# Quantum Hall states for Fractons
## with conserved dipole moment

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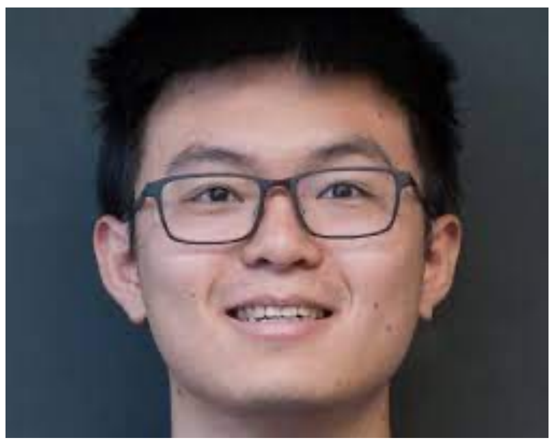
# Acknowledgement



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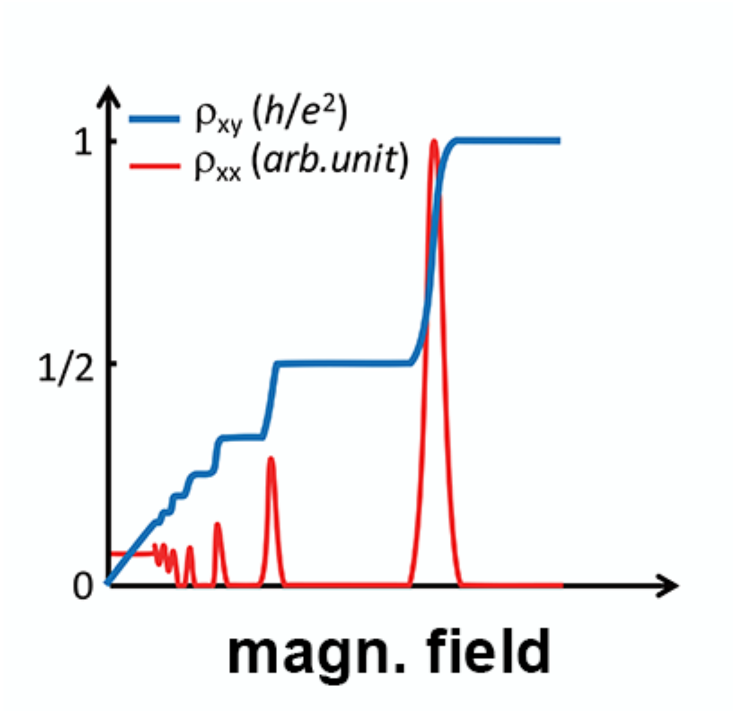
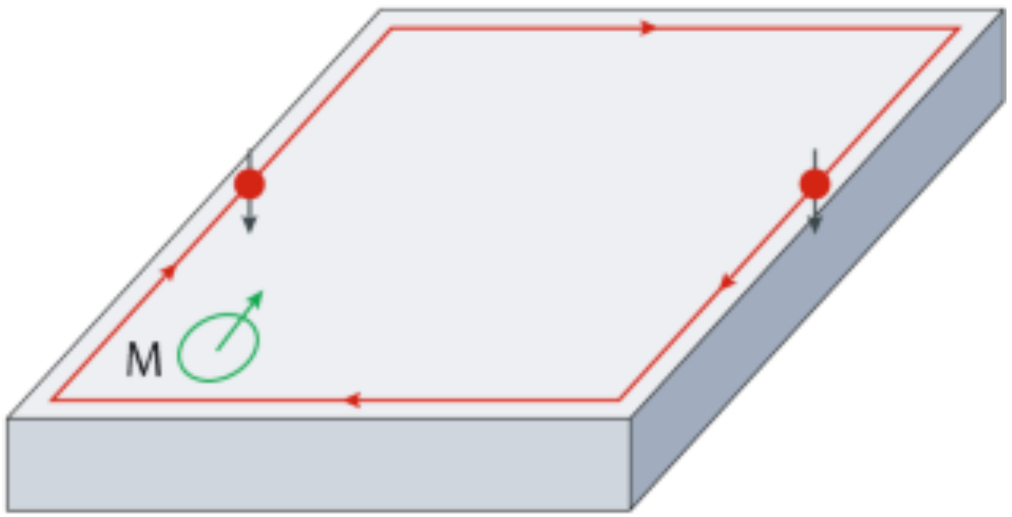
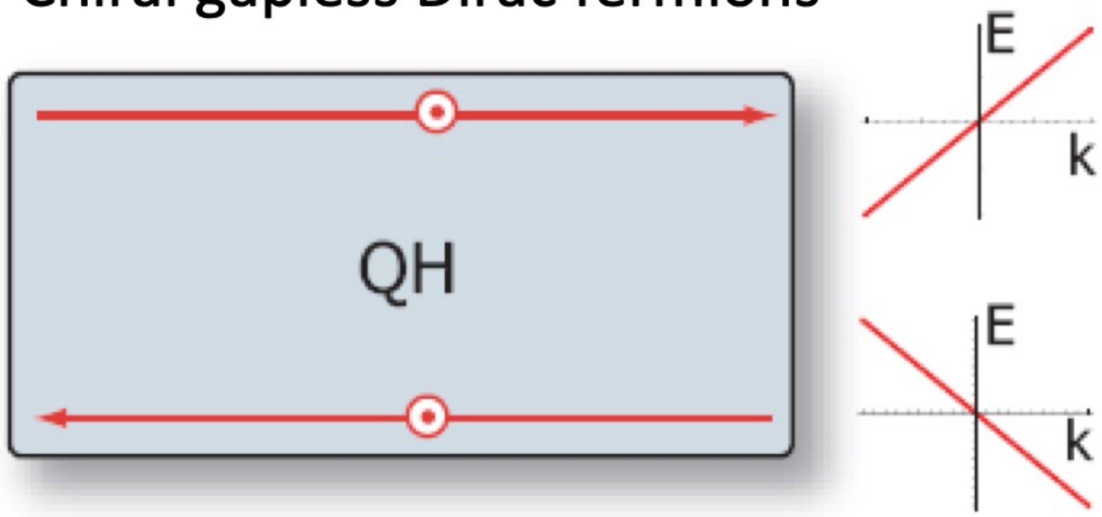
# Quantum Hall Zoology




Chiral gapless Dirac fermions




**✓ QHE: charge insulator**

Edge → chiral charge current
→ Current anomaly

# Generalization of Quantum Hall states?

Quantum Hall zoology

- Bulk is incompressible, charge carrier gapped
- Boundary has a certain type of anomaly → gapless
- Quantized EM response

Today: Generalization to Fracton system

→ With both **charge and dipole** conservation

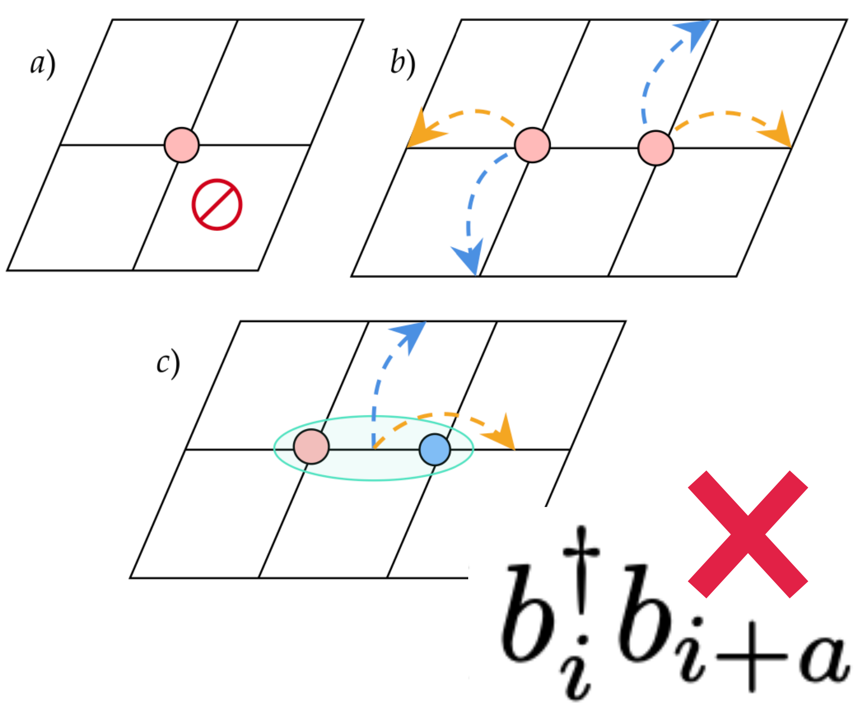
# Dipole U(1) symmetry

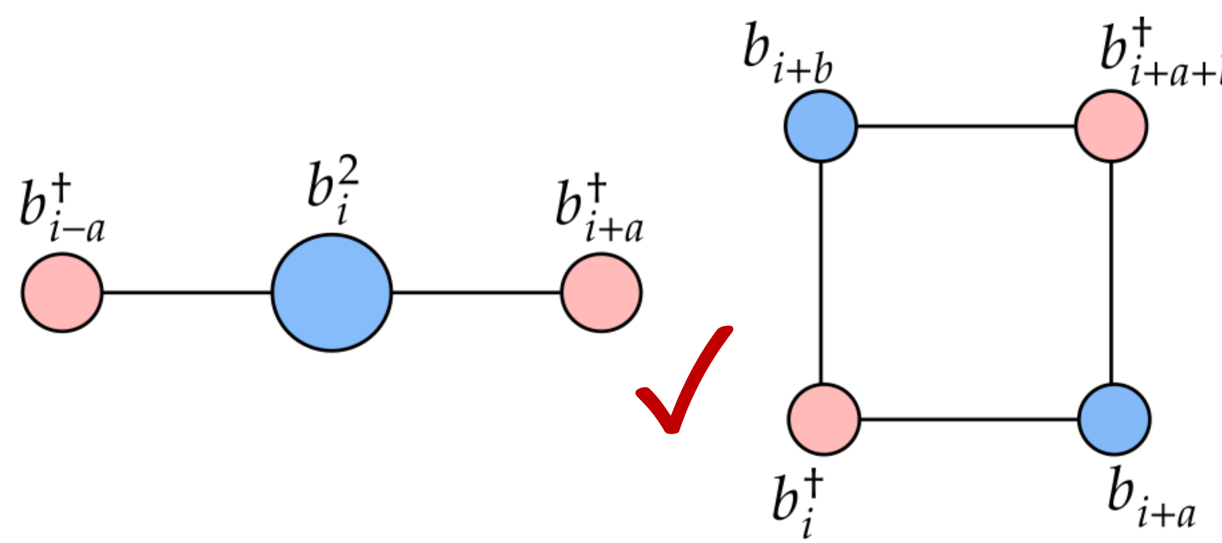
***Definitions***

- Conserved charge $q$ and dipole moment $p^x$ , $p^y$

$$p^i = \sum_{x_i} x_i q(x_i)$$

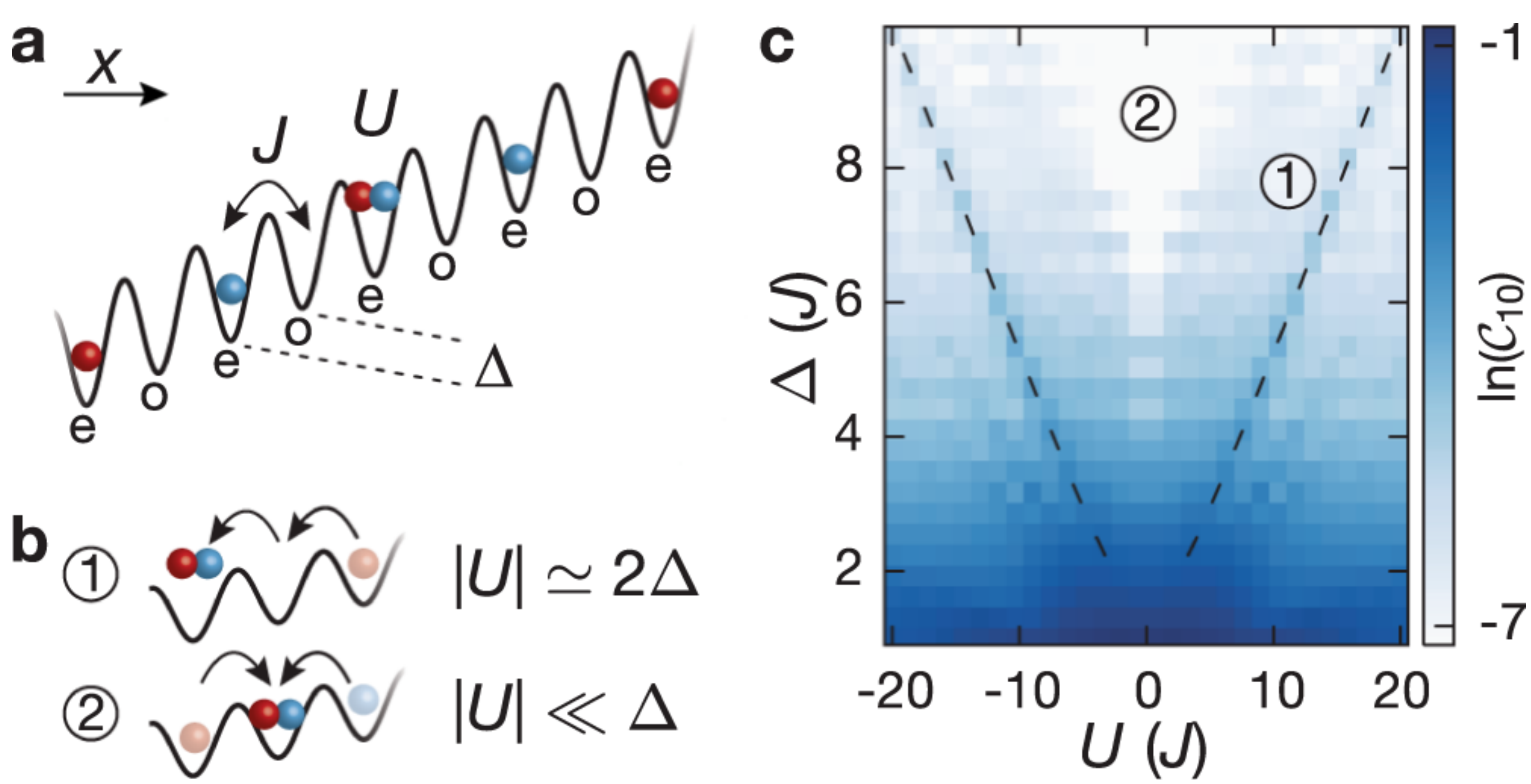
- Give rise to fracton dynamics
- A single charge cannot move



# Dipole U(1) symmetry

## *Experimental relevance*

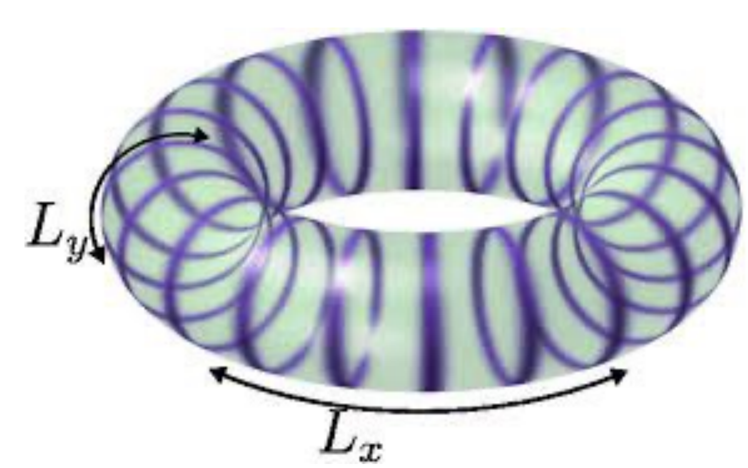



**Tilted Fermi-Hubbard chains with linear potential**

*Aidelsburger, et al., Nature comm*

**Quantum Hall on thin torus in LLL, Momentum locked with position:**

Momentum conservation → dipole moment conservation



# Outline

- What types of ‘quantum Hall’ type state can exist in dipole preserving system?

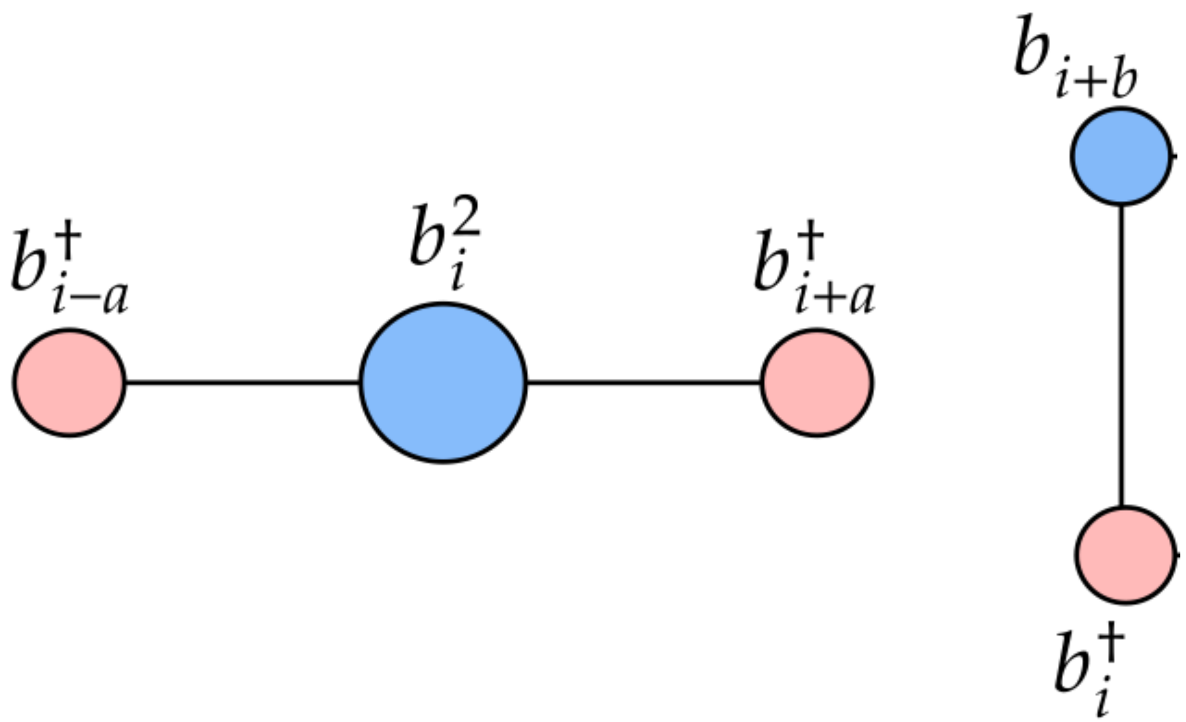
- What types of anomalies emerge on the boundary?

- EM response?

- What are the field theories of dipole QHE state?

- Generalization in other fracton system (with exotic subsystem symmetry) in higher Dim

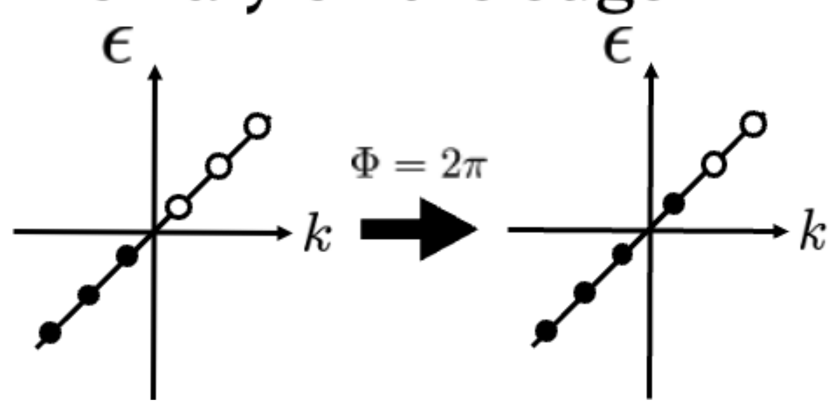
# Aim: QHE with conserved dipole moment $p^x$



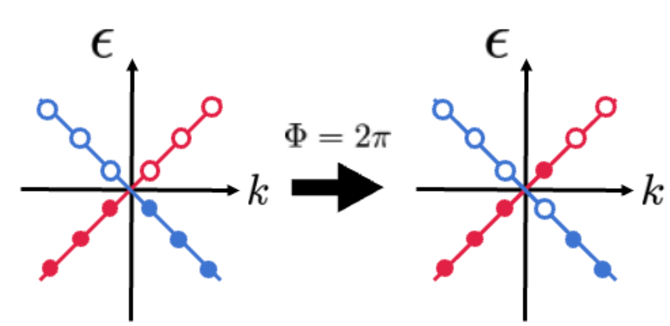

How to imagine 'quantum Hall' state in dipole preserving system?

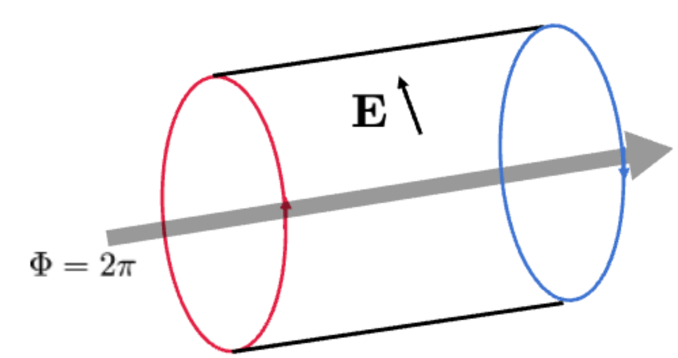
Review of QHE

- Anomaly on the edge



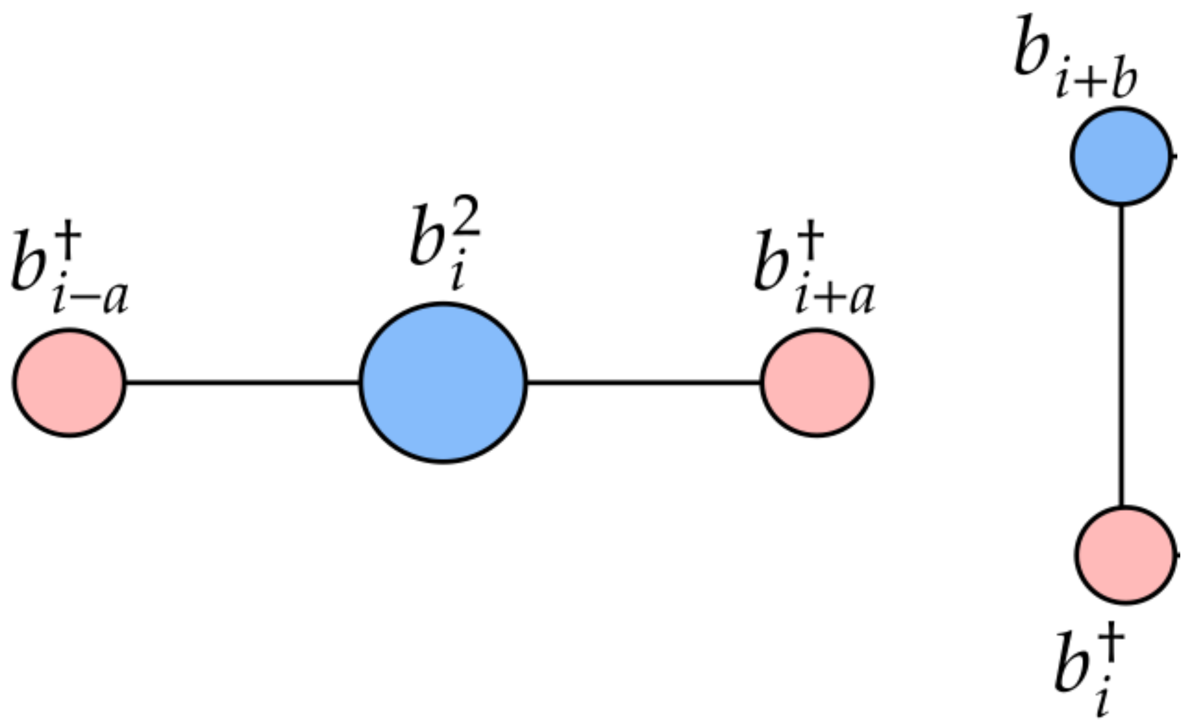

- Combining the two edges




- Opposite edges carry opposite anomalies that cancel each other when brought together

# Aim: QHE with conserved dipole moment $p^x$




How to imagine 'quantum Hall' state in dipole preserving system?

**Hunch?**

- **QHE** → Charge insulator → Edge Chiral charge current
- **Dipole QHE** → Dipole insulator → Edge dipole charge current ??

$$\psi^j_{R/L} \sim e^{i\phi^j_{R/L}}$$

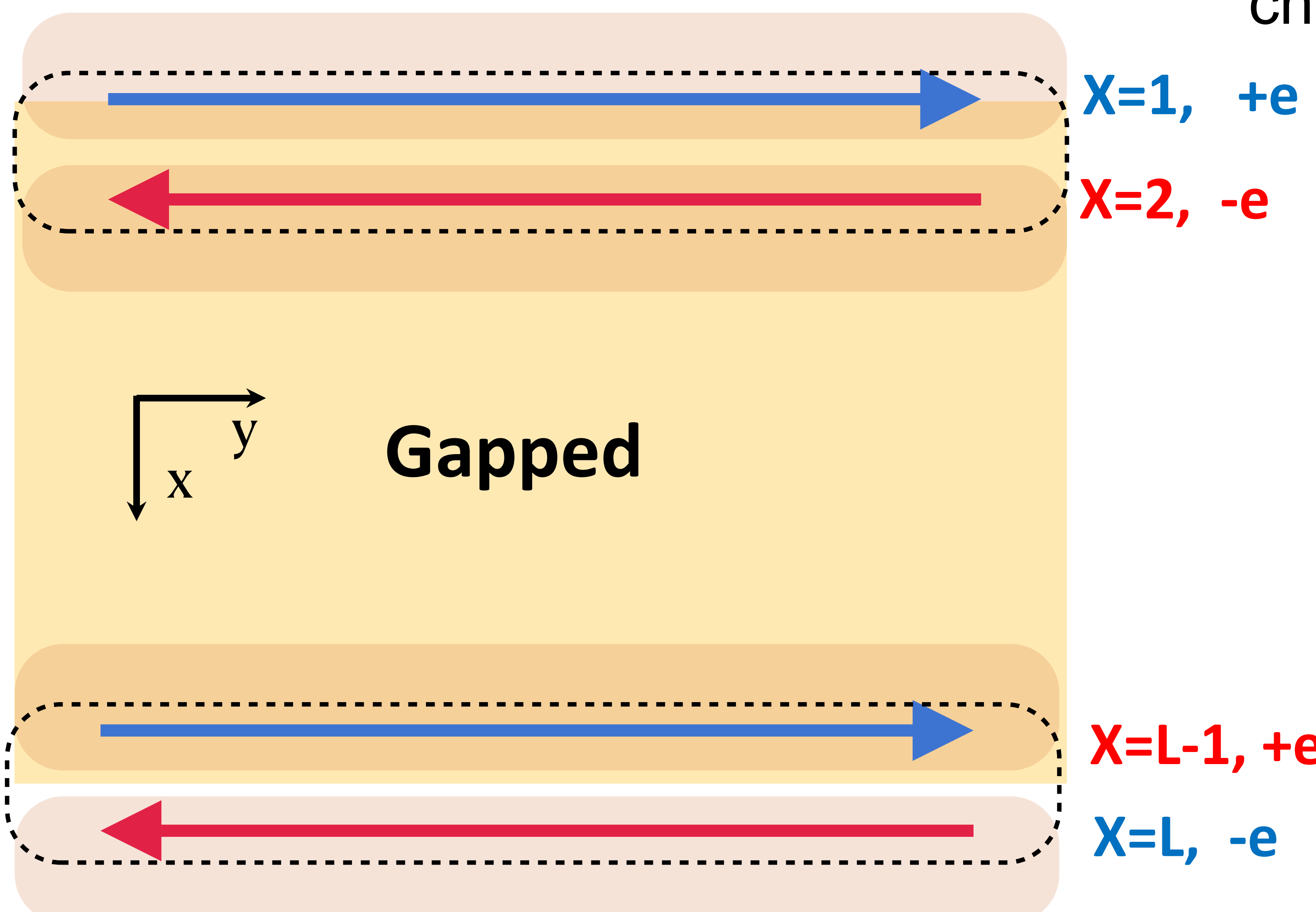
**Dipole U(1):**

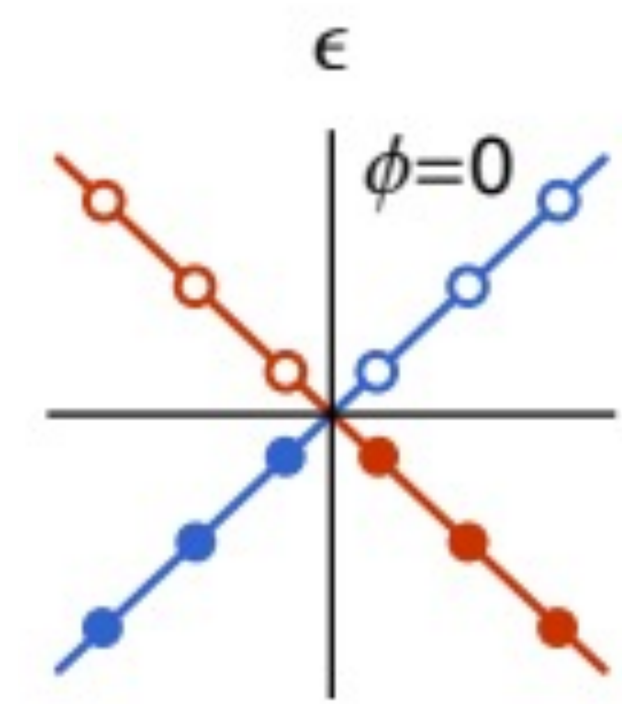
$$\phi^1_{L/R}(r) \to \phi^1_{L/R}(r) + x\alpha$$

**Consequence of Chiral dipole current**

**→Insert flux**
**→charge pattern change**

Flux wrt charge $U(1):\ A_y = \frac{2\pi}{L_y},$



$$\psi^j_{R/L} \sim e^{i\phi^j_{R/L}}$$

**Dipole U(1):**

$$\phi^1_{L/R}(r) \to \phi^1_{L/R}(r) + x\alpha$$

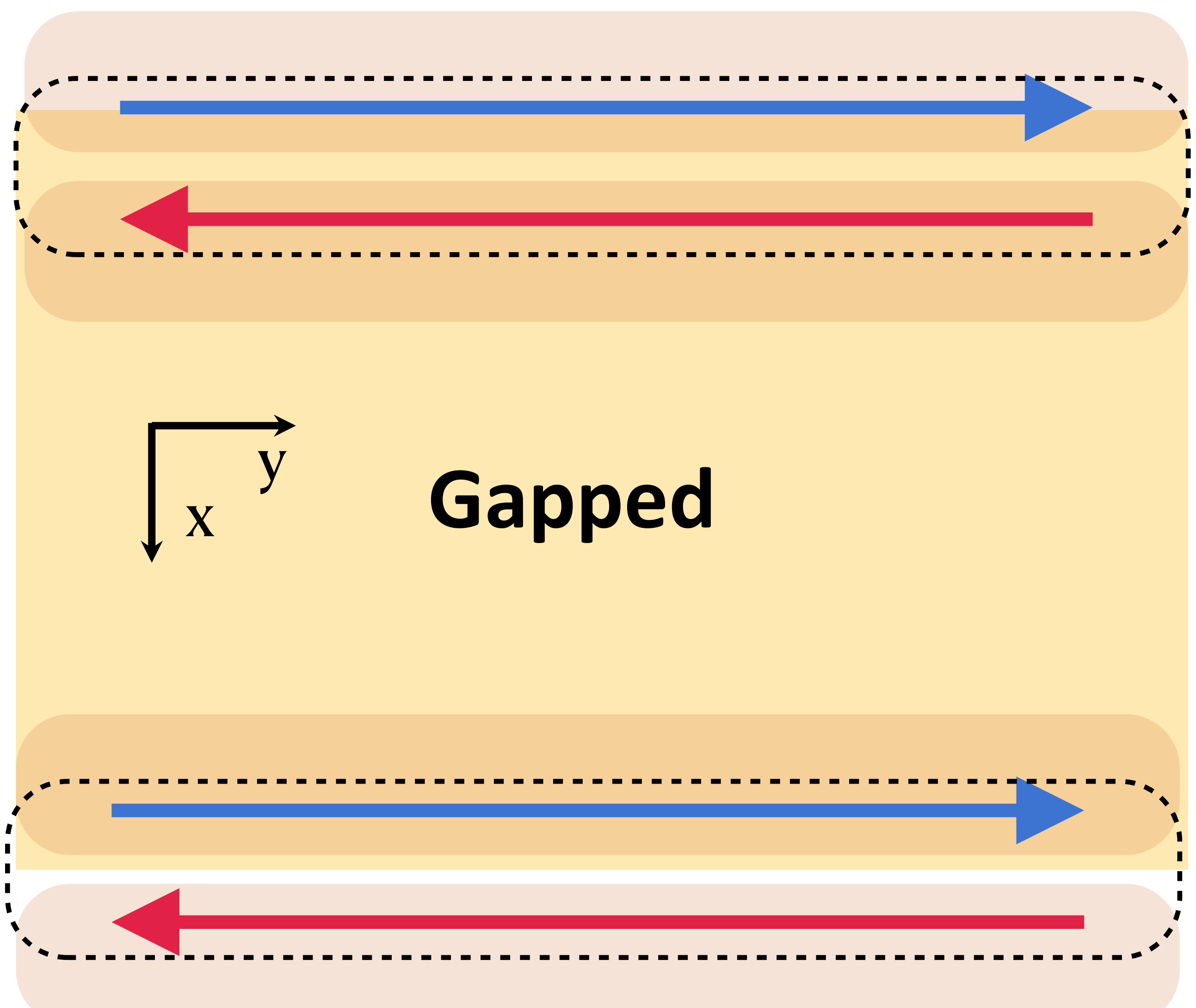
**Consequence?**

Chiral dipole current

→Insert flux
→charge pattern change

Flux wrt dipole

$$U^{dip}(1):\ A_y = \frac{2\pi x}{L_y}$$




X=1, +e — $\phi_y = 2\pi$

X=2, -2e — $\phi_y = 4\pi$

...

X=L-1, +(L-1)e — $\phi_y = 2(L-1)\pi$

X=L, -Le — $\phi_y = 2L\pi$

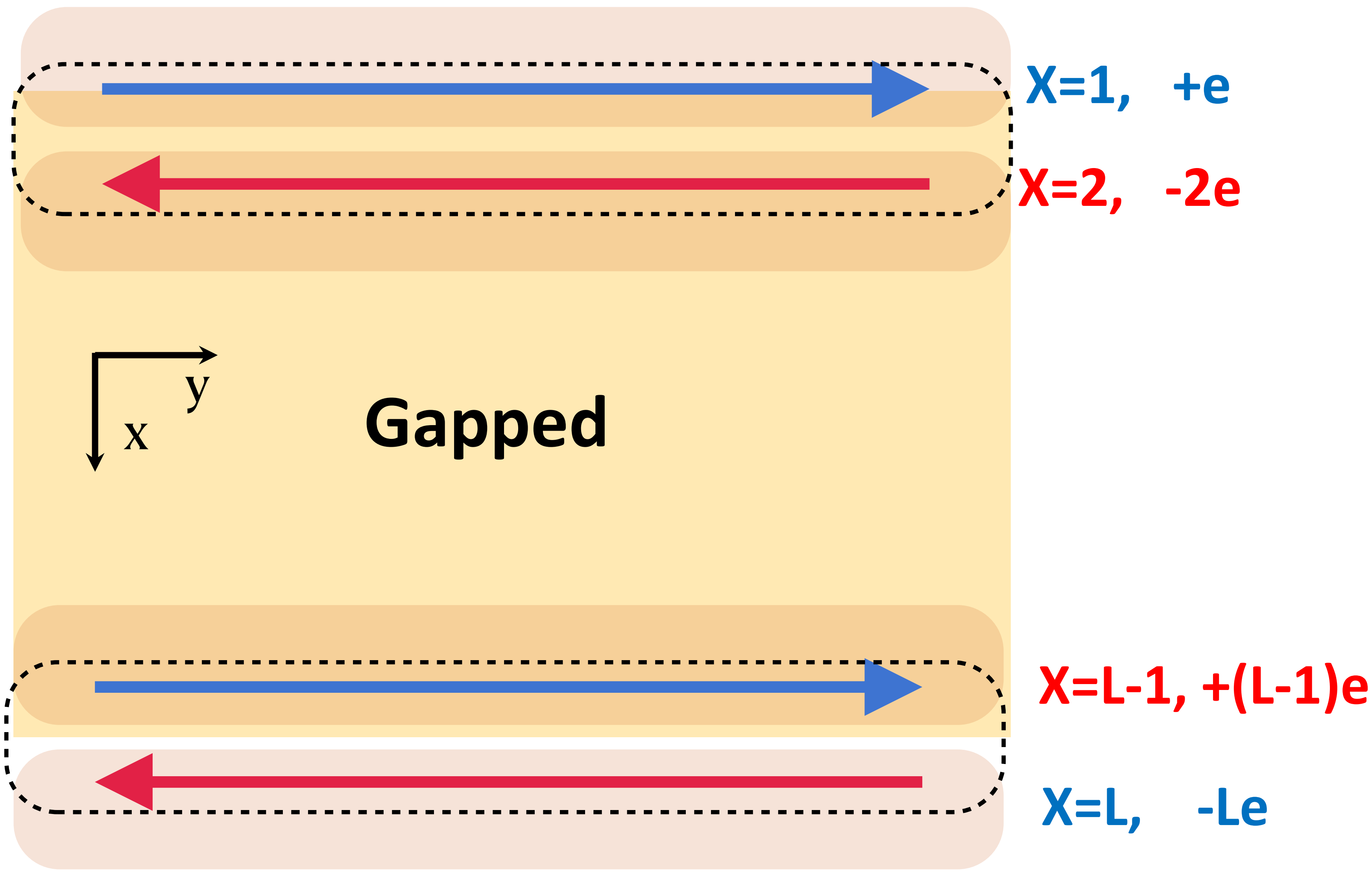
**Consequence?**
→Insert flux

$$U^{dip}(1):\ A_y = \frac{2\pi x}{L_y}$$

Total charge $\delta q = 0$

Total dipole moment ?

$$\delta p^x = e - 4e + (L-1)^2 e - L^2 e \neq 0$$

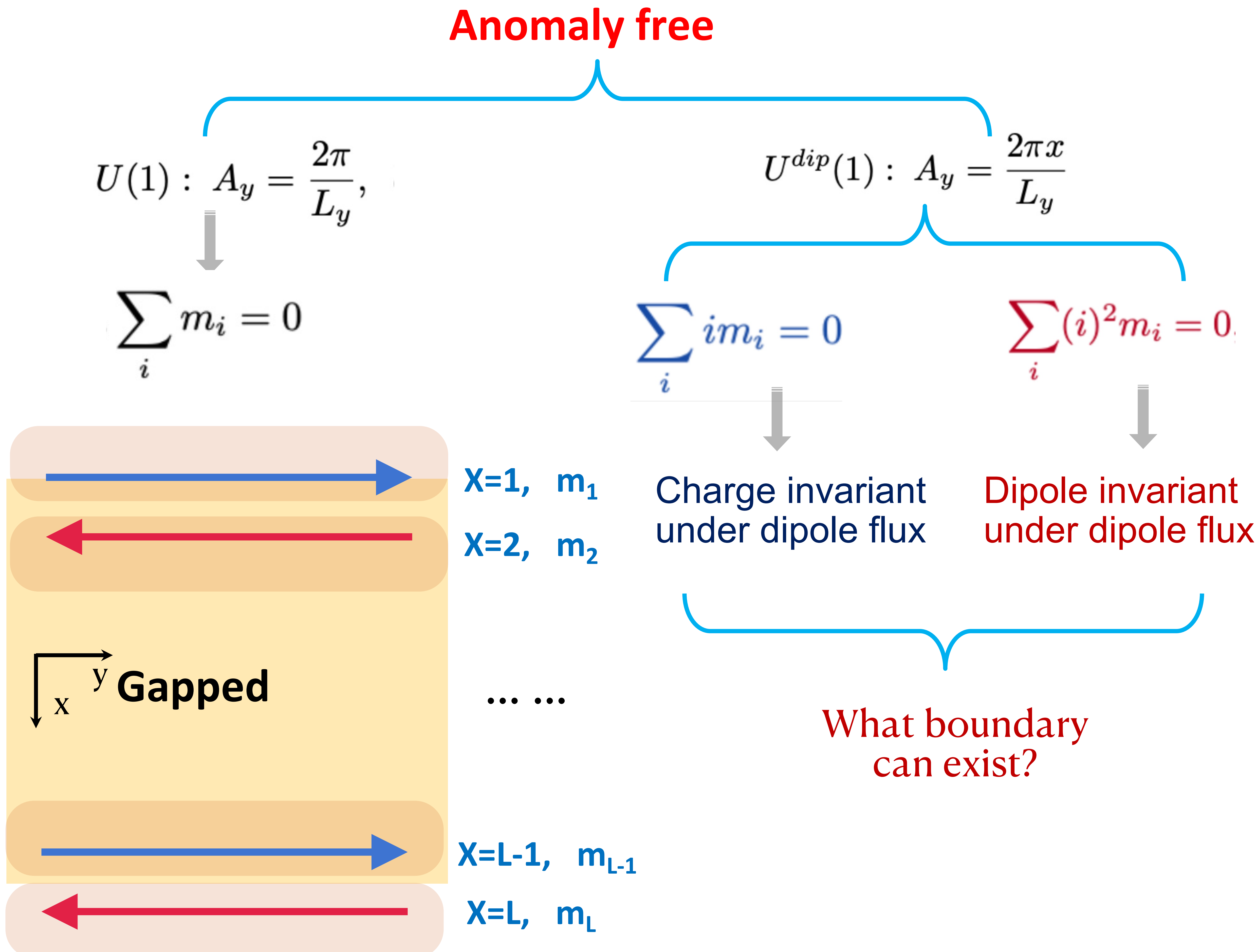



**Anomalous!**

**Cannot exist!**

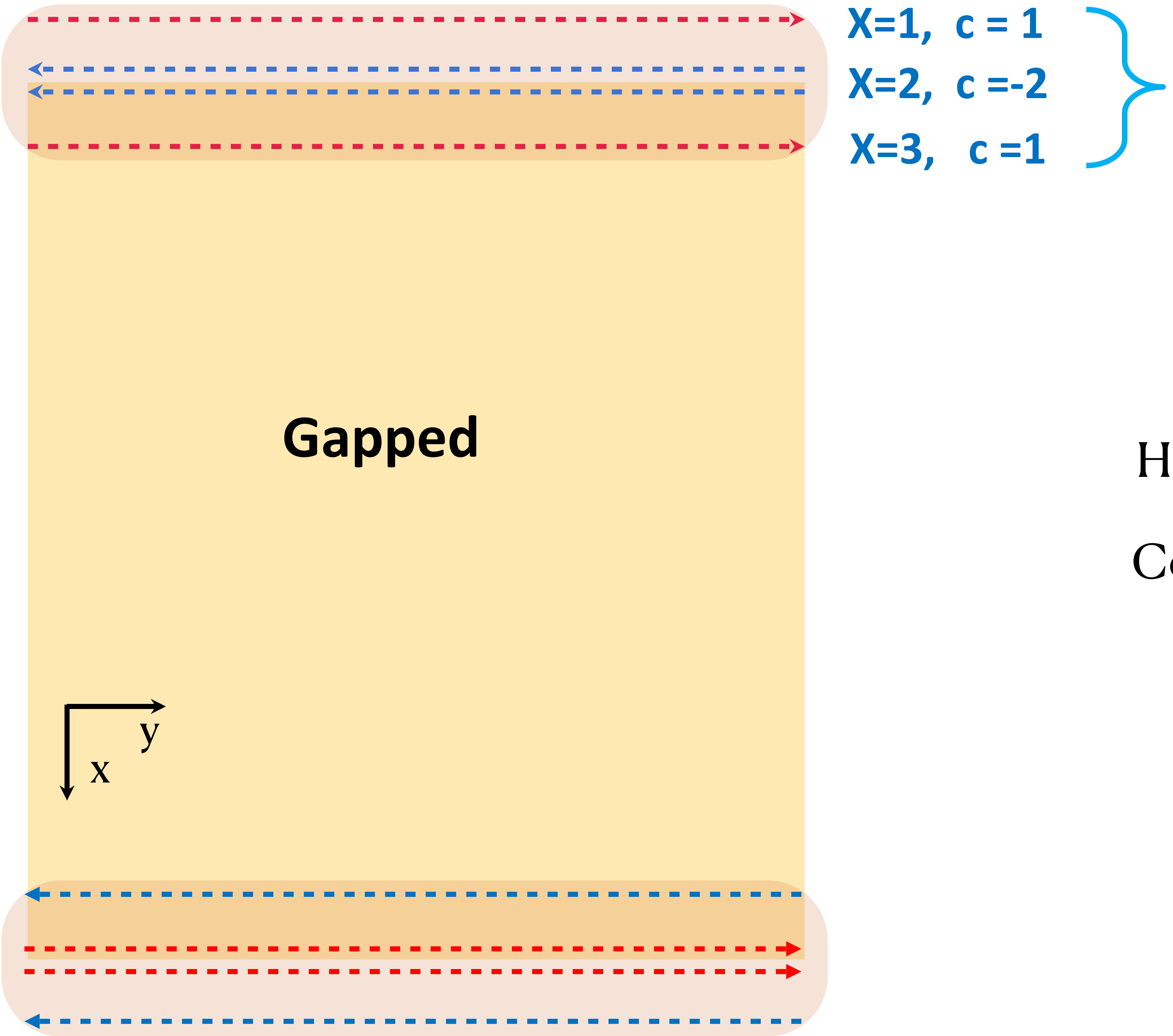
No go theorem:

No chiral dipole current for dipole QHE



Anomaly free
$U(1): \ A_y = \frac{2\pi}{L_y},$
$U^{dip}(1): \ A_y = \frac{2\pi x}{L_y}$
$\sum_i m_i = 0$
$\sum_i i m_i = 0$
$\sum_i (i)^2 m_i = 0.$
Charge invariant under dipole flux
Dipole invariant under dipole flux
What boundary can exist?
X=1, $m_1$
X=2, $m_2$
Gapped
y
x
… …
X=L-1, $m_{L-1}$
X=L, $m_L$

# What boundary can exist?




Quadrupole Current?

How to fulfill ?

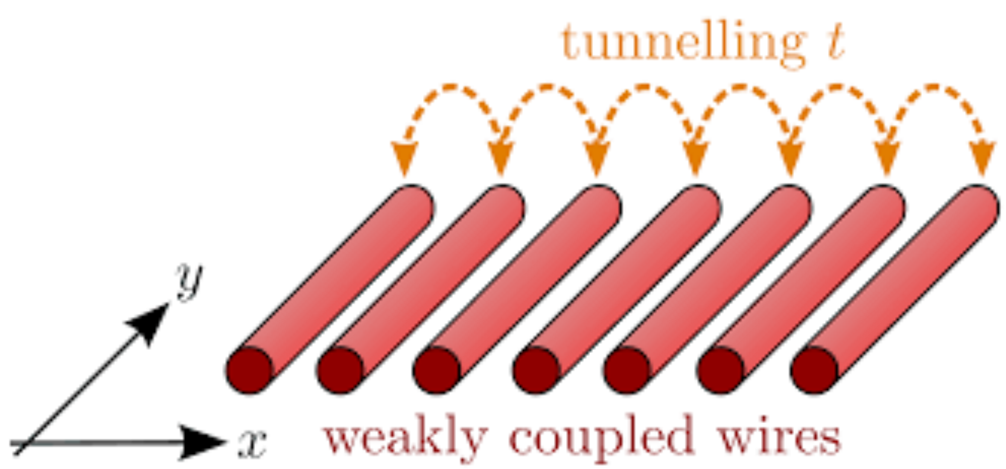
Coupled wires!

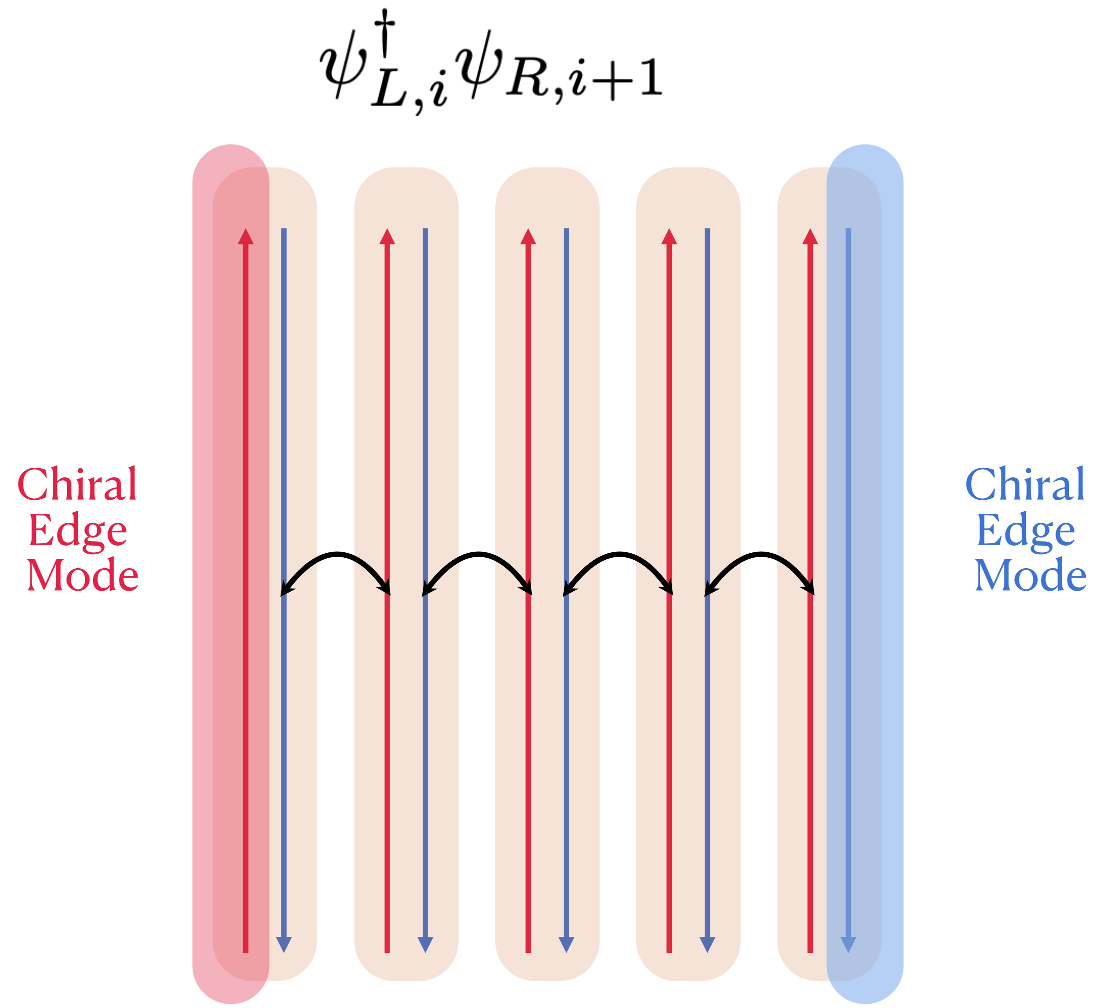
# Review of QHE from coupled wire

- Wire construction: A popular tool to construct gapped topological phases in d>1 from 1+1d Luttinger liquid.

*Example: wire construction of 2d QHE*

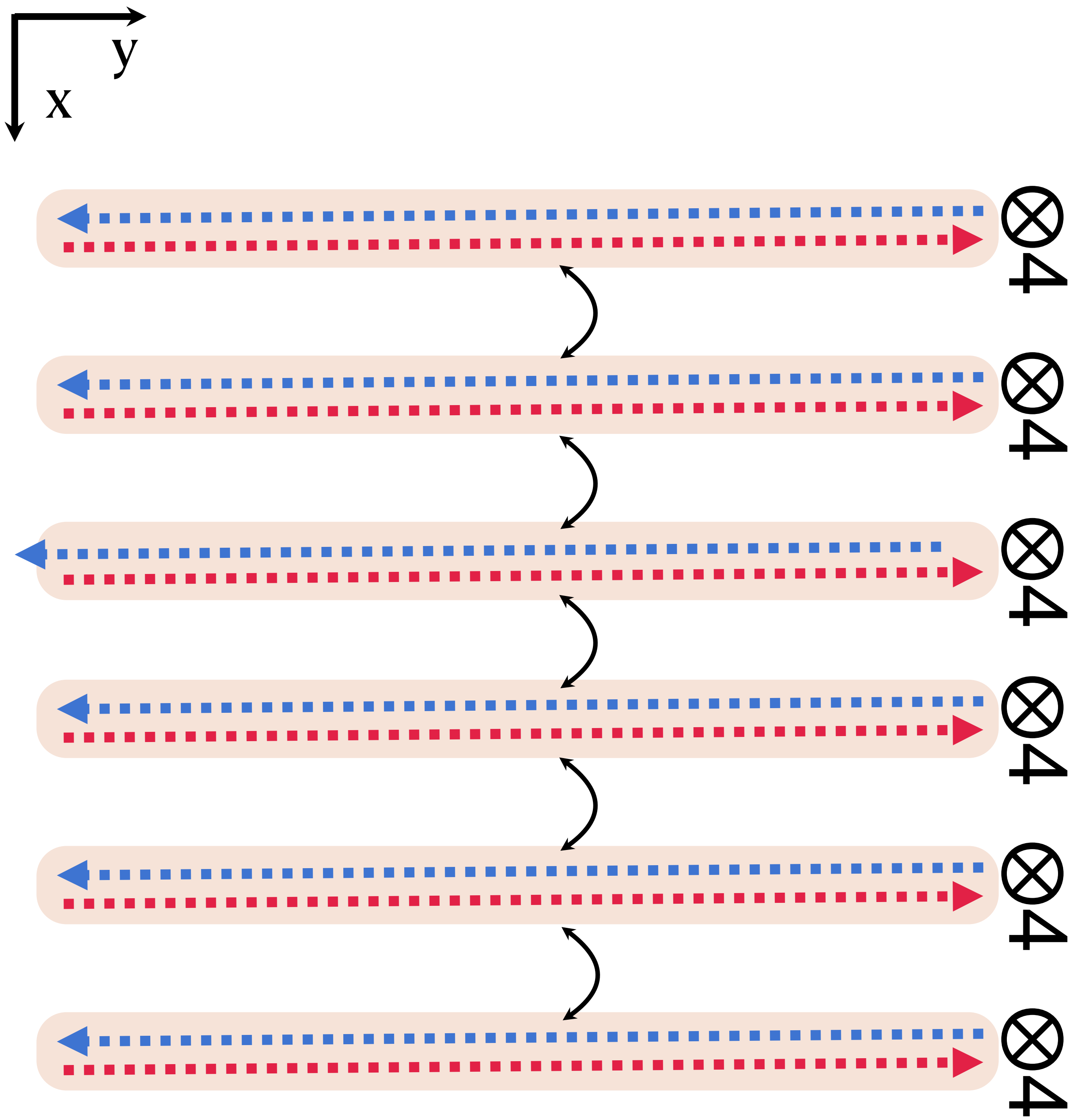
- Pave Luttinger liquid wires along y
- Inter-wire coupling L/R movers
- Left over chiral fermion on the edge




QHE

# Coupled wire setup




$$\mathcal{H}_{\text{wires}} = \sum \boldsymbol{\psi}_{\boldsymbol{r}}^\dagger i\partial_y \tau^{zz0}\boldsymbol{\psi}_{\boldsymbol{r}},$$

$$\boldsymbol{\psi} = (\psi^1_{L/R}, \psi^2_{L/R}, \psi^3_{L/R}, \psi^4_{L/R})$$

$$\psi^j_{R/L} \sim e^{i\phi^j_{R/L}}$$

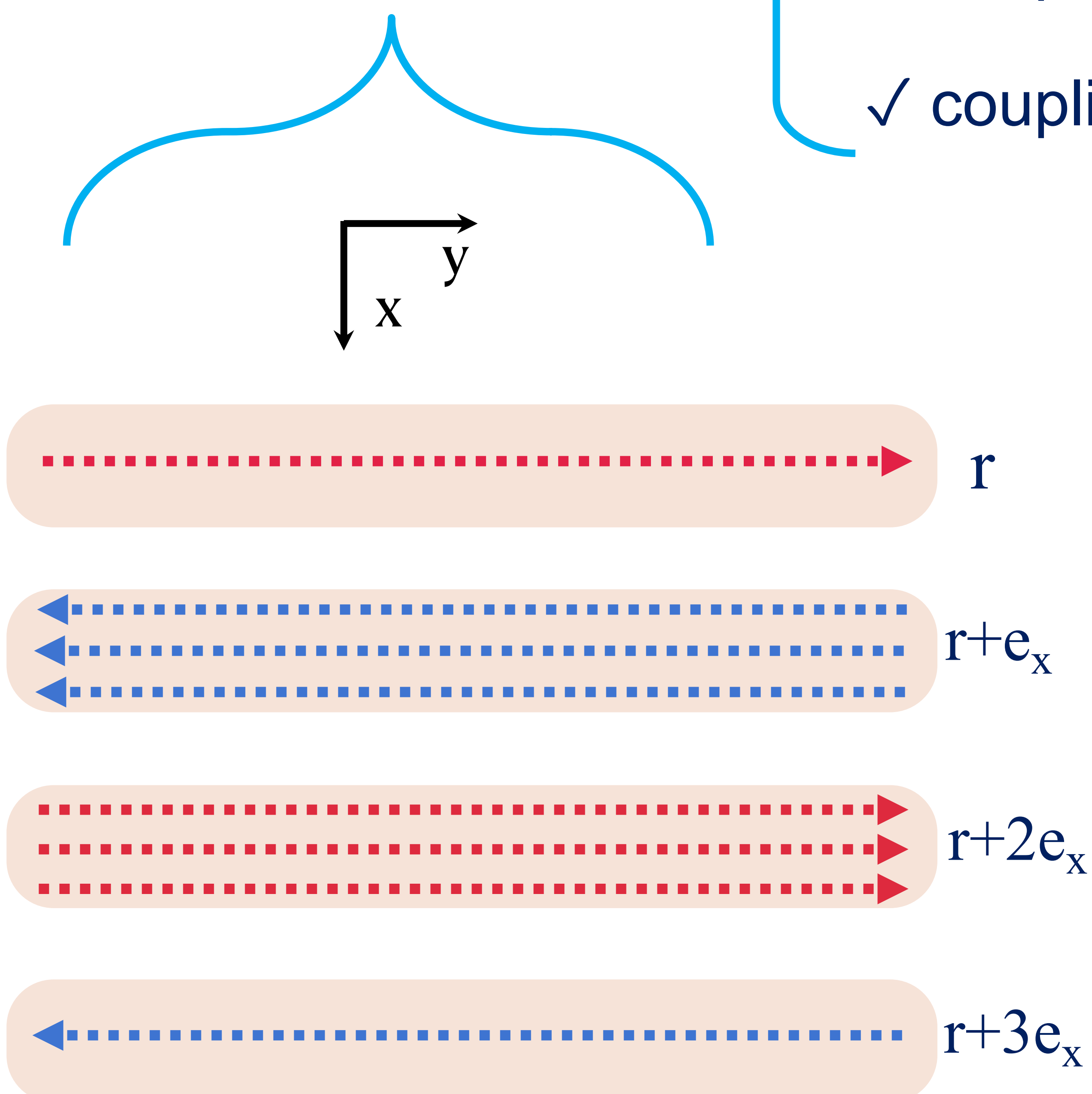
$$U^{dip}(1)$$

$$\phi^1_{L/R}(r) \to \phi^1_{L/R}(r) + x\alpha$$

$\psi^\dagger_{L,i}\psi_{R,i+1}$ ✗ Not allowed by U(1) dipole conservation

## *Building block*

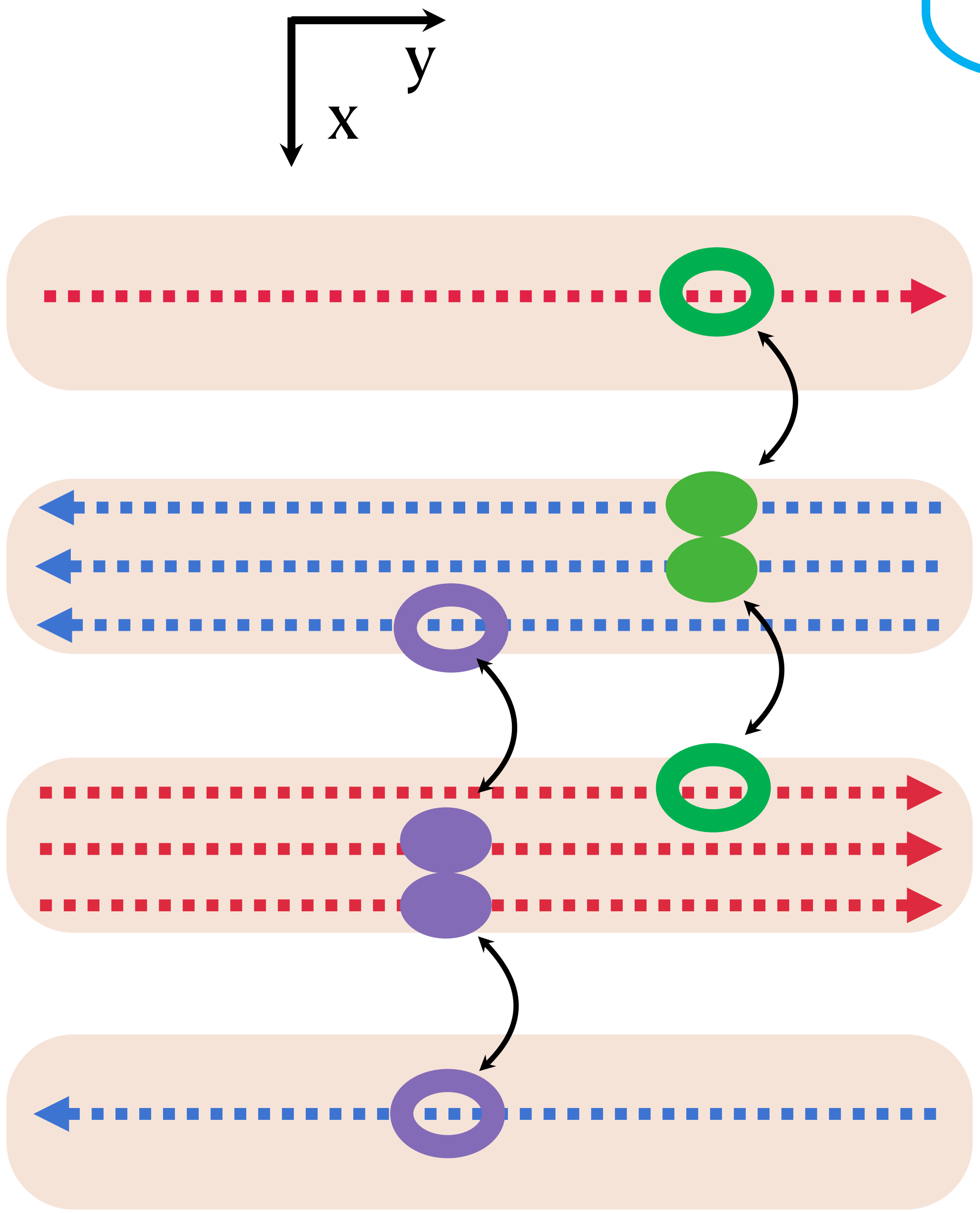
- ✓ building blocks of chiral bosons from 4 rows
- ✓ bulk= translation stacking of building blocks
- ✓ coupling of wire within building block
- ✓ coupling respect charge & dipole symm




| | U(1) | $U^{dip}(1)$ |
|---|---|---|
| $\phi_r$ | 1 | x |
| $\phi_{r+e_x}$ | 1 | x+1 |
| $\phi_{r+2e_x}$ | 1 | x+2 |
| $\phi_{r+3e_x}$ | 1 | x+3 |

***Building block***

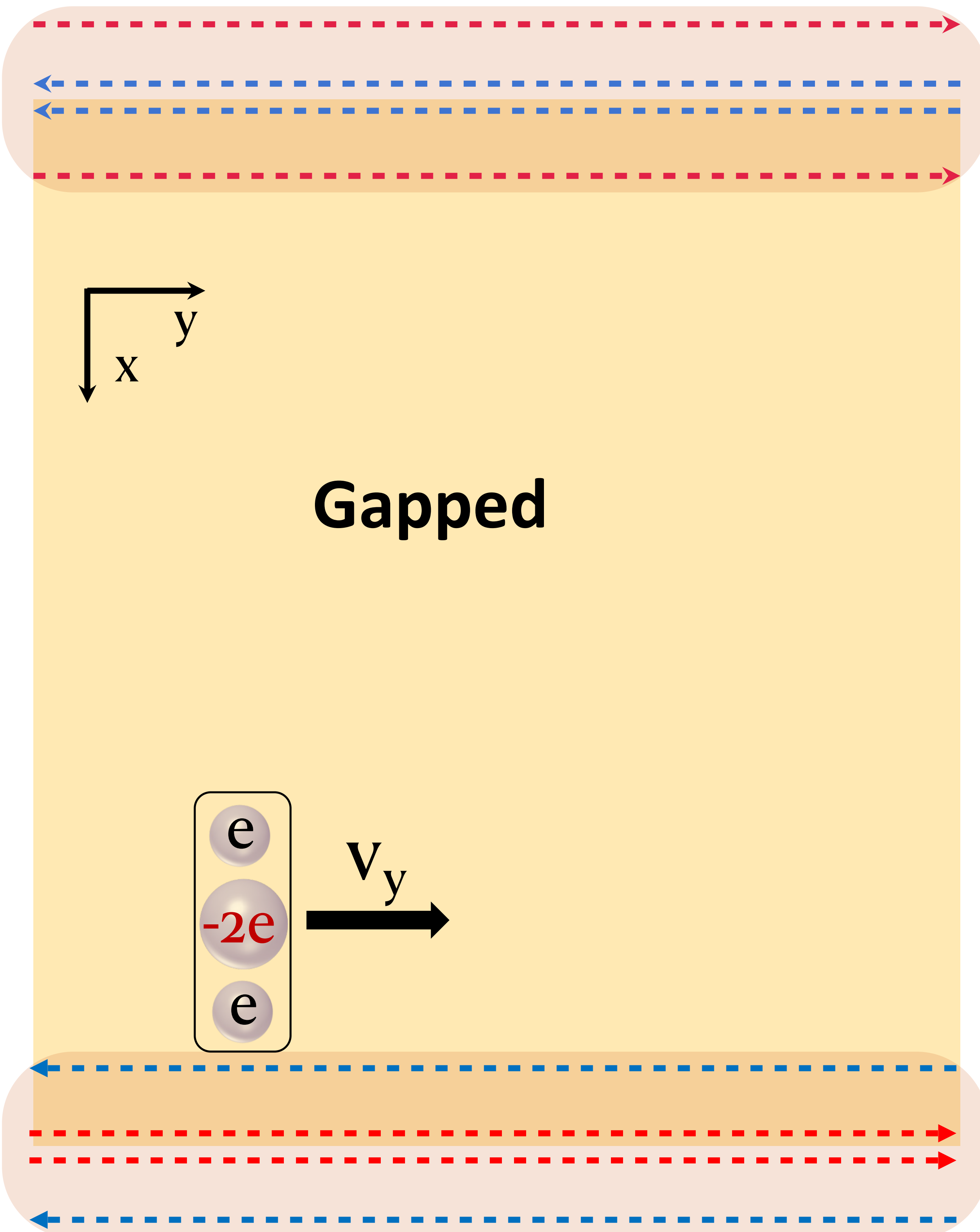
- ✓ building blocks of 4 adjacent wires
- ✓ bulk= translation stacking of building blocks
- ✓ coupling of wire within building block
- ✓ coupling respect charge & dipole symm




$$\mathcal{L} = \sum_{\boldsymbol{r}} \Big[ -g_1 \cos(-\phi^1_{L,\boldsymbol{r}} + \phi^2_{R,\boldsymbol{r}+\boldsymbol{e}_x} + \phi^3_{R,\boldsymbol{r}+\boldsymbol{e}_x} - \phi^4_{L,\boldsymbol{r}+2\boldsymbol{e}_x})$$
$$- g_2 \cos(-\phi^1_{L,\boldsymbol{r}} + \phi^3_{R,\boldsymbol{r}+\boldsymbol{e}_x} + \phi^4_{R,\boldsymbol{r}+\boldsymbol{e}_x} - \phi^2_{L,\boldsymbol{r}+2\boldsymbol{e}_x})$$
$$- g_3 \cos(-\phi^1_{L,\boldsymbol{r}} + \phi^4_{R,\boldsymbol{r}+\boldsymbol{e}_x} + \phi^2_{R,\boldsymbol{r}+\boldsymbol{e}_x} - \phi^3_{L,\boldsymbol{r}+2\boldsymbol{e}_x})$$
$$- g_4 \cos(\phi^1_{R,\boldsymbol{r}+3\boldsymbol{e}_x} - \phi^2_{L,\boldsymbol{r}+2\boldsymbol{e}_x} - \phi^3_{L,\boldsymbol{r}+2\boldsymbol{e}_x} + \phi^4_{R,\boldsymbol{r}+\boldsymbol{e}_x})$$
$$- g_5 \cos(\phi^1_{R,\boldsymbol{r}+3\boldsymbol{e}_x} - \phi^3_{L,\boldsymbol{r}+2\boldsymbol{e}_x} - \phi^4_{L,\boldsymbol{r}+2\boldsymbol{e}_x} + \phi^2_{R,\boldsymbol{r}+\boldsymbol{e}_x})$$
$$- g_6 \cos(\phi^1_{R,\boldsymbol{r}+3\boldsymbol{e}_x} - \phi^4_{L,\boldsymbol{r}+2\boldsymbol{e}_x} - \phi^2_{L,\boldsymbol{r}+2\boldsymbol{e}_x} + \phi^3_{R,\boldsymbol{r}+\boldsymbol{e}_x}) \Big],$$

Four independent mass terms, fully gapped out

# Boundary modes




X=1, c =1
X=2, c =-2
X=3, c =1

Quadrupole Current?

$$U(1): \; A_y = \frac{2\pi}{L_y}, \quad U^{dip}(1): \; A_y = \frac{2\pi x}{L_y}.$$

✓ anomaly under Udip(1)

Edge Dipole moment change under $U^{dip}(1)$ flux insertion

✓ Both edge has chiral quadrupole current

$$\mathcal{L} = \partial_y(\partial_x^2\phi)\partial_t\phi + K(\partial_y\partial_x\phi)^2$$

$$\mathcal{L}_{y=L} = \partial_t\partial_x^2\Phi\partial_x\Phi + K(\partial_x^2\Phi)^2$$

# Classification = Edge Anomaly

$U(1): \; A_y = \frac{2\pi}{L_y},$

$U^{dip}(1): \; A_y = \frac{2\pi x}{L_y}$

$\sum_i m_i \not\doteq 0$

Shift of boundary charge under flux
<u>Self anomaly</u>

$\sum_i i m_i \not\doteq 0$

Shift of boundary charge under dipole flux, <u>mixing anomaly</u>

$\sum_i (i)^2 m_i \not\doteq 0$

Shift of edge Dipole under dipole flux, <u>Self anomaly</u>

| | U(1) | $U^{dip}$ (1) | Mixing |
|---|---|---|---|
| Phase 1 | | ✓ | |
| Phase 2 | | ✓ | ✓ |
| Phase 3 | ✓ | ✓ | ✓ |

# Take home message

- Quantum Hall states in with dipole conservation

Boundary~ chiral quadrupole current

- No-go theorem: No chiral dipole current for dipole QHE

→No chiral N-pole current for N-pole QHE

→chiral $N^2$-pole current for N-pole QHE

- Generalization to fractionalized case : coupled fractional Luttinger liquids

- Other Generalization

✓ 3D ✓ TSC

# 3d FQHE for Fracton
# with subsystem charge conserved on planes

- The side surfaces of the system are gapped.

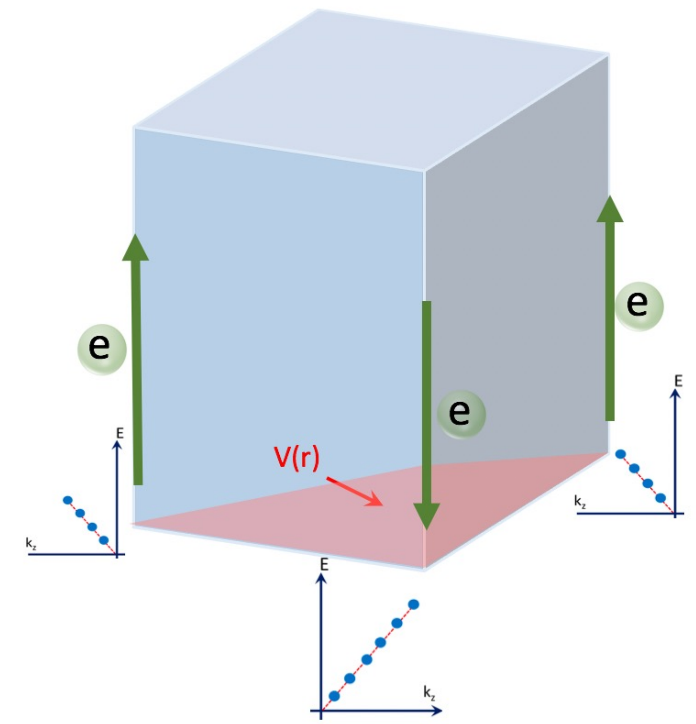
However, on the hinge, there are gapless hinge modes.

May-Mann, You, Hughes, Bi, PRB (2022)

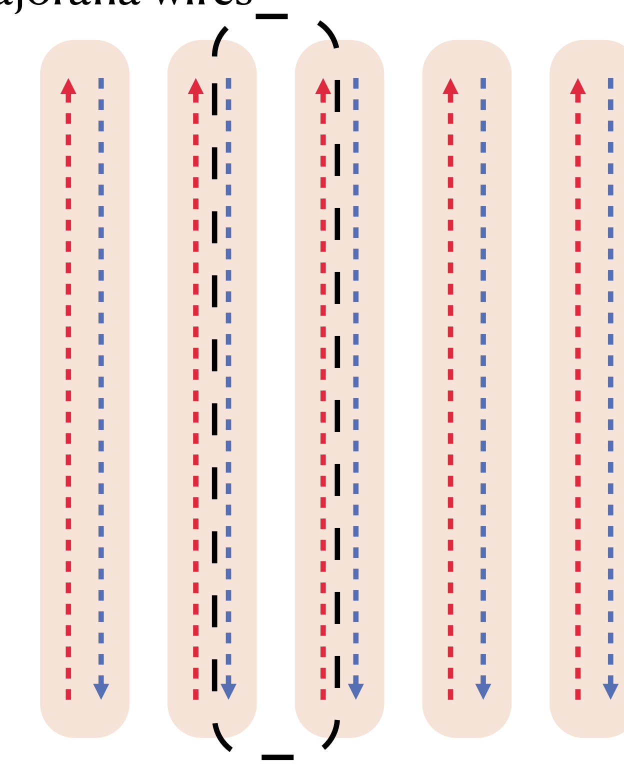
- The side surface are gapless, subsystem U(1) anomaly

Sullivan, Iadecola, Williamson, PRB (2021)
Sullivan, Dua and Cheng, PRR (2021)

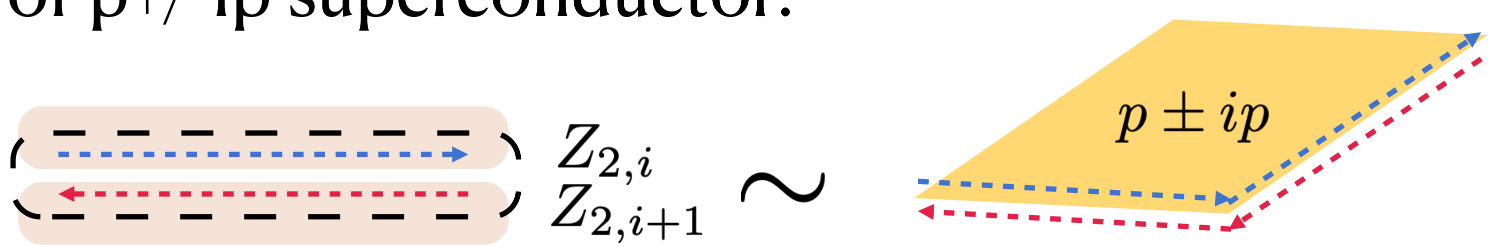
Chiral modes along the z-hinges



# 2d TSC with subsystem $Z_2$ symmetry

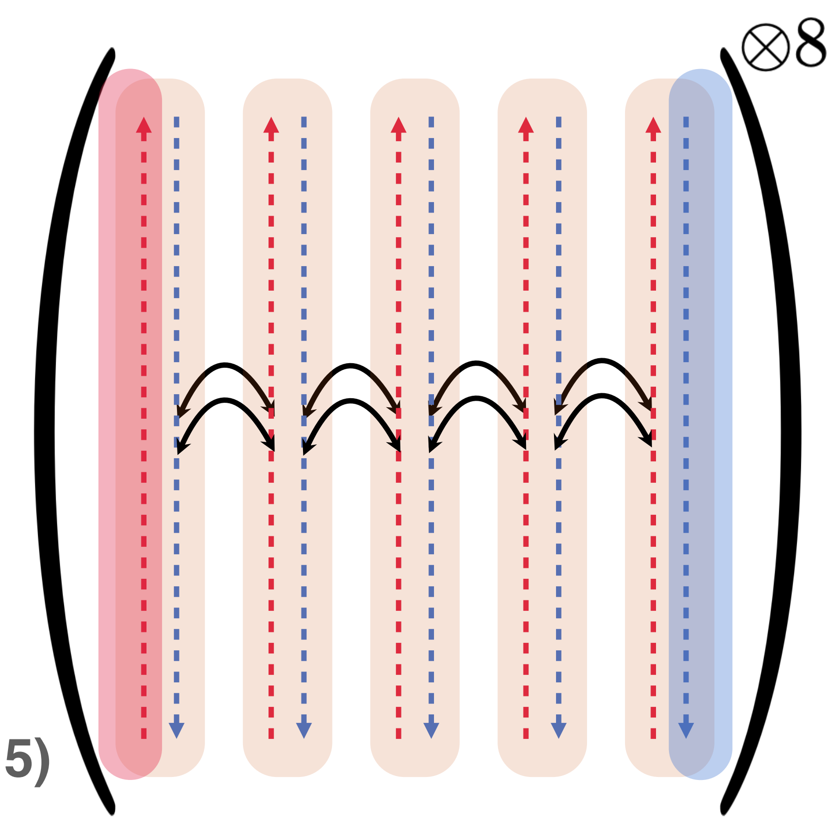



- Majorana wire construction
- Intersection modes are mapped to the boundary of p+/-ip superconductor.




May-Mann, You, Hughes, Bi, PRB (2022)

- Interacting fermion SPT classification tells us *8* copies of (p+/-ip) SC is trivial — the edge could be gapped by interaction.
  - Edge chiral central charge c_=4




**Fidkowski, Kitaev, PRB (2009)**
**Bi, Rasmussen, You,Cheng, Xu, NJP(2015)**

# Open questions?

- EFT for these FQH states. Higher-rank symmetric tensor gauge fields?

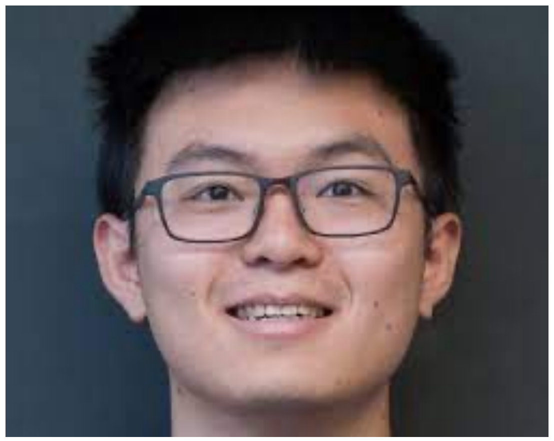
UV-IR mixing: GSD depend on system size!

- The interplay between spatial symmetry with charge multipole symmetry?

Dislocation, disclination carries additional zero mode, projective non-Abelian defects.

- Anyon condensation web: Normal QHE → QHE for fracton

# Thank You



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# Questions?

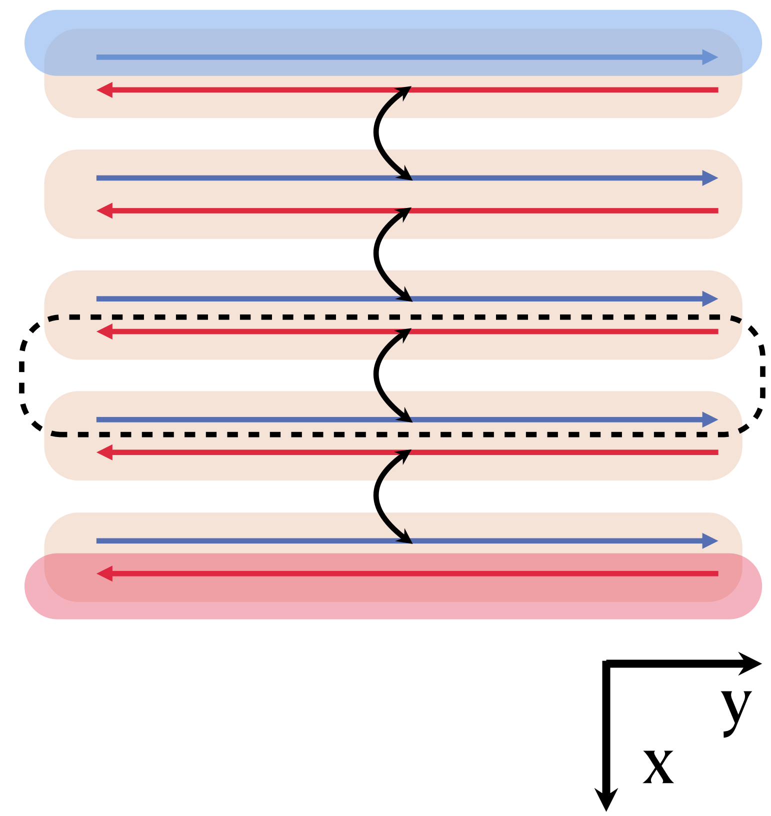
# Aim: QHE with conserved dipole moment $p^x$

### *How to construct?*

- Wire construction
- Pave wires of Luttinger liquid along y

$$\psi^{\dagger}_{L,i}\psi_{R,i+1}$$ ✗

Not allowed, breaks dipole moment $p^x$ conservation




- Interacting terms, no band theory limit

$$\psi^{\dagger}_{L,i}\psi_{R,i+1}\psi_{R,i+2}\psi^{\dagger}_{L,i+3}$$

**Hunch?**

- **QHE** → Charge insulator → Edge Chiral charge current
- **Dipole QHE** → Dipole insulator → Edge dipole charge current ??

# Outline

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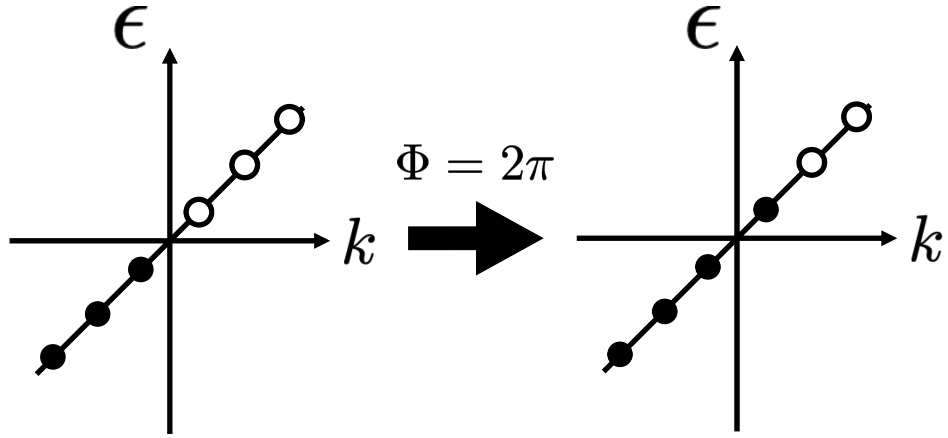
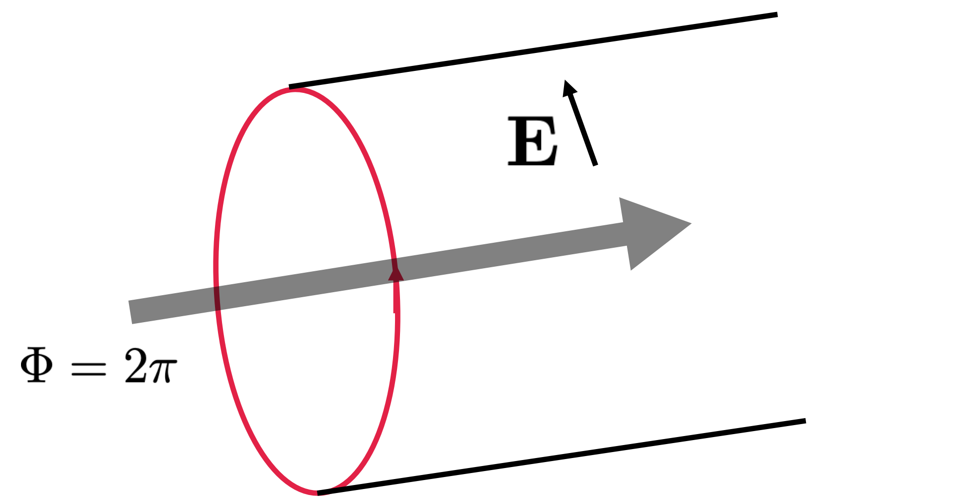
- EM response?
- What are the field theories of dipole QHE state?
- Generalization in other fracton system (with exotic subsystem symmetry) in higher Dim

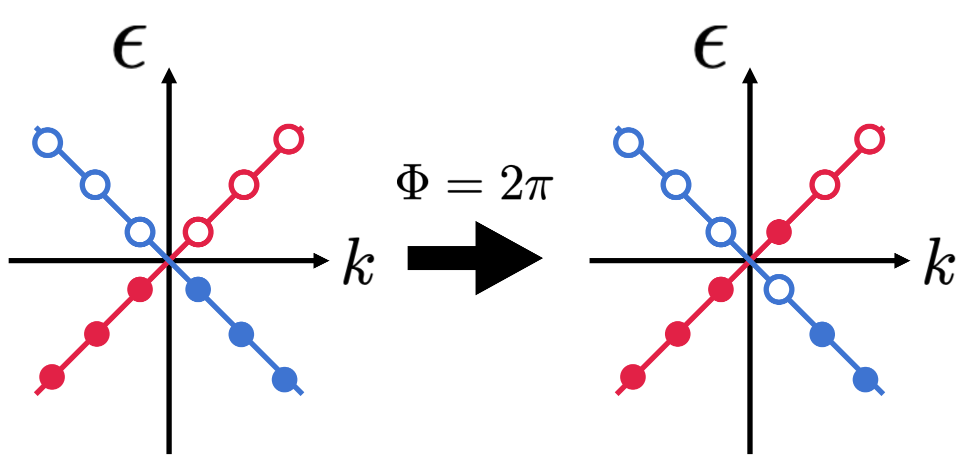
# Current anomalies in QHE

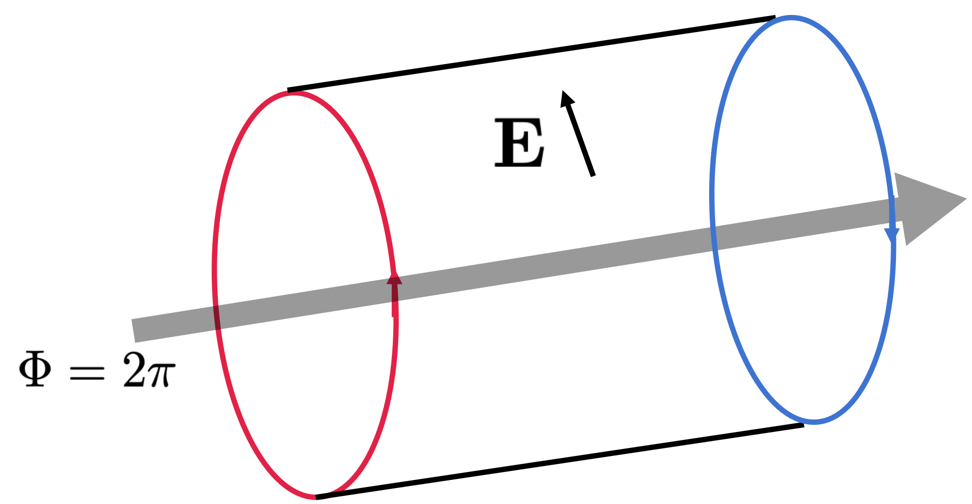
- Anomaly on the edge




- Combining the two edges

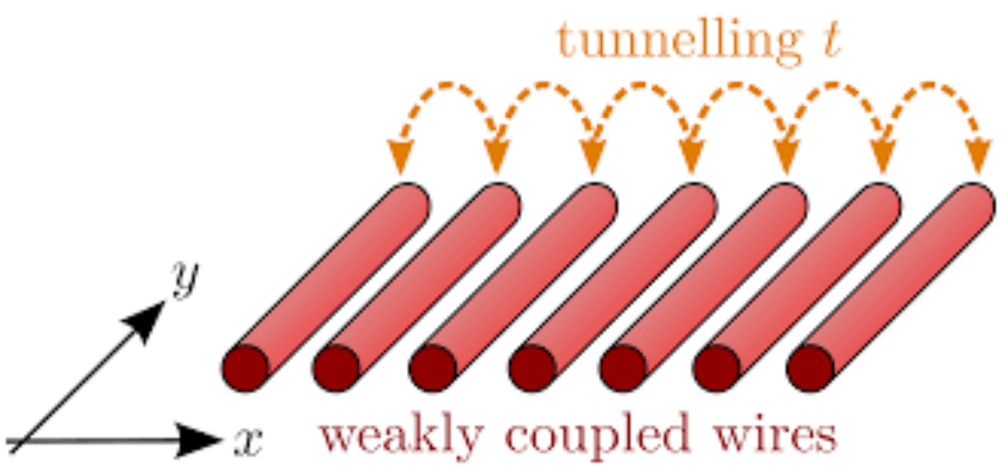


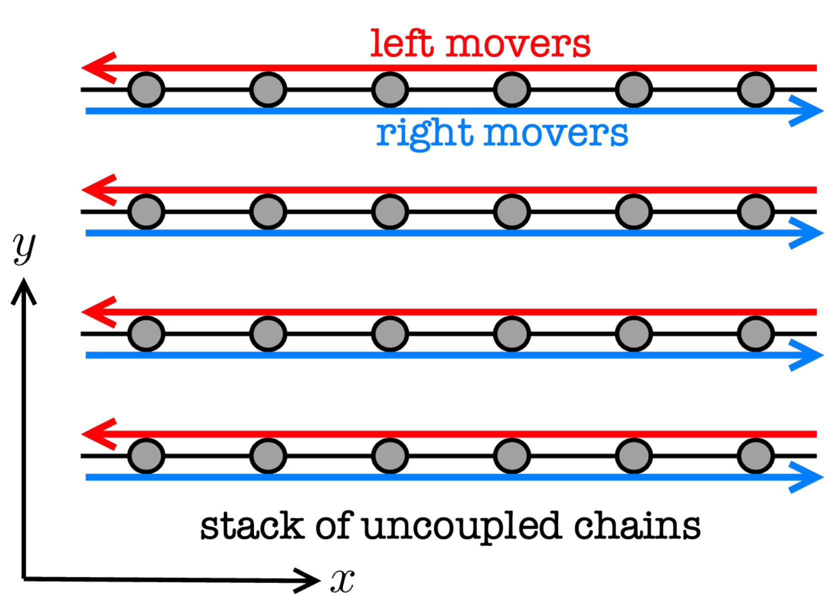

- Opposite edges carry opposite anomalies that cancel each other when brought together

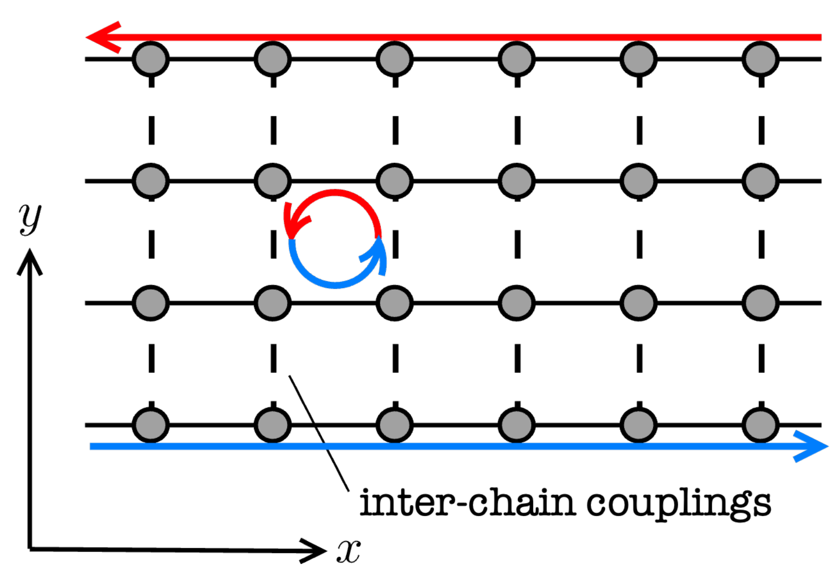
# Coupled wire construction

## *A quick overview*




- ✓ Pave Luttinger liquid wires along y
- ✓ Inter-wire coupling L/R movers wrt $U^{dip}(1)$
- ✓ Left over modes on the edge



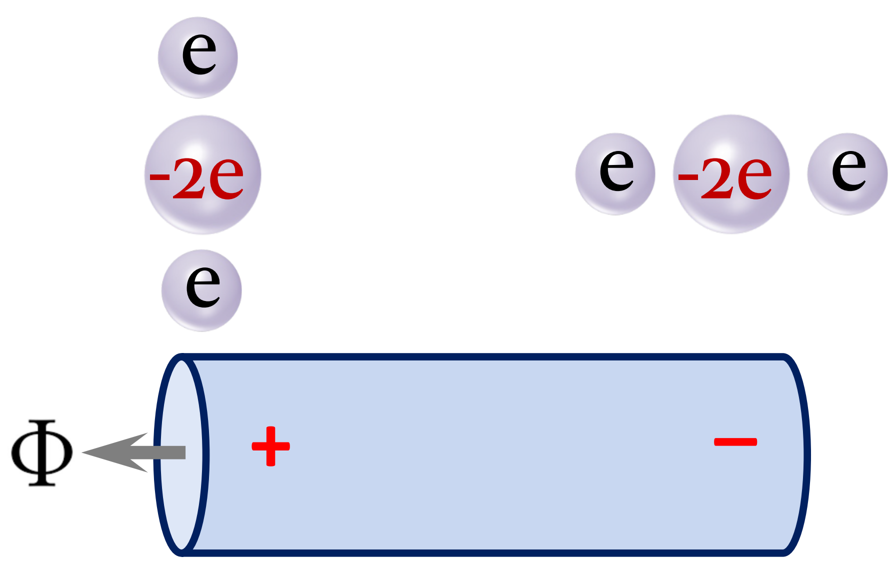
# EM response?

X=0

$$\mathcal{L} = \partial_y(\partial_x^2\phi)\partial_t\phi + K(\partial_y\partial_x\phi)^2$$


$$\mathcal{L}_{y=L} = \partial_t\partial_x^2\Phi\partial_x\Phi + K(\partial_x^2\Phi)^2$$

Y=L




Global Dipole flux → charge pump