Effective Field Theory of Nonrelativistic Fermi Gases

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Pionless Effective Field Theory

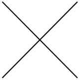
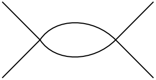
- The most general local Lagrangian satisfying Galilean and time-reversal invariance

$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{\overrightarrow{\nabla}^2}{2m} \right) \psi - \frac{C_0}{2}(\psi^\dagger \psi)^2 + \dots$$
$$+ \frac{C_2'}{8}(\psi \nabla_i \psi)^\dagger (\psi \nabla_i \psi) + \dots - \frac{D_0}{6}(\psi^\dagger \psi)^3 + \dots$$

- $\nabla = \overleftarrow{\nabla} - \overrightarrow{\nabla}$ is the Galilean invariant derivative

Two-Body Scattering

- Resum two-body partial waves to obtain partial wave amplitudes

$i\mathcal{T}_0 =$  $+$  $+ \cdots = \frac{-i}{\frac{1}{V} - iI_0}$

where

$$V = \sum_{i=0}^{\infty} C_{2i} k^{2i}$$

$$iI_0 = m \left(\frac{\mu}{2}\right)^{\varepsilon} \int \frac{1}{k^2 - q^2 + i\delta} \frac{d^d q}{(2\pi)^d}$$

- Effective range expansion parameterized by iI_0, V

Dimensional Compactification

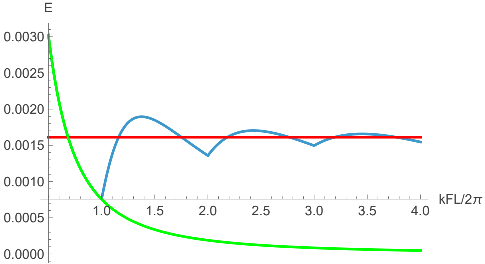
- Atomic physics experiments frequently involve trapping a gas within some confining potential V with characteristic lengths L_i along each dimension.
- Interaction between particles is of finite range ℓ.
- Gases of reduced dimensionality obtained by adjusting the strength of the potential so that $L_i \ll \ell$ along the confined dimensions and $L_i \gg \ell$ along the free dimensions.
- Effects of trapping potential incorporated by loops since

$$iI_0 = G(0,0)$$

- The compactified theory can be related to the free theory by matching the phase shifts to the effective range expansion.

Ground State Energy

- At finite density, non-interacting Fermions build up a Fermi surface due to Pauli exclusion
- EFT facilitates calculation of ground state energy for interacting but dilute Fermions at finite temperature
- Many advantages over model dependent techniques used by AMO theorists



Questions?