# Spin-motion correlation in QCD matter

1. Intro.
2. Hydro. limit: shear-induced polarization and its cousin effects
3. Beyond hydro.
4. Matching to quantum kinetic theory
5. Outlook


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*Based on works Baochi Fu, Shuai Liu, Longgang Pang, Huichao Song, YY and Zonglin Mo:*

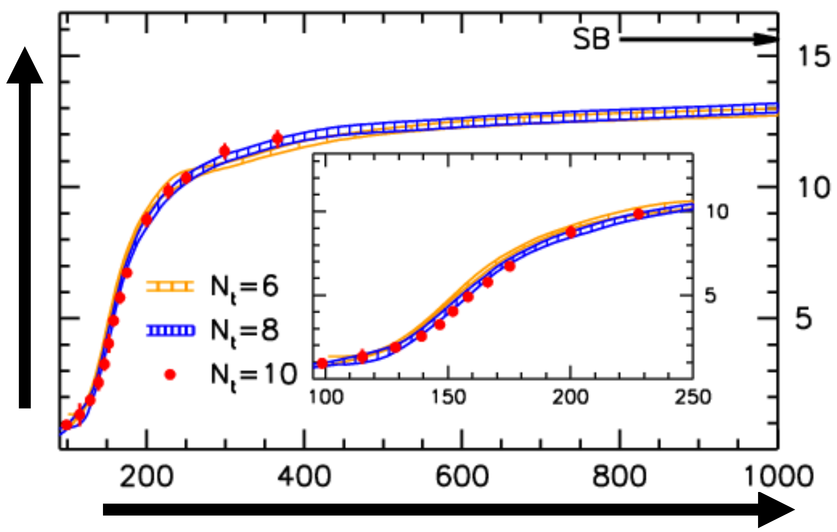
*2006.12421,PRD; 2103.09200, JHEP 21., 2103.10403, PRL 21; 2201.12970 and more*

*Zonglin Mo, Master student from USTC*

# Motivation

# Spin observables probe QCD phase structure.

$\varepsilon/T^4$~# of Degree of freedom

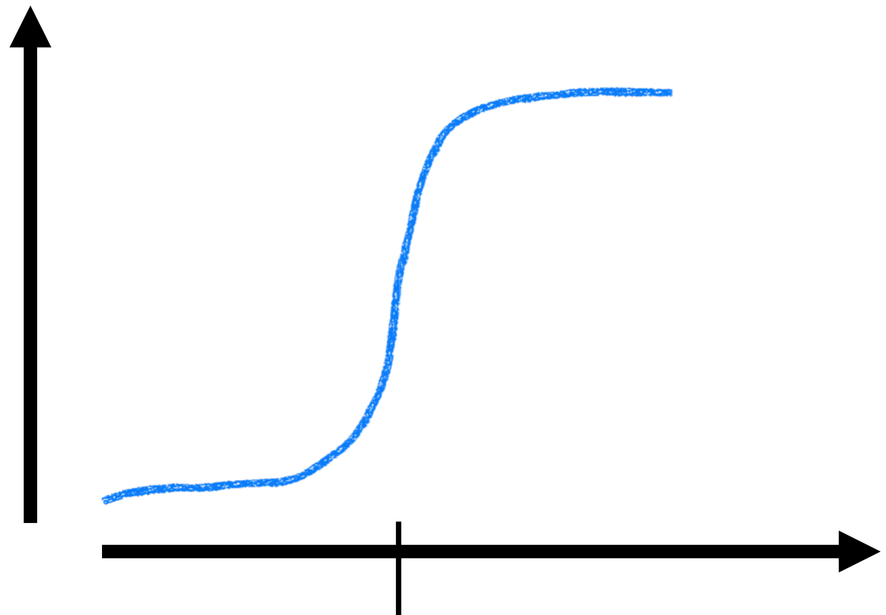



T (MeV)

(Budapest-Marseille-Wuppertal Collaboration, JHEP'10)

Review: Becattini, Lisa, Annals Phys. 2020

# of spin carrier

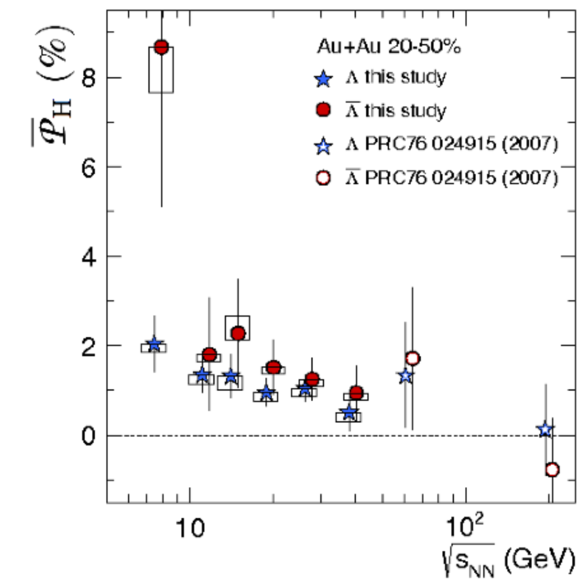


$T_c$ T (MeV)

- Density of spin carriers changes dramatically near Tc.
- Quantum effects may be employed to distinguish different phases.

# Spin observables in HIC

*Review: Becattini, Lisa, Annals Phys. 2020*



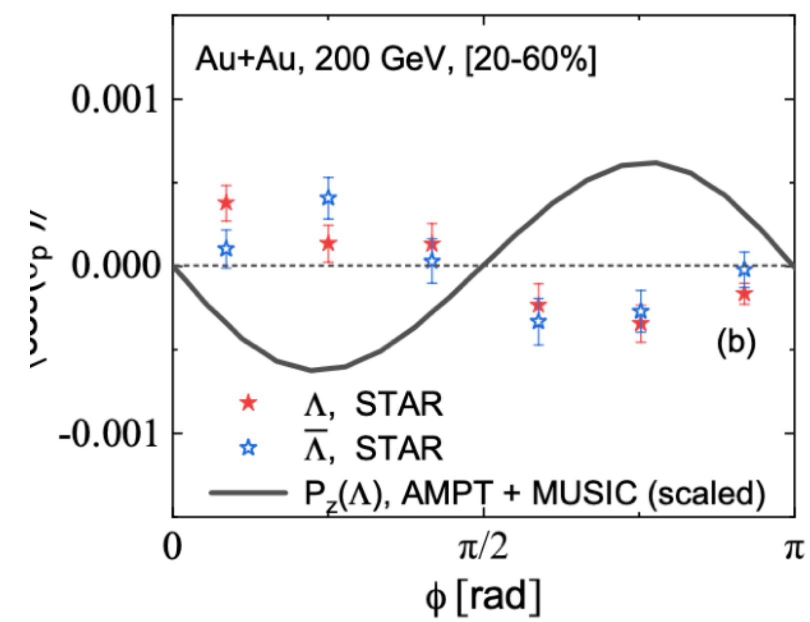

*STAR, Nature 17'.*

- Informative Λ hyperon polarization data
  - Vorticity effect describes the trends of global (phase-space averaged) Λ.

    *Xin-Nian Wang, Zuo-Tang Liang, PRL 05'; Becattini et al, Annals Phys 13'*

  - the differential data, i.e. the flow of spin.: intensive study.

    *STAR PRL 19'; hydro. simulations by many*




*Baochi Fu et. al, PRC21'*

- Intriguing results on vector meson (kaon, ϕ, J/ψ) spin density matrix.

  *e.g. STAR, 2204.02302;ALICE PRL 20', 2204.10171*

## *Mechanism for spin polarization of Fermions*

- Rotation polarizes spin: *Landau-Lifshitz volume 5*

$$\Delta\epsilon = -\,\hat{s}\cdot\overrightarrow{\Omega} \rightarrow \hat{s} \parallel \overrightarrow{\Omega}$$ (similar for B-field)

  - extensively studied. *Xin-Nian Wang, Zuo-Tang Liang, PRL 05'; Becattini et al, 13'*

  - induced-polarization is independent of $\hat{p}$ .

- A different class of mechanisms: spin polarization that is correlated with motion (spin-motion correlation).

# Spin Hall effect (SHE)

$$\hat{s} \propto \vec{v} \times \vec{E}$$

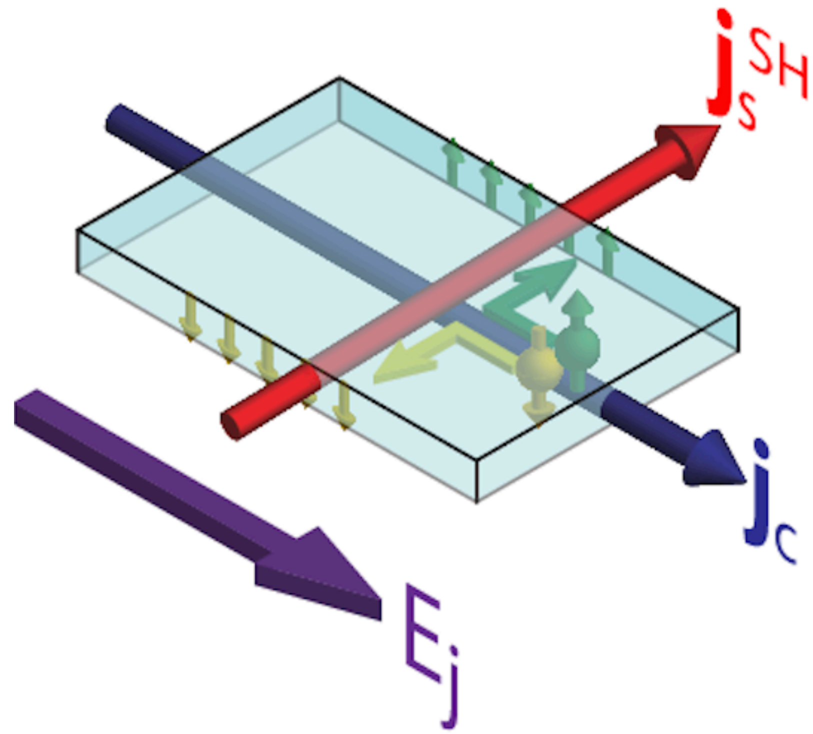



*Fig. from Meyer et al, Nature material 17'*

- implies non-trivial spin current and spin-momentum correlation.
- is instrumental for characterizing topological phases in 2+1 insulators.
  - SHE conductivity is zero for topological matter class A but non-zero for class AII.

*Kane-Mele PRL 05'*

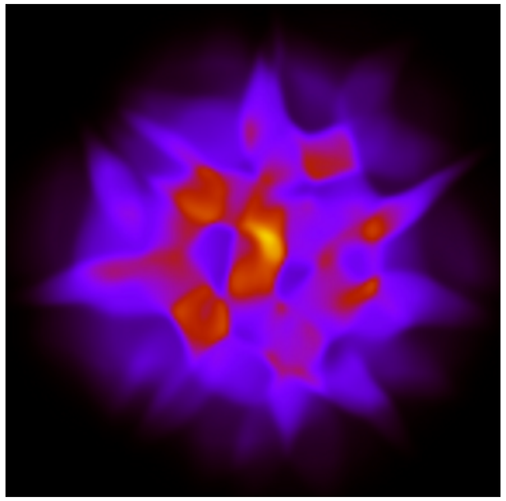
- connects to Berry curvature and angular momentum conservation.

*Murakami, Nagaosa, Shou-Cheng Zhang, Science 2003'*

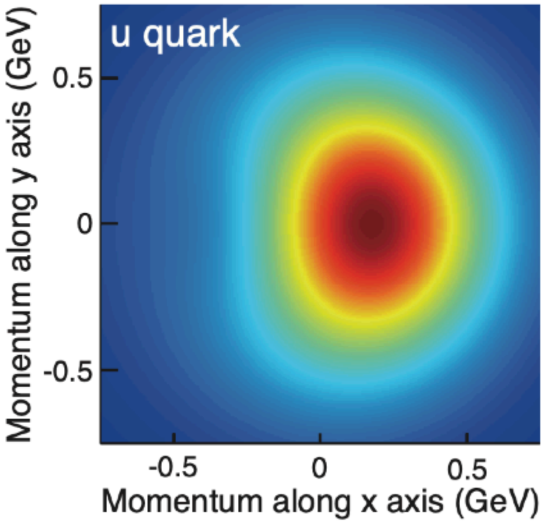
# Heavy-ion collisions

- We may generalize SHE by replacing electric field by the effective force $\vec{F}$ such as $T\nabla(\mu/T)$, T-gradient ….
- Spin transport induced by the gradient of hydro. field (e.g. flow and energy/charge density)?

NB: probing quantum correlation between spin and motion inside proton is one of the main goals of future electron-ion collider.



*Simulation by Schenke et al*




*transverse momentum dependent quark distribution in a polarized proton (fig from EIC white paper)*

# Hydro. limit

## Hydro.(long time and wave-length) limit

- For a generic fluid, conserved densities, e.g, energy and momentum density (related to flow velocity ) are only dynamic d.o.f. in this limit.

- Others observables $O$ (including spin polarization) are
  - slaved to the evolution of hydro. modes;
  - expressible in terms of hydro. fields via gradient expansion.
  - Expansion coefficients can be extracted from correlators $\langle O(x)T^{\mu\nu}(x')\rangle$.

- E.g.: the constitutive relation for viscous stress-tensor

$$(T^{\mu\nu})_{\text{vis}} \propto \eta\sigma^{\mu\nu}$$

## Axial Wigner function

$$\mathscr{A}^\mu(t,\vec{x},\vec{p}) = \int d^3\vec{y}\, e^{-i\vec{y}\cdot\vec{p}} \langle \bar{\psi}(t,\vec{x}-\frac{1}{2}\vec{y})\,\gamma^\mu\gamma^5\psi(t,\vec{x}+\frac{1}{2}\vec{y})\rangle$$

- describes the phase space distribution of spin vector $\vec{s}$.
- Building blocks for the gradient expansion:

$$\theta = \partial_\perp \cdot u\,,$$

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\lambda}u_\nu\partial_\alpha^\perp u_\lambda\,, \qquad \beta^{-1}\partial_\perp^\mu\beta\,,$$

$$\sigma^{\mu\nu} = \frac{1}{2}(\partial_\perp^\mu u^\nu + \partial_\perp^\nu u^\mu) - \frac{1}{3}\Delta^{\mu\nu}\theta\,.$$

*Tensors from gradient (focus on the neutral fluid)*

$$p^\mu = \epsilon u^\mu + p_\perp^\mu,$$

$$Q^{\mu\nu} = -\frac{p_\perp^\mu p_\perp^\nu}{p_\perp^2} - \frac{1}{3}\Delta^{\mu\nu}, \ldots$$

*Tensors formed by single particle momentum*

$$\text{e.g.}: \mathscr{A}^\mu \sim \epsilon^{\mu\nu\alpha\lambda}u_\nu Q_{\alpha\rho}\sigma^\rho{}_\lambda$$

## The derivative expansion

Shuai Liu, YY, 2103.09200, JHEP

- The most general expression consistent with symmetries (for a neutral fluid):

$$u \cdot \mathscr{A} = \tilde{c}_\omega p \cdot \omega \,,$$

$$\mathscr{A}^\mu_\perp = c_\omega \omega^\mu + c_T \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \partial_\lambda \log \beta + g_\sigma \epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma^\rho{}_\lambda + g_\omega Q^{\mu\nu} \omega_\nu$$

vorticity effects — spin Nernst effect — shear-induced polarization

$$\vec{s} \propto \hat{p} \times \nabla \log T$$

Spin-momentum correlation

- Although allowed by symmetry, flow gradient and momentum quadrupole coupling, has been overlook before.

- All above effects might be non-dissipative (associated coefficients are T-even).

## Results in hydro. limit

Shuai Liu, YY, 2103.09200;

see also 2103.10917 by Becattini et al

- For general fermion mass at one-loop:

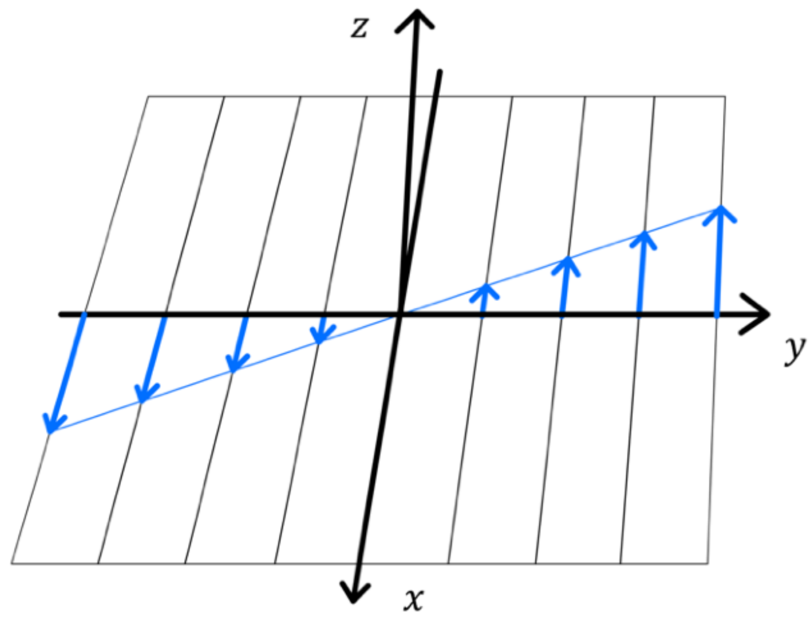
$$\mathscr{A}^{\mu}_{\perp} = (-n'_{FD})\left[\omega^{\mu} + \epsilon^{\mu\nu\alpha\lambda}u_{\nu}p_{\alpha}\partial_{\lambda}\log\beta + \frac{-p_{\perp}^{2}}{(p\cdot u)}\epsilon^{\mu\nu\alpha\lambda}u_{\nu}Q_{\alpha\rho}\sigma^{\rho}{}_{\lambda}\right] + 0\times Q^{\mu\nu}\omega_{\nu}$$

vorticity effects — spin Nernst effect — shear-induced polarization

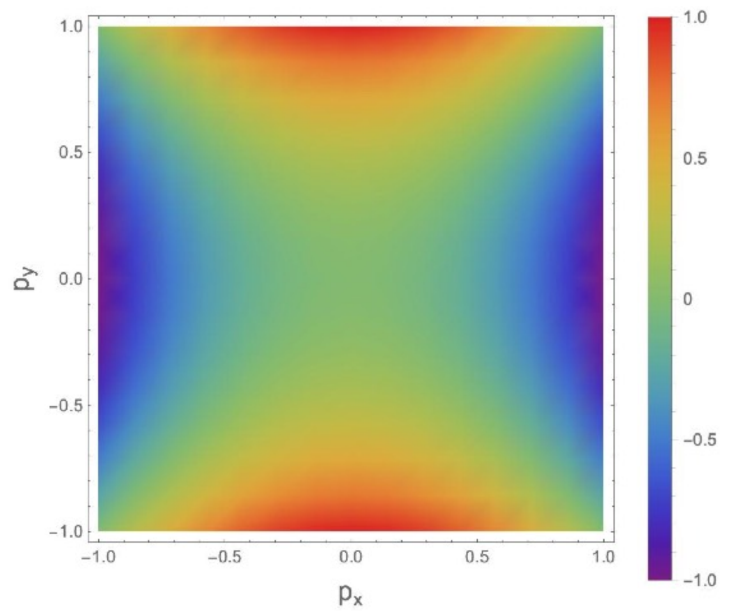
- Consistent with (collisionless) quantum kinetic theory analysis.

Collisional effect: ;Shu Lin & Ziyue Wang, 2206.12573; Wagner, Wickgenannt, Speranza, Rischke, 2208.01955

# Shear-induced polarization: an illustration




A standard shear flow profile:
$\omega^z \neq 0\,, \sigma^{xy} \neq 0$




Spin polarization along z-direction in phase space from SIP.

$$\mathscr{A}^i_{SIP} \propto \epsilon^{ikj} Q_{jl} \sigma^l{}_k\,, \qquad Q_{ij} = \hat{p}_i \hat{p}_j - \frac{1}{3}\delta_{ij}$$

Shear-induced polarization (SIP): imaging anisotropy in a fluid into anisotropy in spin space.

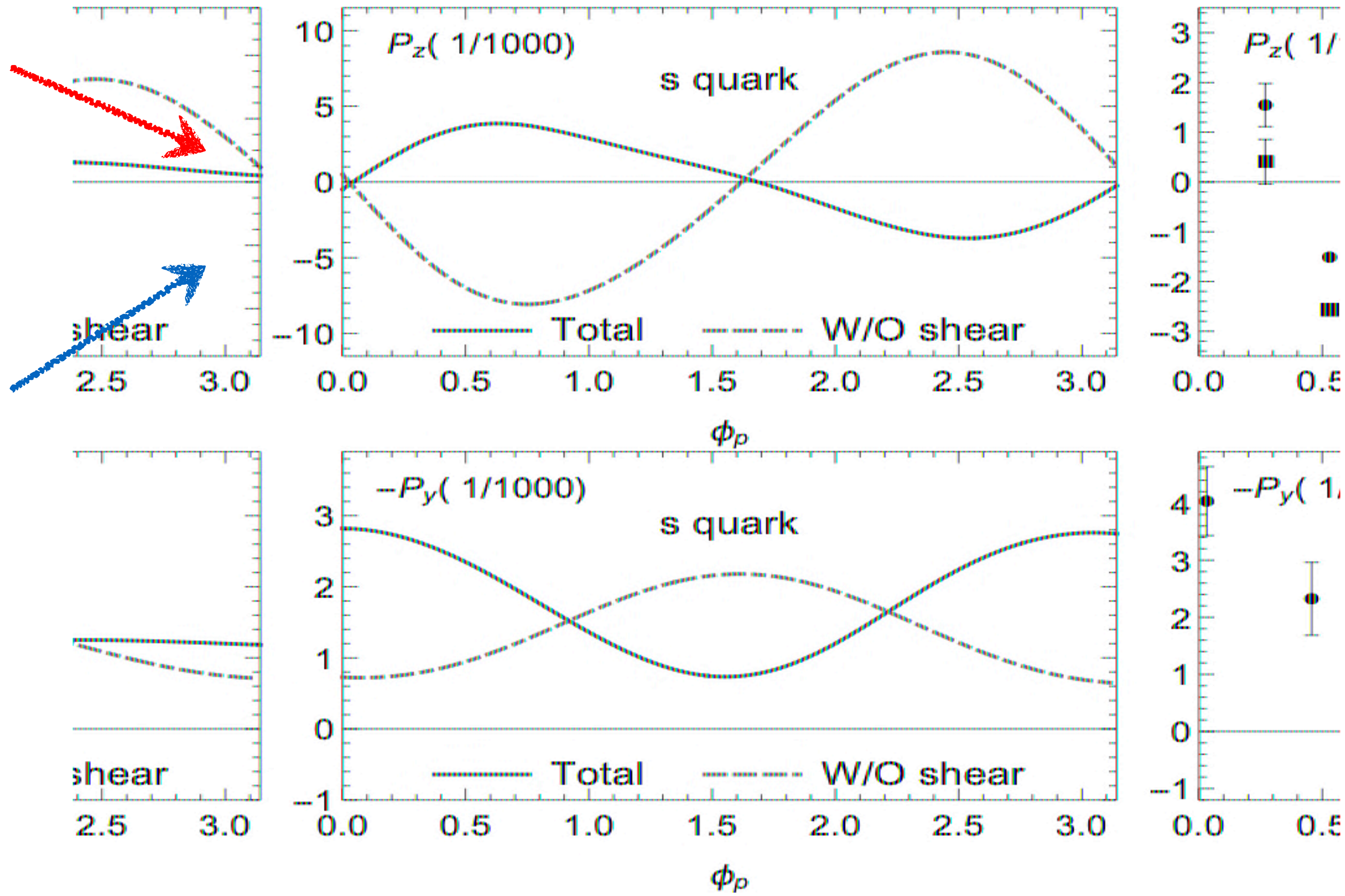
# SIP at RHIC

*Baochi Fu, Shuai Liu, Longgang Pang, Huichao Song, YY, 2103.10403, PRL 21*

*see also 2103.14621, PRL 21 by Becattini et al.*

*With SIP*

*Without SIP*



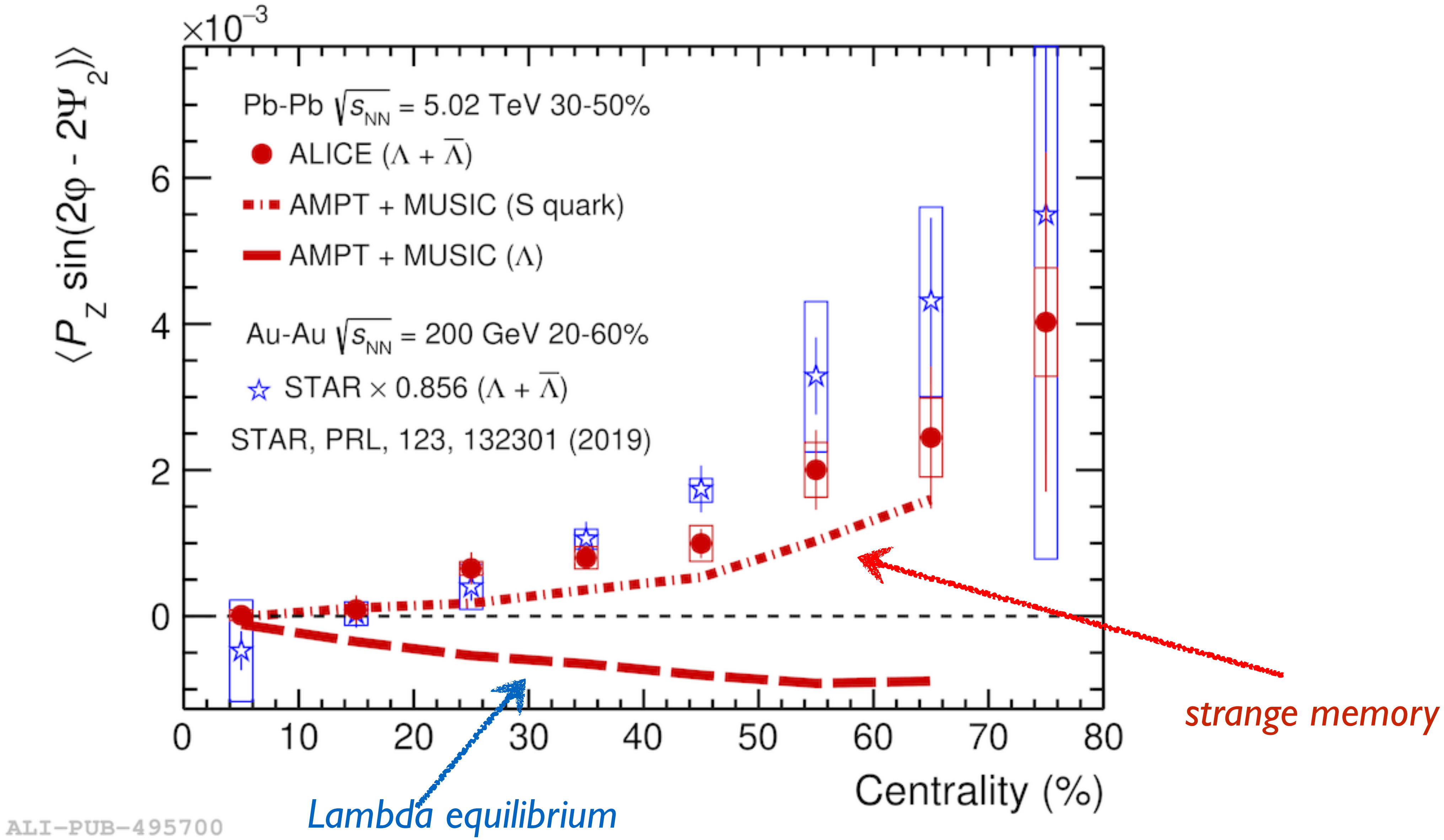

*vs transverse azimuthal angle* $\phi_p$

- determines the qualitative feature of differential polarization.
- Main uncertainties in quantitative study arise from freeze-out prescription for spin and subsequent hadronic evolution.

# SIP at LHC

*ALICE collaboration: 2107.11183 PRL ;*

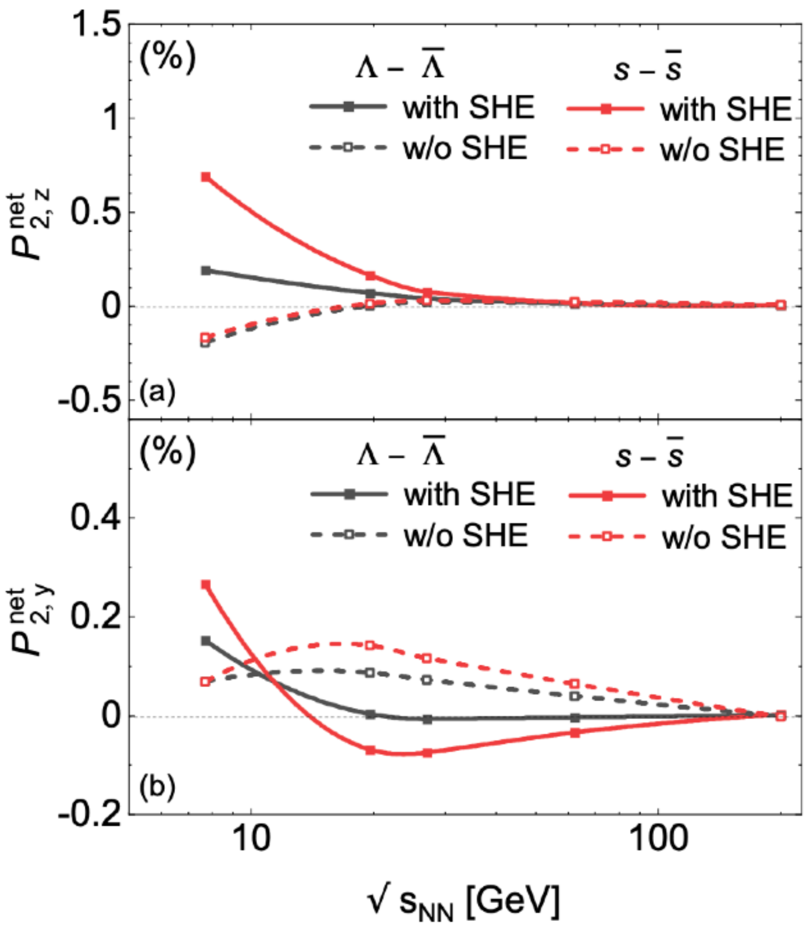
*benchmark calculations by Baochi Fu&Huicaho Song*

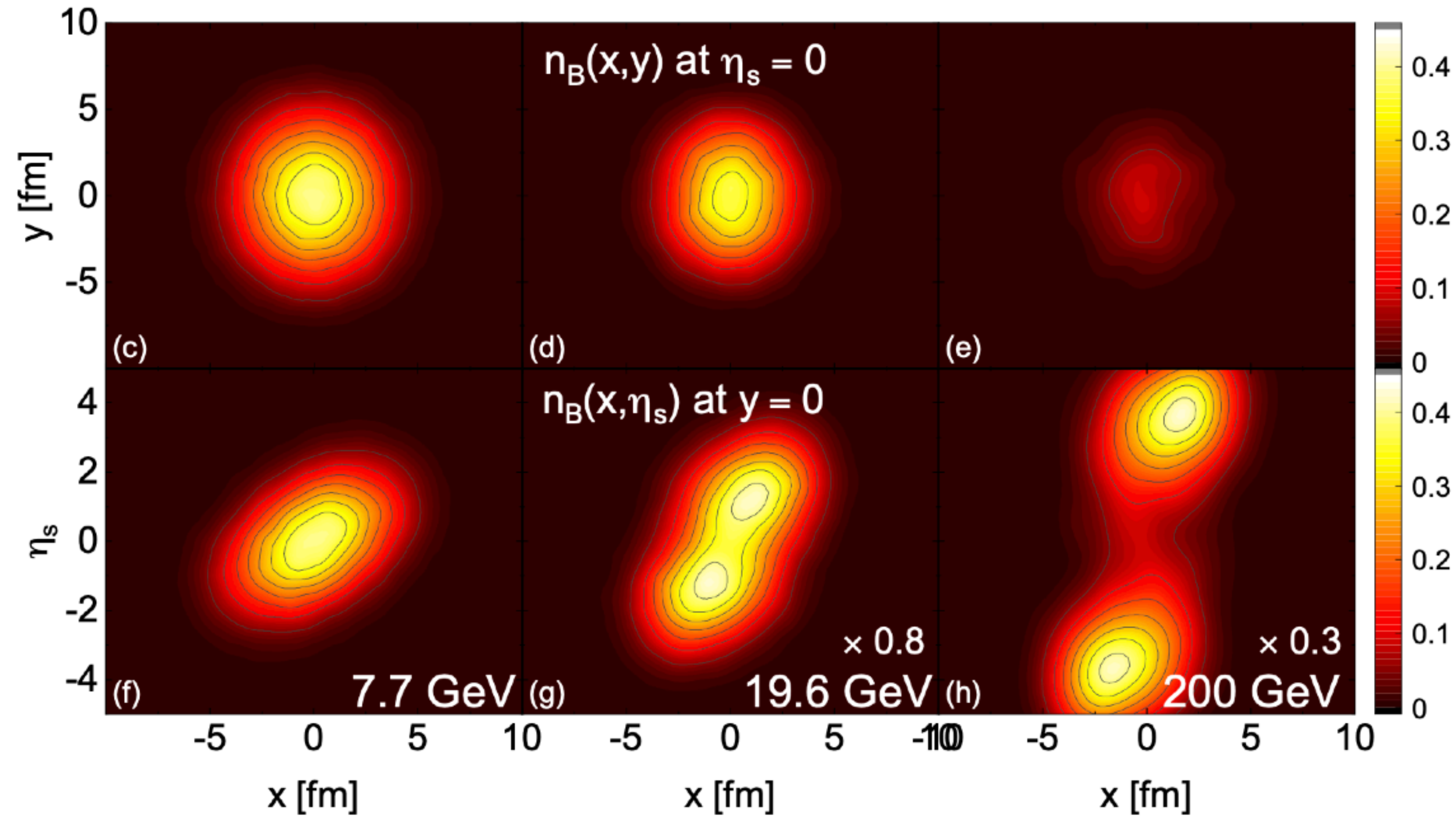



- Tantalizing evidence for SIP; more efforts are needed.

*The observation of SIP in QGP might be its first detection among all kinds of fluid!*

# Baryonic spin Hall effect probes medium created at lower beam energy




$$\hat{s}_{\pm} \propto \pm \hat{p} \times \nabla \mu_B$$

*Analogy to SHE*

*Theory: Son, Yamamoto, PRD 12; Di-Lung Yang, Hattori, Yoshimasa, PRD 19' … ; First phenomenological study: Liu-YY, 2006.12421, PRD21; also Shen et al, 2106.08125*

- The proposed observables is sensitive to initial baryon stopping and EoS at finite $\mu_B$.

*Baochi Fu, Longgang Pang, Huichao Song, YY, 2201.12970.*

- Experimental analysis is under way (hungry for statistics). Studying forward rapidity?

# Beyond hydro. limit

# Motivation


- QGP at mesoscopic scale where typical gradient k is too large for vHydro. and too short for pQCD:
  - Unexplored regime;
  - may feature new emergent properties; *e.g. Weiyao Ke and YY, 2208.01046, PRL 23*
  - new insights from spin effects? (complementary to the study of jet-medium interaction)
- For small colliding systems or at early stages of a collision, the characteristic gradient can be compare to $\tau_{\rm mft}^{-1}, l_{\rm mft}^{-1}$ .

# Retarded correlators

$$G_R \sim \langle \hat{O}\, \hat{T}^{\alpha\beta} \rangle$$

- describes $\delta O$ induced by metric perturbation.

$$\delta O(\omega, q) = G_R(\omega, q)\, h(\omega, q)$$

- gives response to initial perturbation $T^{\mu\nu}(t = 0, q) = T^{\mu\nu}_{init}(q)$ (can be prepared by turning on $h_{\mu\nu}$ from $t = -\infty$ to $t = 0$).

*see Weiyao Ke and YY, in preparation for the quantitative expression*

- Focusing on the induced spin distribution

$$G^{\mu;\alpha\beta} = \int_{\vec{y}} e^{i\vec{y}\cdot\vec{p}} \langle \bar{\psi}(t, \vec{x} - \frac{1}{2}\vec{y})\, \gamma^\mu \gamma^5 \psi(t, \vec{x} + \frac{1}{2}\vec{y})\, T^{\alpha\beta}(0,0) \rangle$$

## Results at finite frequency/gradient

- Correlator $G_R^{\mu;\alpha\beta}(q_0, \vec{q}; \vec{p})$ at one loop

  *Imaginary time formalism: Shuai Liu, YY, 2103.09200, JHEP 21; Complete analysis using Schwinger-Keldysh formalism, Zonglin Mo, YY in preparation.*

e.g. $$G^{i,0j}(q_0, \vec{q}; \vec{p}) = \frac{i}{2}\frac{\partial n_F}{\partial \epsilon_p}\frac{\vec{v}\cdot\vec{q}}{q_0 - \vec{v}\cdot\vec{q} + i0^+}\epsilon^{ijl}\left(q_l - q_0 v_l\right) + \mathcal{O}(\frac{q_0, q}{T})$$

- In hydro. regime: $\vec{s} \propto \overrightarrow{\omega}$ .

- Beyond hydro. regime, new feature emerges: rotation induces spin-motion correlation

$$\tau_{\rm mft}^{-1} \ll q \ll \omega \ll T: s^i \propto \frac{2}{3}\tilde{\omega}^i - (v^i v^l - \frac{1}{3}\delta^{il})\tilde{\omega}_l \qquad \tilde{\omega}^i \equiv \epsilon^{ijk}\partial_j(\frac{T_{0k}}{\epsilon + p})$$

*Preliminary*

*generalized vorticity*

# Quantum kinetic theory

# Quantum kinetic theory

- right tool to describe spin-motion correlation and non-hydro. response.

- Top-down approach (most popular): starting from Dirac action, deriving EOM for independent components of Wigner function

  *see Hidaka et al, 202 1.07644 for the review.*

$$W_{ab}(t,\vec{x},\vec{p}) = \int d^3\vec{y}\, e^{-i\vec{y}\cdot\vec{p}} \langle \psi_a(t,\vec{x}+\frac{1}{2}\vec{y})\, \bar{\psi}_b(t,\vec{x}-\frac{1}{2}\vec{y})\rangle$$

*(4 by 4)*

- "Bottom-up" approach (hydro-like):

Eqn for "slow" components of $W_{ab}$+ constitutive relation for observables of interests

## Chen-Son

*Jingyuan Chen and Dam Son 1604.07857*

- reformulated the kinetic theory of Landau Fermi liquid with "bottom-up" approach.
- verified the matching between the proposed kinetic theory and field theory calculations to all orders in perturbation theory for a large class of interacting field theories for linear response to EM field.
- We extend their formalism to a hot plasma and confirm the matching to field theory at one-loop order for both EM and metric perturbation.

*Zonglin Mo, YY, in progress.*

## Distribution

- We propose the distribution of particle at given spin state as the dynamical d.o.f. of kinetic theory

$$f^s(\vec{p}) = W_{ab}(\vec{p})\, u_{a,s}(\vec{p})\bar{u}_{b,s}(\vec{p})$$

or: $f^V = (f^+ + f^-)/2\,, \qquad f^A = (f^+ - f^-)$

- Kinetic eqn. for $f^s$ takes the Boltzmann form
    - e.g. EM field perturbation (in chiral limit)

$$f^V_{(0)} = \frac{n'_F\, \overrightarrow{E} \cdot \vec{v}}{\omega - \vec{v}\cdot\vec{q} + i0^+} \qquad\qquad f^A_{(1)} = \frac{n'_F\, \vec{v}\cdot\vec{q}\,(\overrightarrow{B}\cdot\vec{v})}{\omega - \vec{v}\cdot\vec{q} + i0^+}$$

## Constitutive relation

- expressing observables in terms of $f$ and external field via gradient expansion, e.g.

$$\mathscr{A}^i_{(1)}(\vec{p}) = v^i f^A_{(1)} + \mu^{ij}(\vec{p})\partial_j f^V_{(0)} + \Delta v^i n_F + \sigma^{i\mu\nu}(n_0, n_0', \vec{p})\, F_{\mu\nu}$$

(c.f. $J^i(x) = n(x)u^i(x)$ ) (c.f. $J^i(x) = D\,\delta^{ij}\partial_j n(x)$ ) CME+ Spin Hall effect

magnetization current

The fact that the gradient expansion exists is remarkable given the $G_R$ is non-analytic in $q_0, q$

- $\mu^{ij}$ is related to magnetic dipole moment and can be extracted from vertex functions.

$$\mu^{ij}(\vec{p}) = \frac{\epsilon^{ijk} v_k}{2\epsilon_p} + \propto (g-2)$$

- Velocity shift: $\Delta\vec{v} = \partial_{\vec{p}}(\mu^{ij}F_{ij})$.

- Generalized conductivity $\sigma^{i\mu\nu}$ leads to CME and spin Hall effect.

# The metric perturbation

*(In "TT" gauge $h_{0\mu} = 0, h_{ii} = 0$. The response to a general $h_{\mu\nu}$ can be obtained via diffeomorphism)*

$$\mathscr{A}^i_{(1)}(\vec{p}) = \boxed{v^i f_{(1)}(\vec{p}) + \mu^{ij}(\vec{p})\,\partial_j f_{(0)}(\vec{p}) + \Delta v^i f_{(0)}} + \boxed{\sigma_g^{ijk}\partial_t h_{ij}}$$

*the same as EM response*

- $\sigma_g$-term gives spin Hall like effect:

$$s^i \propto \epsilon^{ijk} v_j (v^l \partial_t h_{lk}) \propto (\vec{v} \times (d\vec{p}/dt))$$

- $\sigma_g^{ijk}$ can be related to $\mu^{ij}$ by imposing Lorentz covariance. (NB: $\nabla_{(i}u_{j)} = \partial_{(i}u_{j)} + \partial_t h_{ij}$): shear-induce polarization implies spin polarization in the presence of gravitation wave.

## Discussion

- Kinetic constitutive relation
  - observables is a functional of $f^s$ (and its gradient) but not that of hydro. field.
  - the expansion parameter is $q/T (q/\mu)$ suppressing particle-antiparticle mixing (when describing particle distribution, anti-particle contribution can be integrated out).
- In bottom-up formulation, only a few components of the Wigner function are dynamical, other components can be determined via gradient expansion. (following the philosophy of EFT)
- Not covered: kinetic theory from the action and symmetry principle.

# Summary and outlook

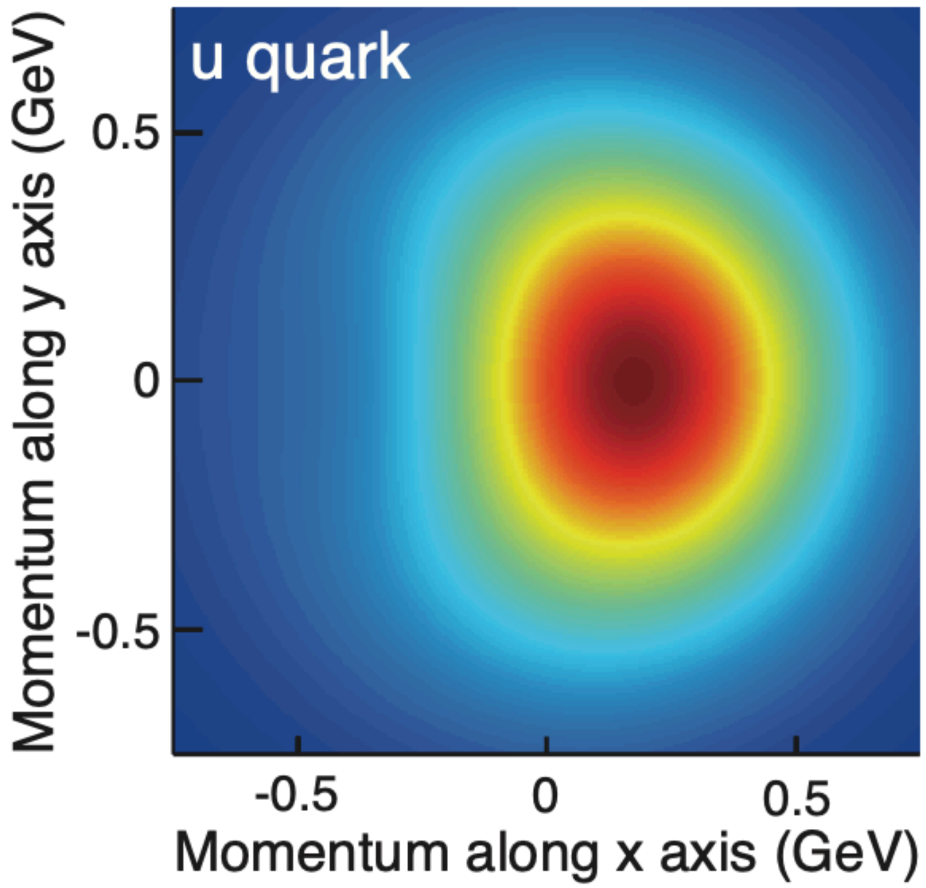
## Summary

- Spin observables probes the QCD phase structure.
  - Shear stress (or more generally hydro. gradient) are important in generating spin-motion correlation.
- New spin effects may emerge beyond hydro. limit.
- "Bottom-up" approach for quantum kinetic theory complements the convention approach.

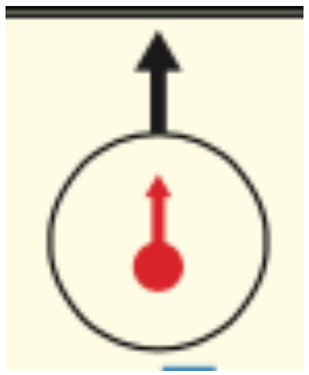
Ultimate goal: spin structure of QCD systems.

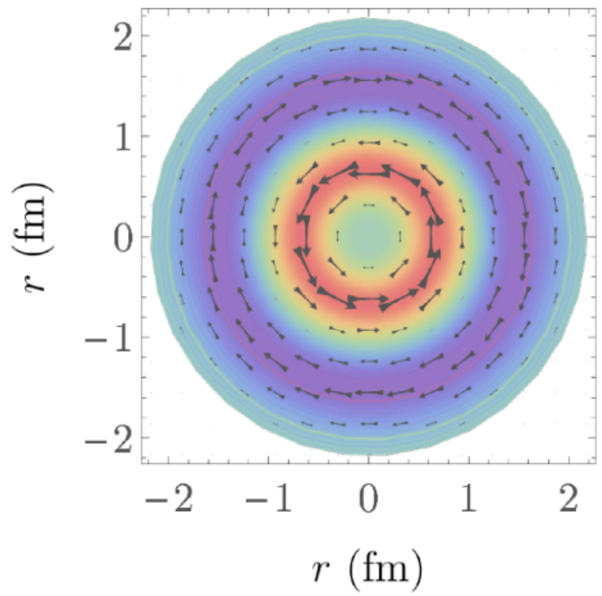
# Back-up

# Spin transport inside nucleons




*transverse momentum dependent quark distribution in a polarized proton (fig from EIC white paper)*






*lattice calculation of shear force inside proton.*

*Shanahan-Detmold, PRL 18*

- Correlation with shear force?

Ultimate goal: spin structure of QCD systems.

## Berry gauge field

- Berry gauge field: average position of an eigenstate.

$$\vec{a}(\hat{p}) = i\langle u(\vec{p},\lambda)\,|\,\partial_{\vec{p}}\,|\,u(\vec{p},\lambda)\rangle > = \langle \hat{x} \rangle_u$$

- Berry curvature: $\vec{b}(\hat{p}) = \nabla \times \vec{a}(\hat{p}) \parallel \lambda\hat{s}$
  - carries topological charge;
  - plays an important role in describing quantum transport.