Gravitational Form Factors at Large Momentum Transfer

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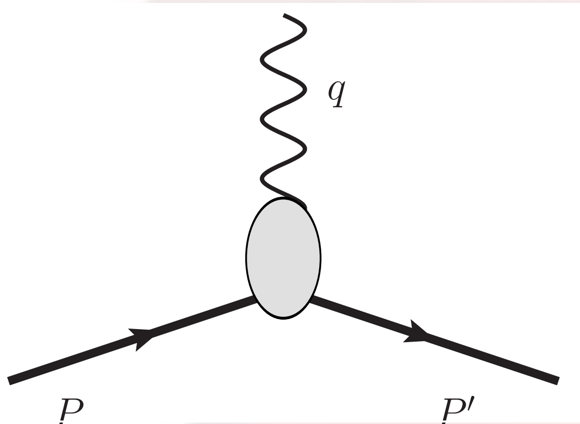
Refs: Tong, Ma, Yuan, Phys.Lett.B 823 (2021) 136751; arXiv: 2203.13493

Outline

- What are the gravitational form factors
- Light-cone wave functions/distribution amplitudes
- Form factor calculations at large t
- Implications
  - Threshold heavy quarkonium production
  - Mass distributions in coordinate space
- Discussions

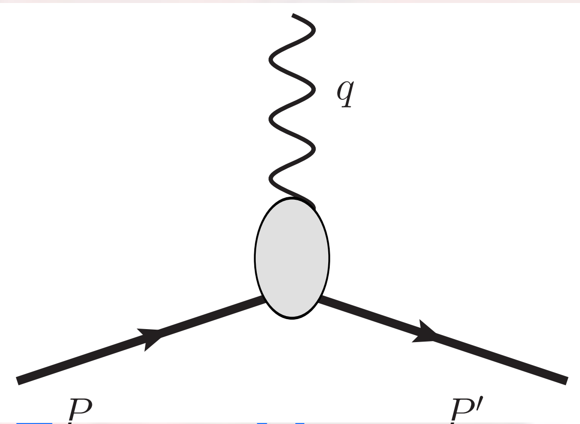
EM vs Gravitational Form Factors

Photon



EM current

Graviton

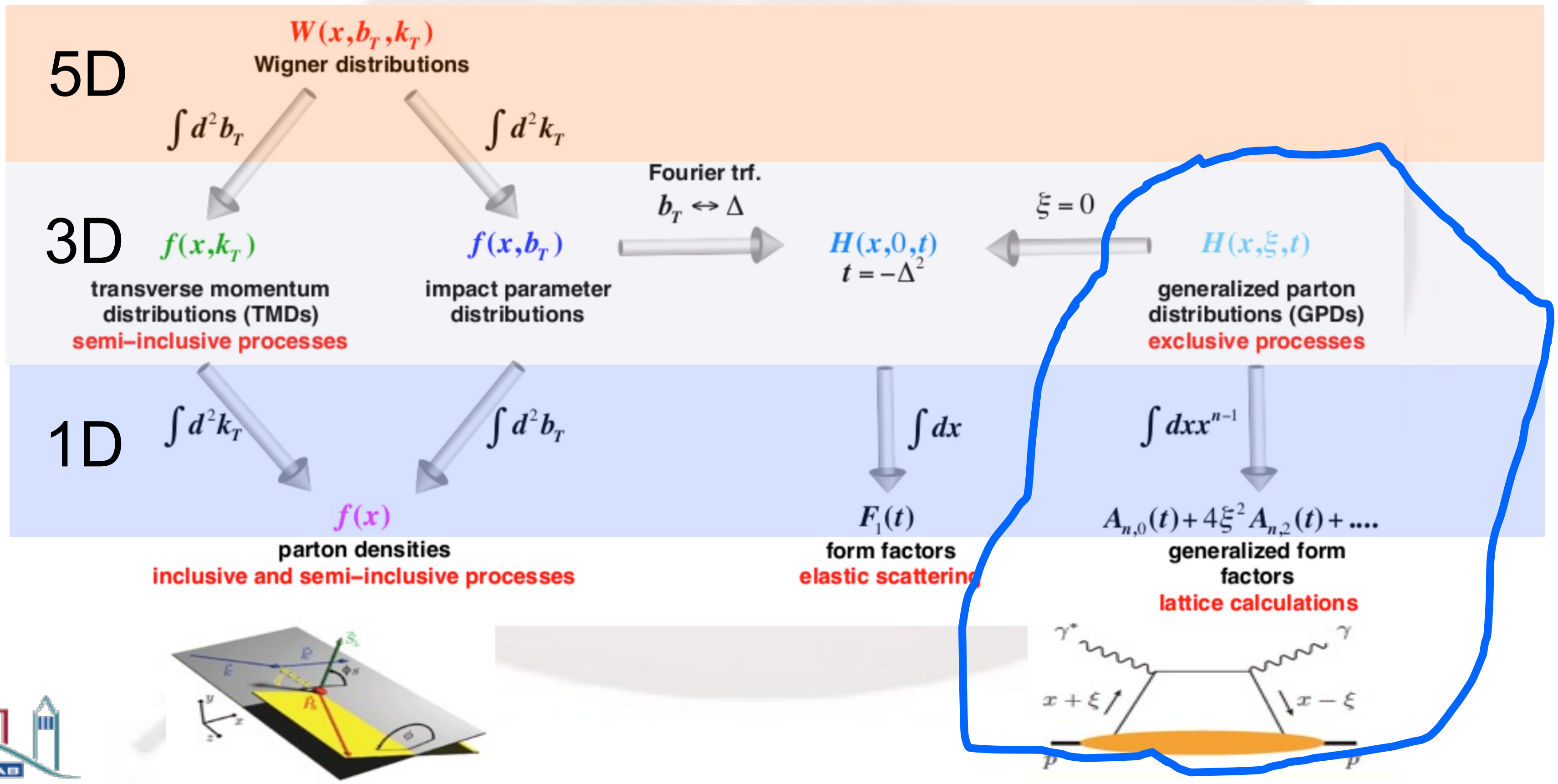


Energy-Momentum current

Kobzarev-Okun 1992; Pagels 1966; Ji 1997

Where to study: through GPDs

- **Wigner distributions (Belitsky, Ji, Yuan)**



What do we learn

- My view: one aspect of the parton tomography in hadrons, because they are part of GPDs
  - Proton spin sum rule is derived from these form factors
- C-form factors
  - Pressure, shear force: Polyakov-Schweitzer 2018
  - Momentum-current gravitation multipoles: Ji-Liu, 2021
- Reconstruct the proton mass
  - Ji 1996; Ji 2021; Ji-Liu 2021
  - Hatta-Rajan-Tanaka 2018;
  - Metz-Pasquini-Rodini 2020

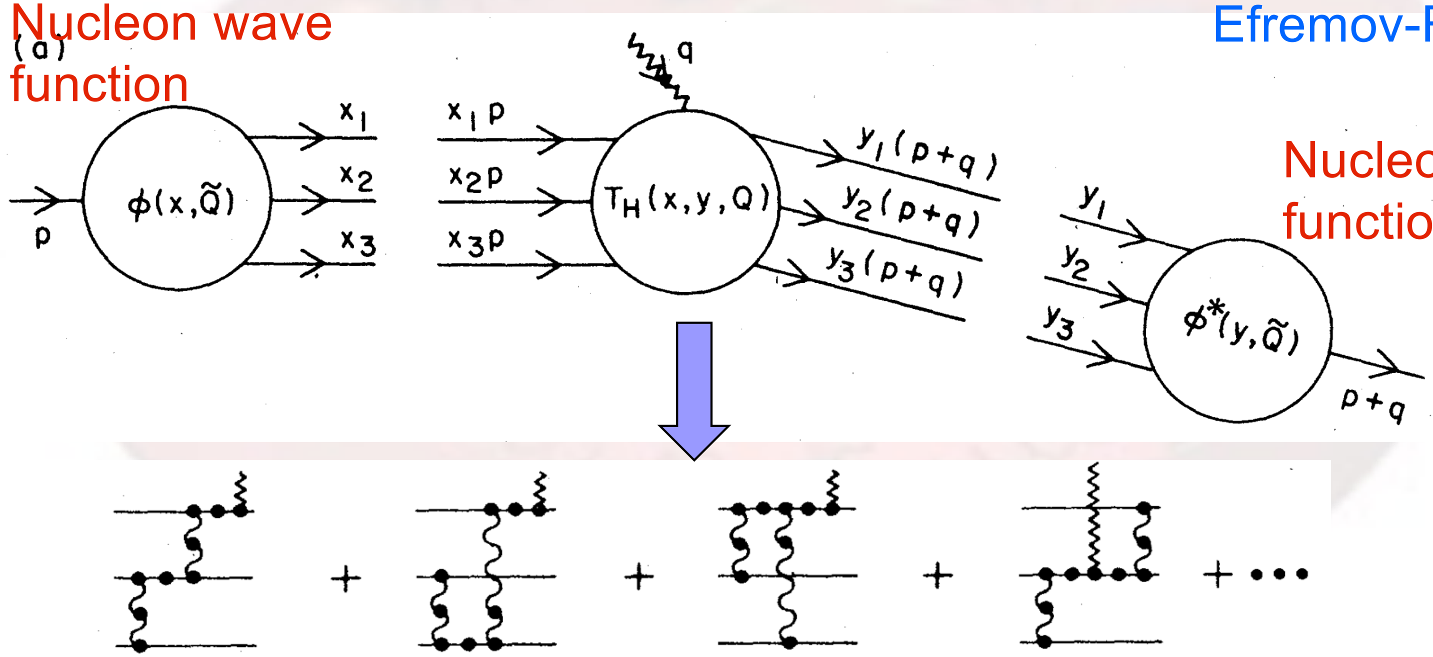
$$\langle P', s'|T_a^{\mu\nu}(0)|P, s\rangle = \bar{u}_s(P')\Big[A_a(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_a(t)\frac{i\bar{P}^{(\mu}\sigma^{\nu)\rho}\Delta_\rho}{2\Lambda} + C_a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{\Lambda} + \bar{C}_a(t)\Lambda g^{\mu\nu}\Big]u_s(P),$$

Form factors at large momentum transfer

Perturbatively, one can compute the form factors at large momentum transfer

Lepage-Brodsky 1980
Efremov-Radyushkin 1980

Nucleon wave function



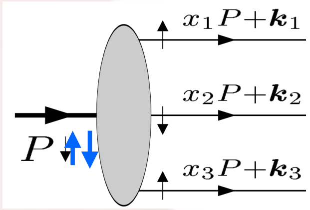
Nucleon wave function

Light-cone Wave Functions

Lepage-Brodsky 1980

- They are building blocks for the hadron structure

$$|P\rangle = \sum_{n,\lambda_i} \int \Pi_i \frac{dx_i d^2k_{\perp i}}{\sqrt{x_i}16\pi^3} \phi_n(x_i, k_{\perp i}, \lambda_i) |n : x_i, k_{\perp i}, \lambda_i\rangle$$



- Fock state of n-partons: momentum fractions, transverse momenta, helicities
- Can be used to calculate the form factors, GPDs, and hard exclusive scattering amplitudes, including near threshold heavy quarkonium production

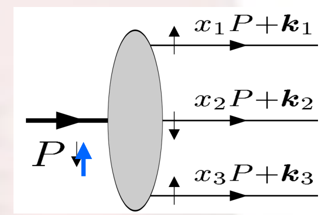
Nucleon's 3-quarks WF

Ji, Ma, Yuan, 2002

- According to the general structure, six independent light-cone wave functions for three quarks component:

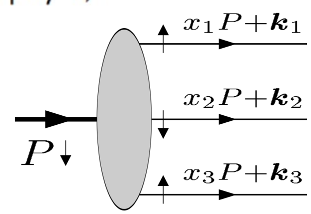
$$|P\uparrow\rangle_{1/2} = \int d[1]d[2]d[3]\left(\tilde{\psi}^{(1)}(1,2,3)\right) \times \frac{\epsilon^{abc}}{\sqrt{6}} u^{\dagger}_{a\uparrow}(1)\left(u^{\dagger}_{b\downarrow}(2)d^{\dagger}_{c\uparrow}(3) - d^{\dagger}_{b\downarrow}(2)u^{\dagger}_{c\uparrow}(3)\right)|0\rangle$$

L_z=0



$$|P\downarrow\rangle_{1/2} = \int d[1]d[2]d[3]\left((k_1^x - ik_1^y)\tilde{\psi}^{(3)}(1,2,3) + (k_2^x - ik_2^y)\tilde{\psi}^{(4)}(1,2,3)\right) \times \frac{\epsilon^{abc}}{\sqrt{6}}\left(u^{\dagger}_{a\downarrow}(1)u^{\dagger}_{b\uparrow}(2)d^{\dagger}_{c\uparrow}(3) - d^{\dagger}_{a\downarrow}(1)u^{\dagger}_{b\uparrow}(2)u^{\dagger}_{c\uparrow}(3)\right)|0\rangle\ ,$$

$|L_z|$=1



Distribution amplitudes

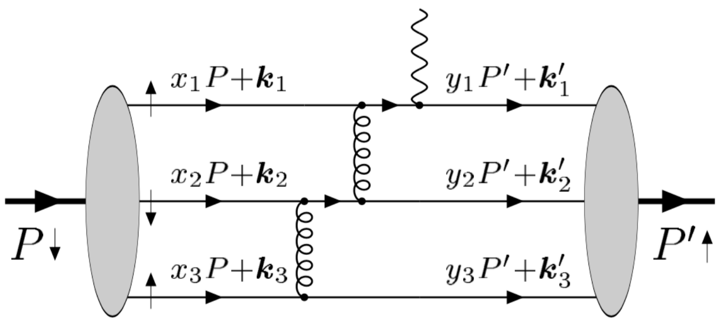
- Integrate out the transverse momentum
  - Twist-three (leading-twist)

$$\Phi_3(y_i) = -2\sqrt{6}\int \frac{d^2\vec{k}'_{1\perp} d^2\vec{k}'_{2\perp} d^2\vec{k}'_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}'_{1\perp}+\vec{k}'_{2\perp}+\vec{k}'_{3\perp})\tilde{\psi}^{(1)}(1,2,3)$$

  - Twist-four (Braun-Fries-Mahnke-Stein 2000)

$$\begin{aligned}\Psi_4(x_1,x_2,x_3) &= -\frac{2\sqrt{6}}{x_2 M}\int \frac{d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp} d^2\vec{k}_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}) \\ &\quad \times \vec{k}_{2\perp}\cdot\left[\vec{k}_{1\perp}\tilde{\psi}^{(3)}(1,2,3)+\vec{k}_{2\perp}\tilde{\psi}^{(4)}(1,2,3)\right] . \\ \Phi_4(x_2,x_1,x_3) &= -\frac{2\sqrt{6}}{x_3 M}\int \frac{d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp} d^2\vec{k}_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}) \\ &\quad \times \vec{k}_{3\perp}\cdot\left[\vec{k}_{1\perp}\tilde{\psi}^{(3)}(1,2,3)+\vec{k}_{2\perp}\tilde{\psi}^{(4)}(1,2,3)\right] .\end{aligned}$$

Form factor calculations



Compute the partonic scattering amplitudes, convert to hadron's Leading-twist: direct integration of k_t, higher-twist: need k_t-expansion

- Two gluon exchanges are needed to generate large momentum transfer
- Helicity-non-flip has power behavior, F_1~1/t^2
- Helicity-flip amplitude has power behavior, F_2~1/t^3

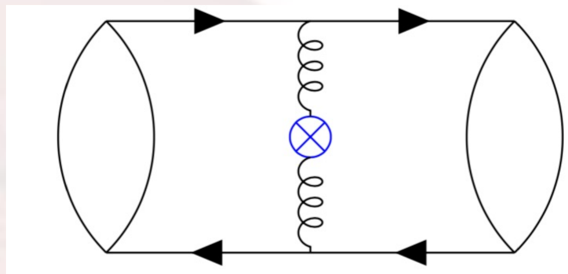
Brodsky-Lepage 1981 for F_1
Belitsky-Ji-Yuan 2002 for F_2

Gravitational form factors: No much difference, only some surprises

- Pion case

$$\langle P'|T_g^{\mu\nu}|P\rangle = 2\bar{P}^\mu\bar{P}^\nu A_g^\pi(t) + \frac{1}{2}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)C_g^\pi(t) + 2m^2 g^{\mu\nu}\overline{C}_g^\pi(t)$$

$$A_g^\pi(t) = C_g^\pi(t) = \frac{4m^2}{t}\overline{C}_g^\pi(t) = \frac{4\pi\alpha_s C_F}{-t}\int dx_1 dy_1 \phi^*(y_1)\phi(x_1)\left(\frac{1}{x_1\bar{x}_1} + \frac{1}{y_1\bar{y}_1}\right)$$



Tong-Ma-Yuan, 2103.12047;
Different from Tanaka, PRD 2018

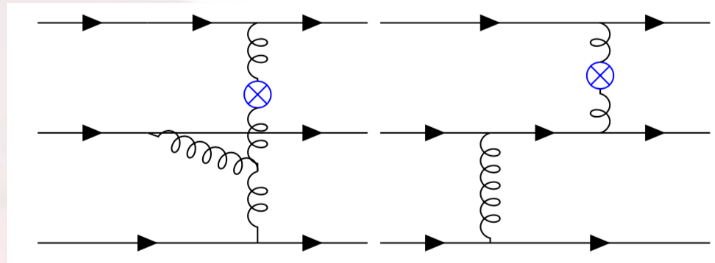
➢ May introduce difficulty in the interpretation, since integral over t is not convergent

Polyakov-Schweitzer 2018
Freese-Miller 2021

- $A_g=C_g$!!
- Cbar cancels between quarks and gluons
  - Quark part from GPD quark at large-t (Hoodbhoy-Ji-Yuan 2003)

$$\langle P', s'|T_a^{\mu\nu}(0)|P, s\rangle = \bar{u}_s(P')\Bigg[A_a(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_a(t)\frac{i\bar{P}^{(\mu}\sigma^{\nu)\rho}\Delta_\rho}{2\Lambda} + C_a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{\Lambda} + \bar{C}_a(t)\Lambda g^{\mu\nu}\Bigg]u_s(P),$$

Nucleon case



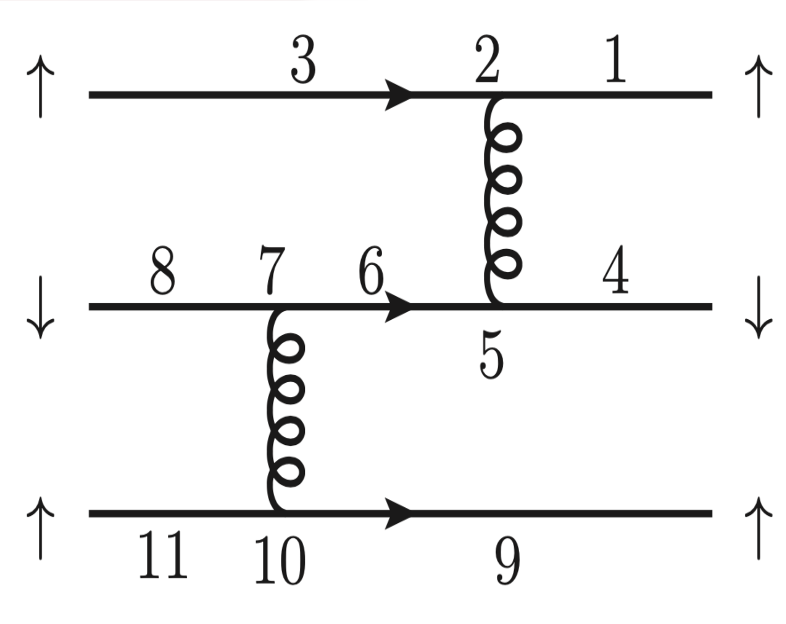
- No contribution from three-gluon vertex diagram
- A_g~1/t^2
- B_g, C_g scale as 1/t^3,Cbar$_g$ scales as 1/t^2

$$\mathcal{A} = \frac{4\pi^2\alpha_s^2 C_B^2}{3t^2}\left(I_{13} + I_{12} + I_{31} + I_{32}\right), \quad I_{ij} = \frac{x_i + y_i}{\bar{x}_i\bar{y}_i x_i x_j y_i y_j}$$

Tong-Ma-Yuan, 2103.12047

Important cross checks

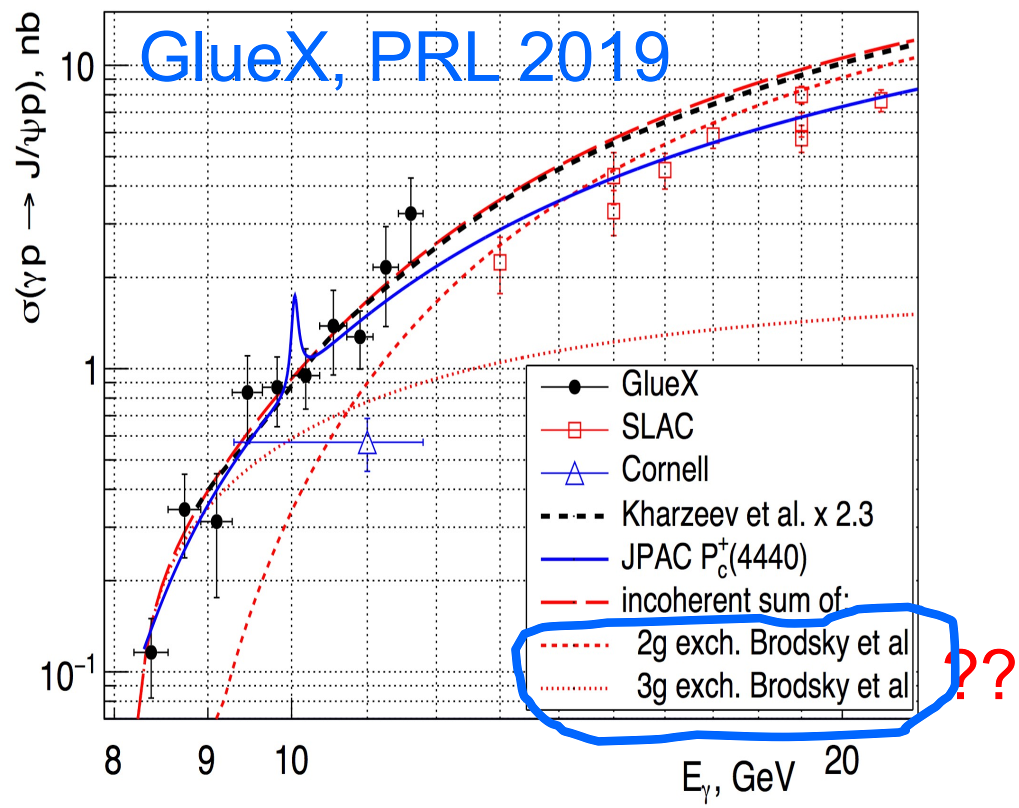
- Cbar from quarks and gluon cancel out
- A_q form factors agree with the GPD calculations at large-t
  - Hoodbhoy-Ji-Yuan, 2003



Applications: Threshold photoproduction of heavy quarkonium

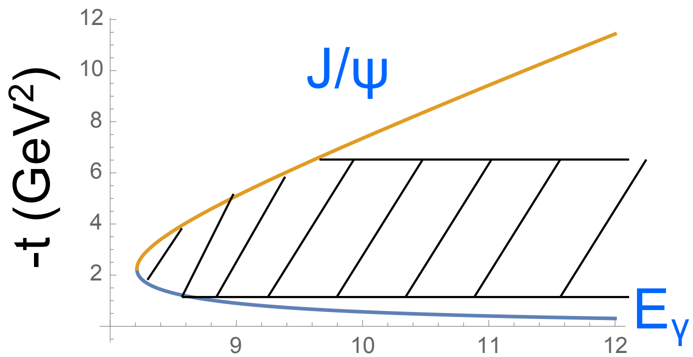
Different methods have been applied

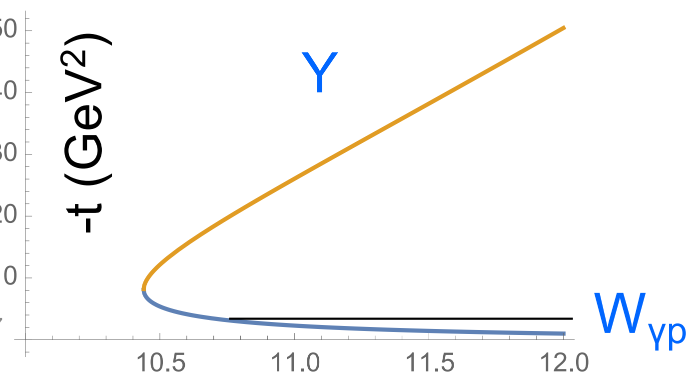
- Which gravitational form factors contribute
  - VDM: scalar gravitational form factor, Kharzeev and others
  - Holographic model and QCD analysis: all form factors, maybe dominated by C-form factor, Hatta et al, Ji et al, Zahed et al
- Two-gluon or three-gluon?



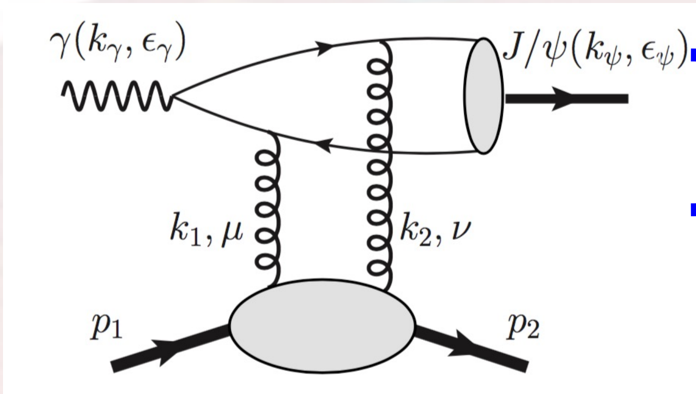
Large momentum transfer is relevant

- We can compute both the cross section and the form factors separately in perturbative QCD, then we can check that if there is/not a direct connection between the near threshold production and the gluonic gravitational form factors (and how)





Near threshold production: kinematics



Photon-J/ψ Part: $\boxed{\mathcal{M}_\psi^{\mu\nu}}$

- Two limits
  - Threshold: $\chi = \frac{M_V^2 + 2M_p M_V}{W_{\gamma p}^2 - M_p^2}$ → 1, (1-χ) a small parameter, Brodsky et al 2001
  - Heavy quark limits

$$\boxed{W_{\gamma p}^2 \sim M_V^2 \gg (-t) \gg \Lambda_{QCD}^2 \ ,}$$

$$p_1 \cdot k_\gamma \sim p_1 \cdot k_\psi \sim M_V^2$$
$$p_2 \cdot k_\gamma \sim p_2 \cdot k_\psi \ll M_V^2$$

$\mathcal{M}_{\psi}^{\mu\nu}$

- NRQCD for heavy quarkonium production   Bodwin-Braaten-Lepage 1995
- Propagators are of order heavy quark mass, $\sim 1/M_V$
- Take transverse polarization for the incoming photon

$$\mathcal{M}_{\psi,ab}^{\mu\nu} = \frac{\delta^{ab} N_\psi \left[\epsilon_\psi^* \cdot \epsilon_\gamma \mathcal{W}_T^{\mu\nu} + \epsilon_\psi^* \cdot k \mathcal{W}_L^{\mu\nu} + \mathcal{W}_S^{\mu\nu}\right]}{k_1 \cdot k_\gamma k_2 \cdot k_\gamma}$$

$$\boxed{\begin{aligned} \mathcal{W}_T^{\mu\nu} &= -k_1 \cdot k_\gamma k_2 \cdot k_\gamma g^{\mu\nu} - k_1 \cdot k_2 k_\gamma^\mu k_\gamma^\nu \\ &\quad + k_1 \cdot k_\gamma k_2^\mu k_\gamma^\nu + k_2 \cdot k_\gamma k_1^\nu k_\gamma^\mu \end{aligned}}$$

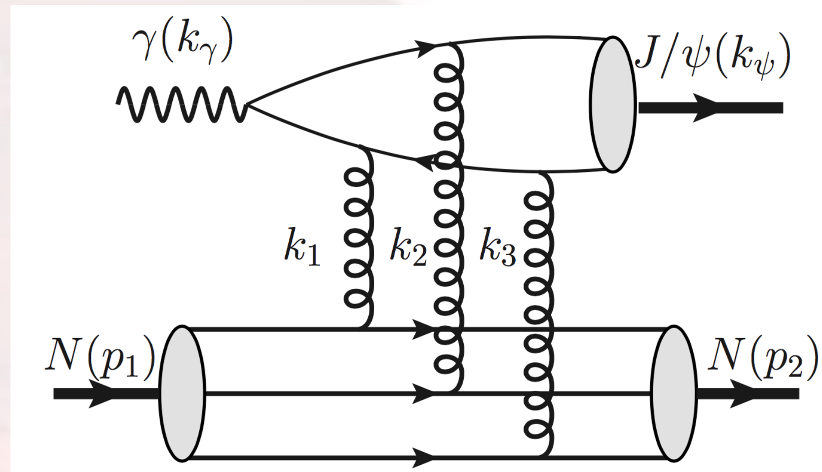
Leading terms

$$\mathcal{W}_L^{\mu\nu} = k_1 \cdot k_\gamma \epsilon_\gamma^\nu k_2^\mu + k_2 \cdot k_\gamma \epsilon_\gamma^\mu k_1^\nu$$

$$\begin{aligned} \mathcal{W}_S^{\mu\nu} &= -k_1 \cdot k_2 \Big(k_1 \cdot k_\gamma \epsilon_\psi^{*\mu} \epsilon_\gamma^\nu + k_2 \cdot k_\gamma \epsilon_\psi^{*\nu} \epsilon_\gamma^\mu \\ &\quad + k_1 \cdot \epsilon_\psi^* k_\gamma^\nu \epsilon_\gamma^\mu + k_2 \cdot \epsilon_\psi^* k_\gamma^\mu \epsilon_\gamma^\nu\Big) \ . \end{aligned}$$

Vanishing of three-gluon exchange

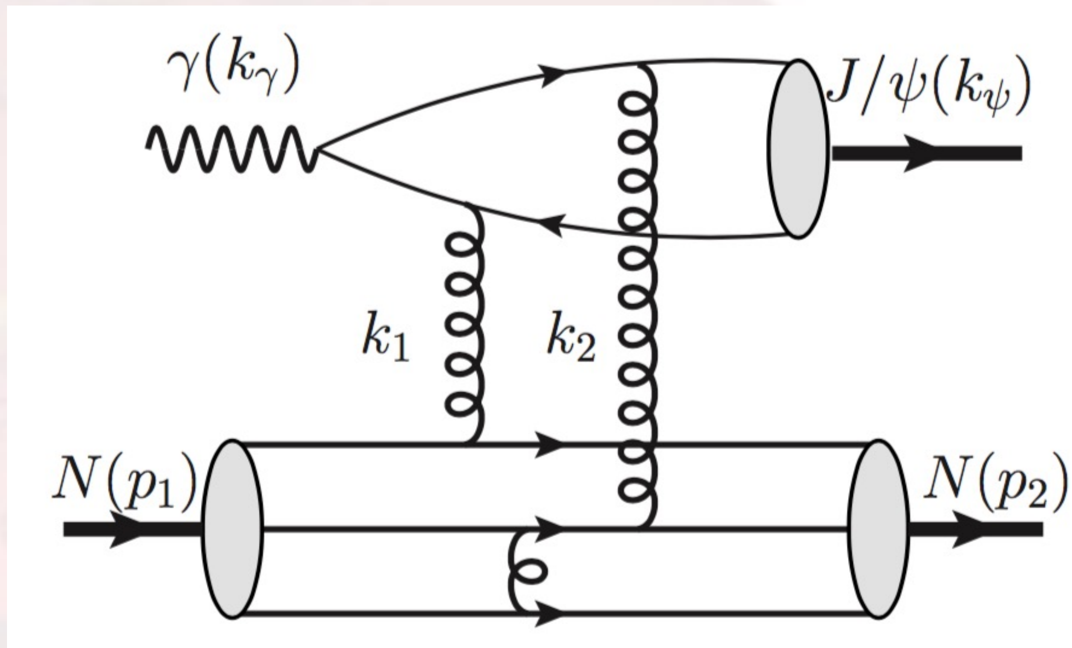
- Suggested by Brodsky et al, 2001, and widely accepted by exp. and claimed that
  - Two-gluon exchange suppressed by $(1\text{-}x)^2$, where three-gluon dominates at threshold
- Due to C-parity conservation, there is no contribution from the three-gluon exchange



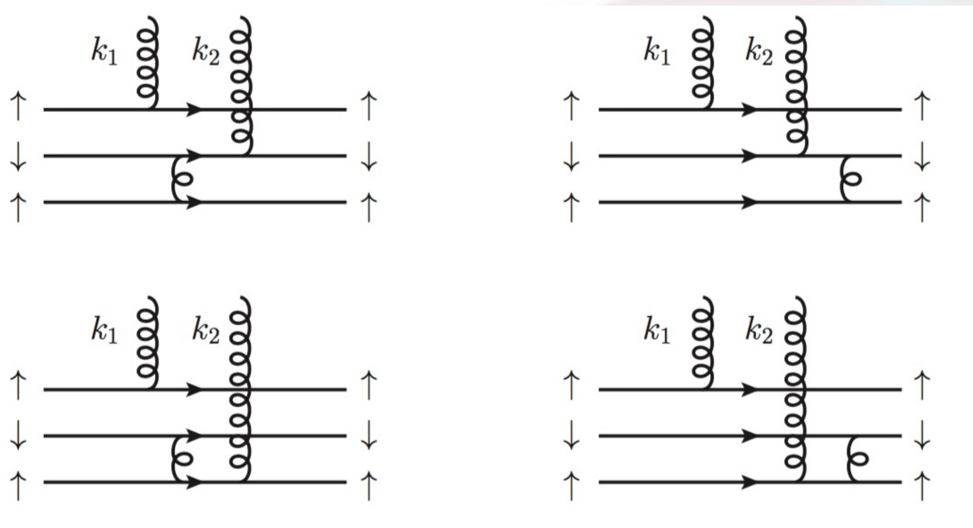
$$\boxed{\epsilon^{ijk}\epsilon^{lmn}T^a_{il}T^b_{jm}T^c_{kn} \propto d^{abc}}$$

Couple to the Nucleon

- Additional gluon exchange to generate large-t
- Nucleon spin configurations
  - Helicity conserved
  - Helicity-flip



Partonic scattering: I

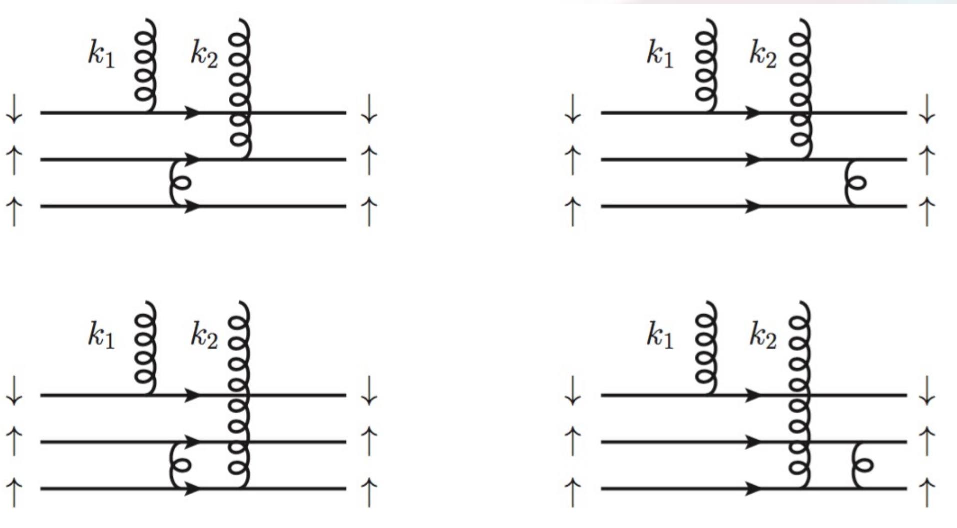


$$\bar{U}(p_2)\gamma^\mu U(p_1)\mathrm{Tr}\left[\frac{1+\gamma_5}{2}\not{p}_2\cdots\gamma^\nu\cdots\frac{1+\gamma_5}{2}\not{p}_1\cdots\right]$$

$$\bar{U}(p_2)\gamma^\mu U(p_1)\bar{P}^\nu$$

- k_1 attaches the helicity-up quark line

Partonic scattering: II



$$\bar{U}(p_2)\gamma^{\rho}U(p_1)\mathrm{Tr}\left[\frac{1+\gamma_5}{2}\not{p}_2\cdots\gamma^{\nu}\cdots\frac{1+\gamma_5}{2}\not{p}_1\cdots\gamma^{\rho}\cdots\right]$$

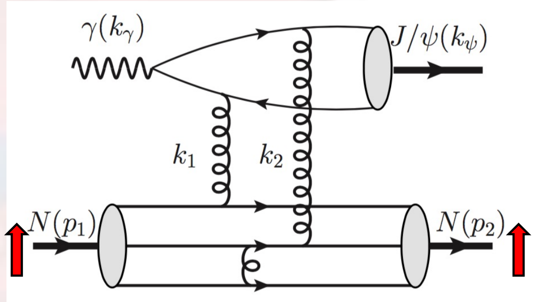
$$\bar{U}(p_2)\gamma^{\mu}U(p_1)\bar{P}^{\nu}$$

- k_1 attaches the helicity-down quark line

Final amplitude

$$\mathcal{A}_3 = \langle J/\psi(\epsilon_\psi), N'_\uparrow | \gamma(\epsilon_\gamma), N_\uparrow \rangle$$
$$= \int [dx][dy]\Phi(x_1, x_2, x_3)\Phi^*(y_1, y_2, y_3)\frac{1}{(-t)^2}$$
$$\times \bar{U}_\uparrow(p_2)\not{k}_\gamma U_\uparrow(p_1)\mathcal{M}_\psi^{(3)}(\epsilon_\gamma, \epsilon_\psi, \{x_i\}, \{y_i\}) \ ,$$

- Similar structure as A_g form factor or GPD H_g contribution



$$\mathcal{M}^{(3)} = \epsilon_\psi^* \cdot \epsilon_\gamma \frac{8e_c e g_s^6}{27\sqrt{3M_\psi^7}} \psi_J(0) \left(2\mathcal{H}_3 + \mathcal{H}'_3\right)$$

$$\mathcal{H}_3 = I_{13} + I_{31} + I_{12} + I_{32}, \quad I_{ij} = \frac{1}{x_i x_j y_i y_j \bar{x}_i^2 \bar{y}_i}$$

Amplitude squared

$$|\overline{\mathcal{A}_3}|^2 = (1-\chi) G_\psi G_{p3}(t) G^*_{p3}(t)$$

$$G_\psi = |N_\psi|^2 = \frac{384\pi^2 e_c^2 \alpha (4\pi\alpha_s)^2}{N_c^2 M_\psi^3} \langle 0 | \mathcal{O}^\psi(^3S_1^{(1)}) | 0 \rangle$$

$$G_{p3}(t) = \frac{8\pi^2 \alpha_s^2 C_B^2}{3t^2} \int [dx][dy] \Phi_3(\{x\}) \Phi_3^*(\{y\}) \left[2\mathcal{H}_3 + \mathcal{H}_3'\right]$$

- Suppressed at the threshold, $\chi \to 1$
- This behavior is similar to H_g contribution to J/ψ production in the GPD formalism with 1-ξ suppression factor
  - Hoodbhoy 1996, see also, Koempel-Kroll-Metz-Zhou 2012, Guo-Ji-Liu 2021
- Power behavior of 1/t^4

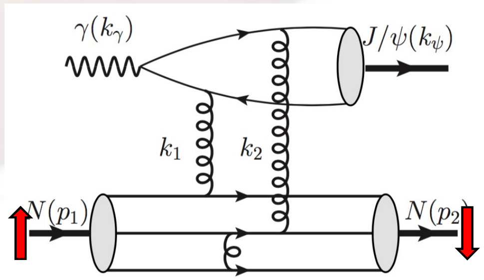
Twist-four contribution

$$\mathcal{A}_4 = \langle J/\psi(\epsilon_\psi), N'_\uparrow | \gamma(\epsilon_\gamma), N_\downarrow \rangle$$
$$= \int [dx][dy] \Psi_4(\{x\}) \Phi_3^*(\{y\}) \mathcal{M}_\psi^{(4)}(\{x\},\{y\})$$
$$\times \bar{U}_\uparrow(p_2) U_\downarrow(p_1) \frac{M_p}{(-t)^3} ,$$

$$|\overline{\mathcal{A}_4}|^2 = \widetilde{m}_t^2 G_\psi G_{p4}(t) G_{p4}^*(t) \quad \widetilde{m}_t^2 = M_p^2/(-t)$$

$$G_{p4}(t) = \frac{C_B^2 (4\pi\alpha_s)^2}{12t^2} \int [dx][dy] \Phi_3(y_1,y_2,y_3)$$
$$\times \{x_3 \Phi_4(x_1,x_2,x_3) T_{4\Phi}(\{x\},\{y\})$$
$$+ x_1 \Psi_4(x_2,x_1,x_3) T_{4\Psi}(\{x\},\{y\})\} ,$$

- Helicity-flip amplitude
- kt-expansion, similar to F_2 form factor
- There is no interference between twist-3 and twist-4
- Power behavior~1/t^5



There is no direct connection to the gluonic gravitational form factors

- Scattering amplitude

$$G_{p3}(t) = \int [dx][dy]\Phi_3(x_1,x_2,x_3)\Phi_3^*(y_1,y_2,y_3) \times [2\mathcal{H}_3 + \mathcal{H}_3'] \ ,$$

$$\mathcal{H}_3 = \frac{8\pi^2\alpha_s^2 C_B^2}{3t^2}(I_{13}+I_{31}+I_{12}+I_{32})$$

$$I_{ij} = \frac{1}{x_i x_j y_i y_j \bar{x}_i^2 \bar{y}_i}$$

- Gluonic Form Factors

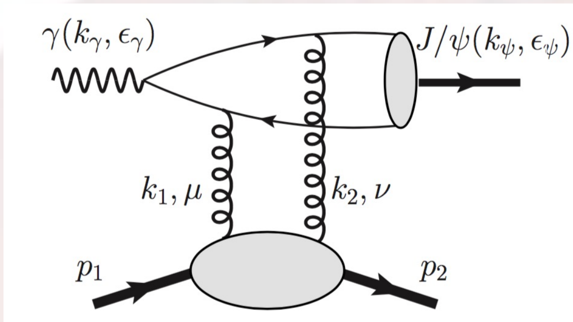
$$A_g(t) = \int [dx][dy]\Phi_3(x_1,x_2,x_3)\Phi_3^*(y_1,y_2,y_3) \times [2\mathcal{A}_3 + \mathcal{A}_3'] \ ,$$

$$\mathcal{A} = \frac{4\pi^2\alpha_s^2 C_B^2}{3t^2}\Big(I_{13}+I_{12}+I_{31}+I_{32}\Big),$$

$$I_{ij} = \frac{x_i + y_i}{\bar{x}_i\bar{y}_i x_i x_j y_i y_j}$$

Discussion: construct the gluon operators

- Take the leading contribution of heavy quark mass limit



Photon-J/ψ Part:

$$\boxed{\mathcal{M}_\psi^{\mu\nu}}$$

$$\mathcal{M}_\psi^{\mu\nu} = N_\psi \epsilon_\psi^* \cdot \epsilon_\gamma \frac{k_{\gamma,\alpha} k_{\gamma,\beta}}{k_1 \cdot k_\gamma k_2 \cdot k_\gamma} \mathcal{W}_T^{\alpha\beta\mu\nu}$$

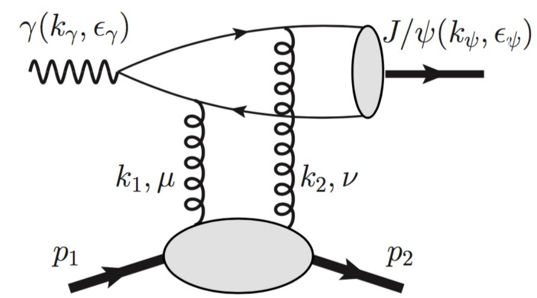
$$\mathcal{W}_T^{\alpha\beta\mu\nu} = -k_1^\alpha k_2^\beta g^{\mu\nu} - k_1 \cdot k_2 g^{\alpha\mu} g^{\beta\nu} + k_1^\nu k_2^\beta g^{\alpha\mu} + k_2^\mu k_1^\alpha g^{\beta\nu} ,$$

Connect to gravitational form factors?

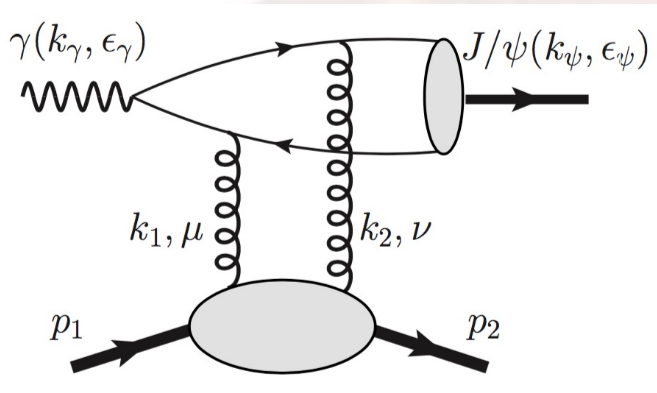
$$\mathcal{A} \propto \int d^4\eta_1 d^4\eta_2 d^4k_1 d^4k_2 e^{ik_1\cdot\eta_1+ik_2\cdot\eta_2}\frac{k_\gamma^\alpha k_\gamma^\beta}{k_1\cdot k_\gamma k_2\cdot k_\gamma} \times\langle N'|F^\alpha{}_\rho(\eta_1)F^{\beta\rho}(\eta_2)|N\rangle \ . \qquad ($$

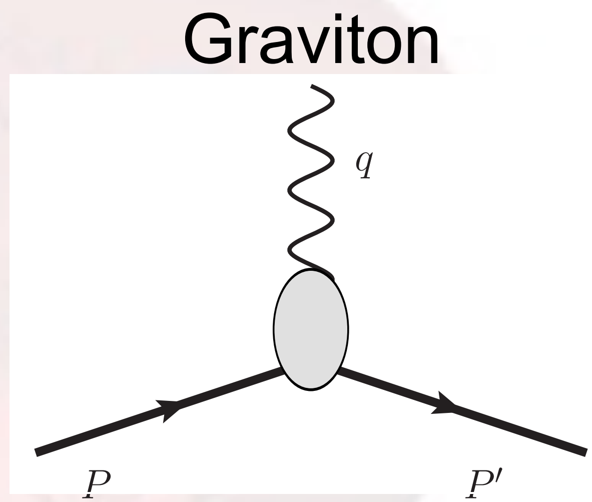
- We have to make approximations: the two gluons in the t-channel carry the same momentum

$$\mathcal{A} \propto \frac{k_\gamma^\alpha k_\gamma^\beta}{\langle k_1\cdot k_\gamma k_2\cdot k_\gamma\rangle}\langle T_g^{\alpha\beta}\rangle$$



It is a long stretch to make this connection





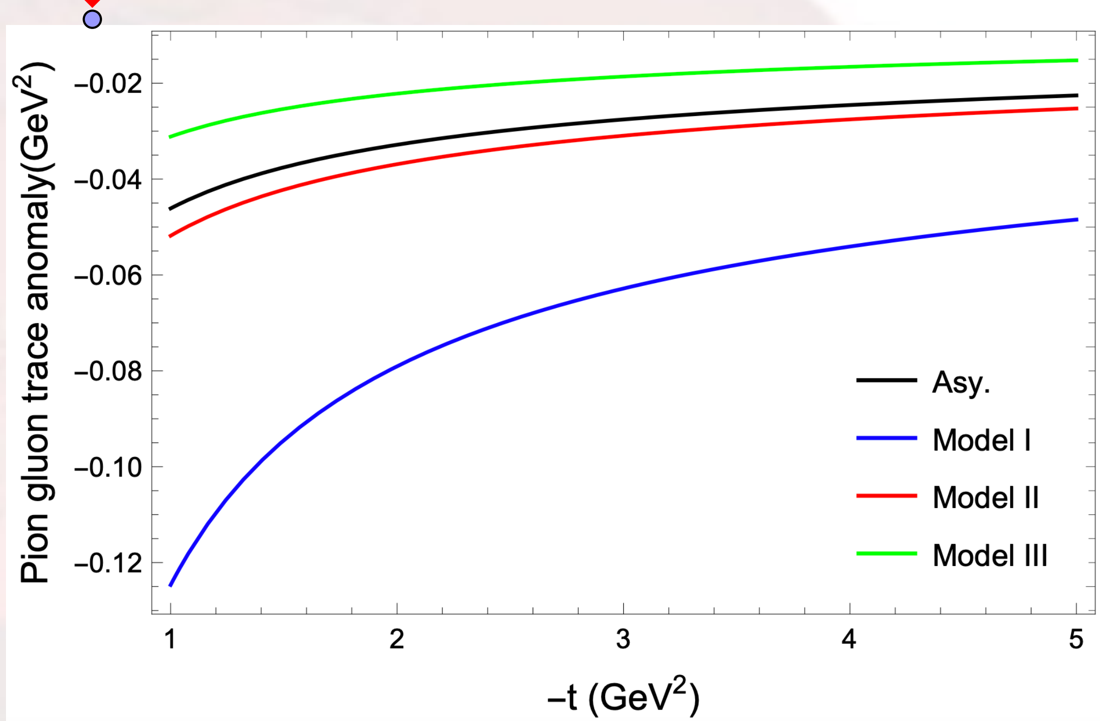
- The QCD dynamics involved in the on-shell photon (massless) transition to a massive heavy quarkonium does not allow a simple interpretation

Implication: Mass distribution of hadrons (F^2 term)

Scalar form factor of Pion

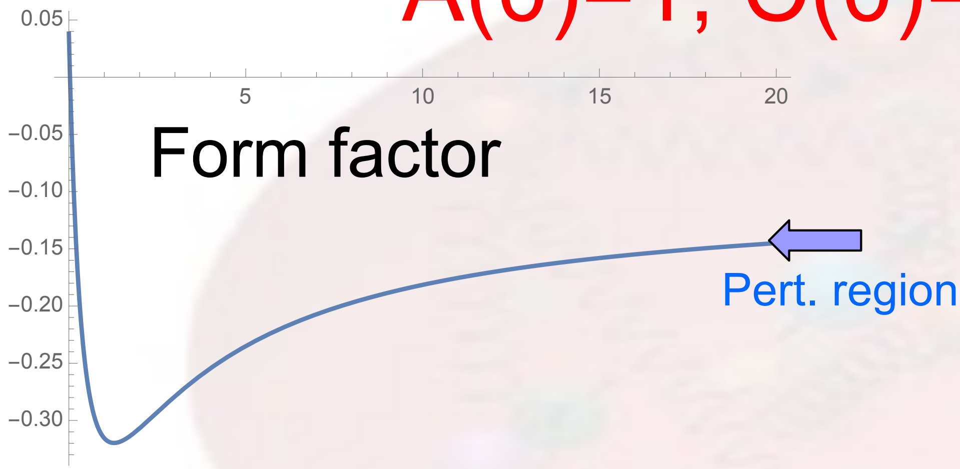
In the perturbative region:

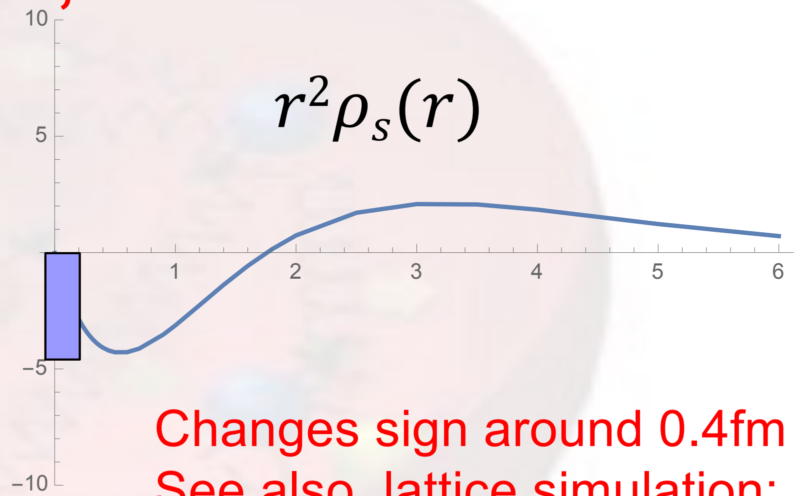
$$\langle P'|\frac{\beta(g)}{2g}F^2|P\rangle = -3\alpha_s^2(-t)C_F\left(11-\frac{2n_f}{3}\right)f_\pi^2$$
$$\times\left[1+\sum_{n=1}^{\infty}a_{2n}(\mu_0^2)L_{2n}^{\gamma_{2n}/b}(-t)\right]^2 ,$$



See also: X. Gao et al, 2206.04084

A simple dipole model for A(t) and C(t): A(0)=1, C(0)=-1, Λ=1GeV





$$\rho_{s,m}(r) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} G_{s,m}(-\vec{\Delta}^2)$$

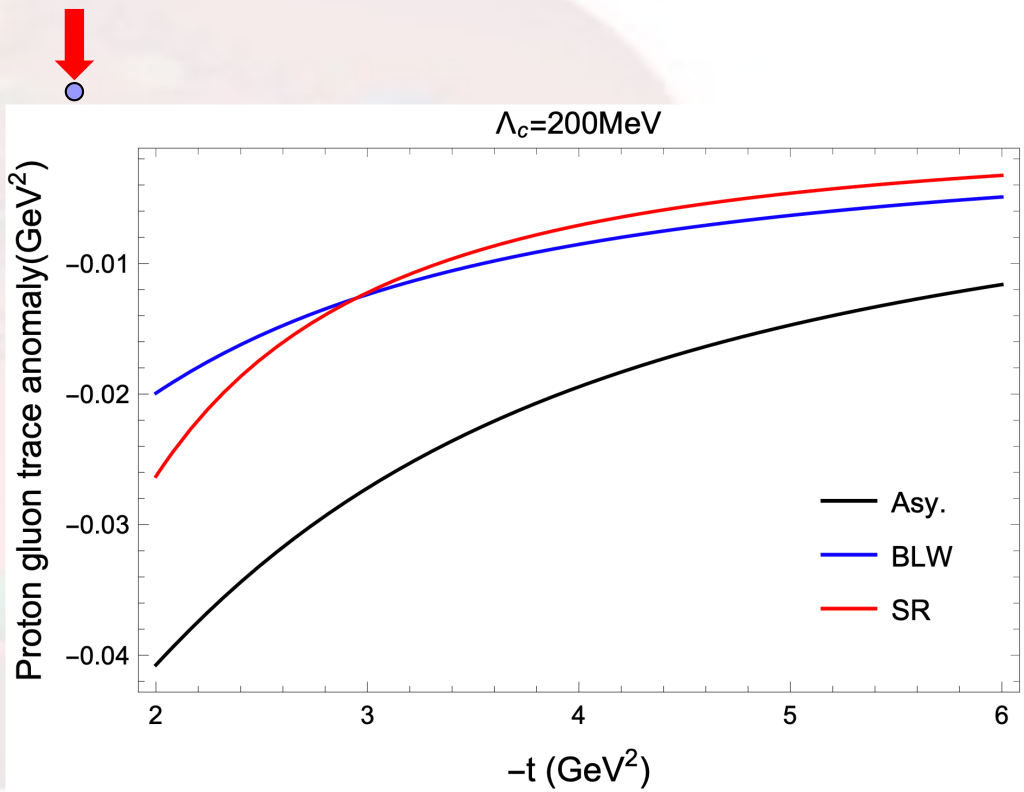
Changes sign around 0.4fm
See also, lattice simulation:
He et al, 2101.04942

Scalar form factor of Nucleon

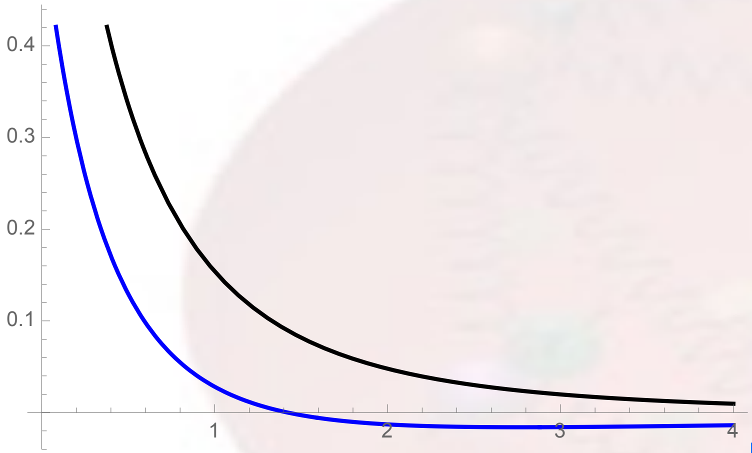
In the perturbative region:

$$G_p(t) = \int [dx][dy]\{x_3\Phi_4(x_1,x_2,x_3)\mathcal{G}_\Phi(\{x\},\{y\}) + x_1\Psi_4(x_2,x_1,x_3)\mathcal{G}_\Psi(\{x\},\{y\})\}\Phi_3(y_1,y_2,y_3)$$

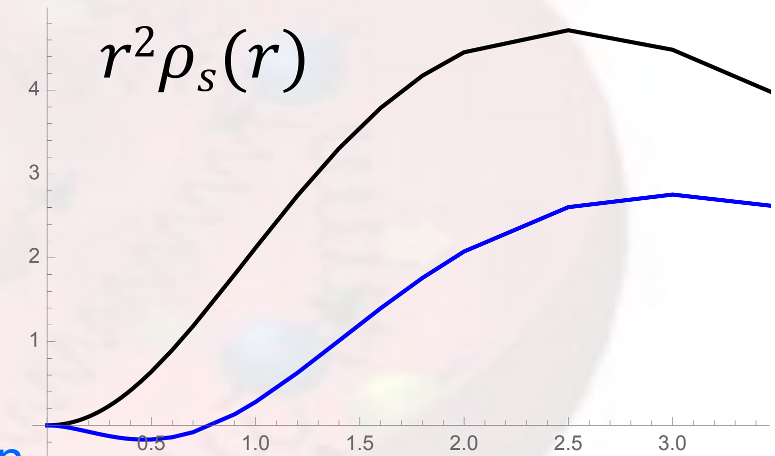
$$\mathcal{H}_\Psi = \frac{C_B^2 M_p}{6t^2}(4\pi\alpha_s)^2\Big[x_3\left((y_1-y_3)\bar{x}_1 + x_1y_2\right)T_1 - T_3 + \bar{x}_3\left(y_3\bar{x}_3 + x_3\bar{y}_3\right)\tilde{T}_1 + x_3\left(y_2\bar{x}_2 + x_2\bar{y}_2\right)(\tilde{T}_2 - T_2) + x_3\left(y_1-\bar{y}_1\right)(T_4+T_5) + \left(y_3\bar{x}_3 + x_3\bar{y}_3\right)(\tilde{T}_4 + \tilde{T}_5)\Big]$$



Simple model for proton
dipole for A(t) and tripole for C(t)



Pert. region



$$\rho_{s,m}(r) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} G_{s,m}(-\vec{\Delta}^2)$$

Summary

- Power behavior of the gravitational form factors derived in perturbative QCD at large momentum transfer
  - Imply that the scalar field distribution may change sign
- It is hard to build a direct connection between the near threshold photoproduction of heavy quarkonium and the gluonic gravitational form factors
- Looking forward: phenomenological study in terms of the gluon GPDs is greatly needed
  - Indirect connection to the gluonic gravitational form factors