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SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Compton amplitude and low moments of nucleon structure functions from lattice QCD

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Parton Distributions and Nucleon Structure
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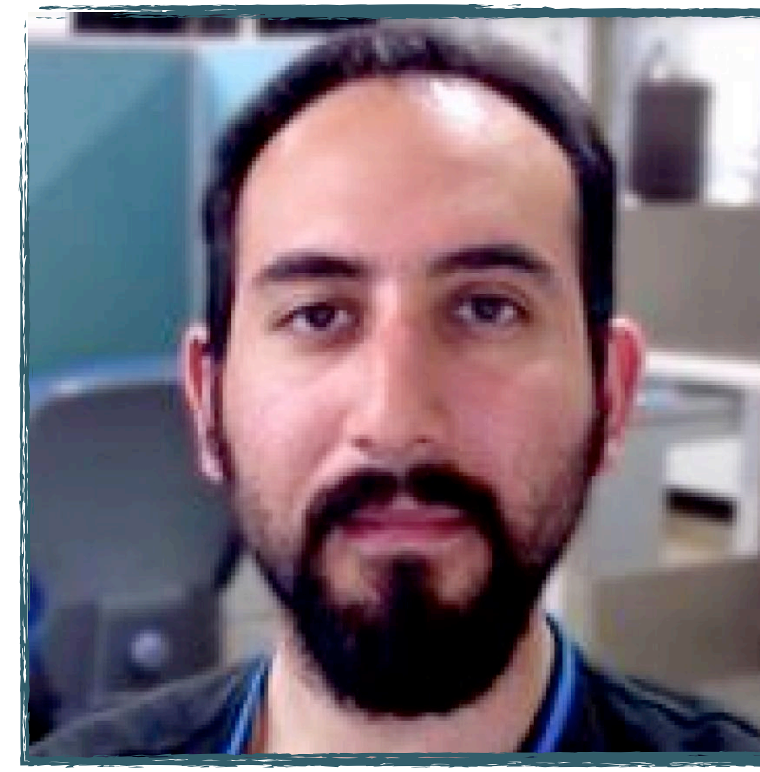

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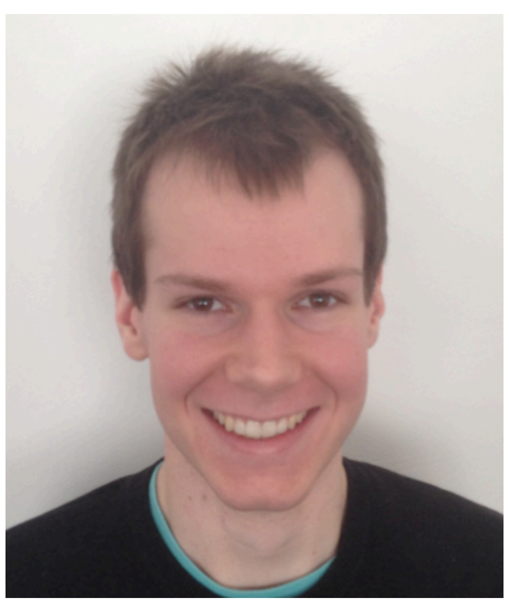

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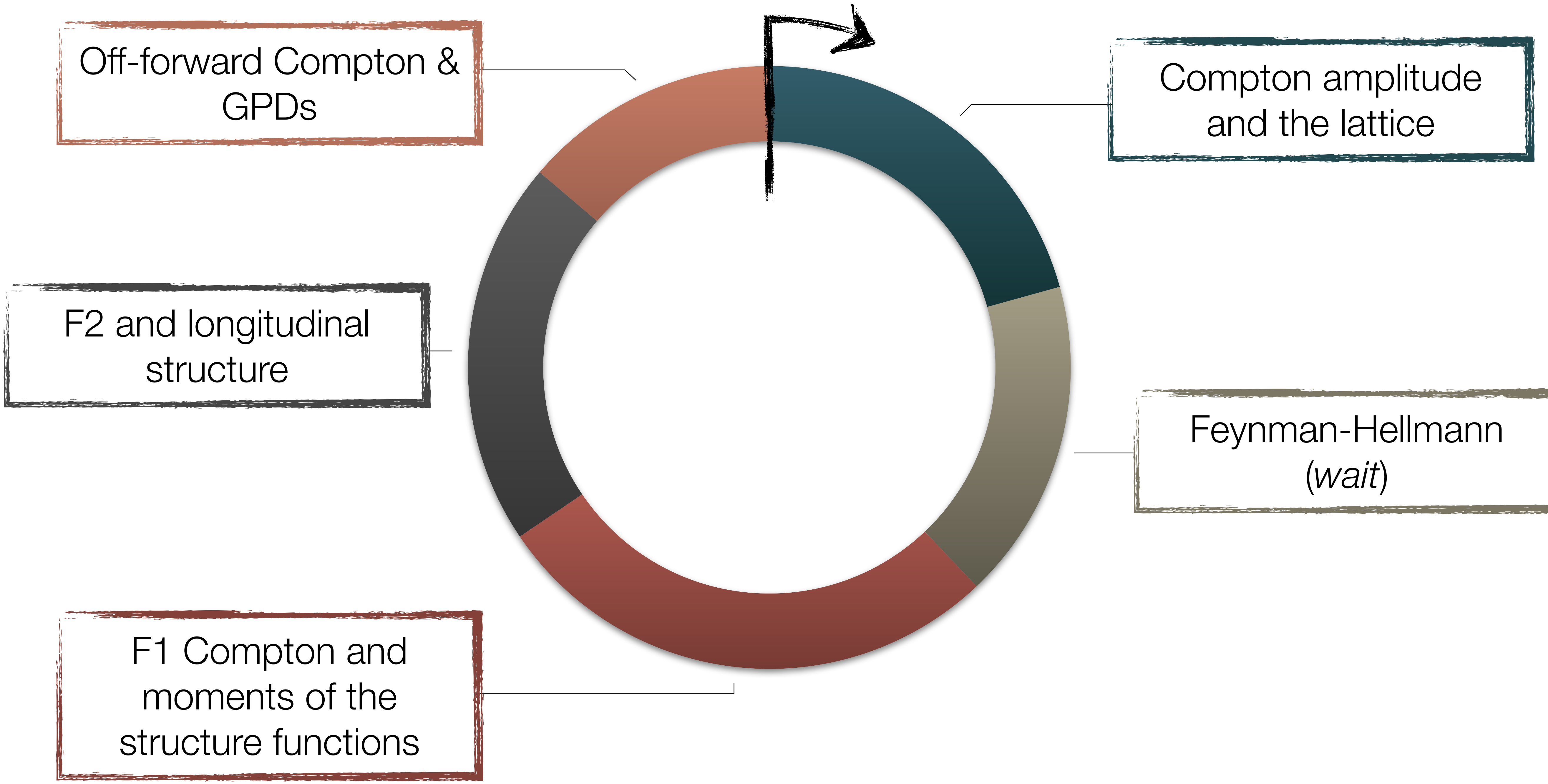
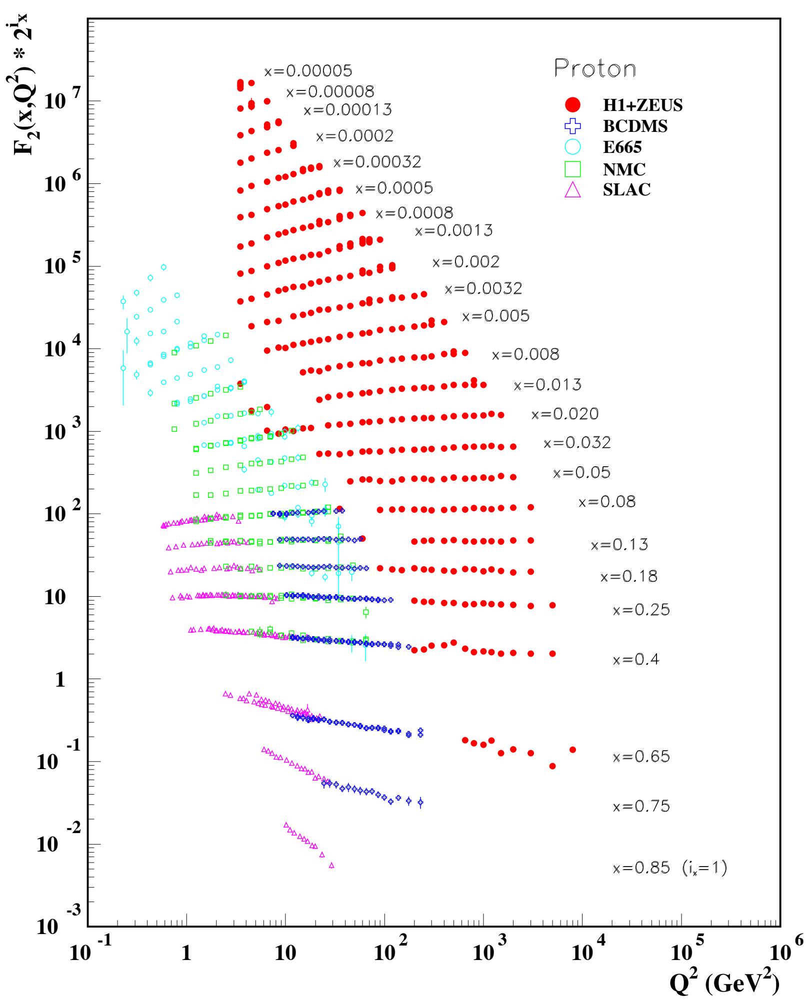
Outline

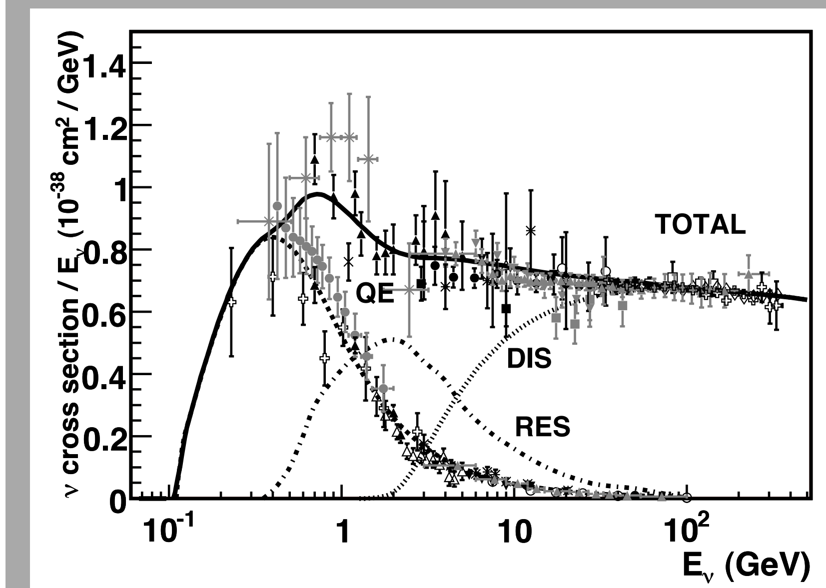


Off-forward Compton & GPDs
Compton amplitude and the lattice
F2 and longitudinal structure
Feynman-Hellmann (*wait*)
F1 Compton and moments of the structure functions

Motivation

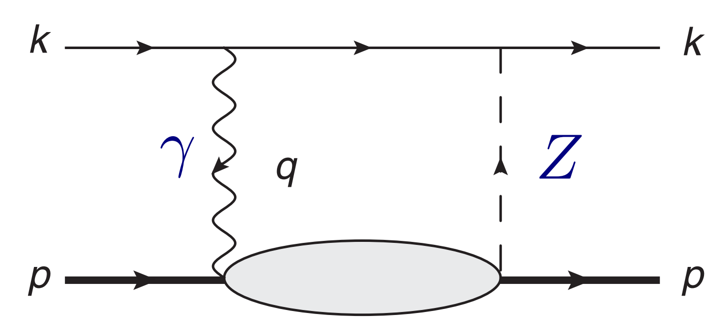
Power corrections
Theoretical foundations to inform Q^2 cuts of empirical parton fits.

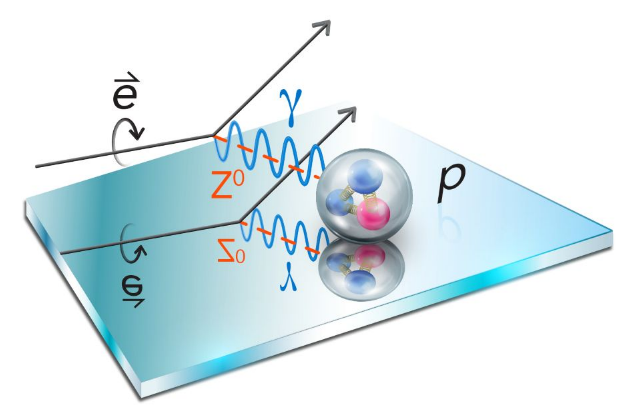




Neutrino-nucleus cross sections
Precise theoretical input required for next-generation neutrino oscillation program

Radiative corrections
Searches for new physics in the proton weak charge.
Require knowledge of gamma-*Z* interference structure functions.

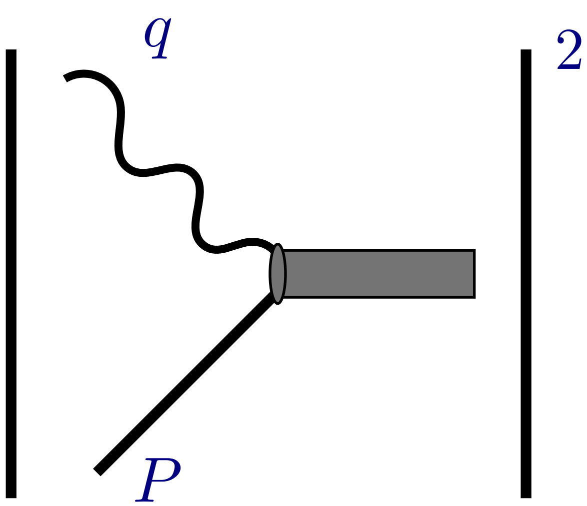




Compton amplitude and structure functions

Optical theorem

Cross section ~ Hadron tensor



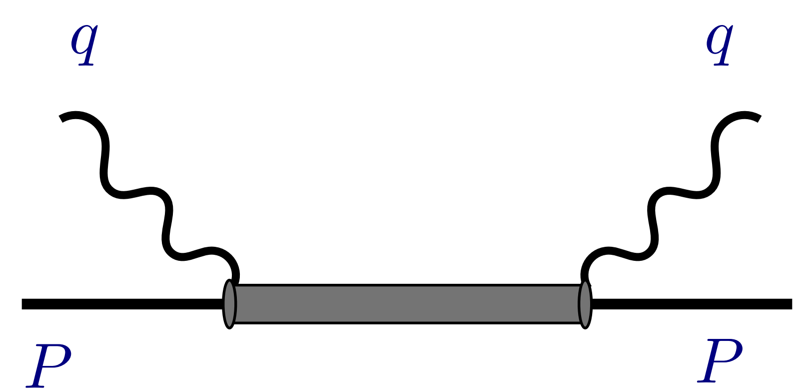
$$W_{\mu\nu} \sim \int d^4x\, e^{iq.x} \langle p|\, [J_\mu(x), J_\nu(0)]\, |p\rangle$$

Structure functions

$$F_{1,2}(P \cdot q, Q^2)$$

spin-averaged

Forward Compton **amplitude**



$$T_{\mu\nu} \sim \int d^4x\, e^{iq.x} \langle p|\mathrm{T} J_\mu(x) J_\nu(0)|p\rangle$$

(Compton) structure functions

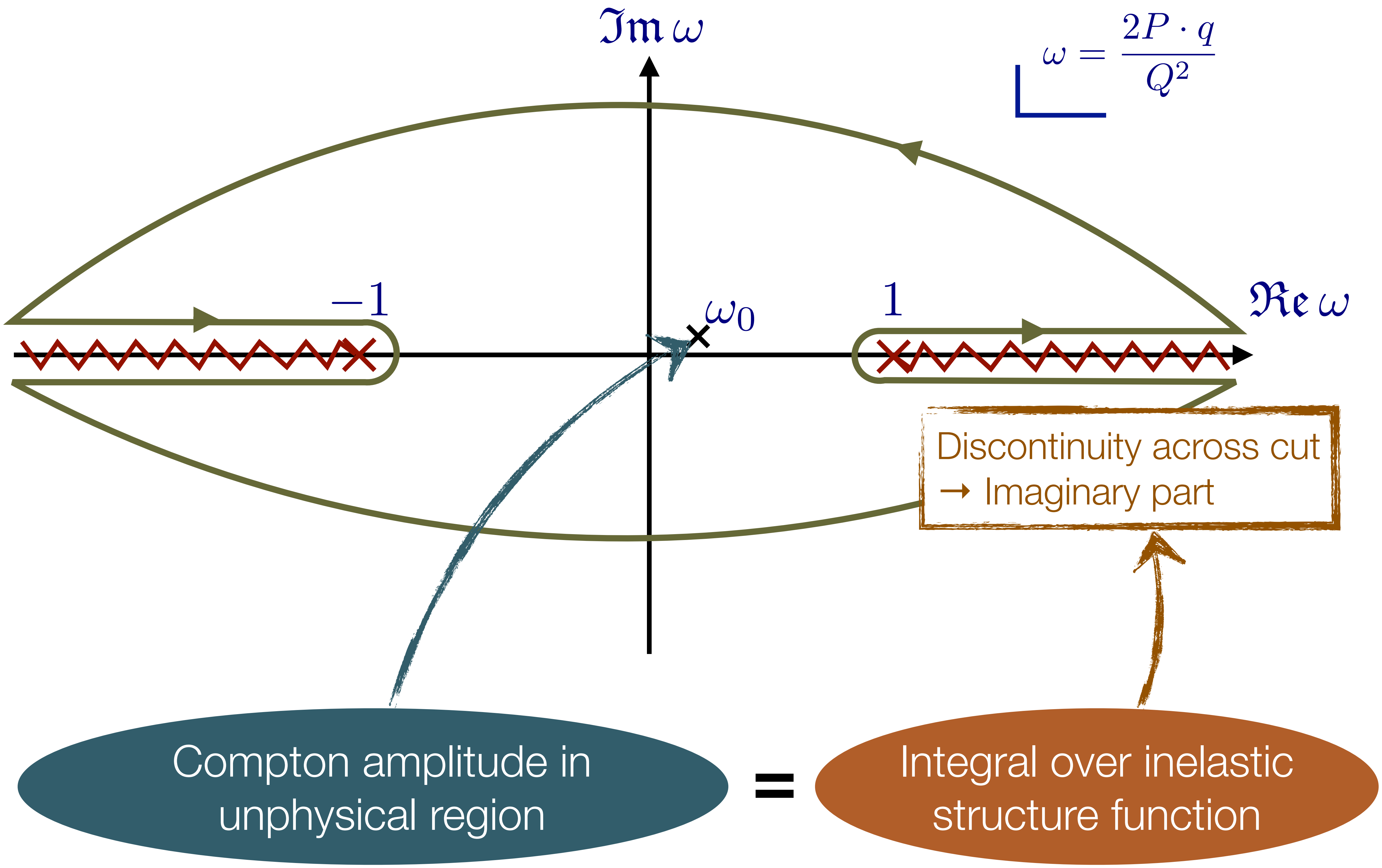
$$\mathcal{F}_{1,2}(P \cdot q, Q^2)$$

Optical theorem: $F_i = \dfrac{1}{2\pi} \mathrm{Im}\, \mathcal{F}_i$

Dispersion relation for Compton amplitude

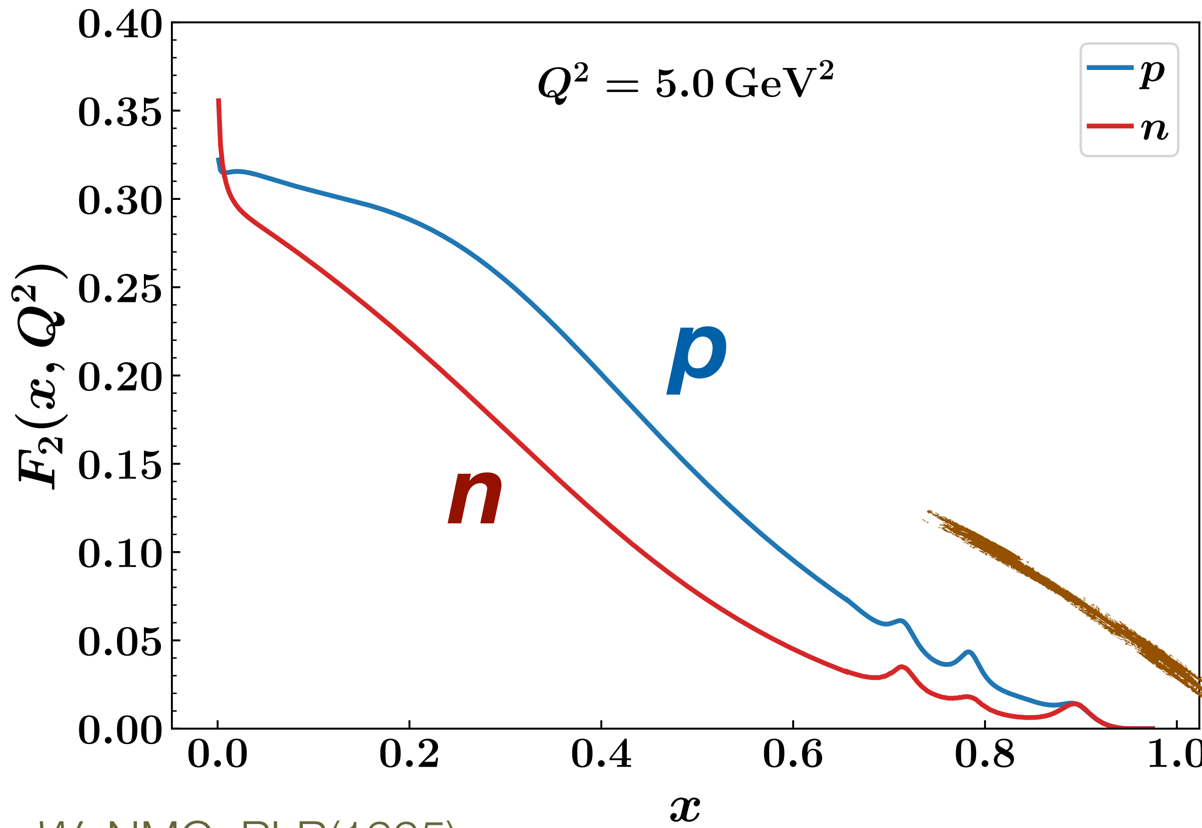
Inverse Bjorken variable

$$\omega = \frac{2P\cdot q}{Q^2}$$

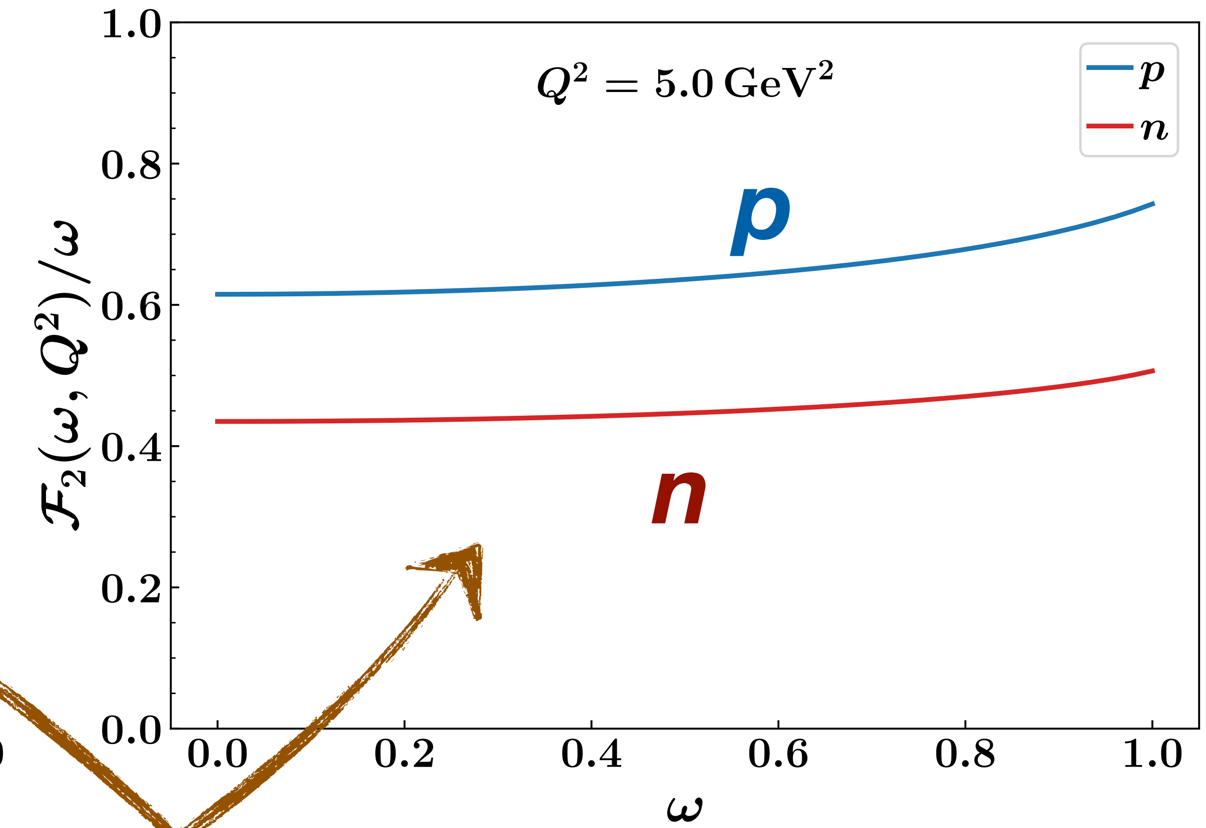


What does Compton look like? F2

Structure functions



Compton amplitude



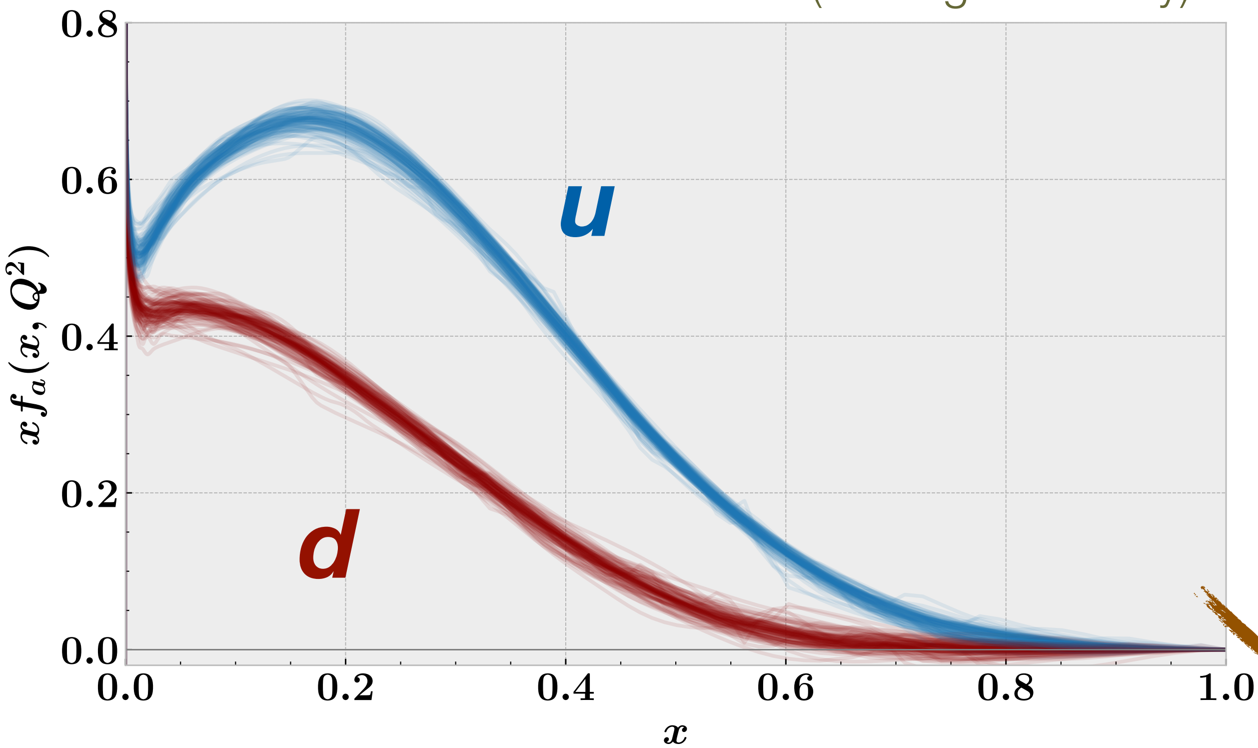
high-W: NMC, PLB(1995)
low-W: Bosted-Christy, PRC(2010)

$$\frac{\mathcal{F}_2(\omega, Q^2)}{\omega} = 4\int_0^1 dx\, \frac{F_2(x, Q^2)}{1-(x\omega)^2}$$

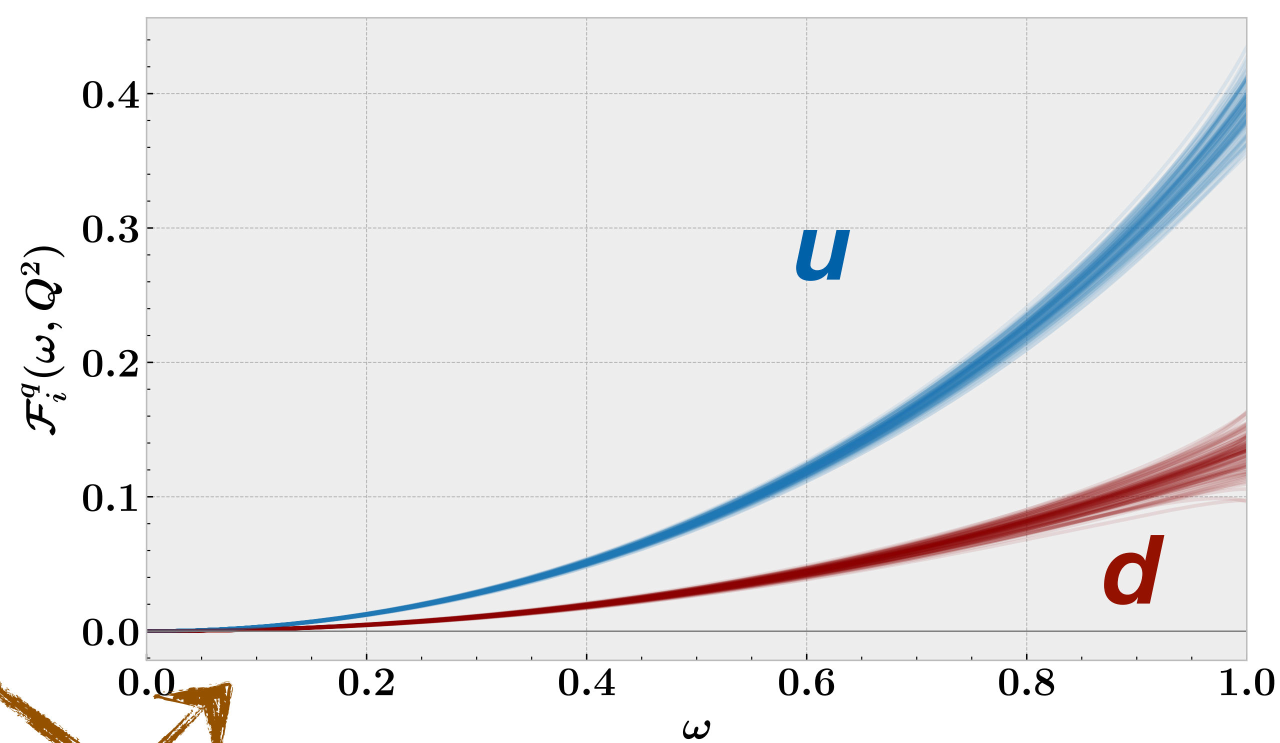
What does Compton look like? F1

Structure functions

NNPDF (leading-twist only)



Compton amplitude



$$\overline{\mathcal{F}}_1(\omega, Q^2) = 2\omega^2 \int_0^1 dx\, \frac{2xF_1(x, Q^2)}{1-(x\omega)^2}$$

But the lattice is Euclidean??

Minkowski Compton

(spin, Lorentz suppressed)

$$T(p,q) = i\int d^4z\, e^{iq\cdot z}\langle p|\mathrm{T}\{J(z)J(0)\}|p\rangle$$

$$= \sum\!\!\!\!\!\!\int_X \int_0^\infty dt\, i\, e^{i(q_0+E_p-E_X+i\epsilon)t}\langle p|J(0)|X(\mathbf{p}+\mathbf{q})\rangle\langle X(\mathbf{p}+\mathbf{q})|J(0)|p\rangle + (q\to -q)$$

$$\frac{1}{(E_X - E_p - q_0 - i\epsilon)}$$

Euclidean hadron tensor

Unintegrated form

Inversion problem, see Keh-Fei Liu

Euclidean Compton

$$T^{\mathcal{E}}(p,q) = \int_0^\infty d\tau \int d^3z e^{-i\mathbf{q}\cdot\mathbf{z}}\langle p|J(\mathbf{z},\tau)J(0)|p\rangle$$

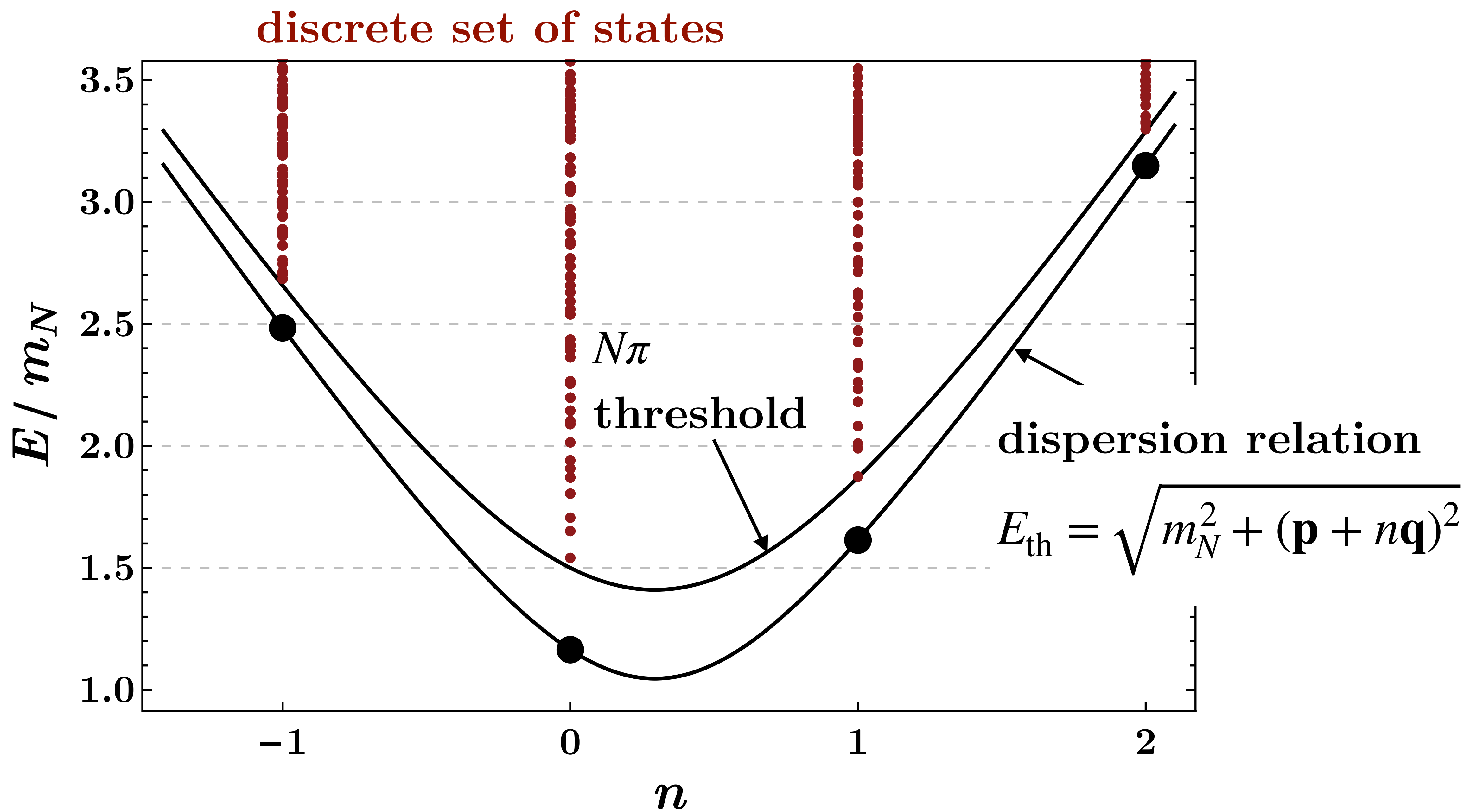
$$= \sum\!\!\!\!\!\!\int_X \int_0^\infty d\tau\; e^{(q_0+E_p-E_X)\tau}\langle p|J(0)|X(\mathbf{p}+\mathbf{q})\rangle\langle X(\mathbf{p}+\mathbf{q})|J(0)|p\rangle + (q\to -q)$$

$$\frac{1}{(E_X - E_p - q_0)} \qquad \textbf{if } E_X > E_p + q_0$$

and if $E_X > E_p + q_0$ there are no singularities in $\int_{E_{\rm th}}^\infty dE_X$

if $E_{X(\mathbf{p}\pm\mathbf{q})} > E_p \pm q_0 \;\Rightarrow T^{\mathcal{E}}(p,q) = T(p,q)$

Kinematic restriction



Must **only** consider nucleon momenta to correspond to lowest energy connected by discrete multiples of **q**

(First) numerical results: $\mathscr{F}_1 \to$ moments of F_1

Can, RDY *et al*. PRD(2020)

Compton on the lattice

- Forward spin-averaged Compton amplitude

$$\omega = \frac{2p\cdot q}{Q^2}$$

$$T^{\mu\nu}(p,q) = \rho_{ss'}\int d^4x\, e^{iq\cdot x}\langle p,s'|\mathrm{T}\left\{J^\mu(x)J^\nu(0)\right\}|p,s\rangle$$
$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)\mathcal{F}_1(\omega,Q^2)+\frac{1}{p\cdot q}\left(p^\mu-\frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu-\frac{p\cdot q}{q^2}q^\nu\right)\mathcal{F}_2(\omega,Q^2)$$

- Choose simplest kinematics to directly isolate F1

$$J^3J^3,\ \text{and}\ q_3=p_3=0$$

$$T^{33}(p,q)\to\mathcal{F}_1(\omega,Q^2)$$

Obligatory slide on lattice specs

QCDSF/UKQCD configurations

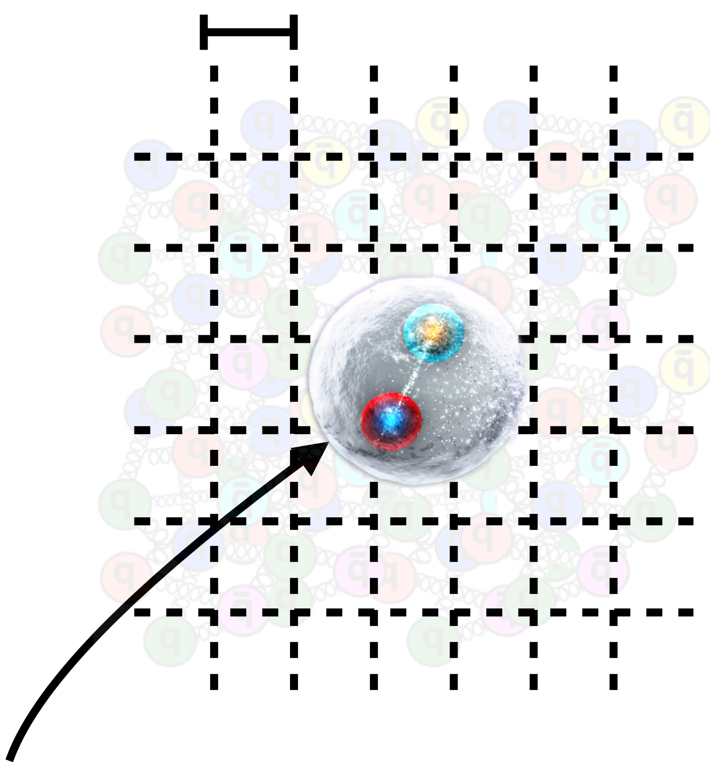
$$\begin{pmatrix} 32^3 \times 64 \\ 48^3 \times 96 \end{pmatrix}, \text{ 2+1 flavor (u/d+s)}$$

$$\beta = \begin{pmatrix} 5.50 \\ 5.65 \end{pmatrix}, \text{ NP-improved Clover action}$$

Phys. Rev. D 79, 094507 (2009), arXiv:0901.3302 [hep-lat]

$$m_\pi \sim \begin{bmatrix} 470 \\ 420 \end{bmatrix} \text{MeV, ~SU(3)}$$

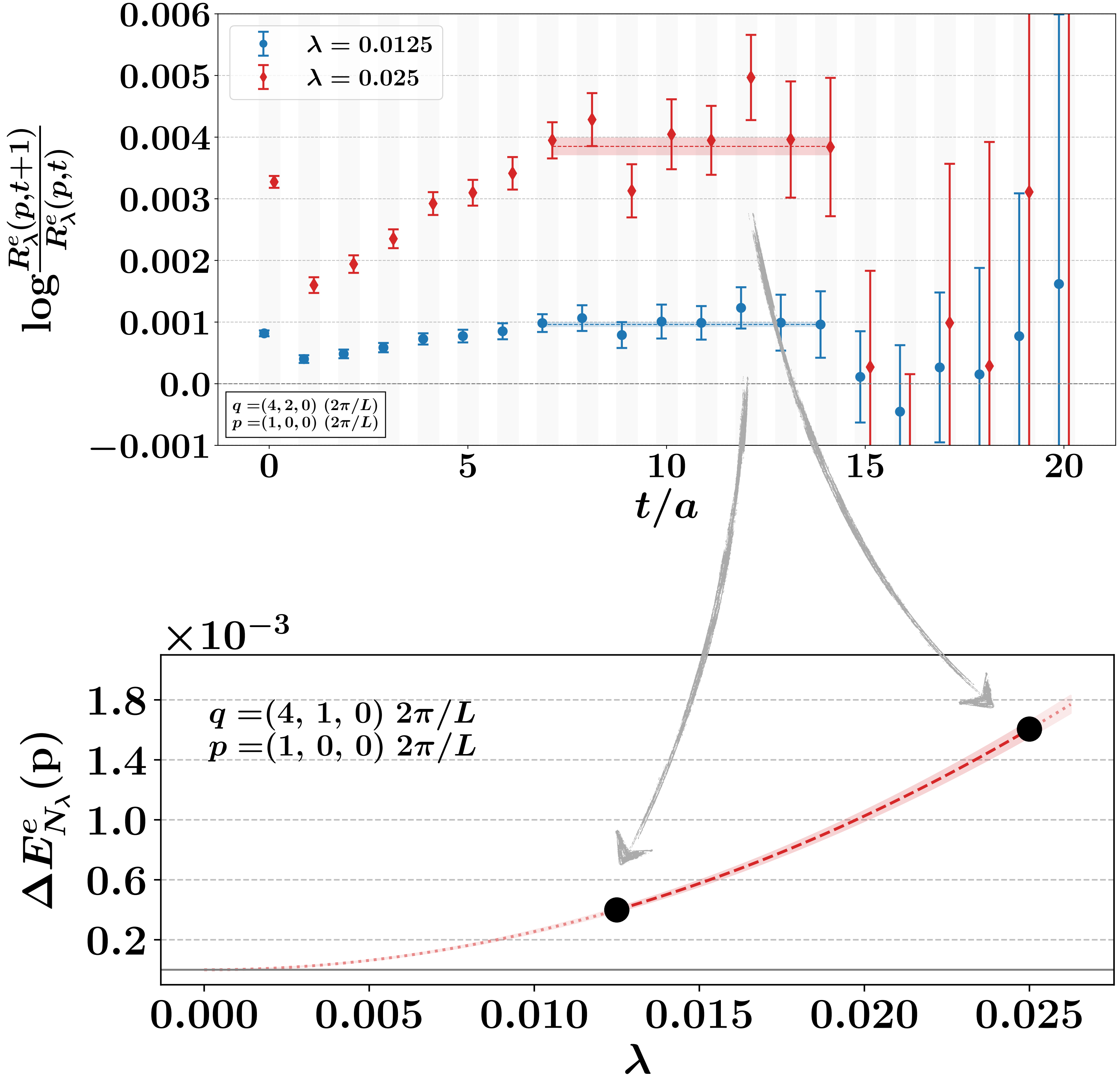
$$m_\pi L \sim \begin{bmatrix} 5.6 \\ 6.9 \end{bmatrix} \qquad a = \begin{bmatrix} 0.074 \\ 0.068 \end{bmatrix} \text{ fm}$$



Unmodified
QCD background

- Local EM current insertion, $J_\mu(x) = Z_V \bar{q}(x)\gamma_\mu q(x)$ (valence only)
- Feynman–Hellmann propagators at 4 field strengths, $\lambda = [\pm 0.0125, \pm 0.025]$
- Up to $\mathcal{O}(10^4)$ measurements for each pair of Q^2 and λ

Energy shifts



Effective energies

2 external field strengths

$$\Delta E = E(\lambda) - E_0$$

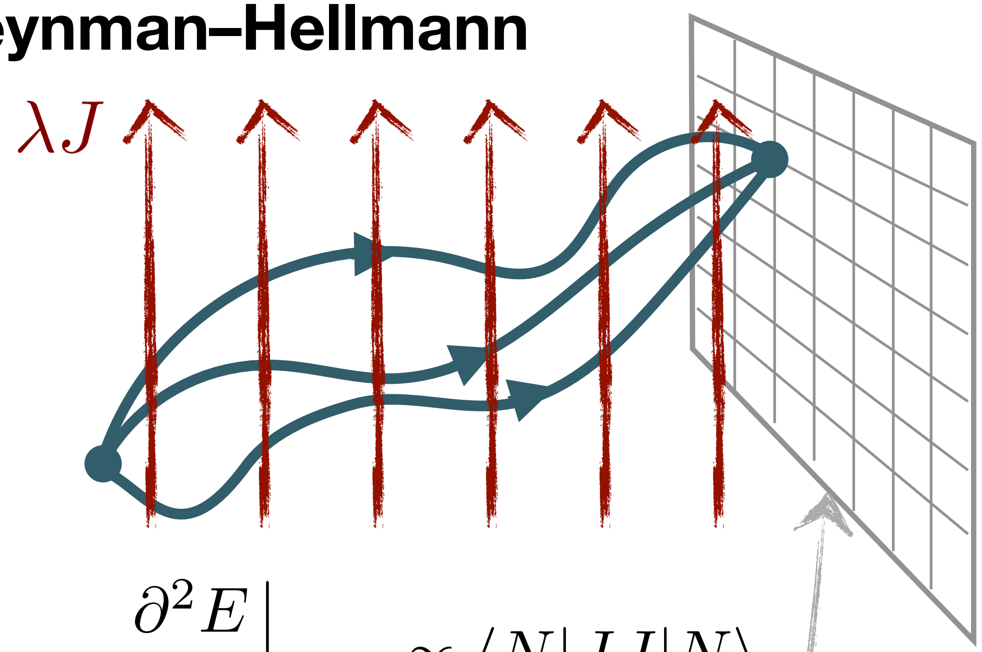
Isolate 2nd derivative

(almost “exact” quadratic)

$$\Delta E = \tfrac{1}{2}\lambda^2 \frac{\partial^2 E}{\partial \lambda^2} + \ldots$$

Kinematic coverage

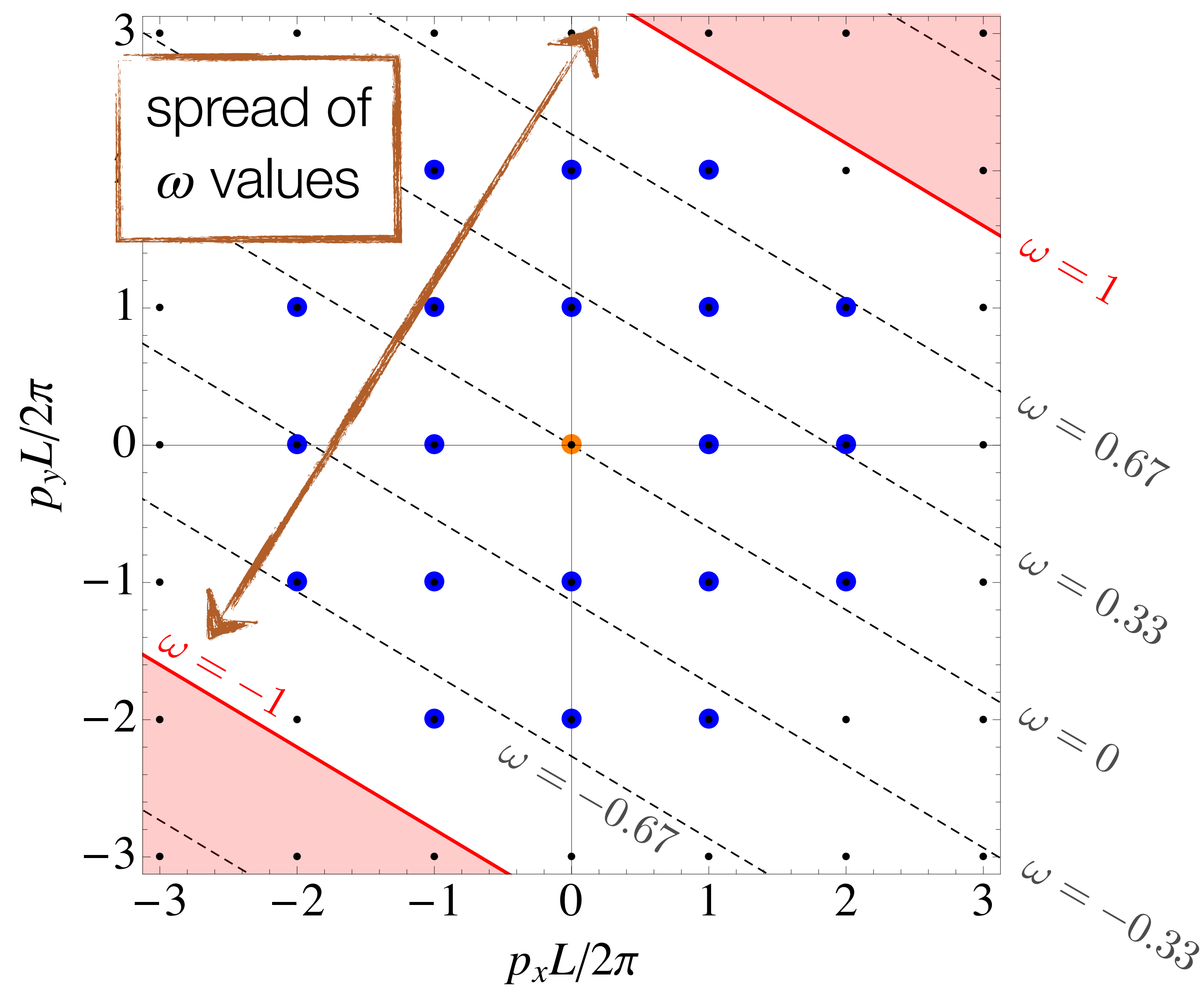
Feynman–Hellmann



$$\left.\frac{\partial^2 E}{\partial \lambda^2}\right|_{\lambda\to 0} \propto \langle N|JJ|N\rangle$$

$\mathbf{q}$ fixed for each propagator evaluation (costly)

freedom to choose Fourier projection at hadron sink (cheap)

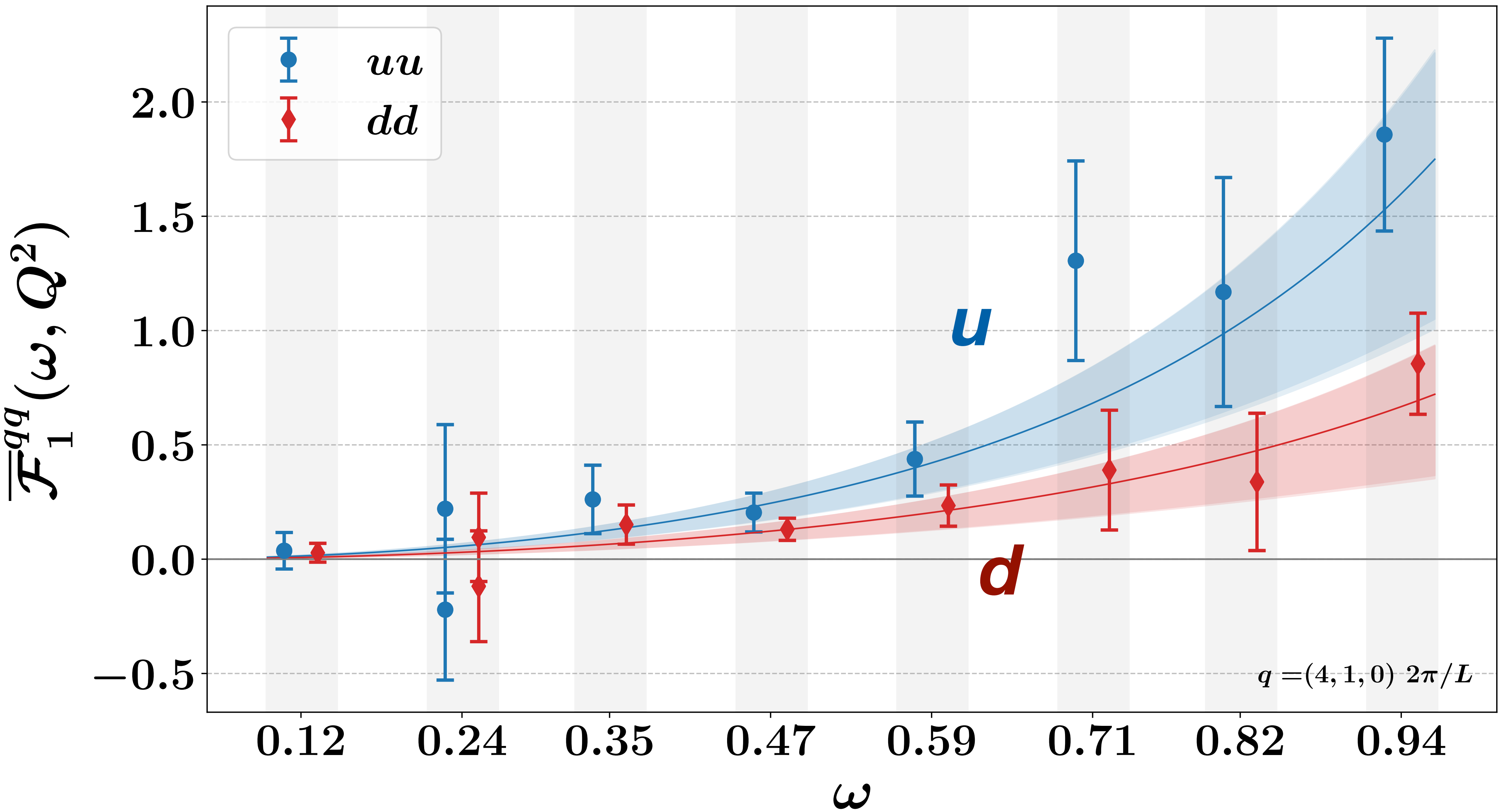


$$\omega = \frac{2p\cdot q}{Q^2} = \frac{2\mathbf{p}\cdot\mathbf{q}}{\mathbf{q}^2}$$

$$\mathbf{q} = \frac{2\pi}{L}(3,5,0)$$

$$q_4 = 0$$

Compton



uu
dd
2.0
1.5
1.0
0.5
0.0
−0.5
$\overline{\mathcal{F}}_1^{qq}(\omega, Q^2)$
u
d
$q = (4, 1, 0)\ 2\pi/L$
0.12
0.24
0.35
0.47
0.59
0.71
0.82
0.94
ω

Moments

- Recall dispersion integral:

$$\overline{\mathcal{F}}_1(\omega, Q^2) = 2\omega^2 \int_0^1 dx \, \frac{2xF_1(x, Q^2)}{1-(x\omega)^2} = 2\sum_{n=1}^{\infty} \omega^{2n} M_{2n}^{(1)}(Q^2)$$

Moments

$$M_{2n}^{(1)}(Q^2) = 2\int_0^1 dx\, x^{2n-1} F_1(x, Q^2)$$

- Positivity constraint:

$$M_2 \geq M_4 \geq M_6 \geq M_8 \geq M_{10} \geq \ldots > 0$$

- Use Bayesian fit enforcing monotonicity of moments

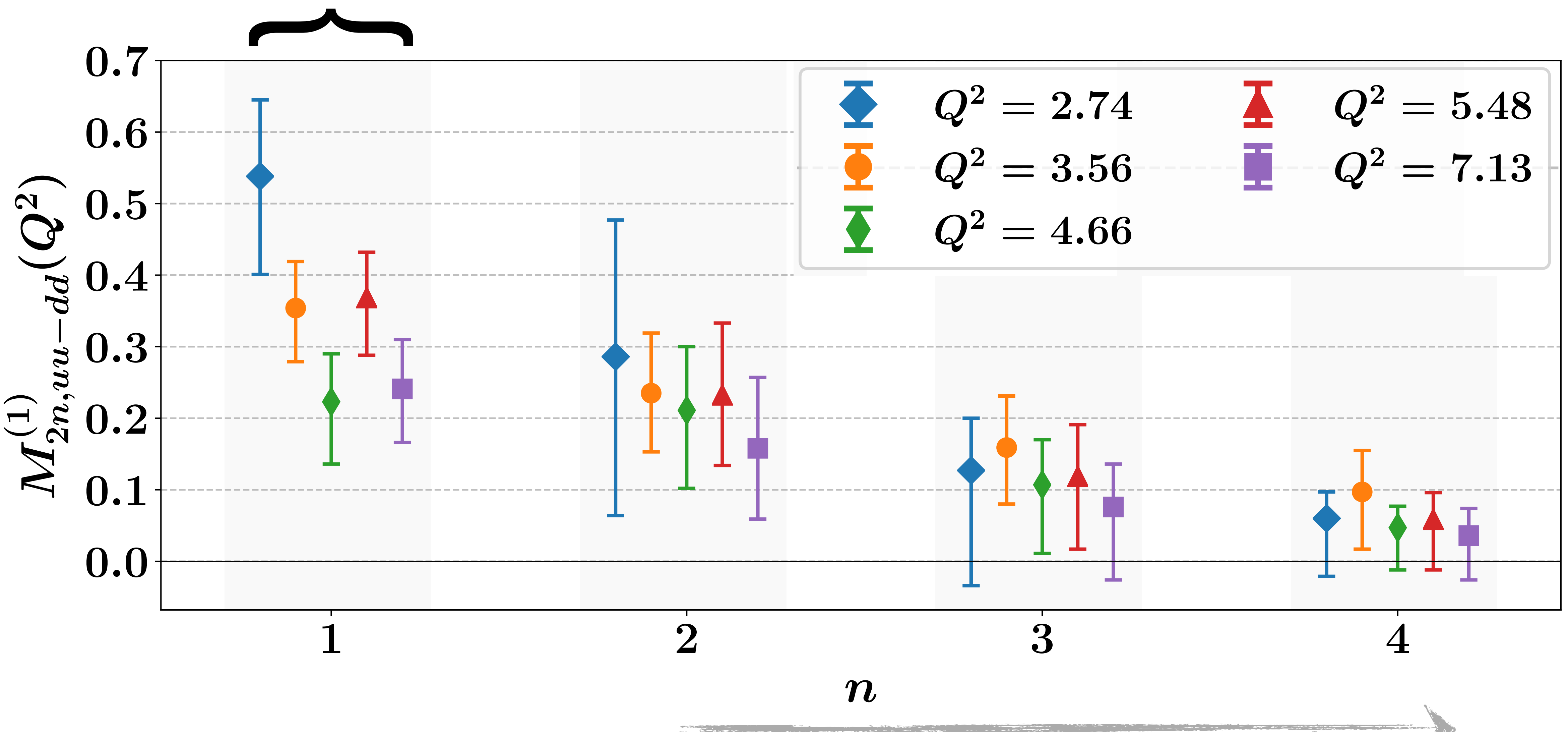
Priors: $M_{2n+2} \in [0, M_{2n}]$ (uniform sampling)

low moments insensitive to truncation order

Low moments

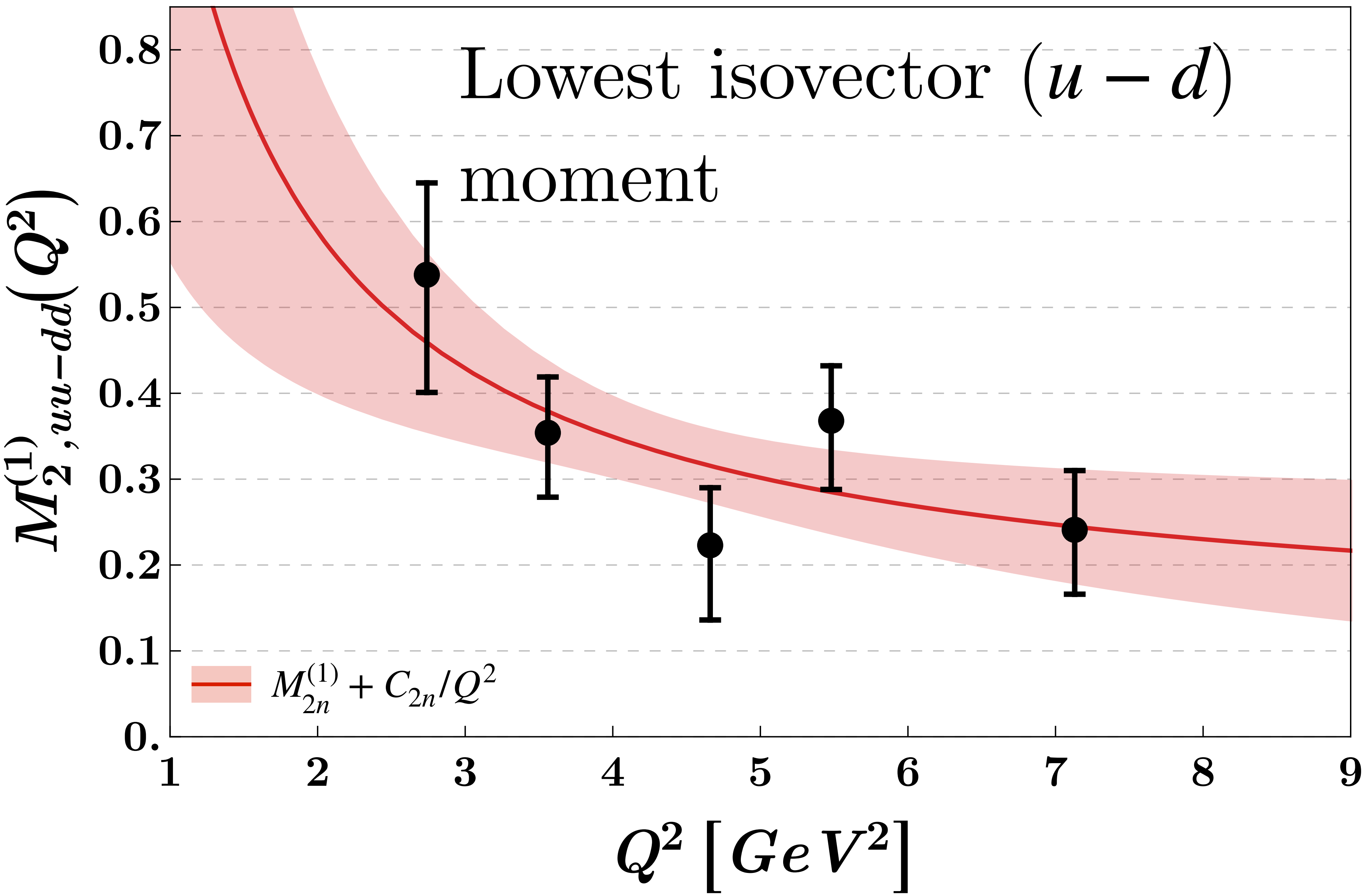
different Q^2

Isovector: *uu–dd*



higher moments

A hint of power

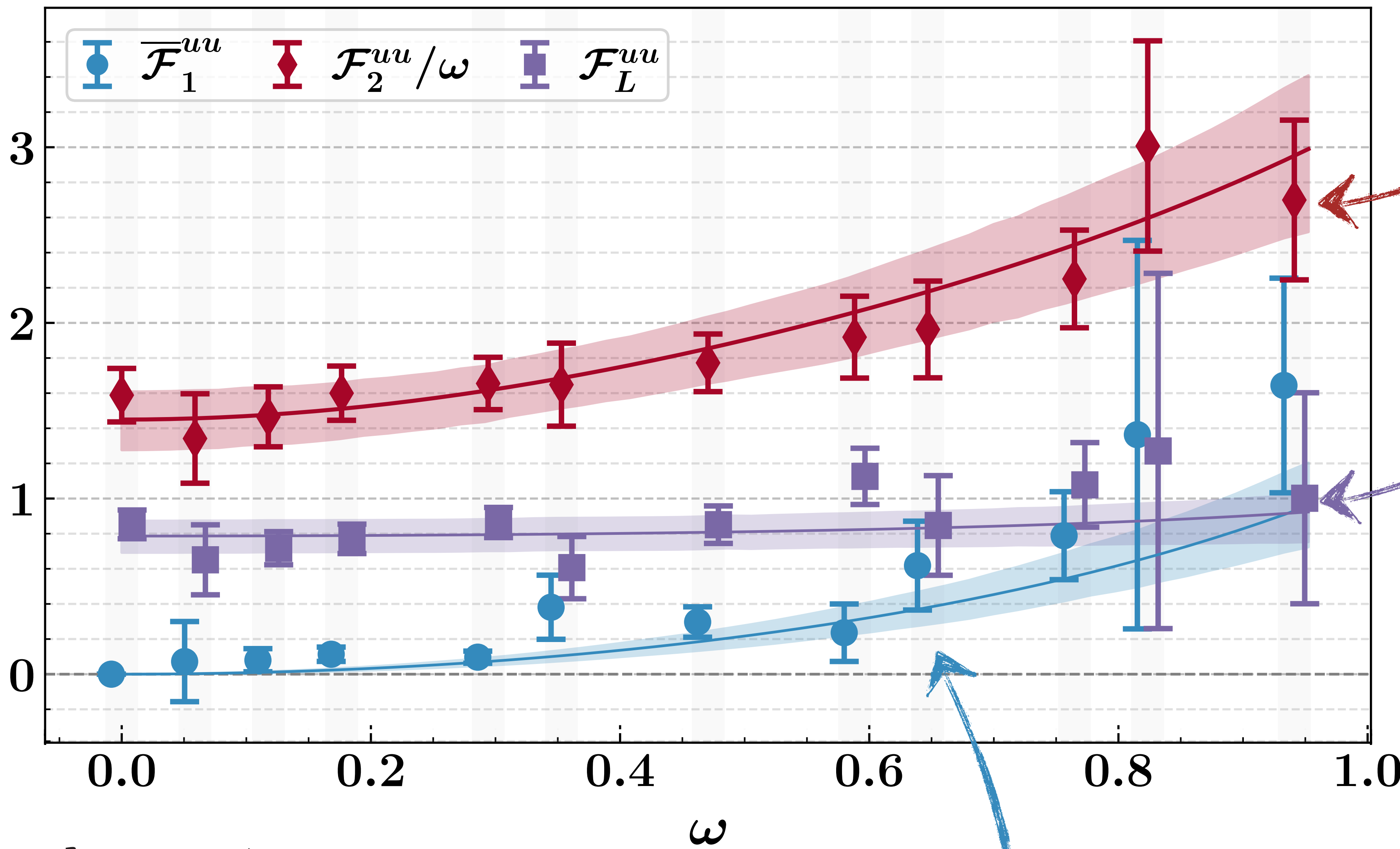


almost a power correction in (lowest moment of) $\mathscr{F}_1$

$\mathscr{F}_2$ and the longitudinal structure function

Can, RDY *et al*. arXiv:2209:04141

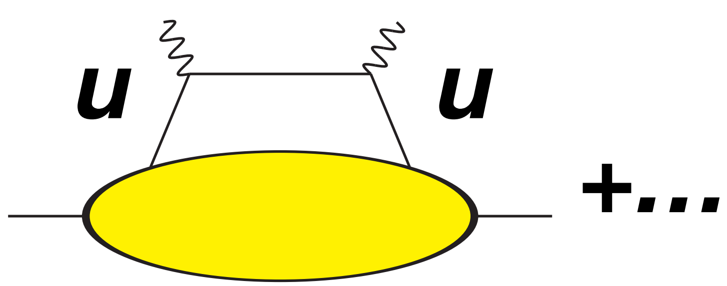
Compton structure functions



$\mathcal{F}_2(\omega, Q^2)$

$\overline{\mathcal{F}}_L(\omega, Q^2)$

$\overline{\mathcal{F}}_1(\omega, Q^2)$



$$\overline{\mathcal{F}}_L(\omega, Q^2) = -\overline{\mathcal{F}}_1(\omega, Q^2) + \left(\frac{\omega}{2} + \frac{2M_N^2}{\omega Q^2}\right) \mathcal{F}_2(\omega, Q^2)$$

Longitudinal Compton amplitude

Moments: Simultaneous fits

- Dispersion relation for FL:

$$\overline{\mathcal{F}}_L(\omega, Q^2) = \frac{8M_N^2}{Q^2}\int_0^1 dx\, F_2(x,Q^2) + 2\omega^2 \int_0^1 dx \frac{F_L(x,Q^2)}{1 - x^2\omega^2 - i\epsilon}$$

- Parameterise in terms of moments of F1 and FL

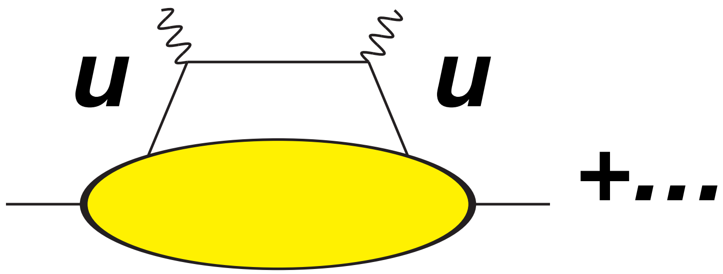
$$M_2^{(1)}, M_4^{(1)}, M_6^{(1)}, \ldots,$$
$$M_0^{(L)}, M_2^{(L)}, M_4^{(L)}, \ldots$$

independently positive definite

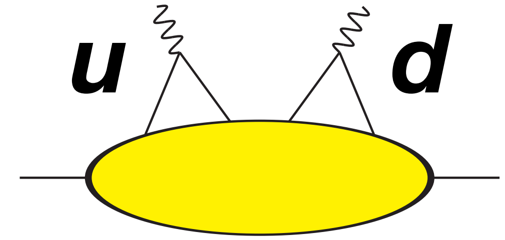
- Fit to two independent amplitudes F1 and F2

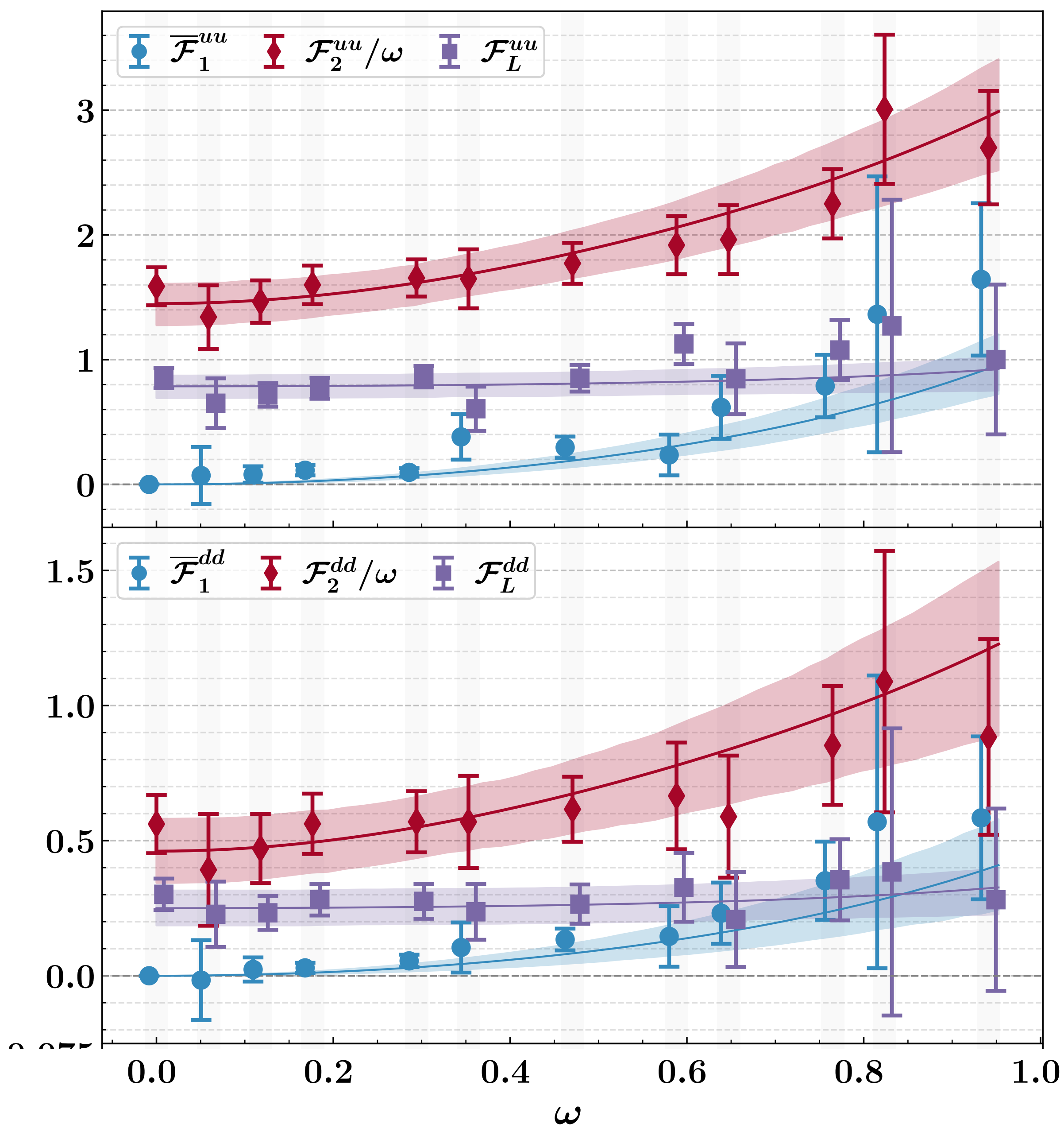
$$\overline{\mathcal{F}}_1(\omega, Q^2) = 2\sum_{n=1} \omega^{2n} M_{2n}^{(1)}(Q^2)$$

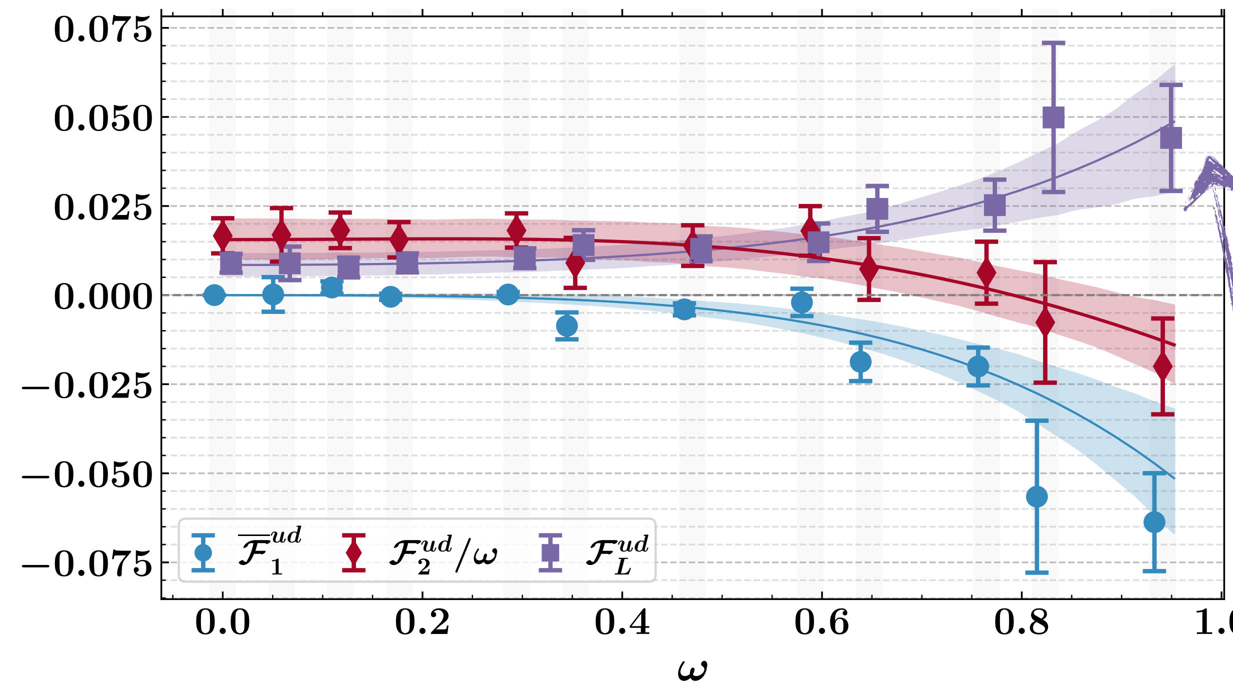
$$\frac{\mathcal{F}_2(\omega, Q^2)}{\omega} = \frac{\tau}{1+\tau\omega^2}\sum_{n=0} 4\omega^{2n}\left[M_{2n}^{(1)} + M_{2n}^{(L)}\right]$$



$Q^2 = 4.9\,\mathrm{GeV}^2$



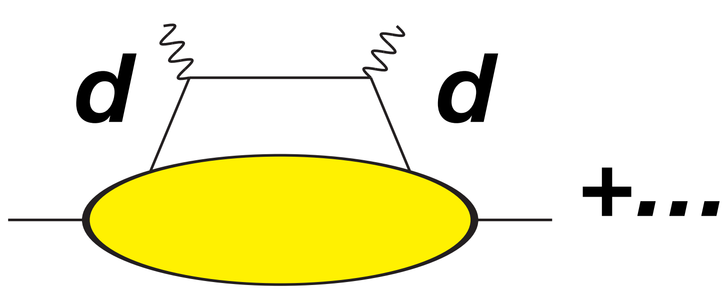




flavour-interference structure functions

* *small* in magnitude

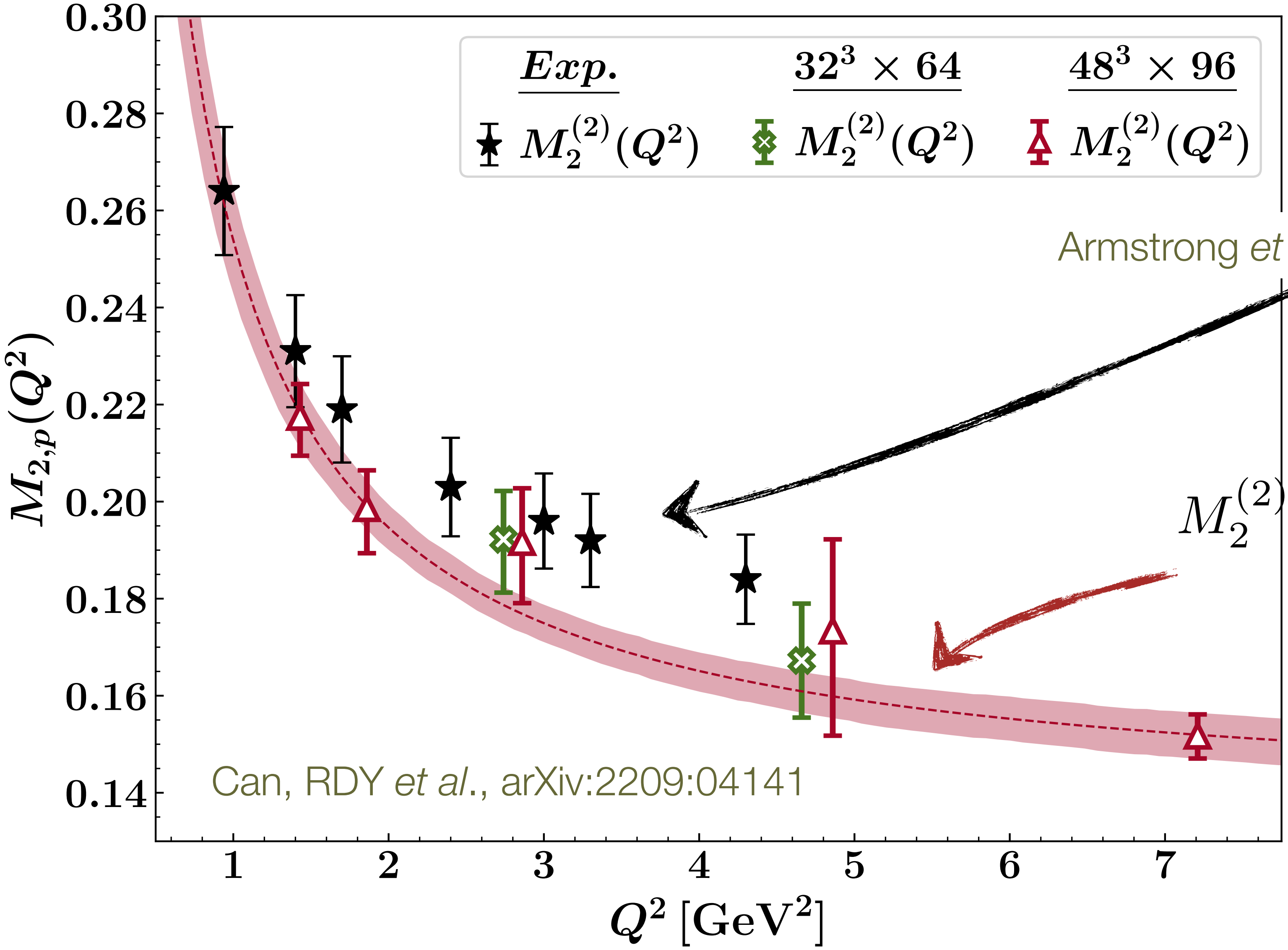
* non-trivial signal for longitudinal structure



Lowest moment of F2 (proton)

Comparison with experiment

48^3x96, 2+1 flavour
$a = 0.068$ fm
$m_\pi \sim 420\,\text{MeV}$

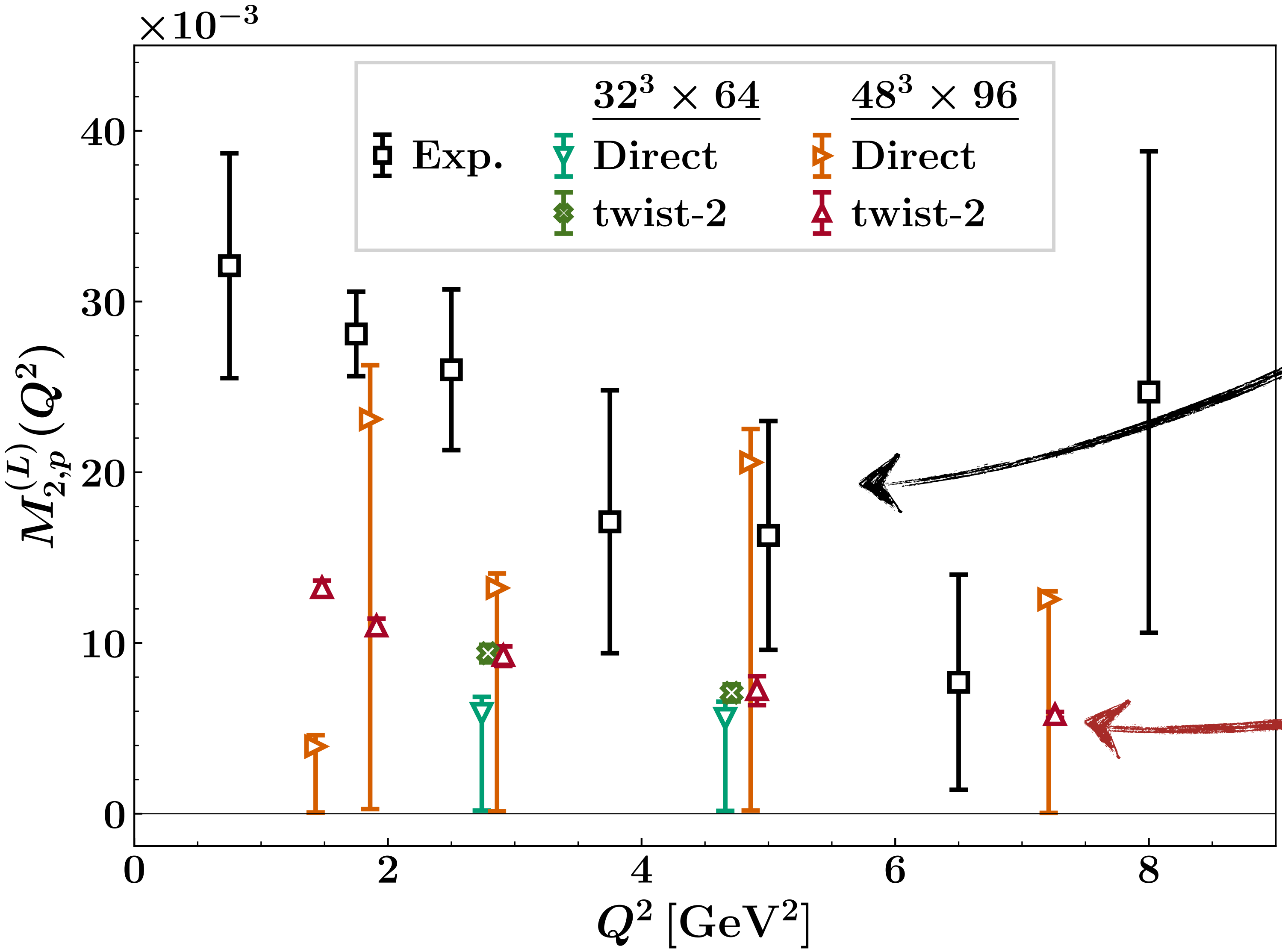


$$M_2^{(2)}(Q^2) = M_2^{(2)} + \frac{C_2}{Q^2}$$

Clear evidence for power corrections!

Compatible with phenomenological trend

Longitudinal moments

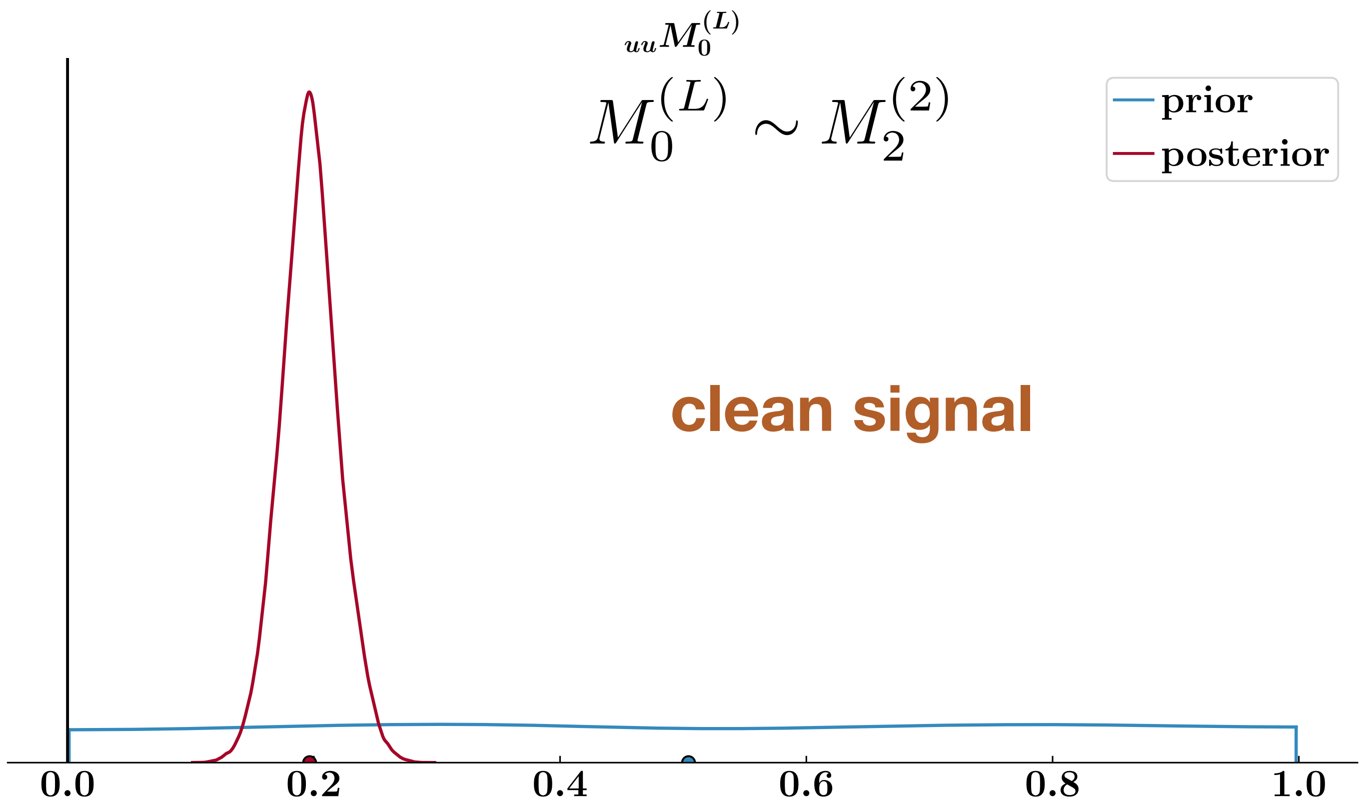


Monaghan *et al.*, PRL110,152002(2013)

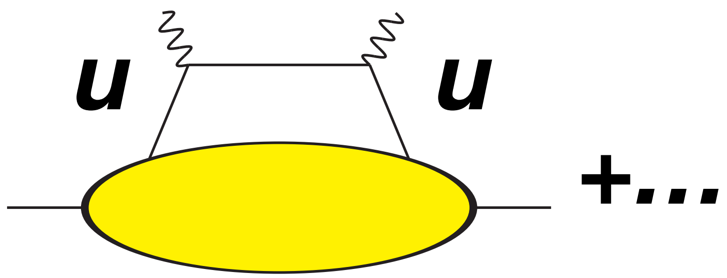
Leading-twist part

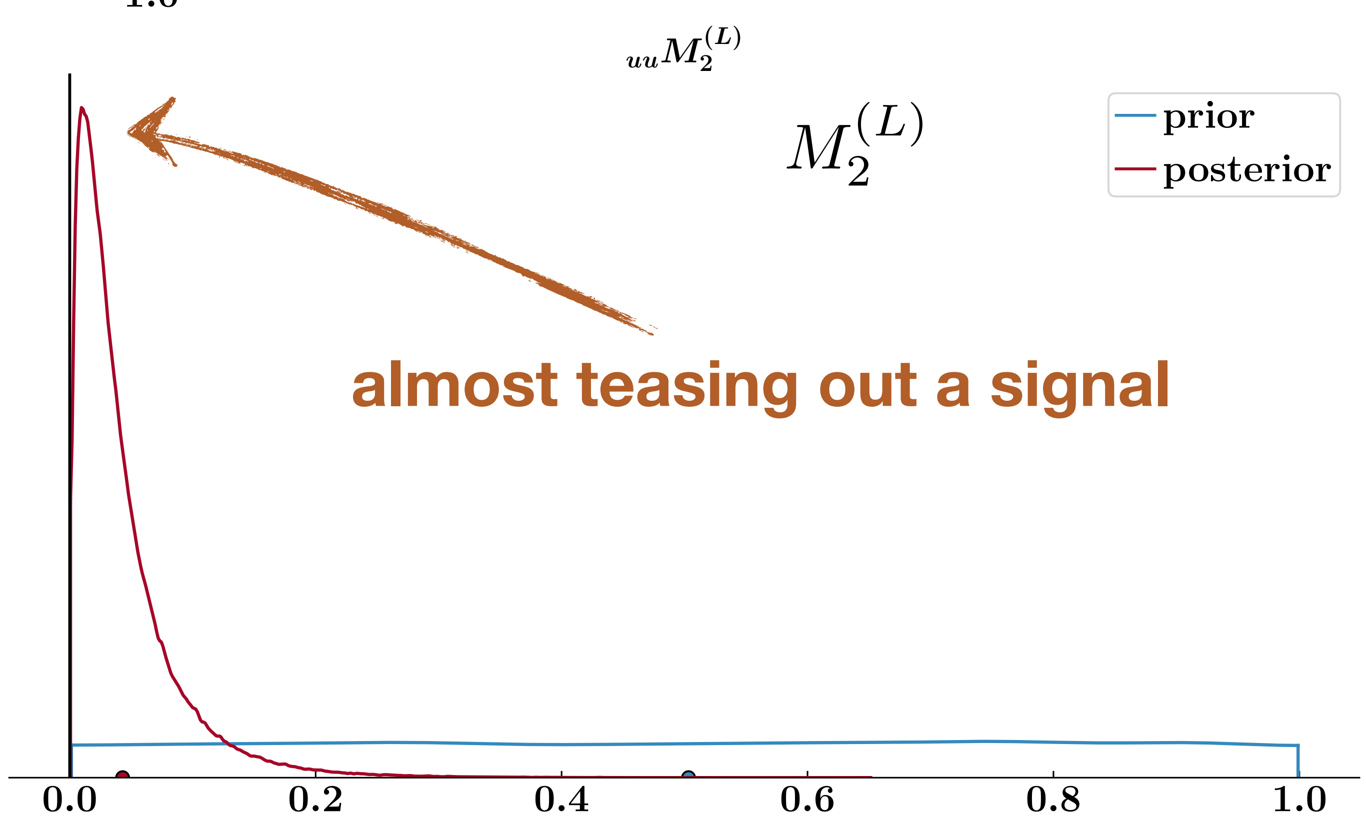
$$M_{2,p}^{(L),\text{twist-2}} = \frac{4}{9\pi}\alpha_s M_{2,p}^{(2),\text{twist-2}}$$

Moment posteriors — longitudinal SF



clean signal





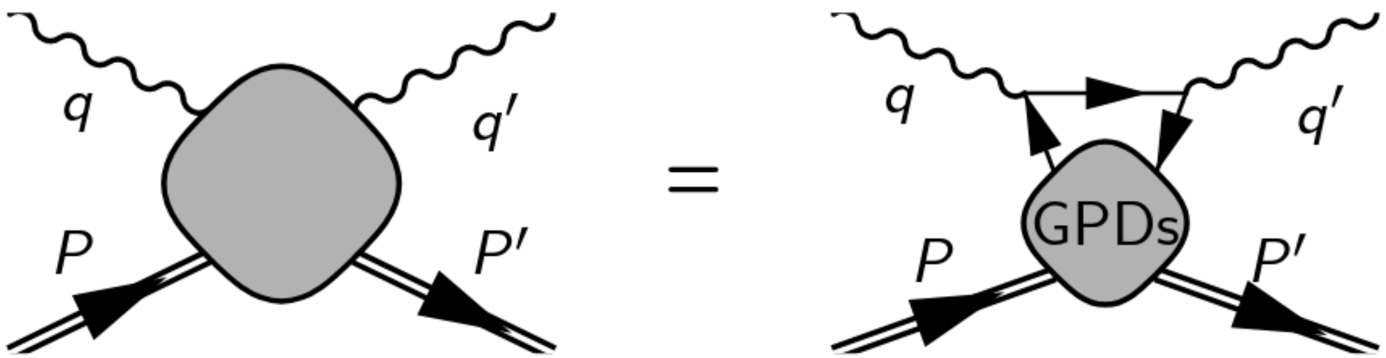
almost teasing out a signal

Off-forward Compton and GPDs

Hannaford-Gunn, RDY *et al*. PRD(2022)

Off-forward Compton amplitude

$$T^{\mu\nu} \equiv i \int d^4 z e^{\frac{i}{2}(q+q')\cdot z} \langle P' | T\{ j^\mu(z/2) j^\nu(-z/2) \} | P \rangle$$



$$+ \frac{\Lambda_{\mathrm{QCD}}^2}{Q^2} + S_1$$

power corrections

subtraction "constant"

$S_1(t, Q^2)$

$$T^{\mu\nu}(P, q, q') = \sum_{i=1}^{18} \mathcal{A}_i(\overline{\omega}, \theta, t, \overline{Q}^2) L_i^{\mu\nu}$$

18 tensor structures

$$\overline{P} = \tfrac{1}{2}(P + P'), \quad \overline{q} = \tfrac{1}{2}(q + q'), \quad \Delta = P' - P$$

$$t = \Delta^2, \quad \overline{Q}^2 = -\overline{q}^2,$$

$$\overline{\omega} = \frac{2\overline{P}\cdot\overline{q}}{\overline{Q}^2}, \quad \theta = -\frac{\Delta\cdot\overline{q}}{\overline{Q}^2}$$

Tensor decomposition

$$\bar{T}_{\mu\nu} = \frac{1}{2\bar{P}\cdot\bar{q}}\Big[-\Big(h\cdot\bar{q}\mathcal{H}_1 + e\cdot\bar{q}\mathcal{E}_1\Big)g_{\mu\nu} + \frac{1}{\bar{P}\cdot\bar{q}}\Big(h\cdot\bar{q}\mathcal{H}_2 + e\cdot\bar{q}\mathcal{E}_2\Big)\bar{P}_\mu\bar{P}_\nu + \mathcal{H}_3 h_{\{\mu}\bar{P}_{\nu\}}\Big]$$
$$+\frac{i}{2\bar{P}\cdot\bar{q}}\epsilon_{\mu\nu\rho\kappa}\bar{q}^\rho\Big(\tilde{h}^\kappa\tilde{\mathcal{H}}_1 + \tilde{e}^\kappa\tilde{\mathcal{E}}_1\Big) + \frac{i}{2(\bar{P}\cdot\bar{q})^2}\epsilon_{\mu\nu\rho\kappa}\bar{q}^\rho\Big[(\bar{P}\cdot\bar{q}\tilde{h}^\kappa - \tilde{h}\cdot\bar{q}\bar{P}^\kappa)\tilde{\mathcal{H}}_2 + (\bar{P}\cdot\bar{q}\tilde{e}^\kappa - \tilde{e}\cdot\bar{q}\bar{P}^\kappa)\tilde{\mathcal{E}}_2\Big]$$
$$+\Big(\bar{P}_\mu q'_\nu + \bar{P}_\nu q_\mu\Big)\Big(h\cdot\bar{q}\mathcal{K}_1 + e\cdot\bar{q}\mathcal{K}_2\Big) + \Big(\bar{P}_\mu q'_\nu - \bar{P}_\nu q_\mu\Big)\Big(h\cdot\bar{q}\mathcal{K}_3 + e\cdot\bar{q}\mathcal{K}_4\Big) + q_\mu q'_\nu(h\cdot\bar{q} - e\cdot\bar{q})\mathcal{K}_5$$
$$+h_{[\mu}\bar{P}_{\nu]}\mathcal{K}_6 + \Big(h_\mu q'_\nu + h_\nu q_\mu\Big)\mathcal{K}_7 + \Big(h_\mu q'_\nu - h_\nu q_\mu\Big)\mathcal{K}_8 + \bar{P}_{\{\mu}\bar{u}(P')i\sigma_{\nu\}\alpha}u(P)\bar{q}^\alpha\mathcal{K}_9,$$

$\mathcal{K}$ vanish at leading twist

Diehl, EPJC(2001)
Belitsky, Müller, Kirchner, NPB(2002)
Belitsky, Müller, Ji, NPB(2014)

$$h^\mu = \bar{u}'\gamma^\mu u, \quad e^\mu = \bar{u}'\frac{i\sigma^{\mu\alpha}\Delta_\alpha}{2m_N}u,$$
$$\tilde{h}^\mu = \bar{u}'\gamma^\mu\gamma_5 u, \quad \tilde{e}^\mu = \frac{\Delta^\mu}{2m_N}\bar{u}'\gamma_5 u.$$

simple mapping to forward limit

$$\mathcal{H}_1 \xrightarrow{t\to 0} \mathcal{F}_1, \quad \mathcal{H}_2 + \mathcal{H}_3 \xrightarrow{t\to 0} \mathcal{F}_2,$$
$$\tilde{\mathcal{H}}_1 \xrightarrow{t\to 0} \tilde{g}_1, \quad \tilde{\mathcal{H}}_2 \xrightarrow{t\to 0} \tilde{g}_2,$$

Off-forward Compton amplitude

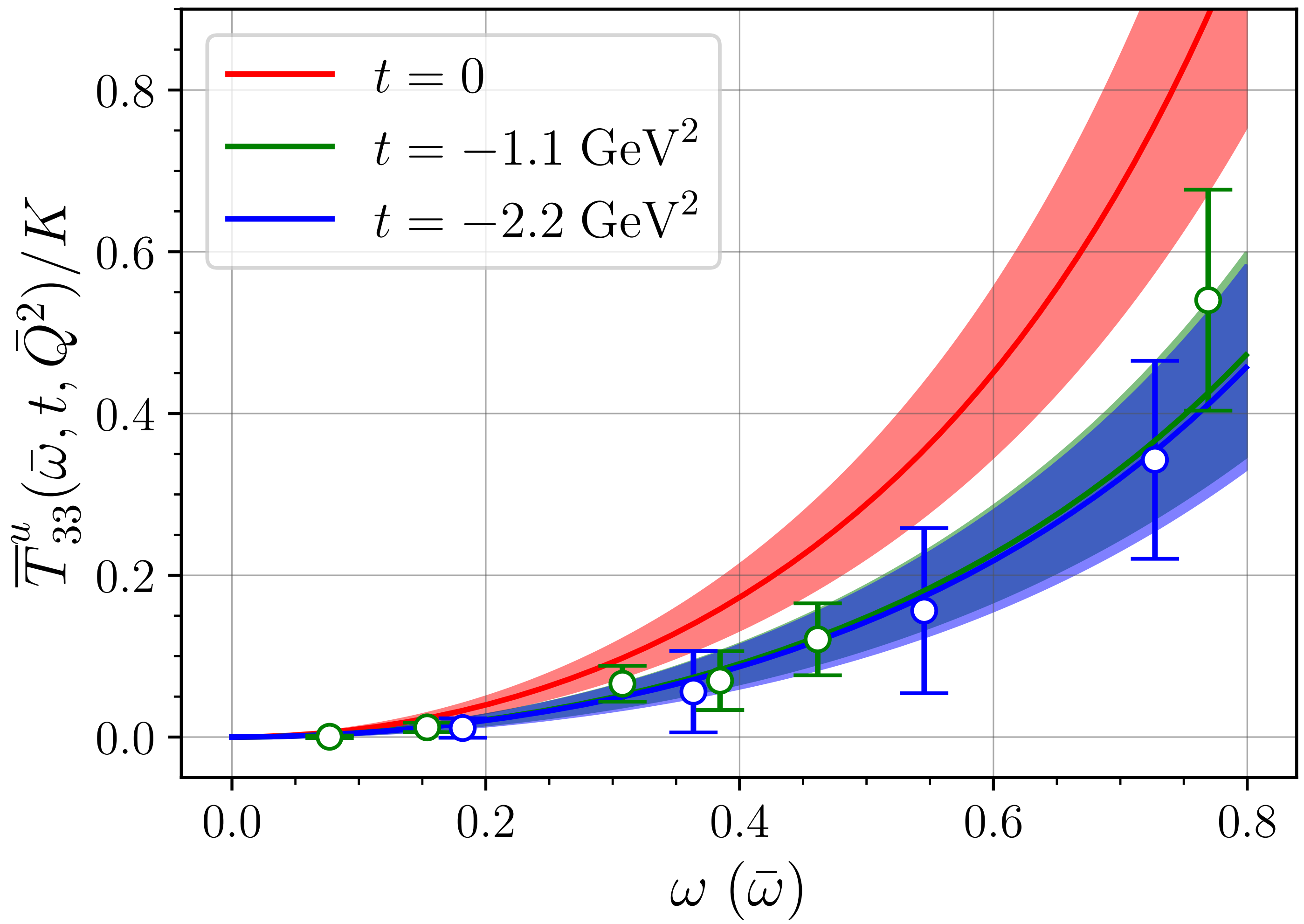
$$T_{\mu\nu} = \frac{1}{2\bar{P}\cdot\bar{q}}\left[-\left(h\cdot\bar{q}\mathcal{H}_1 + e\cdot\bar{q}\mathcal{E}_1\right)g_{\mu\nu} + \frac{1}{\bar{P}\cdot\bar{q}}\left(h\cdot\bar{q}\mathcal{H}_2 + e\cdot\bar{q}\mathcal{E}_2\right)\bar{P}_\mu\bar{P}_\nu + \mathcal{H}_3 h_{\{\mu}\bar{P}_{\nu\}}\right] + \ldots$$

Expand CFF in moments

$$\mathrm{CFF}(\bar{\omega}, t, \bar{Q}^2) = 2\sum_n \bar{\omega}^n M_n(t, \bar{Q}^2)$$

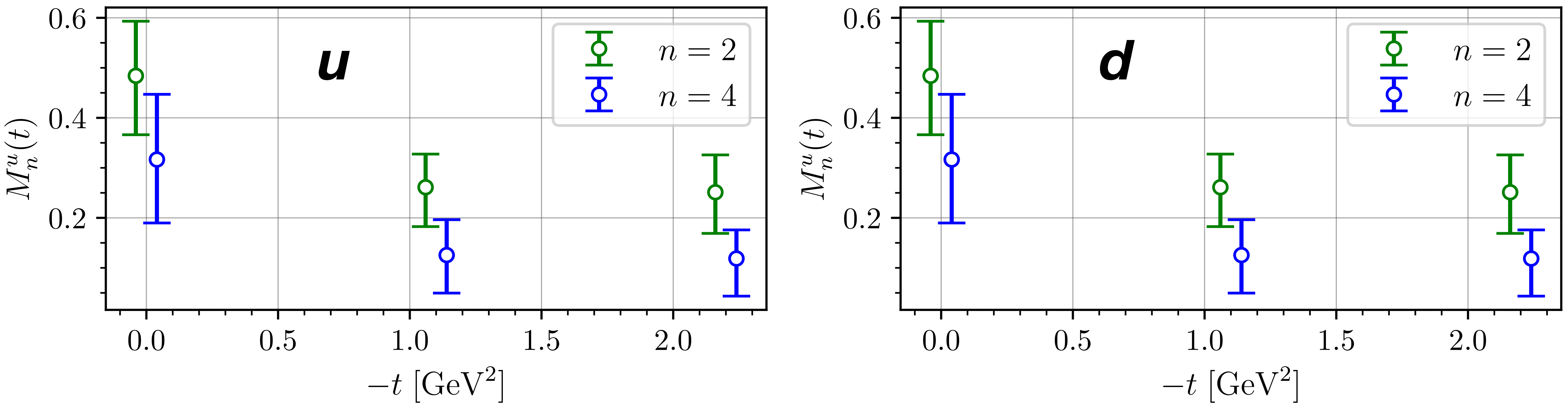
Moments match onto Mellin moments of GPDs

$$M_n(t, \bar{Q}^2) \overset{\bar{Q}^2\to\infty}{\longrightarrow} \int_{-1}^{1} dx x^{n-1}\mathrm{GPD}$$



assume off-forward “Callan-Gross” relations

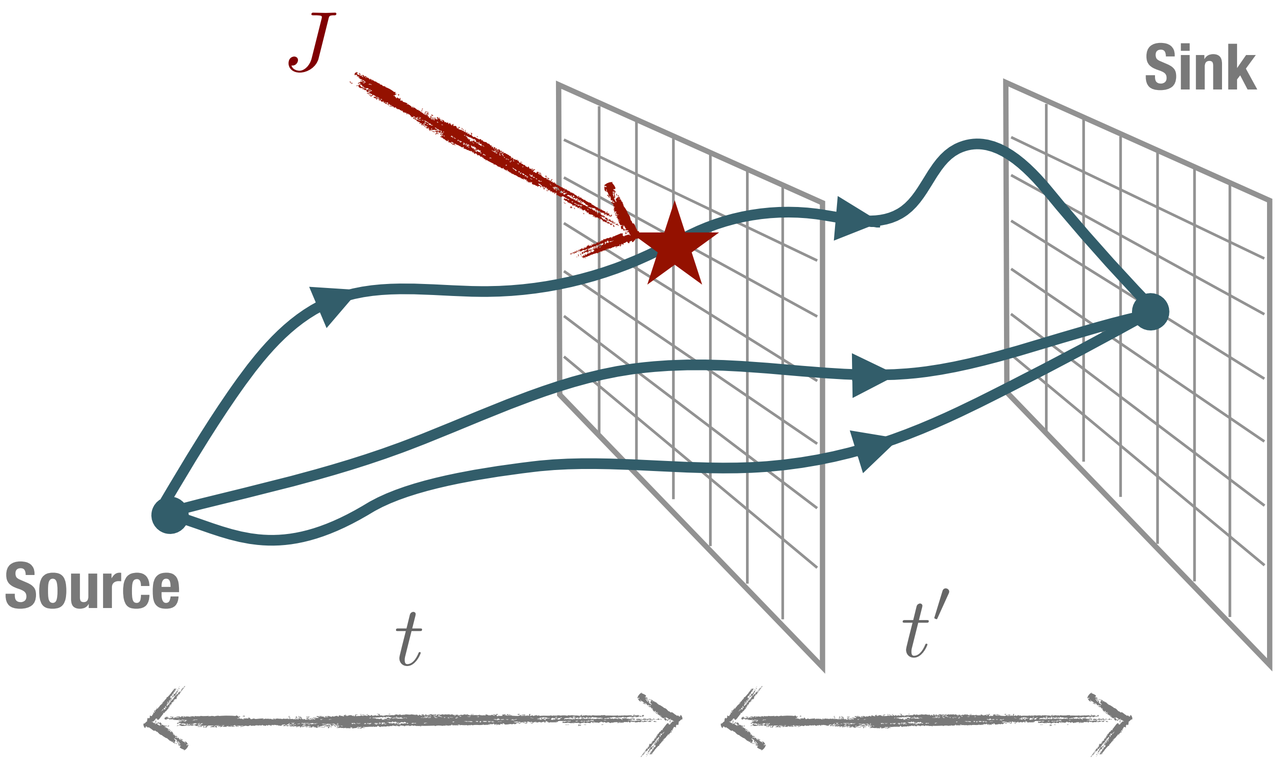
t-dependence of leading moments



$$\sum_{n=2,4,6}^{\infty} \bar{\omega}^n \left[A_{n,0}^q(t) + \frac{t}{4m_N(E_N + m_N)} B_{n,0}^q(t)\right],$$

Resolving t-dependence of leading and sub-leading moment!

Feynman-Hellmann in lattice QCD



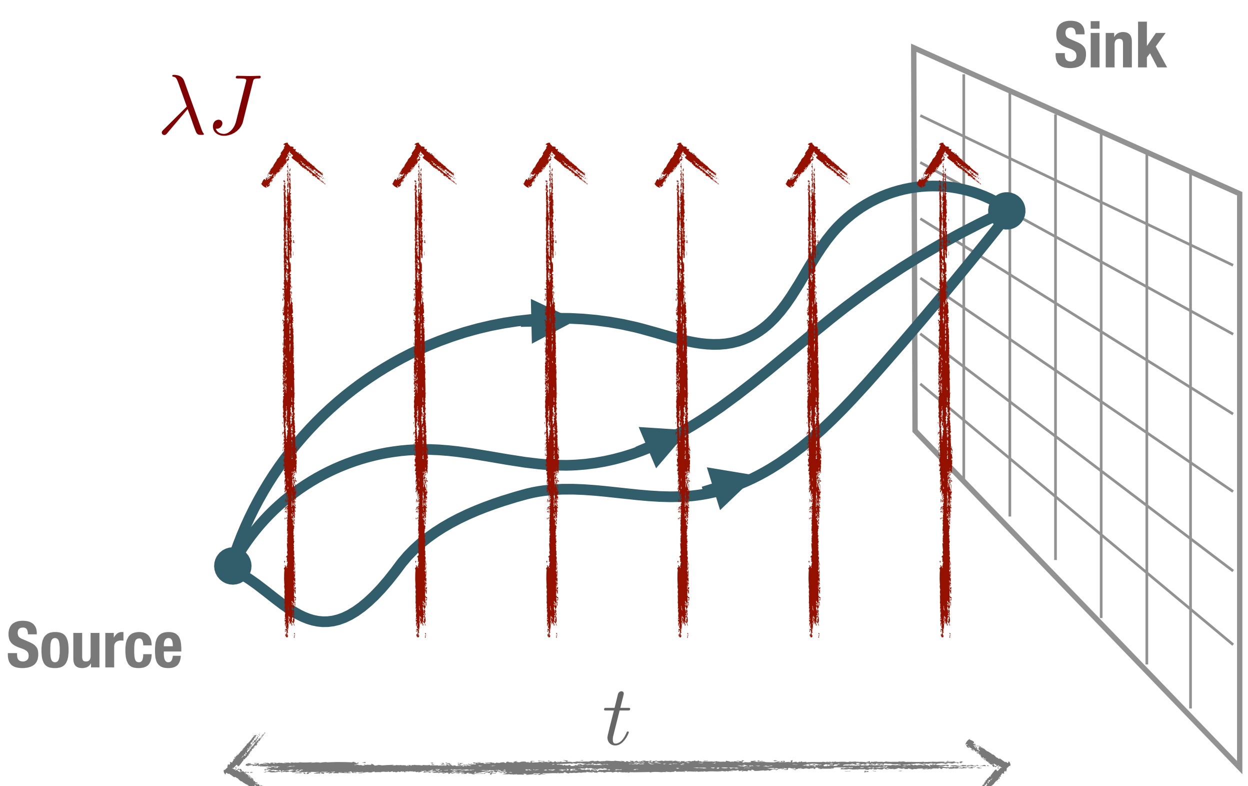
3-pt functions

$$t, t' \gg \frac{1}{\Delta E}$$

energy gap to lowest excitation

$$\frac{\langle C_3(t,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N'|J|N\rangle$$

Matrix elements on the lattice



Feynman–Hellmann

$$t \gg \frac{1}{\Delta E}$$

$$\left.\frac{\partial E}{\partial \lambda}\right|_{\lambda\to 0} \propto \langle N|J|N\rangle$$

Matrix elements from Feynman–Hellmann

- Feynman–Hellmann in quantum mechanics:

$$\frac{dE_n}{d\lambda} = \langle n|\frac{\partial H}{\partial \lambda}|n\rangle$$

  - matrix elements of the derivative of the Hamiltonian determined by derivative of corresponding energy eigenvalues

- Lattice QCD: evaluate energy shifts with respect to weak external fields

- Modify action with external field:

$$S \to S + \lambda \int \mathrm{d}^4x\, \mathcal{O}(x)$$

real parameter

local operator, e.g. $\bar{q}(x)\gamma_5\gamma_3 q(x)$

- Calculation of matrix element hadron spectroscopy [2-pt functions only]

$$\frac{\partial E_H(\lambda)}{\partial \lambda} = \frac{1}{2E_H(\lambda)}\langle H|\mathcal{O}|H\rangle$$

Spin content [connected]

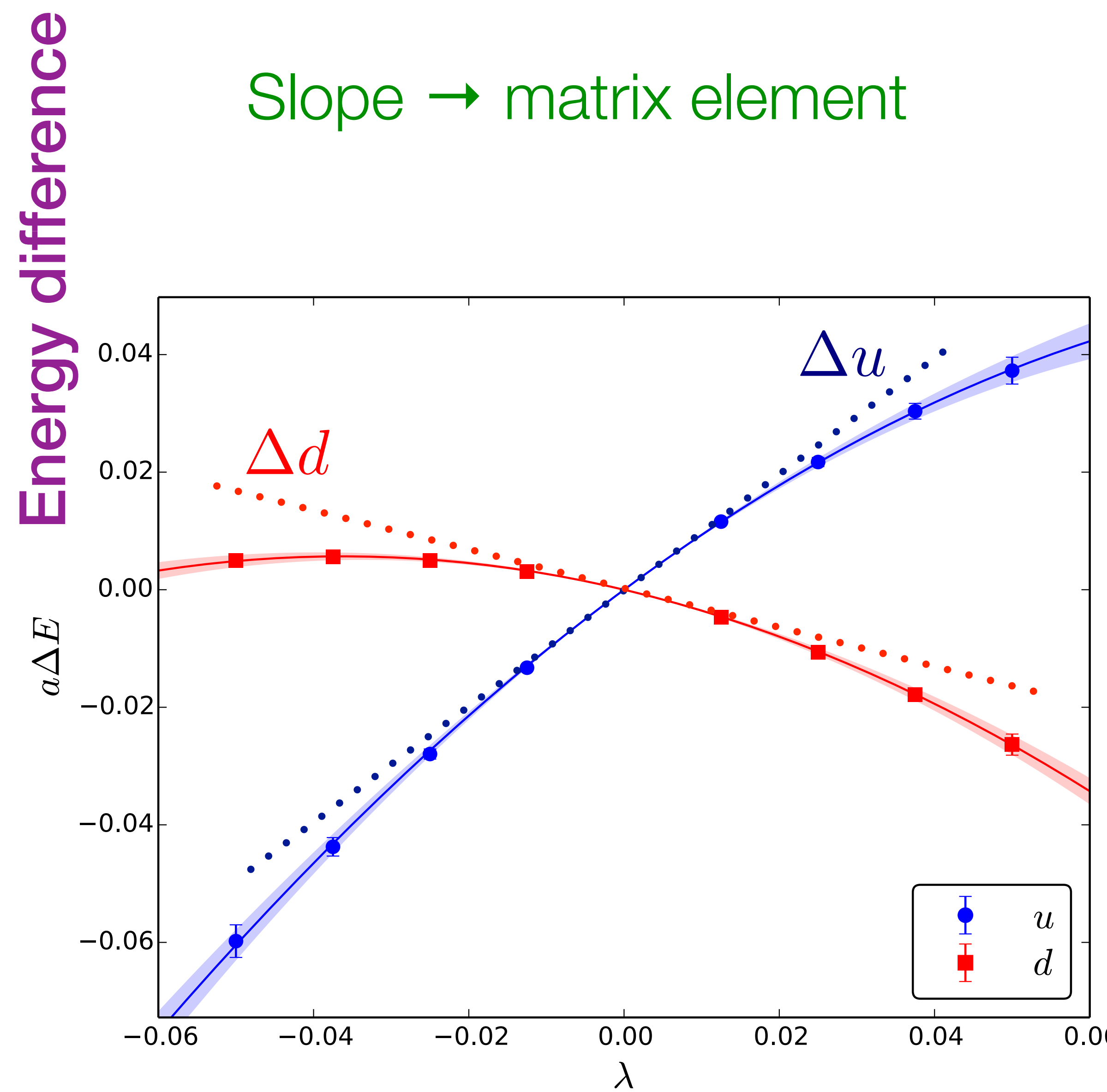
- Modify action

$$S \to S + \lambda \sum_x \bar{q}(x) i\gamma_5\gamma_3 q(x)$$

- Nucleon energy shift isolates spin content

$$\frac{\partial E_N(\lambda)}{\partial \lambda} = \frac{1}{2M_N}\langle N|\bar{q}i\gamma_5\gamma_3 q|N\rangle$$
$$= \Delta q$$

Slope → matrix element

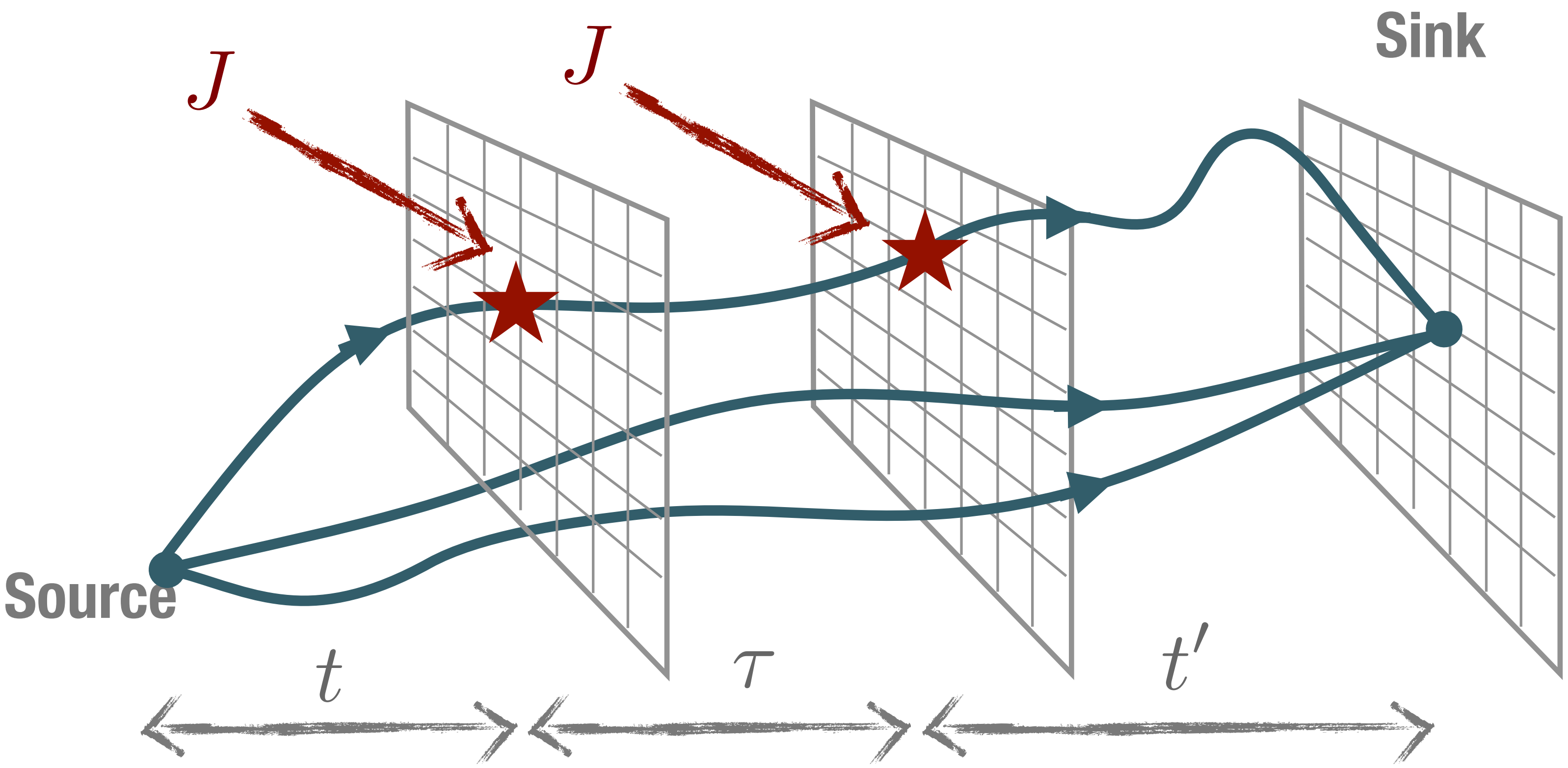


Strength of external field

[Chambers *et al*. PRD(2014)]

3-pt function → 2-pt function

4-pt functions



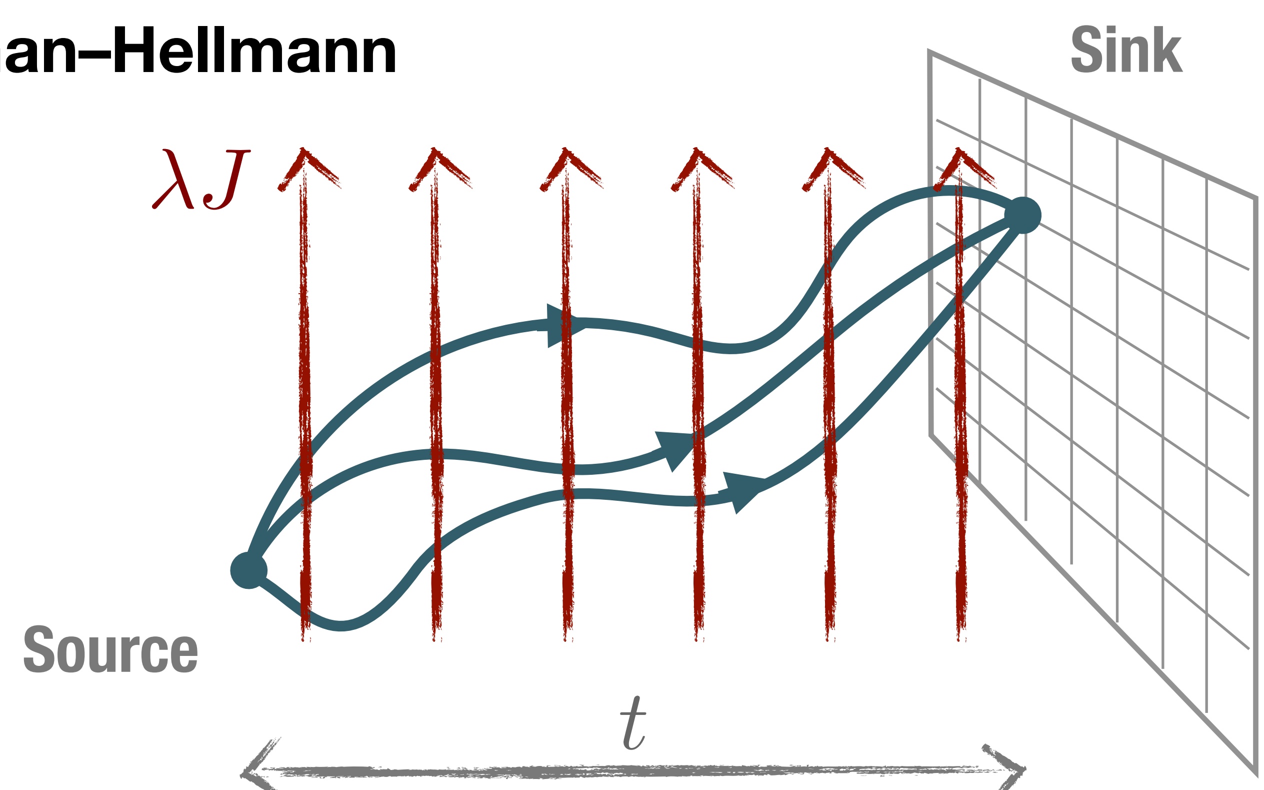
$$t, t' \gg \frac{1}{\Delta E}$$

$$\frac{\langle C_4(t,\tau,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N'|J(\tau_E)J|N\rangle$$

$$\int_0^\infty d\tau_E \to \langle N|JJ|N\rangle$$

Compton on the lattice

Feynman–Hellmann



$$t \gg \frac{1}{\Delta E}$$

$$\left.\frac{\partial^2 E}{\partial \lambda^2}\right|_{\lambda\to 0} \propto \langle N|JJ|N\rangle$$

Feynman–Hellman (2nd order)

- Field theory version of 2nd order perturbation theory:

$$E = E_0 + \lambda\langle N|V|N\rangle + \lambda^2 \sum_{X\neq N} \frac{\langle N|V|X\rangle\langle X|V|N\rangle}{E_0 - E_X} + \ldots$$

Only get a linear term for elastic case $\boldsymbol{\omega}=1$

$E_0 < E_X$
Intermediate states cannot go on-shell for $\boldsymbol{\omega}<1$

- Final result. We study second-order perturbation on the lattice

$$\frac{\partial^2 E_{\mathbf{p}}}{\partial\lambda^2} = -\frac{1}{2E_{\mathbf{p}}}\int d^4\xi \left(e^{iq.\xi} + e^{-iq.\xi}\right)\langle\mathbf{p}|\mathrm{T}J(\xi)J(0)|\mathbf{p}\rangle$$

see Can, RDY *et al*. PRD(2020)

Perspective

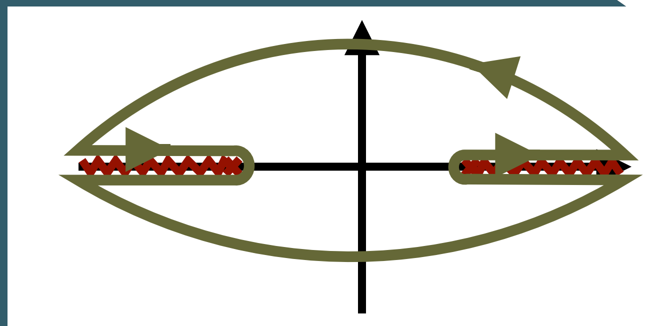
Compton on the lattice provides clean determination of integrated quantities

$$\frac{\mathcal{F}_2(\omega, Q^2)}{\omega} = 4\int_0^1 dx\, \frac{F_2(x, Q^2)}{1-(x\omega)^2}$$

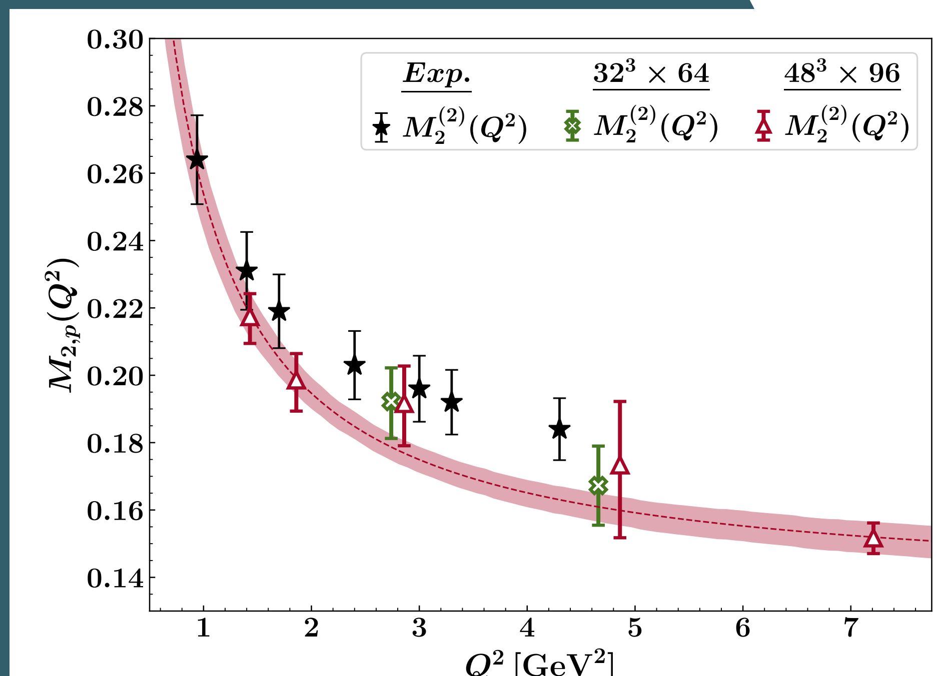
Moments provide useful benchmarking tool

Ultimately, would wish to use Compton constraints directly with phenomenological analyses

Recap

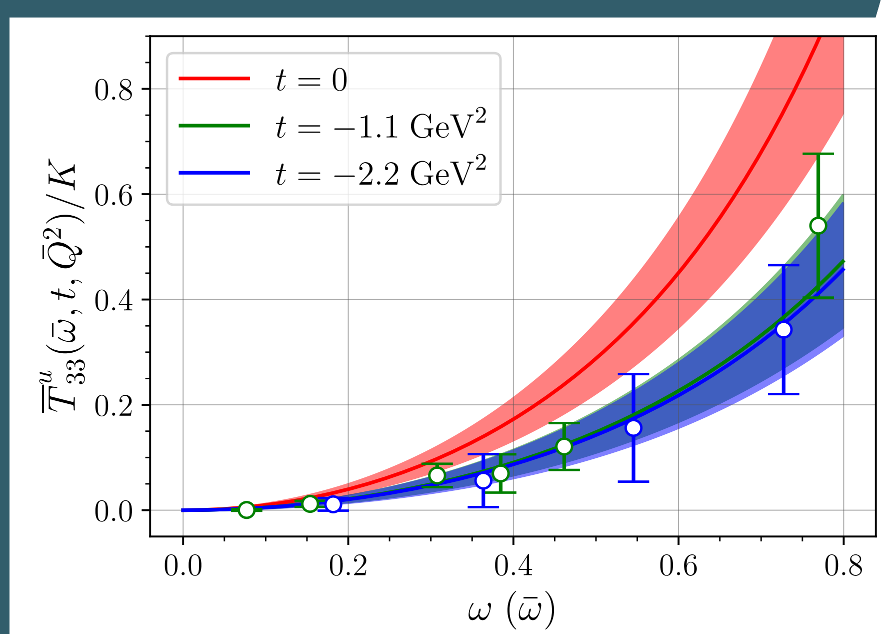


Compton in unphysical region is spacelike
⇒ can study on the lattice



First look at the Q^2 dependence of leading moments

Emerging signal for longitudinal structure



Can extend to off-forward Compton (GPDs),
… spin coming soon