Impact of the EOS on f-, p- and g- mode oscillations of neutron stars

Presenter: Tianqi Zhao
Institute of Nuclear & Particle Physics
Ohio University

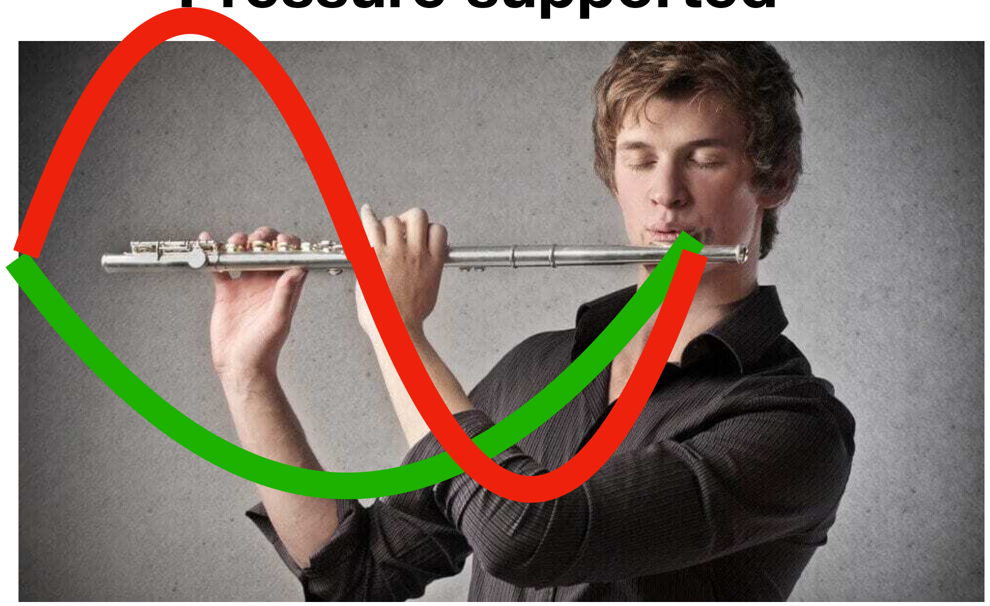
Collaborators: Madappa Prakash, James Lattimer, Constantinos Constantinou, Prashanth Jaikumar, Athul K P, Bharat Kumar, Bijay K Agrawal

Research supported by the US DOE

Oscillation modes

$$A(r)e^{i\omega t} \quad \omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$$

Pressure supported



Standing sound wave of order n:

$$\omega^2 \approx \frac{dp}{d\varepsilon}k^2 \quad k = \frac{\sqrt{l(l+1)}n}{2\pi R}$$

n=0 f-mode (fundamental)

n=1 p-mode (pressure)

Gravity & interface

Stratified fluid in uniform gravity g:

$$\omega^2 = \frac{(\varepsilon_+ - \varepsilon_-)gk}{\varepsilon_+/\tanh(kd_+) + \varepsilon_-/\tanh(kd_-)}$$

Discontinuity g-mode (interface gravity mode)

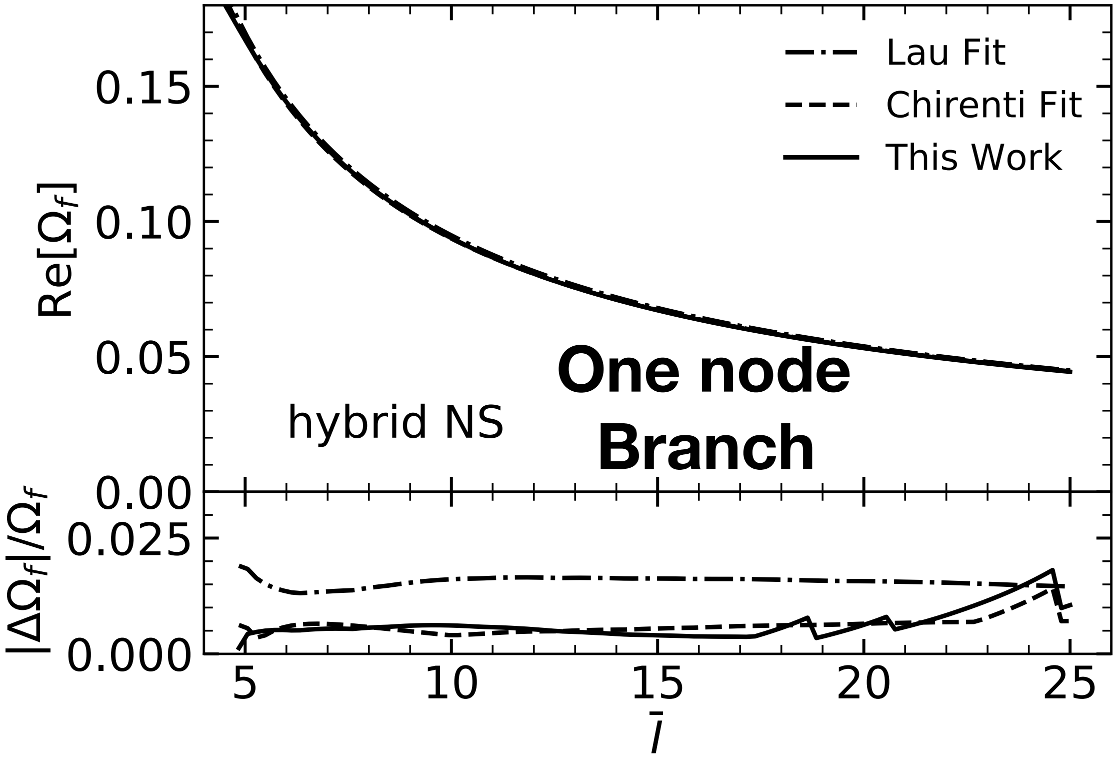
Gravity & x gradient

Buoyancy oscillation in uniform gravity g:

$$\omega^2 \approx \mathcal{N}^2 = -\frac{g}{\varepsilon}\left(\frac{\partial\varepsilon}{\partial x}\right)_p \frac{dx}{dr} = g^2\left(\frac{1}{c_{eq}^2} - \frac{1}{c_{ad}^2}\right) \quad x = \{\frac{n_p}{n_B}, \frac{n_p}{n_B}, T, \ldots\}$$

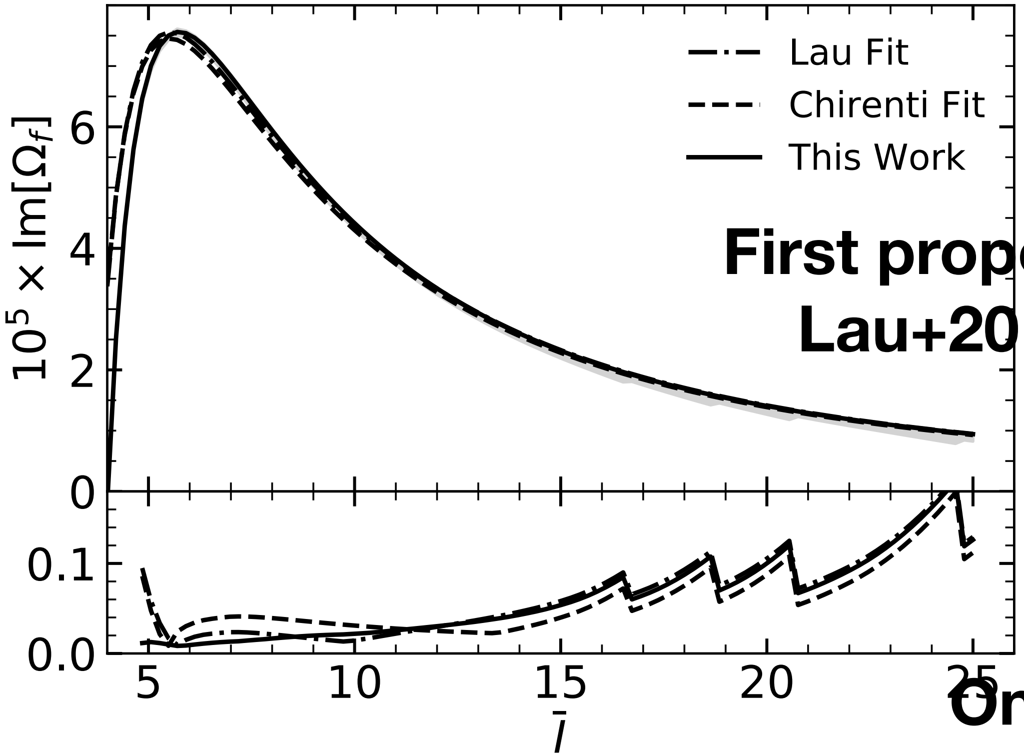
Chemical g-mode (gravity with composition gradient)

f-mode universal relations



One node Branch

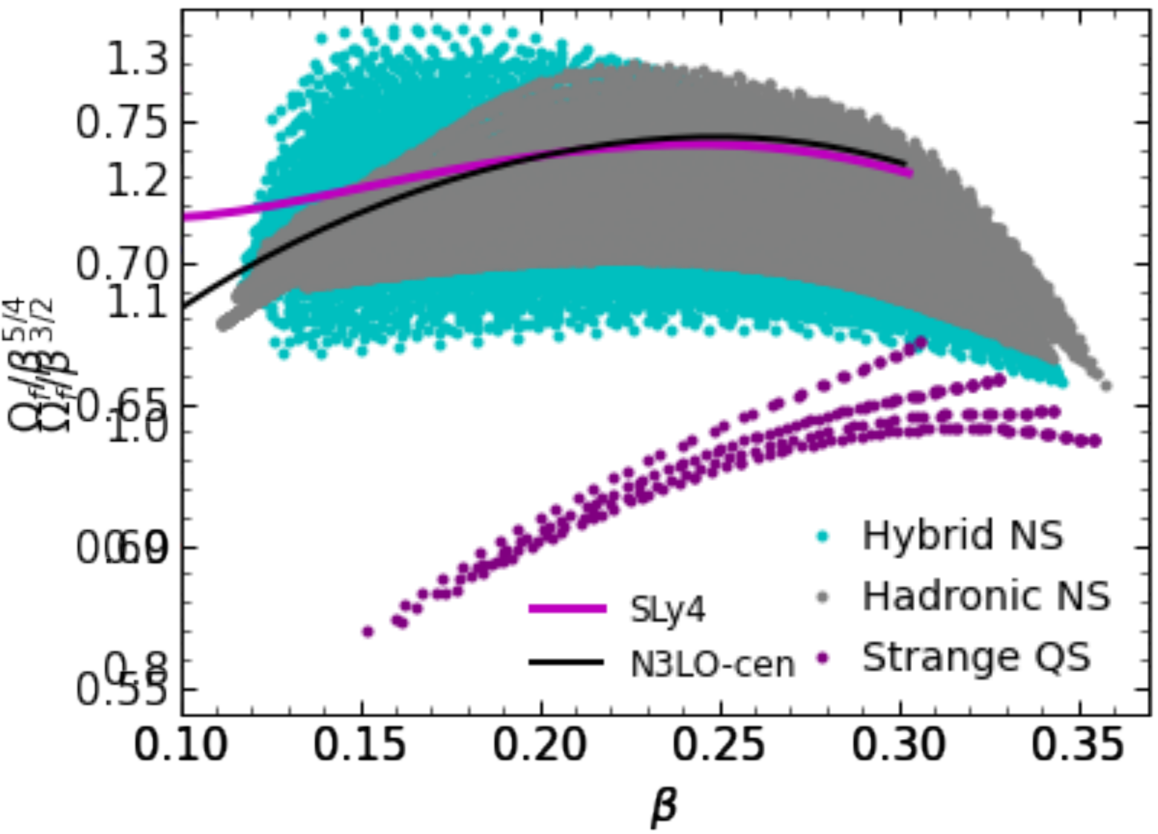
0.1% deviation except for one-node branches



First proposed Lau+2010

One-node Branch

1% deviation except for one-node branches



$$\Omega_f = GM\omega_f/c^3 \qquad (\propto \beta^{3/2} \text{ in Newtonian})$$

1. $\Omega_f - \Lambda$ **is slightly weaker than** $\Omega_f - \bar{I}$
2. Ω_f **is close related to compactness** β

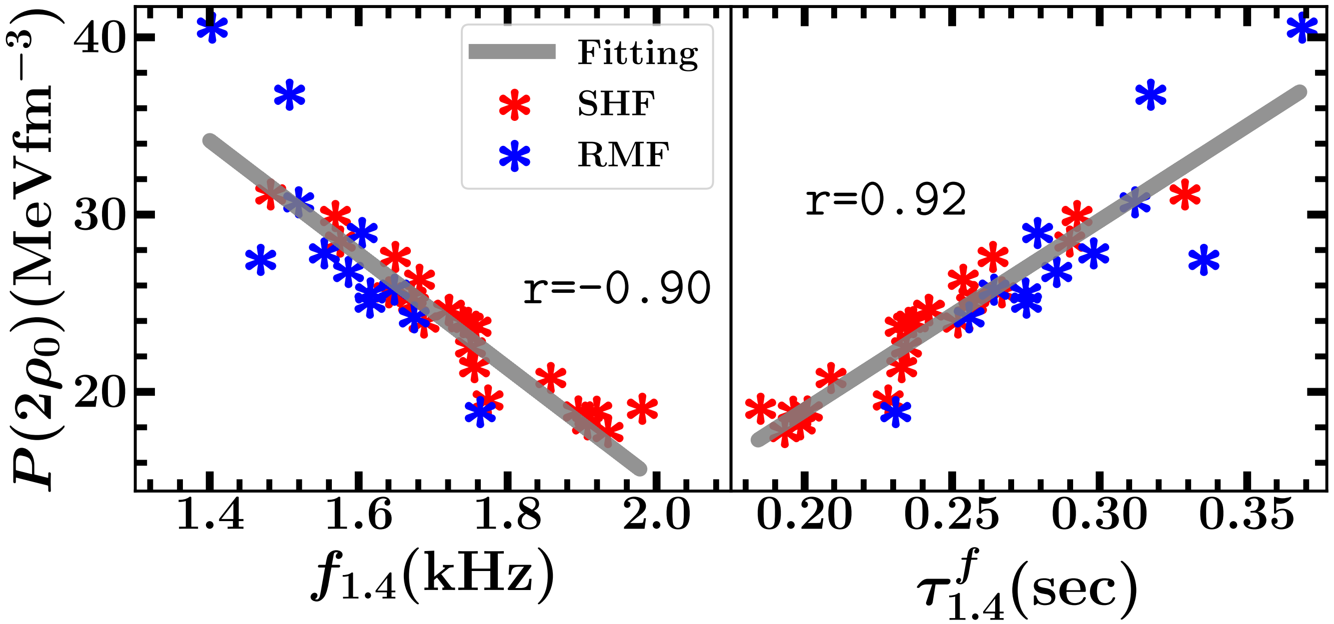
$\Omega_f = (0.887 \pm 0.061)\ \beta^{3/2}$ **Quark star**

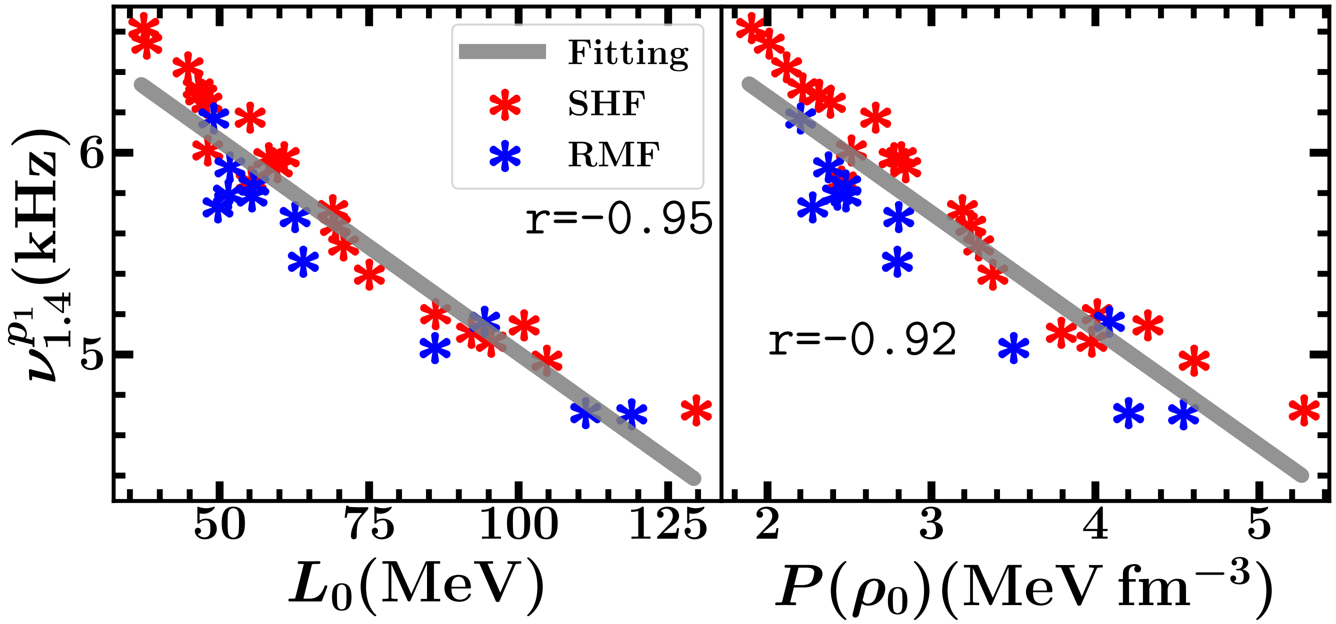
$\Omega_f = (0.714 \pm 0.056)\ \beta^{5/4}$ **Hadronic & hybrid NS**

Zhao & Lattimer 2022

BACK

p-modes with SHF and RMF EOSs

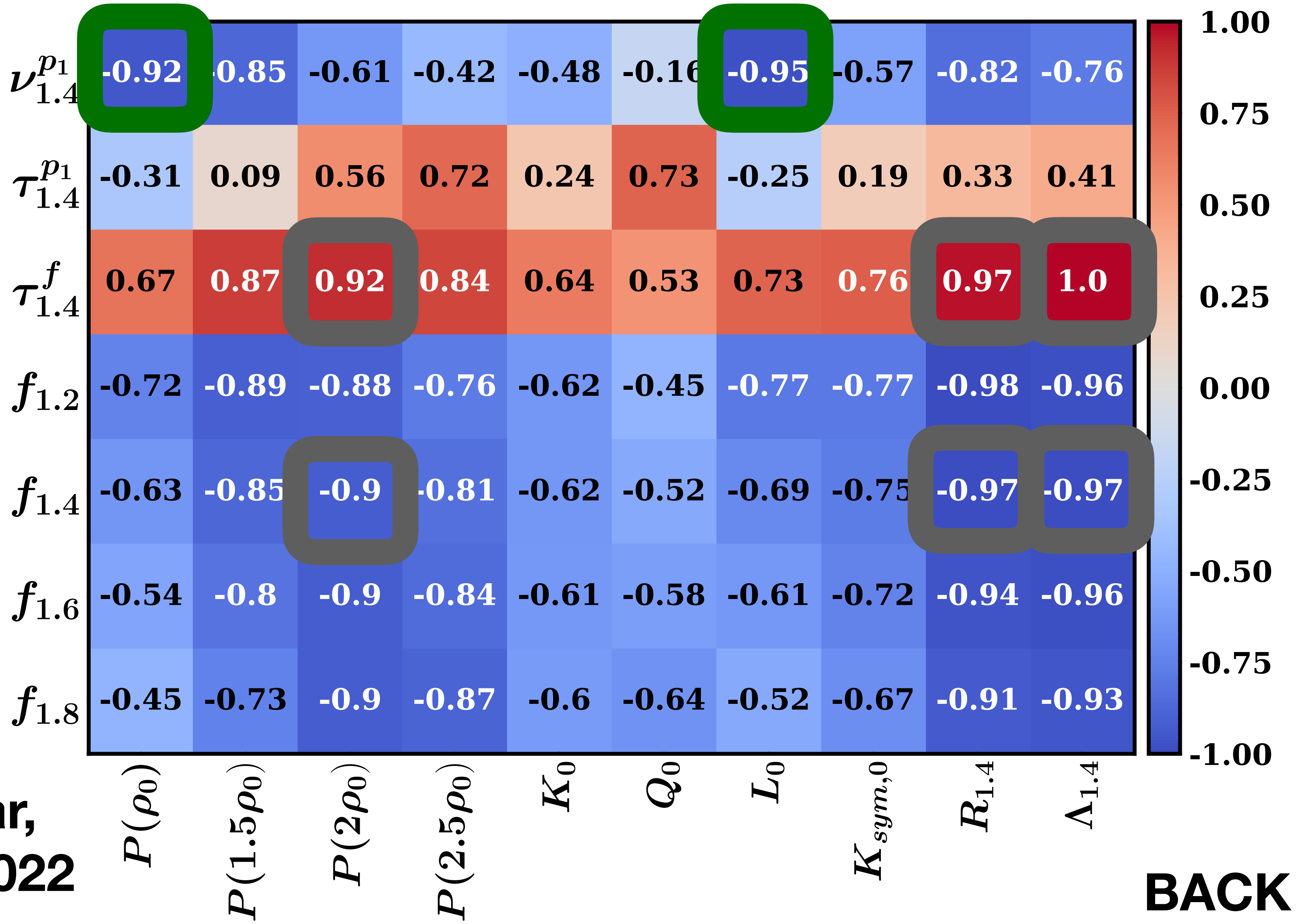




$\nu^{p_1}_{1.4}$ is sensitive to EOS around saturation

$f_{1.4}$ and $\tau^f_{1.4}$ are sensitive to EOS around twice saturation density

Higher order p-mode is sensitive to EOS at lower density



	$P(\rho_0)$	$P(1.5\rho_0)$	$P(2\rho_0)$	$P(2.5\rho_0)$	K_0	Q_0	L_0	$K_{sym,0}$	$R_{1.4}$	$\Lambda_{1.4}$
$\nu^{p_1}_{1.4}$	-0.92	-0.85	-0.61	-0.42	-0.48	-0.16	-0.95	-0.57	-0.82	-0.76
$\tau^{p_1}_{1.4}$	-0.31	0.09	0.56	0.72	0.24	0.73	-0.25	0.19	0.33	0.41
$\tau^f_{1.4}$	0.67	0.87	0.92	0.84	0.64	0.53	0.73	0.76	0.97	1.0
$f_{1.2}$	-0.72	-0.89	-0.88	-0.76	-0.62	-0.45	-0.77	-0.77	-0.98	-0.96
$f_{1.4}$	-0.63	-0.85	-0.9	-0.81	-0.62	-0.52	-0.69	-0.75	-0.97	-0.97
$f_{1.6}$	-0.54	-0.8	-0.9	-0.84	-0.61	-0.58	-0.61	-0.72	-0.94	-0.96
$f_{1.8}$	-0.45	-0.73	-0.9	-0.87	-0.6	-0.64	-0.52	-0.67	-0.91	-0.93

Athul, Zhao, Kumar, Agrawal, Prakash 2022

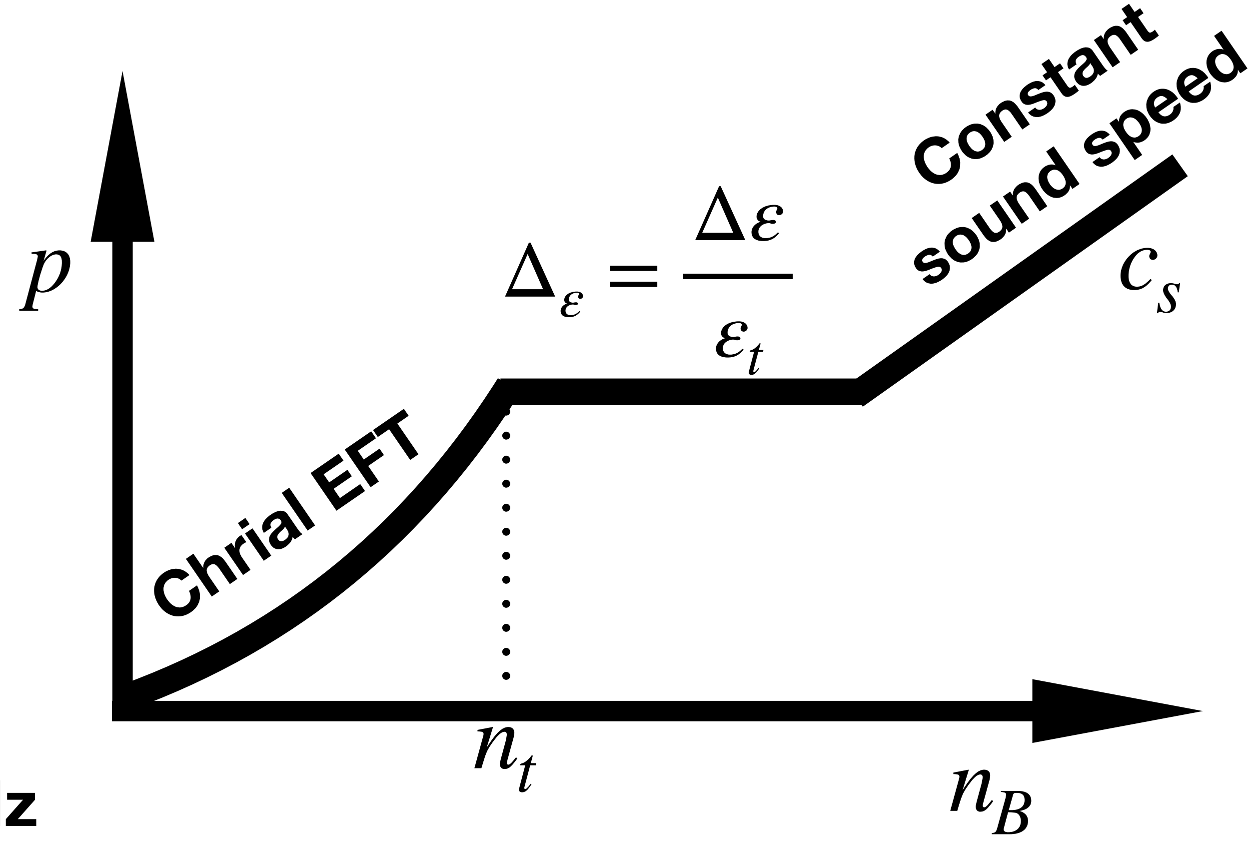
BACK

Discontinuity g-mode

First order transition

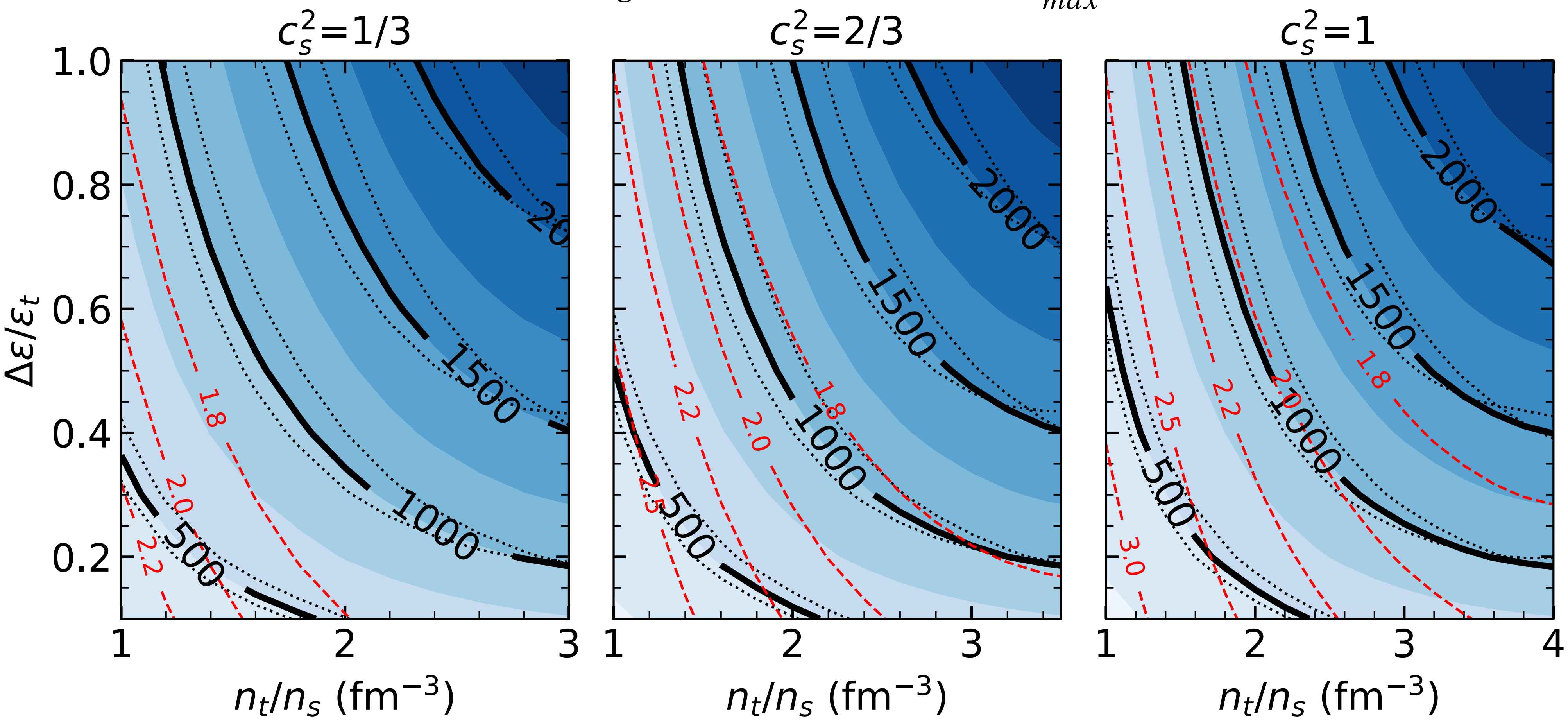
$$\Omega_g^2 \approx \frac{\beta^3 (M_t/M)(R/R_t)^3(\Delta\varepsilon/\varepsilon_t) D \tanh[D]}{1+\Delta\varepsilon/\varepsilon_t + \tanh[D]/\tanh[D(R/R_t-1)]}$$

Ω_g **is sensitive to structure factors,** $\frac{R_t}{R}, \frac{M_t}{M}, \frac{\Delta\varepsilon}{\varepsilon}$



$$\Delta_\varepsilon = \frac{\Delta\varepsilon}{\varepsilon_t}$$

Contour of $\nu_g(n_t,\ \Delta_\varepsilon,\ c_s^2)\,|_{M=M_{max}}$ **Hz**



Dotted: Chrial EFT Uncertanty 5%

Dashed: maximum mass

Frequency: $\nu_g < 0.8\ (1.5)$ kHz for $c_s^2 = c^2/3\ (c^2)$

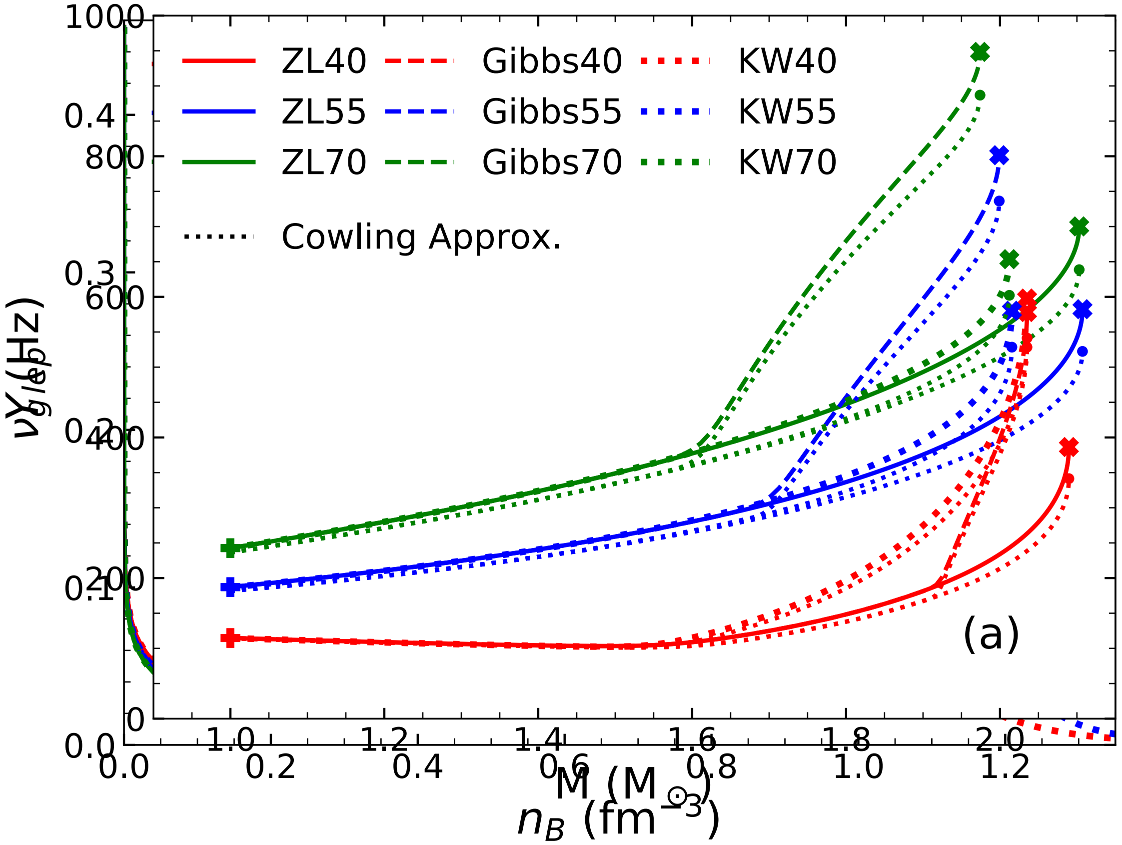
Zhao & Lattimer 2022

Damping time: $\tau > 100\ (10000)$ s

BACK

Compositional g-mode universal relation

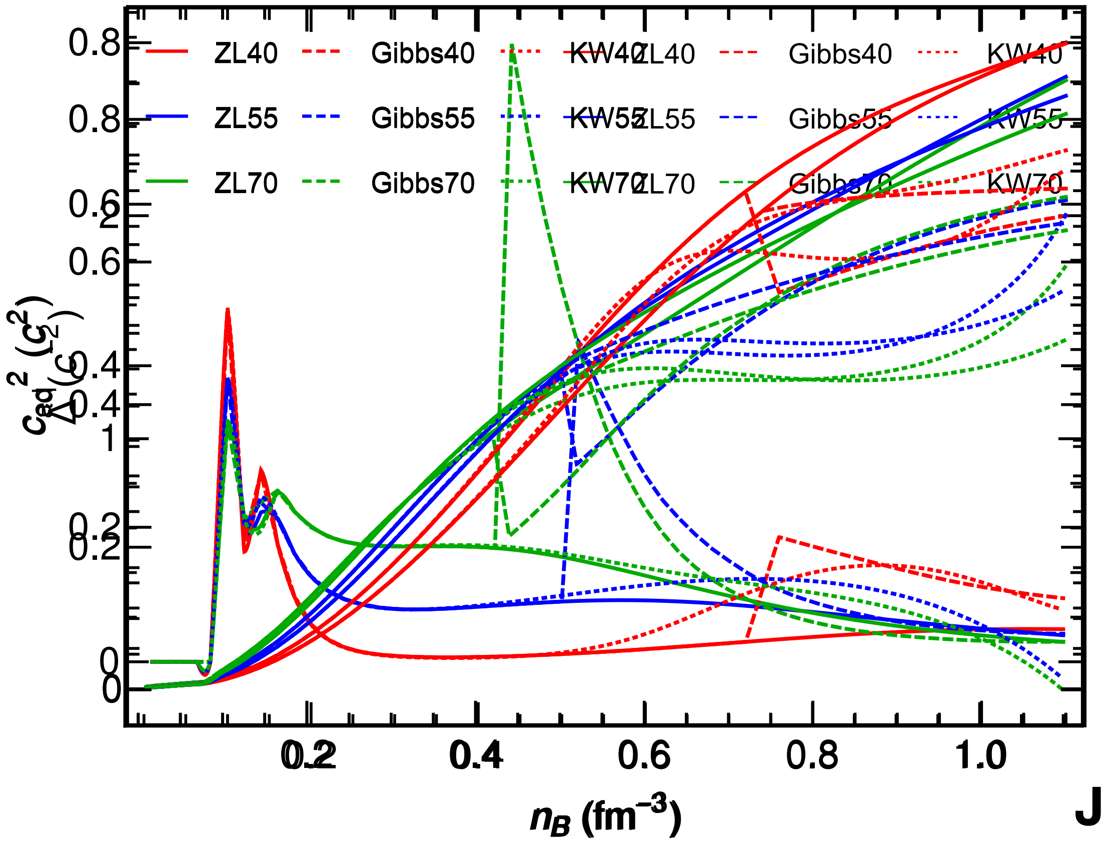
Hadronic: ZL **First-order: Gibbs** **Crossover: KW**

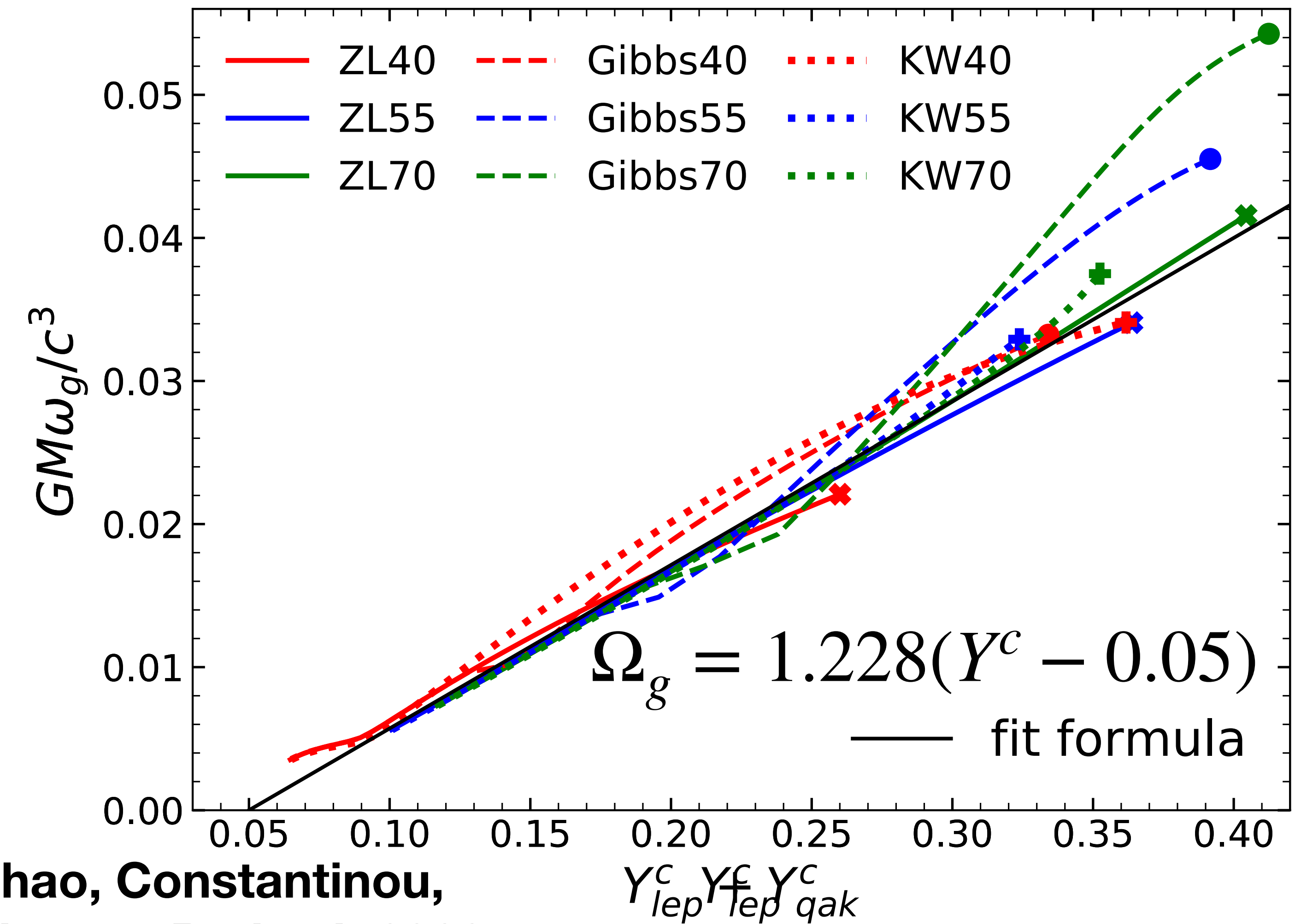


$$\nu_g^2 = g^2 e^{\nu-\lambda}\Delta(c^{-2}) \qquad \Delta(c^{-2}) = g^2\left(\frac{1}{c_{ad}^2} - \frac{1}{c_{eq}^2}\right)$$

is sensitive to:

1. **Symmetry energy S(n)**
2. **Number of particle species**
3. **Proto-NS neutrino emission**
4. **Bulk viscosity**





Zhao, Constantinou, Jaikumar, Prakash 2022

BACK

BACK UP SLIDES

Gravitational radiation of NS oscillation

- The amplitude of observed oscillations is

$$h(t) = h_0 e^{-t/\tau} \cos \omega t \qquad A(r)e^{i\omega t} \quad \omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$$

- The observed GW energy flux is

$$F(t) = \frac{c^3\omega^2 h_0^2}{16\pi G} e^{-2t/\tau} = 3.17 e^{-2t/\tau} \left(\frac{\nu}{\textbf{kHz}}\right)^2 \left(\frac{h_0}{10^{-22}}\right)^2 \textbf{ ergs cm}^{-2}\textbf{s}^{-1}$$

- The total GW energy is

$$E = \frac{c^3\omega^2 h_0^2 \tau D^2}{8G} = 4.27 \times 10^{49} \left(\frac{\nu}{\textbf{kHz}}\right)^2 \left(\frac{h_0}{10^{-23}}\right)^2 \left(\frac{\tau}{0.1\textbf{ s}}\right) \left(\frac{D}{\textbf{15 Mpc}}\right)^2 \textbf{ ergs}$$

• supernovae remnant: 10^44-10^47 ergs	$D < 20$ **kpc**	$D < 200$ **kpc** **A few per century**
merger remnant: 10^51-10^52 ergs	$D \lesssim 20 - 45$ **Mpc**	$D \lesssim 200 - 450$ **Mpc** **0.06-4 per year**
	aLIGO	**3G**

Observation of Oscillations of NS

- Direct observation:

matter motion

1. BNS merger remnant
2. Core-collapse SNe
3. Star quake (glitches)
4. NS close encounter

spacetime variation

gravitational wave radiation

- Indirect observation:

Binary NS inspiring

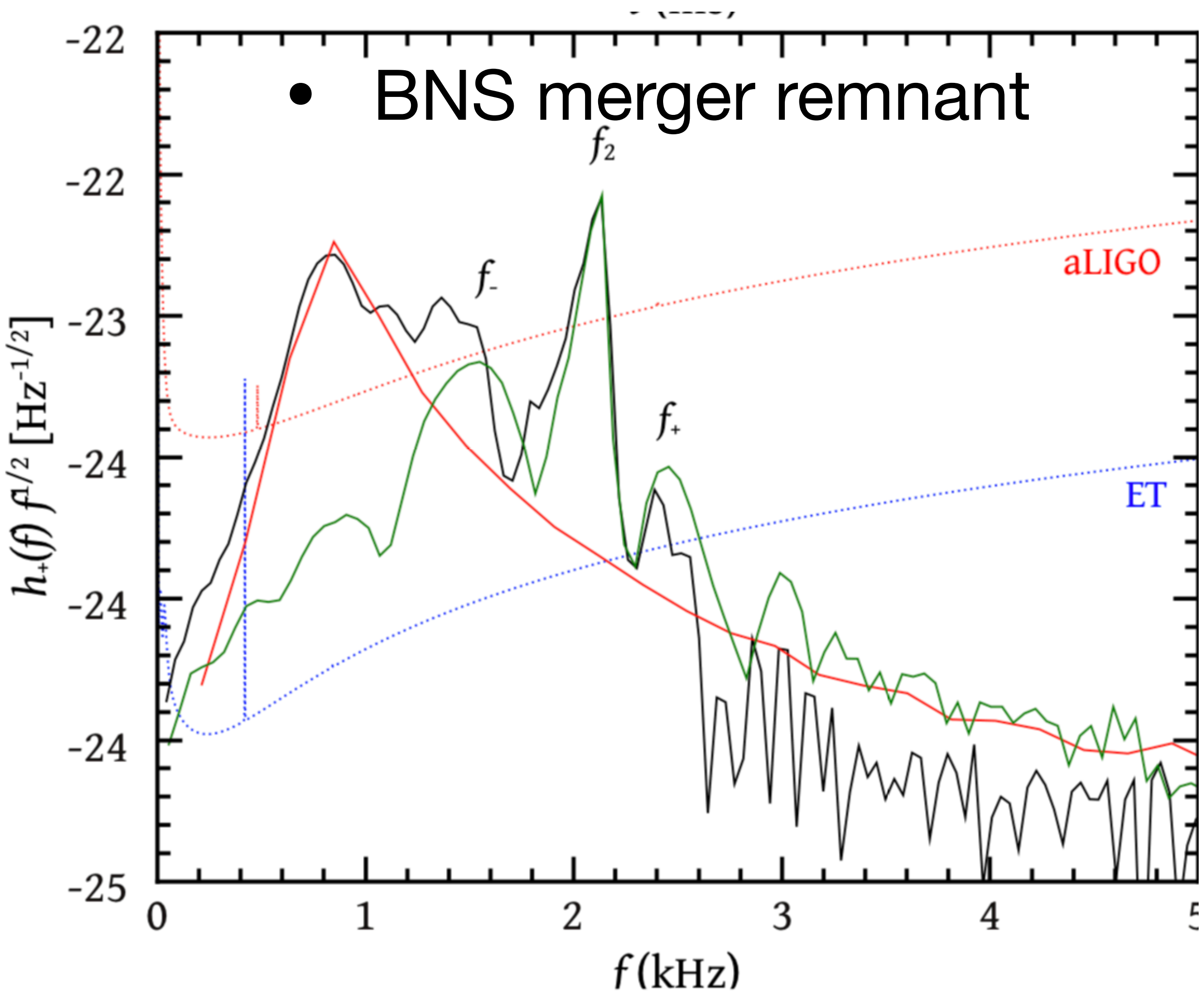
Orbital angular momentum transfer

gravitational wave form information

- Instrument:
  Comic explore (US)
  Einstein Telescope (Europe)

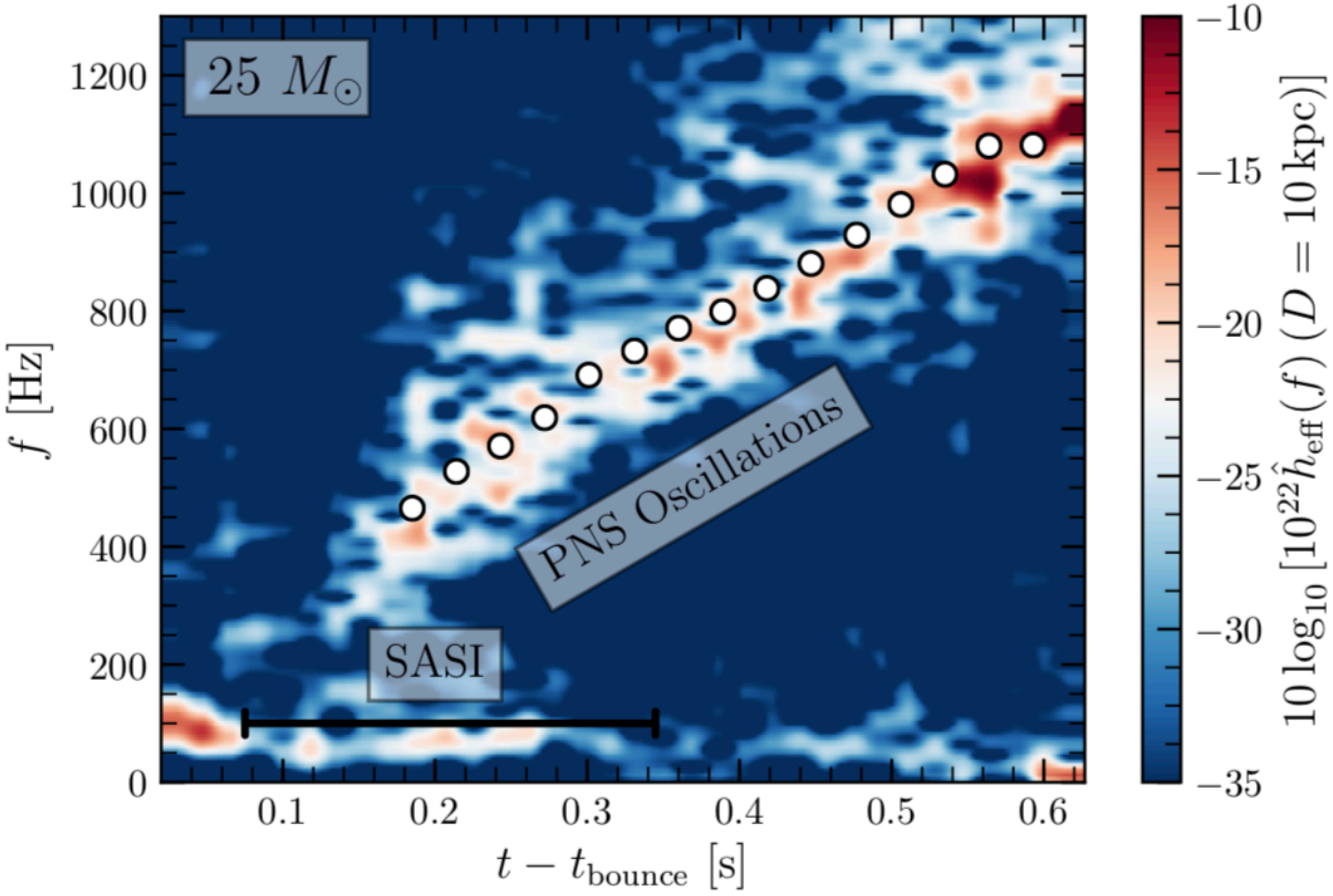
Oscillations of NS in simulation

- BNS merger remnant



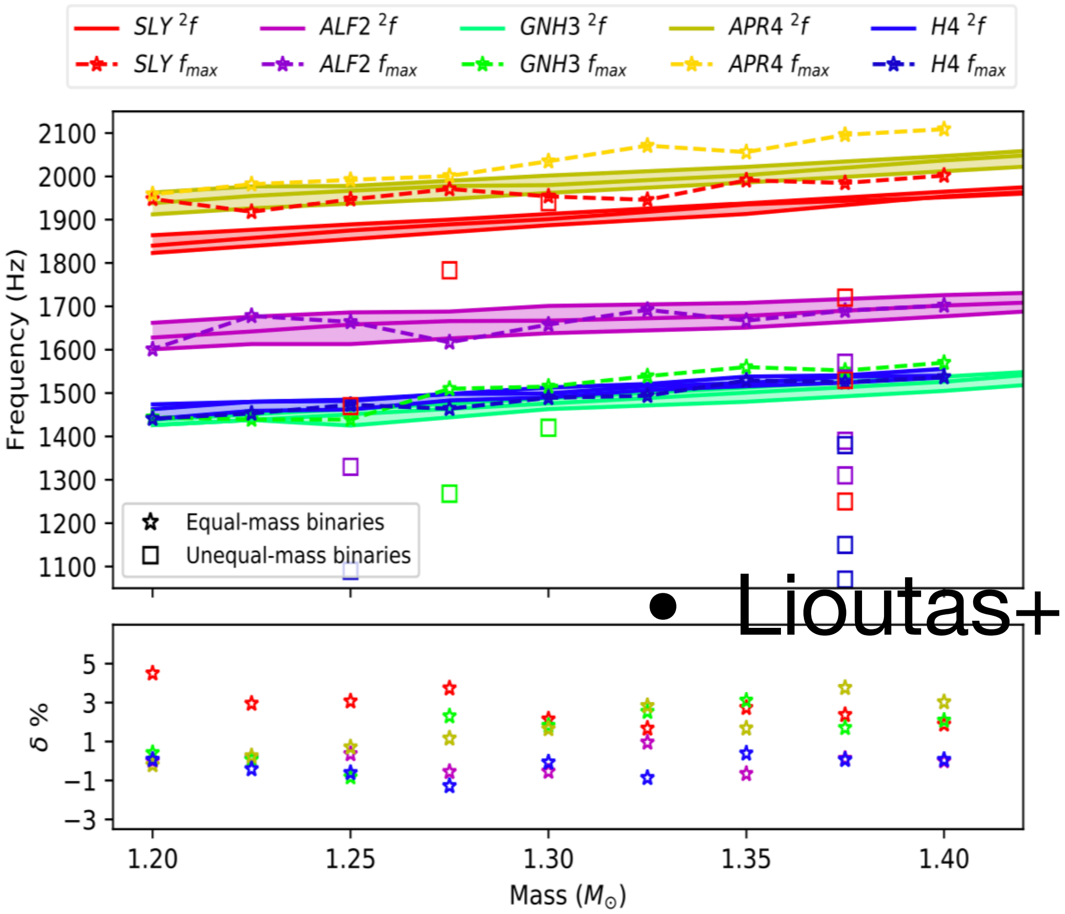
- Stergioulas+2011

- Core-collapse SNe

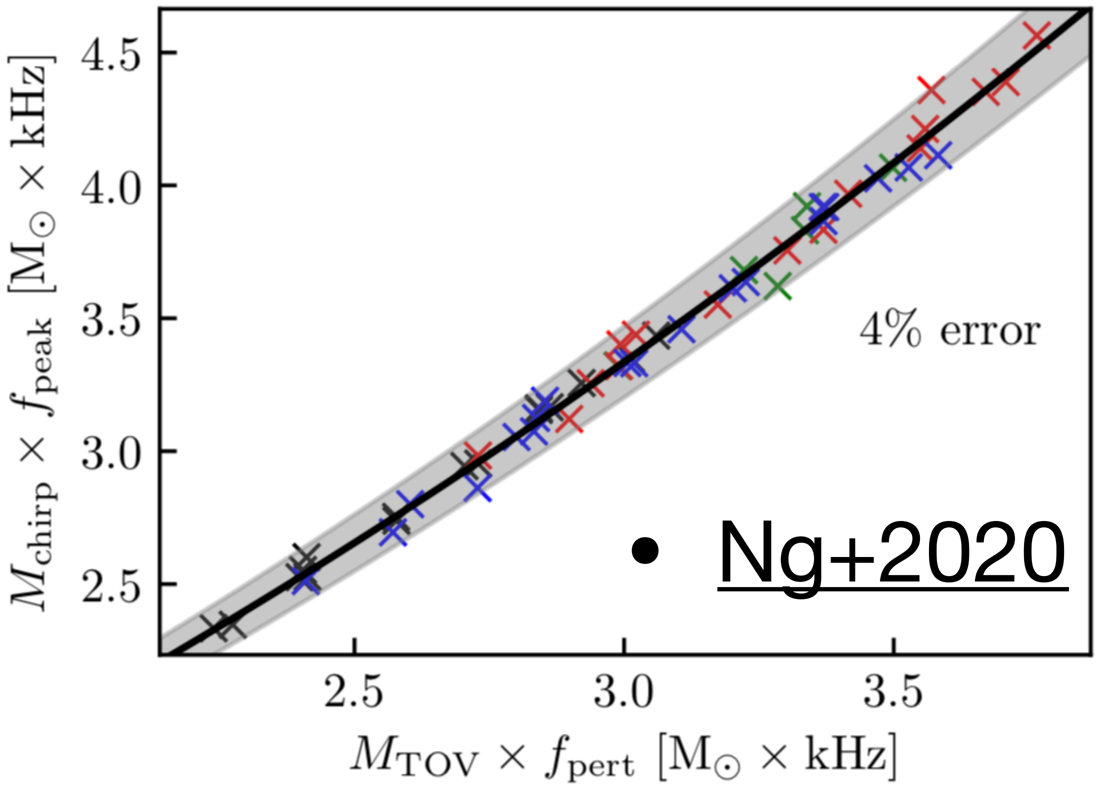


- Radice+2019

Isolated oscillation VS merger remnant



- Lioutas+2021

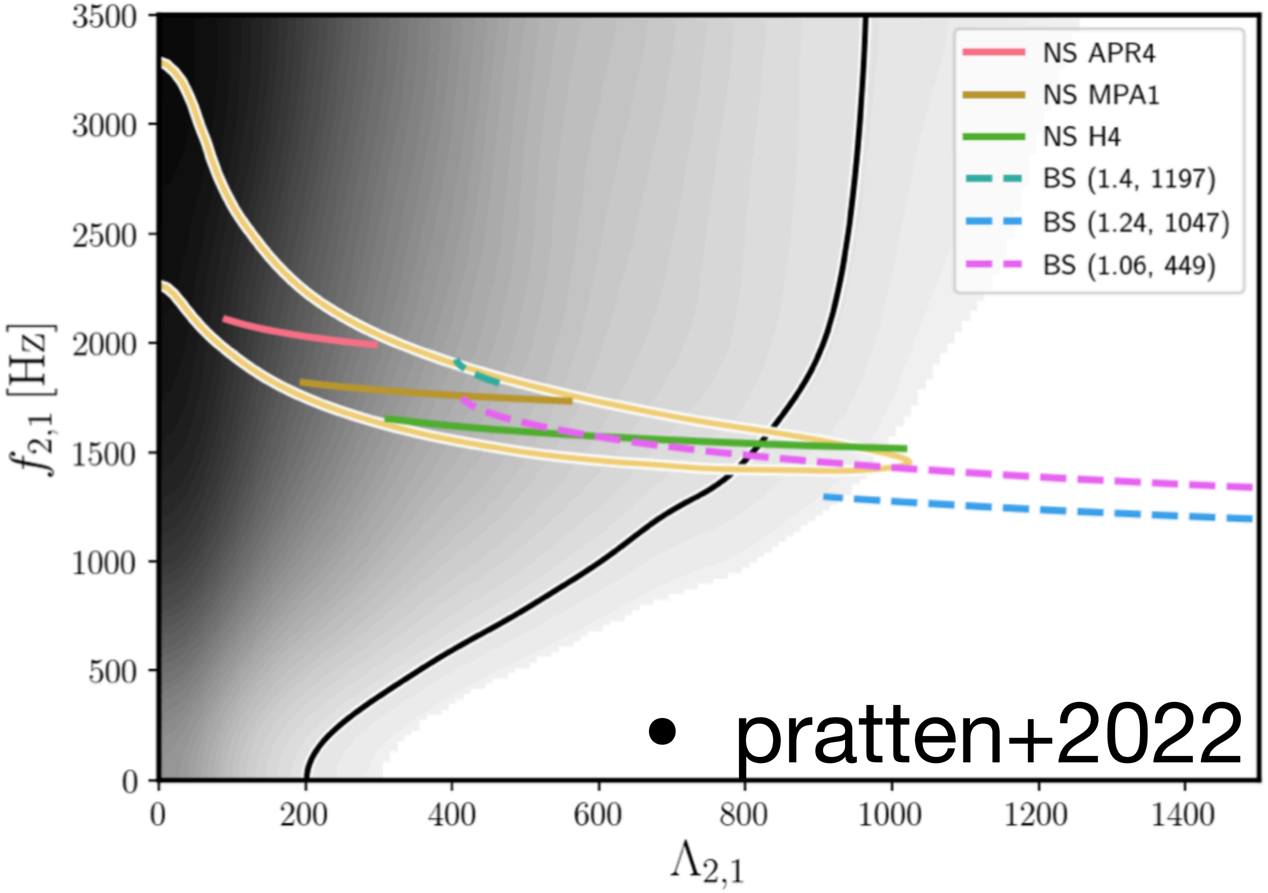


- Ng+2020

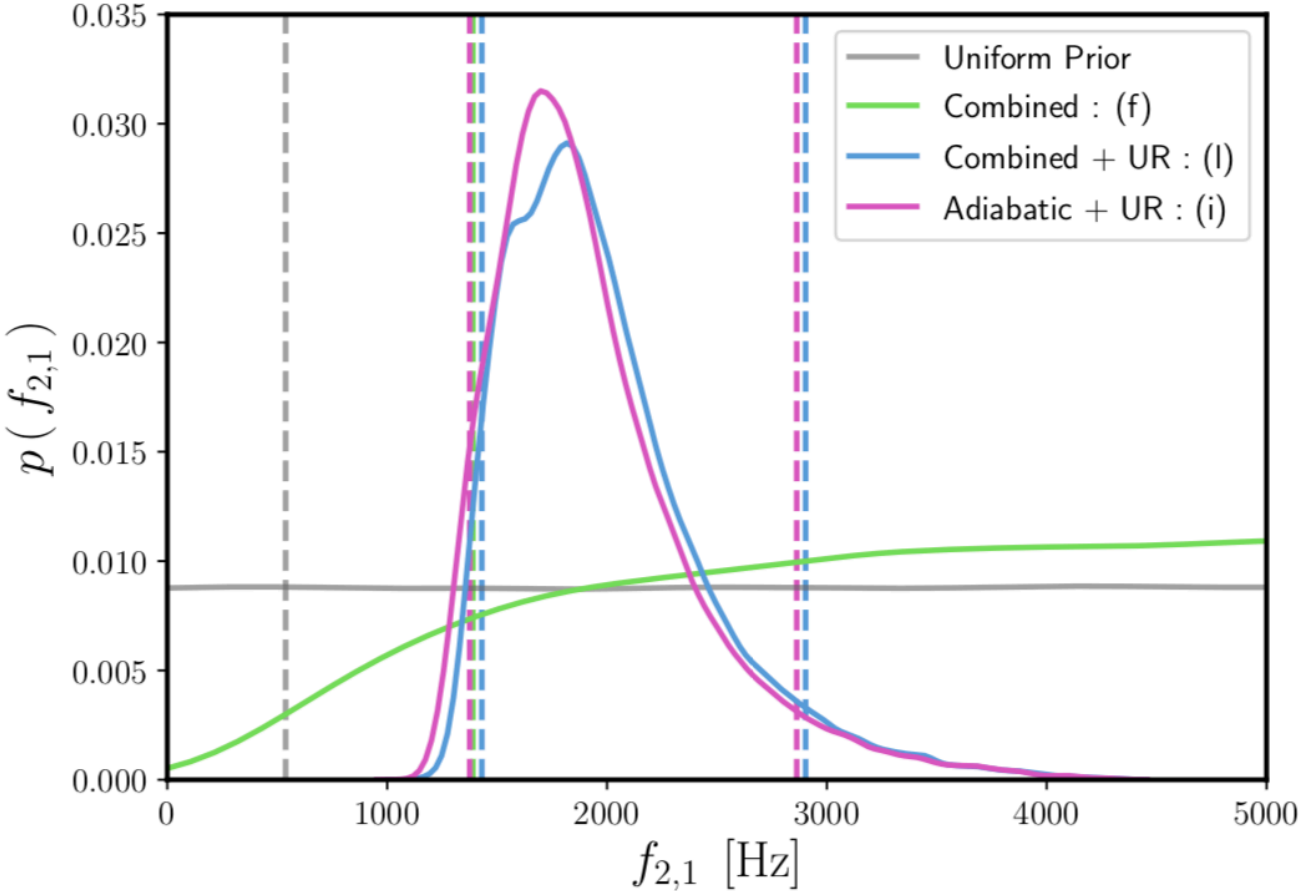
- Strong correlation with the isolated NS f-mode frequency and the peak frequency in post merger.

- case of equal-mass mergers, the peak frequency in supramassive NSs is almost equal to that of the non-rotating f-mode frequency of isolated NSs with the same mass as each of the merging components

Dynamical tidal effect of GW170817



- pratten+2022



- 90% credible interval of f-mode frequency for GW170817:
  1.43 kHz ~ 2.90 kHz for the more massive star
  1.48 kHz ~ 3.18 kHz for the less massive star

Oscillations of NS

Fluid perturbations

- Radial oscillation (l=0): $\varepsilon^{\boxed{r}} = R_n^r(r)e^{\mathbf{i}\omega t}$
  don't couple to gravitational waves
- Non-radial oscillation (l>=2):
  $$\varepsilon^{\boxed{r,\,\theta}\,\boxed{\phi}} = \partial_{r,\,\theta,\,\phi}\left(R_n^{r,\,\theta,\,\phi}(r)Y_m^l(\theta,\phi)e^{\mathbf{i}\omega t}\right)$$
  f-mode (fundamental n=0) (even),
  p-modes (pressure n=1,2,…) (even)
  g-modes (gravity n=1,2,…) (even)
  r-modes (rotation m=+-1,+-2,…) (odd)

$$\omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$$

	ν **(kHz)**	τ **(s)**
f-mode	1.3-2.8	0.1-1
g-mode	<0.8	>100
p-mode	>2.7	1-1000
r-mode	~ spin	<0
w-mode	~10	~1E-5

- **Spacetime perturbations:**
  Family I w-modes (even)
  Family II w-modes (odd)
  important for BBH ring-down

even-parity
(polar mode)

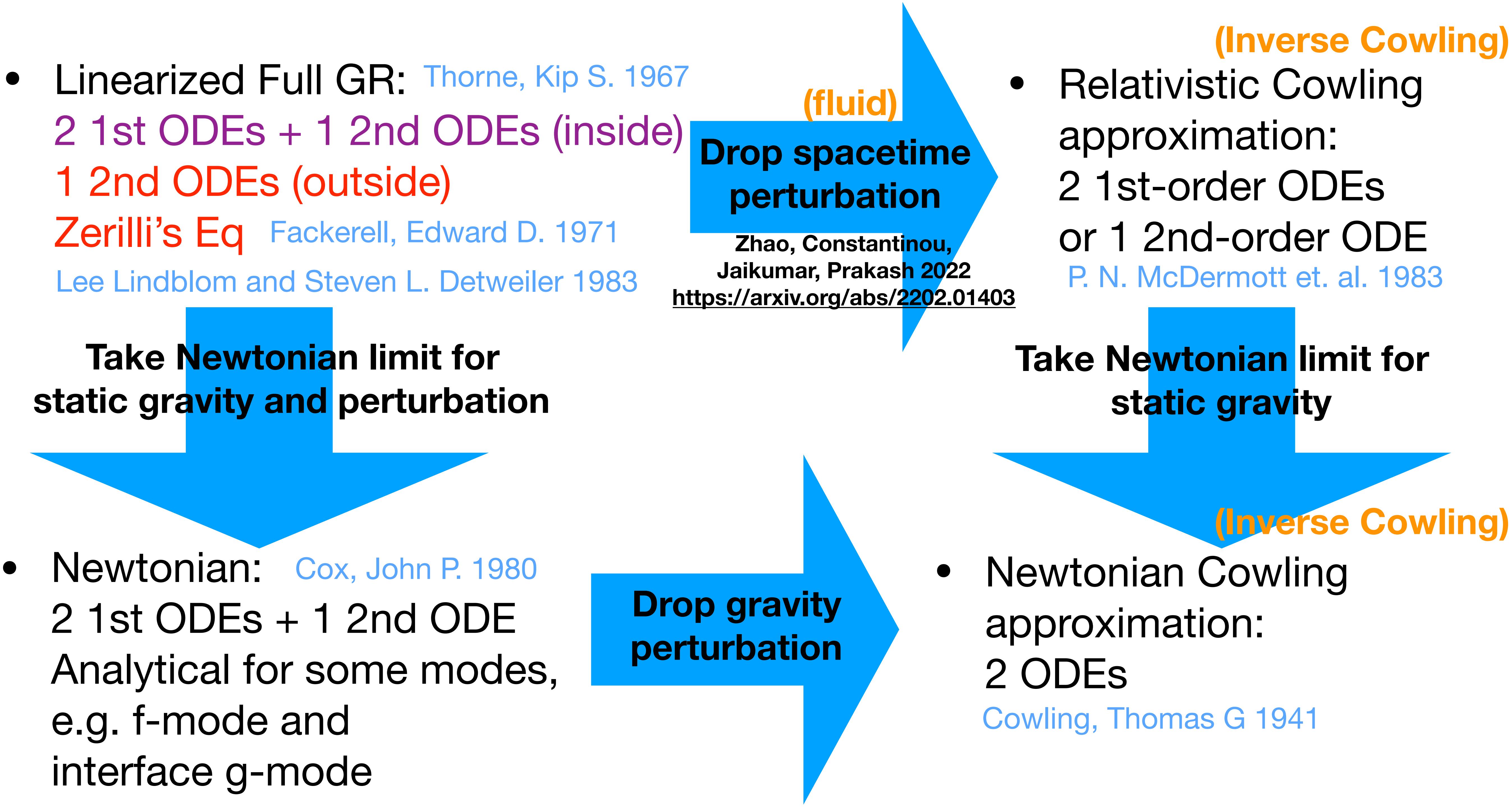
$$h_{\mu\nu}^{even} = \begin{pmatrix} H_0e^{\nu} & H_1 & 0 & 0 \\ H_1 & H_2 & 0 & 0 \\ 0 & 0 & r^2K & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta K \end{pmatrix} P_l(\cos\theta)$$

odd-party
(axial modes)

$$h_{\mu\nu}^{odd} = \begin{pmatrix} 0 & 0 & 0 & H_0' \\ 0 & 0 & 0 & H_1' \\ 0 & 0 & 0 & 0 \\ H_0' & H_1' & 0 & 0 \end{pmatrix} \sin\theta\partial_\theta P_l(\cos\theta)$$

ODEs of Non-radial Adiabatic Oscillation

Eigen value problem of even quasi-normal modes



- Linearized Full GR: Thorne, Kip S. 1967
  2 1st ODEs + 1 2nd ODEs (inside)
  1 2nd ODEs (outside)
  Zerilli’s Eq Fackerell, Edward D. 1971
  Lee Lindblom and Steven L. Detweiler 1983

(fluid)
Drop spacetime perturbation

Zhao, Constantinou, Jaikumar, Prakash 2022
https://arxiv.org/abs/2202.01403

(Inverse Cowling)

- Relativistic Cowling approximation:
  2 1st-order ODEs
  or 1 2nd-order ODE
  P. N. McDermott et. al. 1983

Take Newtonian limit for static gravity and perturbation

Take Newtonian limit for static gravity

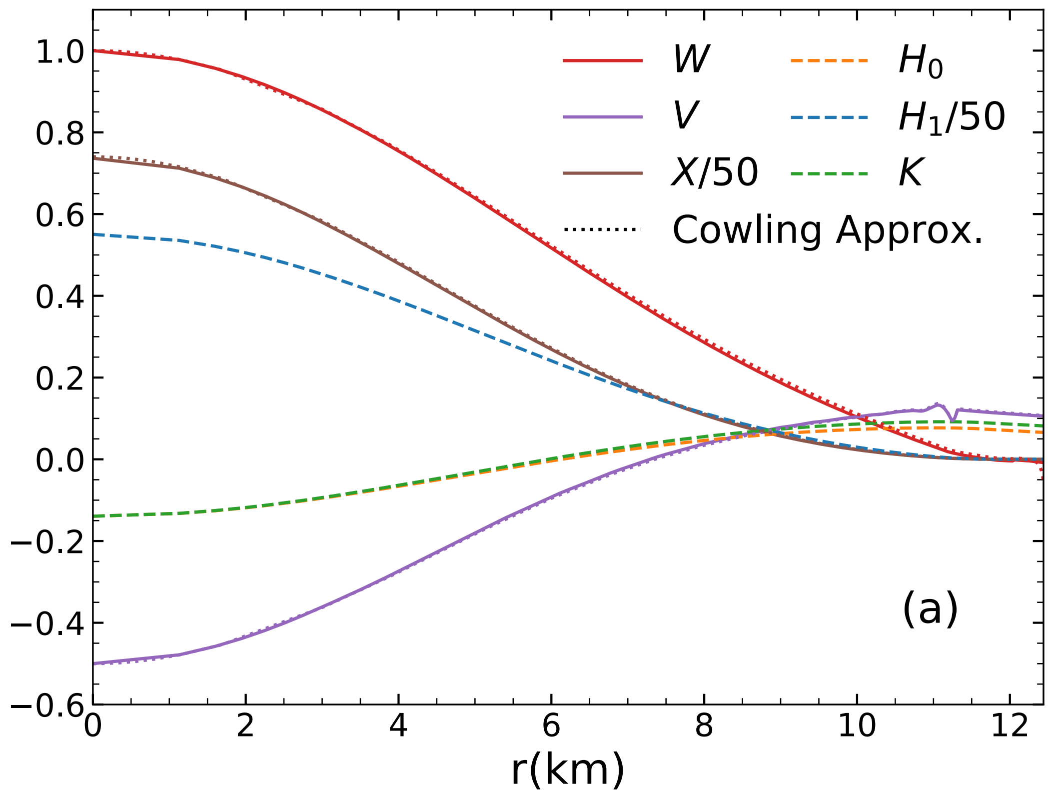
- Newtonian: Cox, John P. 1980
  2 1st ODEs + 1 2nd ODE
  Analytical for some modes,
  e.g. f-mode and
  interface g-mode

Drop gravity perturbation

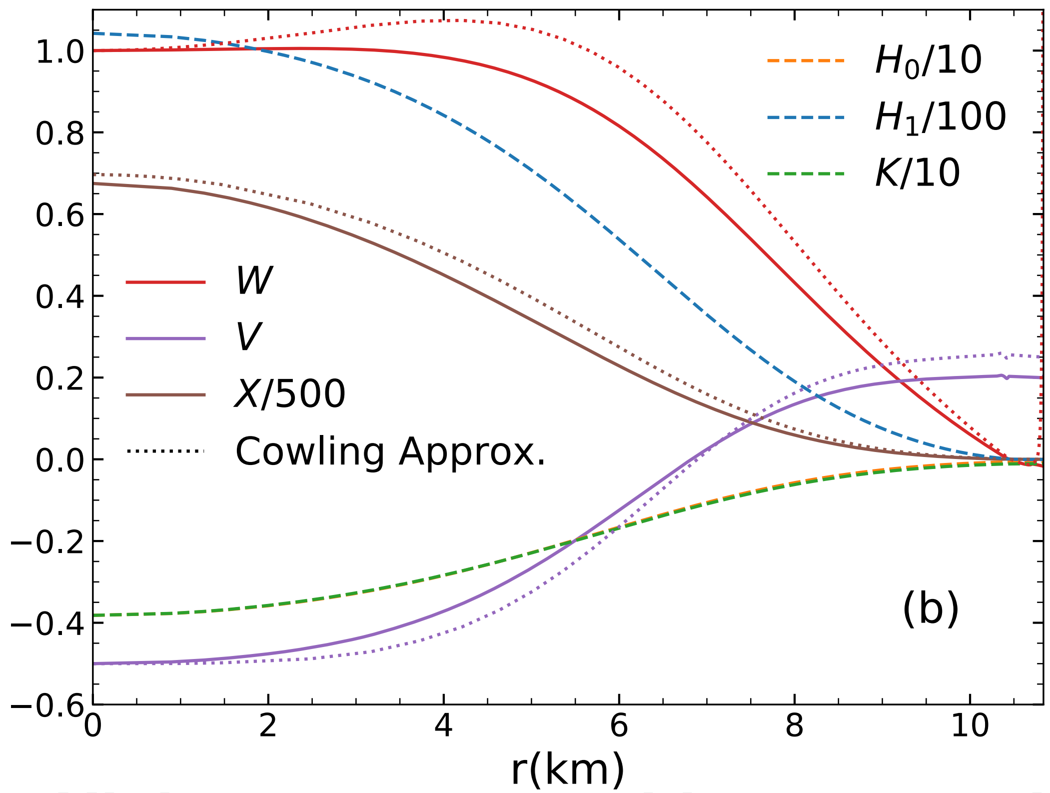
(Inverse Cowling)

- Newtonian Cowling approximation:
  2 ODEs
  Cowling, Thomas G 1941

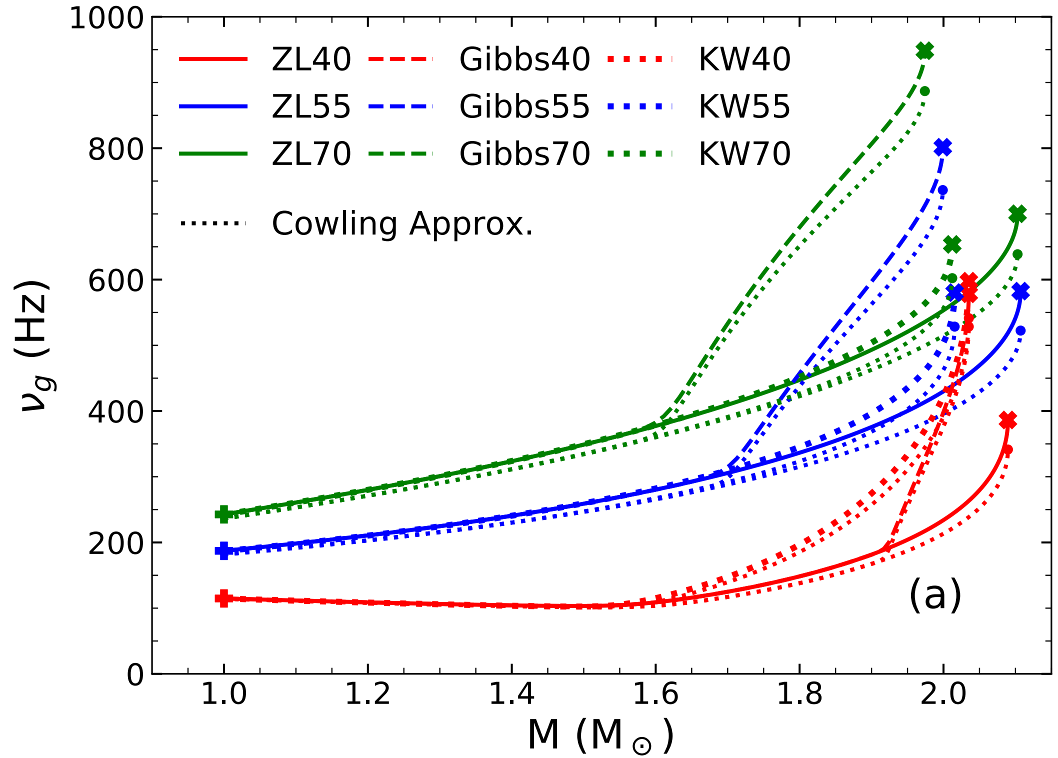
Cowling Approximation in Compositional g-modes



Low mass compositional g-mode



High mass compositional g-mode



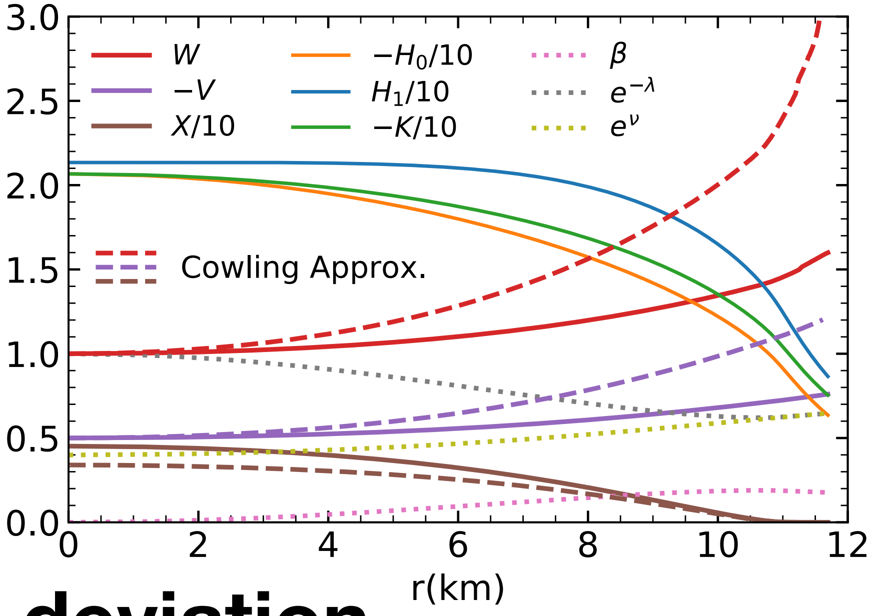
Cowling approximation: up to 10% deviation from the linearized full GR

Zhao, Constantinou,
Jaikumar and Prakash 2022
https://arxiv.org/abs/2202.01403

f-mode with Analytical TOV Solutions

- Dimensionless frequency:

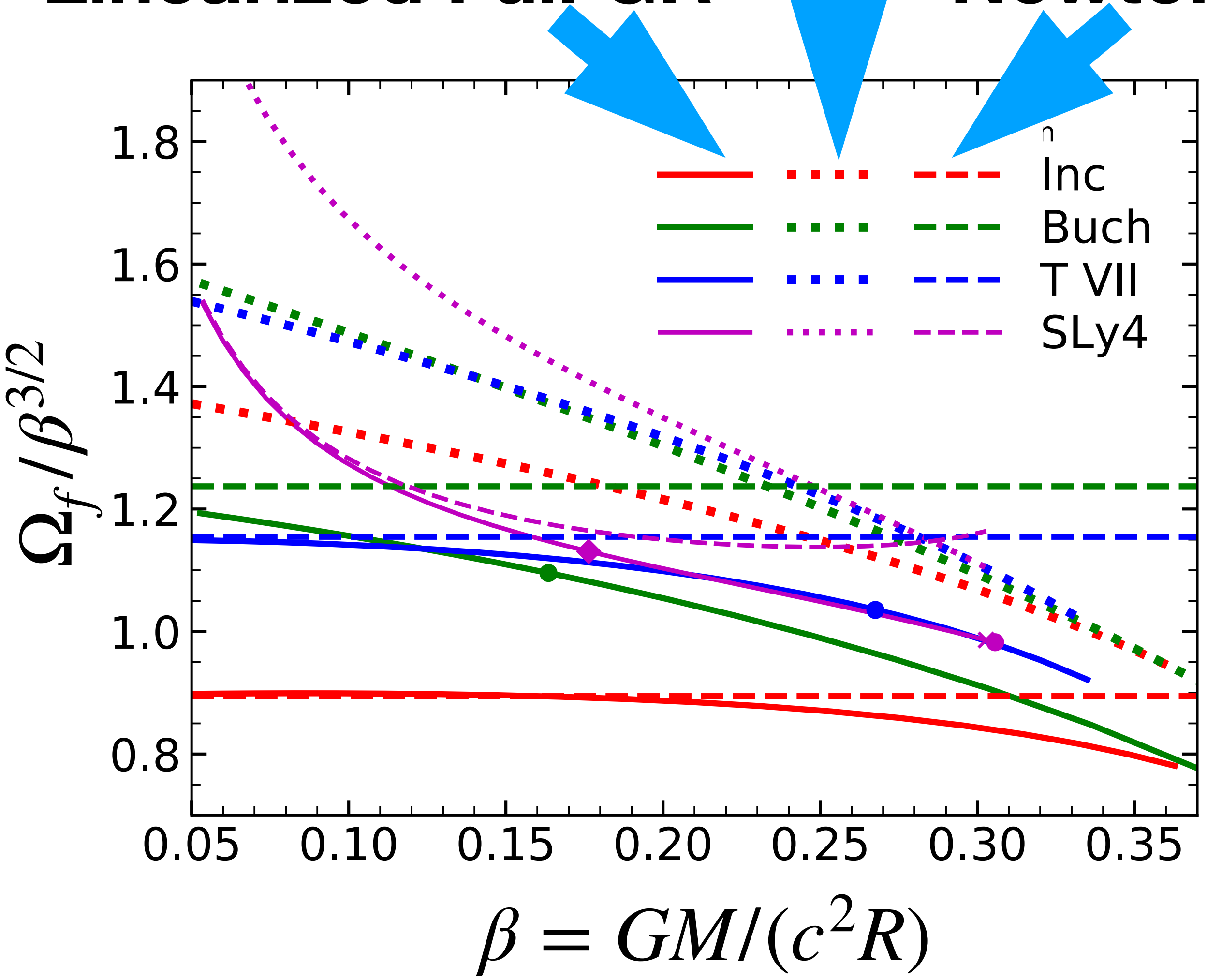
$$\Omega_f = GM\omega_f/c^3 \qquad (\propto \beta^{3/2} \text{ in Newtonian})$$

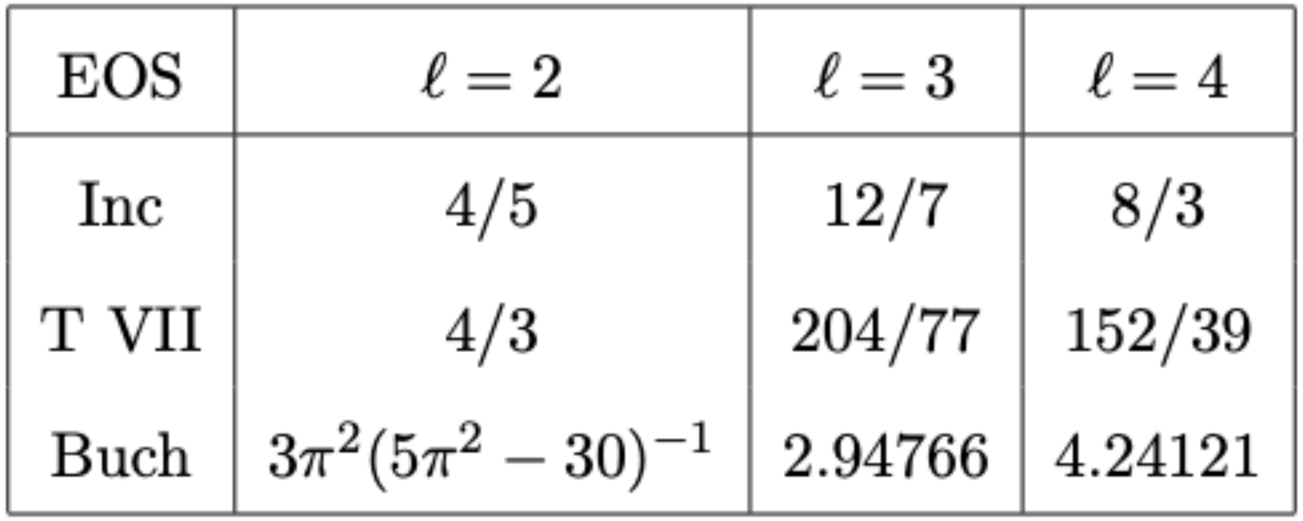


Cowling approximation: up to 30% deviation

Linearized Full GR

Newtonian: up to 15% deviation





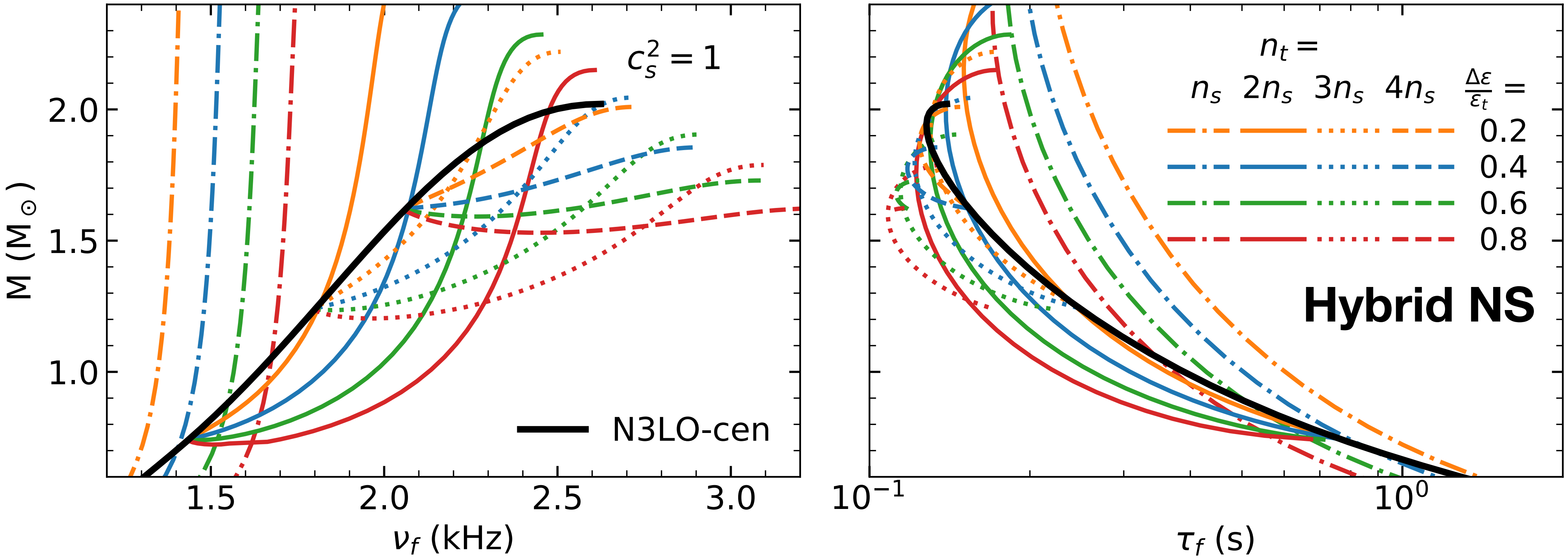
EOS	$\ell = 2$	$\ell = 3$	$\ell = 4$
Inc	4/5	12/7	8/3
T VII	4/3	204/77	152/39
Buch	$3\pi^2(5\pi^2 - 30)^{-1}$	2.94766	4.24121

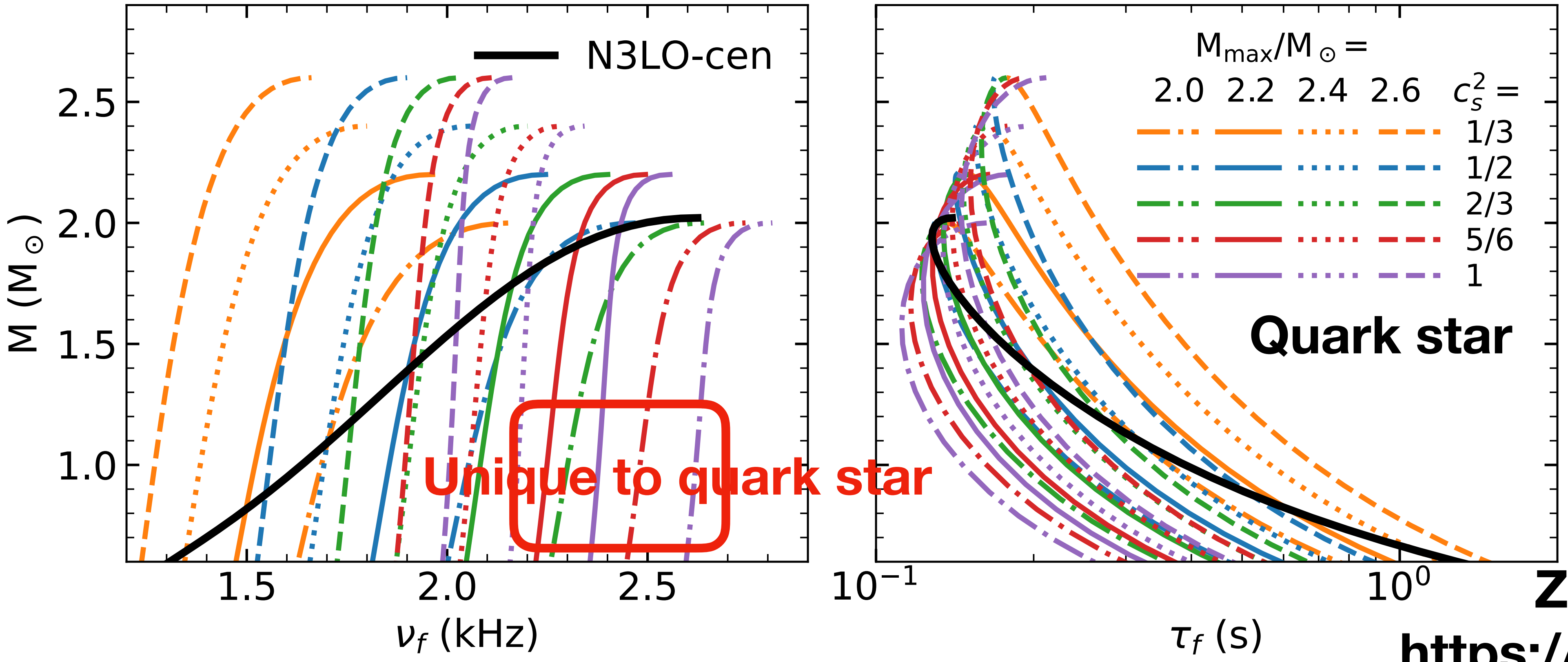
Table: Newtonian $\Omega_f/\beta^{3/2}$

Zhao & Lattimer 2022

https://arxiv.org/abs/2204.03037

f-mode with Hybrid and Quark EOS





Unique to quark star

Frequency:

$$\nu_f \in (1.3 - 2.8) \text{ kHz}$$

Damping time:

$$\tau \in (0.1, 1) \text{ s}$$

Zhao & Lattimer 2022

https://arxiv.org/abs/2204.03037

One node branch

- Lowest order pressure mode have zero node which is named as f-mode (fundamental).

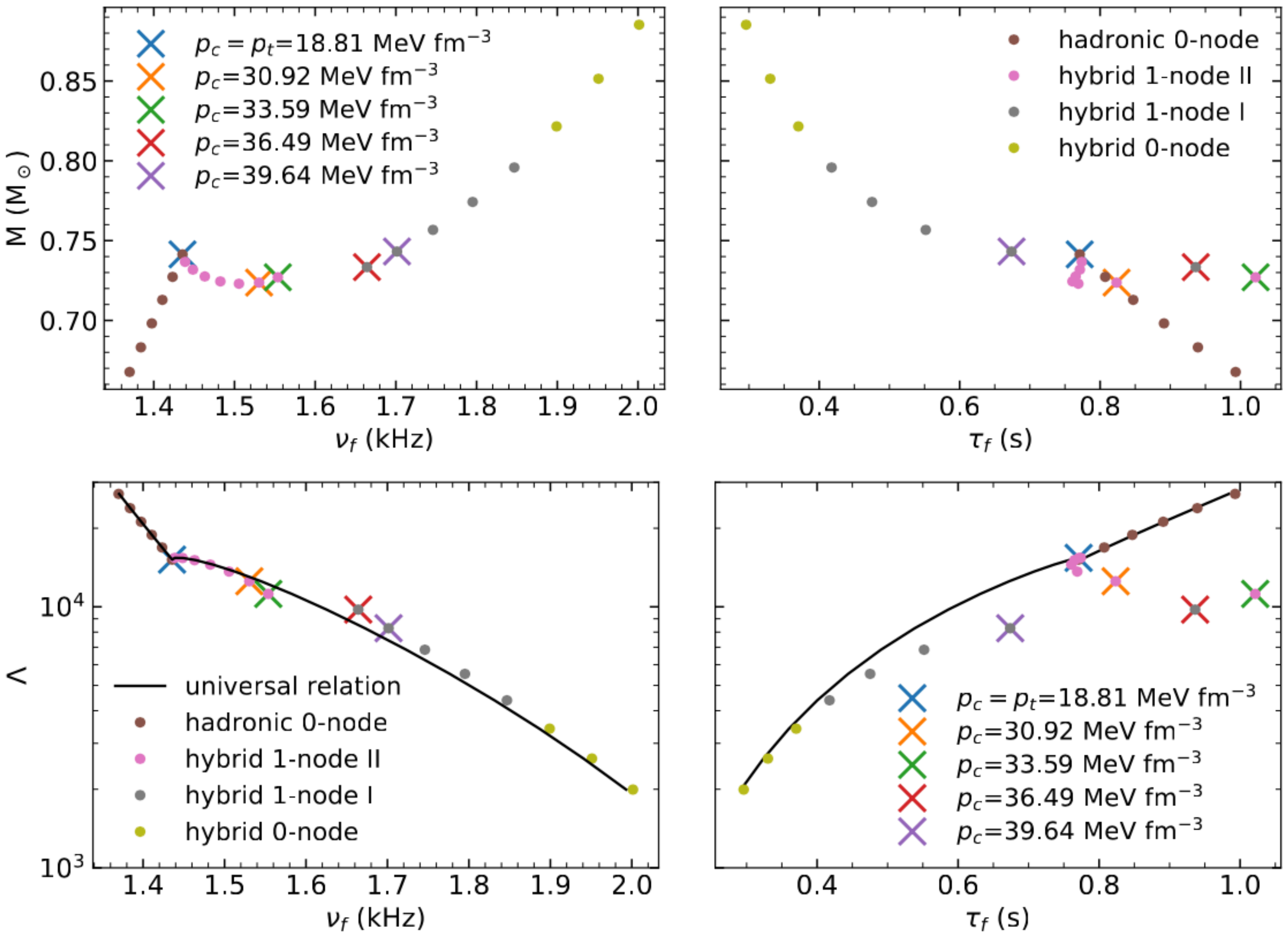
- However, in case of hybrid NS lowest order pressure mode sometimes have one node due to strong density discontinuity.

- Stars with radial nodes in V only we refer to as 1-node I.

- hybrid stars have a radial node (zero) in the fluid and metric perturbation amplitudes X, W, H_0, H_1, K (but not V, which, however discontinuously changes sign) at a radius slightly larger than the phase transition radius R_t. We will call this type of behavior 1-node II

One node branch

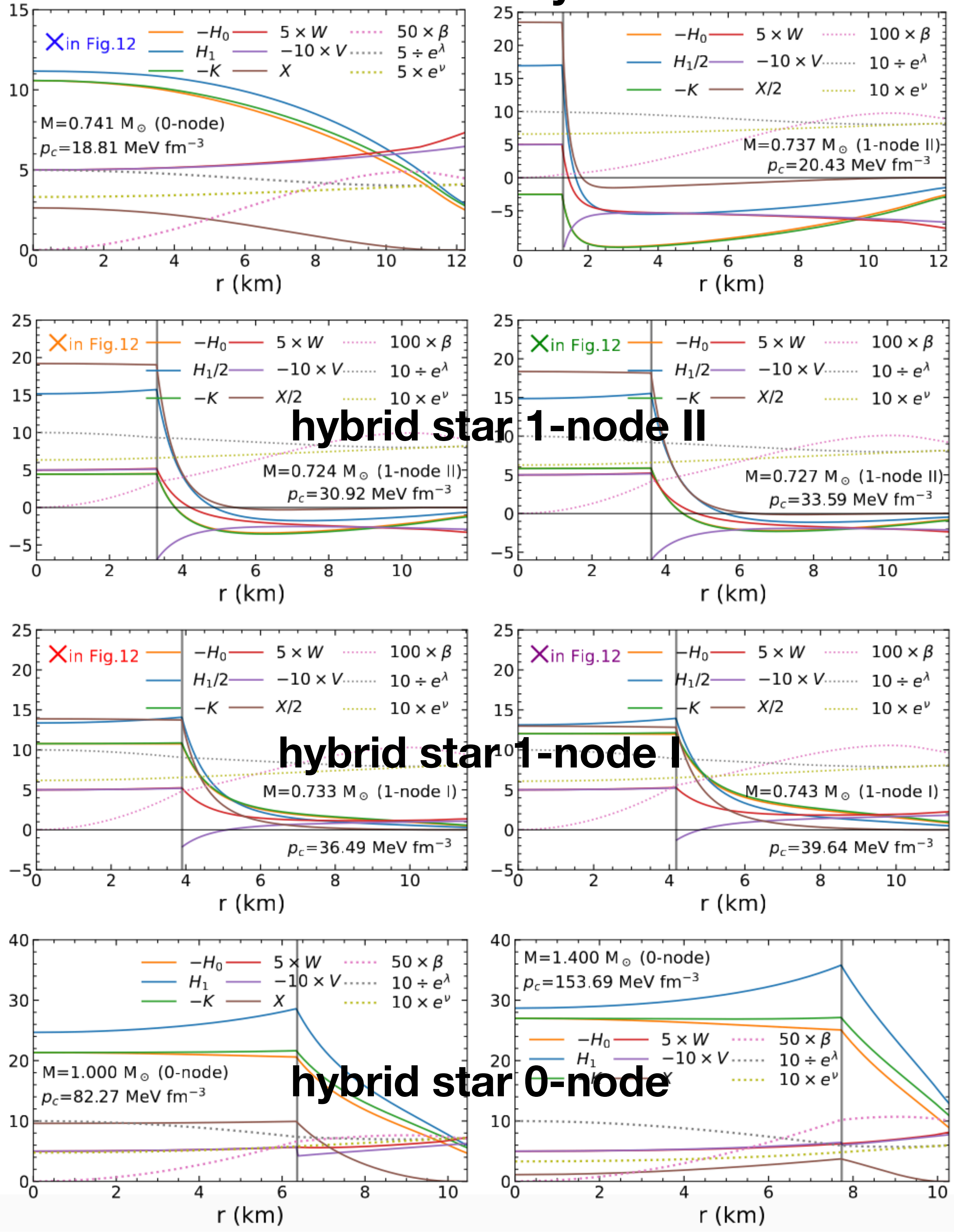
hybrid star with 1-node deviates away from f-I-love-Q relation



Zhao & Lattimer 2022

hadronic star 0-node

hybrid star 1-node II



hybrid star 1-node II

hybrid star 1-node I

hybrid star 0-node

Typical f-mode oscillation

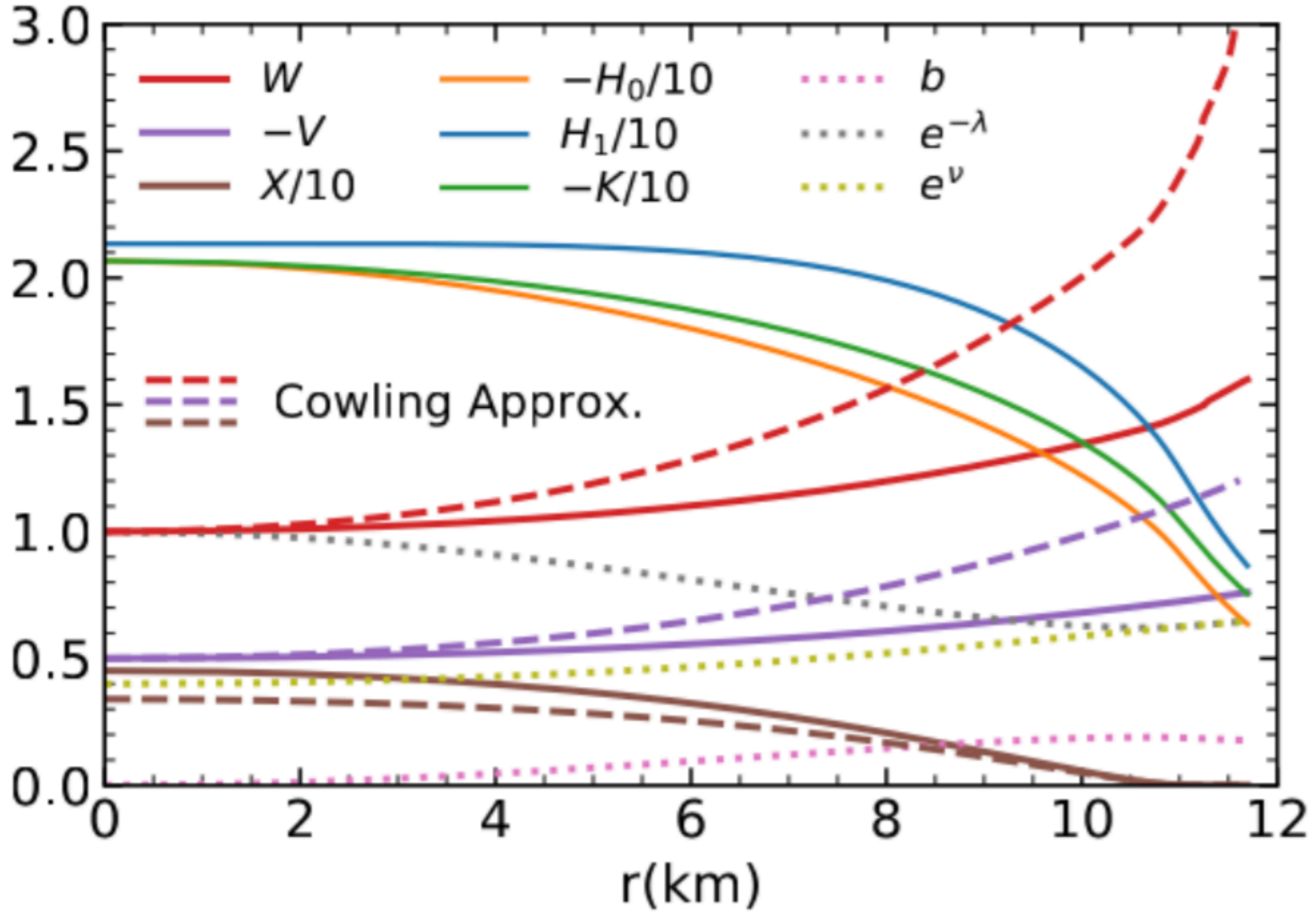
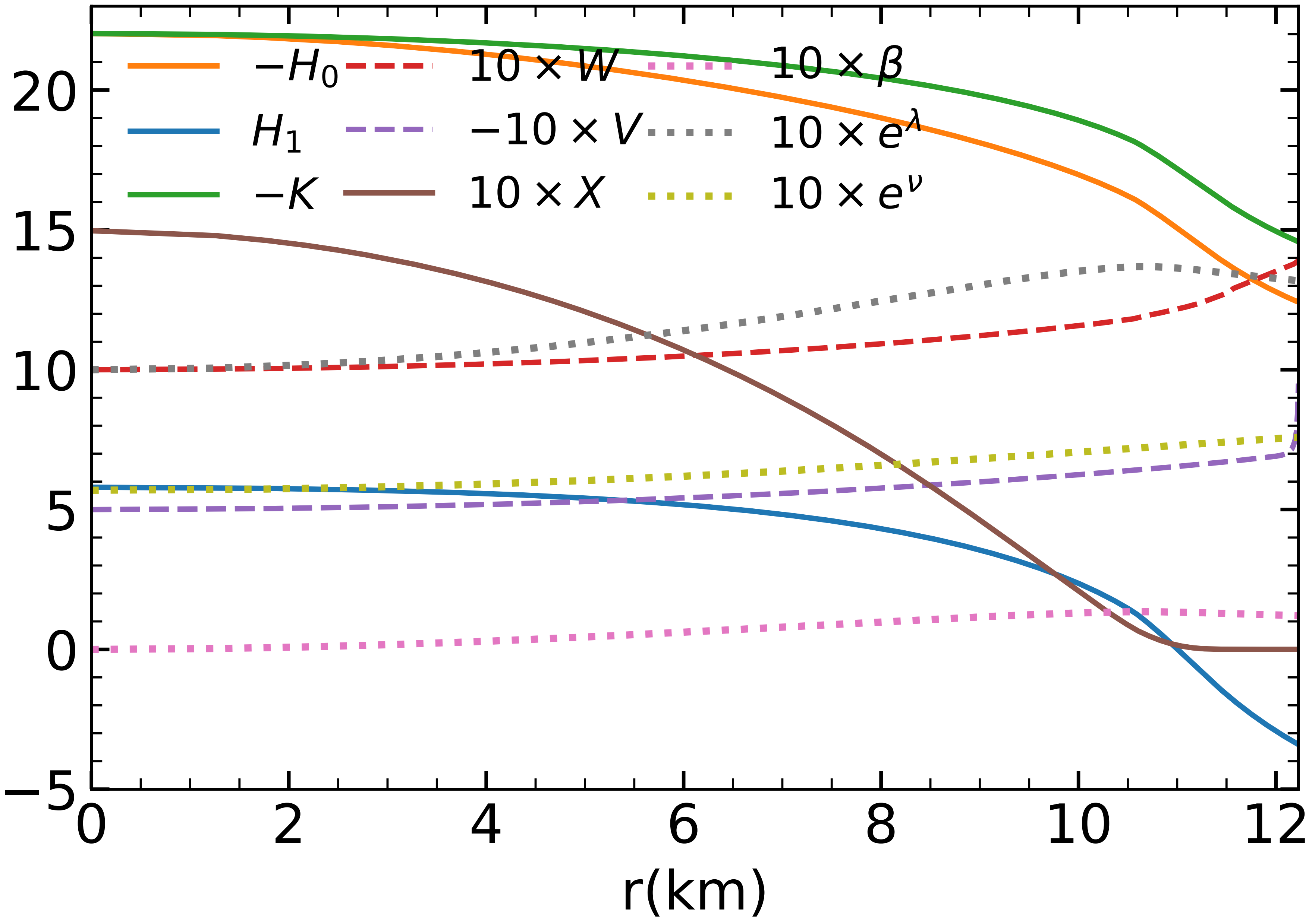
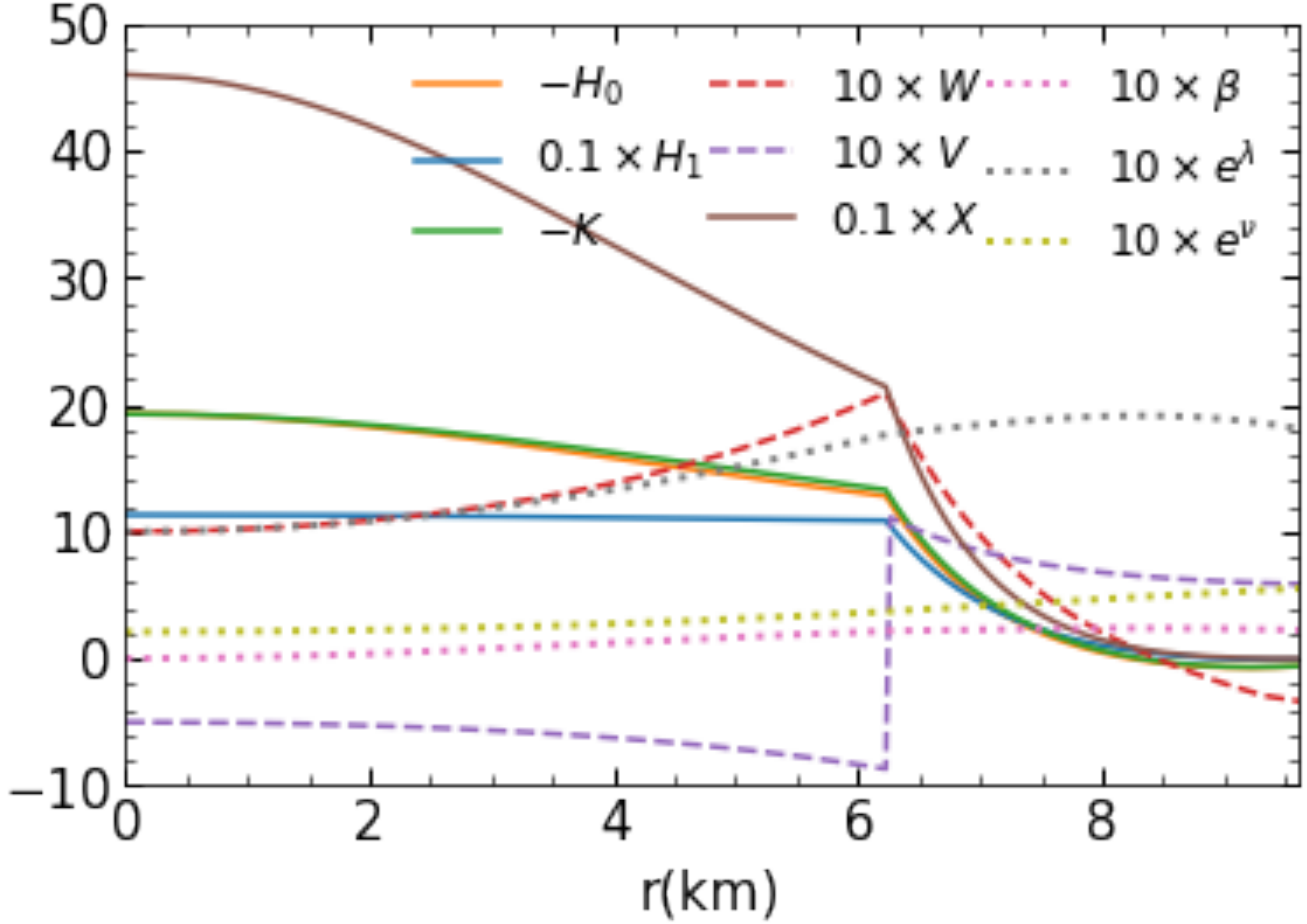


FIG. 14. Metric perturbation amplitudes, fluid perturbation amplitudes for non-radial oscillations with $\ell = 2$ with (dashed curves) and without (solid curves) the Cowling approximation, and static metric functions (dotted curves) inside a $1.4M_\odot$ NS computed with the Sly4 EOS [85]. H_0, H_1 and K are in units of $\varepsilon_s = 152.26$ MeV fm^{-3}, X is in units of ε_s^2, and W, V, ν and λ are dimensionless. Only real parts of the perturbation amplitudes are plotted.

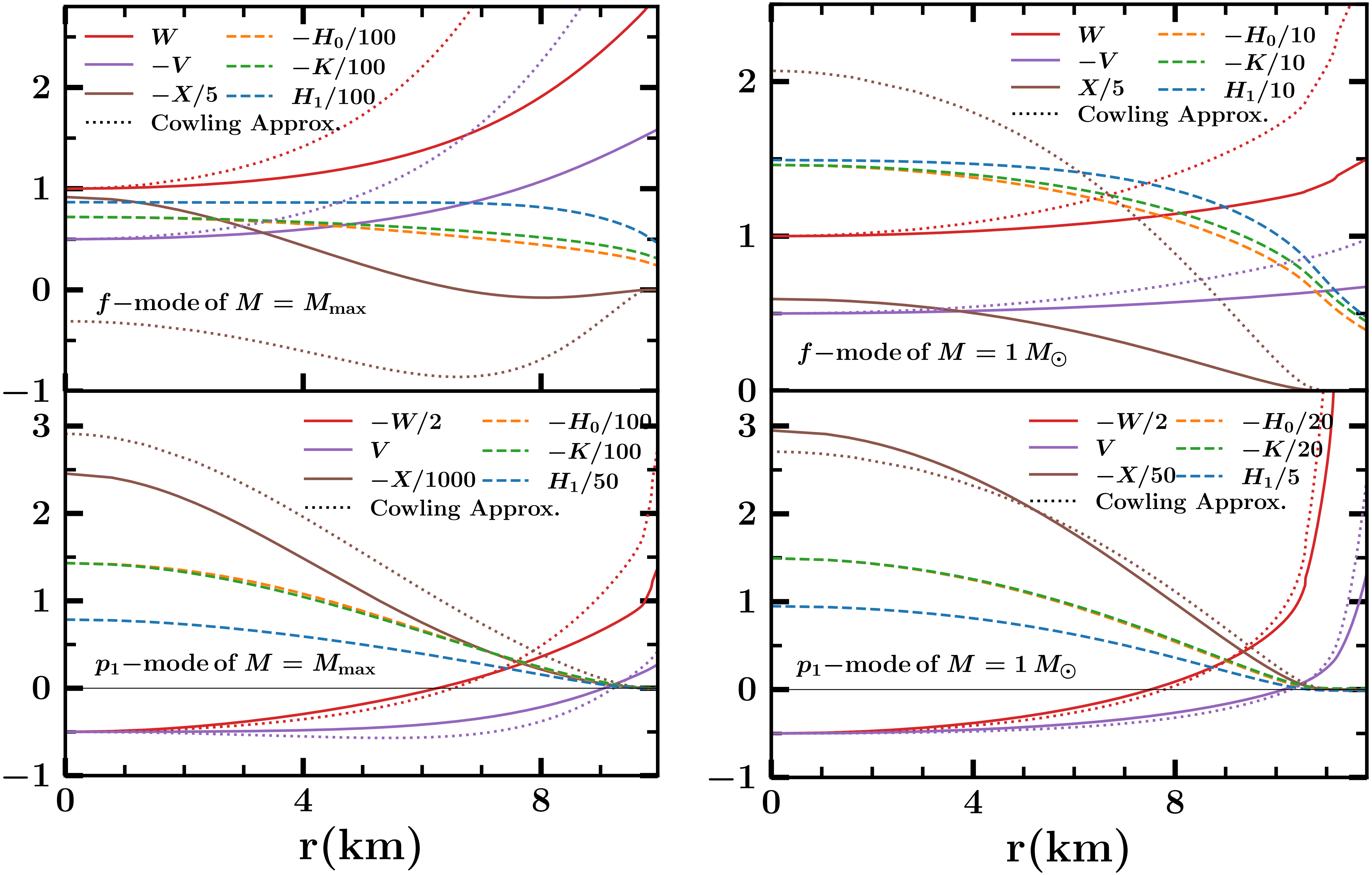
Compositional g-mode of hadronic NS



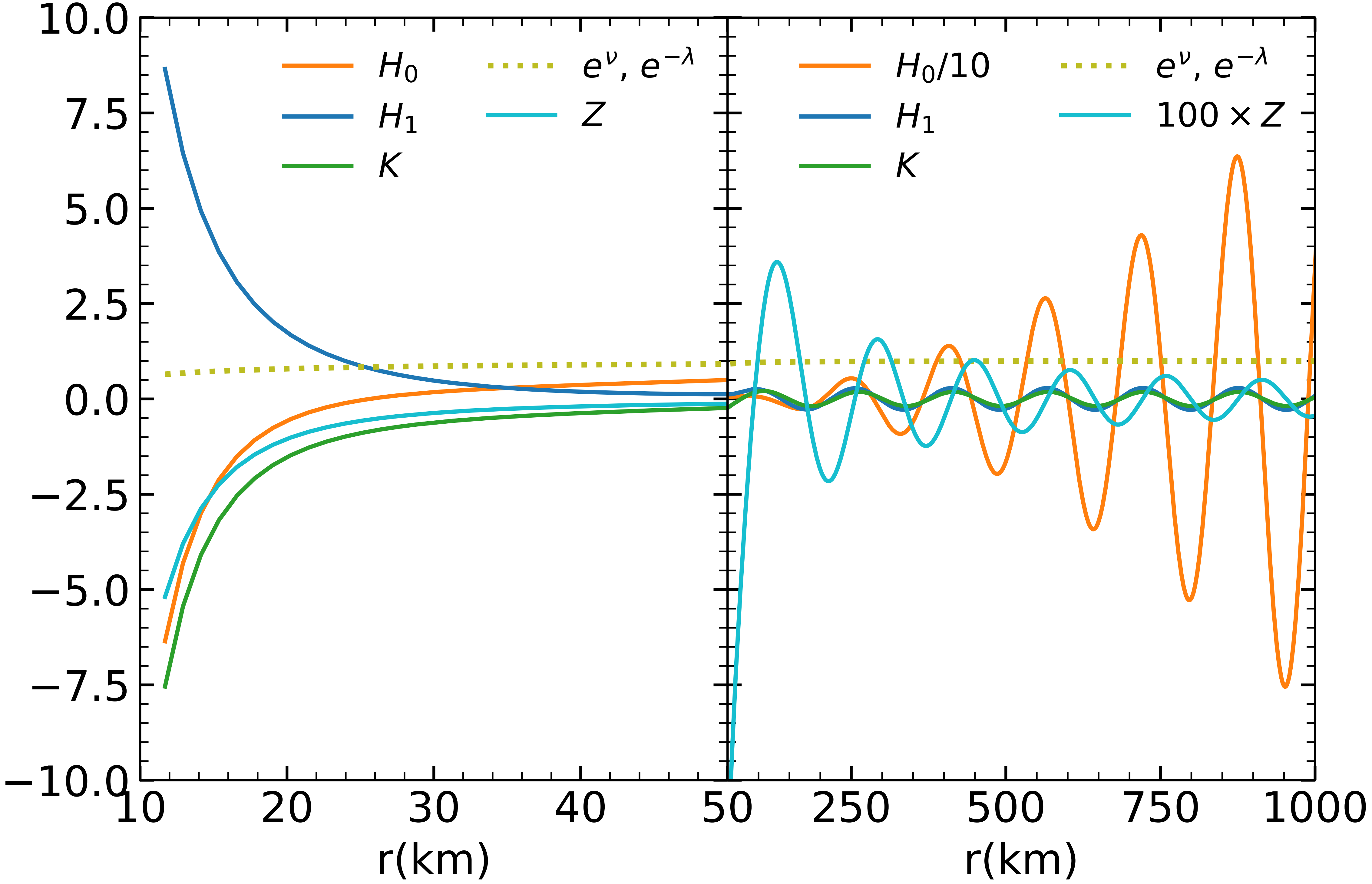
Discontinuity g-mode of hadronic NS



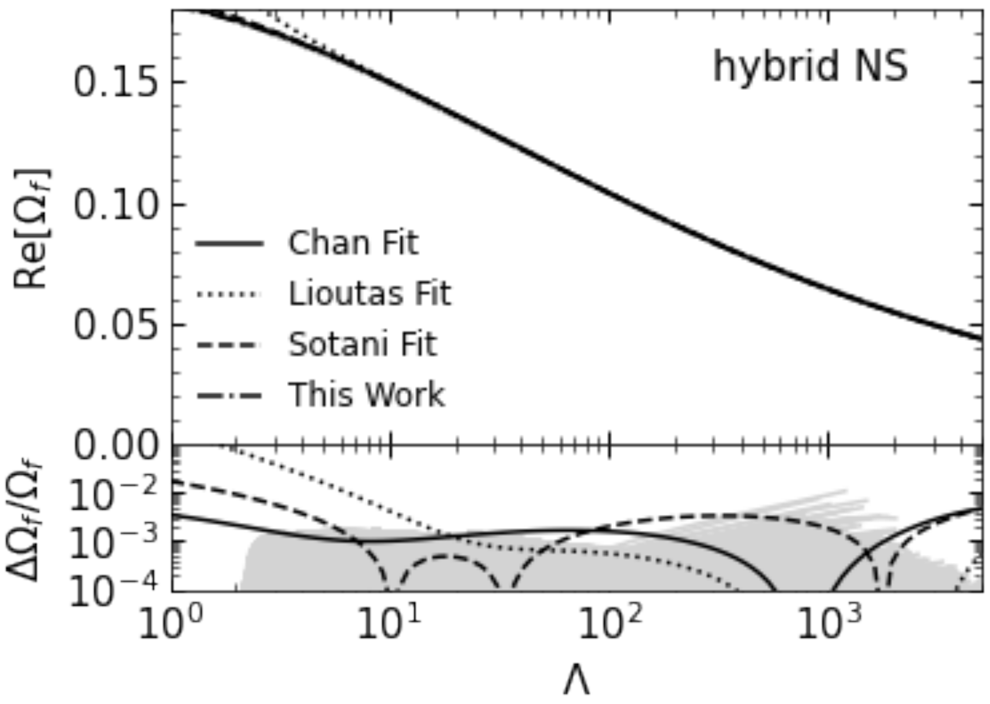
f-mode vs p-mode oscillations



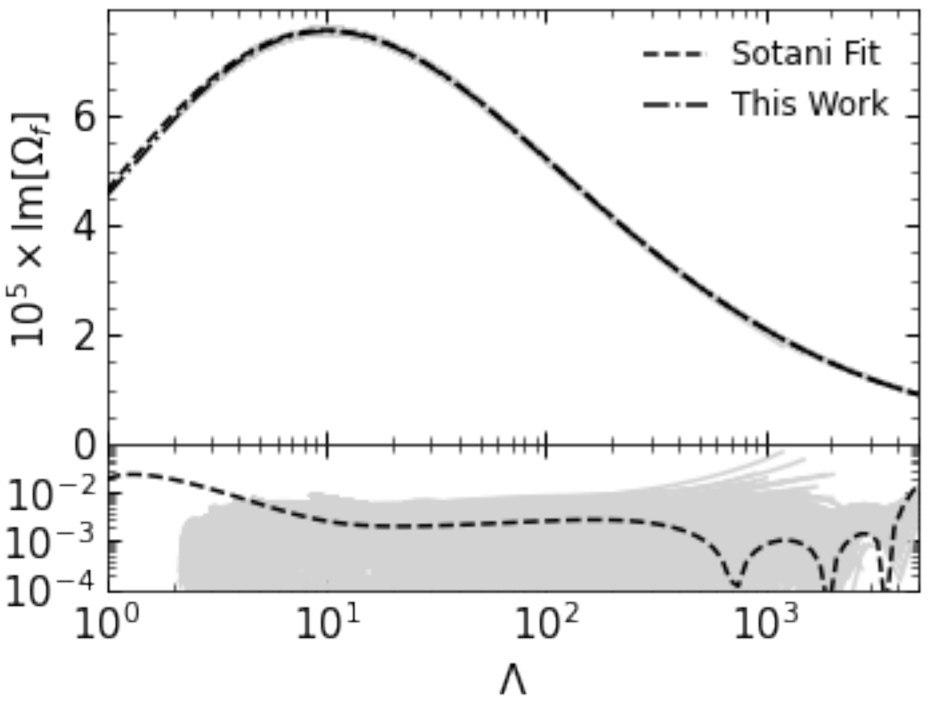
Outside metric perturbation (f-mode as example)



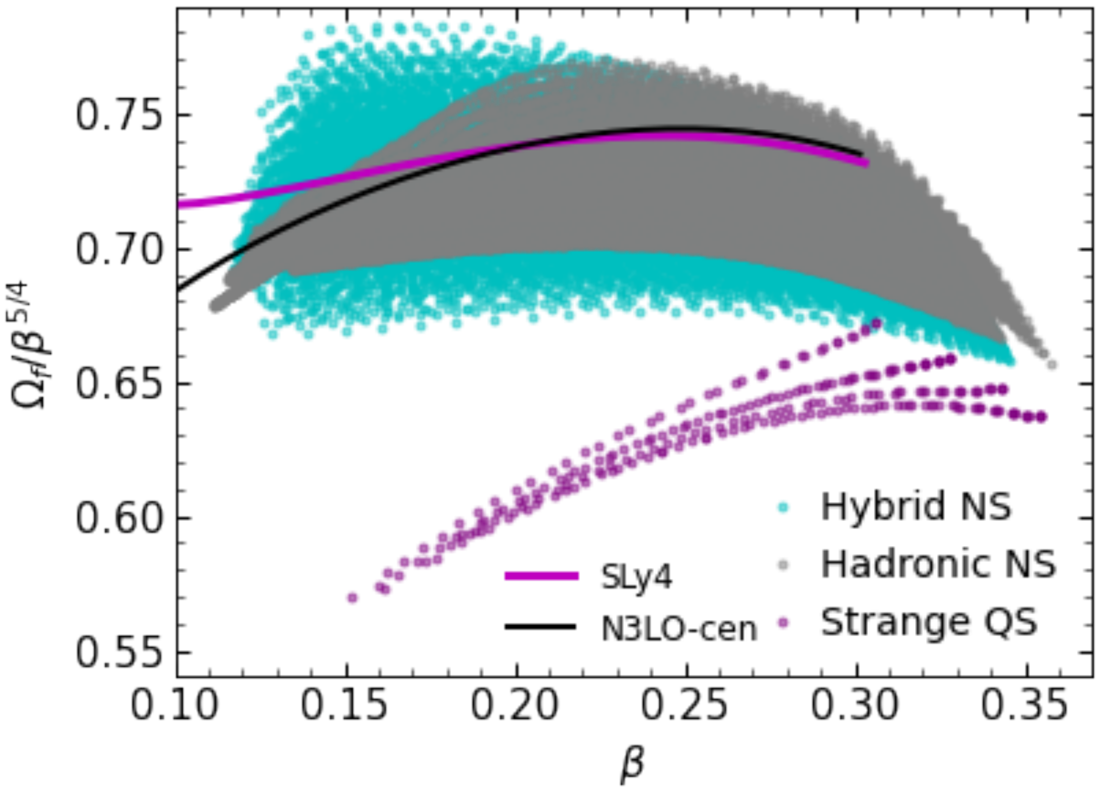
f-mode universal relations



0.1% deviation except for one-node branches



1% deviation except for one-node branches

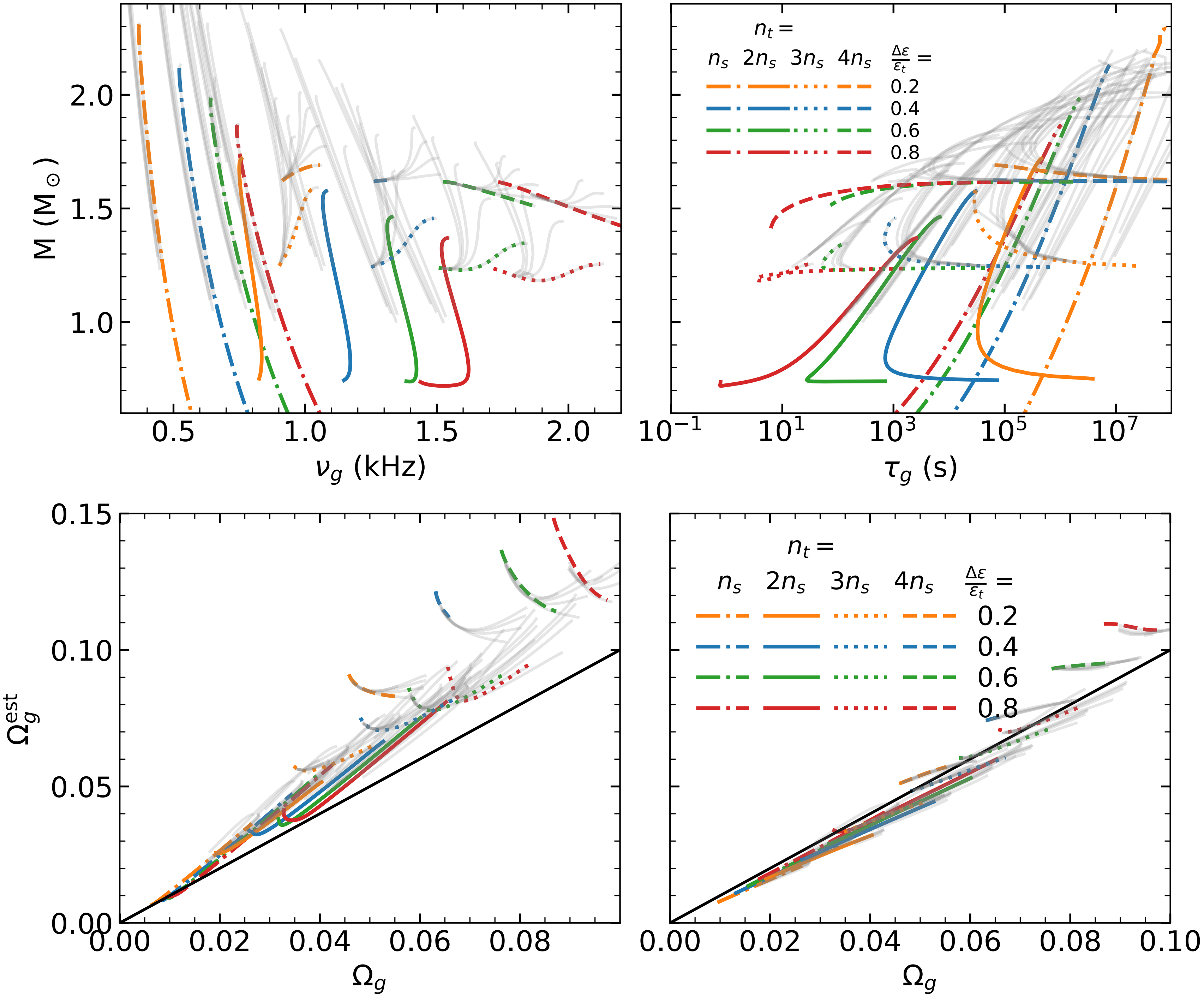


Slightly weaker for $\Omega_f - \bar{I}$, see in the paper

$$\Omega_f = (0.714 \pm 0.056)\ \beta^{5/4}$$

Zhao & Lattimer 2022

Discontinuity g-mode for hybrid NS



Newtonian 1D gravity wave:

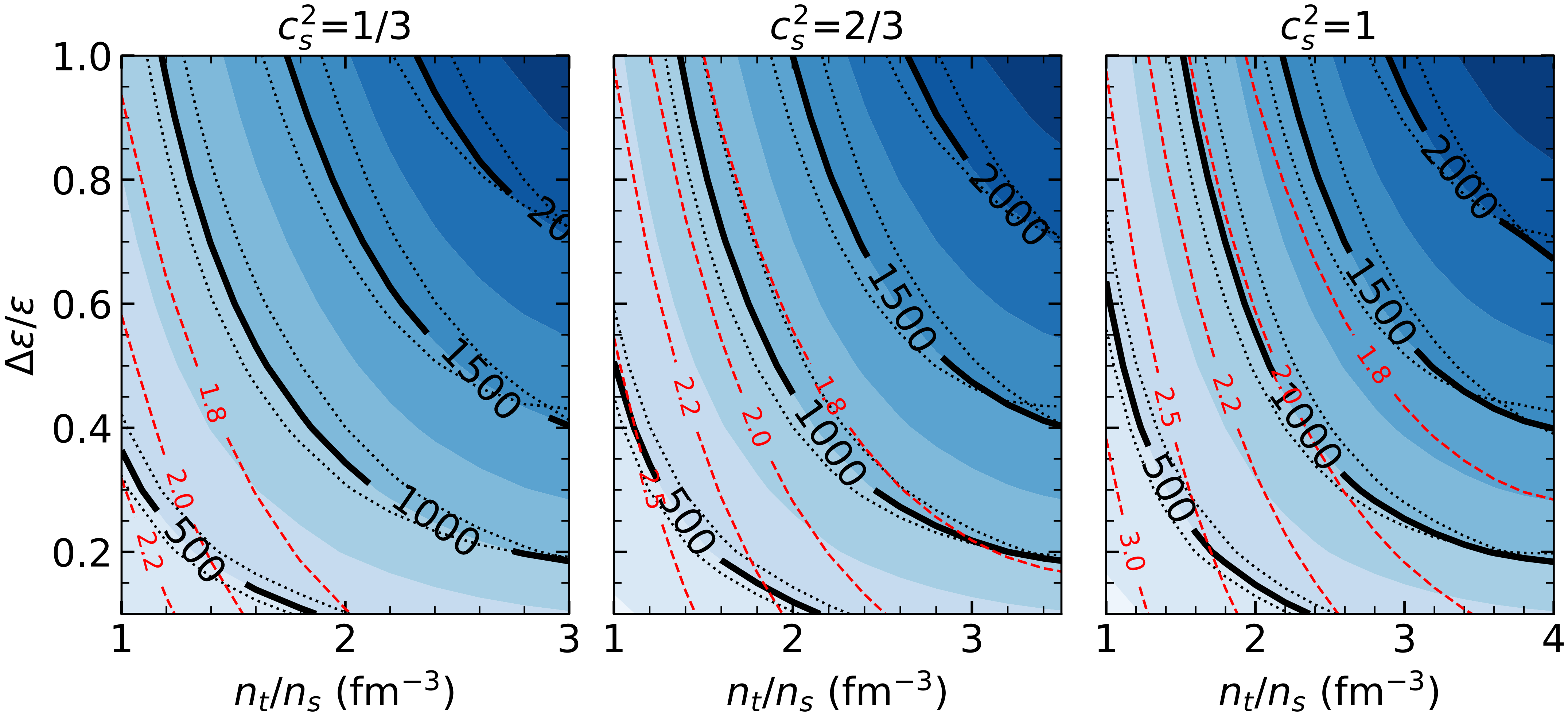
$$\omega_g^2 = \frac{(\varepsilon_+ - \varepsilon_-)gk}{\varepsilon_+/\tanh(kd_+) + \varepsilon_-/\tanh(kd_-)}$$

Landau and Lifshitz 1960

Zhao & Lattimer 2022

Discontinuity g-mode semi-universal relation

Contour of $\nu_g(n_t,\ \Delta_\varepsilon,\ c_s^2)\,|_{M=M_{max}}$



Dotted: Chrial EFT Uncertanty 5%

Dashed: maximum mass

$$\nu_g = (326.4 \pm 36.1\,\textbf{Hz})\left(\frac{p_t}{\textbf{MeV fm}^{-3}}\right)^{0.268+0.146c_s/c}\sqrt{\frac{\Delta\varepsilon}{\varepsilon_t}\left(\frac{c}{c_s}\right)}$$

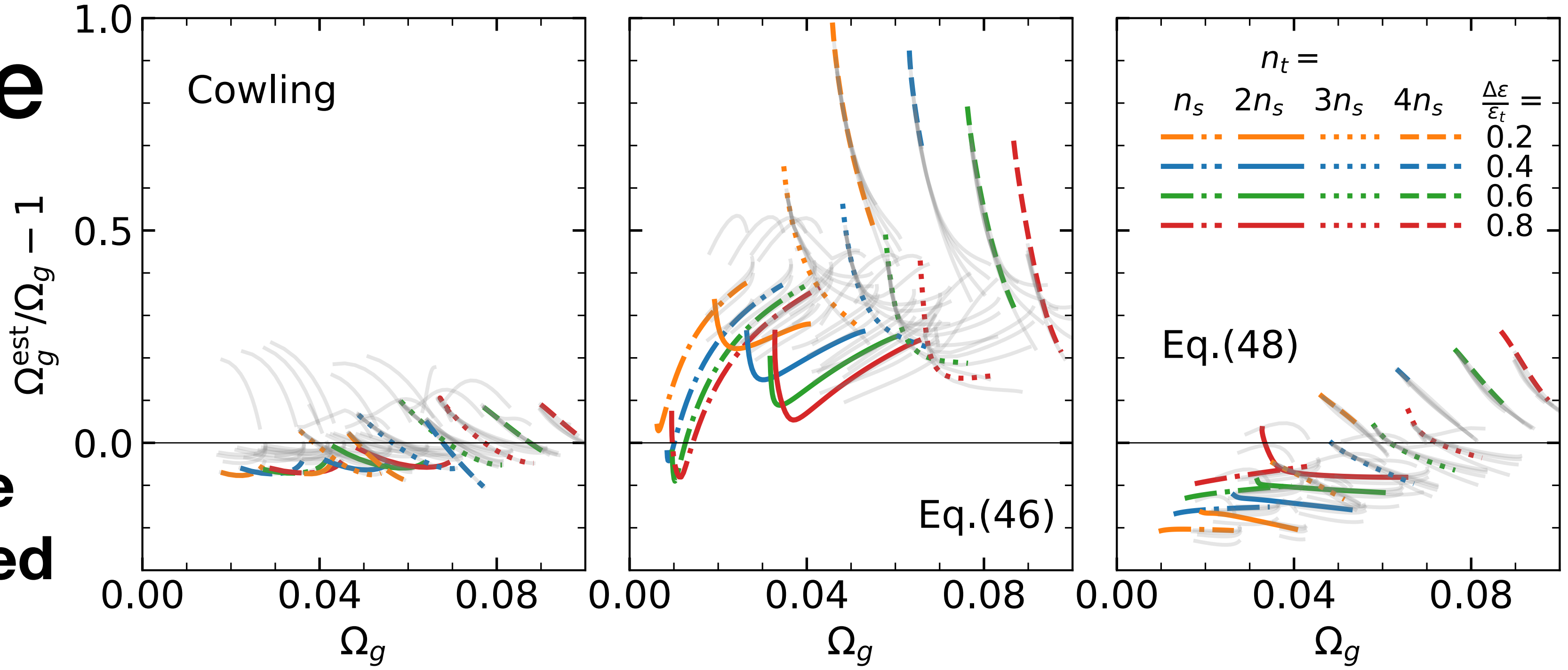
Frequency: $\nu_g < 0.8\ (1.5)$ kHz for $c_s^2 = c^2/3\ (c^2)$

Damping time: $\tau > 100\ (10000)$ s

Zhao & Lattimer 2022

Discontinuity g-mode

First order transition

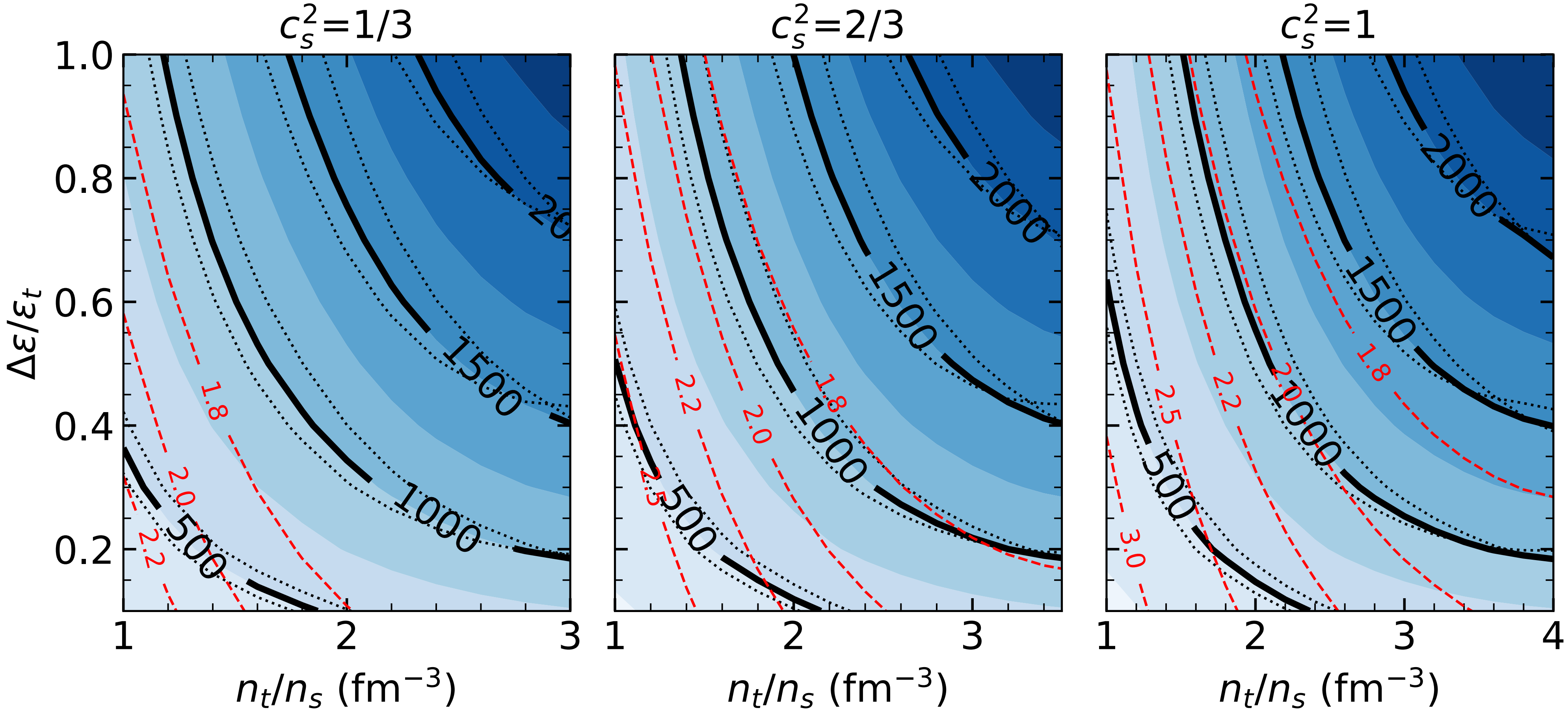


density discontinuity

transition density

Inner core sound speed

Contour of $\nu_g(n_t,\ \Delta_\varepsilon,\ c_s^2)|_{M=M_{max}}$ **Hz**



Dotted: Chrial EFT Uncertanty 5%

Dashed: maximum mass

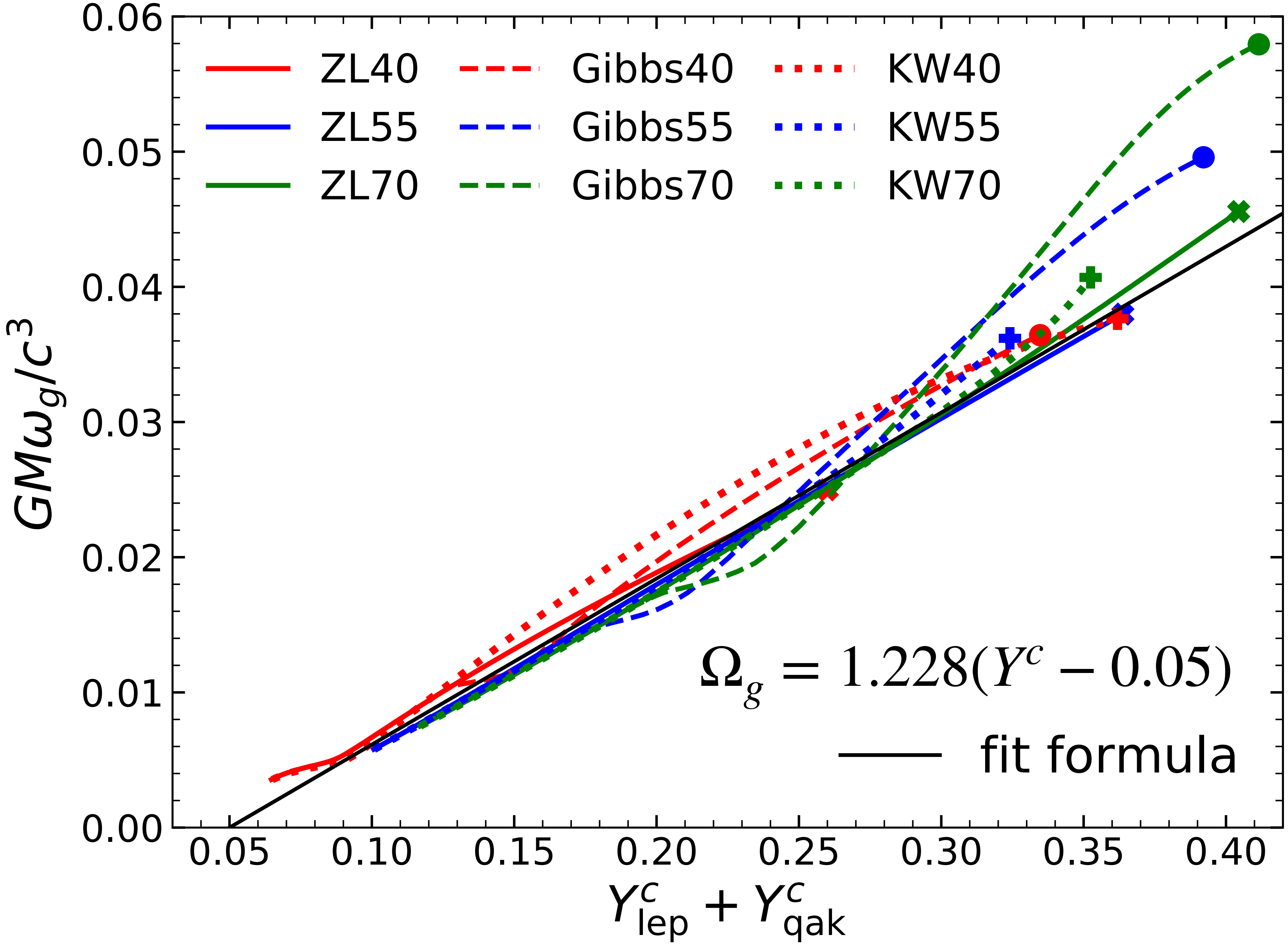
Frequency: $\nu_g < 0.8\ (1.5)$ kHz for $c_s^2 = c^2/3\ (c^2)$

Damping time: $\tau > 100\ (10000)$ s

Zhao & Lattimer 2022

BACK

Compositional g-mode universal relation



Y_{lep} impact Proto-NS neutrino emission

symmetry energy S(n)

G-mode frequency linearly correlated with lepton fraction and quark fraction at center of NS

Zhao, Constantinou, Jaikumar and Prakash 2022
https://arxiv.org/abs/2202.01403

Thank you!