Nonlinear dynamical tides in coalescing binary neutron stars

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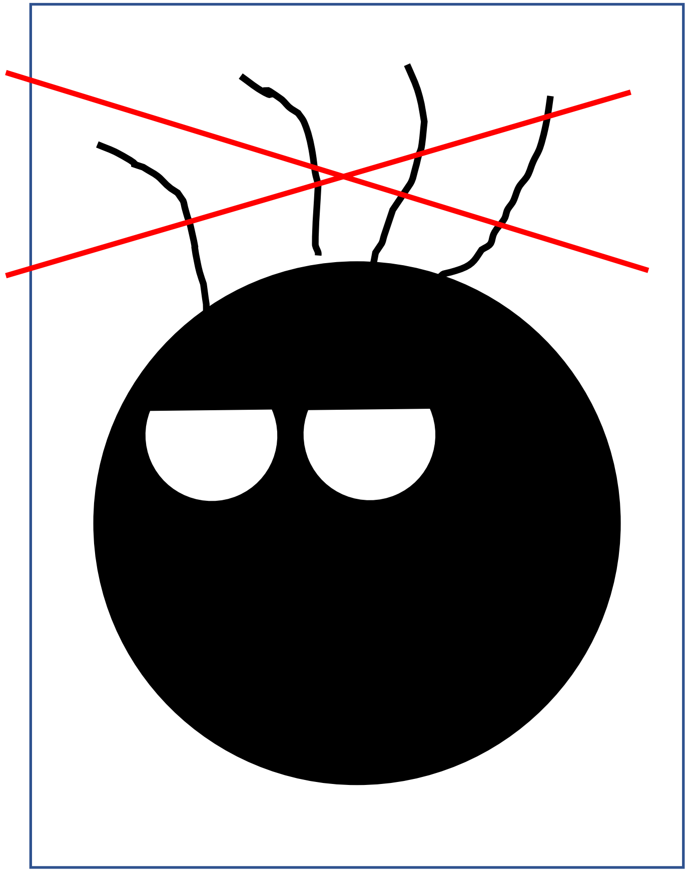
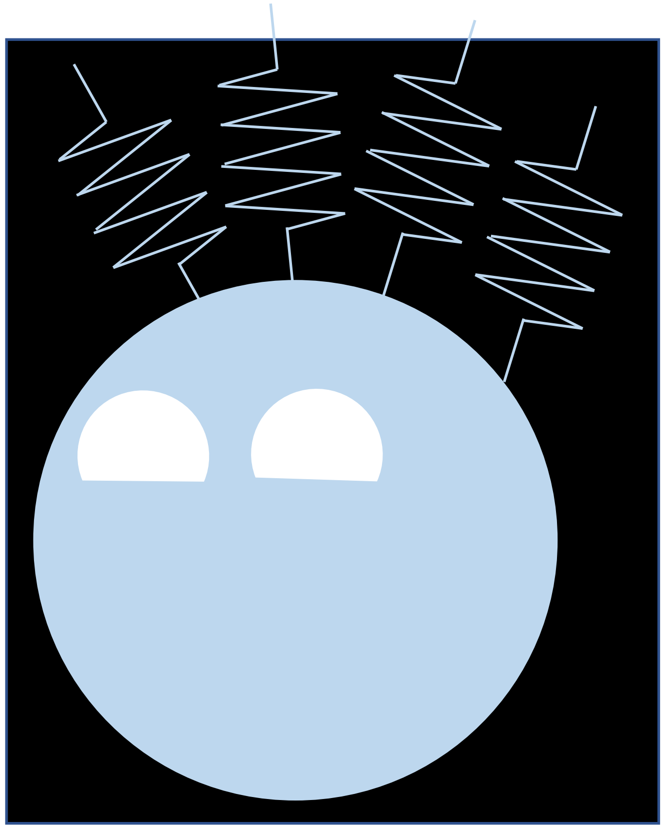
Black holes are boring (not really)
No hair; only mass and spin



Artist's impression of a black hole (BH)

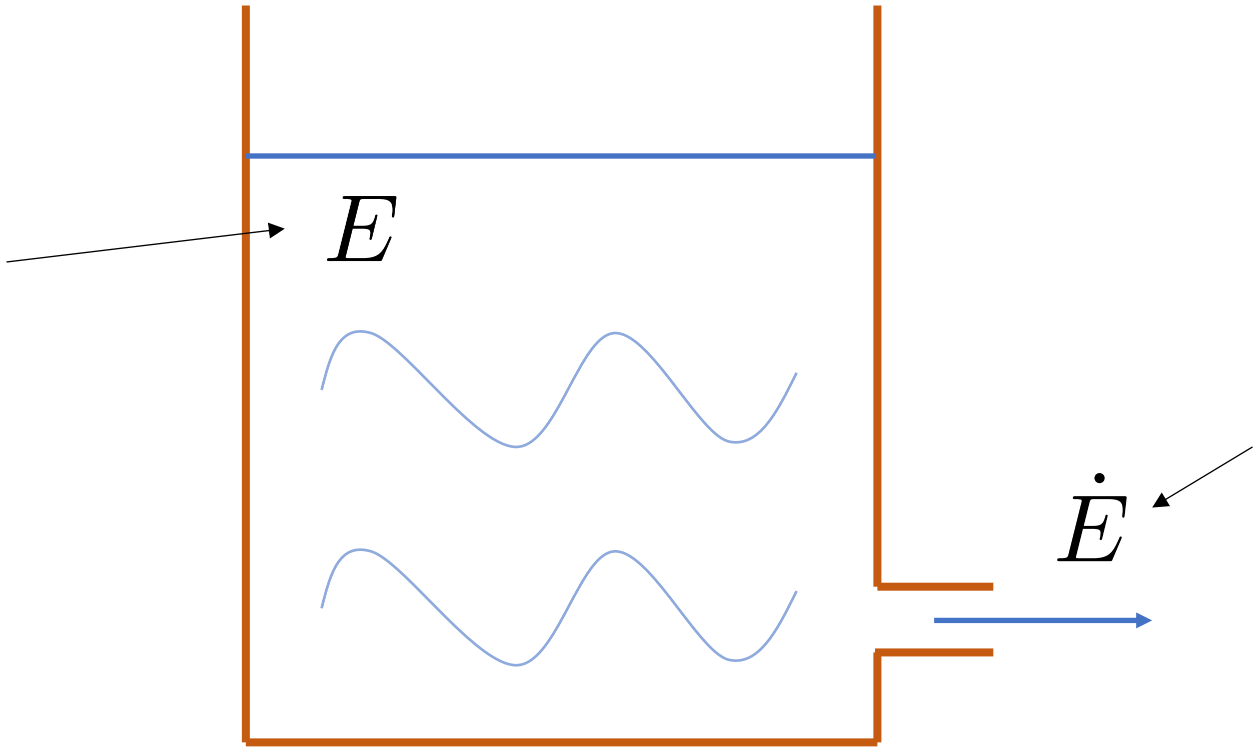
Neutron stars are exciting (really)!
Mass, spin, matter effects!
Probes to neutron star interior & extreme gravity



And a neutron star (NS)

Tidal effects

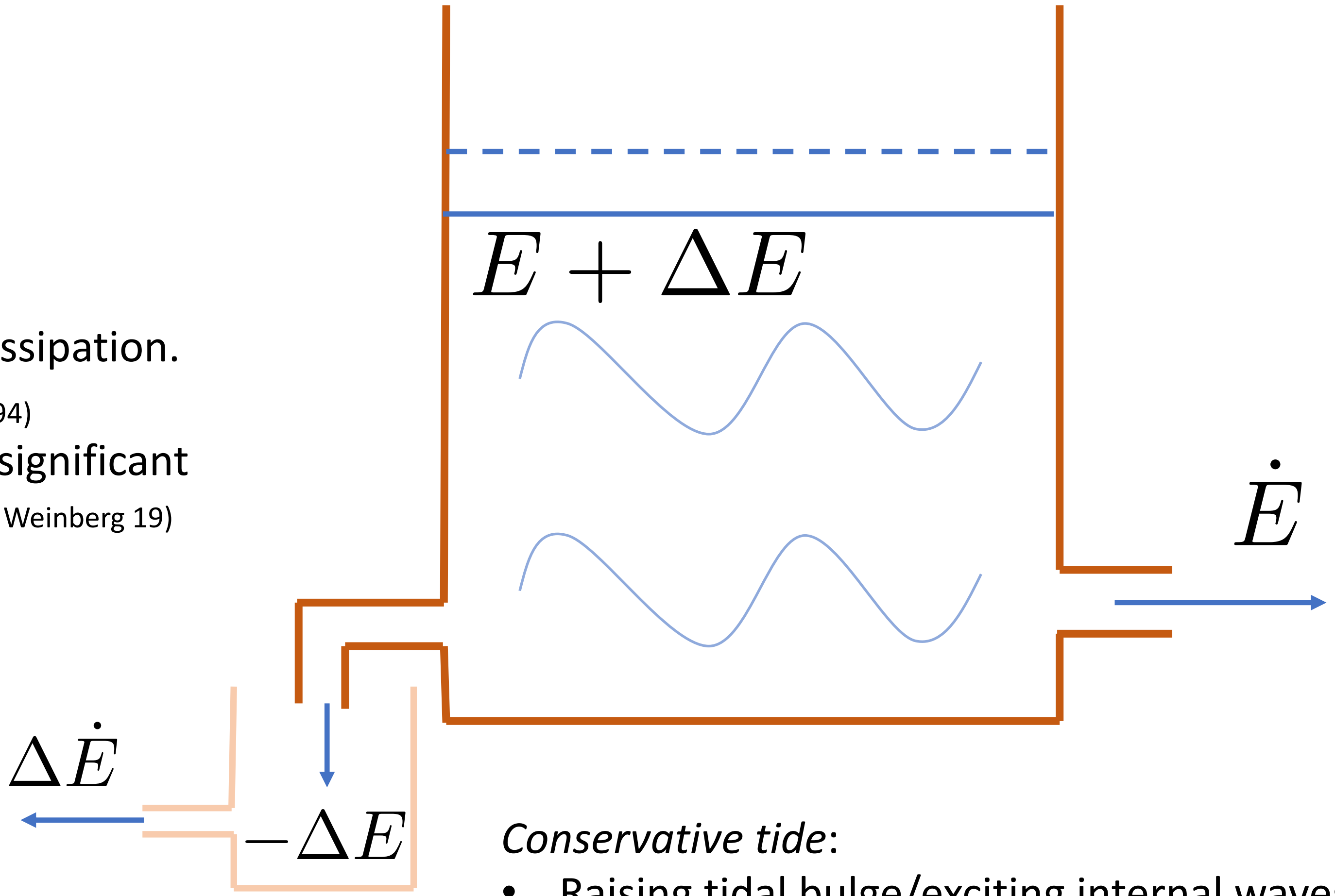
Orbital energy:
Tank of water



GW radiation:
Hole on the tank

What we measure from the GW signal:
Accumulated phase of the waveform
-> time to empty the tank

Tidal effects



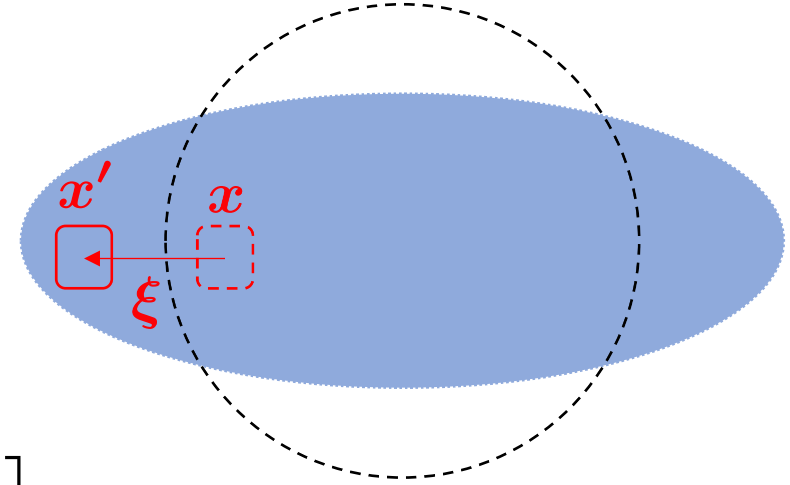
Dissipative tide:

- Internal fluid dissipation.
- Linear: tiny (Lai 94)
- NL: potentially significant (Weinberg 16; Arras & Weinberg 19)

Conservative tide:

- Raising tidal bulge/exciting internal waves
- Dominant effect

Nonlinear Hamiltonian—modal expansion



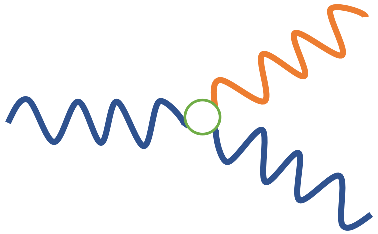
- Lagranian displacement $\boldsymbol{\xi} = \boldsymbol{x}' - \mathbf{x}$
- Nonlinear in $\boldsymbol{\xi}$
- Phase space expansion $\begin{bmatrix} \boldsymbol{\xi} \\ \dot{\boldsymbol{\xi}} \end{bmatrix} (t, \boldsymbol{x}) = \sum_a q_a(t) \begin{bmatrix} \boldsymbol{\xi}_a(\boldsymbol{x}) \\ -i\omega_a \boldsymbol{\xi}_a(\boldsymbol{x}) \end{bmatrix}$

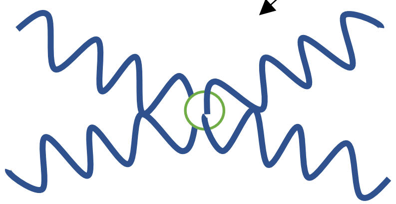
Mode freq corrected by Coriolis force

- Hamiltonian $H_{\text{mode}} = E_1 \left(\frac{1}{2} \sum_a \frac{\omega_a}{\omega_{a0}} q_a q_a^* - \frac{1}{3} \sum_{abc} \kappa_{abc} q_a q_b q_c - \frac{1}{4} \sum_{abcd} \zeta_{abcd} q_a q_b q_c q_d \right)$

Three wave:

- NL enhanced f-mode resonance





Four wave:

- NL g-mode resonance locking
- pg-instability

Why modes?—Fluid response

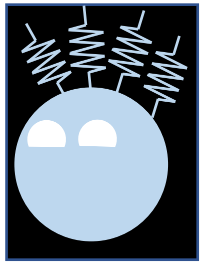
- Largely Newtonian description
- Face challenges in GR (Poisson 21, Pitre & Poisson 24)
- (but see, e.g., Gittins+ 25, Yin+ 25, Hegade+ 25, etc. on recent progress)
- *Best model of mode resonance!*

- Pitre & Poisson 2024: relativistic, modeless "dynamical" tide
- Low-frequency expansion & resummed to a Lorentzian form
- Only works if the NS has one eigenmode (f-mode only)
- *Lorentzian is insufficient in the dynamical regime!*

$$\tilde{k}_\ell(\omega) = \frac{\tilde{k}_\ell(0)}{1 - \omega^2/\omega_{*\ell}^2}, \qquad \omega_{*\ell}^2 := \frac{GM}{R^3}\frac{k_\ell}{\ddot{k}_\ell},$$

Why modes?—Fluid response

Spring-like hair



- Driven harmonic oscillator

$$\ddot{q}_a + \omega_a^2 q_a = \omega_a^2 V_a e^{-im\phi}$$

- Solution:

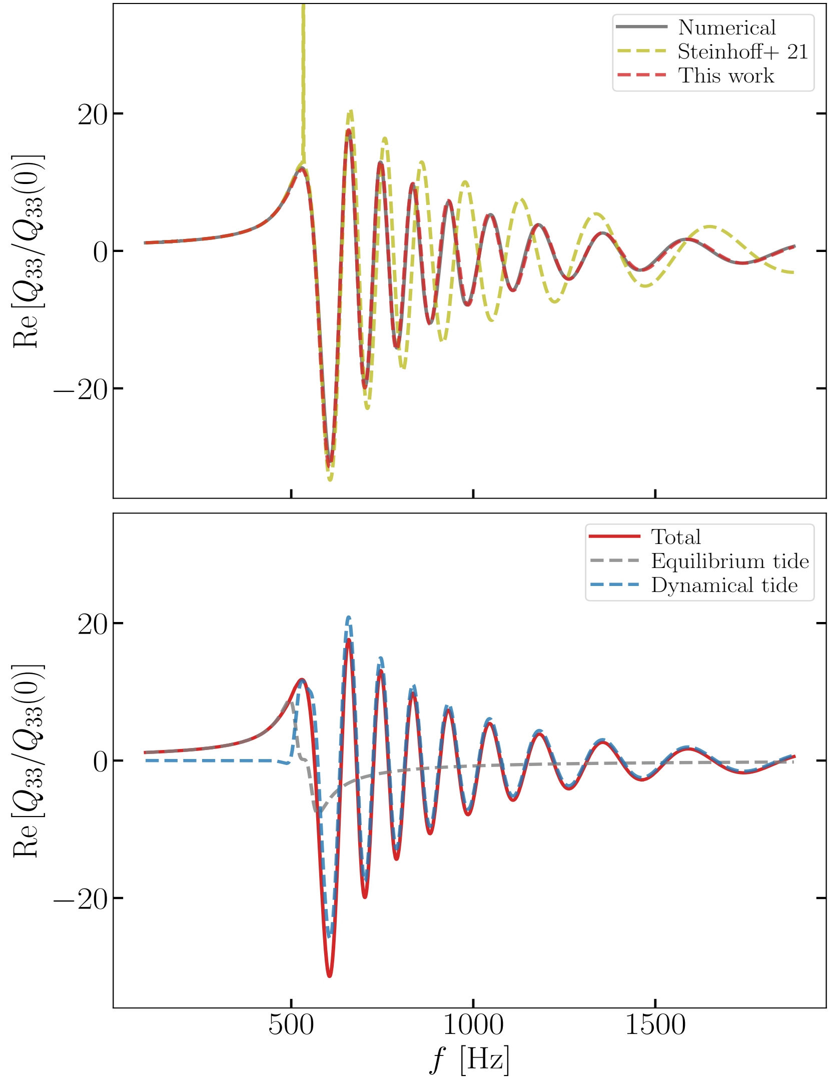
$$q_a = (\text{eq. tide}) e^{-im\phi} + (\text{dyn. tide}) e^{-i\omega_a t}$$

- In-phase with the drive
- Lorentzian

- Excited at resonance
- Varies at natural freq
- History-dependent
- Necessary for NL resonance locking, etc.

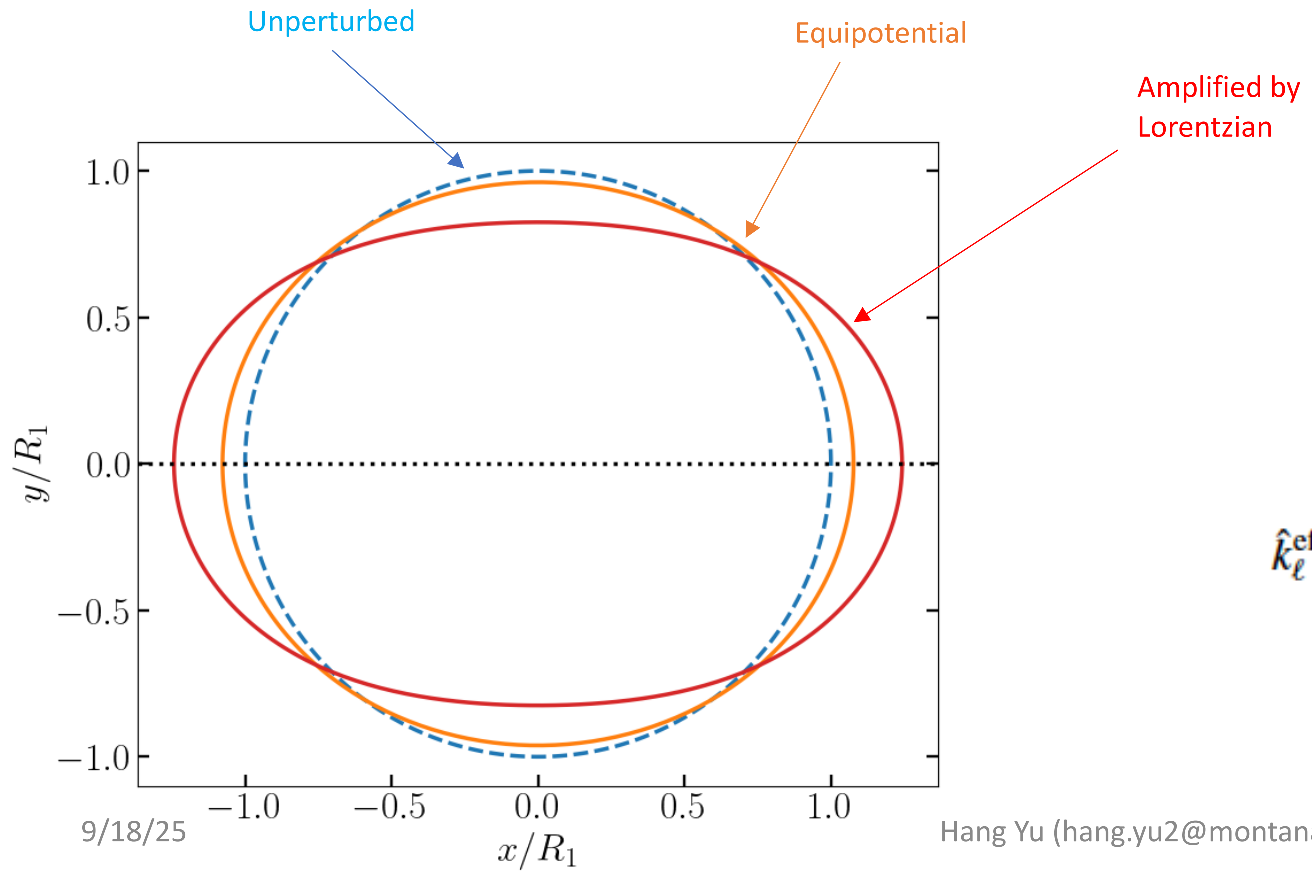
- Insufficient: orbit described by a single Fourier component
- Hinderer+ 16, Steinhoff+ 16, & 21: more than Lorentzian; inaccurate beyond resonance
- Corrected ones: Ma+ 20; Yu+ 24; Pnigouras+ 25

(Yu, Arras, & Weinber 2024, arXiv:2404.00147)



Why modes?—Tidal back-reaction

- Commonly used approach—effective Love number
- Captures only the radial interaction

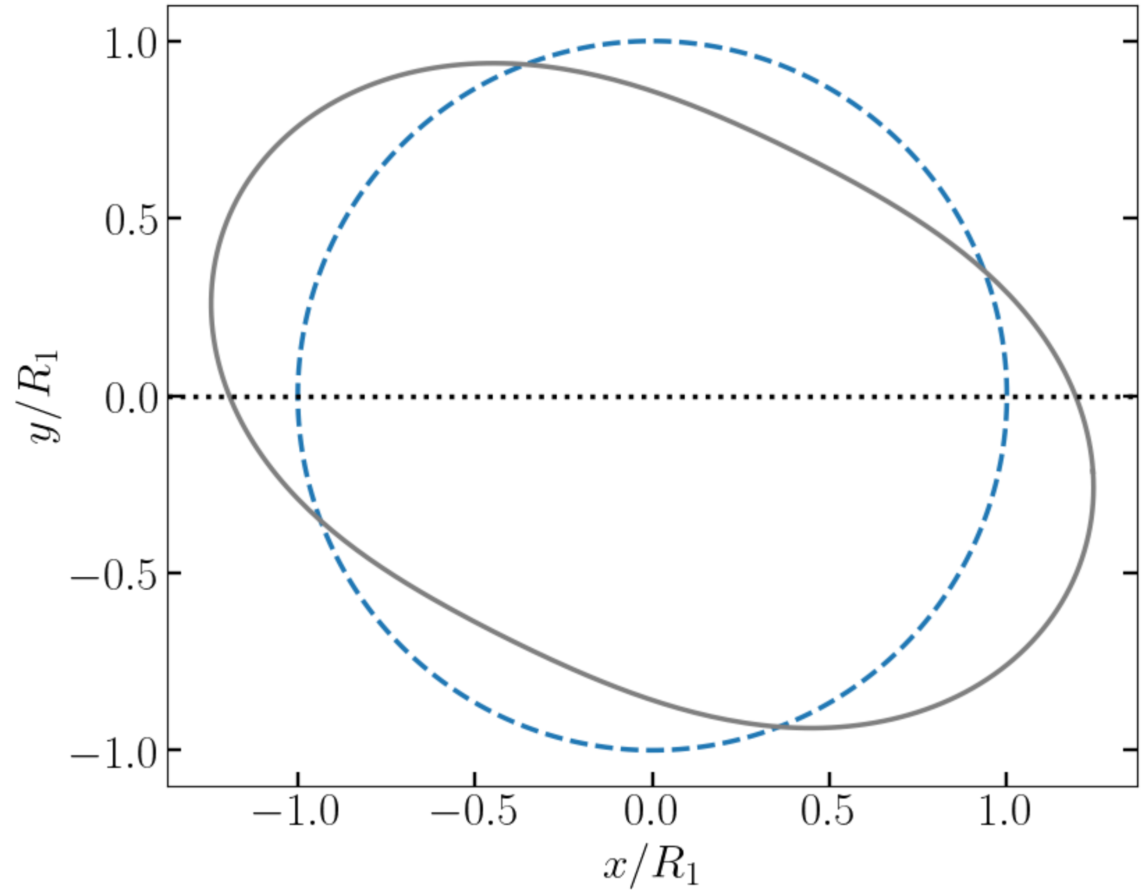


$$\lambda \rightarrow \hat{\kappa}^{\text{eff}} \lambda$$ Steinhoff+ (2021)

$$\hat{k}_\ell^{\text{eff}} = a_\ell + b_\ell \left(1 - \frac{(\Delta\omega_{0\ell})^2}{\omega_{0\ell}^2}\right)\left\{\frac{\omega_{0\ell}^2}{\omega_{0\ell}^2 - (\ell\omega_{\text{orb}} - \Delta\omega_{0\ell})^2}\right.$$

+ (other terms)}

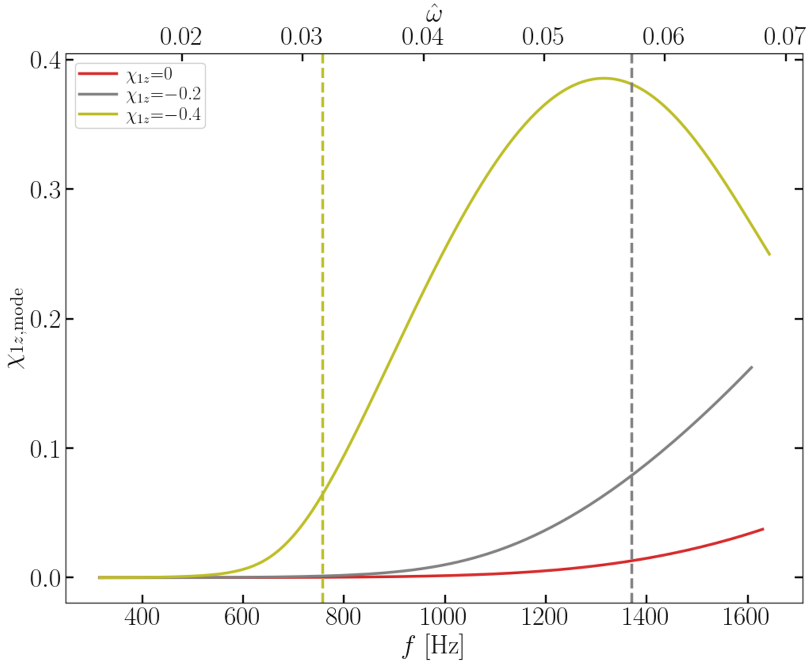
The actual bulge



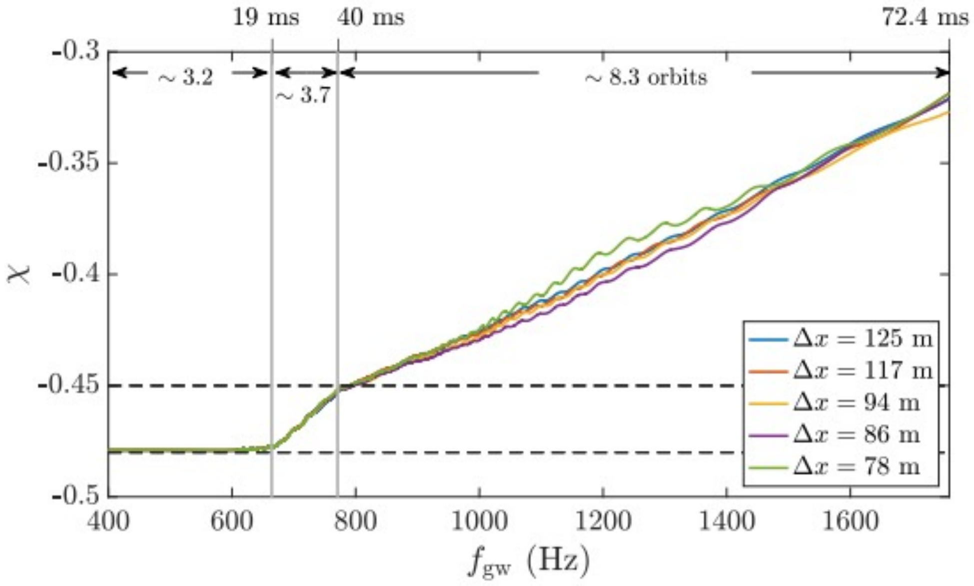
- GW-induced orbital decay creates a lag in the tidal bulge!
- "Effective" damping >> linear fluid dissipation
  (Lai 94; Yu+ 24; Yu & Lau 25)
- Both radial (eff. Love) and tangential (torque) components!
- Why eff. Love is incorrect:
  Reducing the mode amplitudes (canonical variables) in terms of (r, ϕ)

(Yu & Lau 2025, arXiv:2501.13064)

Tidal spin

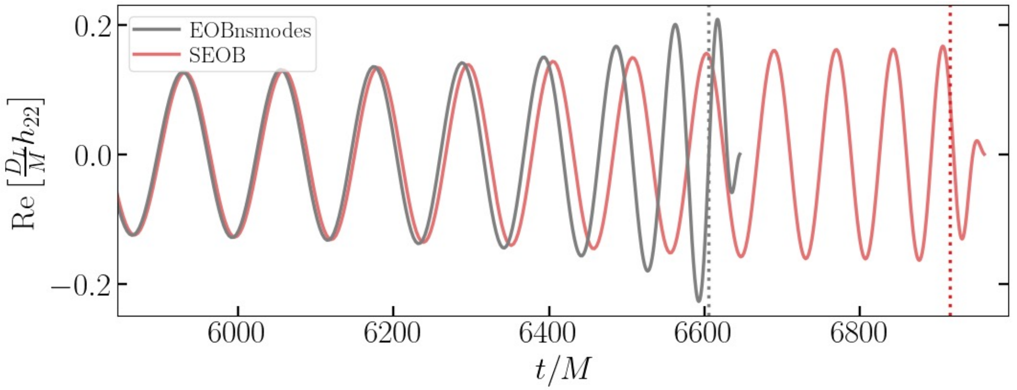


Theoretically predicted in Yu & Lau 25

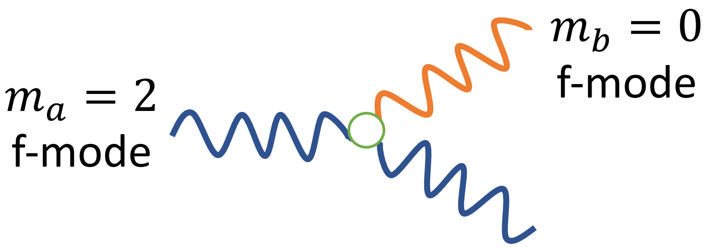


Numerically seen in Kuan+ 24

- Captured by treating mode amplitudes as canonical variables
- x2 diff due to canonical vs. physical spin
- Large phase error if missed (e.g., as in SEOB)



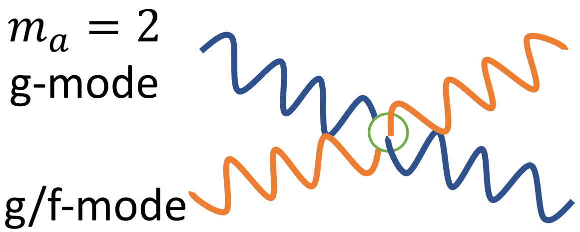
Nonlinear hydrodynamics as frequency shifts



$$\ddot{q}_a + \omega_a^2 q_a = \omega_a^2 \kappa q_a q_b,$$

$$\ddot{q}_a + \underbrace{\omega_a^2 [1 - \kappa q_b]}_{\omega_{a,\mathrm{eff}}^2} q_a = 0.$$

- Frequency shift produced by perturbed eq.
- $\kappa q_b > 0,\ \omega_{a,\mathrm{eff}}^2 \downarrow$
- Enhanced f-mode resonance!



$$\ddot{q}_a + \omega_a^2 q_a = \omega_a^2 \zeta (q_b q_b^*) q_a,$$

$$\ddot{q}_a + \underbrace{\omega_a^2 [1 - \zeta (q_b q_b^*)]}_{\omega_{a,\mathrm{eff}}^2} q_a = 0.$$

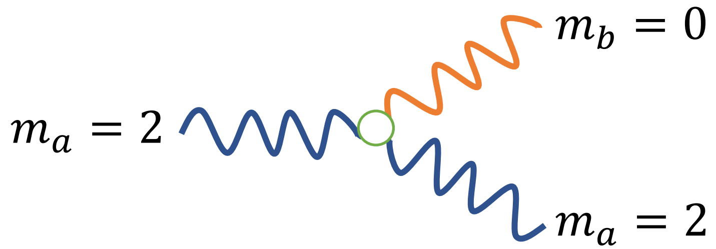
- Frequency shift produced by anharmonicity.
- Important for the g-mode!
- $\omega_{a,\mathrm{eff}}^2 \uparrow$ —resonance locking (low-order g-modes)
- $\omega_{a,\mathrm{eff}}^2 < 0$ —p-g instability (high-order g-modes)

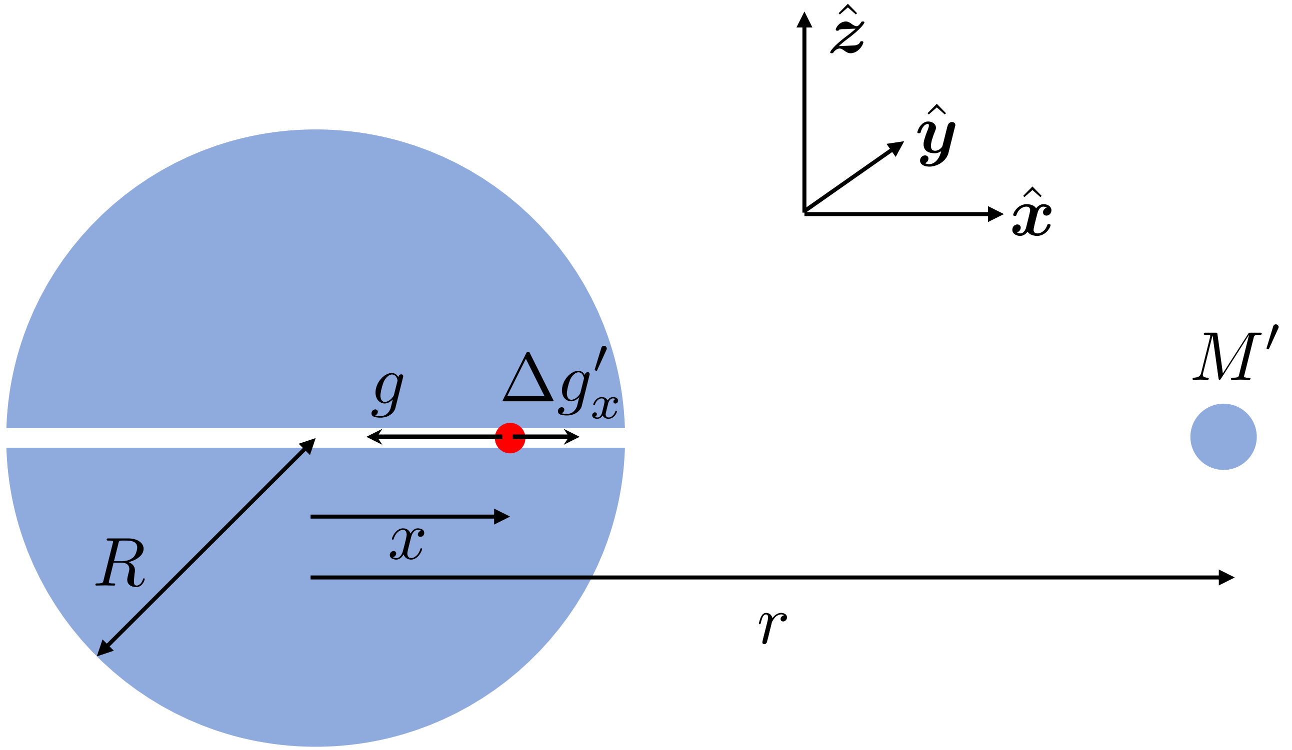
Nonlinearly enhanced f-mode resonance

Yu et al. 2023

arXiv: 2211.07002

Origin of the f-mode frequency shift

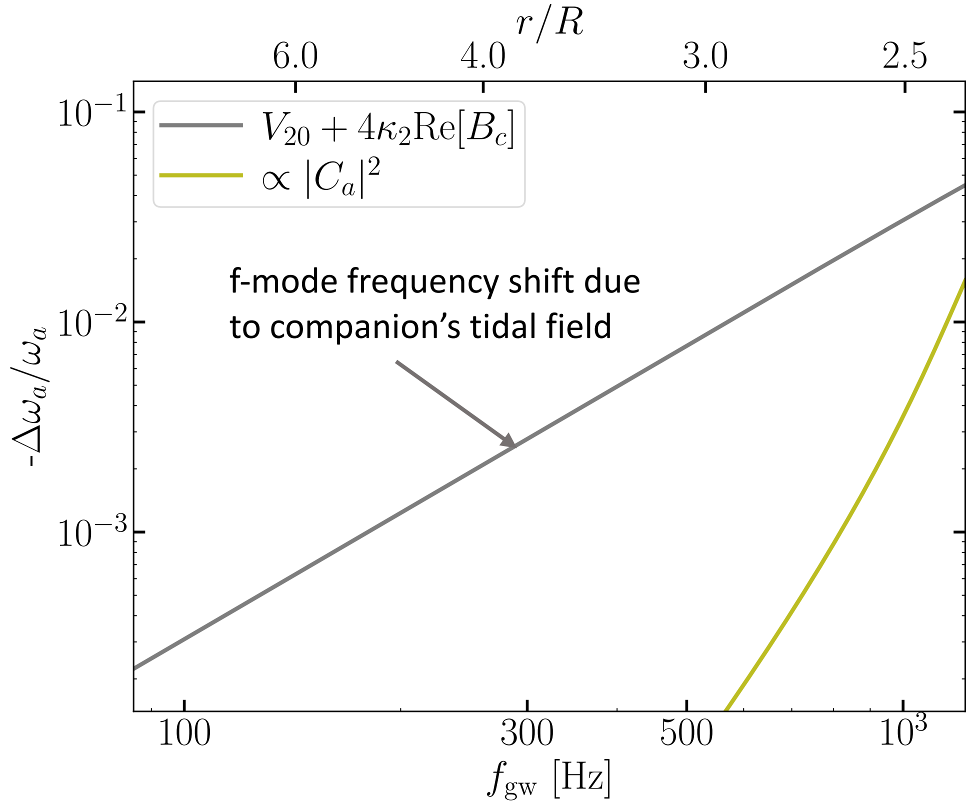


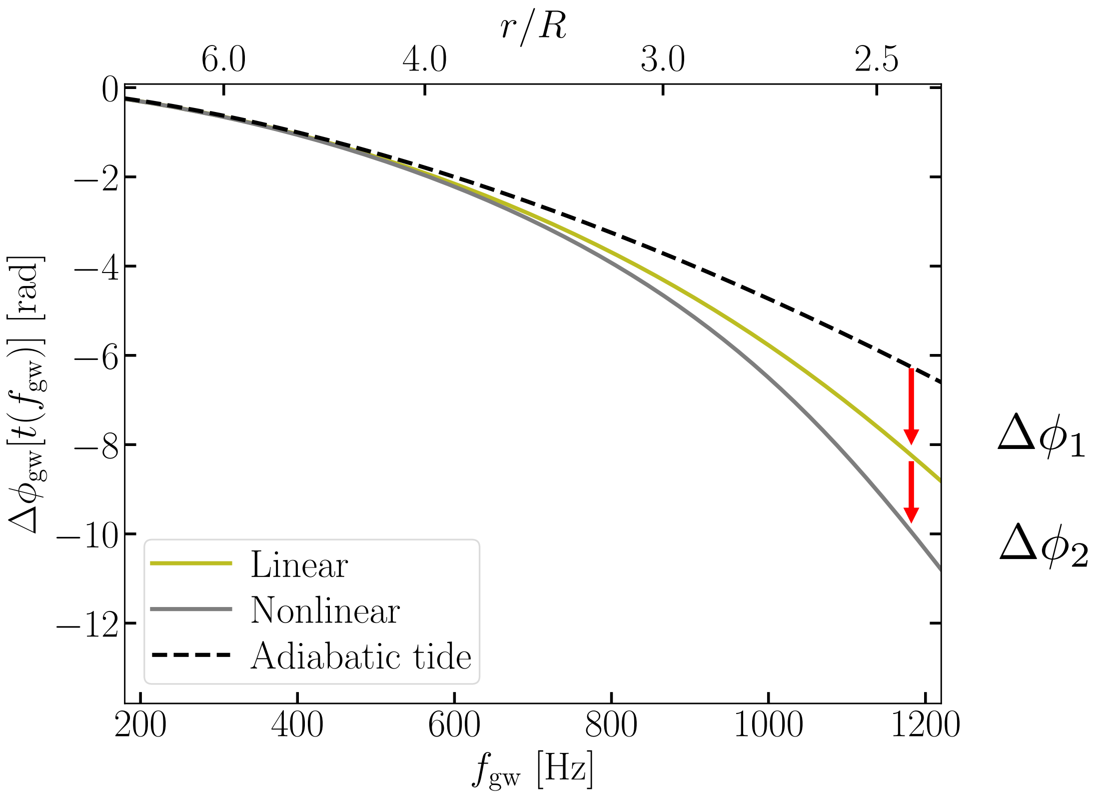


- Test particle in a small hole through the star:
- $\ddot{x} = -g \simeq -\rho x = -\omega^2 x$
- ~ eigenfrequency of the f-mode

- Companion's tidal gravity reduces inward acc. along x
- Fractional frequency decrease ~ (R/r)^3~ω^2
- ($\Delta\omega_y = -\Delta\omega_x/2,\ (\Delta\omega_x + \Delta\omega_y)/2 = \Delta\omega_x/4 < 0.$)

Enhanced mode resonance

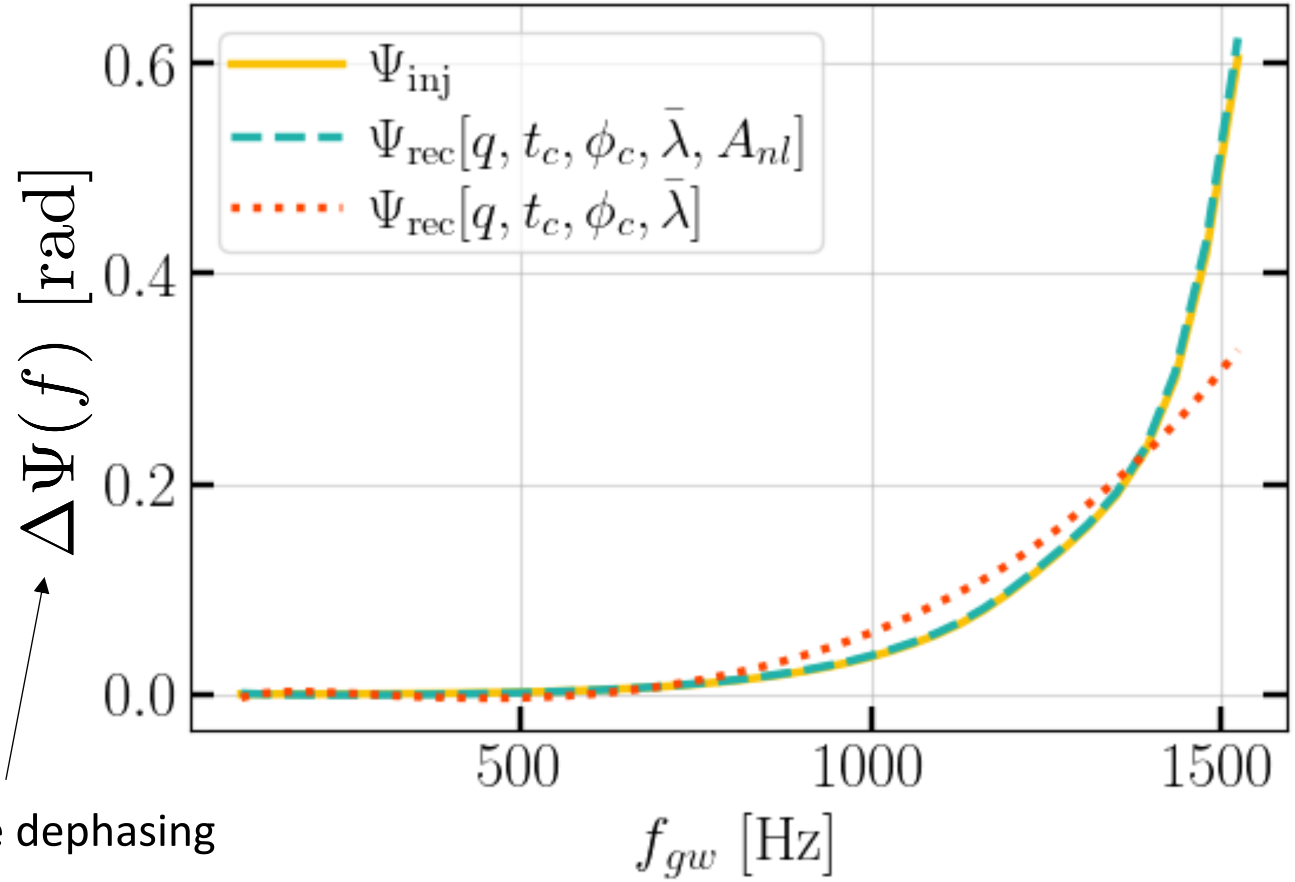




- The nonlinear f-mode frequency shift can be ~ 5% @ 1 kHz
- Confirmed by Pitre & Poisson 25 in GR (w/ low-freq approximation)
- Impact amplified by the Lorentzian!

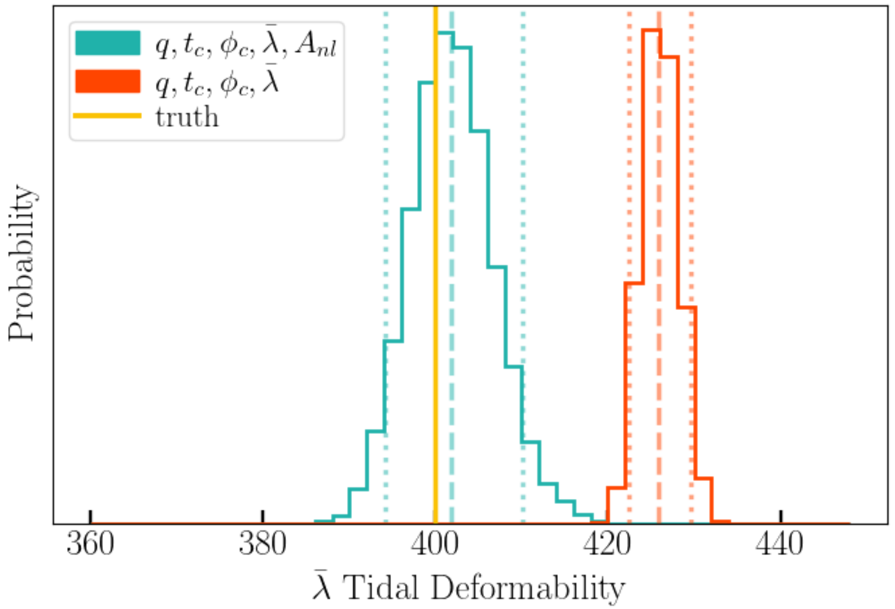
$\Delta\phi_1$~ linear dyn. – ad. tide
$\Delta\phi_2$~ nonlinear – lin. dyn. Tide
$\Delta\phi_2 \simeq \Delta\phi_1$!
Affecting every BNS!

Biased EOS inference (J. Bretz & HY, in prep)

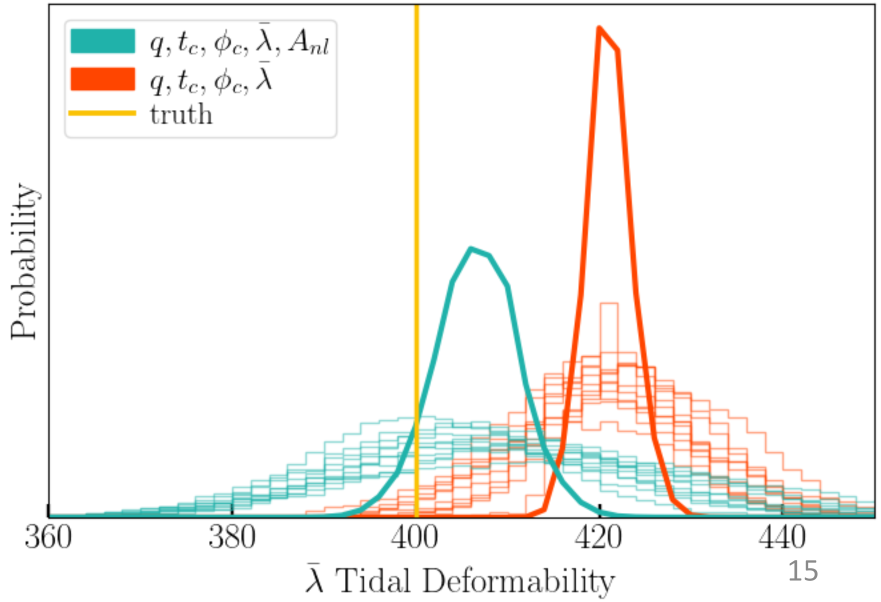


NL tide dephasing

Single SNR=2,000 event

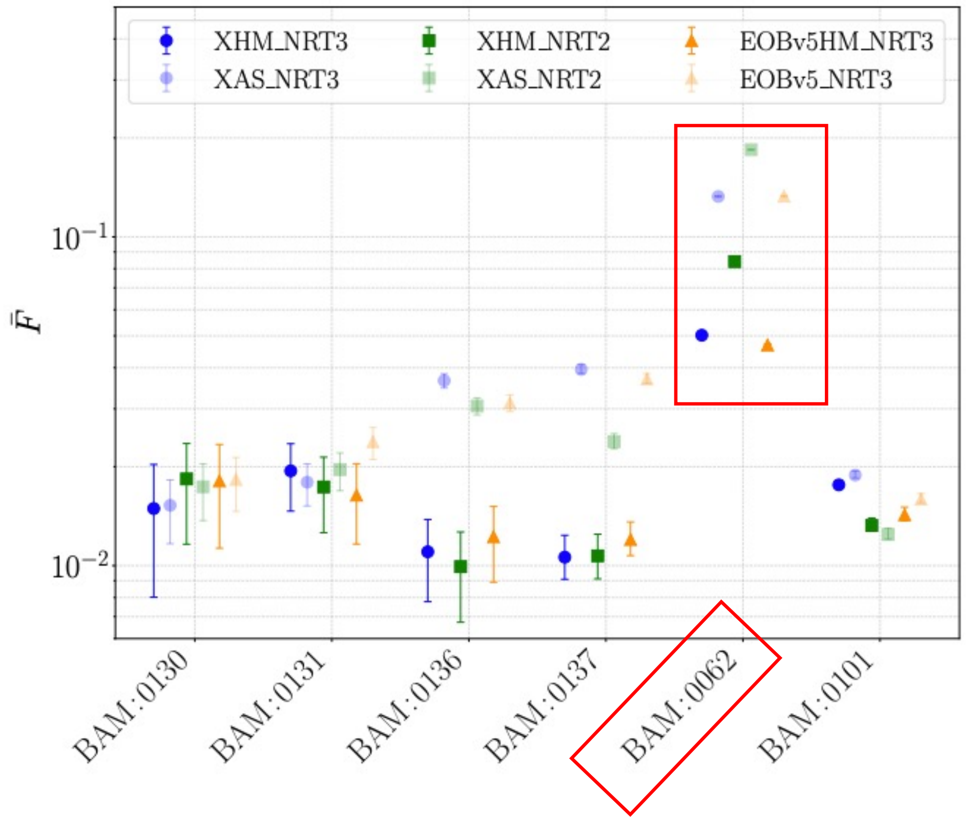


11 SNR=500 events



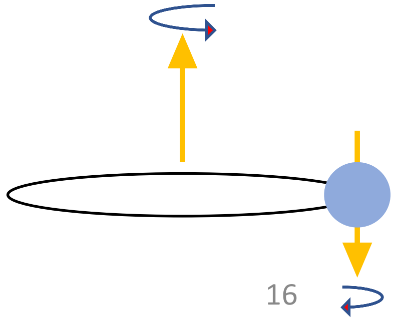
Can NR surrogates/calibrations capture it?

- Yes, but…
- Does it extrapolate?
  - Taylor expansion vs Lorentzian
  - True dynamical tide is way more complicated than simple Lorentzian
  - NRTidalv3 vs BAM:0062 (Abac+ 25)
- Calibration parameters?
  - Are deformability, mass, and spin sufficient?
  - Nonlinear coupling coefficient?
  - Can we explore new physics (thermal effects, neutrino transportation, etc) with NL tide?



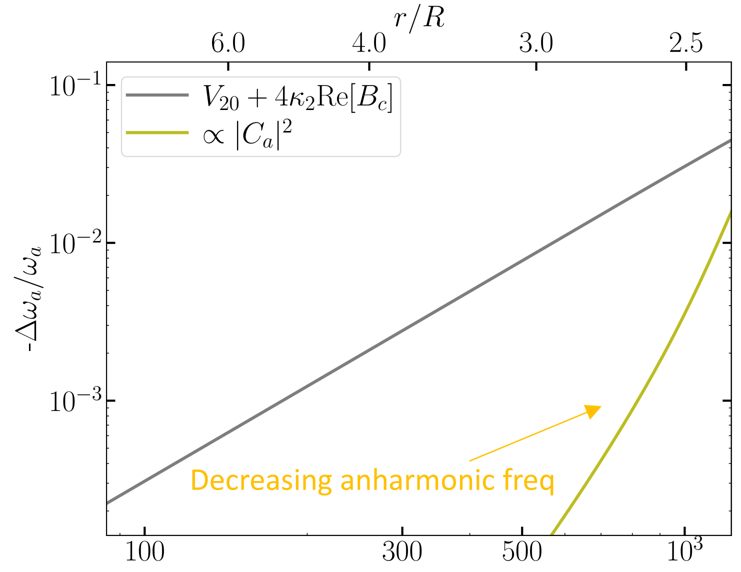
Name	EOS	χ_A	χ_B
BAM:0062	MS1b	−0.10	−0.10

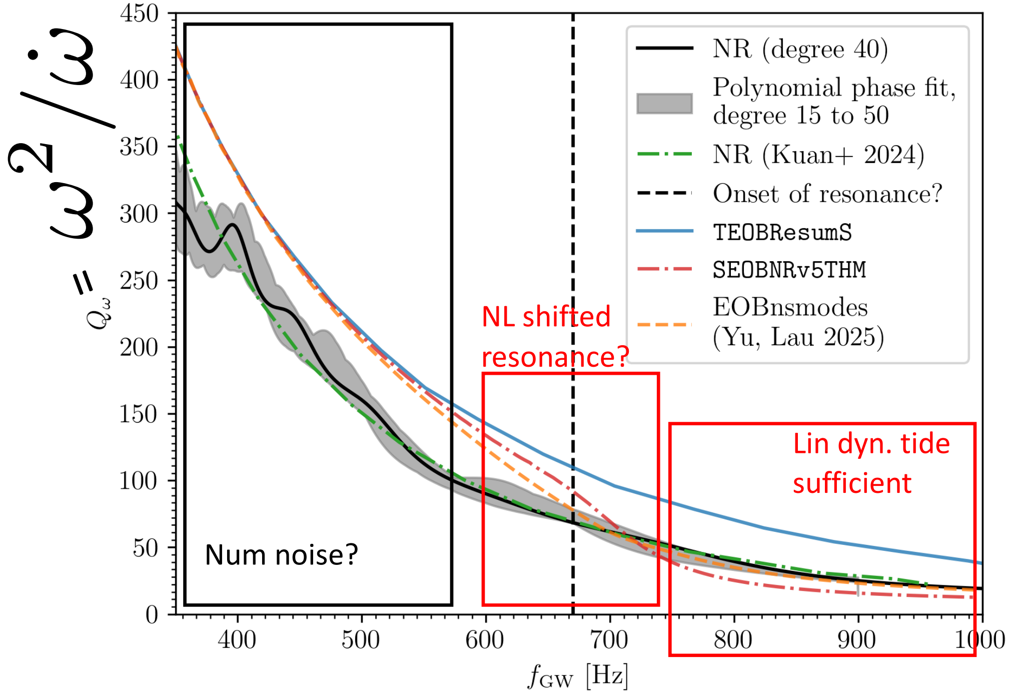
Anti-aligned spin enhances mode resonance



How to interpret NR results?

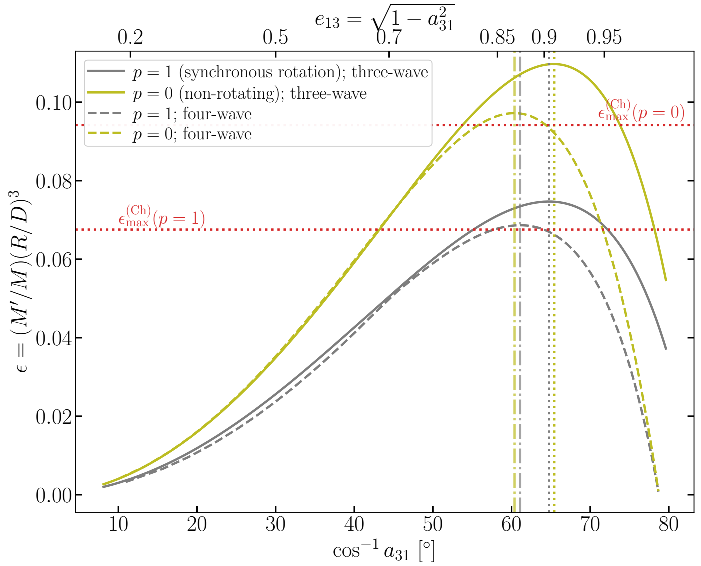
- Kuan+ 24:
  - NR simulation of BNSs with rapid, anti-aligned spins (χ=-0.48)
  - Claimed there is resonance locking of the f-mode.
- Likely incorrect.
  - Theory predicts decreasing f-mode freq w/ increasing energy (Yu+ 23)
  - EOB model by Yu & Lau 25 w/ linear f-mode resonance explains NR largely
- Need theoretical model to properly explain!



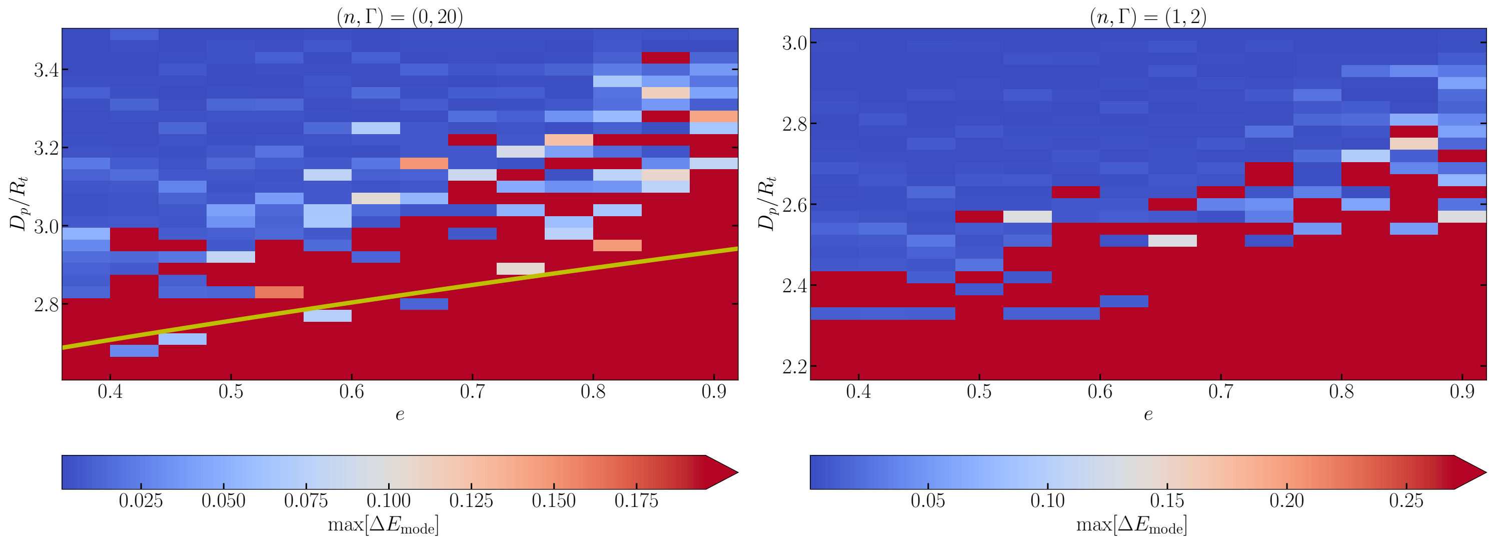


(Figure Credit: Marcus Haberland, AEI)

Aside: Roche limit (Yu+ 25, arXiv:2508.20183)



- When the nonlinear tide destabilizes the f-mode, Roche limit!
- Reproducing Chandrasekhar's hydrostatic result (synchronous rotation, circular orbit, & homogenous ellipsoids)
- Applicable to real stars w/ arbitrary eccentricities/rotation
- Dynamical tide significantly modifies the Roche limit!
- Terminal frequency of BNSs? (Lai 93)

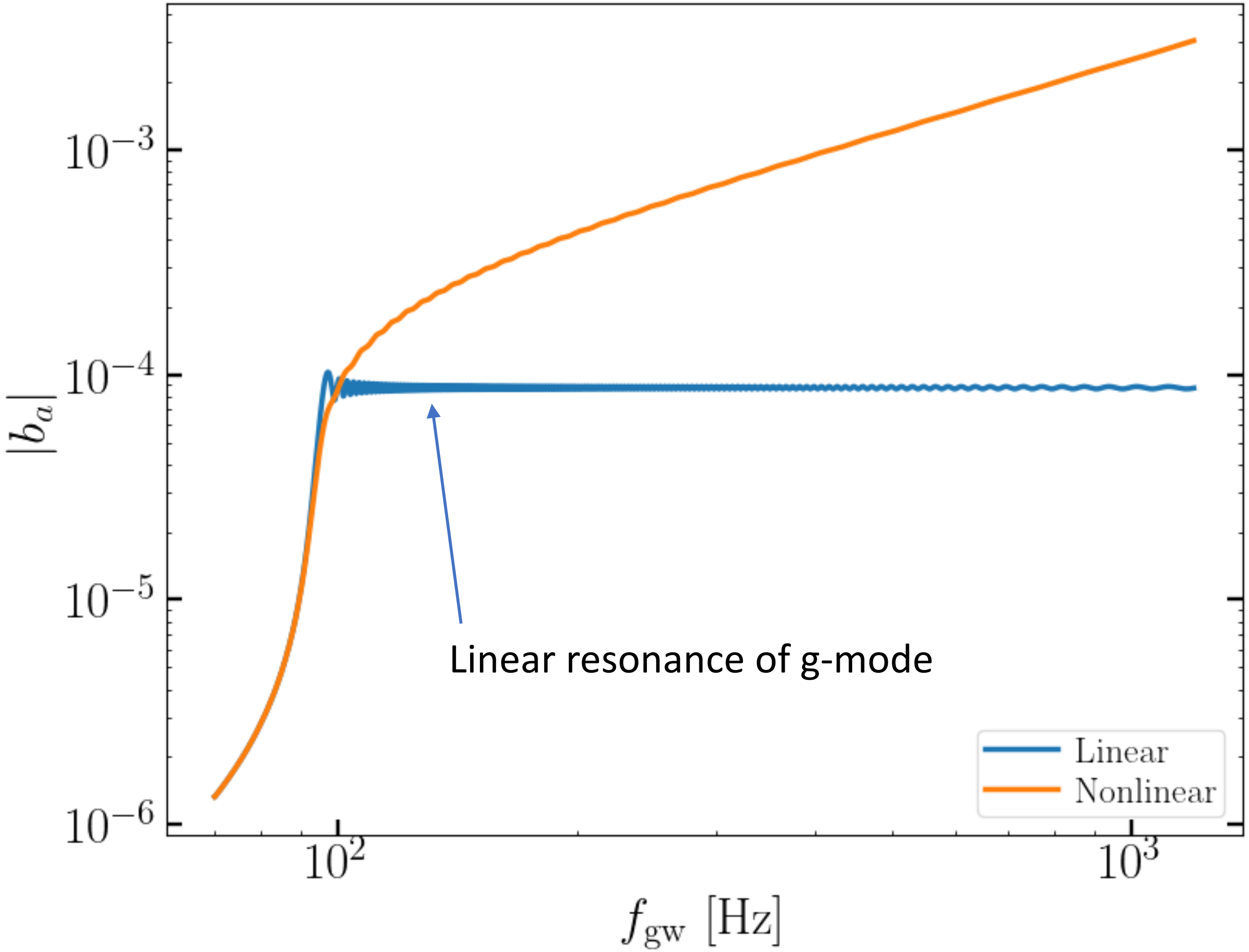


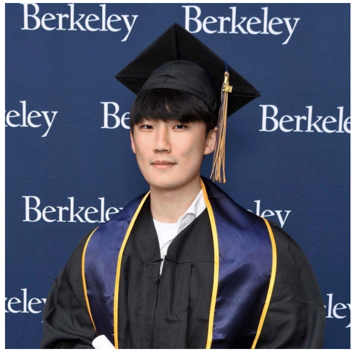
Resonance locking of low-order g-modes

Kwon, Yu, & Venumadhav, 2025

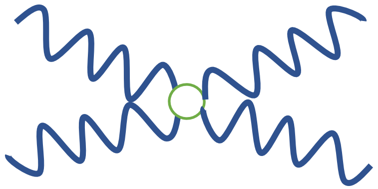
arXiv: 2410.03831, 2503.11837

Resonance locking of g-modes



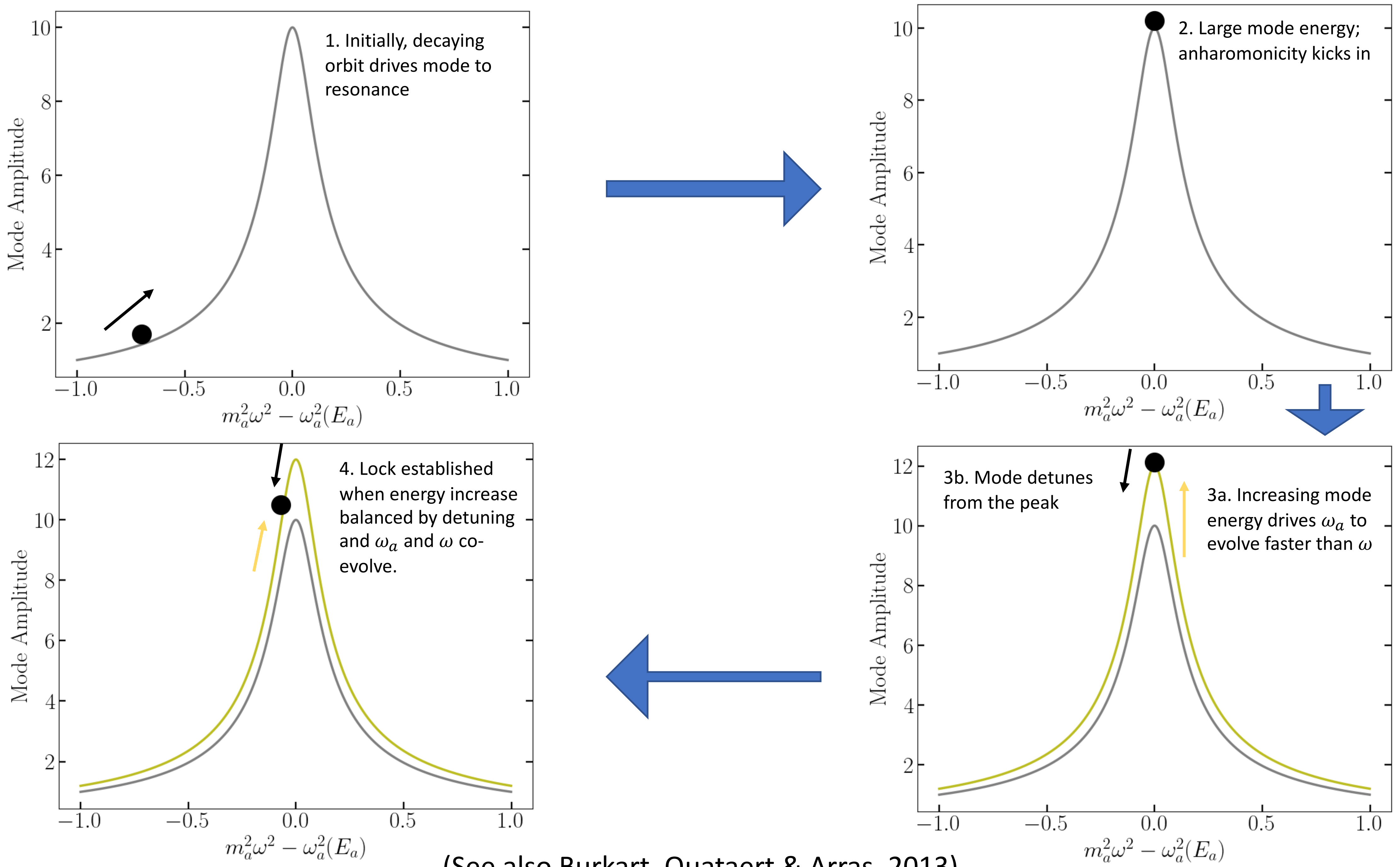


$m_a = 2$

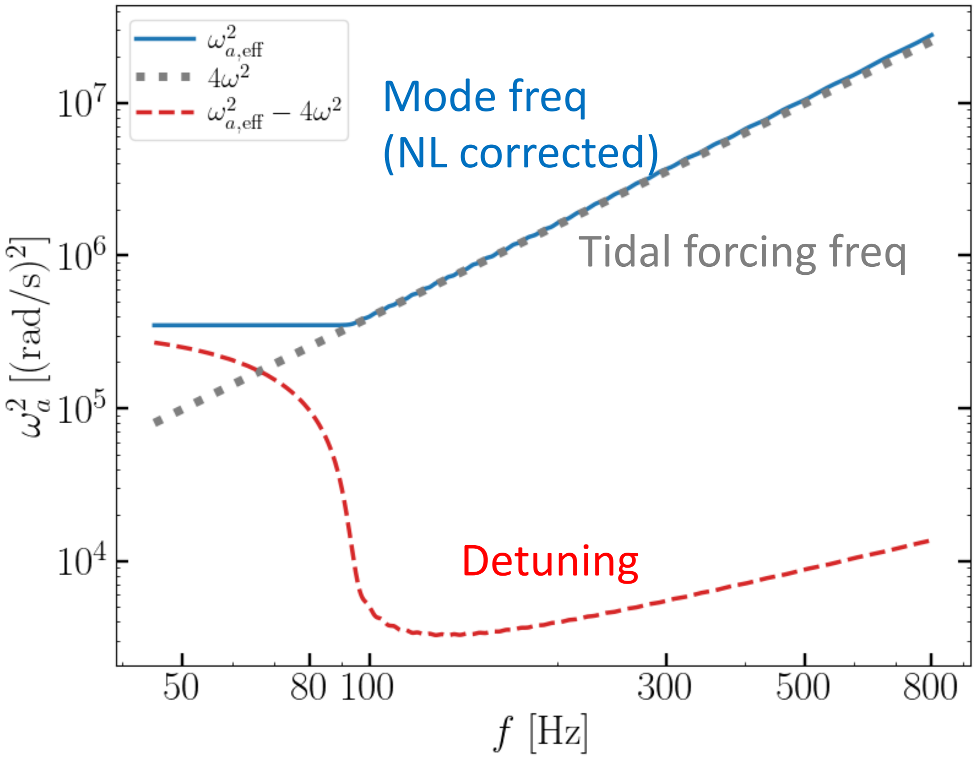


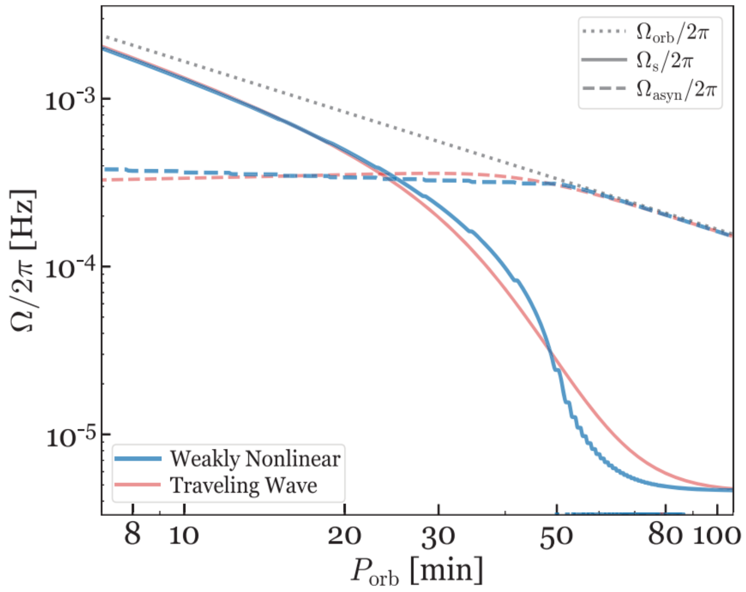
Linear case:
Decoupled with the orbit afterward.
Small phase shift < 0.01 rad (Lai 94; HY & Weinberg 17a, b)

Nonlinear case:
Continuously amplified by resonance and coupled with the orbit.
Significant phase correction ~ 3 rad!



(See also Burkart, Quataert & Arras, 2013)





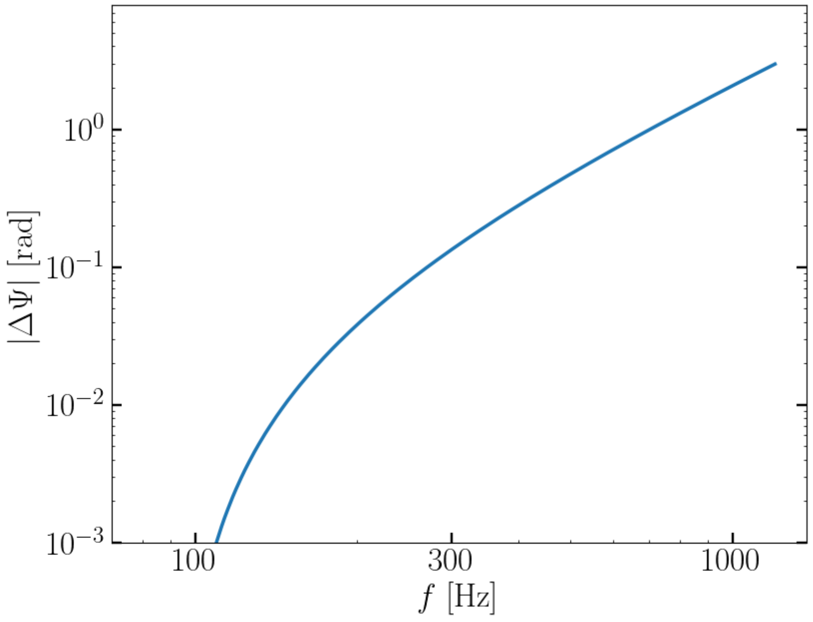
Cf. Resonance locking in double white dwarfs

(HY, N. N. Weinberg, J. Fuller 2021)

WD: freq shift due to spin (Doppler)

NS: freq shift due to anharmonicity

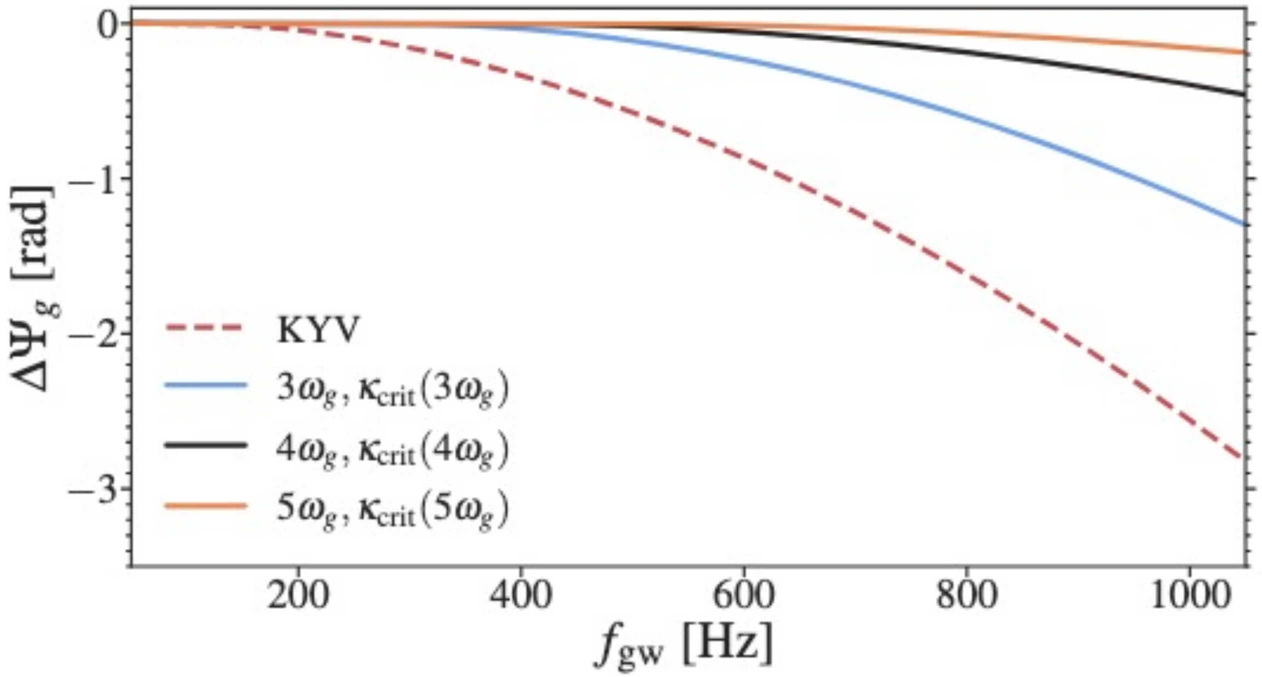
(Also Burkart et al 13; Fuller & Lai 13)



Phase shift ~ 3 rad from the g-mode!

Caveat/Opportunity—superfluid g-modes?

- Core of cold NSs is a superfluid
  - Very different g-mode spectra (x3-x5 higher frequencies) (Kantor & Gusakov 14, Passamonti, Andersson, & Ho 16, Yu & Weinberg 17)
- Assuming optimal coupling strength, effect suppressed by increasing frequency
- Test NS superfluidity?



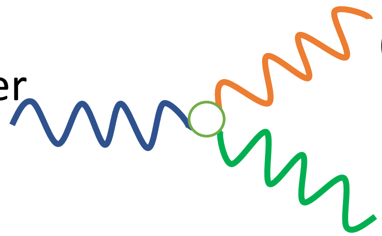
Hydrodynamical instability of high-order p/g modes

Weinberg 2016

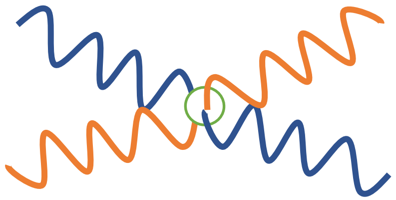
arXiv:1509.06975

Physics

- Competition between (three-wave)^2 and four-wave



vs.



$$
\begin{aligned}
q_p &\simeq \kappa q_f q_g, \\
\ddot{q}_g + \omega_g^2 q_g &= \omega_g^2 \kappa q_f^* q_p^* \simeq \omega_g^2 \kappa^2 (q_f^* q_f) q_g, \\
\text{or } \ddot{q}_g + &\underbrace{\omega_g^2 (1 - \kappa^2 q_f^* q_f)}_{\omega^2_{g,\mathrm{eff},(3\mathrm{m})^2}} q_g = 0
\end{aligned}
$$

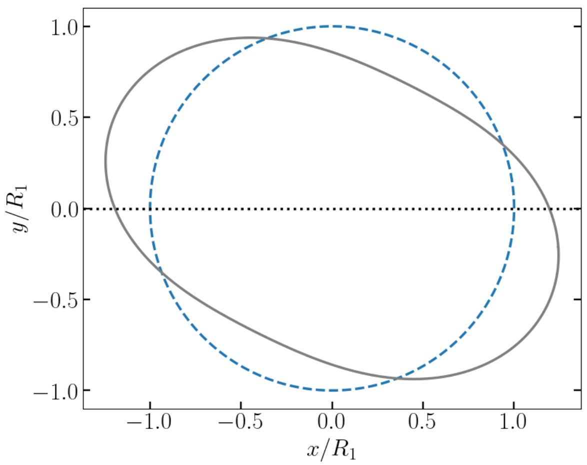
- Three-wave coupling used twice
- p-mode as a mediator

$$
\begin{aligned}
\ddot{q}_g + \omega_g^2 q_g &= \omega_g^2 \zeta q_f^* q_f q_g \\
\text{or } \ddot{q}_g + &\underbrace{\omega_g^2 (1 - \zeta q_f^* q_f)}_{\omega^2_{g,\mathrm{eff},4\mathrm{m}}} q_g = 0
\end{aligned}
$$

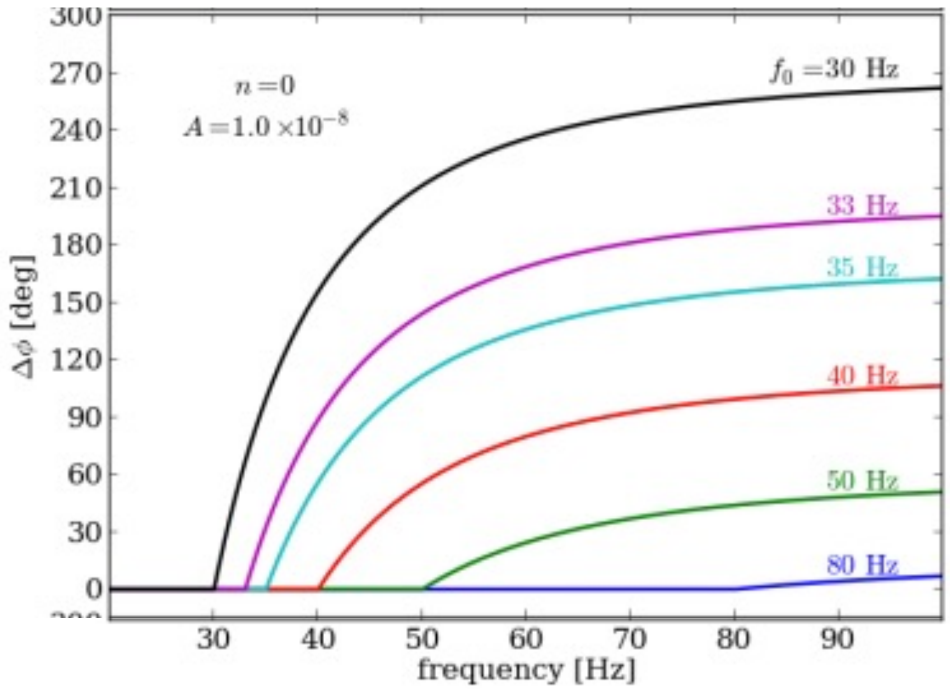
- Direct four-wave (ggff) coupling enters at the same order
- $\kappa^2 \sim$ +++ +; $\zeta \sim$ −− −;
- $(\kappa^2 + \zeta) \sim +$

Dissipative effect

- High-order modes have large shear viscosities
- Accelerating orbital decay
- Important in the 30-80 Hz band
- Dwarfed by GW-induced lag at higher-frequencies

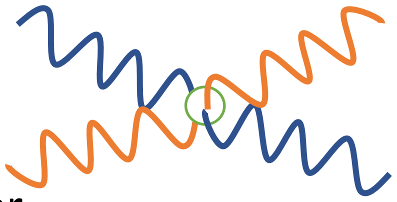


(Essick+ 16)

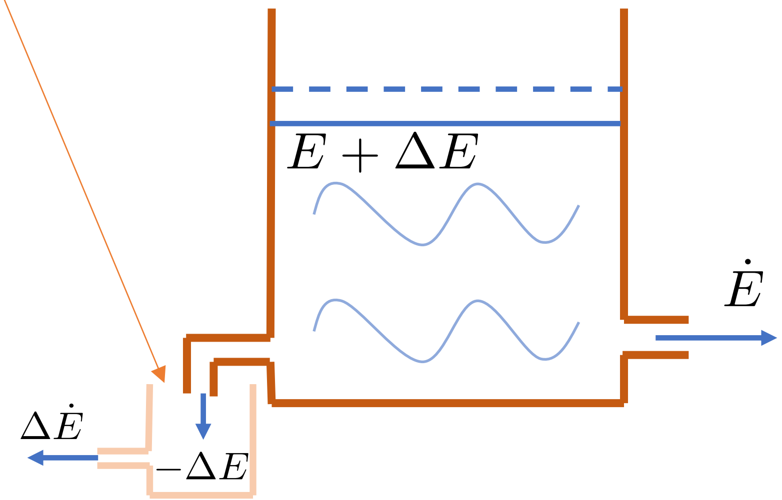


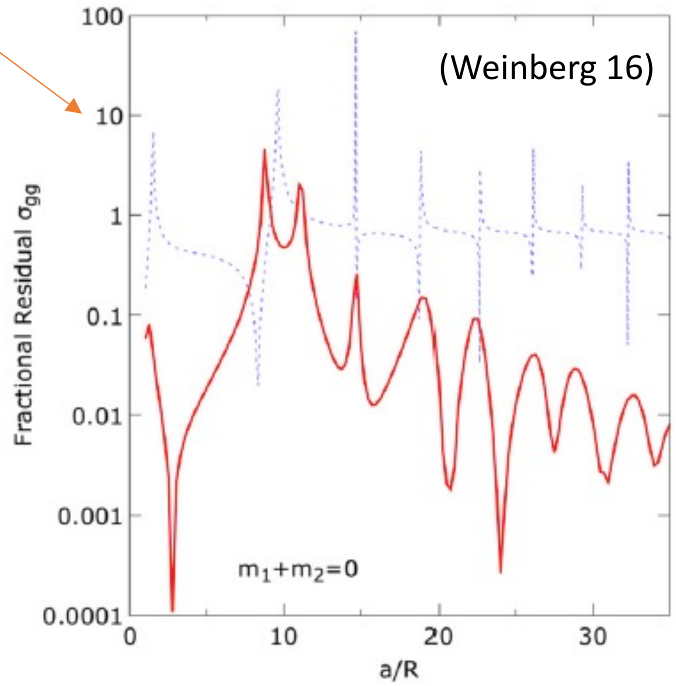
Caveat—dynamical tide

- Only considered equilibrium tide (low-order modes given by simple Lorentzian)
  - How about resonantly locked g-mode?



- Ignored energy transfer to grow the high-order modes





Other nonlinear effects?

- A NS hosts various modes beyond just f/g/p modes
  - Inertial modes (including r-modes) in rotating NSs (Ho & Lai 99; Flanagan & Racine 07; Poisson 20; Ma, Yu, & Chen 21; Gupta+ 21; Gittins & Andersson 23)
  - Interface modes (Core-crust: Tsang+ 12; Pan+ 20; Passamonti+ 21; Phase-transition: Counsell+ 25)
- Many can reach >~ 0.1 rad phase shift in the linear theory
- How are they affected by nonlinear couplings?

Summary

- Modal description is still an important tool to capture the dynamical tide
- Nonlinear hydrodynamics—shifts of natural frequencies

Modes	Order	Conservative?	Effects	Frequency of significance	Comment	Reference
f-modes	Three-wave	Conservative	Enhanced resonance	>500 Hz	Most robust	Yu+ 23 2211.07002
Low-order g-modes self-coupling	Four-wave	Conservative	Resonance locking	100-1,000 Hz	Most sizable (caveat: superfluidity)	Kwon, Yu, & Venumadhav 2410.03831, 2503.11837
High-order g-modes and low-order f/g-modes	Four-wave	dissipative	pg-instability & dissipation	30-80 Hz	Most complicated	Weinberg 16 1509.06975

- Faithful waveform models
- New probe of NS physics beyond just deformability