Local and global simulations of collective neutrino oscillations

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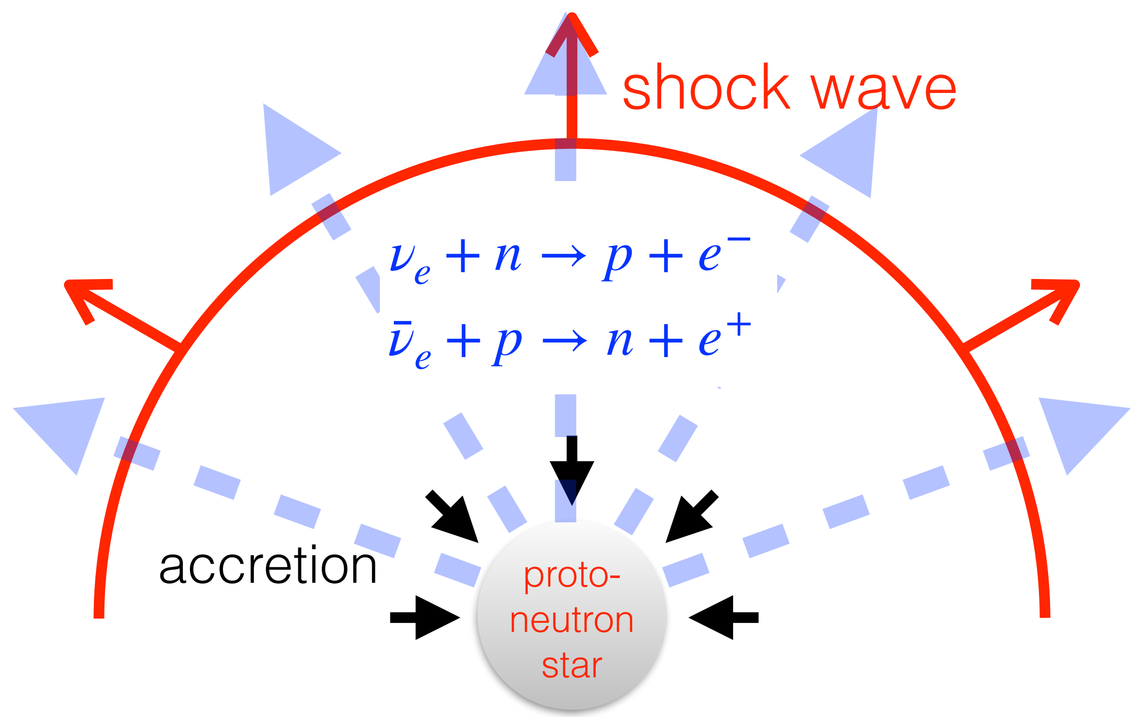
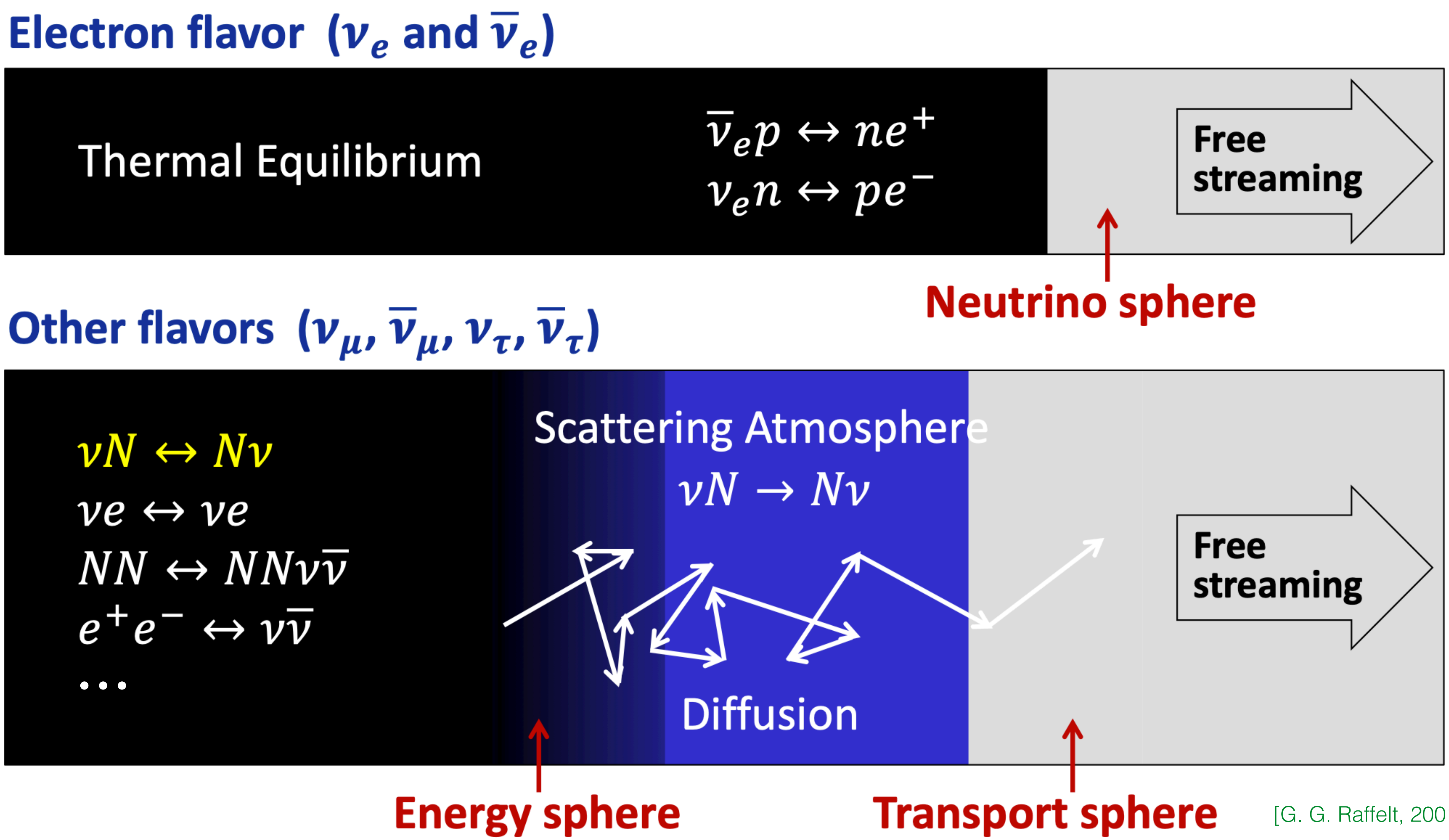
Neutrinos in supernovae



- intense sources of neutrinos in all flavors
- Neutrinos play important roles in dynamics and nucleosynthesis.

Neutrino collisional processes

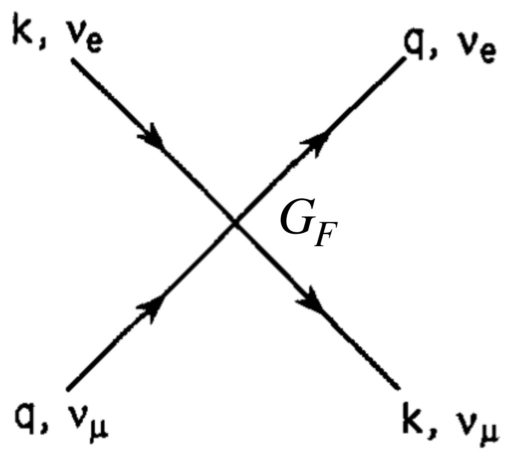


[G. G. Raffelt, 2001]

Collective neutrino oscillations

$$\mathbf{H}_{\nu\nu} = \sqrt{2}G_F \int d\mathbf{p}' \,(1 - \hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')[\varrho(\mathbf{p}') - \bar{\varrho}^*(\mathbf{p}')]$$

$$\varrho = \begin{pmatrix} f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho^*_{e\mu} & f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho^*_{e\tau} & \varrho^*_{\mu\tau} & f_{\nu_\tau} \end{pmatrix}$$



- Flavor mixing in vacuum
- Mikheyev–Smirnov-Wolfenstein (MSW) matter effect
- Collective phenomena:
  - Matter-neutrino resonances
  - Slow instability: different vacuum oscillation frequencies for different energies
  - Fast instability: angular distributions of different neutrino species cross over each other.
  - Collisional instability: asymmetric collisional rates (small Y_e near proto-neutron star).

[Abbar, Balantekin, Bhattacharyya, Burrows, Capozzi, Carlson, Chakraborty, Cirigliano, Dasgupta, Duan, Ehring, Fernandez, Fischer, Foucart, Friedland, Froustey, Fuller, George, Grohs, Hansen, Janka, Johns, Just, Kato, Kneller, Li, Lin, Liu, Lunardini, Martin, Martínez-Pinedo, McLaughlin, Morinaga, Nagakura, Padilla-Gay, Patwardhan, Qian, Raffelt, Richers, Roggero, Rrapaj, Sasaki, Sawyer, Siegel, Sigl, Shalgar, Takiwaki, Tamborra, Vlasenko, Volpe, Willcox, Wu, Xiong, Yamada, Zaizen, Zhu (and many others)]

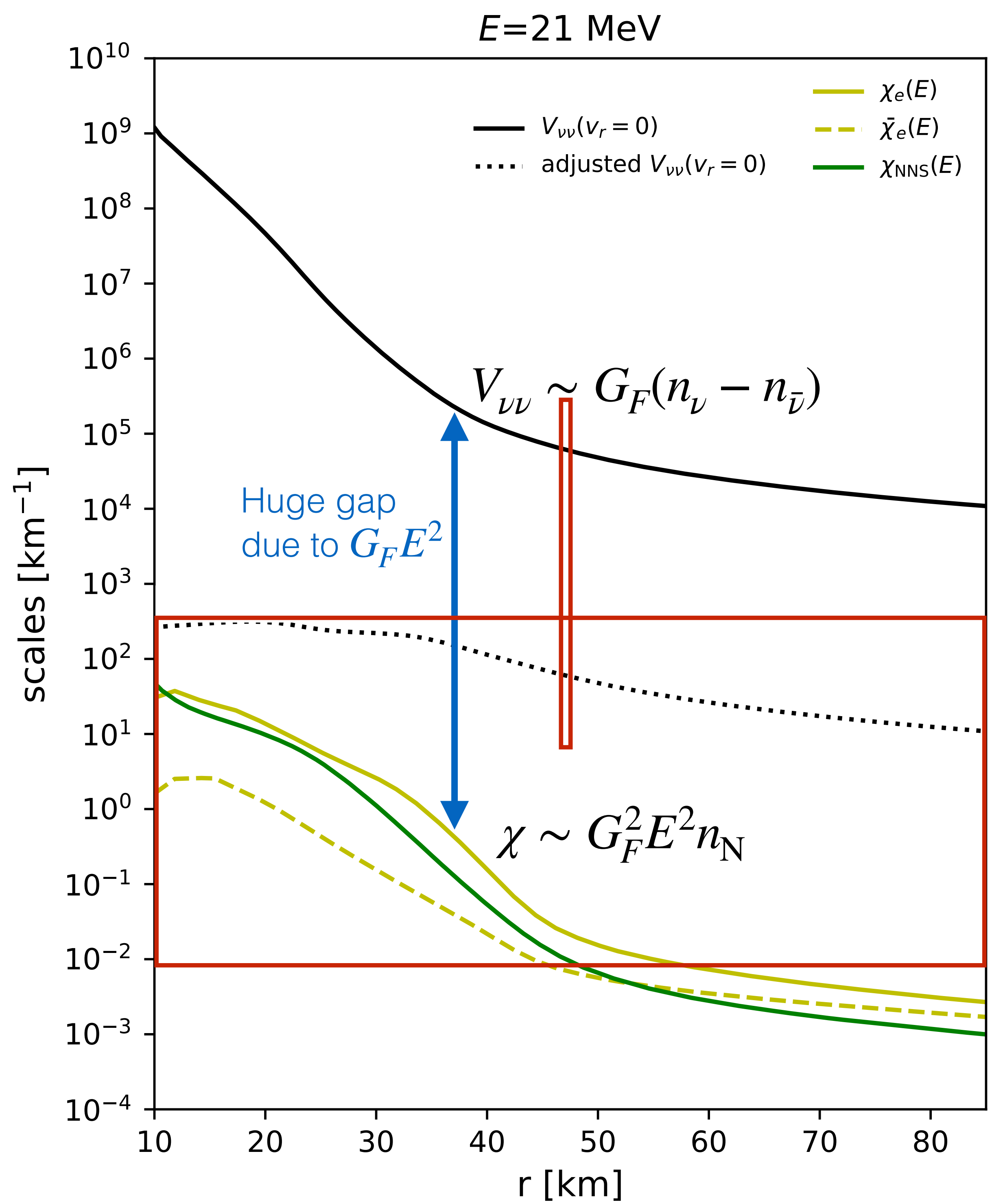
A challenging problem

$$\frac{D\varrho}{Dt} = -i[\mathbf{H}_{\rm vac} + \mathbf{H}_{\rm mat} + a_{\nu\nu}\,\mathbf{H}_{\nu\nu}, \varrho] + \mathbf{C}(\varrho)$$

- Methods:
  - discrete-ordinate
  - moment-based
  - Monte-Carlo

see talks by Evan, Sherwood & Julien

- Strategies:
  - Local box with a periodic boundary condition, without collisions or with enhanced collisions
    [J. Martin, C. Yi, H. Duan, 2020; J. Martin+, 2021]
  - Global advection, attenuation on $\mathbf{H}_{\nu\nu}$
    [H. Nagakura, M. Zaizen, 2022]



Fast flavor conversions (FFCs) in a periodic 1-D box

[M.-R. Wu, M. George, C.-Y. Lin, ZX, PRD 105 103002 (2021), arXiv: 2108.09886]

[ZX, M.-R. Wu, S. Abbar, S. Bhattacharyya, M. George, C.-Y. Lin, (2023), arXiv: 2307.11129]

Simulation set-up

- Solve the neutrino flavor evolution equation in COSEν [M. George+, 2023]

$$(\partial_t + v_z\partial_z)\varrho(v_z) = -i[\mathbf{H}_{\nu\nu}(v_z), \varrho(v_z)]$$

with $\mathbf{H}_{\nu\nu} = \mu \int_{-1}^{1} dv_z'(1 - v_z v_z')[g_\nu \varrho - g_{\bar{\nu}}\bar{\varrho}^*]$ and normalized $\varrho \equiv \begin{pmatrix} \varrho_{ee} & \varrho_{ex} \\ \varrho_{ex}^* & \varrho_{xx} \end{pmatrix}$

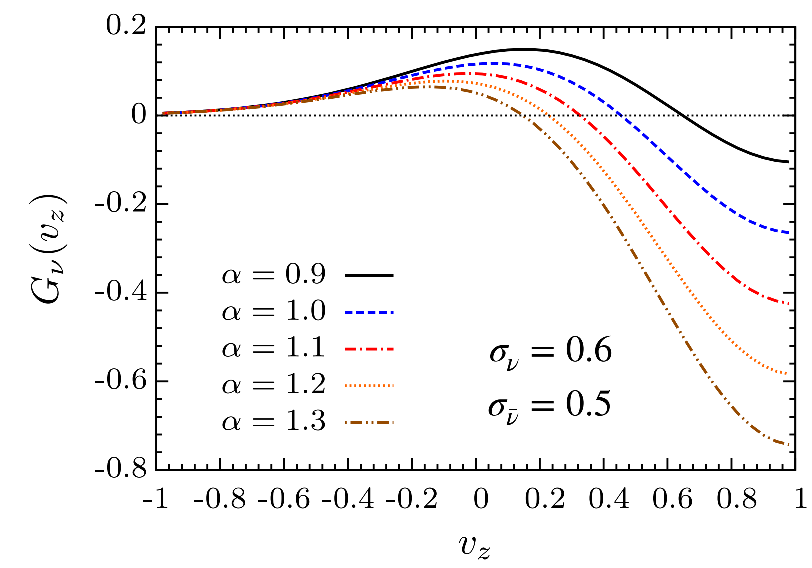
- Box size: $L = 1200\,\mu^{-1}$ (with $\mu = \sqrt{2}G_F n_{\nu_e}$) and can be $\sim \mathcal{O}(\mathrm{m})$

- Initial Gaussian distributions: $g_{\nu(\bar{\nu})}(v_z) \propto \exp[-(v_z - 1)^2/(2\sigma^2_{\nu(\bar{\nu})})]$

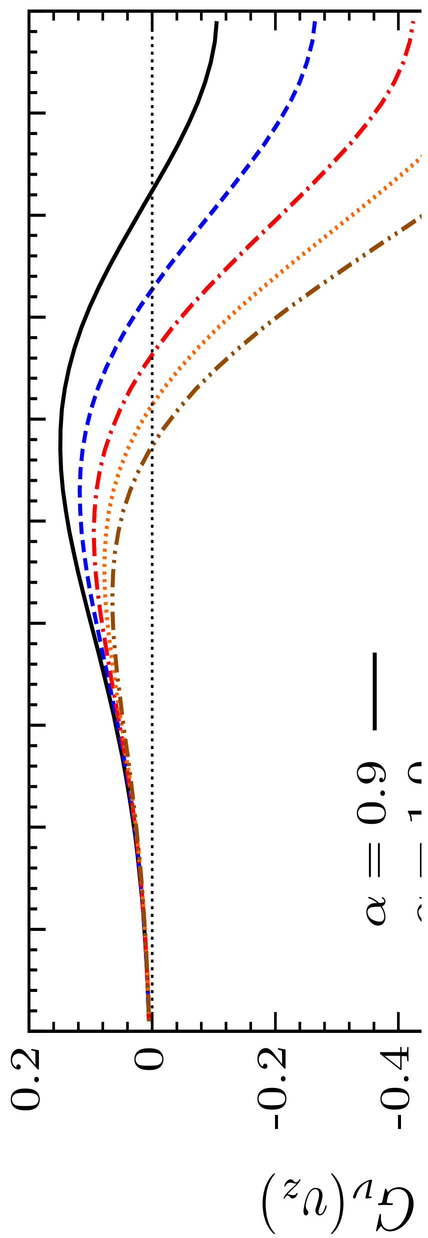
- Neutrino electron lepton number (νELN): $G_\nu(v_z) \equiv g_\nu(v_z) - g_{\bar{\nu}}(v_z)$

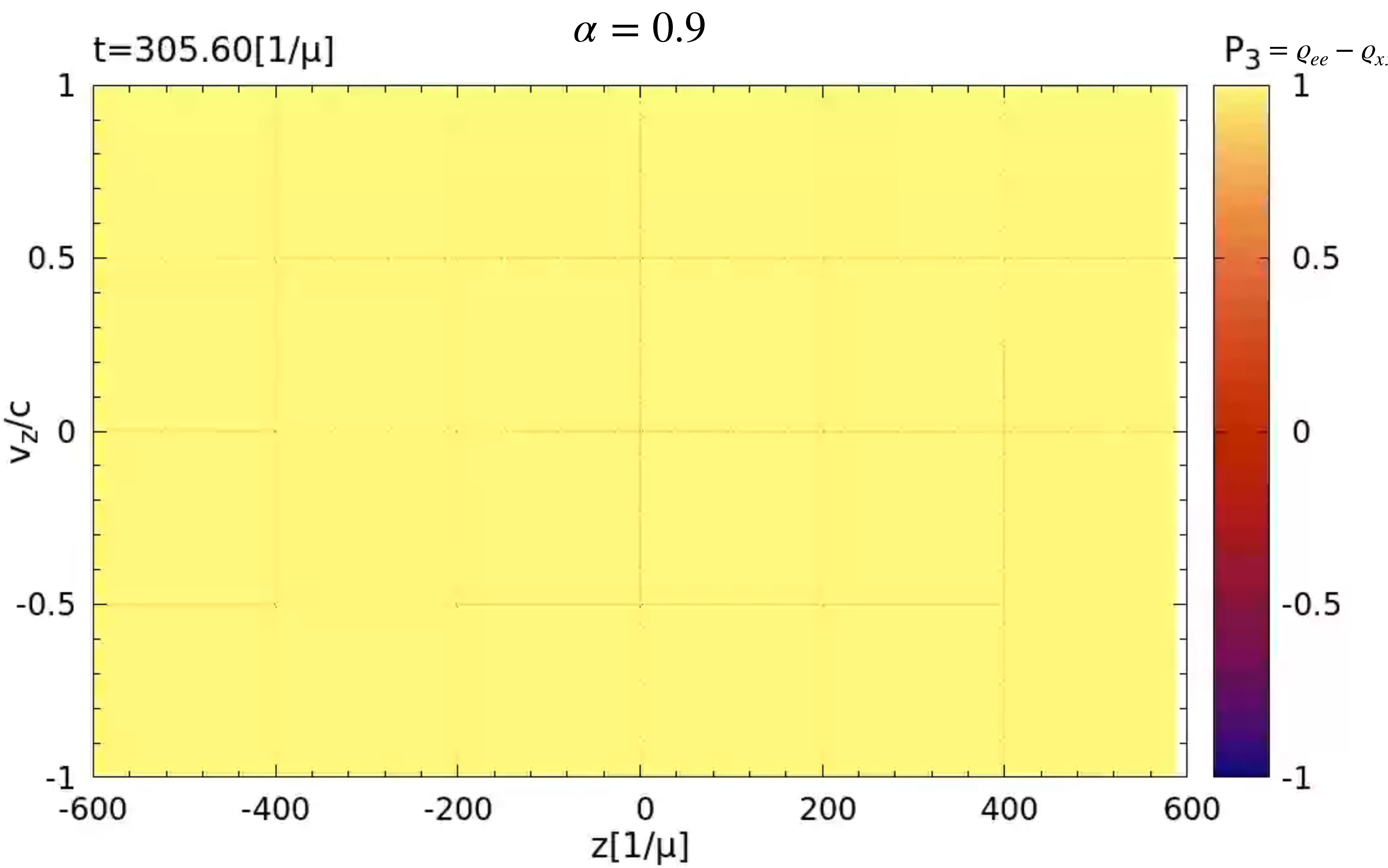
- Asymmetry factor $\alpha = \int dv_z\, g_{\bar{\nu}}(v_z) \;/\; \int dv_z\, g_\nu(v_z)$

- Trigger the instabilities by random perturbations



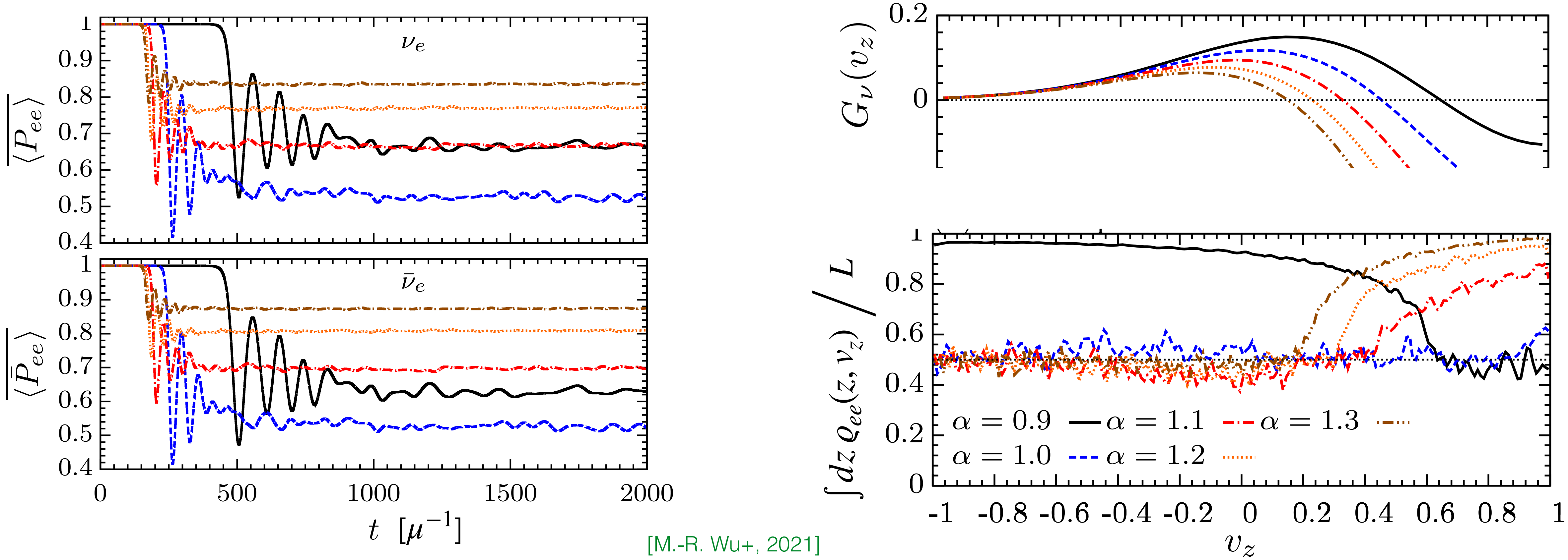
Evolution of FFCs





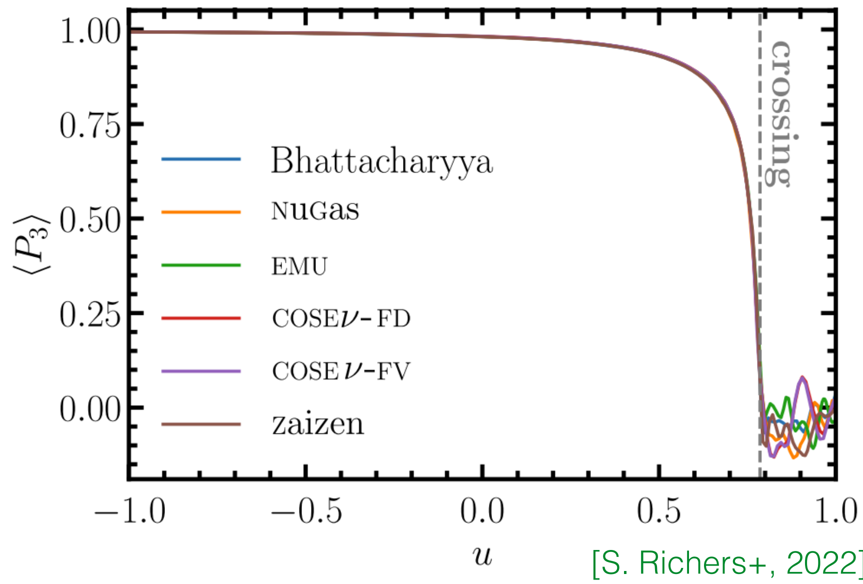
Asymptotic state

$\overline{\langle P_{ee} \rangle} = \int dz dv_z\, g_\nu(v_z) \varrho_{ee}(z, v_z) \Big/ \int dz dv_z\, g_\nu(v_z)$



[M.-R. Wu+, 2021]

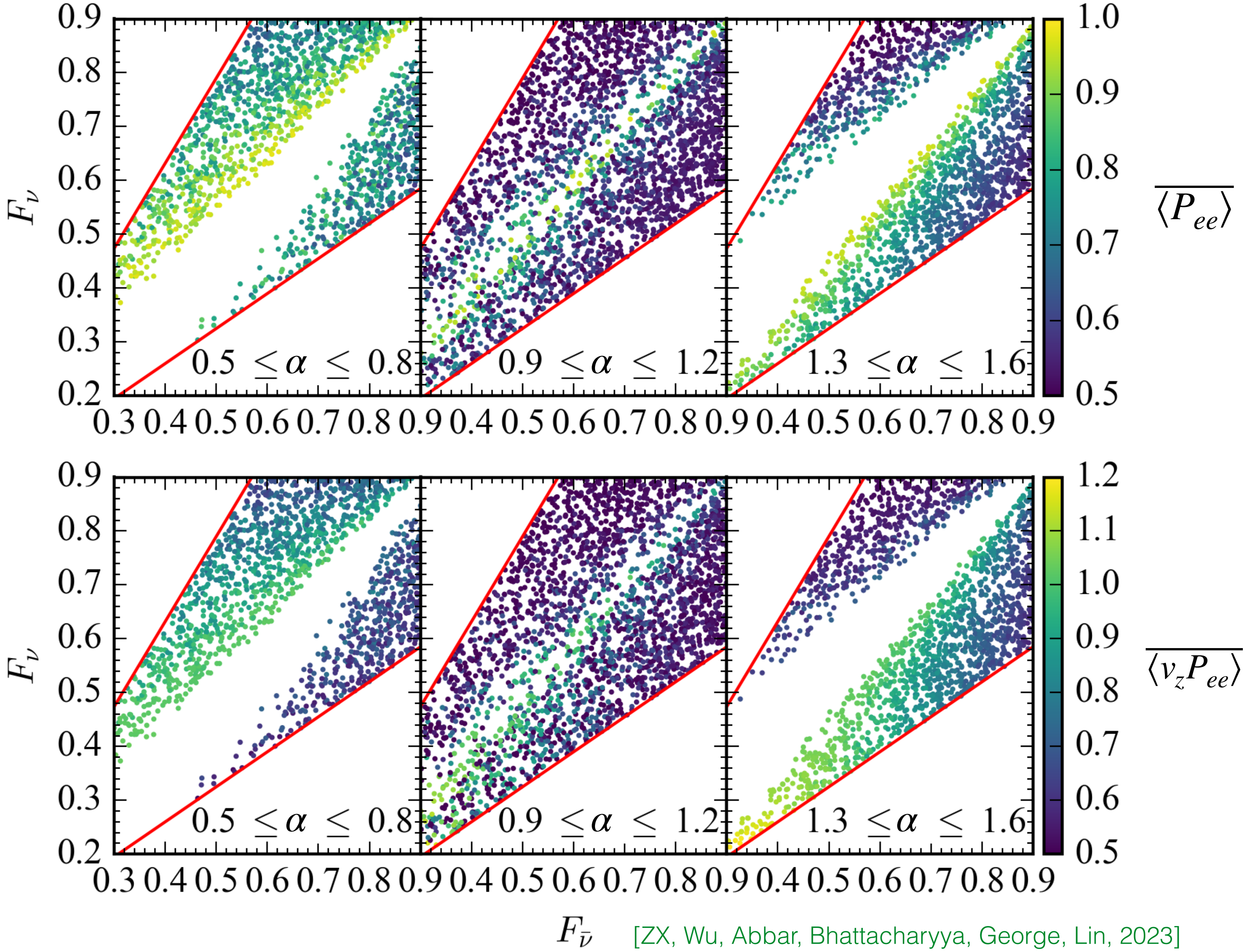
- Strong angular dependence
- For $\alpha = 1.0$, complete flavor equilibration in the entire angular range
- For $\alpha \neq 1.0$, only part of angular range reaches complete flavor equilibration, due to the conservation of vELN



[S. Richers+, 2022]

Dependence on the initial distributions

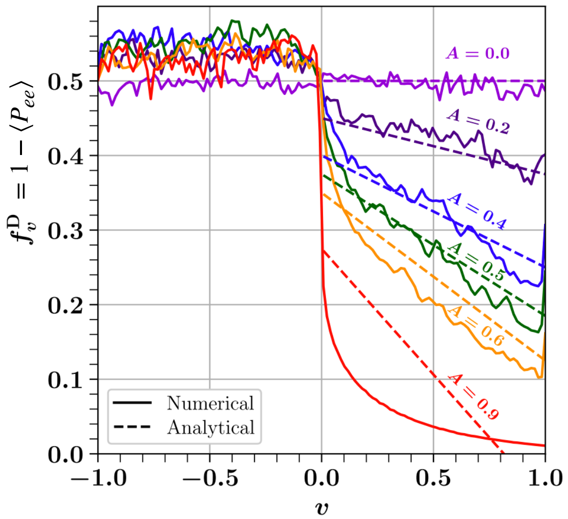
- Survey over ~8000 sets of initial distributions characterized by the asymptotic factor α, and the flux factors for ν_e and $\bar{\nu}_e$.
- The dataset is public in Zenodo: 10.5281/zenodo.8167253



[ZX, Wu, Abbar, Bhattacharyya, George, Lin, 2023]

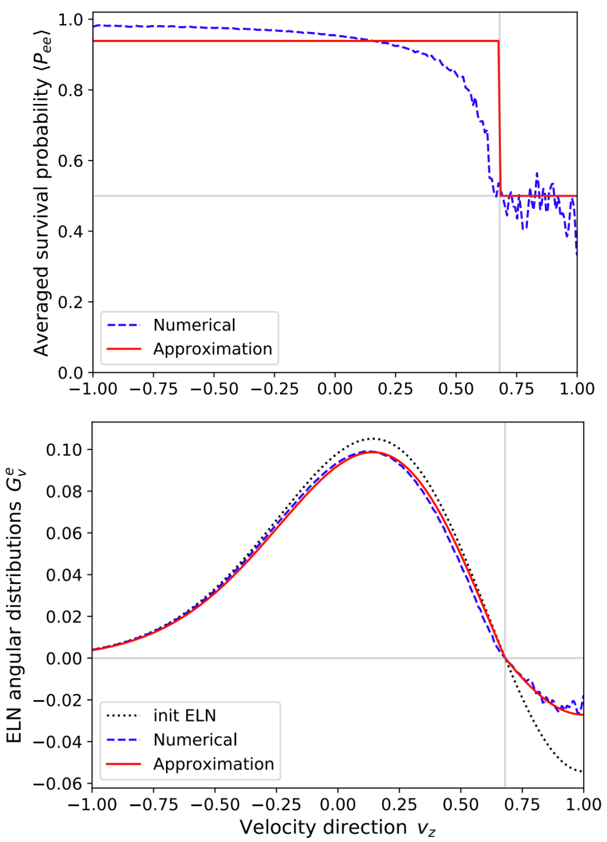
Prescriptions for asymptotic distributions

Linear prescription



[S. Bhattacharyya, B. Dasgupta, 2021]

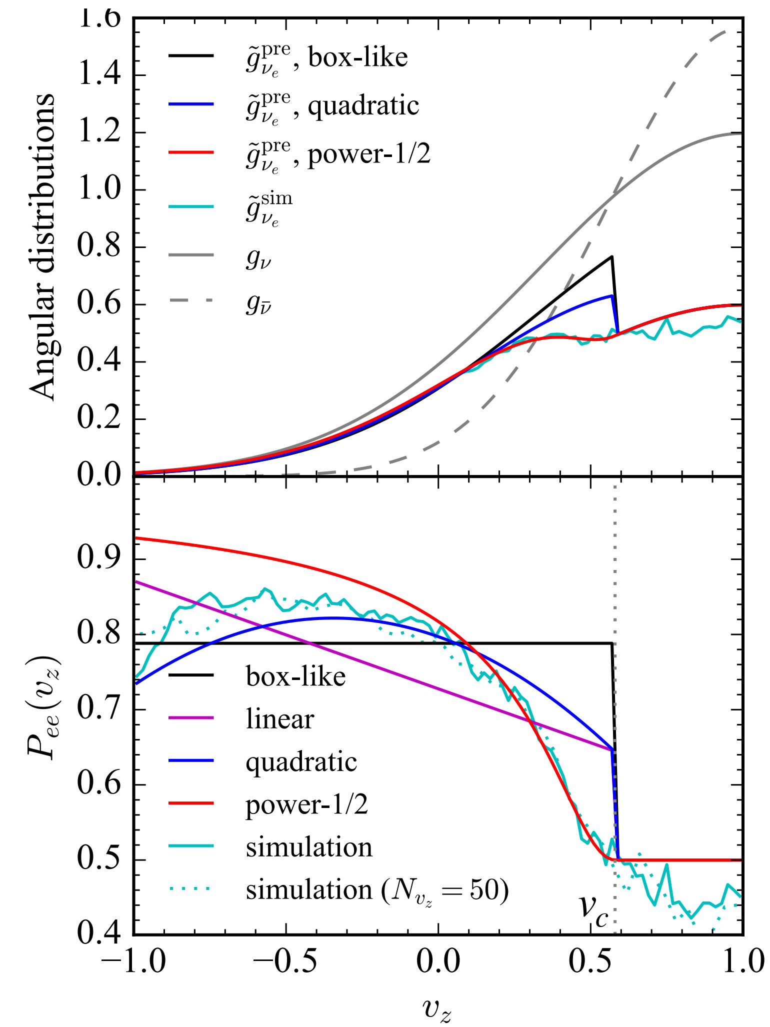
Box-like prescription



[M. Zaizen, H. Nagakura, 2023]

- We propose new prescriptions without abrupt transition at the crossing point

$$P_{ee} = 1 - \frac{1}{2} h(|v_z - v_c|/a)$$
with $h(x) = (x^2+1)^{-1/2}$



[ZX, Wu, Abbar, Bhattacharyya, George, Lin, 2023]

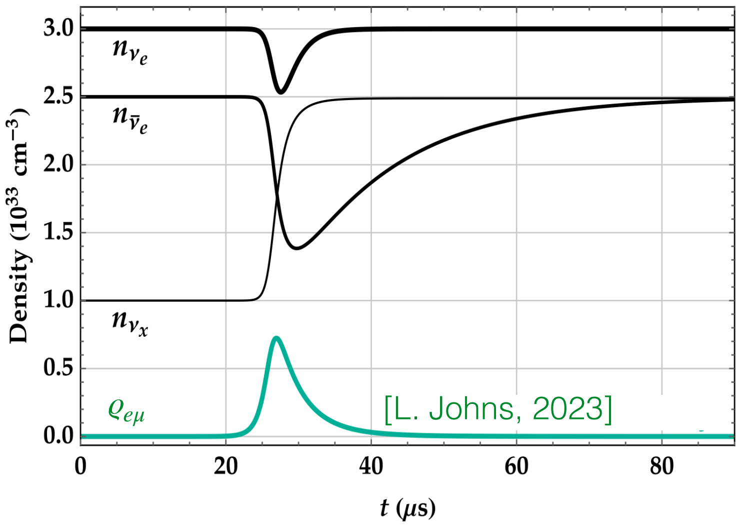
- We evaluate the performance based on our simulation dataset. The prescriptions with continuous transition achieve a great improvement.

Collisional flavor instabilities in spherically symmetric supernova models

[ZX, M.-R. Wu, G. Martínez-Pinedo, T. Fischer, M. George, C.-Y. Lin, L. Johns, PRD 107 083016 (2023), arXiv: 2210.08254]

Simulation set-up

- Background matter profiles from $\texttt{AGILE}-\texttt{BOLTZTRAN}$, $18M_\odot$ progenitor, and the post-bouncing time $t_{\text{pb}} \approx 250$ ms



- Solve the neutrino flavor evolution equation

[similar setups with angular advection by Irene's and Hiroki's groups]

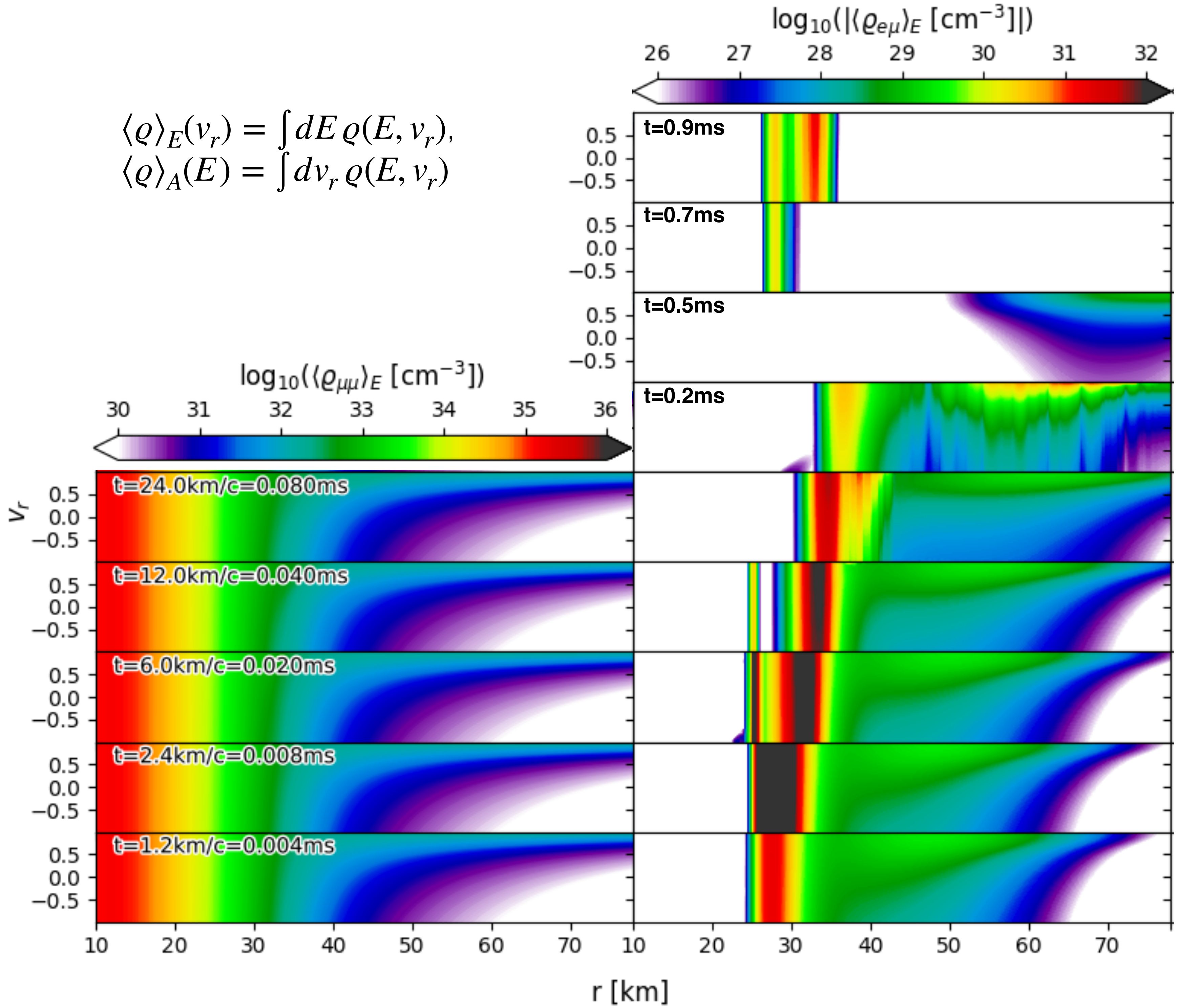
$$(\partial_t + v_r\partial_r + \frac{1-v_r^2}{r}\partial_{v_r})\varrho(E, v_r) = -i[a_{\nu\nu}\mathbf{H}_{\nu\nu}, \varrho(E, v_r)] + \mathbf{C}_{\text{CC}} + \mathbf{C}_{\text{NNS}}$$

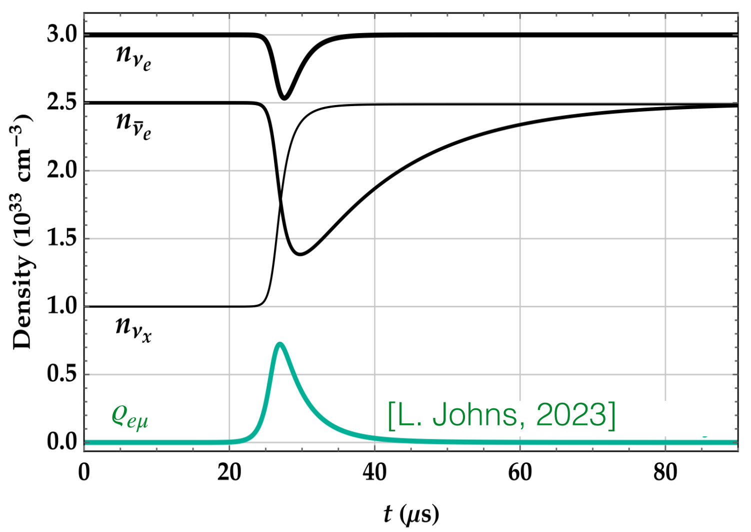
  - Multi-energy & multi-angle, two flavors ν_e and ν_μ
  - in the absence of fast flavor instability
  - Resolutions: $N_r = 25000$, $N_{v_r} = 50$, $N_E = 20$
  - Including charge-current (CC) interactions and iso-energetic neutrino-nucleon scattering (NNS) in QKE formalism [A. Vlasenko, G. Fuller, V. Cirigliano, 2014; C. Volpe, 2015 ...]
  - Inelastic scatterings and pair reactions are more computationally expensive because of $R(E, E', v_r, v_r')$
  - Inner boundary: neutrinos in thermal equilibrium with matter between 10 and 16 km to mimic pair reactions
  - Outer boundary: freely stream out at 85 km

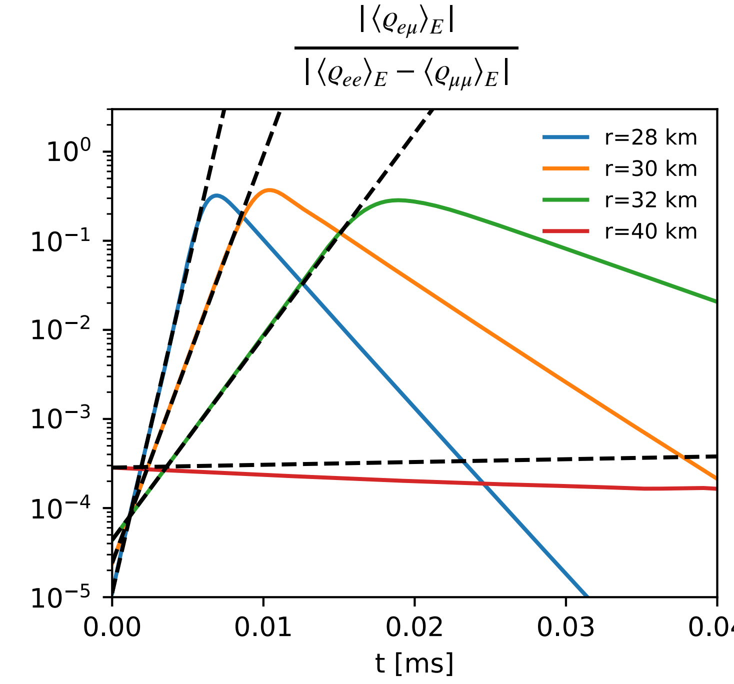
- Initial perturbation (flavor mixing seed): radial-dependent perturbation in Gaussian function

Evolution of collisional flavor instability

$$\langle \varrho \rangle_E(v_r) = \int dE\, \varrho(E, v_r),$$
$$\langle \varrho \rangle_A(E) = \int dv_r\, \varrho(E, v_r)$$



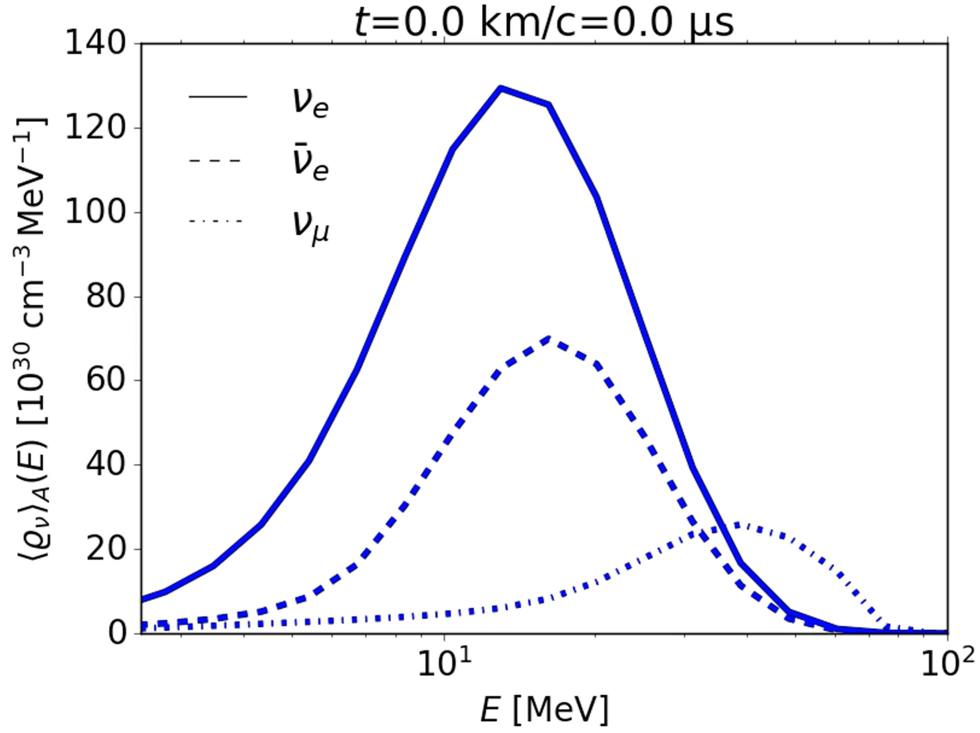


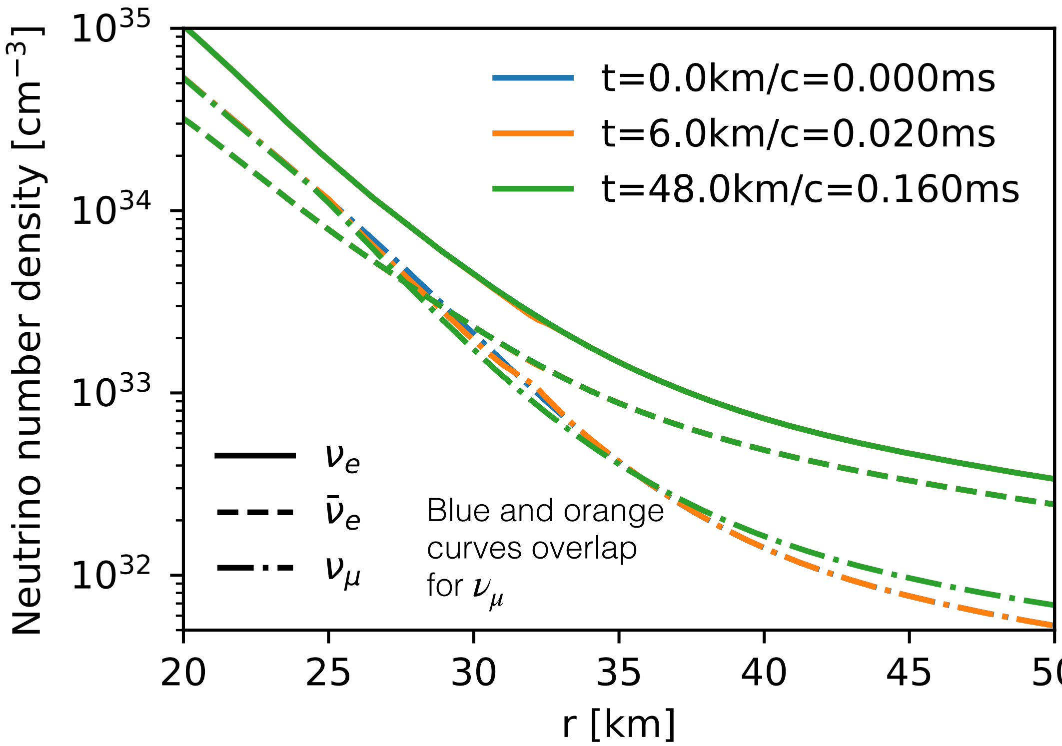


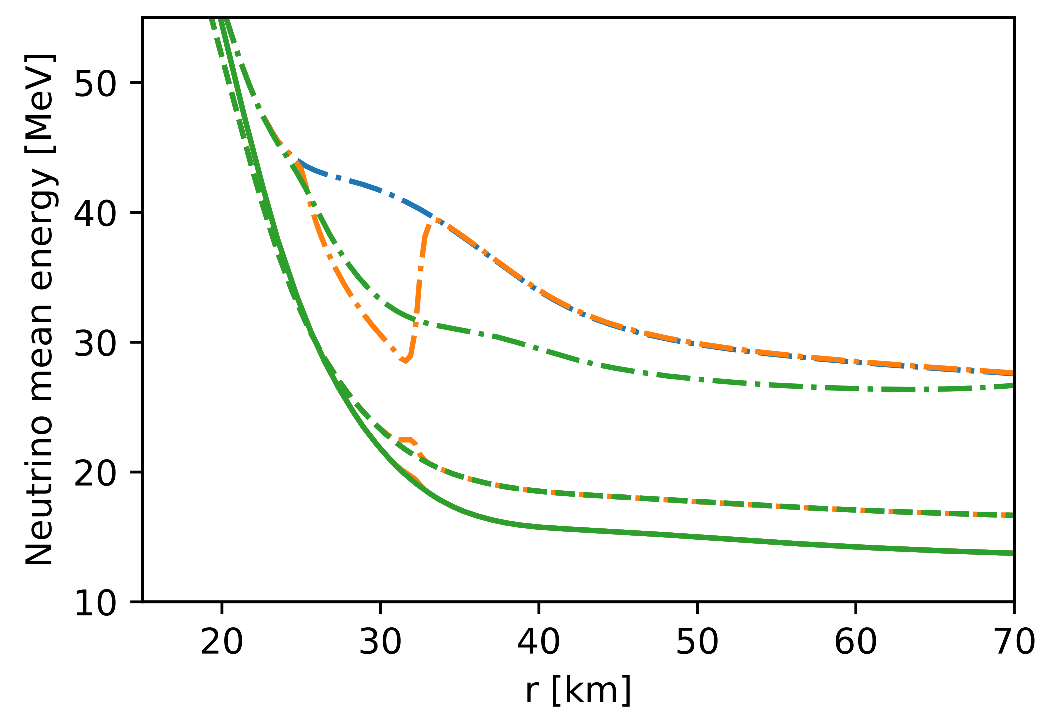
Evolution of collisional flavor instability

- distributions of ν_e and $\bar{\nu}_e$ are affected at the onset of the flavor conversion, but quickly restored by CC interactions
- leave imprints in the spectra of heavy-lepton (anti)neutrinos at the free-streaming regime
- Different from the homogeneous model, spectrum of heavy-lepton (anti)neutrinos does not converge to that of electron flavor

r = 32 km







Summary

- Local simulation of fast flavor conversions in a periodic 1D box.
  - evolves into an asymptotic state in a coarse-grained level
  - complete flavor equilibration occurs for $\alpha \approx 1$
  - Otherwise, a strong angular dependence with specific angular ranges reaching a complete flavor equilibration.
  - We proposed new prescriptions to predict this angular-dependent asymptotic state and performed comprehensive evaluation based on simulations with a large sample of initial distributions in 10.5281/zenodo.8167253.
- Global simulation in multi-energy and multi-angle treatment
  - to probe the dynamic property of collisional instability in the absence of fast flavor conversions
  - It mainly affects the energy spectra of heavy-lepton flavor neutrinos.
  - inelastic scatterings and pair reactions? attenuation on $\mathbf{H}_{\nu\nu}$? dynamic evolution and matter feedback?