Isovector nucleon charges from lattice QCD

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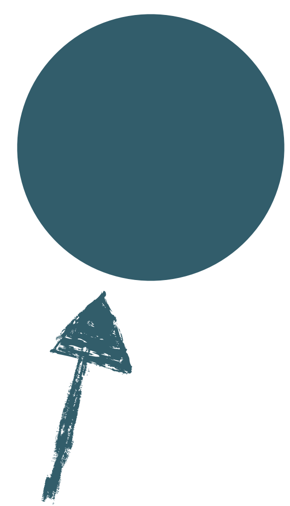
University of Adelaide

with QCDSF/UKQCD/CSSM

INT Workshop
"New physics searches at the precision frontier"
9 May 2023

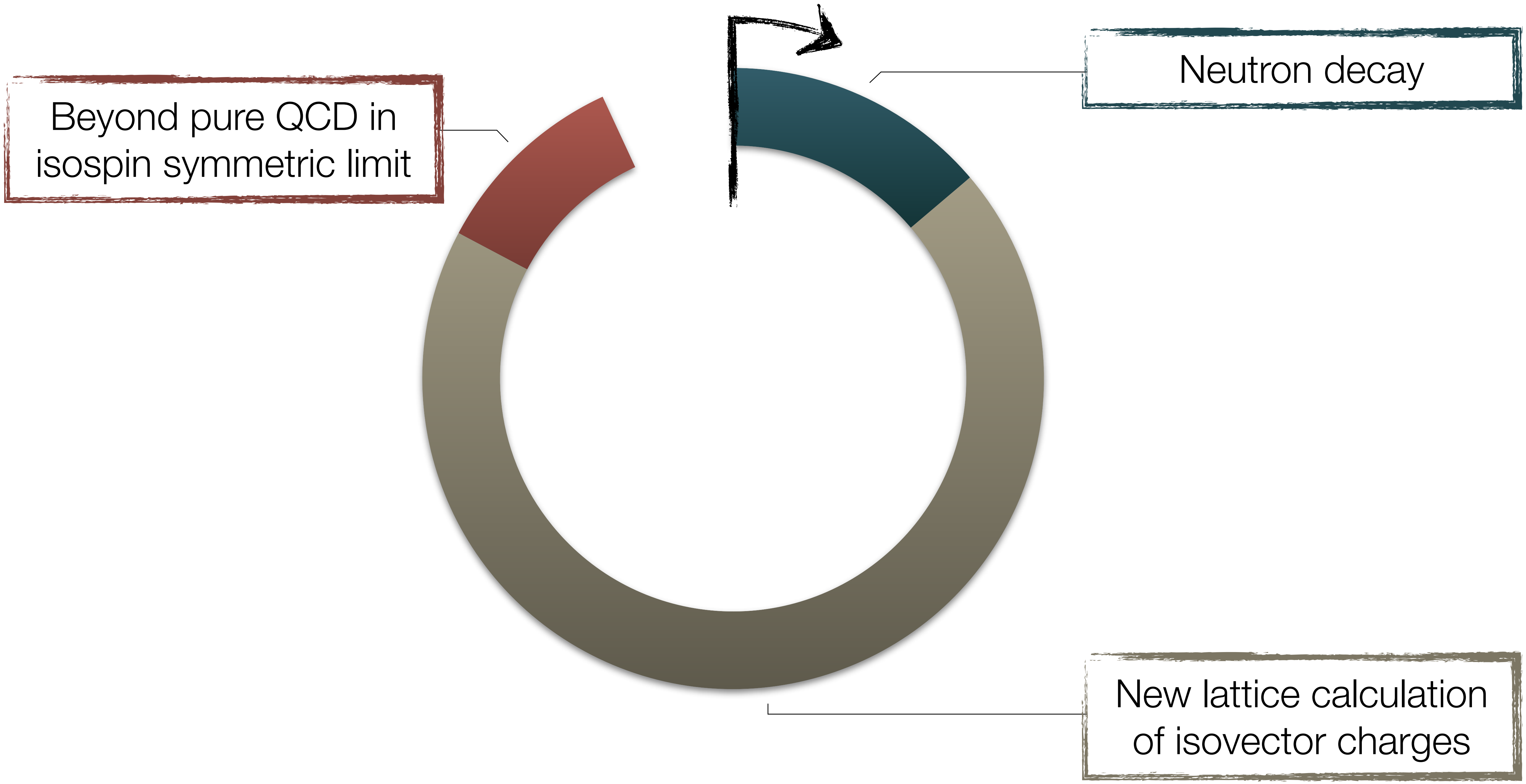
The neutron decays

15 minutes!!



Neutron
(artist impression)

Outline

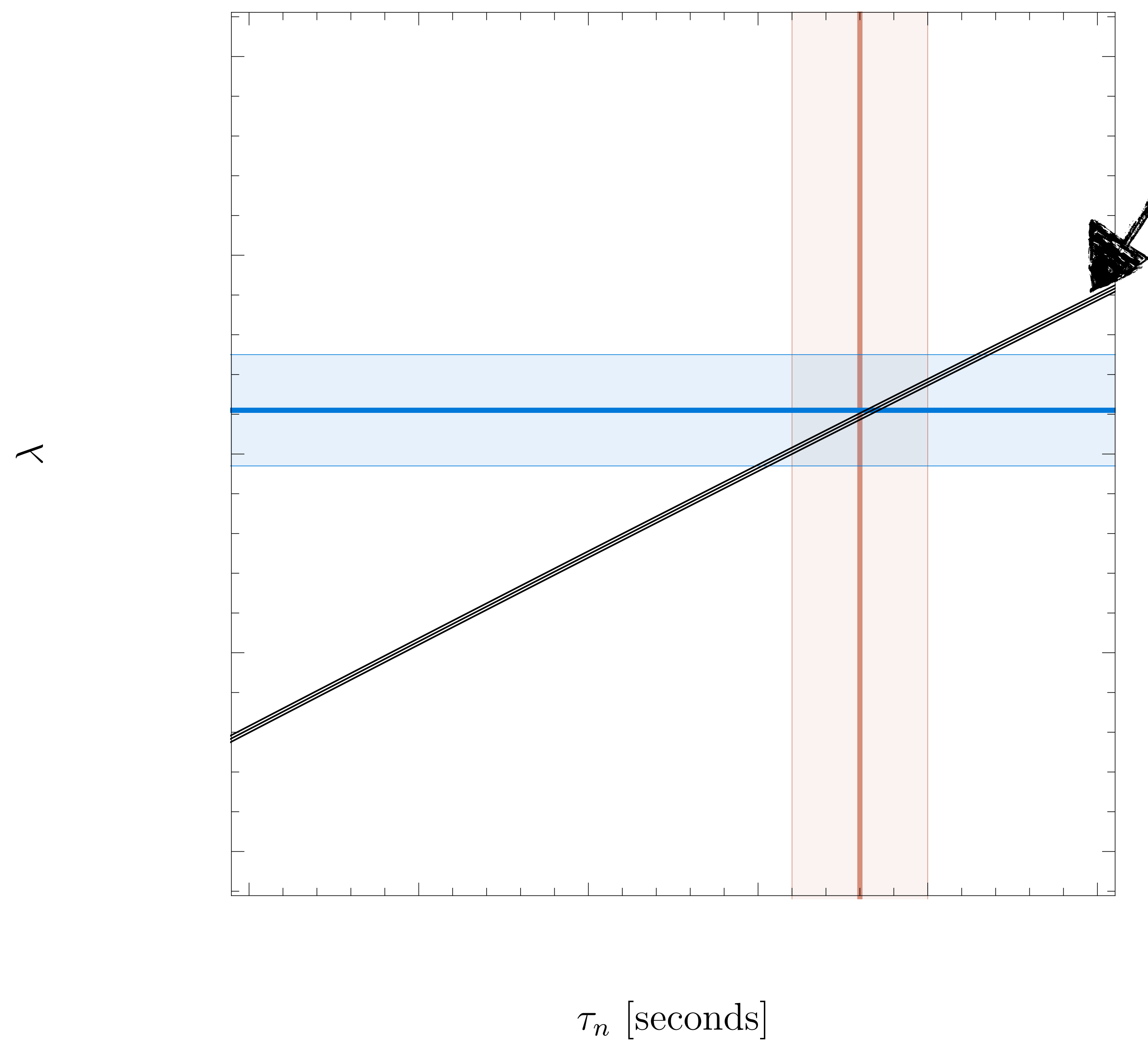


Beyond pure QCD in isospin symmetric limit
Neutron decay
New lattice calculation of isovector charges

Neutron decay

Neutron lifetime and the axial charge

$$\lambda = \frac{G_A}{G_V}$$



$|V_{ud}|^2 \tau_n (1 + 3\lambda^2) = \text{const.}$

$V_{ud}\ [0^+ \to 0^+]$

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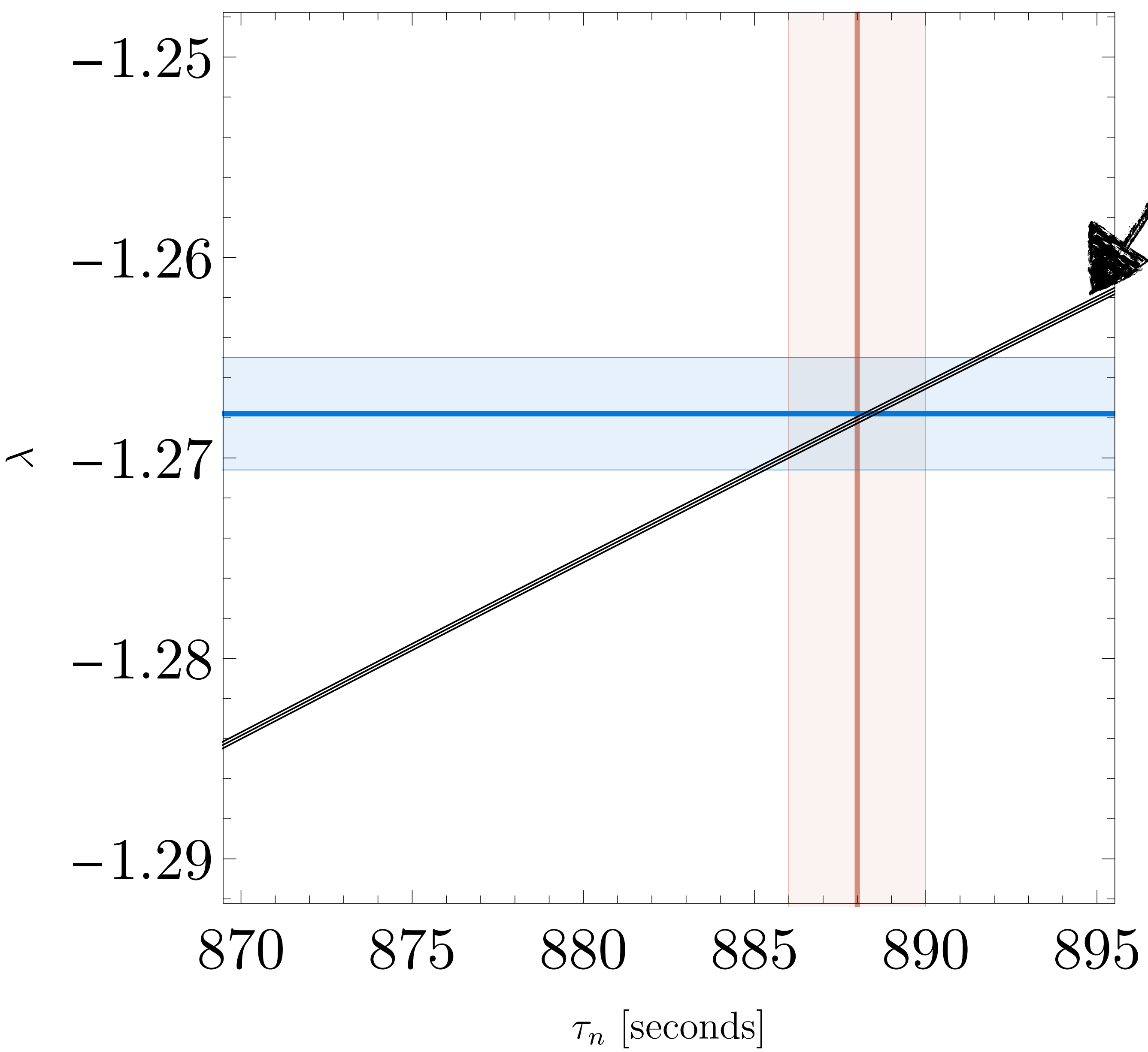

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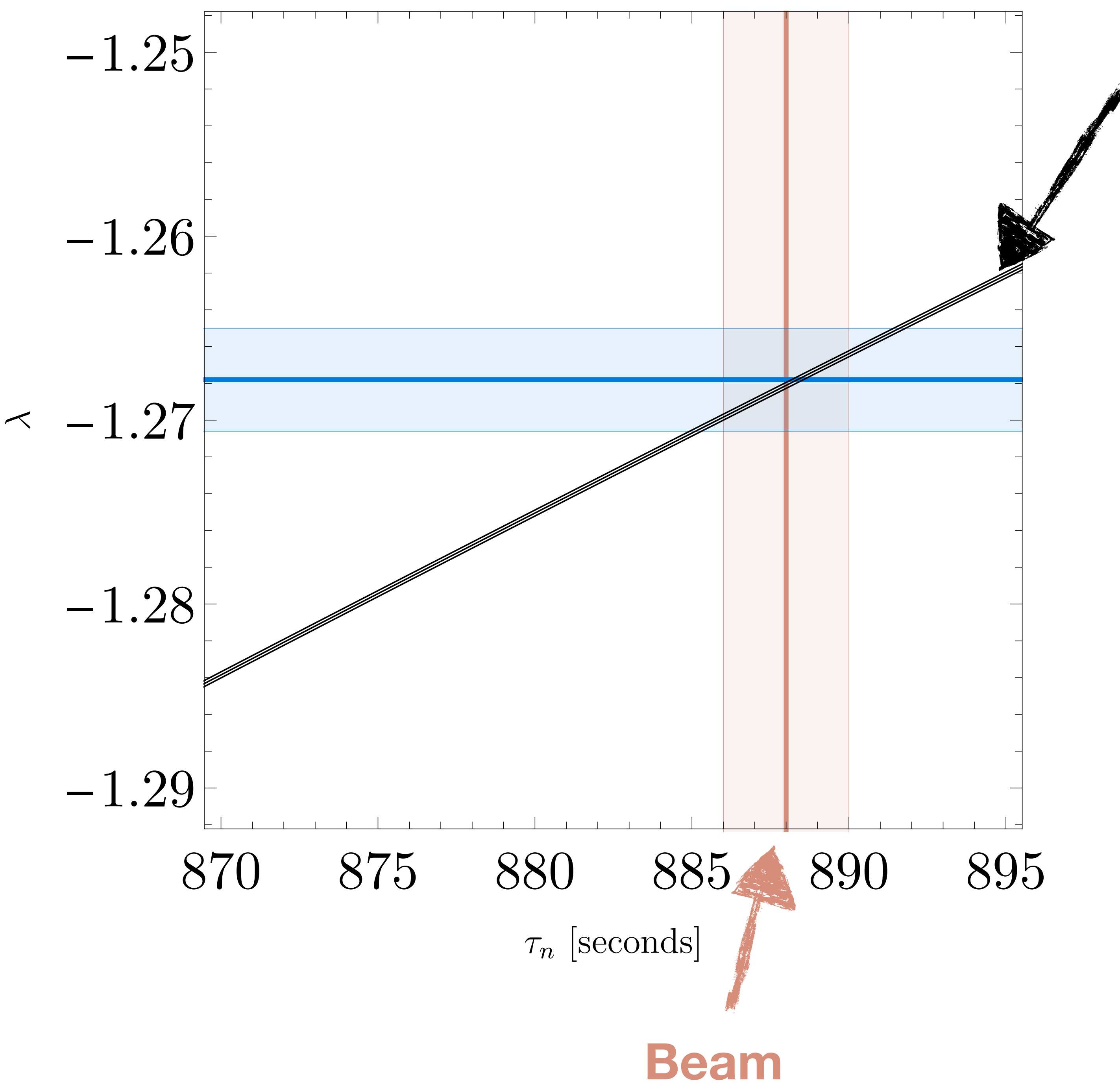

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a coefficient: $\vec{p}_e \cdot \vec{p}_\nu$

aSPECT, Beck *et al*. PRC(2020)

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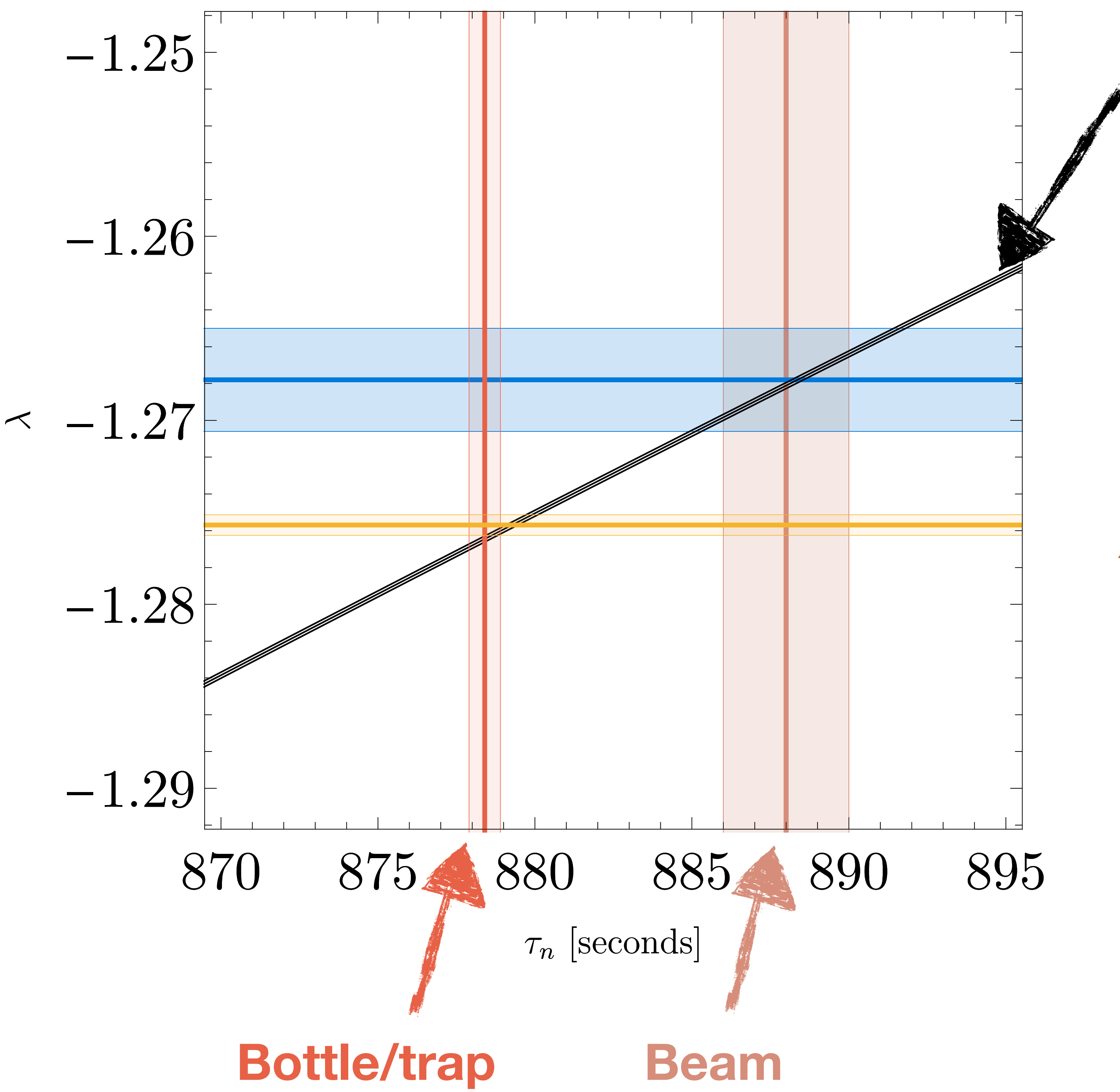

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aSPECT, Beck *et al*. PRC(2020)

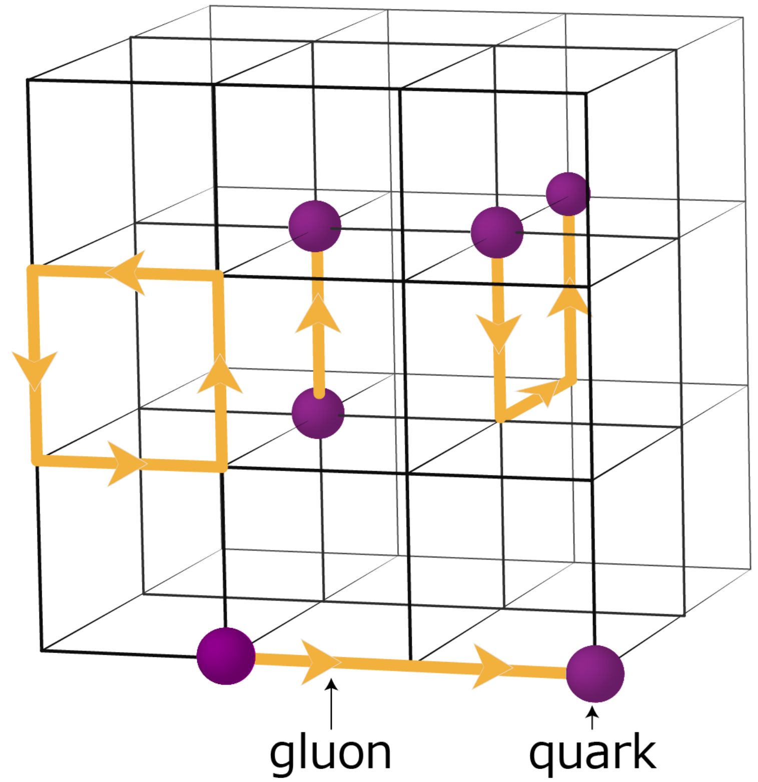
A coefficient: $\overrightarrow{\sigma}_n \cdot \overrightarrow{p}_e$

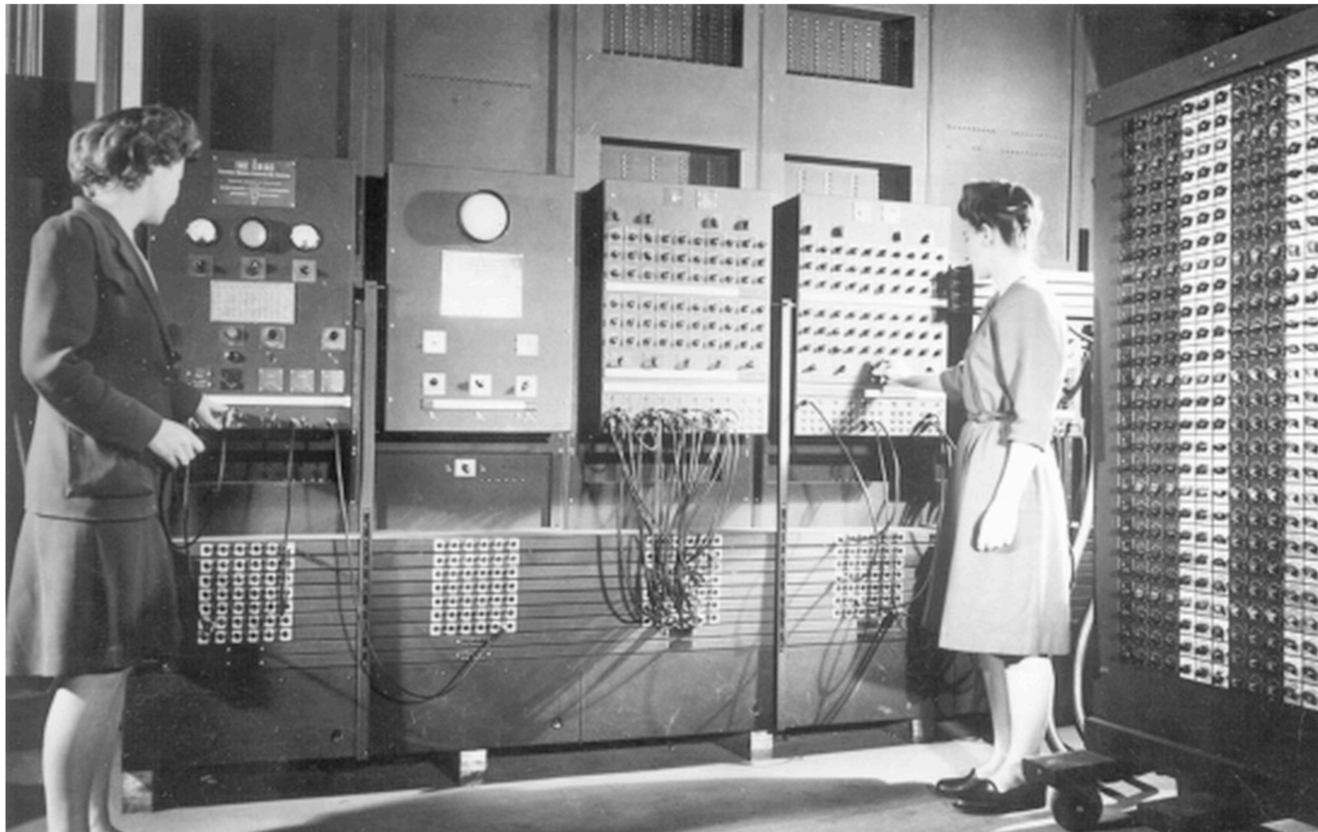
Lattice QCD

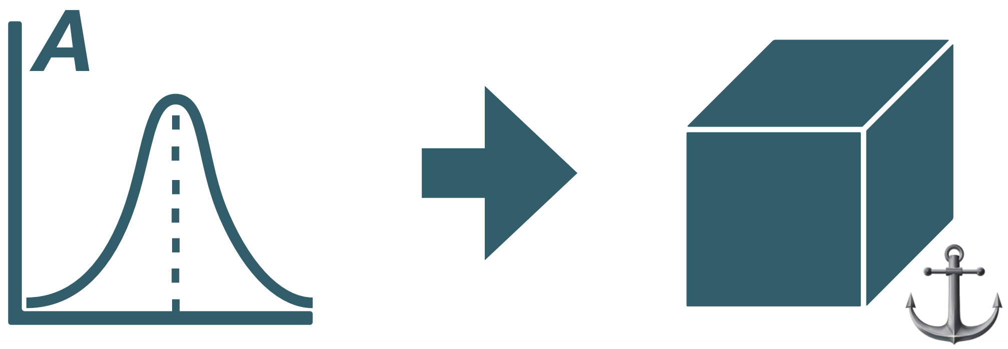
Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Discretise QCD onto 4D space-time
- Approximate path integral by Monte-Carlo methods
    - Computationally intensive, large-scale supercomputing
- Controlled systematics
    - Lattice spacing, lattice volume, quark masses





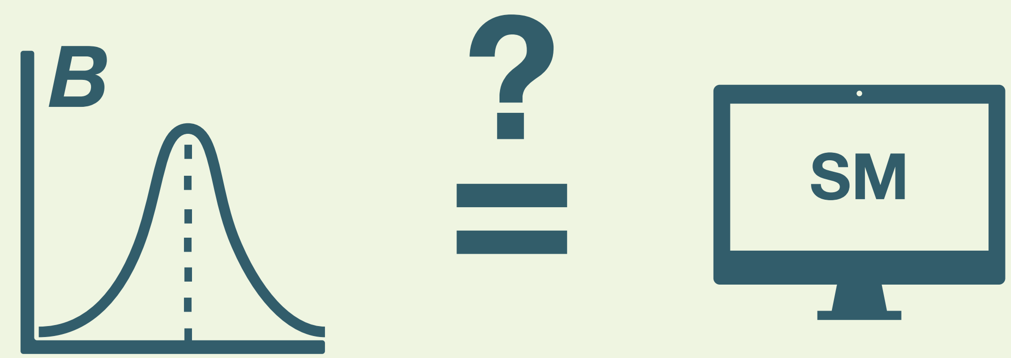


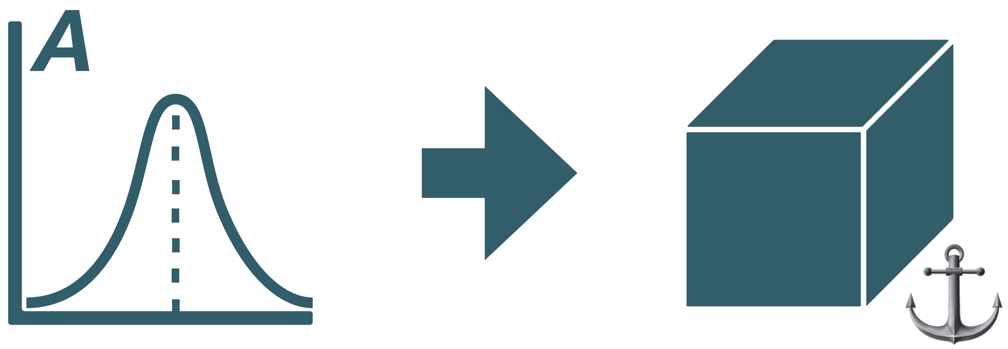
Experimental observations anchor the input parameters of QCD

QCD: Theory of strong-interaction dynamics with zero free parameters

	QCD parameters	Observed masses
quark masses	$m_d + m_u$	π
	$m_d - m_u$	🤔
	m_s	K
	m_c	D_s
	m_b	B_s
QCD scale	Λ_{QCD}	Ω

Predictions test the standard model





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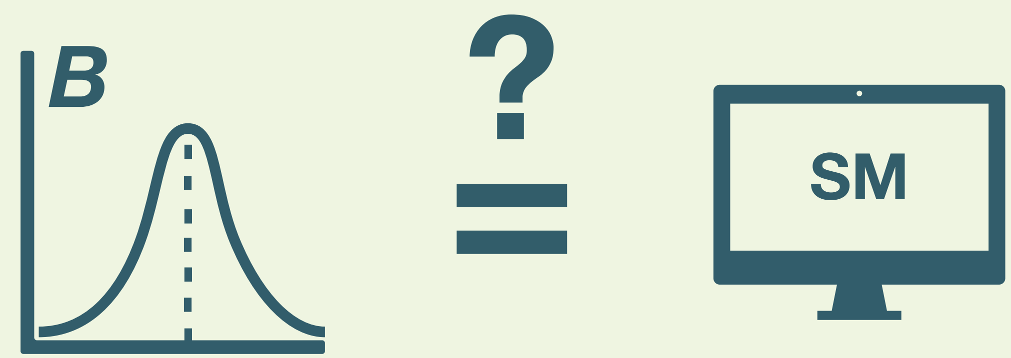
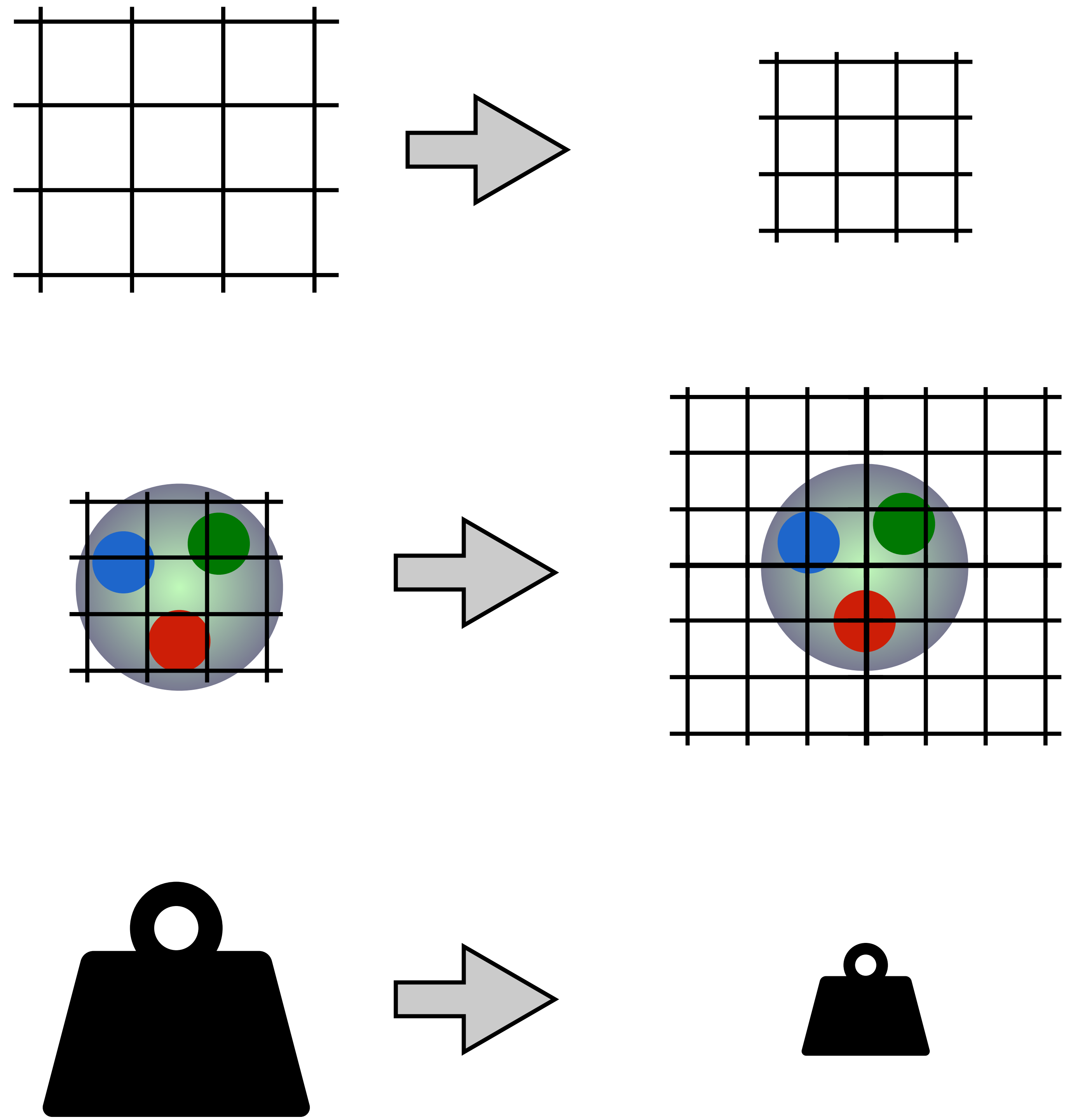


Hadrons in the Standard Model:
QCD + other non-QCD bits

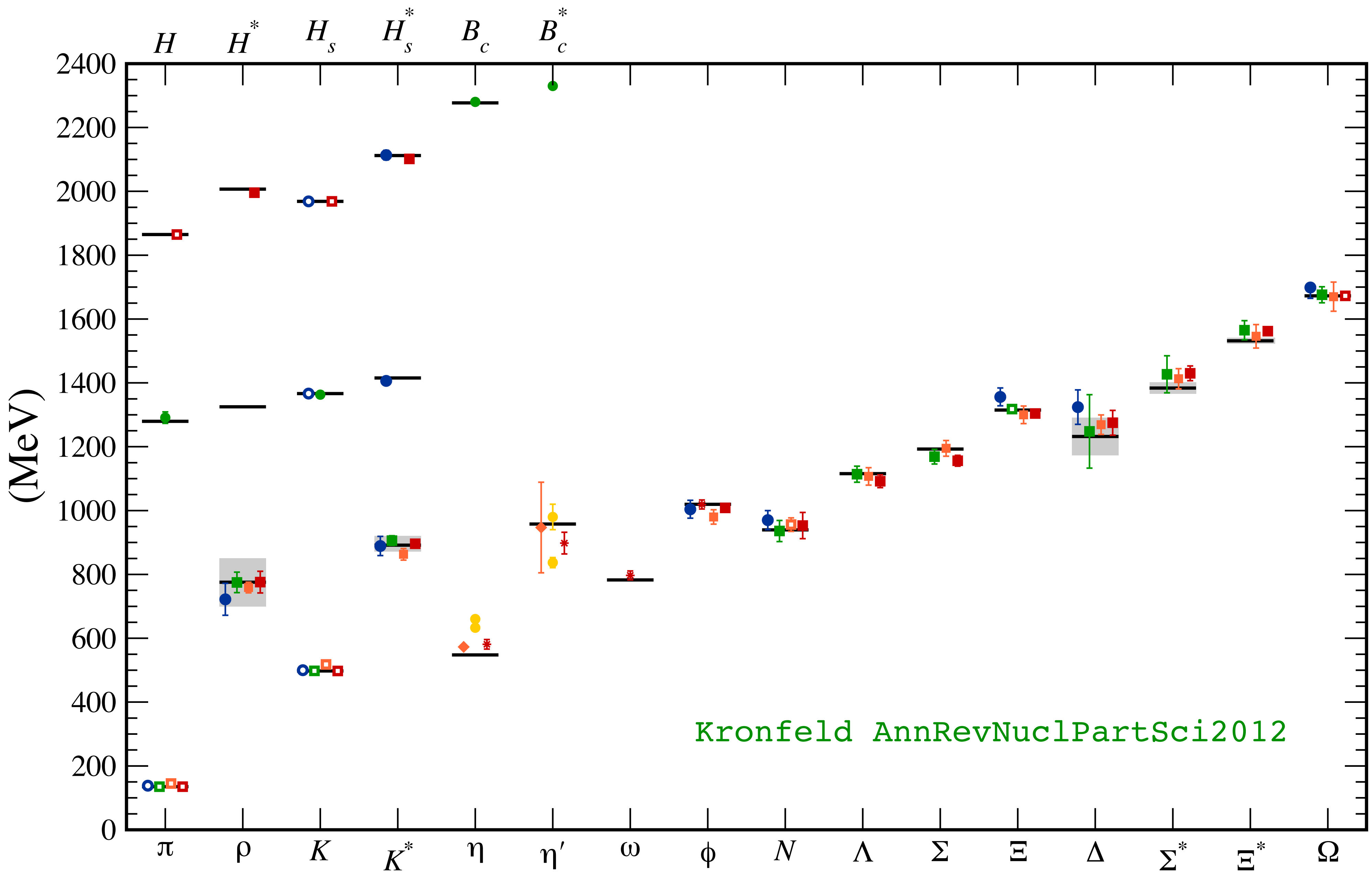
Controlling systematics

- **Finer spacing**
  - Improved actions
  - Extrapolation
- **Bigger volume**
  - Exponentially suppressed
  - Extrapolation
- **Lighter quark masses**
  - Extrapolation
    (or directly at physical point / *interpolation*)



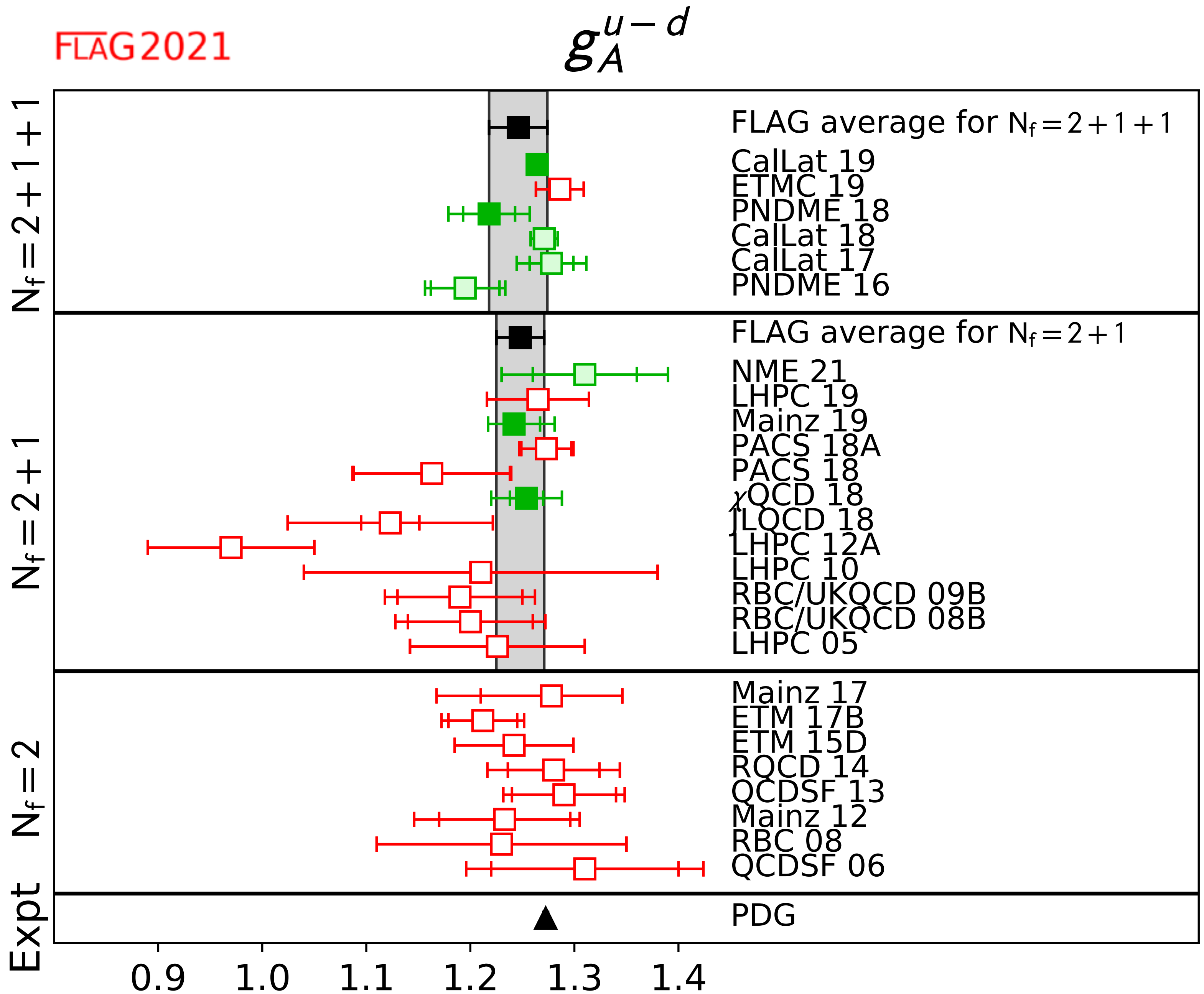
Hadron spectrum in lattice QCD

Low-lying (mostly) stable states



Collage of benchmark spectrum results by different collaborations

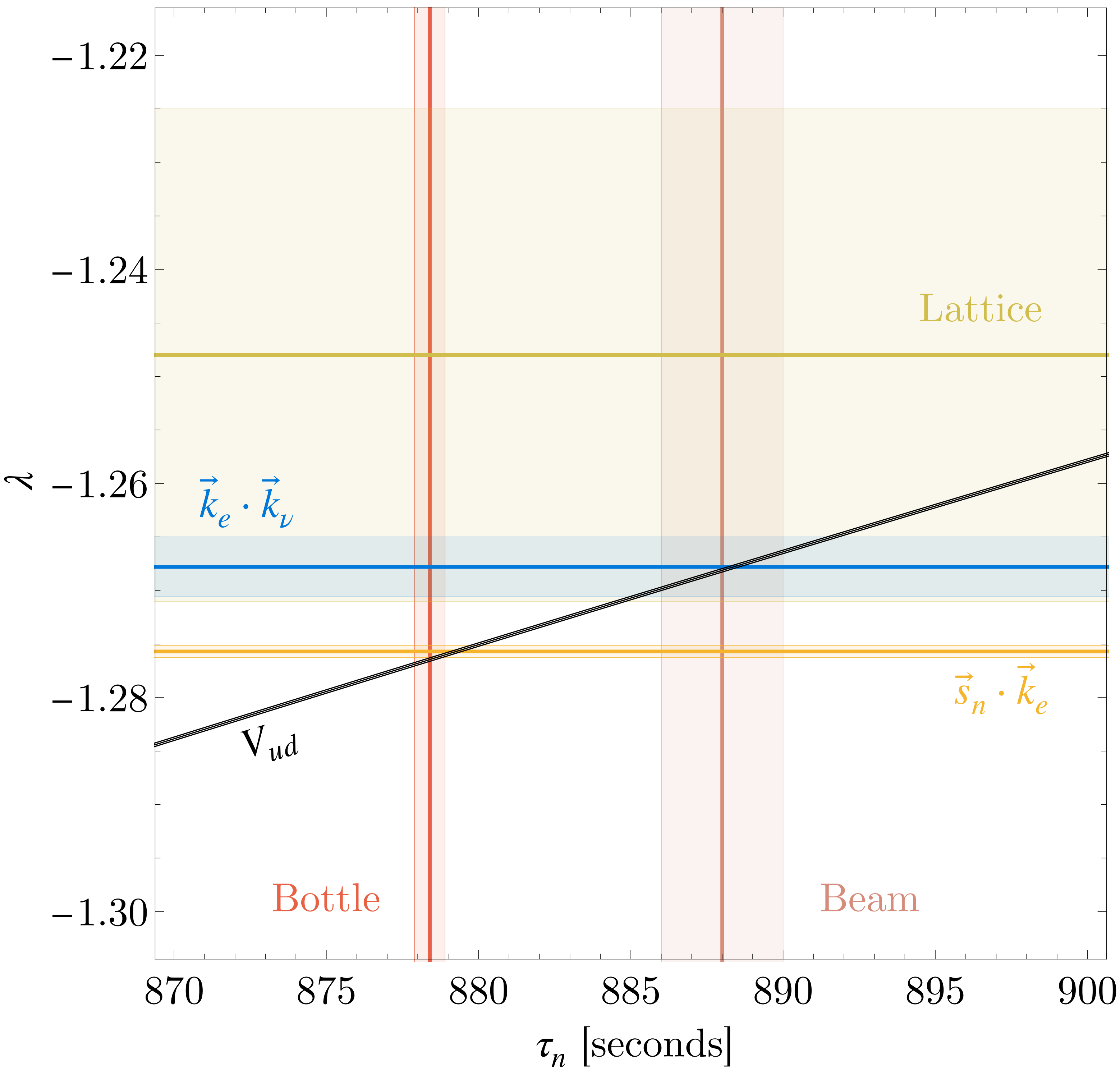
Nucleon axial charge



FLAG: Flavour Lattice Averaging Group

Nucleon matrix elements included for first time in 2019

Lattice and the lifetime

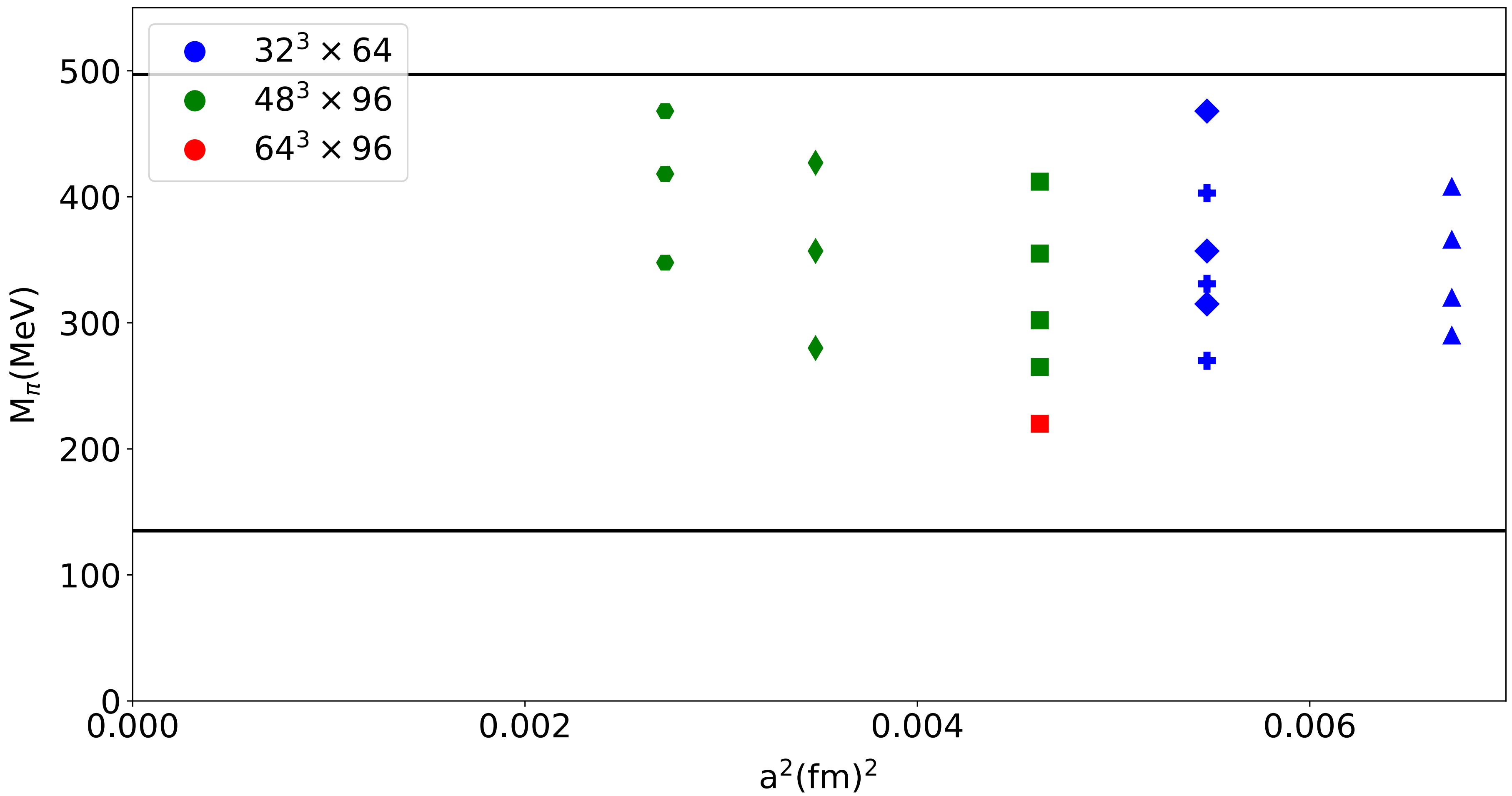


Our *new* results
arXiv: 2304.02866

Lattice calculation

2+1 flavour, NP-improved Wilson fermions

3 volumes



pion masses

$220 \leftrightarrow 500\,\mathrm{MeV}$

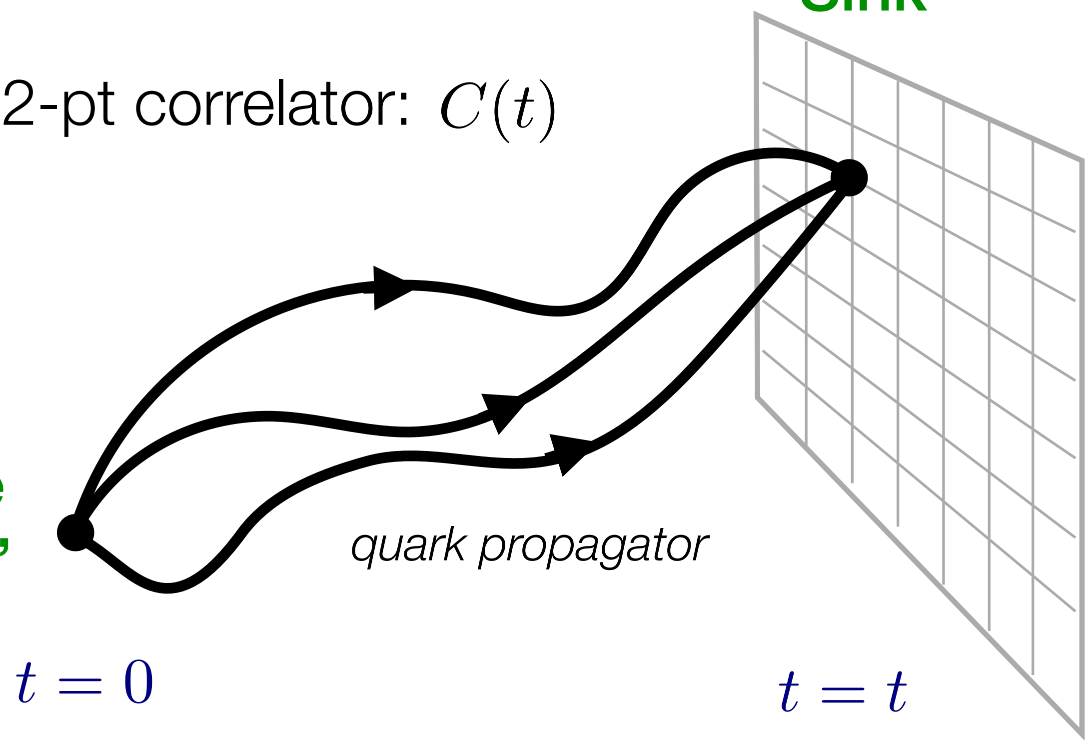
5 lattice spacings

$0.052 \leftrightarrow 0.082\,\mathrm{fm}$

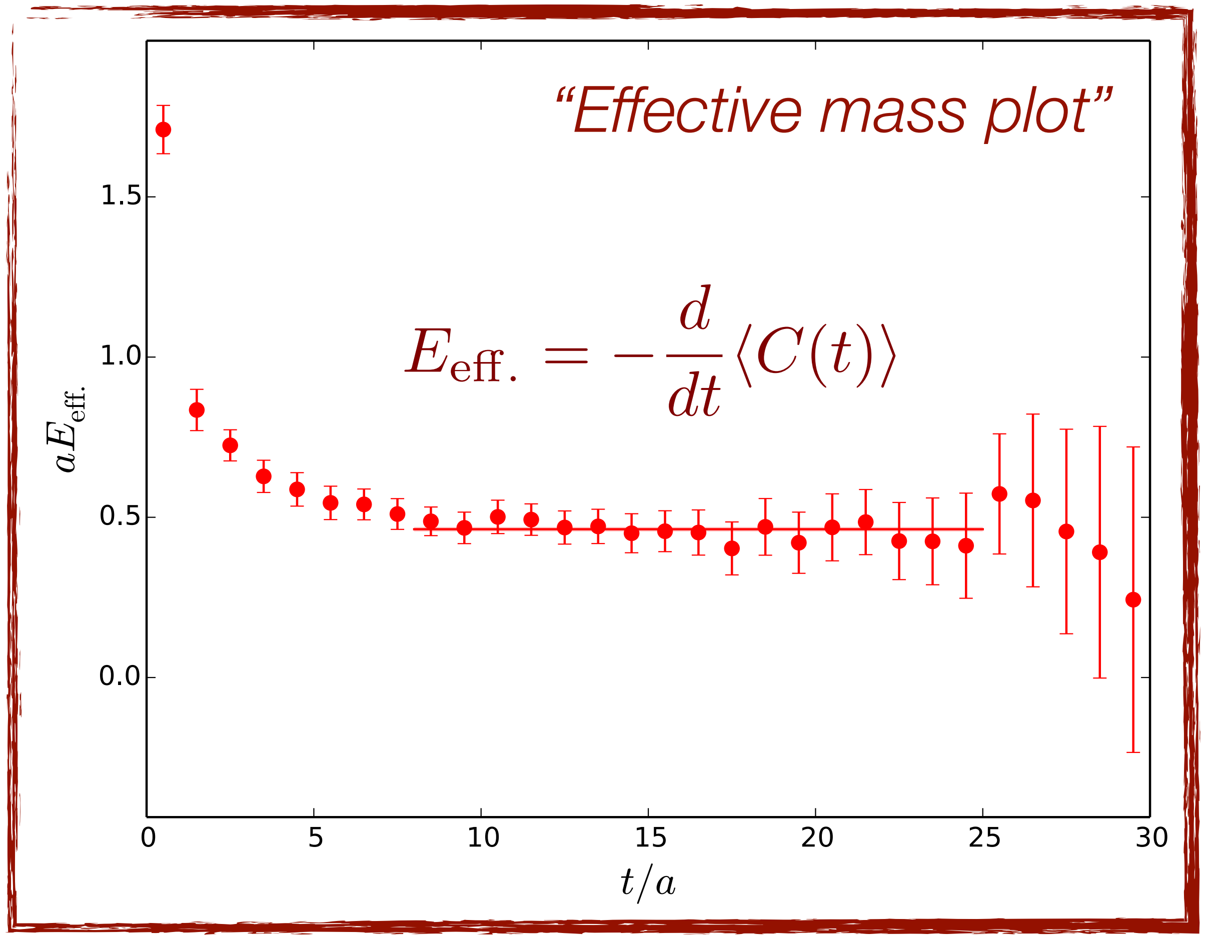
2-pt correlation function ⇒ energy eigenstates

- Stochastic sampling of vacuum gluon configurations
- For each arrangement of the gluons, we compute propagation amplitude

e.g. 2-pt correlator: $C(t)$



Fourier project onto 3-volume at sink
⇒ definite 3-momentum; e.g. $\mathbf{p}' = 0$

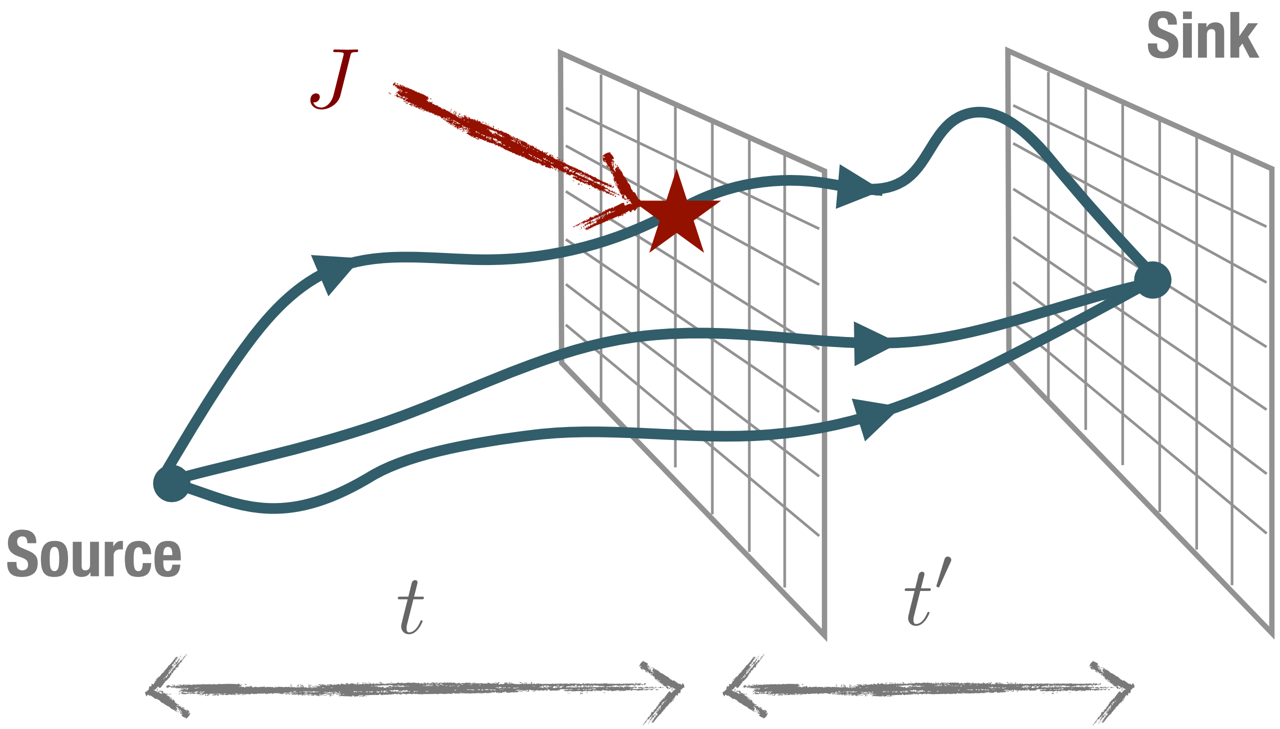


Euclidean time evolution: $\exp(-Ht)$

$$C(t) = \sum_n \left|\langle n|\overline{\psi}(0)|\text{vac.}\rangle\right|^2 e^{-E_n t}$$

lowest energy state dominates at large *t*

3-pt functions

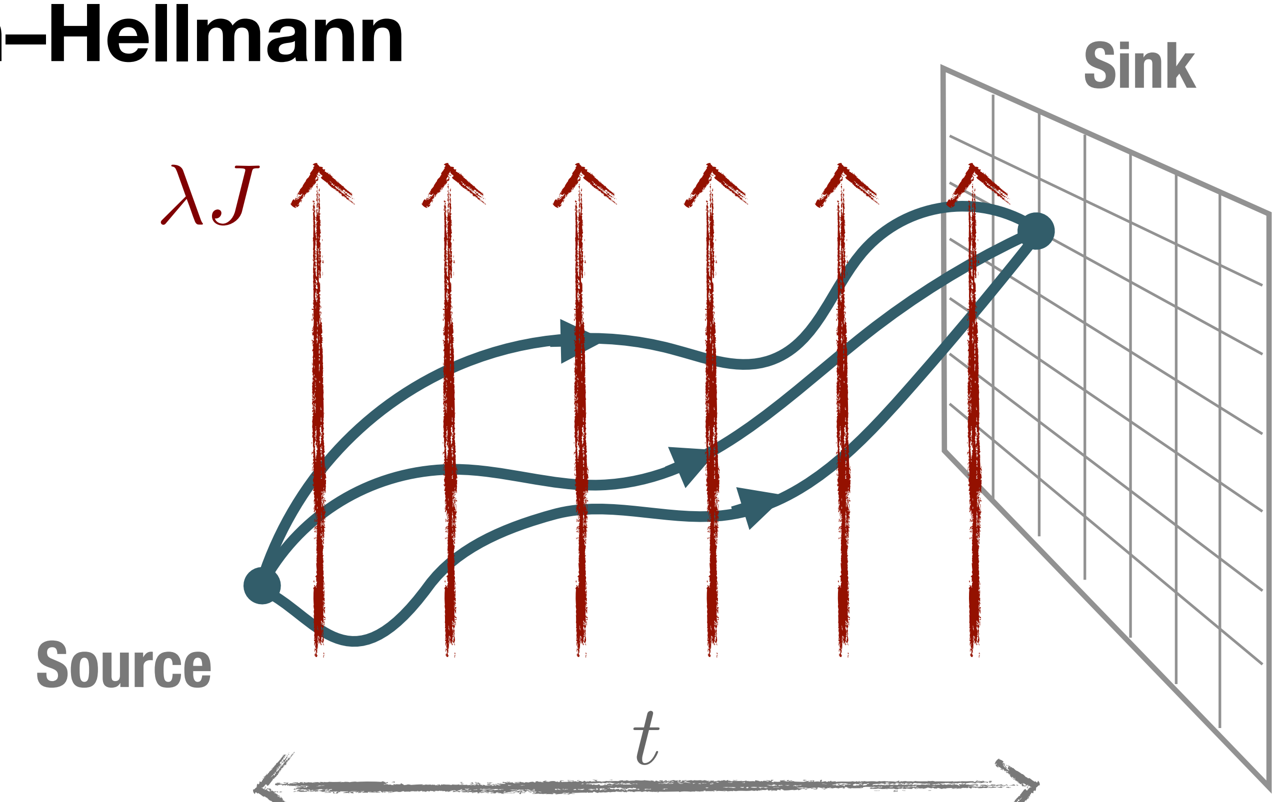


$$t, t' \gg \frac{1}{\Delta E}$$ energy gap to lowest excitation

$$\frac{\langle C_3(t,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N'|J|N\rangle$$

Matrix elements on the lattice

Feynman–Hellmann



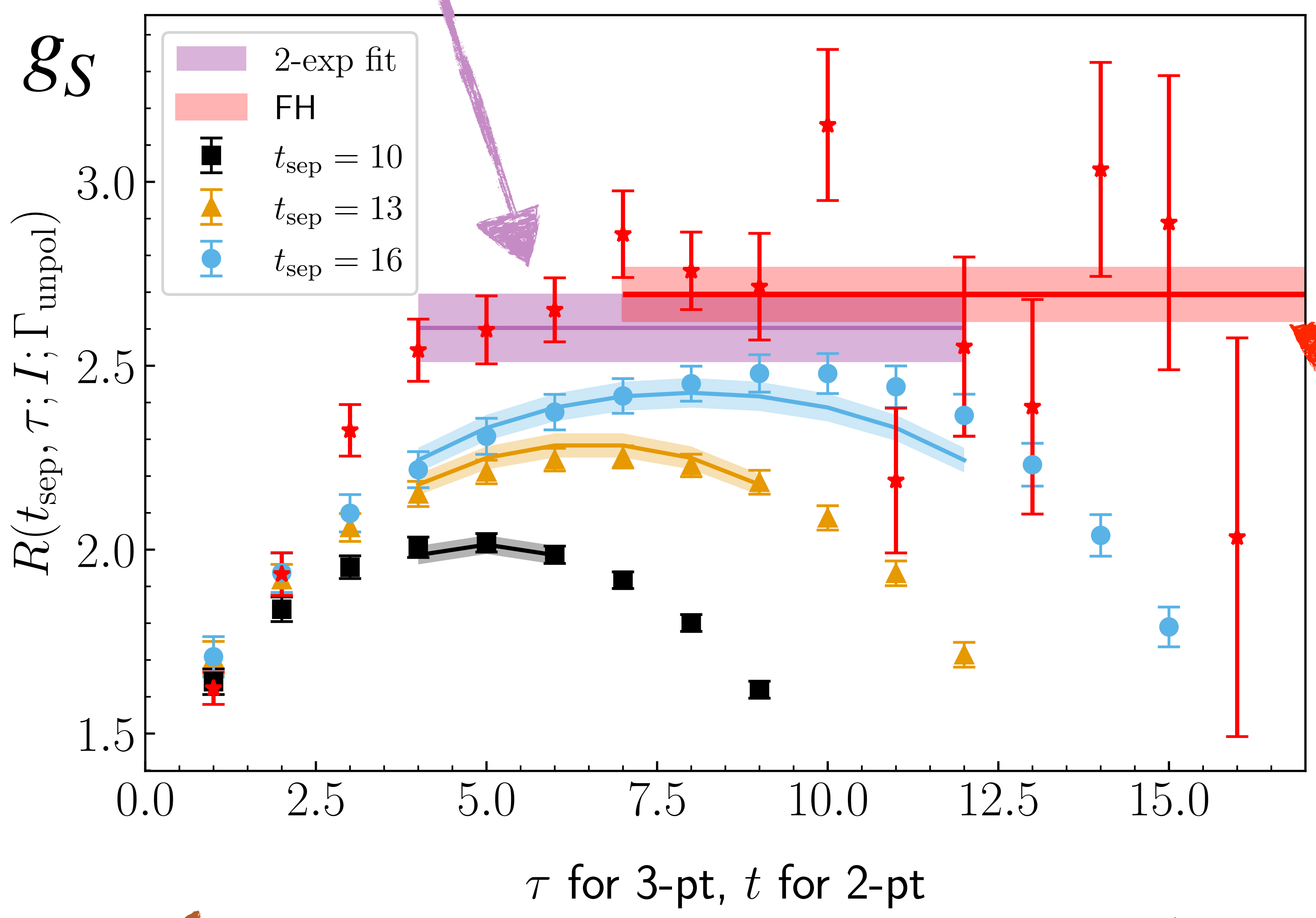
$$t \gg \frac{1}{\Delta E}$$

$$\left.\frac{\partial E}{\partial \lambda}\right|_{\lambda\to 0} \propto \langle N|J|N\rangle$$

3-pt functions and Feynman-Hellmann

2-exponential fit

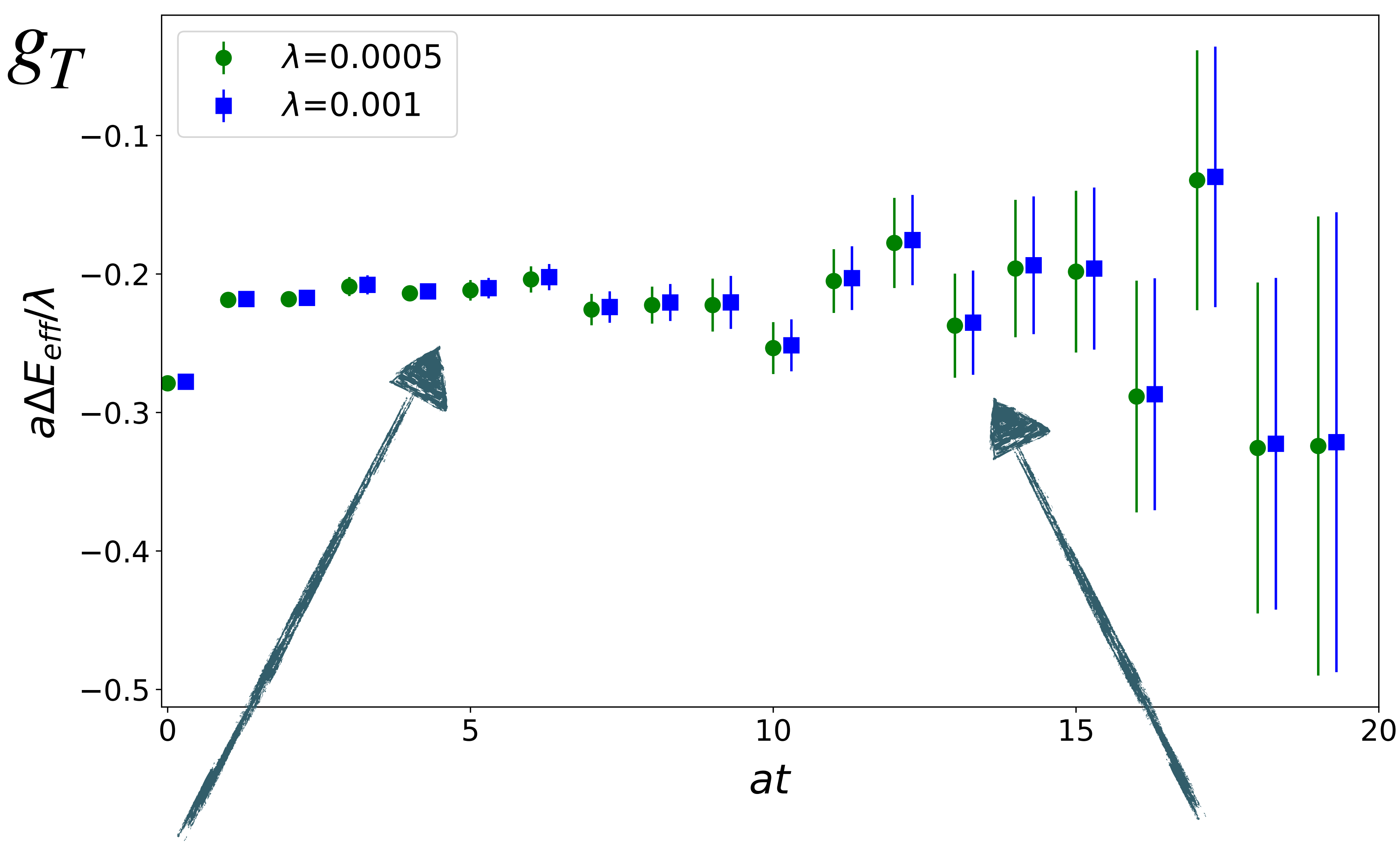
If you were a lattice audience, you would want to talk forever about excited-state contamination



FH: just wait for ground-state saturation

3-pt: 3 source–sink separations

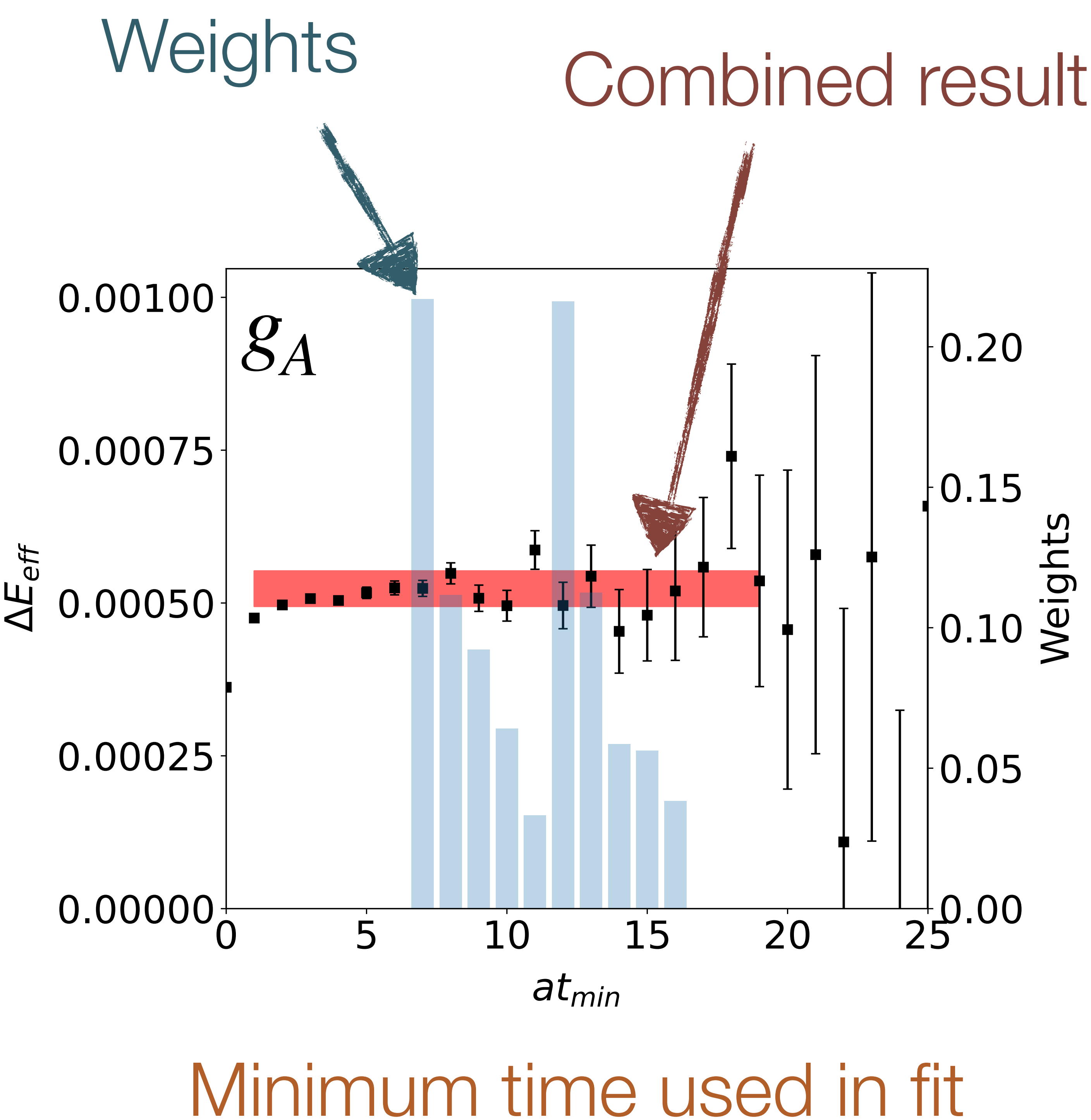
Ground-state saturation?



Early times:
great precision,
but excited-state
contamination.

Late times:
ground-state dominance,
but loss of statistical
signal.

Weighted averages



(Non-normalised) weights:

$$\tilde{w}_f = \frac{p_f}{\sigma_f^2}$$

fit p-value

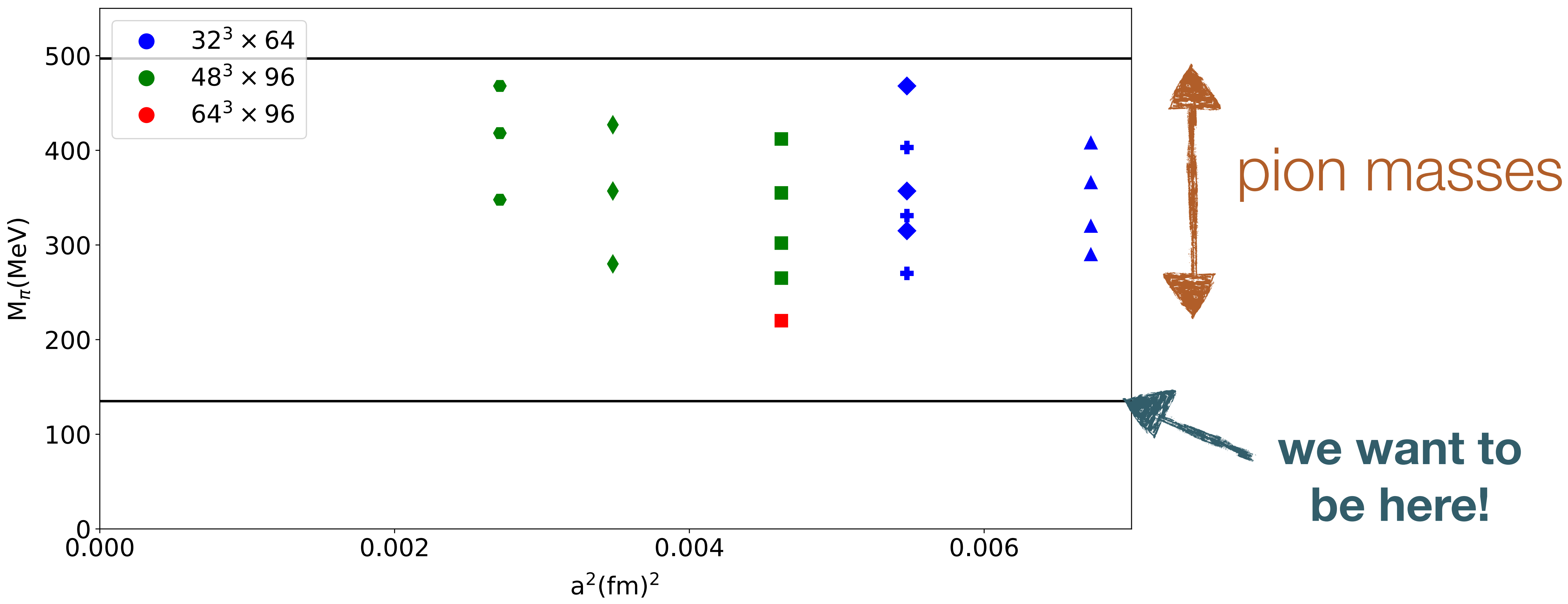
result uncertainty

see also:

Beane *et al*. NPLQCD/QCDSF, PRD(2021)

Rinaldi *et al*., PRD(2019)

Unphysical quark masses on lattice



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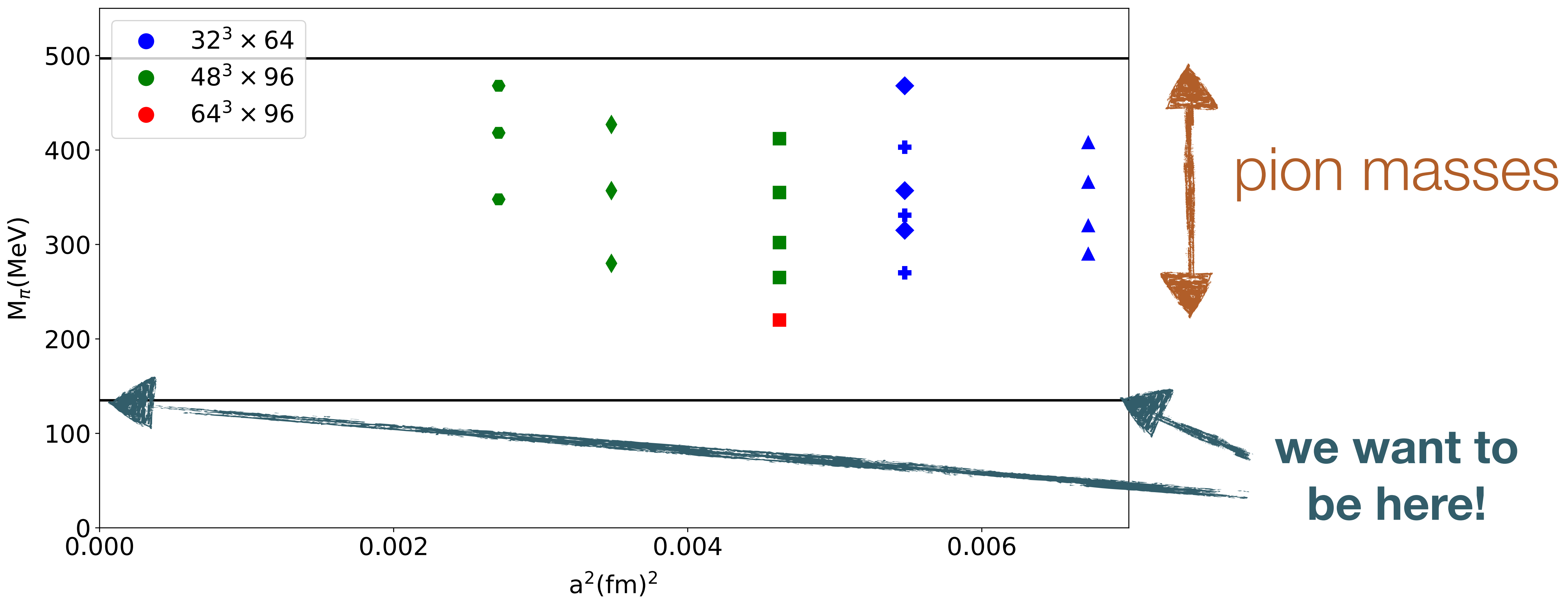
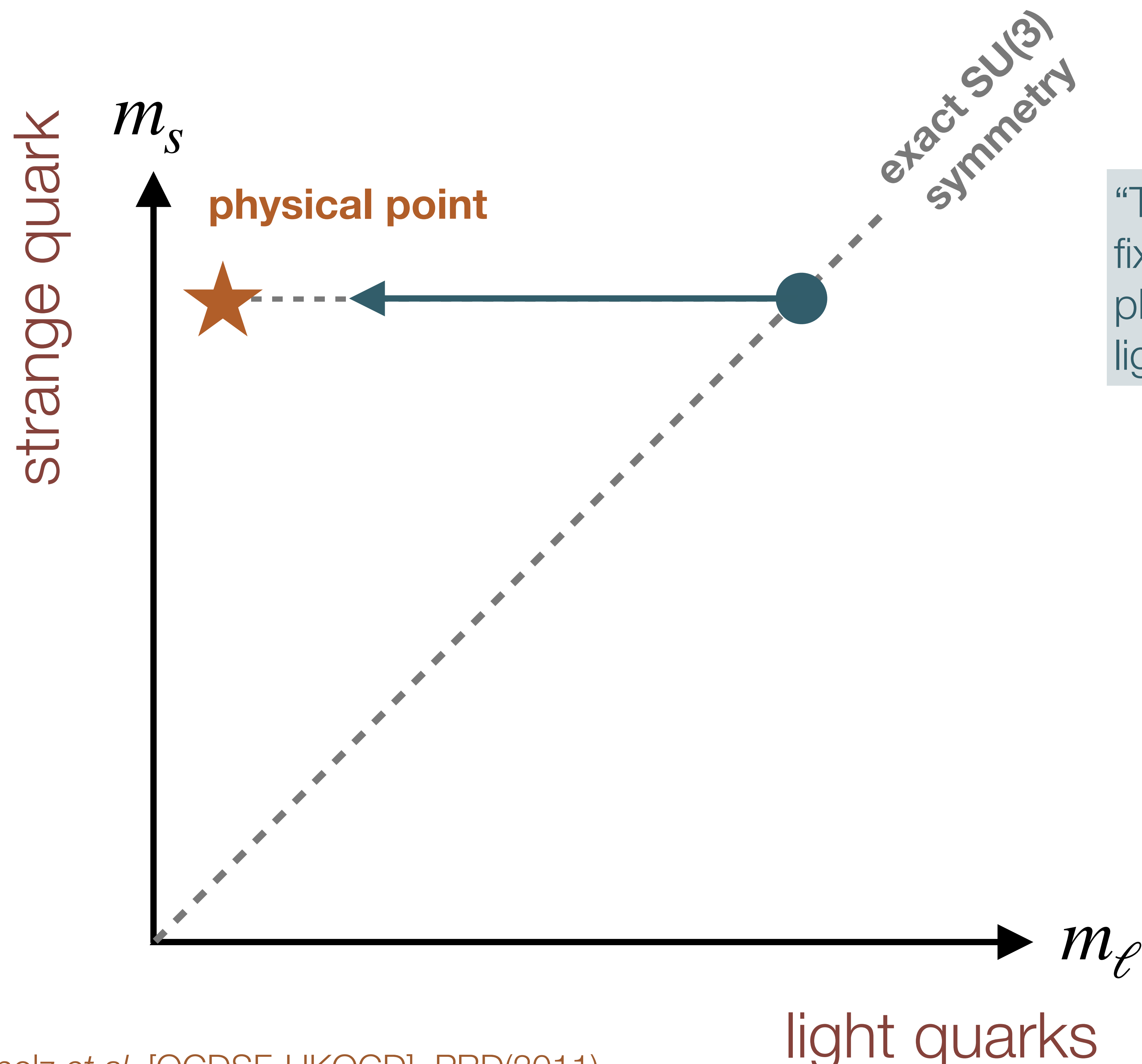


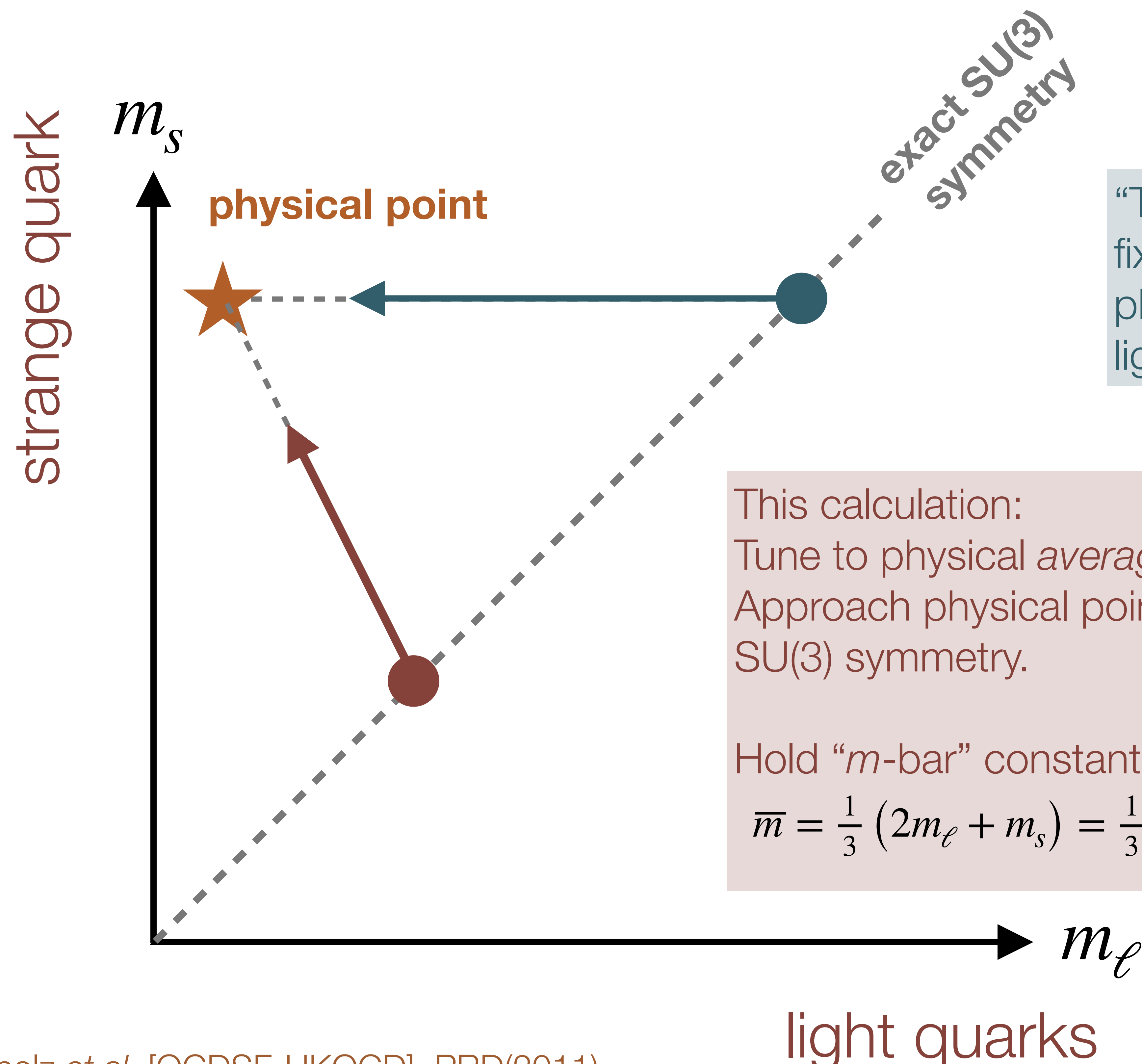
2+1-flavour quark-mass plane



"Typical" calculation:
fix strange quark mass to physical point and lower light quark mass

Bietenholz *et al*. [QCDSF-UKQCD], PRD(2011)

2+1-flavour quark-mass plane



"Typical" calculation:
fix strange quark mass to physical point and lower light quark mass

This calculation:
Tune to physical *average* quark mass.
Approach physical point by breaking SU(3) symmetry.

Hold "*m*-bar" constant:

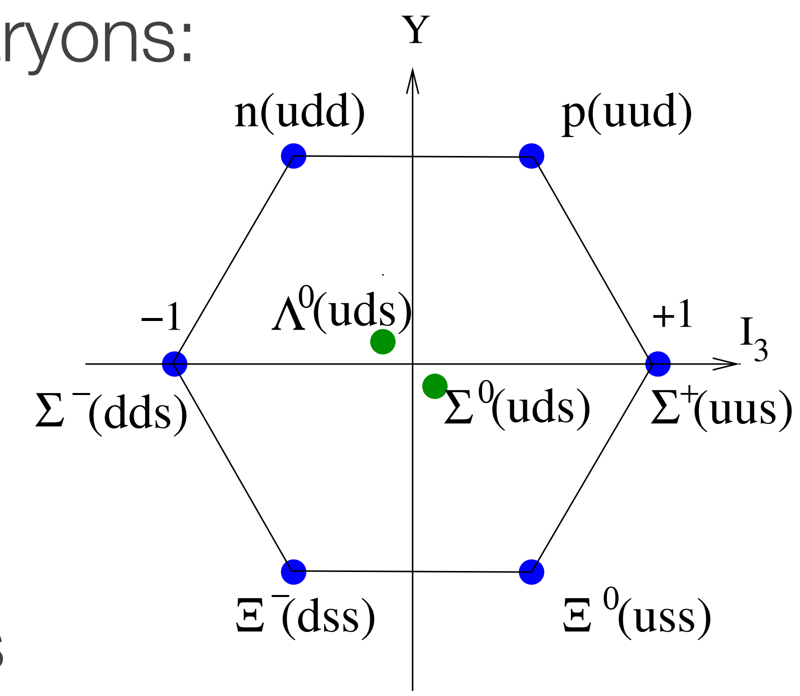
$$\overline{m} = \frac{1}{3}\left(2m_\ell + m_s\right) = \frac{1}{3}\left(2m_\ell^{\text{phys}} + m_s^{\text{phys}}\right)$$

Bietenholz *et al*. [QCDSF-UKQCD], PRD(2011)

Baryon matrix elements

- Consider general flavour matrix elements of octet baryons:

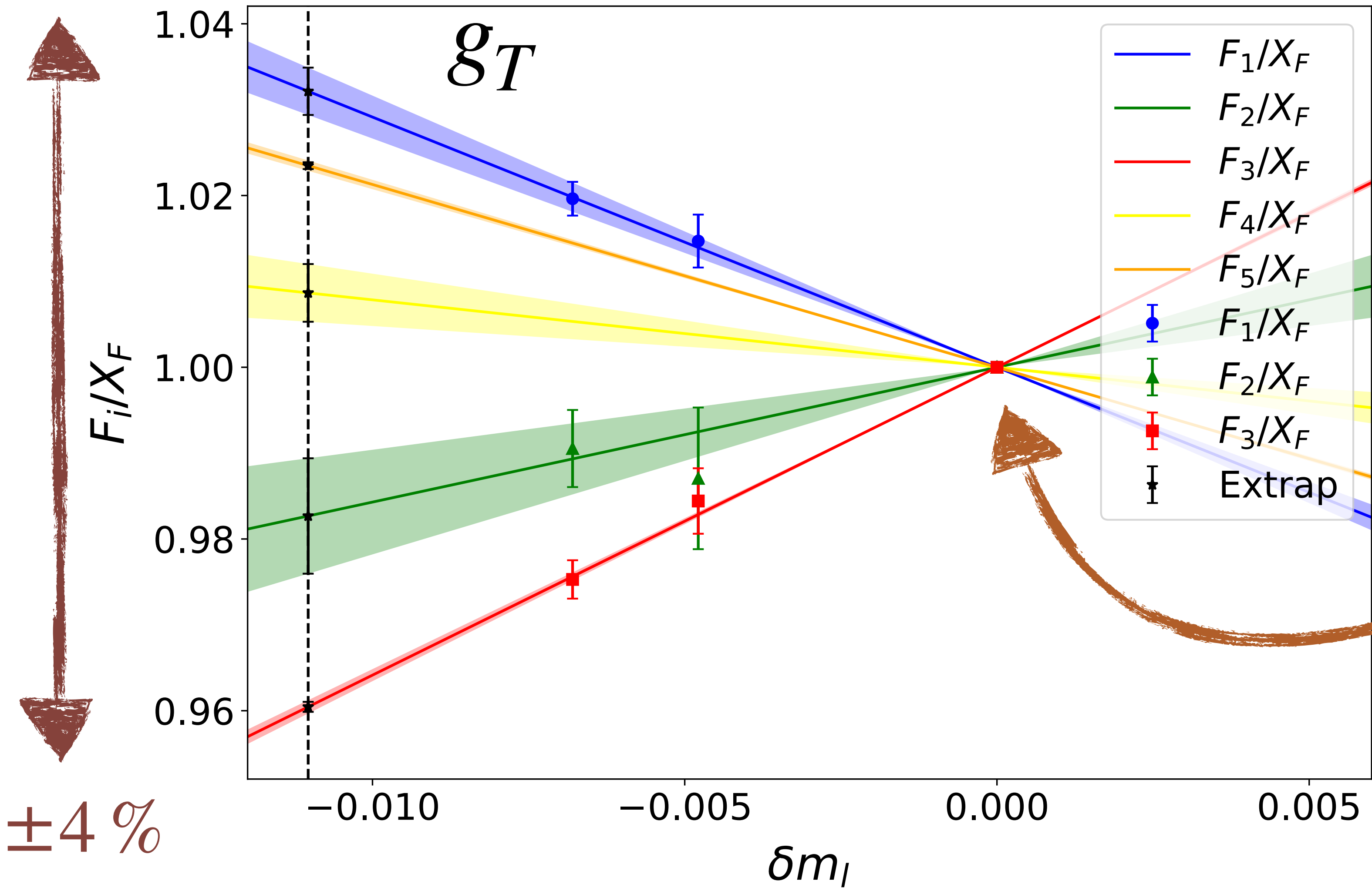
$$\langle B' | J^F | B \rangle = A_{B'FB}$$



- In exact SU(3) limit, just 2 independent constants
  - *F*- and *D*-type couplings
- At linear order in SU(3) breaking: 5 slope parameters (3 *D*'s & 2 *F*'s)
  - # of parameters (polynomials/operators) reduced by restricting to m-bar constant line

Index	Baryon (B)	Meson (F)	Current (J^F)
1	n	K^0	$\bar{d}\gamma s$
2	p	K^+	$\bar{u}\gamma s$
3	Σ^-	π^-	$\bar{d}\gamma u$
4	Σ^0	π^0	$\frac{1}{\sqrt{2}}\left(\bar{u}\gamma u - \bar{d}\gamma d\right)$
5	Λ^0	η	$\frac{1}{\sqrt{6}}\left(\bar{u}\gamma u + \bar{d}\gamma d - 2\bar{s}\gamma s\right)$
6	Σ^+	π^+	$\bar{u}\gamma d$
7	Ξ^-	K^-	$\bar{s}\gamma u$
8	Ξ^0	$\bar{K}^0$	$\bar{s}\gamma d$
0		η'	$\frac{1}{\sqrt{6}}\left(\bar{u}\gamma u + \bar{d}\gamma d + \bar{s}\gamma s\right)$

Bickerton, Horsley *et al*. [CSSM/QCDSF/UKQCD], PRD(2019)



F fan

Can form a "singlet" combination

$$X_F = \frac{1}{6}(3F_1 + F_2 + 2F_3) = 2f + \mathcal{O}(\delta m_\ell^2)$$

Normalisation from singlet

General result: Singlet quantities only vary at 2nd-order in SU(3) breaking.

$$
\begin{aligned}
F_1 \equiv \frac{1}{\sqrt{3}}(A_{\bar{N}\eta N} - A_{\bar{\Xi}\eta\Xi}) &= 2f - \frac{2}{\sqrt{3}} s_2 \delta m_l, \\
F_2 \equiv (A_{\bar{N}\pi N} + A_{\bar{\Xi}\pi\Xi}) &= 2f + 4 s_1 \delta m_l, \\
F_3 \equiv A_{\bar{\Sigma}\pi\Sigma} &= 2f + (-2s_1 + \sqrt{3} s_2)\delta m_l, \\
F_4 \equiv \frac{1}{\sqrt{2}}(A_{\bar{\Sigma}K\Xi} - A_{\bar{N}K\Sigma}) &= 2f - 2 s_1 \delta m_l, \\
F_5 \equiv \frac{1}{\sqrt{3}}(A_{\bar{\Lambda}K\Xi} - A_{\bar{N}K\Lambda}) &= 2f + \frac{2}{\sqrt{3}}(\sqrt{3} s_1 - s_2)\delta m_l.
\end{aligned}
$$

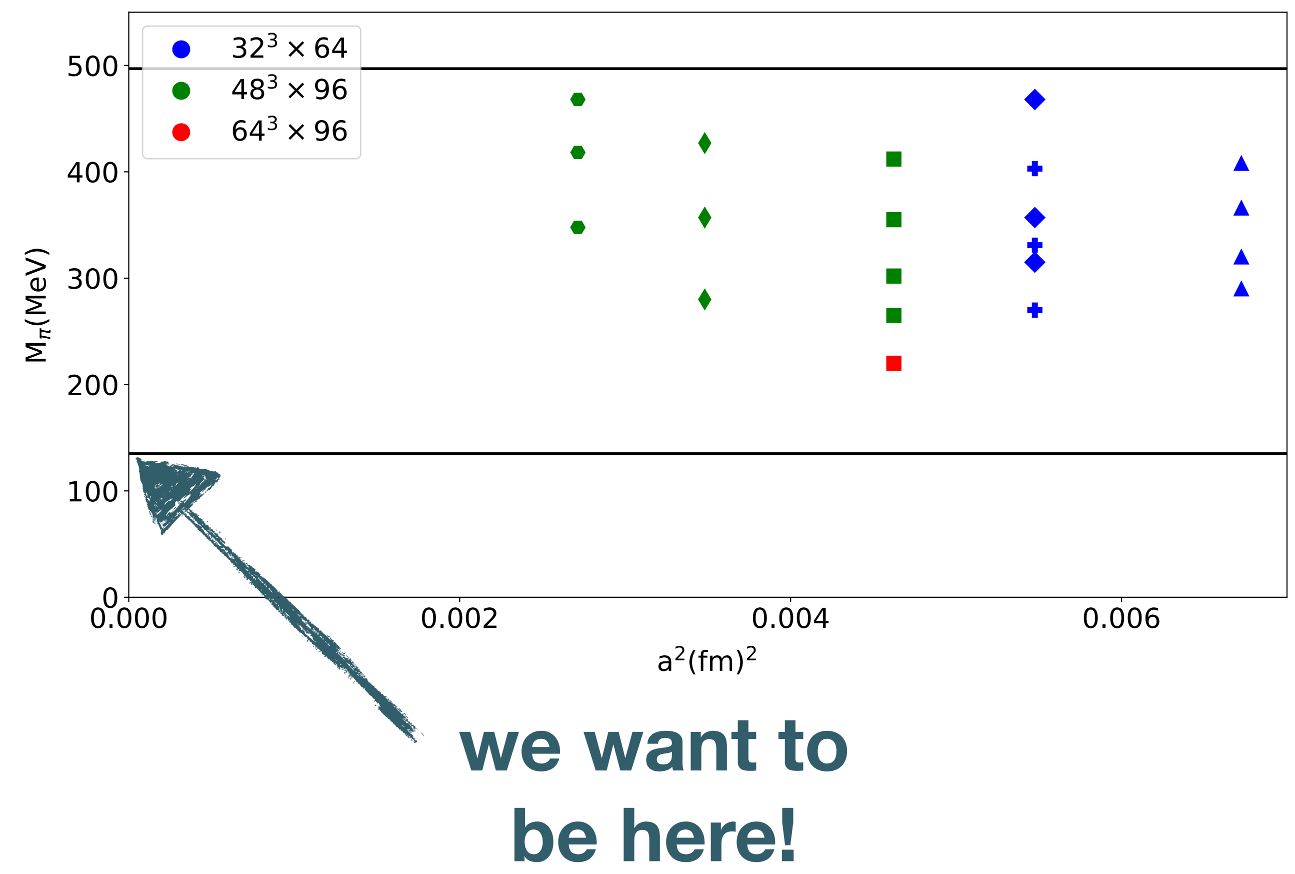
All matrix elements identical in the SU(3) symmetric limit

We want the result in the continuum and infinite-volume limit

- **Global fit**
  - Include O(a) or O(a^2) terms in X (singlet) and slope parameters
  - Free parameter to encode leading finite-volume correction on *singlet*:

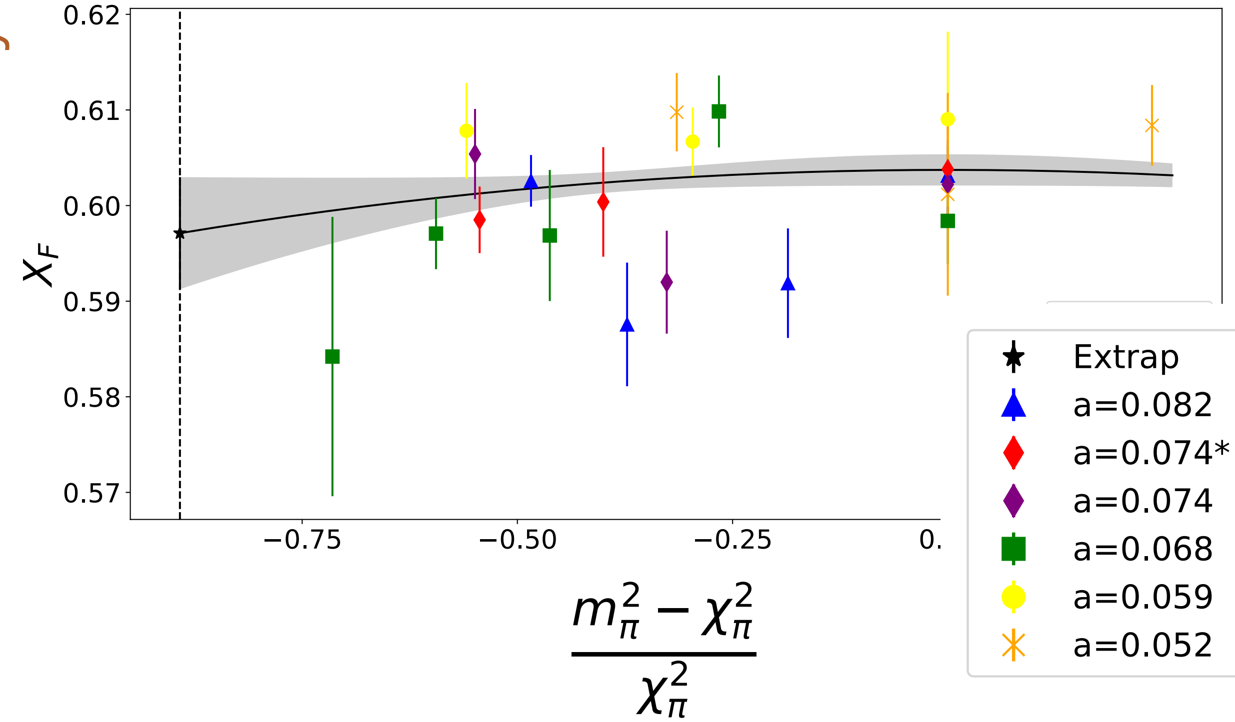
$$f_L(m) = \left(\frac{m}{X_\pi}\right)^2 \frac{e^{-mL}}{\sqrt{mL}}$$

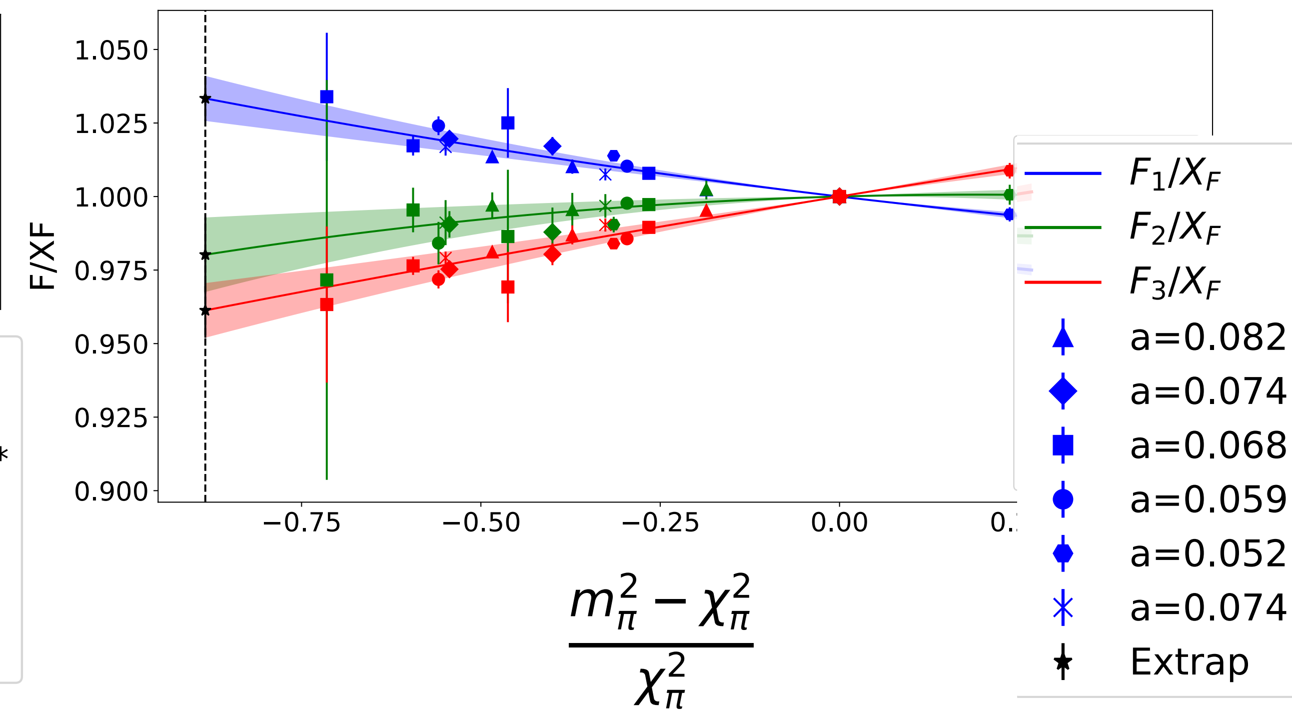
[functional form from chiral EFT, see Beane & Savage PRD(2004)]



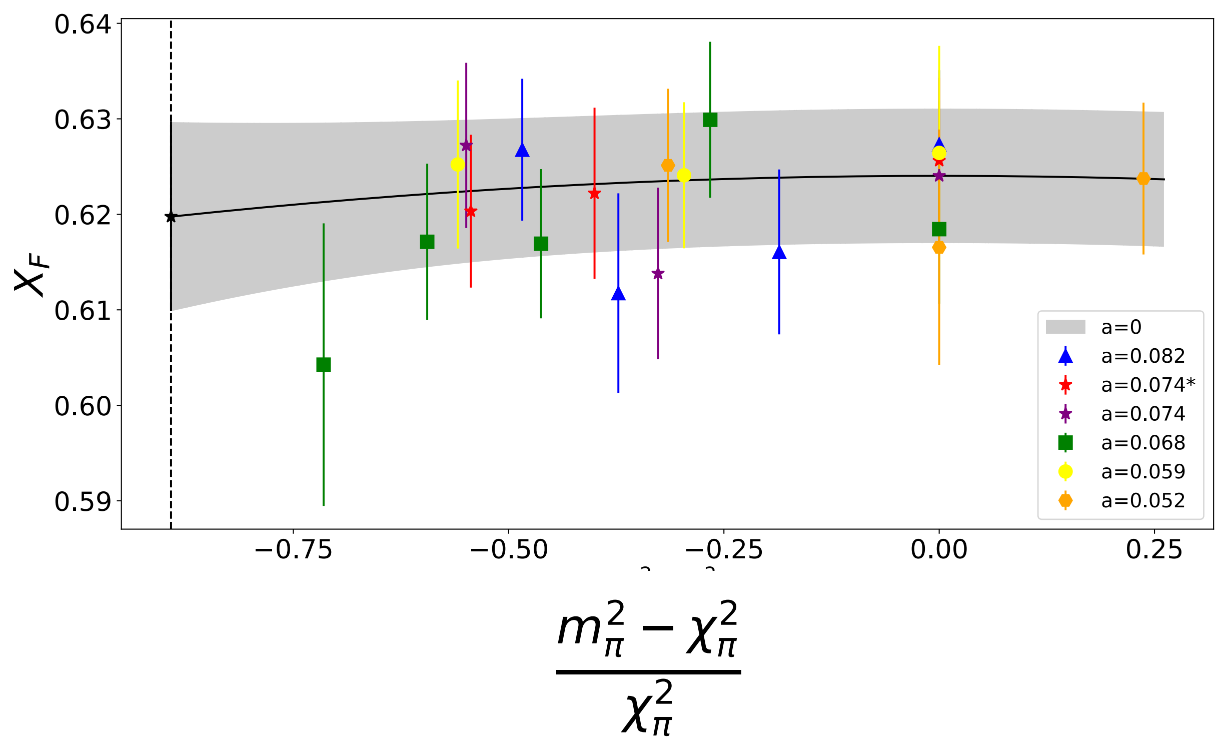
Singlet X_F | F slope parameters

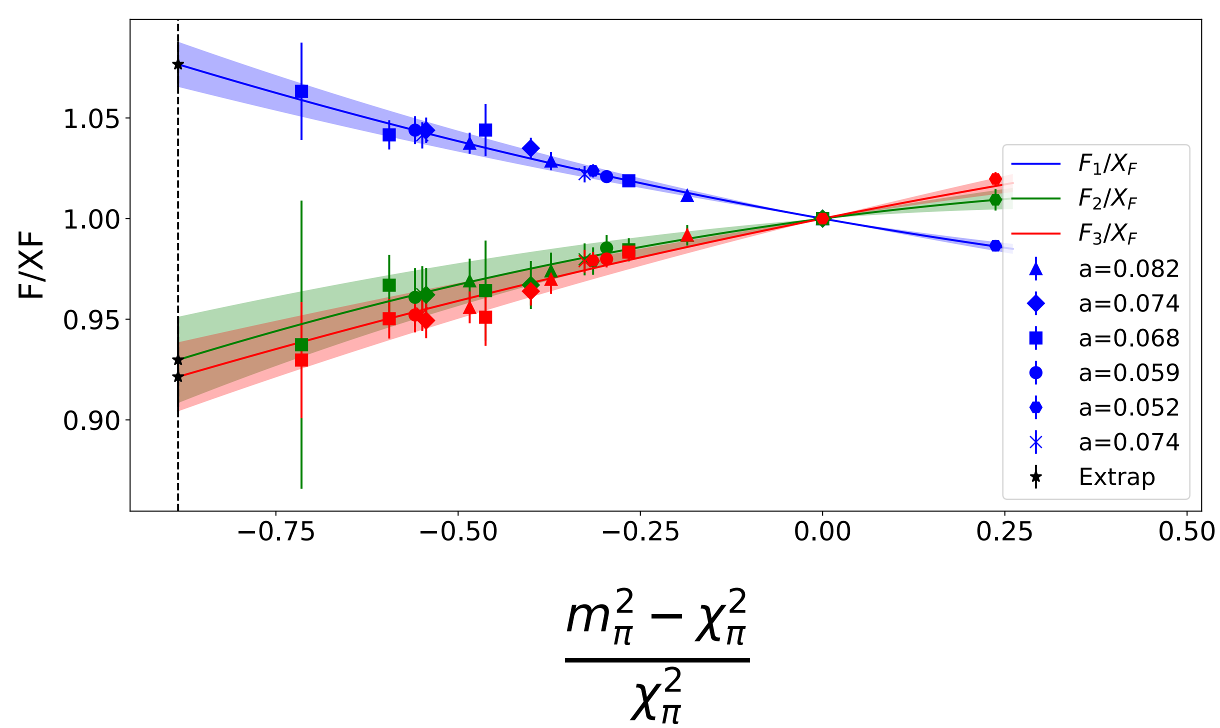
Quark mass only





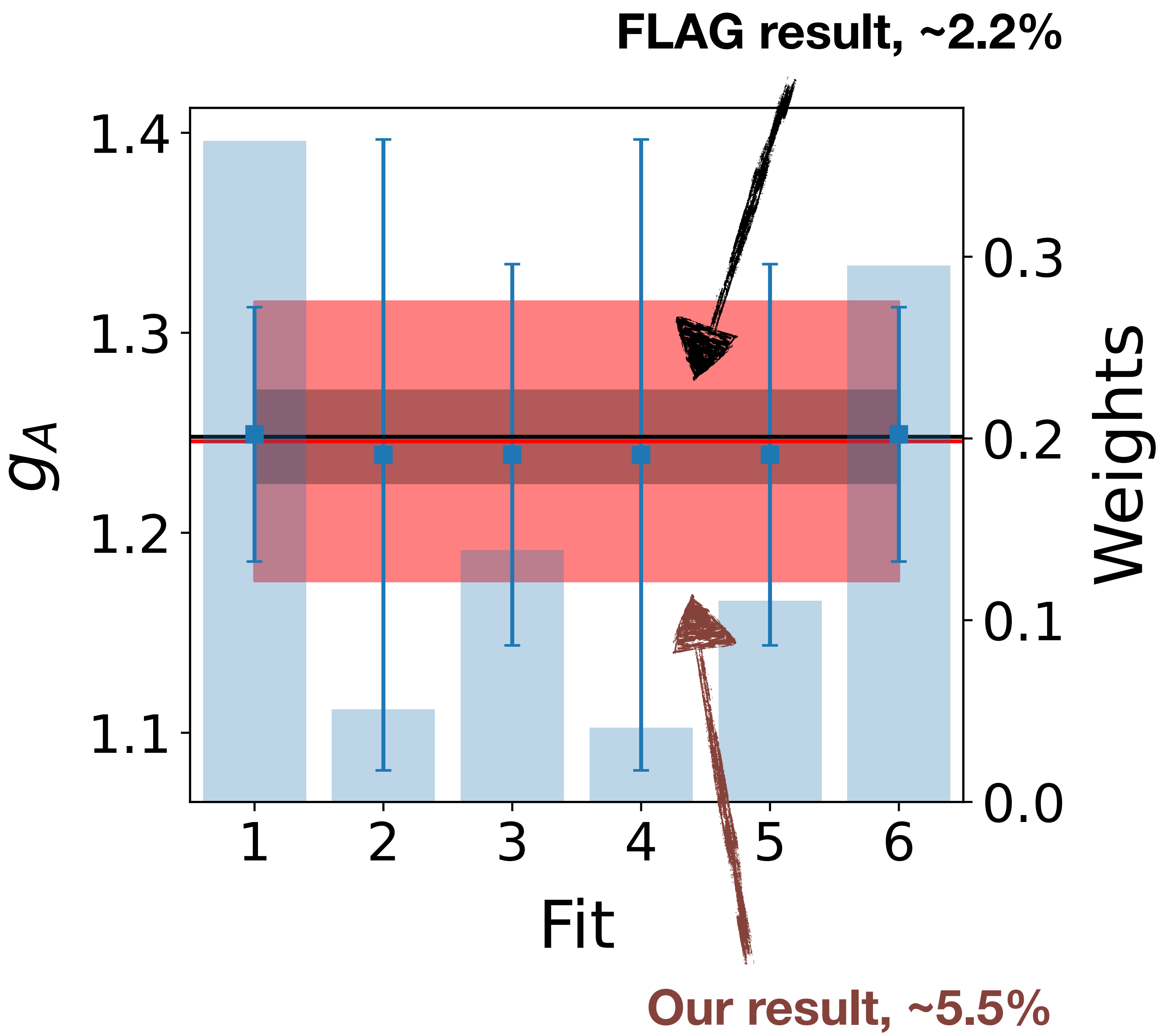
With O(a) and FV





Displayed results extrapolated $a \to 0$, $L \to \infty$

Final result: g_A (isovector)



Different model parameterisations

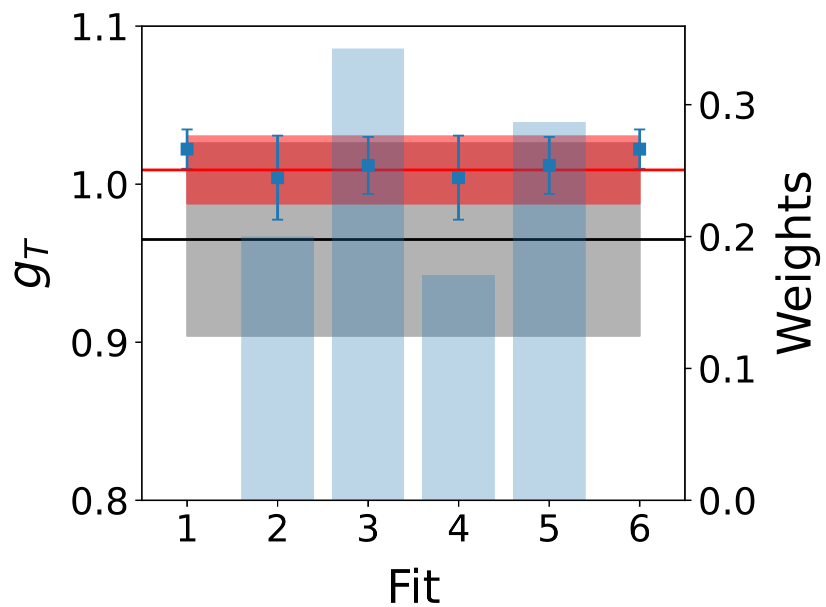
1. δm_l^2
2. $a,\ \delta m_l^2$
3. $a^2,\ \delta m_l^2$
4. $a,\ \delta m_l^2,\ m_\pi L$
5. $a^2,\ \delta m_l^2,\ m_\pi L$
6. $\delta m_l^2,\ m_\pi L$

weighted average among models (as above)

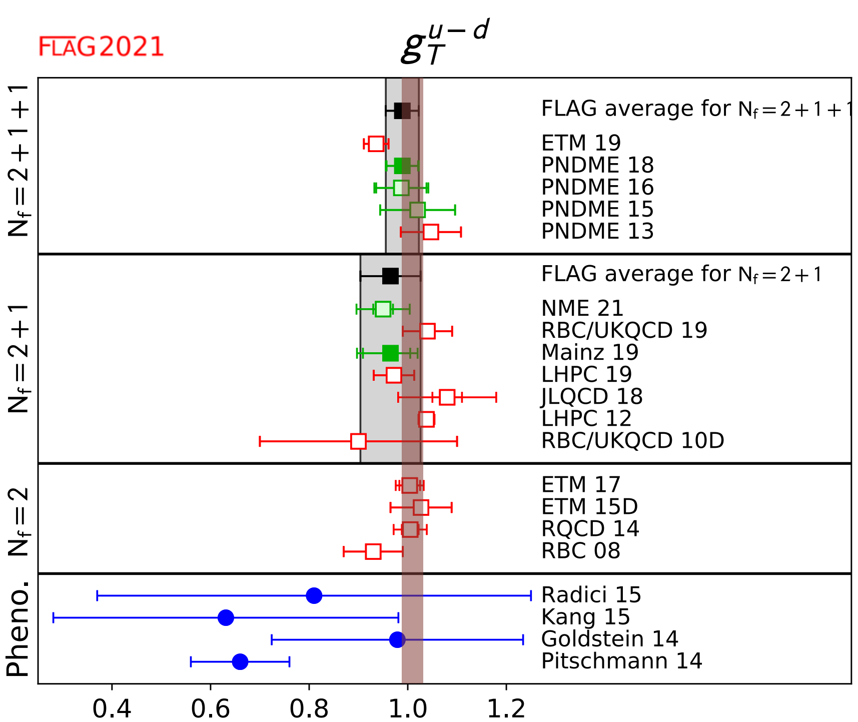
$\overline{\mathrm{MS}}$, $\mu = 2\,\mathrm{GeV}$

Isovector charges (Nf=2+1)

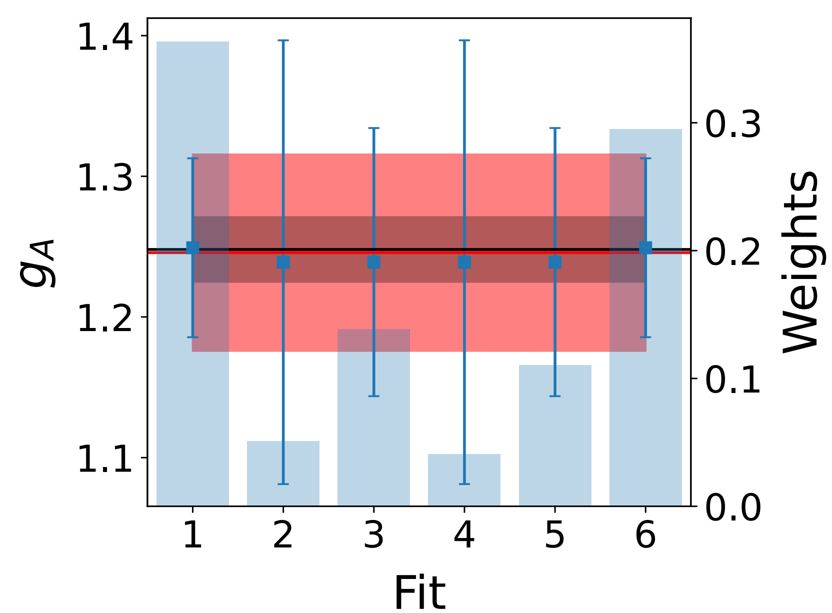
Tensor



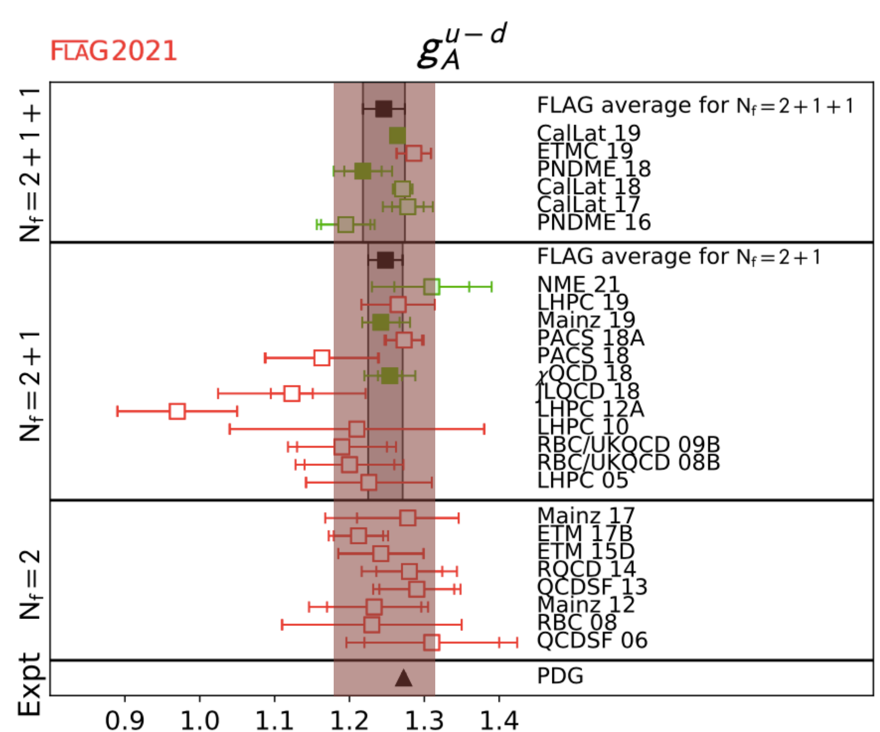
FLAG 2+1: ~6%
FLAG 2+1+1: ~3%
Our result: ~2%



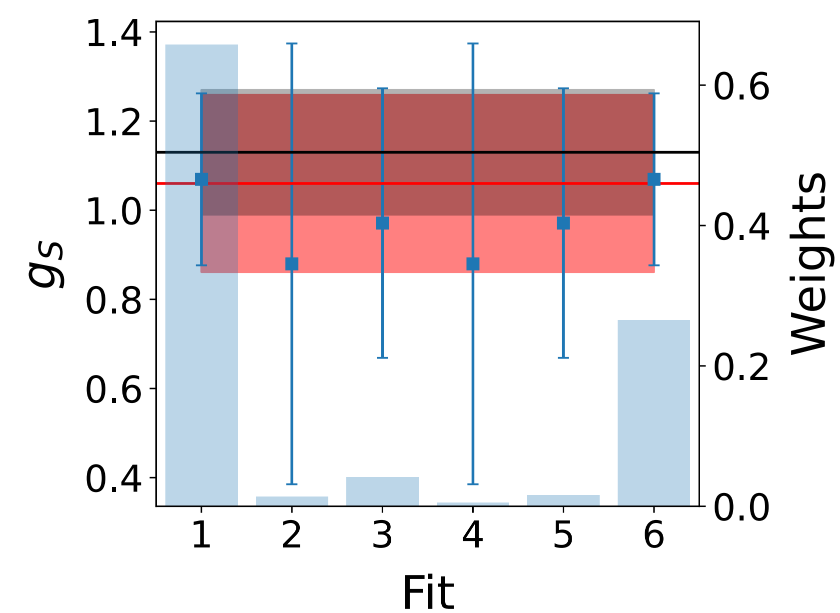
Axial



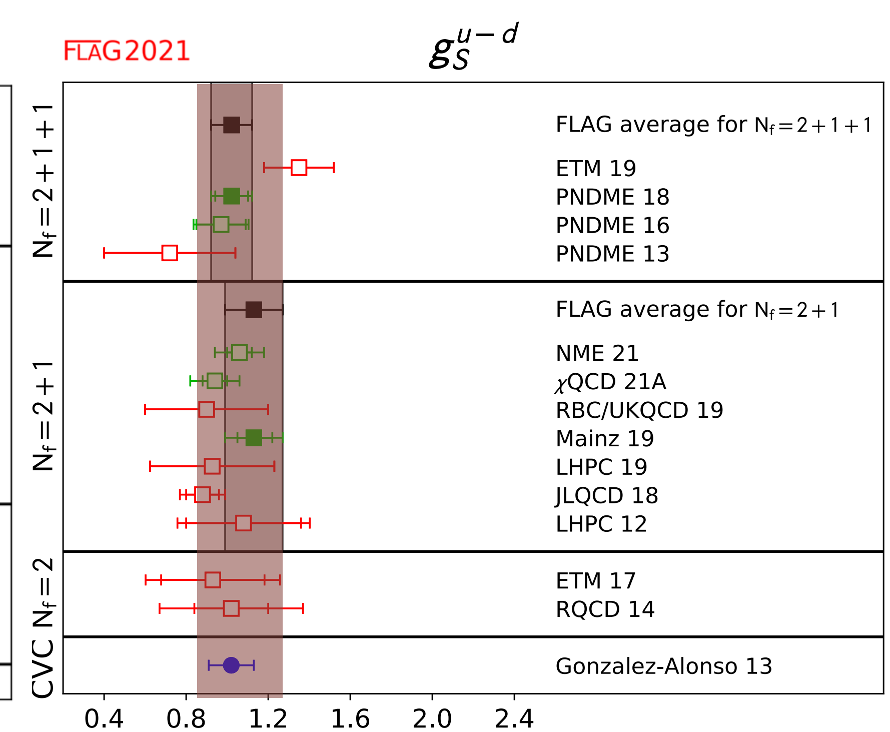
FLAG 2+1: ~2.2%
Our result: ~5.5%



Scalar



FLAG 2+1: ~12%
Our result: ~19%



Beyond the vanilla calculation of gA

Beyond pure QCD in isospin symmetric limit

- Data-driven phenomenological analyses seek to measure the connection between the neutron lifetime and the axial coupling

$$\tau_n \leftrightarrow \lambda$$

  - To state the obvious: the axial charge includes all standard model (and beyond) effects

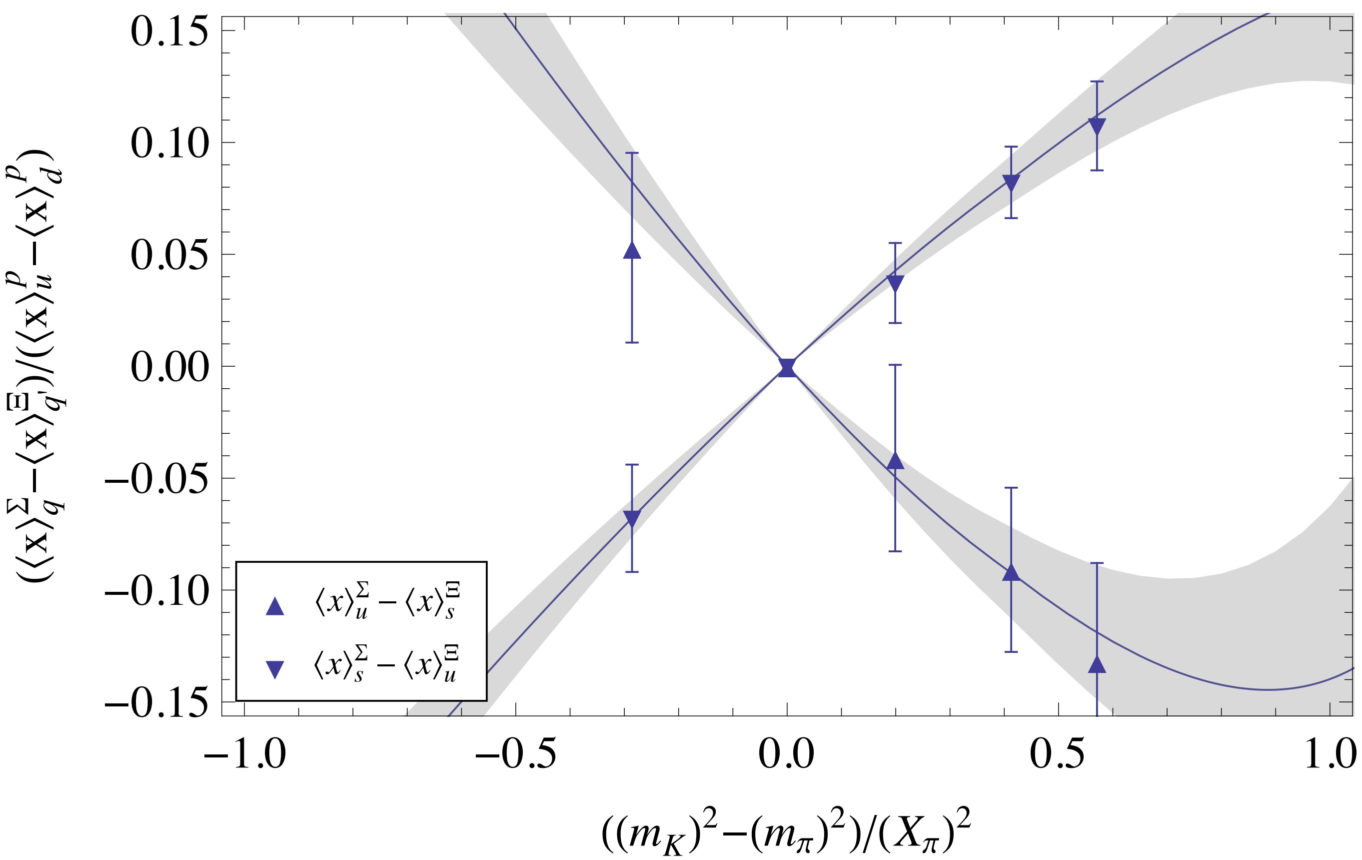
$$\langle p \,|\, A_\mu^- \,|\, n \rangle \sim g_A s_\mu$$

- Theory: **need** electromagnetic and strong isospin breaking in fundamental matrix element

$$g_A = g_A^0(1 + \Delta_{QED} + \Delta_m + \ldots)$$

Strong isospin breaking

Shanahan, Thomas & RDY, PRD(2013)



up quark

down quark

$$\delta\Delta u \simeq -0.0061(13)$$

$$\delta\Delta d \simeq -0.0018(6)$$

$$\delta\Delta(u-d) \simeq -0.0043(14?)$$

~0.5%-ish
Heavy quark masses (~1/3 * ms)

Compton on the lattice

gamma-W box

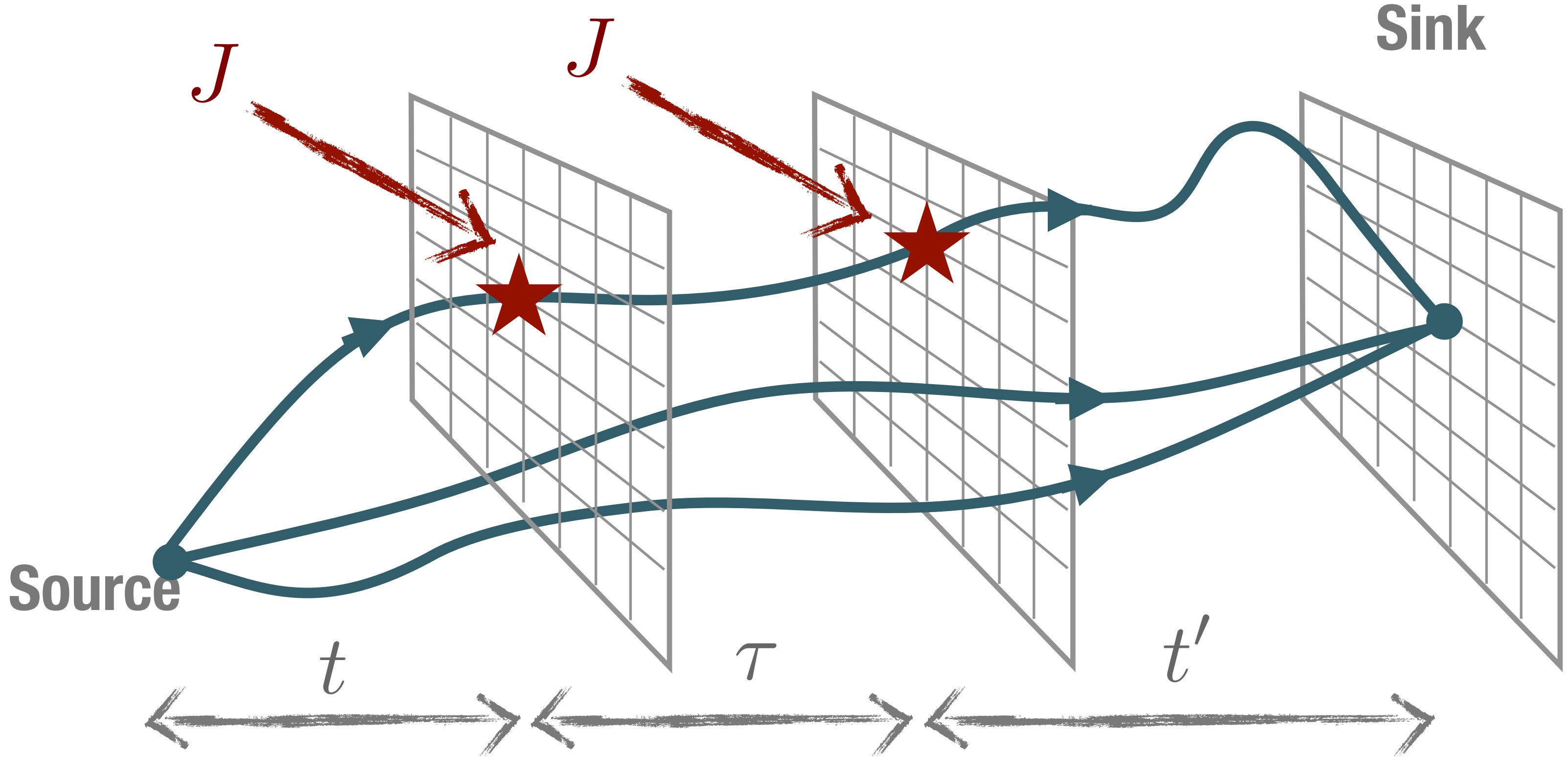
- Dispersive evaluation of gamma-W boxes
- Integration over Nachtmann moments:

M. Gorshteyn (yesterday)
Chien-Yeah Seng
…

$$\Box^b_{\gamma W}(E_e) = \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\left[M_{3,-}(1,Q^2)+\frac{8E_eM}{9Q^2}M_{3,+}(2,Q^2)\right]+\mathcal{O}(E_e^2)$$

- Towards moments (and their Q^2 dependence) from lattice QCD?

4-pt functions



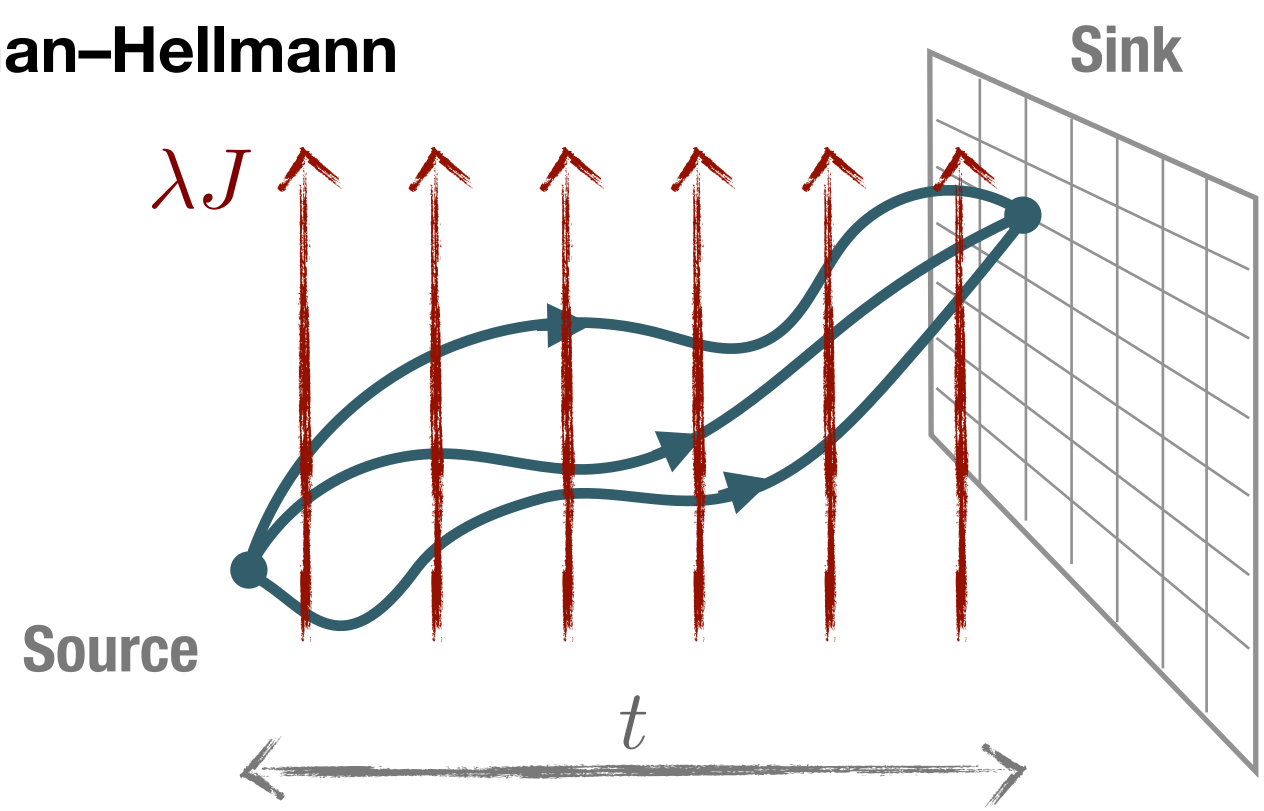
$$t, t' \gg \frac{1}{\Delta E}$$

$$\frac{\langle C_4(t,\tau,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N'|J(\tau_E)J|N\rangle$$

$$\int_0^\infty d\tau_E \to \langle N|JJ|N\rangle$$

Compton on the lattice

Feynman–Hellmann

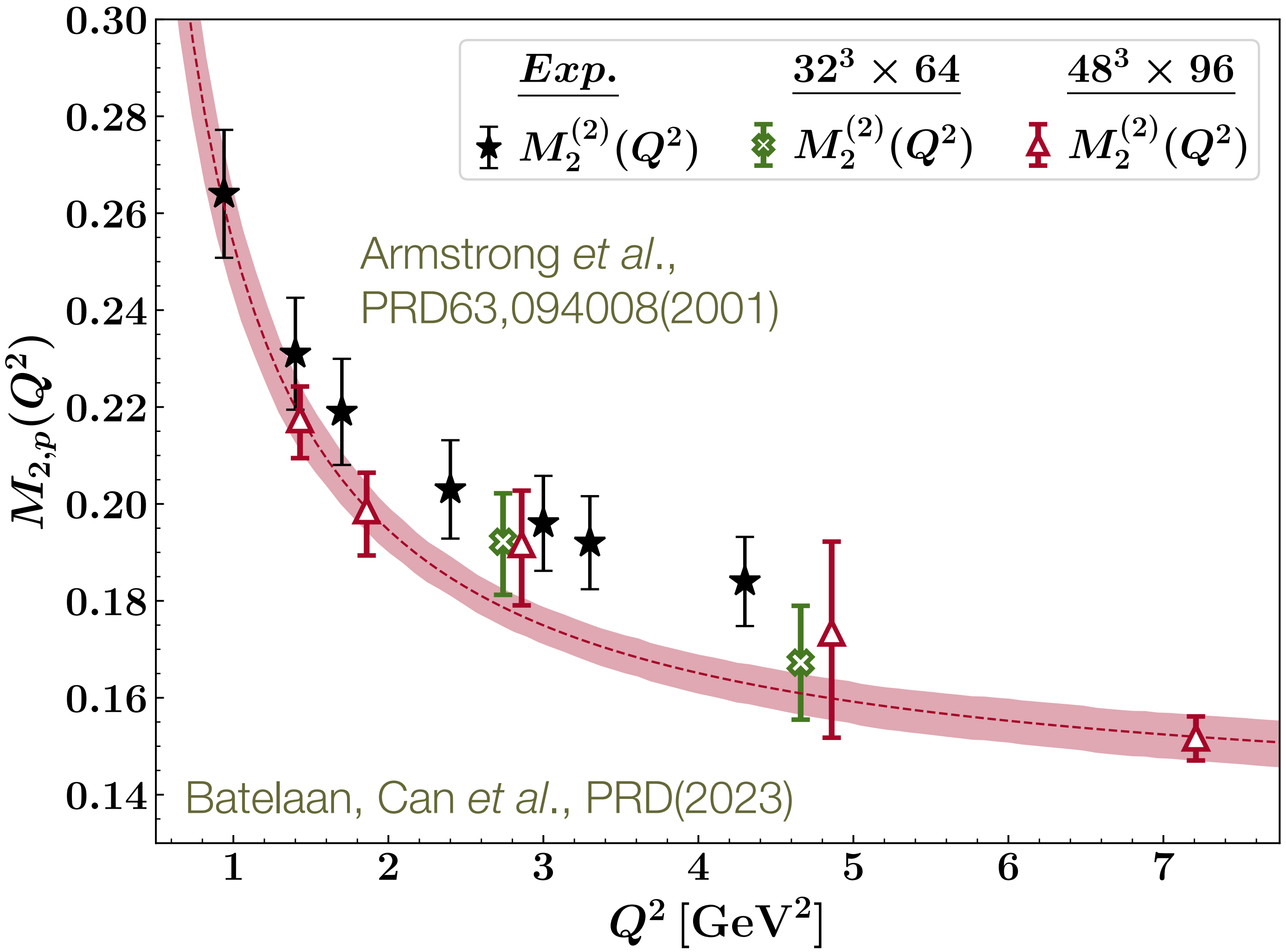


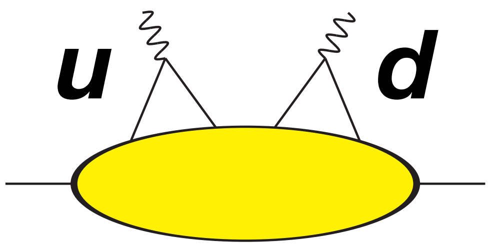
$$t \gg \frac{1}{\Delta E}$$

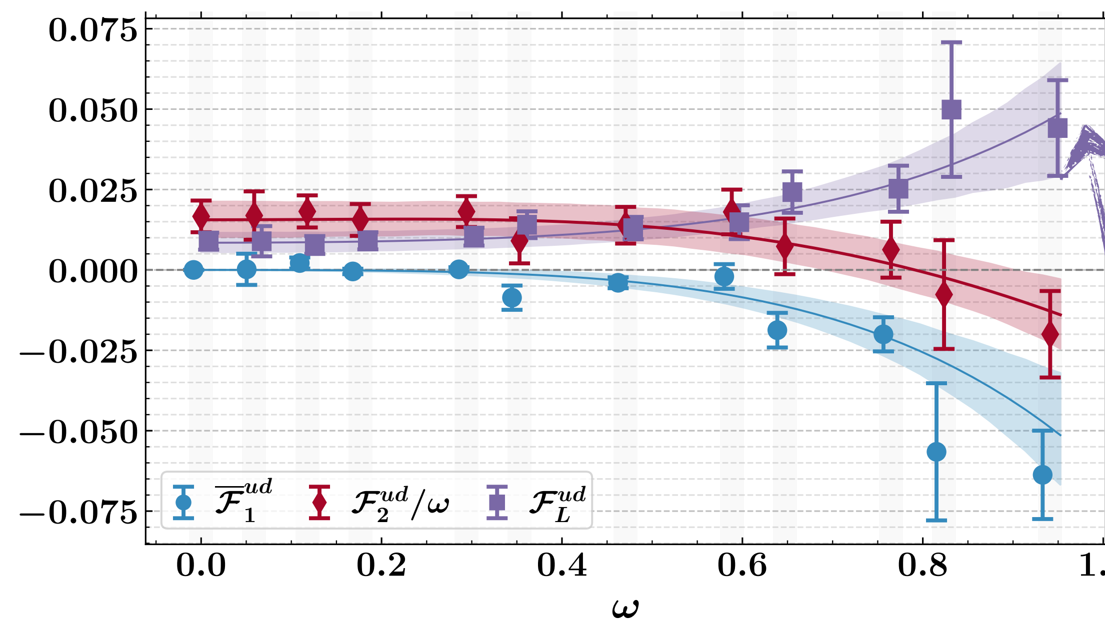
$$\left.\frac{\partial^2 E}{\partial \lambda^2}\right|_{\lambda\to 0} \propto \langle N|JJ|N\rangle$$

Lowest moment of F_2 (proton)

48^3x96, 2+1 flavour
$a = 0.068$ fm
$m_\pi \sim 420$ MeV







Clear evidence for power corrections!

Compatible with phenomenological trend

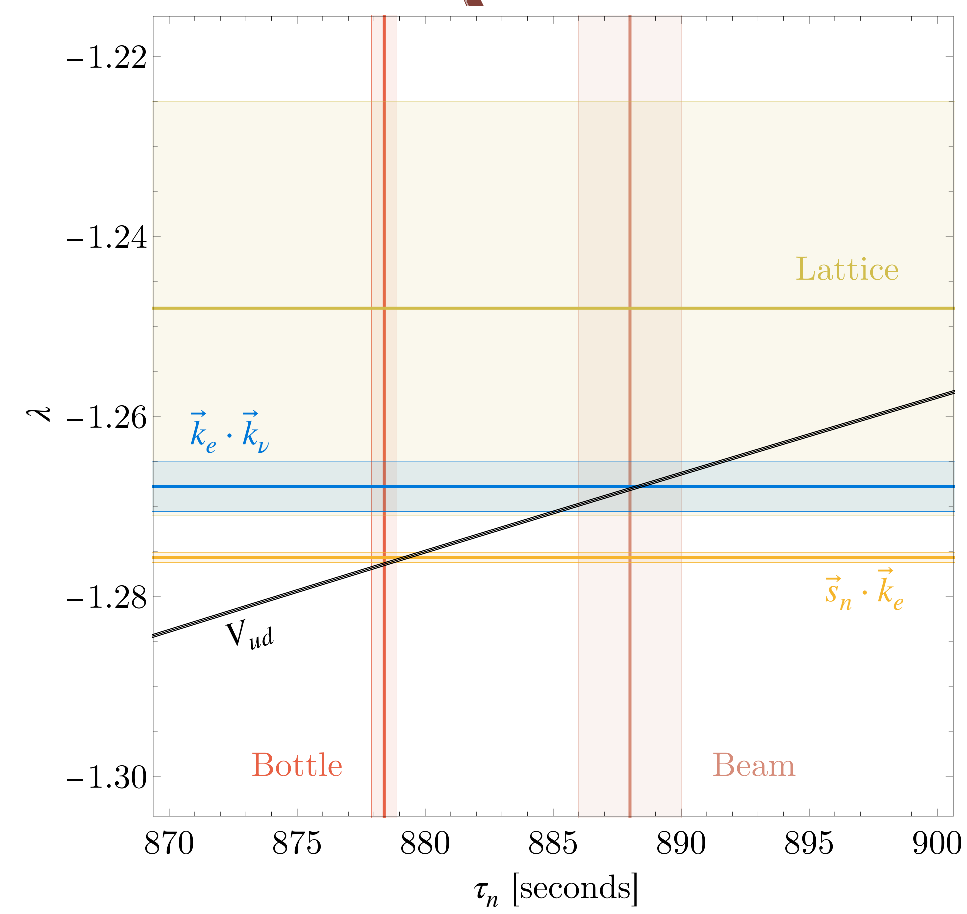
$$M_2^{(2)}(Q^2) = M_2^{(2)} + \frac{C_2}{Q^2}$$

flavour-interference structure functions

* *small* in magnitude
* non-trivial signal for longitudinal structure

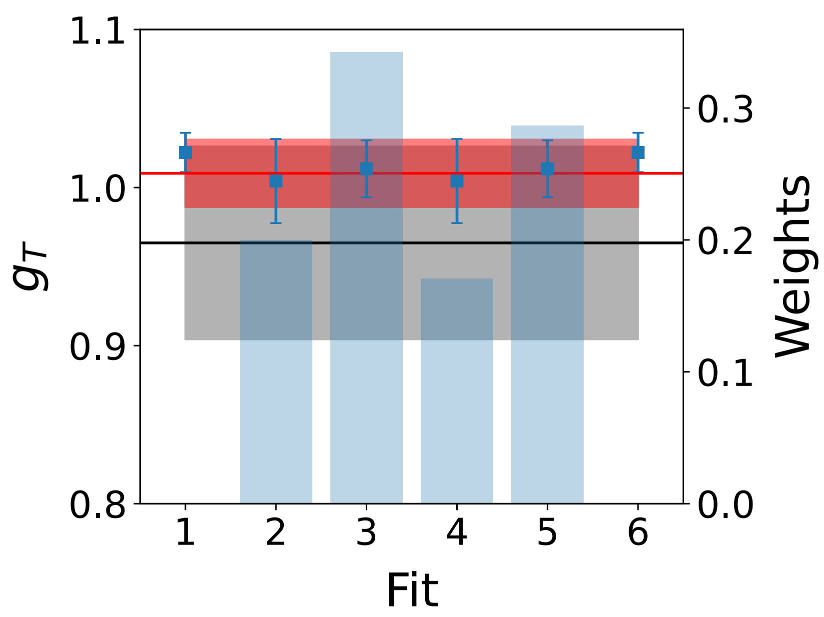
Summary

There's a lot of excitement at the moment over the neutron lifetime



Comprehensive study of isovector matrix elements

Precise determination of tensor matrix element



Looking forward to improved calculations beyond the vanilla gA

… and improved constraints for radiative corrections

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