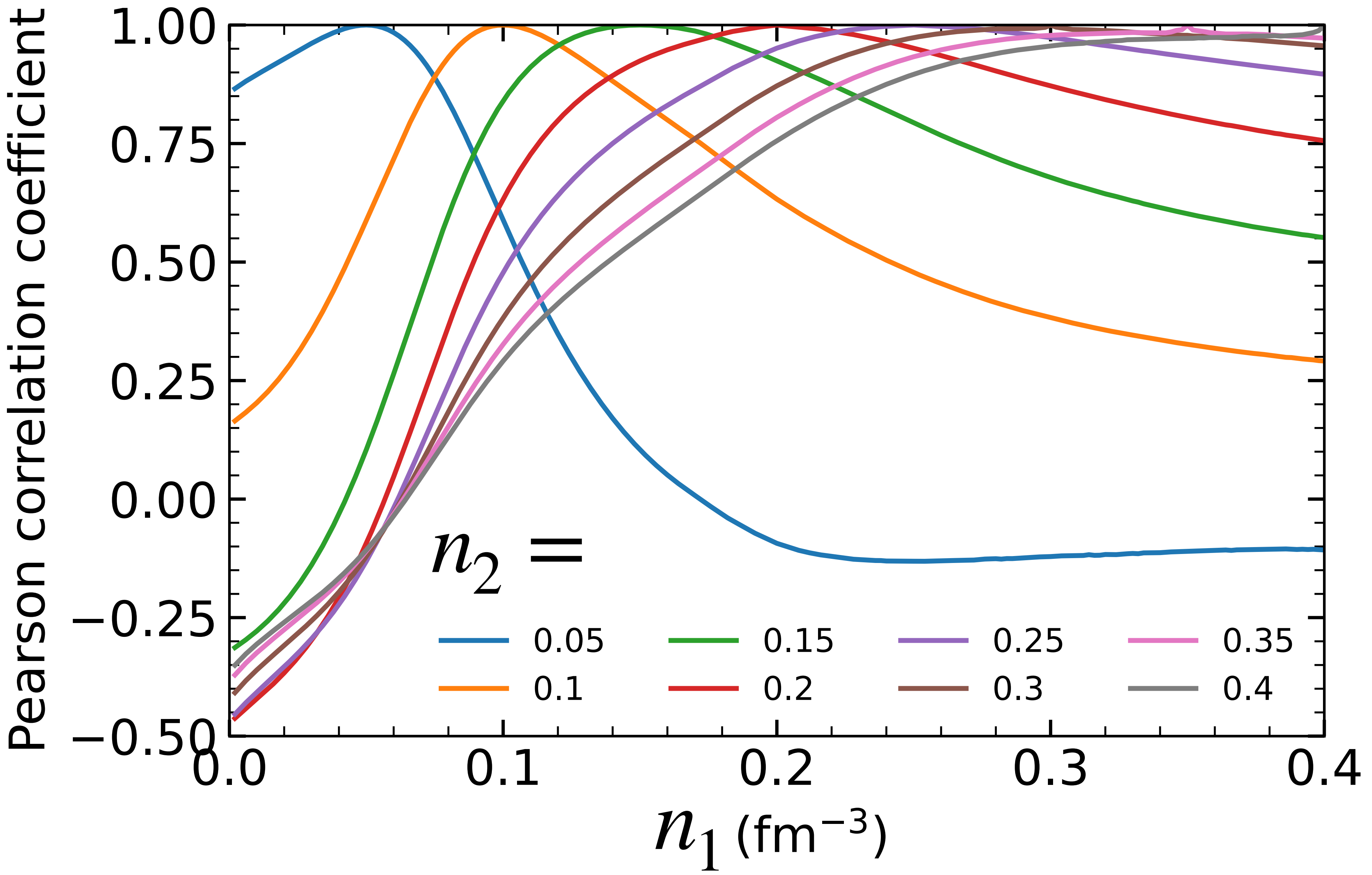
$$S(X, n_1) = \frac{r_X}{\sqrt{2\pi}\sigma_X}\exp[-\frac{(n_1 - \mu_X)^2}{2\sigma_X^2}]$$

[fm]	Fch-Fw	Skin	Fch-Fw	Skin
μ_X	0.147	0.0932	0.121	0.113
σ_X	6E-04	7E-04	0.0522	0.0616
r_X	0.00214	0.00197	0.00250	0.00268



$$\mathrm{cov}[\mathrm{X}, \mathrm{p}(\mathrm{n_2})] = \int \mathrm{cov}[\mathrm{p}(\mathrm{n_1}), \mathrm{p}(\mathrm{n_2})] S(\mathrm{X}, \mathrm{n_1}) d\mathrm{n_1}$$





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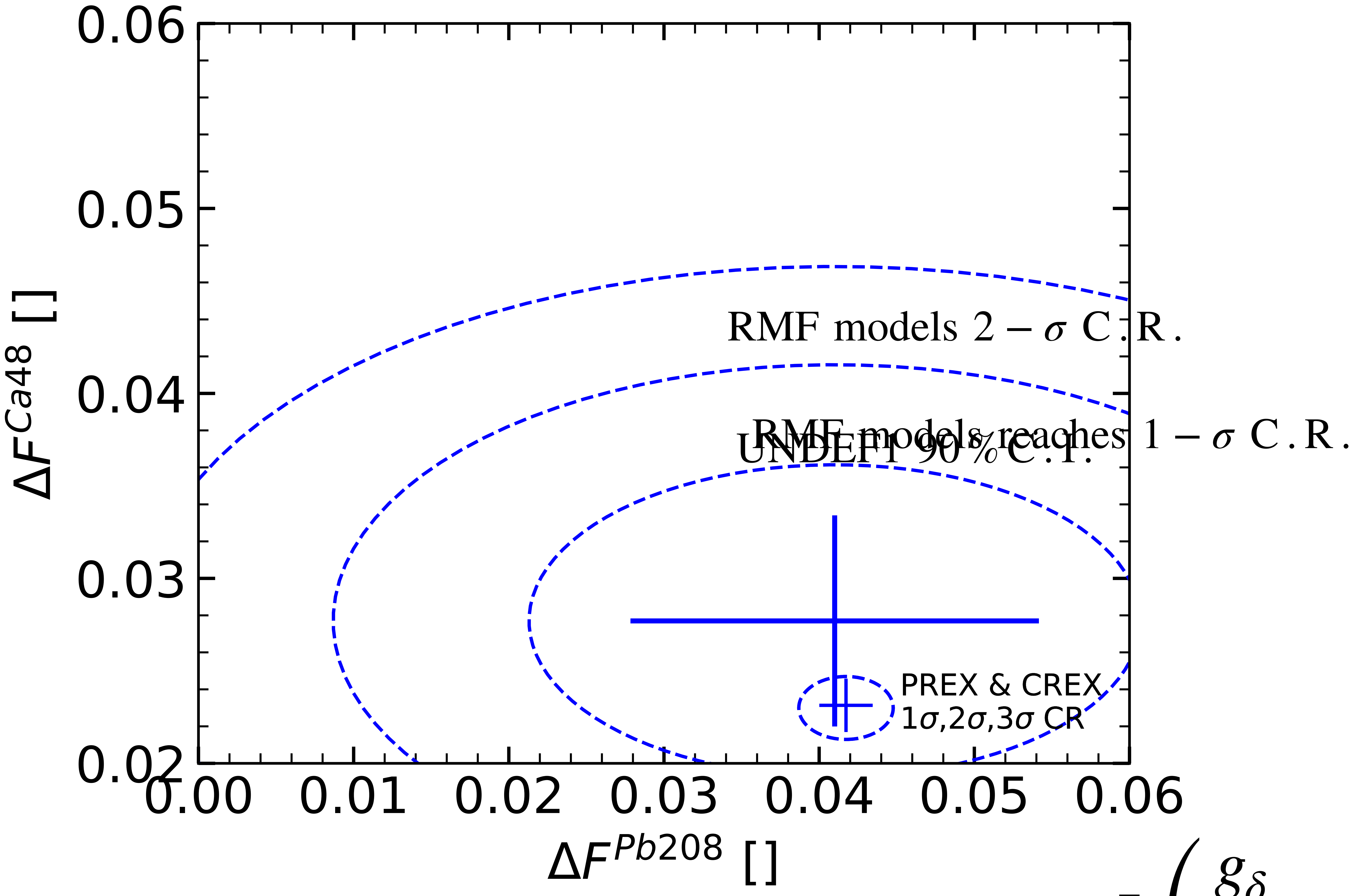
Pre PREX-CREX era



$$\bar{\psi}\left(\frac{g_\delta}{2}\tau\cdot\delta\right)\psi$$

Post PREX-CREX era

- What bulk nuclear properties can we learn from the experiment?
- Why are Skyrme models more compatible than RMF models?
- How may the mean-field model improve in the future?

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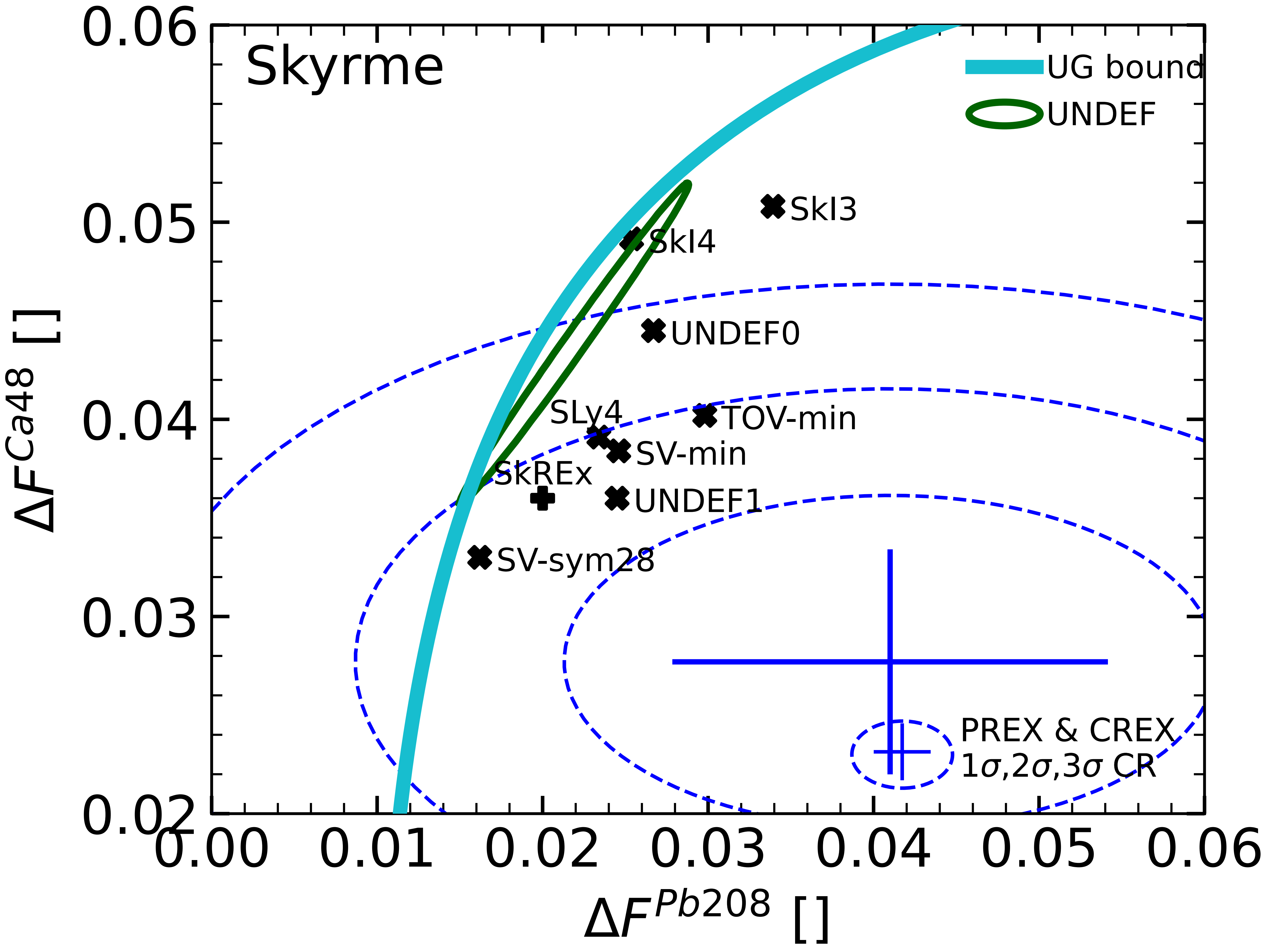
Skyrme and RMF samples



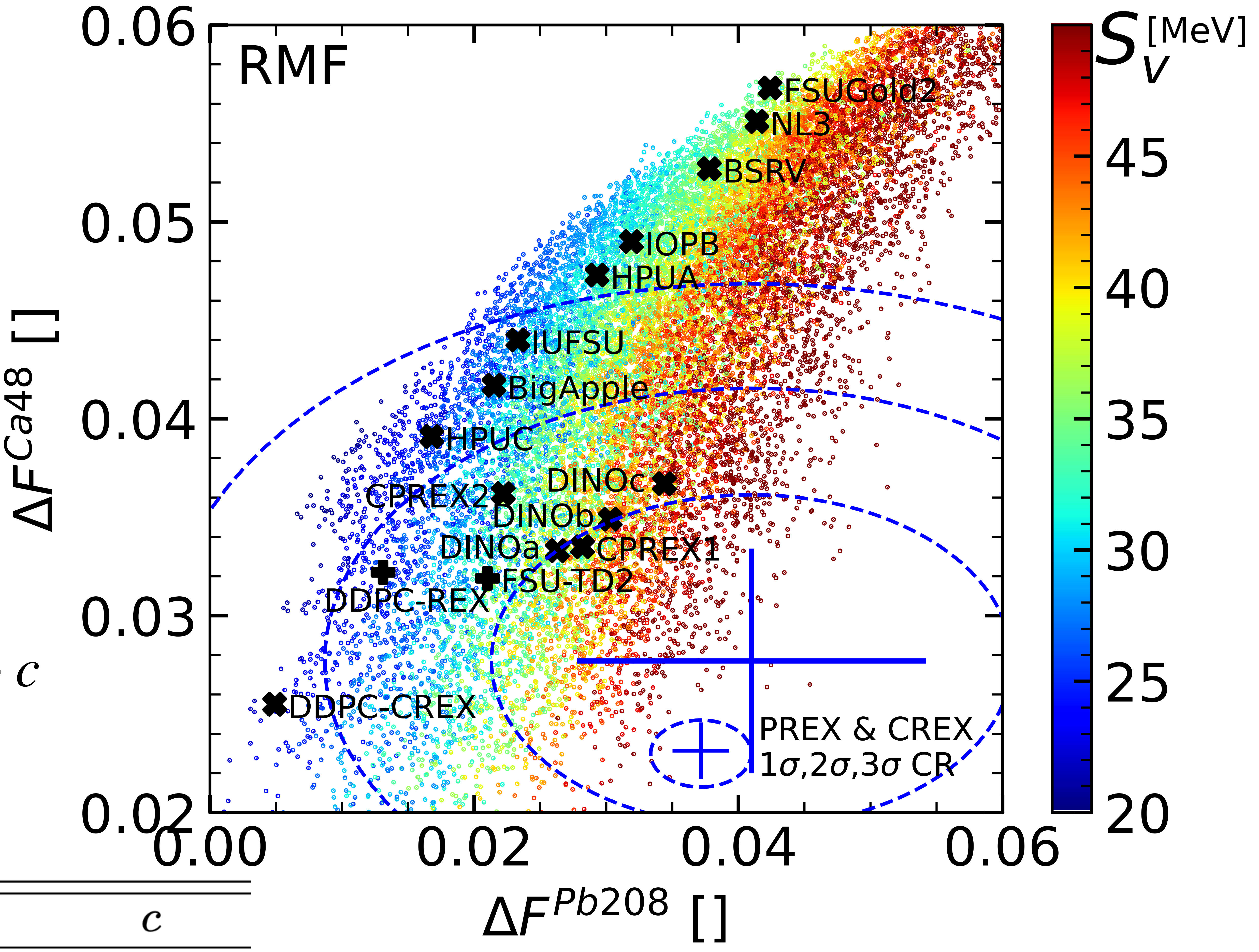
Skyrme models

Symmetry energy S_V

$$S(n) = S_V + \frac{L}{3}(\frac{n}{n_S} - 1) + \cdots$$

- ΔF^{Ca48} and ΔF^{Pb208} are positively correlated for nuclear models with fixed S_V
- The correlation is linear: $S_v = a\Delta F^{Ca48} + b\Delta F^{Pb208} + c$
- Fitting parameter for RMF (Skyrme) models:

	a	b	c
RMF	-575.2 ± 5.1	916.3 ± 4.6	32.2 ± 3.7
Skyrme	-503.2 ± 7.8	945.2 ± 5.5	31.9 ± 2.9

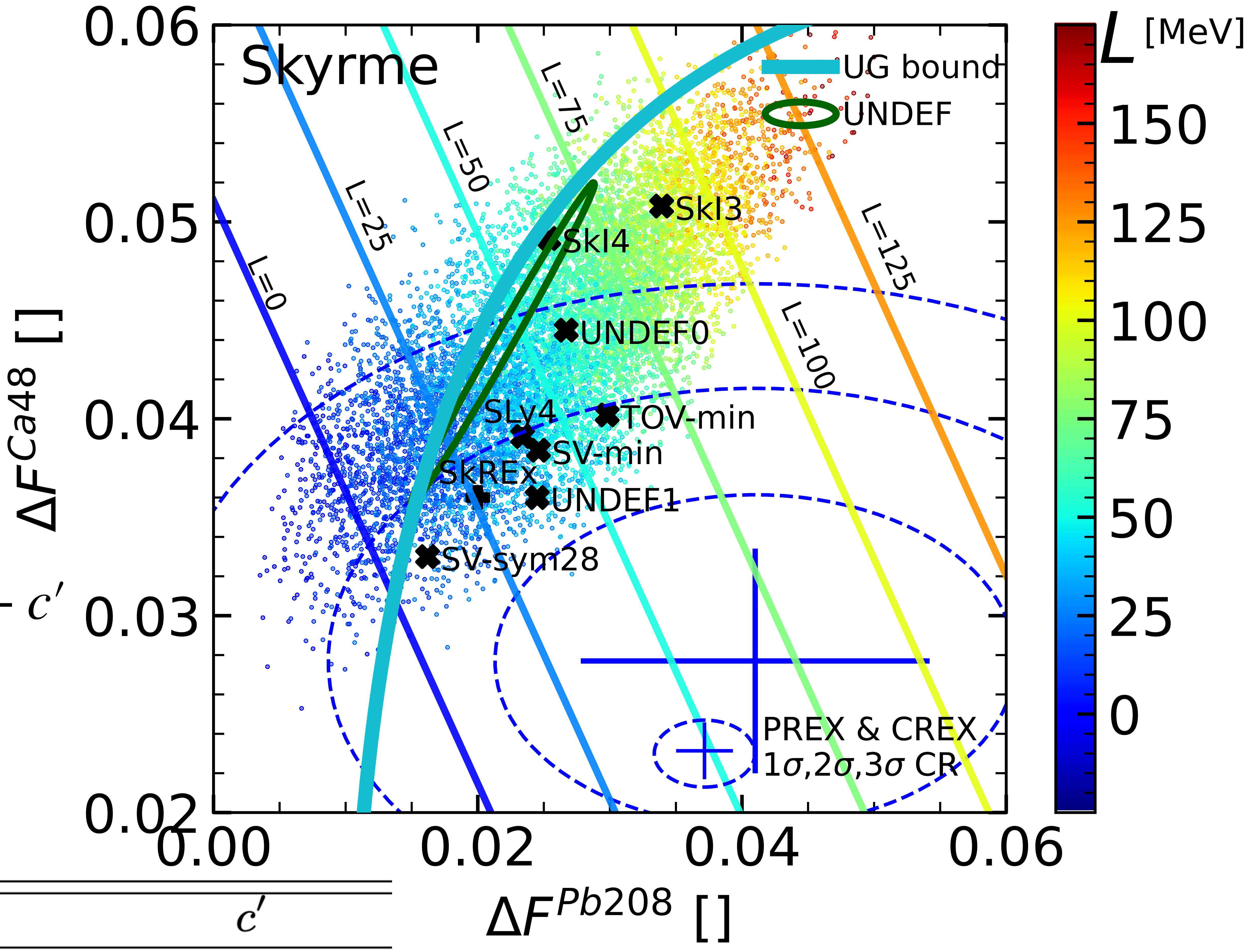


Form factor difference for 48Ca and 208Pb

Symmetry energy slope L

$$S(n) = S_V + \frac{L}{3}\left(\frac{n}{n_S} - 1\right) + \cdots$$

- A similar correlation for L has an opposite slope!
- The correlation is linear: $L = a'\Delta F^{Ca48} + b'\Delta F^{Pb208} + c'$
- Fitting parameter for RMF (Skyrme) models:



	a'	b'	c'
RMF	2938.7 ± 43.5	2420.6 ± 33.9	-149.8 ± 25.6
Skyrme	1791.2 ± 27.2	2652.0 ± 19.0	-91.5 ± 10.1

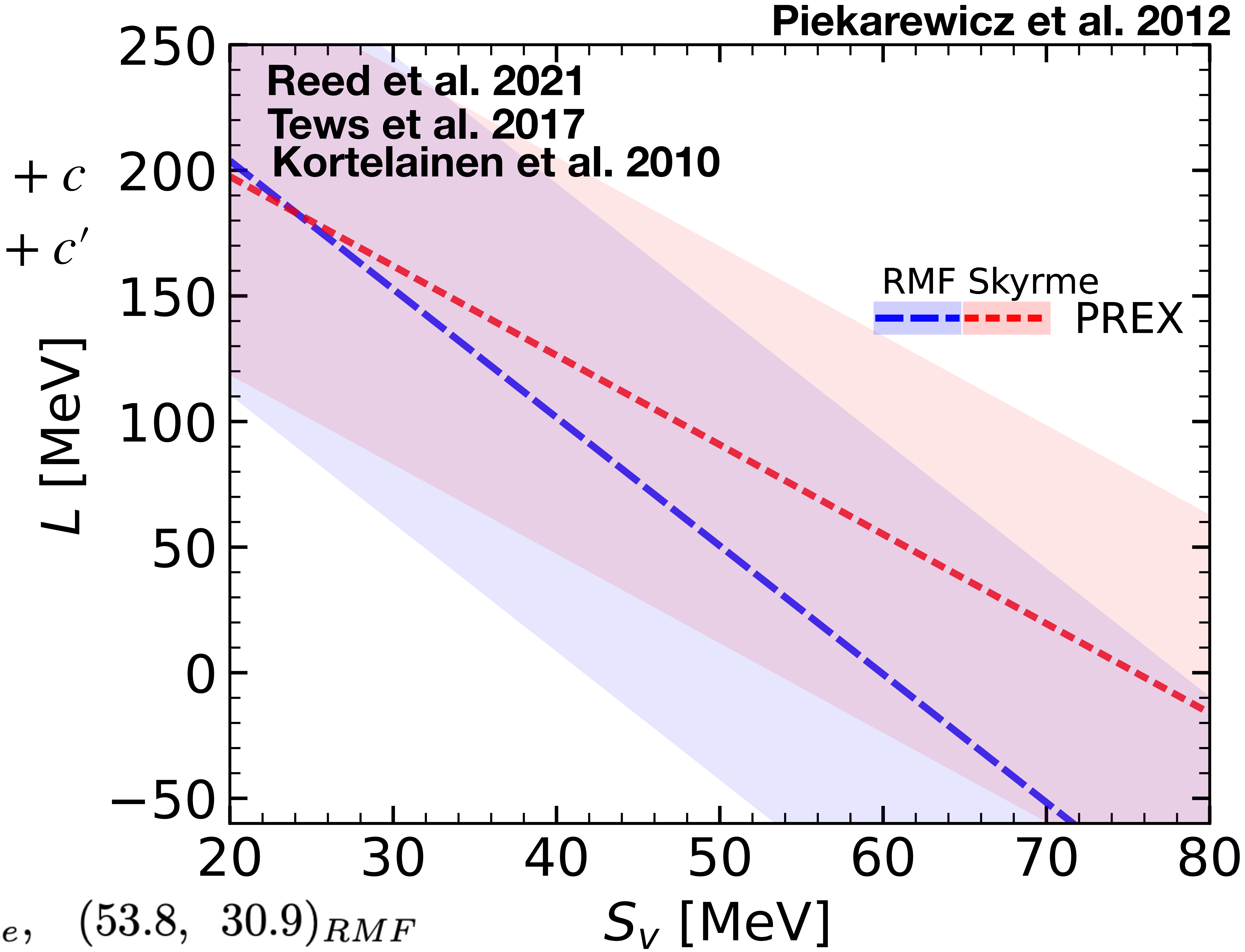
Linear correlation of form factor difference

Constraints on (S_V , L) from (ΔF^{Ca48},ΔF^{Pb208})

- S_V and L can be fixed by ΔF^{Ca48} and(or) ΔF^{Pb208}:
  $S_V = a\Delta F^{Ca48} + b\Delta F^{Pb208} + c$
  $L = a'\Delta F^{Ca48} + b'\Delta F^{Pb208} + c'$
- PREX:
  ΔF^{Pb208}=0.041
  ±0.013(exp)±0.001(theo)
- CREX:
  ΔF^{Ca48}=0.0277
  ±0.0052(stat)±0.002(syst)
- PREX+CREX:

$$(\bar{S}_V, \bar{L}) = (56.7,\ 66.8)_{Skyrme}, \quad (53.8,\ 30.9)_{RMF}$$

$$\sqrt{\text{cov}} = \begin{pmatrix} 19.6 & 31.2 \\ 31.2 & 56.5 \end{pmatrix}_{Skyrme}, \quad \begin{pmatrix} 19.5 & 31.0 \\ 31.0 & 66.3 \end{pmatrix}_{RMF}$$



1-sigma band of Sv and L from PREX

Bayesian Prior & Likelihood

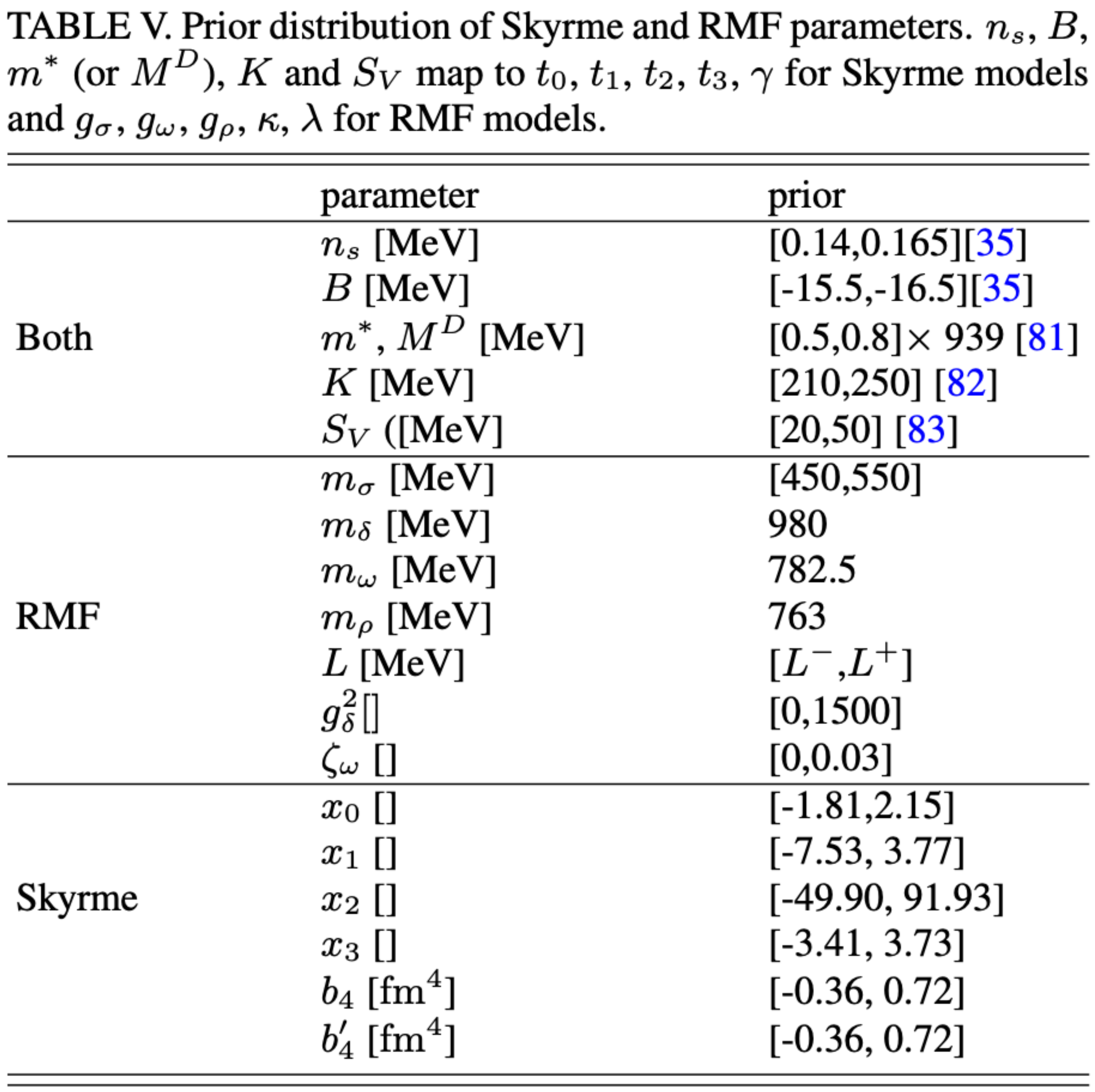


TABLE V. Prior distribution of Skyrme and RMF parameters. n_s, B, m^* (or M^D), K and S_V map to t_0, t_1, t_2, t_3, γ for Skyrme models and g_σ, g_ω, g_ρ, κ, λ for RMF models.

	parameter	prior
Both	n_s [MeV]	[0.14,0.165][35]
	B [MeV]	[-15.5,-16.5][35]
	m^*, M^D [MeV]	[0.5,0.8]× 939 [81]
	K [MeV]	[210,250] [82]
	S_V ([MeV]	[20,50] [83]
RMF	m_σ [MeV]	[450,550]
	m_δ [MeV]	980
	m_ω [MeV]	782.5
	m_ρ [MeV]	763
	L [MeV]	[L^-,L^+]
	g_δ^2[]	[0,1500]
	ζ_ω []	[0,0.03]
Skyrme	x_0 []	[-1.81,2.15]
	x_1 []	[-7.53, 3.77]
	x_2 []	[-49.90, 91.93]
	x_3 []	[-3.41, 3.73]
	b_4 [fm^4]	[-0.36, 0.72]
	b_4' [fm^4]	[-0.36, 0.72]

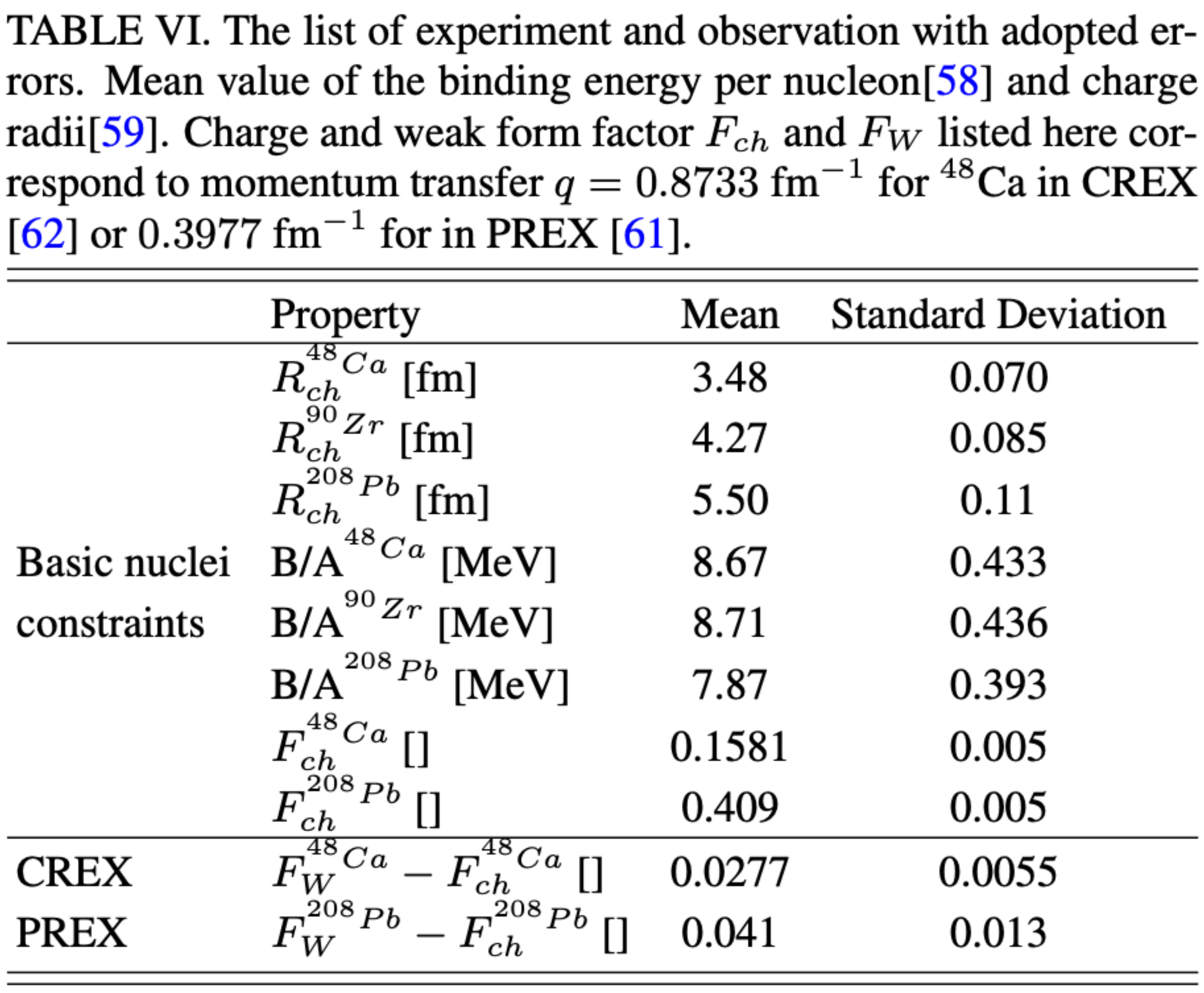
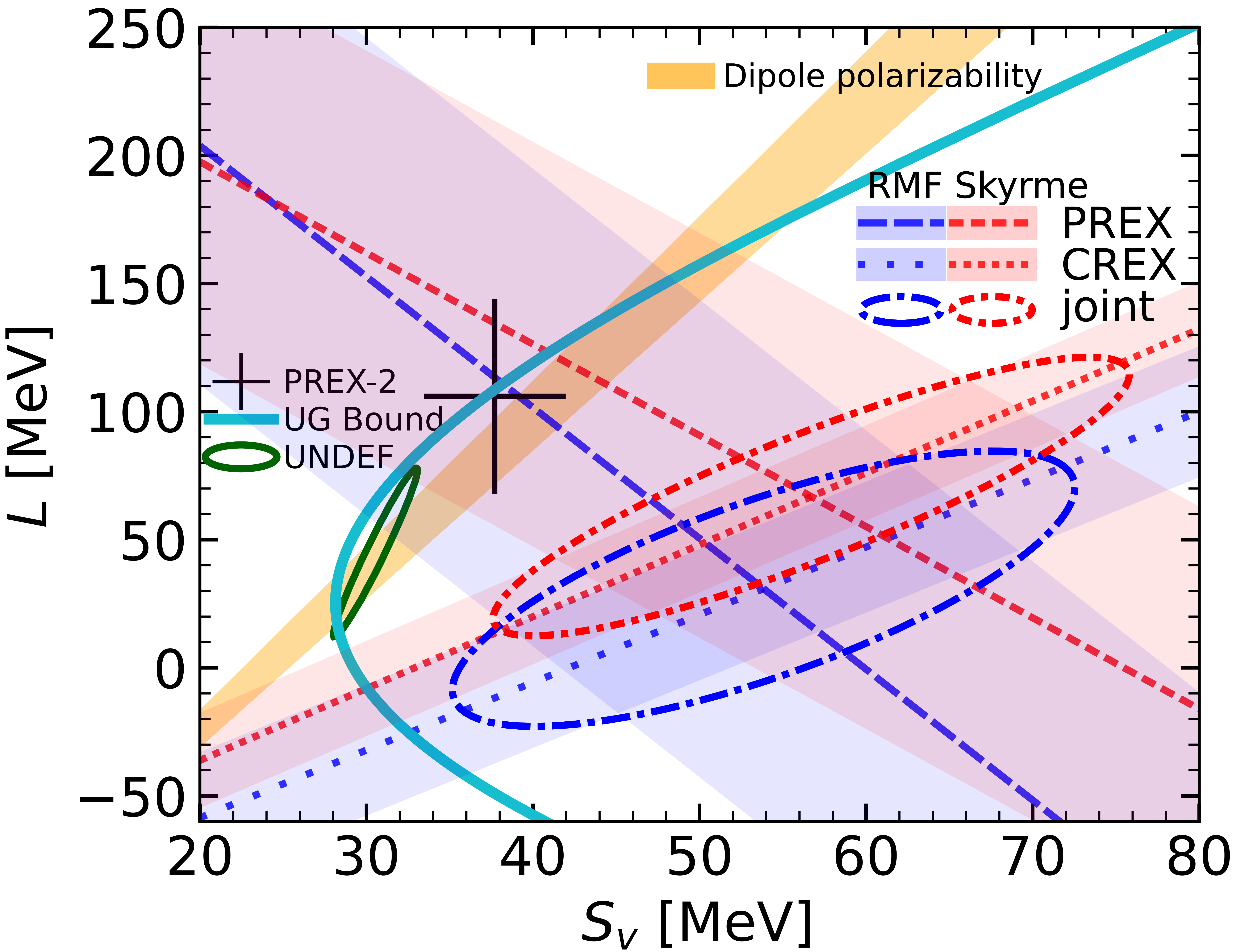


TABLE VI. The list of experiment and observation with adopted errors. Mean value of the binding energy per nucleon[58] and charge radii[59]. Charge and weak form factor F_{ch} and F_W listed here correspond to momentum transfer $q = 0.8733$ fm^{-1} for ^{48}Ca in CREX [62] or 0.3977 fm^{-1} for in PREX [61].

	Property	Mean	Standard Deviation
Basic nuclei constraints	R_{ch}^{48Ca} [fm]	3.48	0.070
	R_{ch}^{90Zr} [fm]	4.27	0.085
	R_{ch}^{208Pb} [fm]	5.50	0.11
	B/A^{48Ca} [MeV]	8.67	0.433
	B/A^{90Zr} [MeV]	8.71	0.436
	B/A^{208Pb} [MeV]	7.87	0.393
	F_{ch}^{48Ca} []	0.1581	0.005
	F_{ch}^{208Pb} []	0.409	0.005
CREX	$F_W^{48Ca} - F_{ch}^{48Ca}$ []	0.0277	0.0055
PREX	$F_W^{208Pb} - F_{ch}^{208Pb}$ []	0.041	0.013

Bayesian posterior



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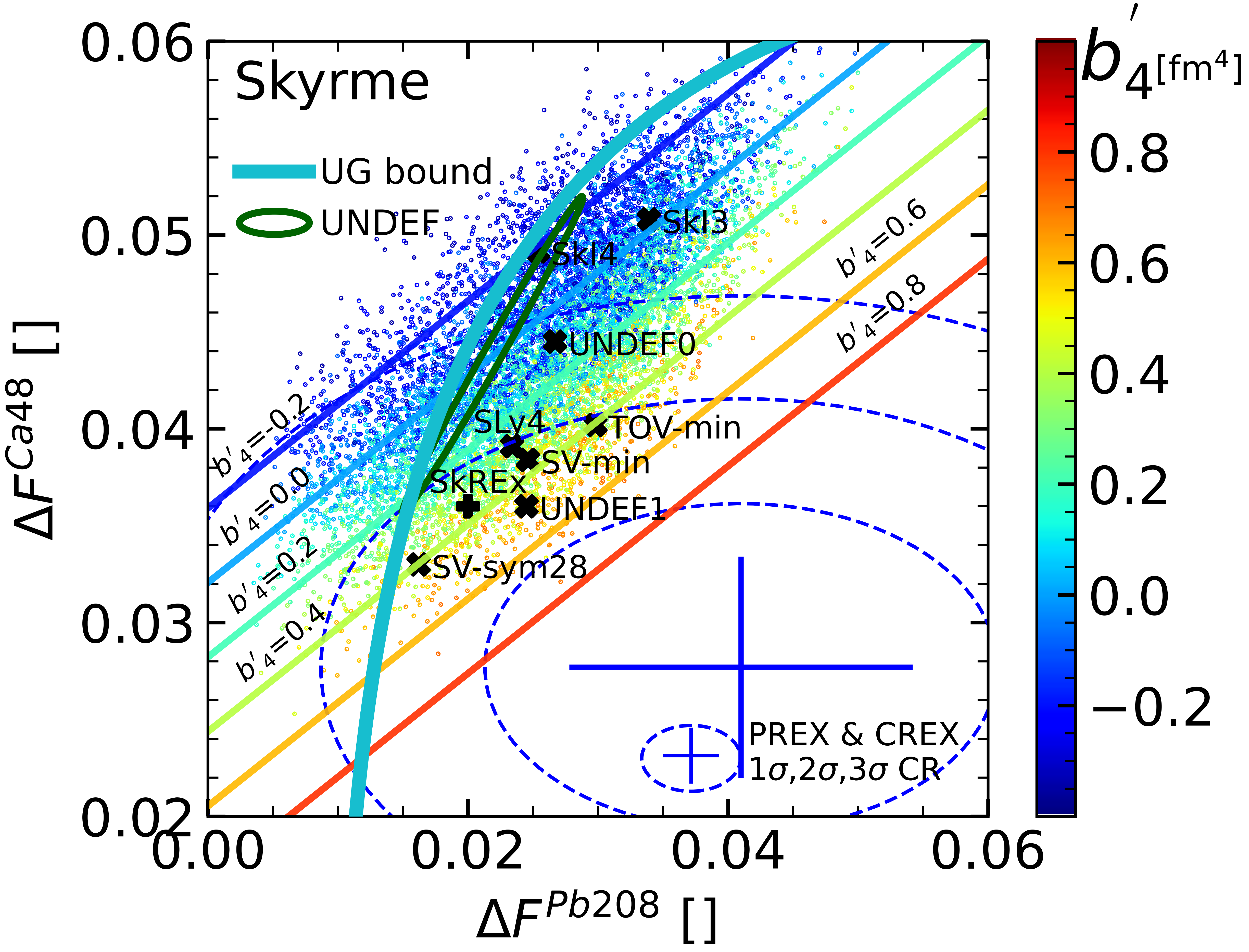
Isovector spin-orbit force

- Isovector spin-orbit force is independent of S_V and L in Skyrme (not in RMF) model.

- Spin-orbit force in Skyrme model:
  $H_{\mathrm{SO}} = b_4 \mathrm{J} \cdot \nabla n$
  $+ b_4'(\mathrm{J}_n \cdot \nabla n_n + \mathrm{J}_p \cdot \nabla n_p)$
  The freedom b_4' improves the Skyrme model performance.

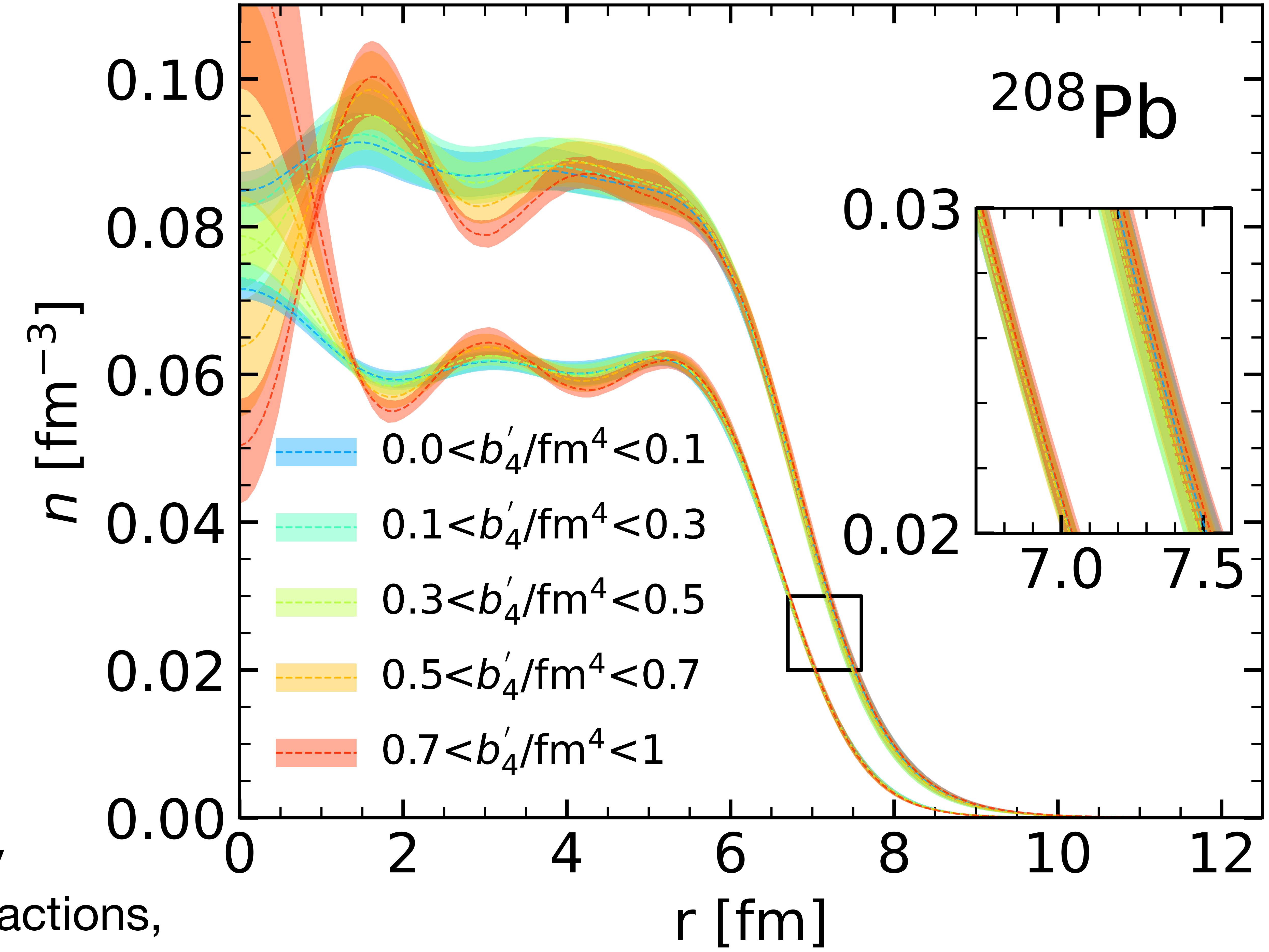
- $v \ll c$ limit of RMF model:
  $$b_4' \approx \frac{1}{8m^2}\left(\frac{g_\delta^2}{m_\delta^2} + \frac{g_\rho^2}{m_\rho^2}\right)$$
  large δ-meson coupling improves the RMF models.



Linear correlation of form factor difference

Impact of b_4' on neutron skin ΔR_{np}

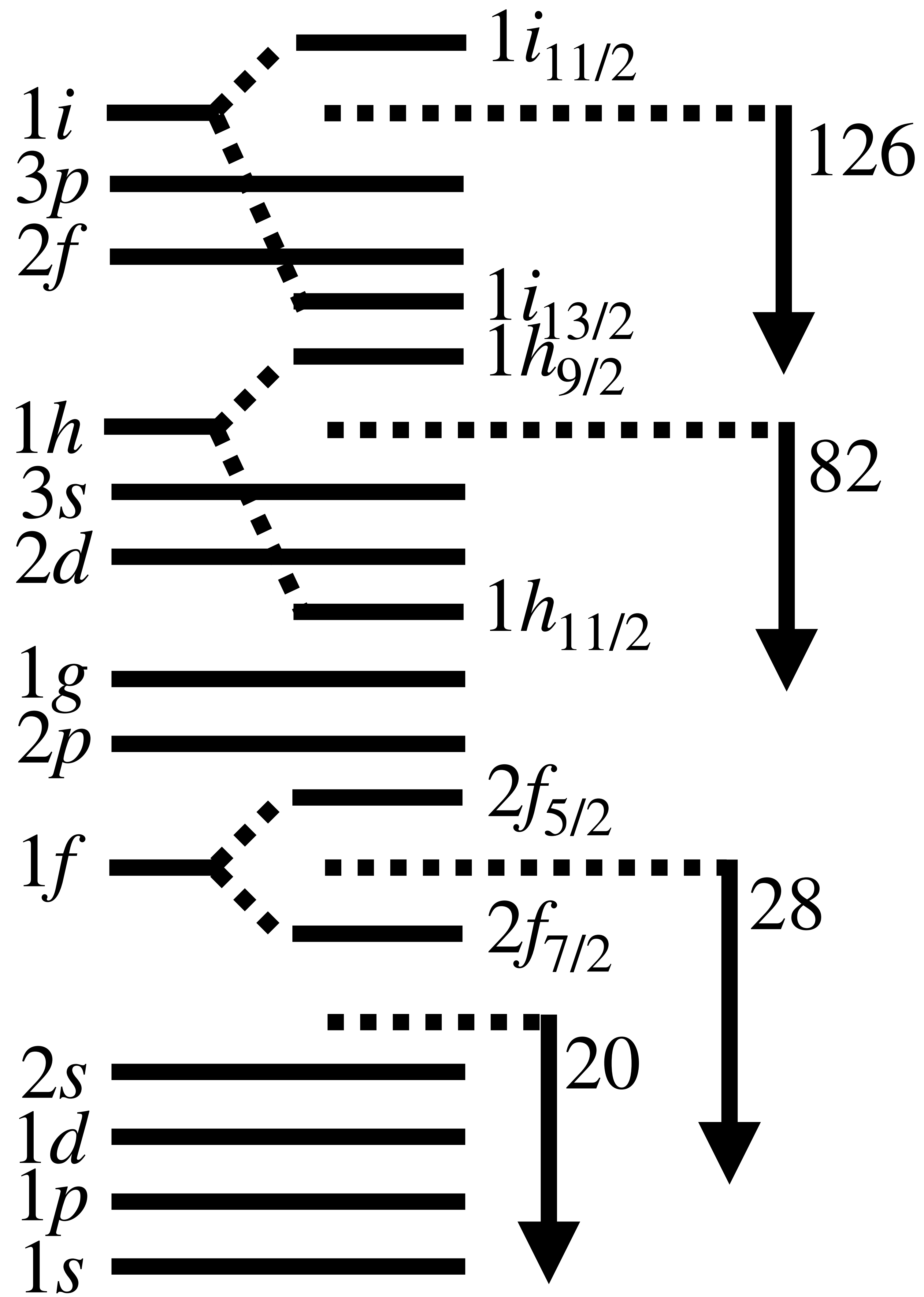
- ΔR_{np} of ^{208}Pb increases with b_4'
- ΔR_{np} of ^{48}Ca decreases with b_4'
- Large b_4' reduces the tension between PREX and CREX.
- 90% lower bound of b_4':
  $b_4' \gtrsim 0.74\ \mathrm{fm}^4$ (Skyrme)
  $b_4' \gtrsim 0.54\ \mathrm{fm}^4$ (RMF)
- The large density fluctuation inside nuclei may be reduced by introducing addition tensor interactions, see M. Salinas and J. Piekarewicz 2024 (arXiv:2312.13474)

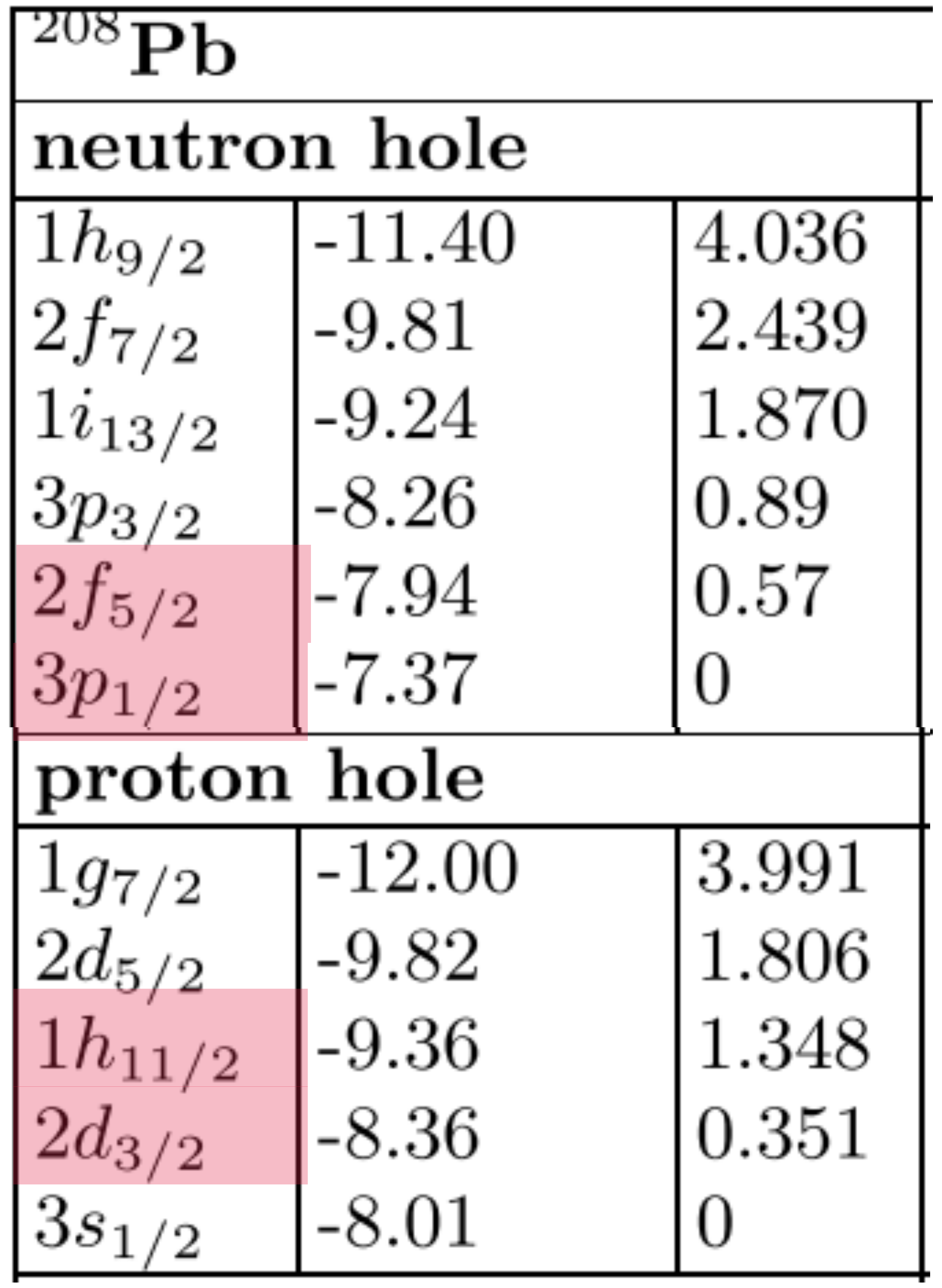


Radial density profile of proton and neutron for 208Pb

Impact of b_4' on neutron skin ΔR_{np}

- Larger b_4' =
  Larger(smaller) splitting for n(p).

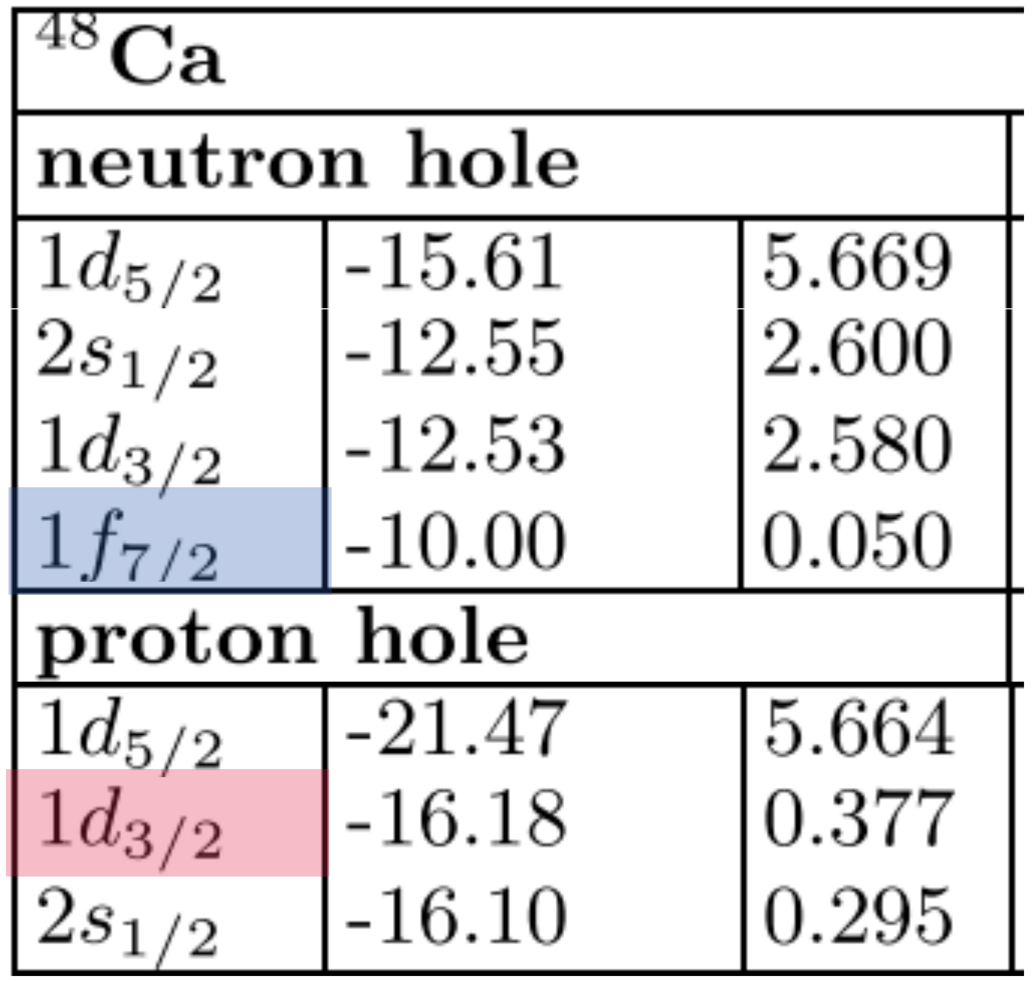




^{208}**Pb**		
neutron hole		
$1h_{9/2}$	-11.40	4.036
$2f_{7/2}$	-9.81	2.439
$1i_{13/2}$	-9.24	1.870
$3p_{3/2}$	-8.26	0.89
$2f_{5/2}$	-7.94	0.57
$3p_{1/2}$	-7.37	0
proton hole		
$1g_{7/2}$	-12.00	3.991
$2d_{5/2}$	-9.82	1.806
$1h_{11/2}$	-9.36	1.348
$2d_{3/2}$	-8.36	0.351
$3s_{1/2}$	-8.01	0

Less bound
Less bound

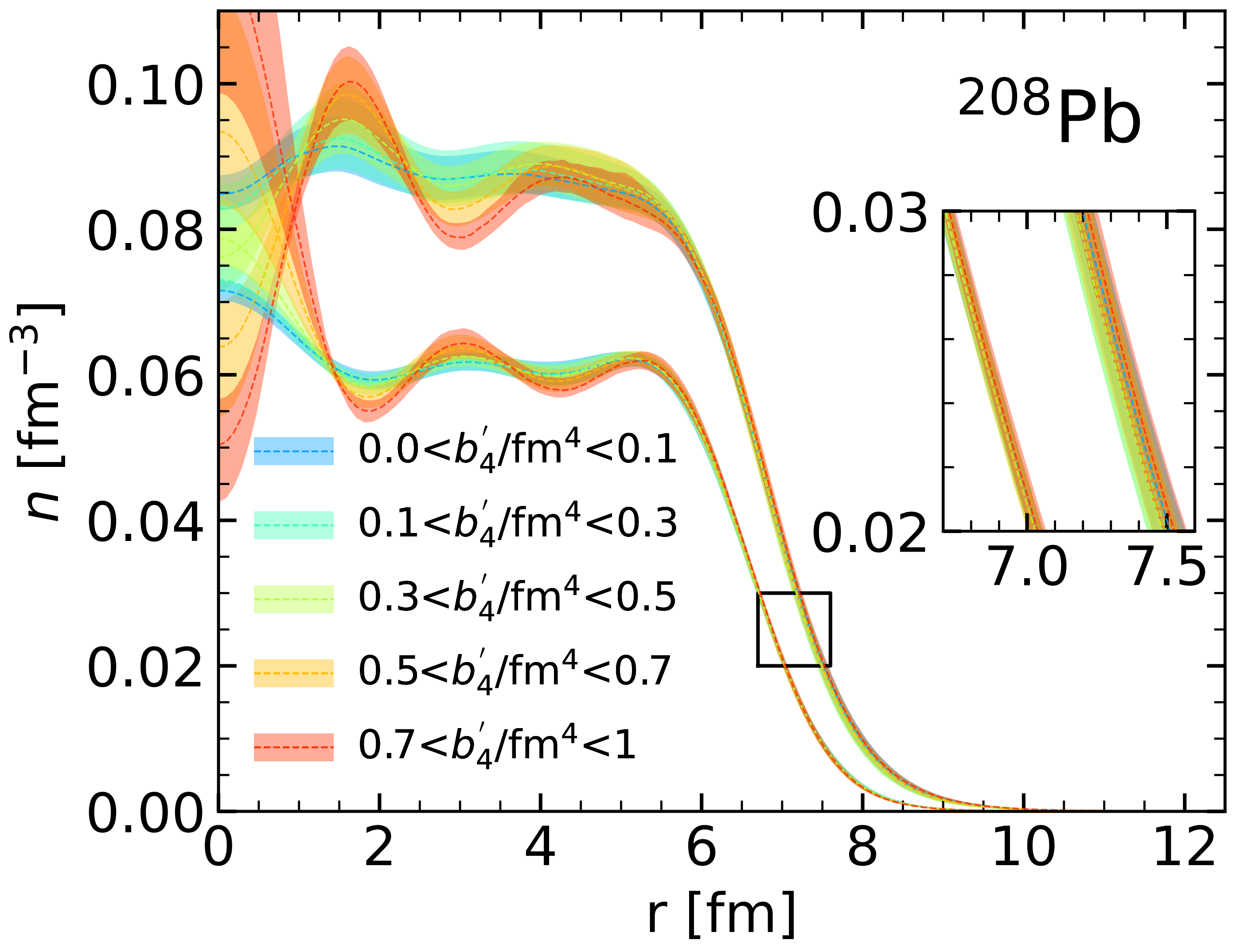
Less bound
Less bound

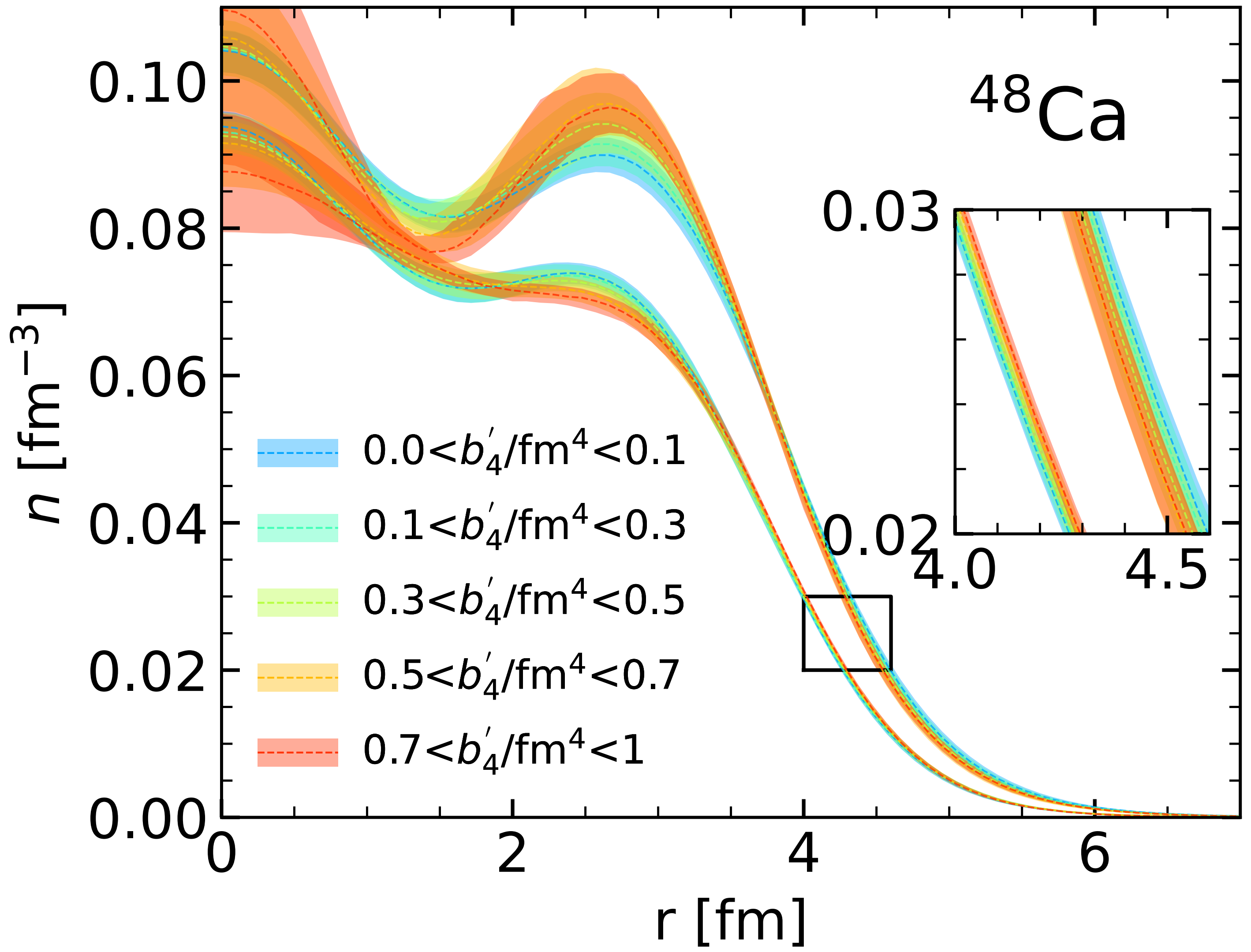


^{48}**Ca**		
neutron hole		
$1d_{5/2}$	-15.61	5.669
$2s_{1/2}$	-12.55	2.600
$1d_{3/2}$	-12.53	2.580
$1f_{7/2}$	-10.00	0.050
proton hole		
$1d_{5/2}$	-21.47	5.664
$1d_{3/2}$	-16.18	0.377
$2s_{1/2}$	-16.10	0.295

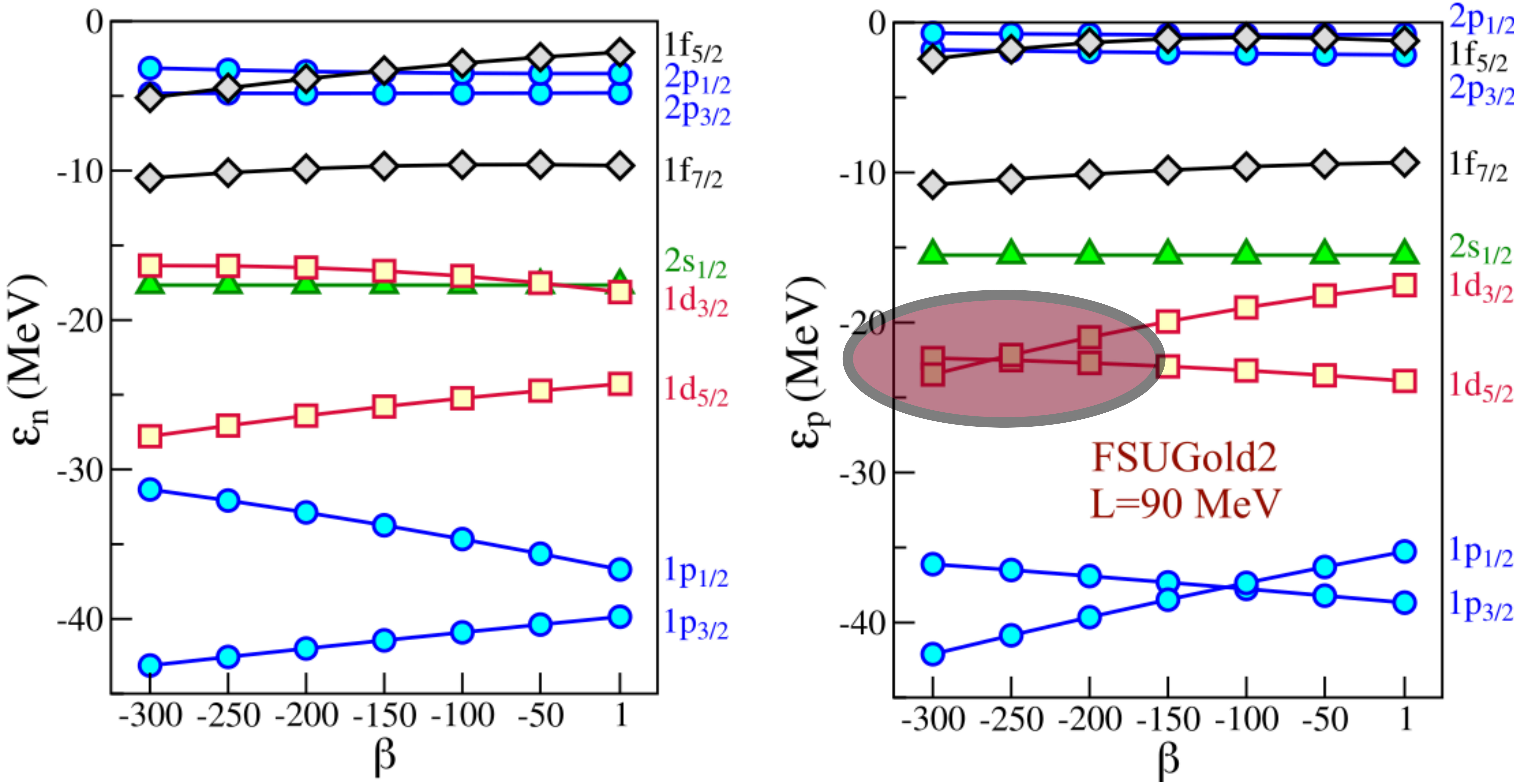
More bound

Less bound





Issue of varying only isovector spin-orbit potential

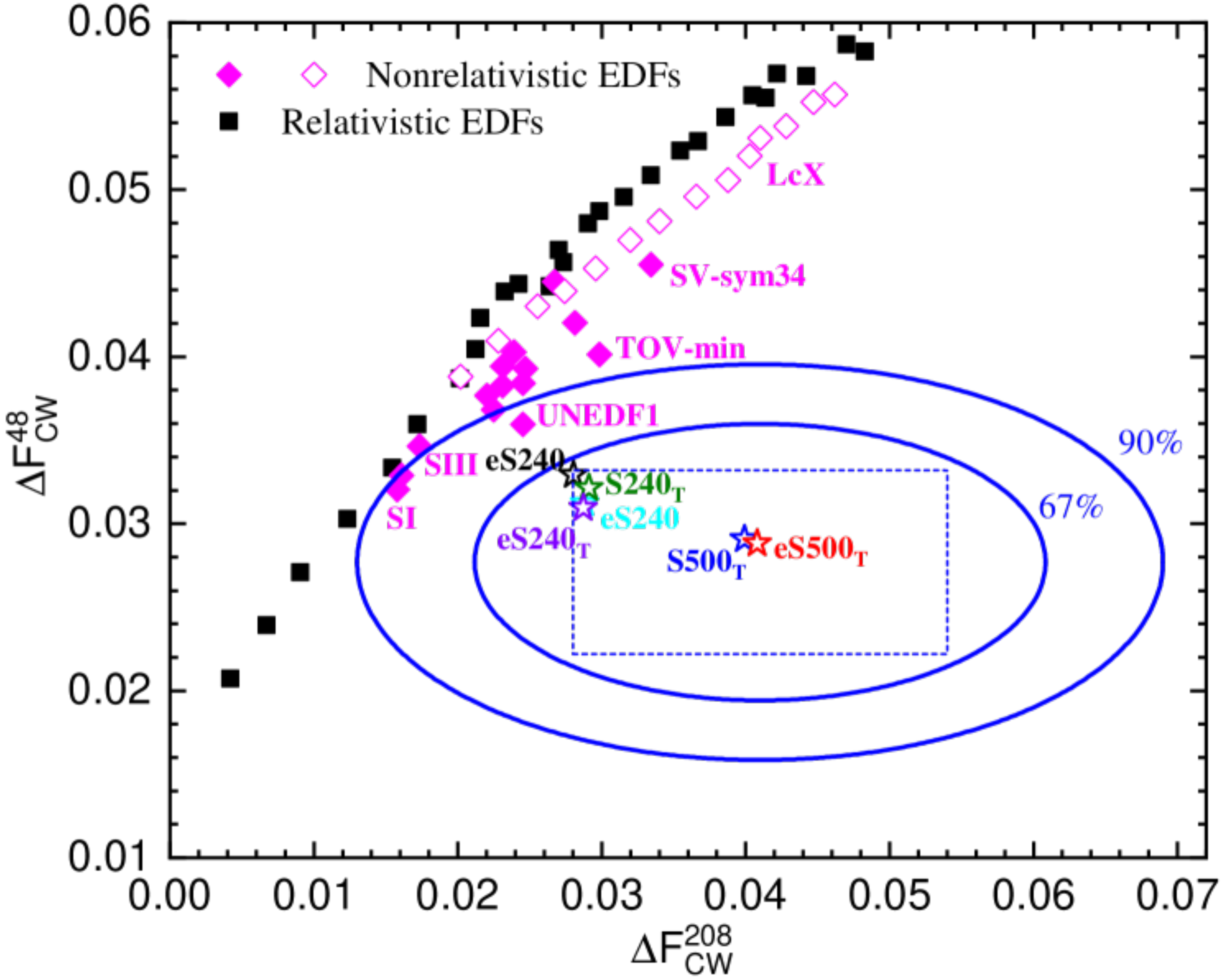


- A. Kunjipurayil, M. Salinas and J. Piekarewicz 2024 (arXiv:2503.07405)

Free Tensor Interaction

- Spin-orbit force in Skyrme model:
  $H_{\rm SO} = b_4 \mathrm{J} \cdot \nabla n$
  $+ b_4'(\mathrm{J}_n \cdot \nabla n_n + \mathrm{J}_p \cdot \nabla n_p)$
  Tensor force in Skyrme model:
  $H_{\rm T} = b_J \mathrm{J}^2 + b_J'(\mathrm{J}_n^2 + \mathrm{J}_p^2)$
  The freedom b_4', b_J' and b_J' improve the Skyrme model performance, see arXiv.2406.03844:
  S240 and eS240: $b_4' = 0.6\ \mathrm{fm}^{-4}$
  S500 and eS500: $b_4' = 1.3\ \mathrm{fm}^{-4}$

- which is consistent with our analysis:
  $b_4' = 1.37 \pm 0.49\ \mathrm{fm}^{-4}$
  and 90% lower bound:
  $b_4' \gtrsim 0.74\ \mathrm{fm}^4$ (Skyrme)
  $b_4' \gtrsim 0.54\ \mathrm{fm}^4$ (RMF)



T.G. Yue, Z. Zhang, L.W. Chen arXiv.2406.03844

Take away

- What nuclear properties can we learn from the experiment?
  PREX+CREX prefers much Larger S_V than expected.
- Why are Skyrme models more compatible than RMF models?
  The freedom in isovector spin-orbit interaction b_4'.
- How may the mean-field model improve in the future?
  Increase the degree of freedom on surface-related isovector interactions, e.g. isovector spin-orbit interaction, isovector tensor interaction.

see arXiv.2406.05267

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