

Nuclear matrix elements of $0\nu\beta\beta$ decay: from MR-CDFT to MR-IMSRG

Jiangming Yao

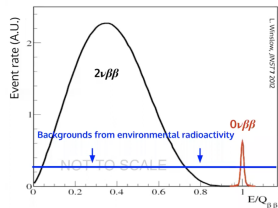
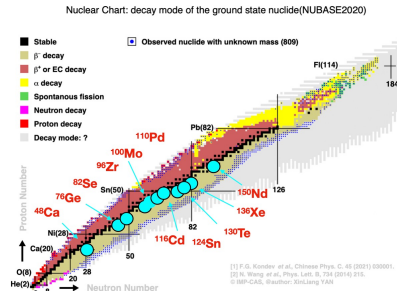
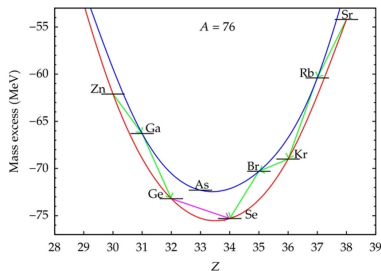
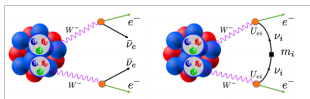
School of Physics and Astronomy, Sun Yat-sen University, China

INT-26-2a program: Recommended Values for Neutrinoless Double-Beta Decay
Nuclear Matrix Elements, July 22, 2026, Seattle, WA

- ① Introduction
- ② MR-CDFT for the NMEs of $0\nu\beta\beta$ decay
 - A brief introduction to the MR-CDFT
 - The NMEs of different decay channels
 - Uncertainty quantification and correlation analysis
- ③ MR-IMSRG for the NMEs of $0\nu\beta\beta$ decay
 - A brief introduction to the IM-GCM
 - The first-wave ab initio studies of ^{48}Ca
 - Uncertainty quantification and correlation analysis
- ④ Summary and outlook

- The two modes of $\beta^-\beta^-$ decay:

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + (2\bar{\nu}_e)$$

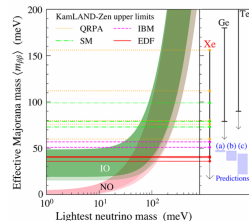


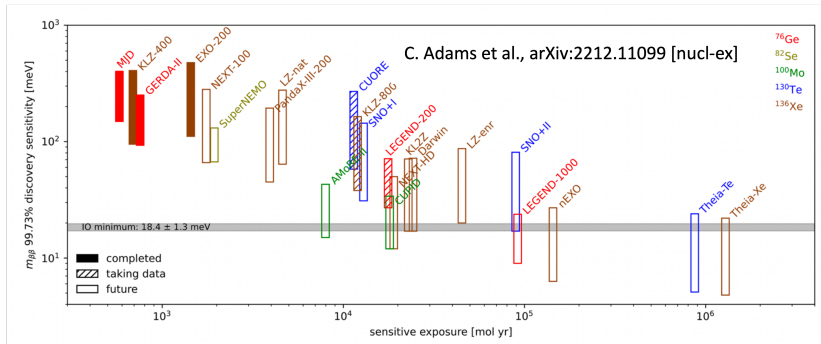
Isotope	$G_{0\nu}$ [10^{-14} yr^{-1}]	$M^{0\nu}$ [min, max]	$T_{1/2}^{0\nu}$ [yr]	$\langle m_{\beta\beta} \rangle$ [meV]	Experiments with best sensitivity References
^{48}Ca	2.48	[0.85, 2.94]	$> 5.8 \cdot 10^{22}$	[2841, 9828]	CANDLES: PRC78, 058501 (2008)
^{76}Ge	0.24	[2.38, 6.64]	$> 1.9 \cdot 10^{26}$	[75, 200]	LEGEND-200: PRL136, 022701 (2026)
^{82}Se	1.01	[2.72, 5.30]	$> 4.6 \cdot 10^{24}$	[277, 540]	CUPID-0: PRL129, 111801 (2023)
^{96}Zr	2.06	[2.86, 6.47]	$> 9.2 \cdot 10^{21}$	[3557, 8047]	NPA847, 168 (2010)
^{100}Mo	1.59	[3.84, 6.59]	$> 2.9 \cdot 10^{24}$	[210, 610]	AMoRE-I: PRL134, 082501 (2025)
^{116}Cd	0.48	[3.29, 5.52]	$> 2.2 \cdot 10^{23}$	[1766, 2963]	PRD 98, 092007 (2018)
^{130}Te	1.42	[1.37, 6.41]	$> 3.5 \cdot 10^{25}$	[70, 250]	CUORE: Science (2025)
^{136}Xe	1.46	[1.11, 4.77]	$> 3.8 \cdot 10^{26}$	[28, 122]	KamLAND-Zen: PRL135, 262501 (2025)
^{150}Nd	6.30	[1.71, 5.60]	$> 2.0 \cdot 10^{22}$	[1593, 5219]	NEMO-3: PRD 94, 072003 (2016)

If $0\nu\beta\beta$ decay is driven by exchanging light Majorana neutrinos,

$$\langle m_{\beta\beta} \rangle \equiv \left| \sum_{j=1}^3 U_{ej}^2 m_j \right| = \left[\frac{m_e^2}{g_A^4 G_{0\nu} T_{1/2}^{0\nu} |M^{0\nu}|^2} \right]^{1/2}$$

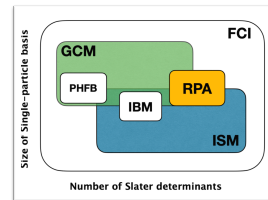
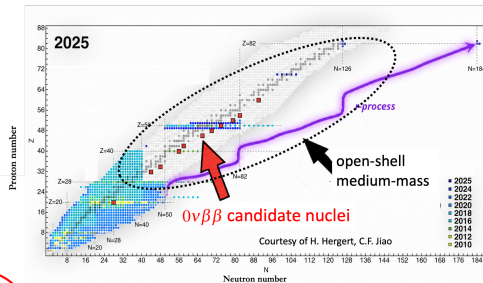
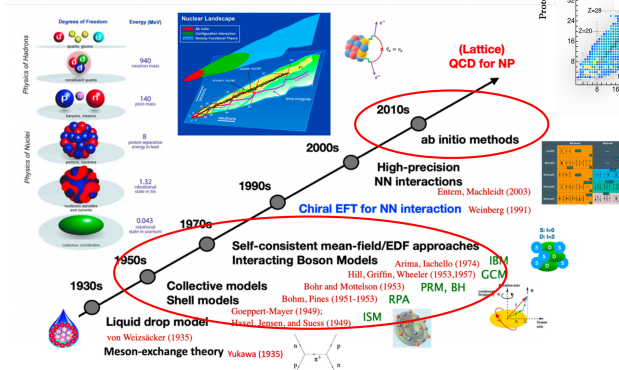
Accurate values of the NMEs $M^{0\nu} = \langle 0_F^+ | \hat{O}^{0\nu} | 0_I^+ \rangle$ are crucial for designing and interpreting those experiments, as they link the observed decay rate to the effective neutrino mass $\langle m_{\beta\beta} \rangle$.





- Lifetime sensitivity of the future ton-scale experiments: $T_{1/2}^{0\nu} > 10^{28} \text{ yr}$.
- Whether or not the ton-scale experiments are able to cover the entire parameter space for the IO case depends strongly on the employed NME, with a large model dependence (**uncertainty of each model is not quantified!**).

- Geometric models
- Phenomenological models (ISM, RPA, EDF, etc)
- Ab initio methods



MR-CDFT: a relativistic EDF based quantum-number projected generator coordinate method (PGCM)

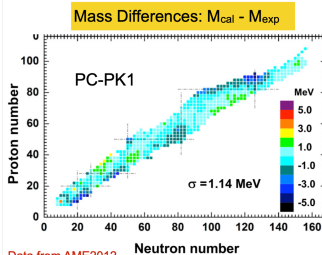
★Point-coupling models

$$(\bar{\psi}\Gamma\psi)^2 \quad \textcolor{red}{D} \quad (\bar{\psi}\psi)\Delta(\bar{\psi}\psi)$$



$$\begin{aligned}
 \mathcal{E}^{\text{int}}(\mathbf{r}) = & \frac{\alpha_S}{2} \rho_S^2 + \frac{\beta_S}{3} \rho_S^3 + \frac{\gamma_S}{4} \rho_S^4 + \frac{\delta_S}{2} \rho_S \Delta \rho_S \\
 & + \frac{\alpha_V}{2} j_\mu j^\mu + \frac{\gamma_V}{4} (j_\mu j^\mu)^2 + \frac{\delta_V}{2} j_\mu \Delta j^\mu \\
 & + \frac{\alpha_{TV}}{2} \vec{j}_{TV}^\mu \cdot (\vec{j}_{TV})_\mu + \frac{\delta_{TV}}{2} \vec{j}_{TV}^\mu \cdot \Delta (\vec{j}_{TV})_\mu
 \end{aligned}$$

~10 parameters fitted to 60 spherical nuclei ...

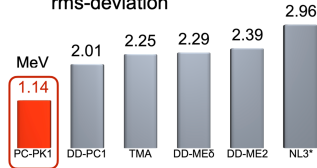


Data from AME2012

PWZ, Li, Yao, Meng, PRC 82, 054319 (2010)

Lu, Li, Li, Yao, Meng PRC 91, 027304 (2015)

rms-deviation



Agbemava PRC 2014
Geng PTP 2005

A slide from P. W. Zhao

Best density-functional description for nuclear masses so far!

- The GCM wave function is expanded in a set of nonorthogonal basis states for a given energy density functional (EDF): P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (1980)

$$|\Psi_\nu^J\rangle = \sum_{\mathbf{a}, K} f_\nu^{JK}(\mathbf{a}) |JK; \mathbf{a}\rangle, \quad |JK; \mathbf{a}\rangle \equiv \hat{P}_{MK}^J \hat{P}^N \hat{P}^Z |\Phi(\mathbf{a})\rangle$$

- The variational principle leads to the Hill–Wheeler–Griffin (HWG) equation:

$$\sum_{\mathbf{a}', K'} \left[\mathcal{H}_{KK'}^J(\mathbf{a}, \mathbf{a}') - E_\nu^J \mathcal{N}_{KK'}^J(\mathbf{a}, \mathbf{a}') \right] f_\nu^{JK'}(\mathbf{a}') = 0.$$

The Hamiltonian and norm kernels are given by

$$\mathcal{H}_{KK'}^J(\mathbf{a}, \mathbf{a}') = \langle JK; \mathbf{a} | \hat{H} | JK'; \mathbf{a}' \rangle, \quad \mathcal{N}_{KK'}^J(\mathbf{a}, \mathbf{a}') = \langle JK; \mathbf{a} | JK'; \mathbf{a}' \rangle.$$

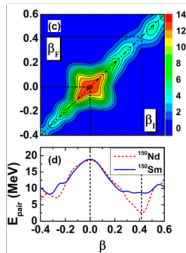
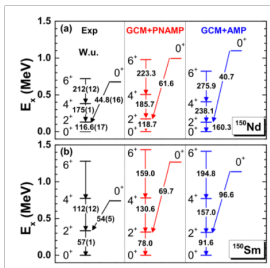
JMY, Meng, Ring, Vretenar, PRC 81, 044311 (2010); JMY, Hagino, Li, Meng, Ring, PRC 89, 054306 (2014)

Transition operators

$$\hat{O}_{0\nu} = \frac{4\pi R}{g_A^2(0)} \iint d^3x_1 d^3x_2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q}\cdot(\mathbf{x}_1-\mathbf{x}_2)}}{q} \times \sum_m \frac{\mathcal{J}_\mu^\dagger(\mathbf{x}_1)|m\rangle\langle m|\mathcal{J}_\mu^\dagger(\mathbf{x}_2)}{q + E_m - (E_I + E_F)/2},$$

Charge-exchange (one-body) nucleon current

$$\mathcal{J}_L^\mu = \bar{\psi}_N \gamma^\mu [g_V(q^2) - g_A(q^2)\gamma_5] \tau^+ \psi_N + \bar{\psi}_N [g_P(q^2)q^\mu \gamma_5 - ig_W(q^2)\sigma^{\mu\nu} q_\nu] \tau^+ \psi_N$$



Editors' Suggestion

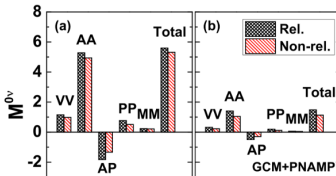
Relativistic description of nuclear matrix elements in neutrinoless double- β decay

L. S. Song, J. M. Yao, P. Ring, and J. Meng

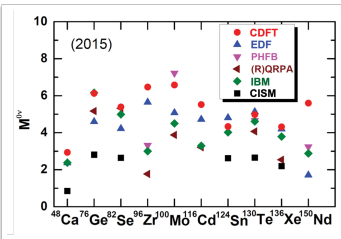
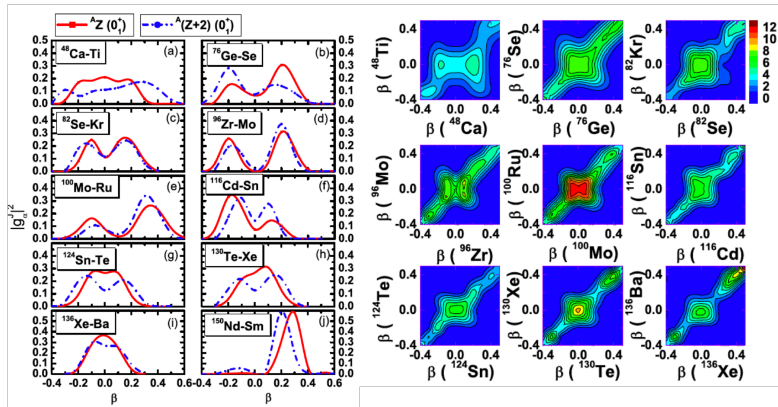
Phys. Rev. C **90**, 054309 – Published 10 November 2014



The authors report a fully relativistic description of the $0\nu\beta\beta$ -decay matrix element from ^{150}Nd to ^{150}Sm based on a state-of-the-art nuclear structure model. The calculations reproduce the low-energy spectra and electric quadrupole transitions in both nuclei well. The results are promising for next-generation double- β decay experiments.

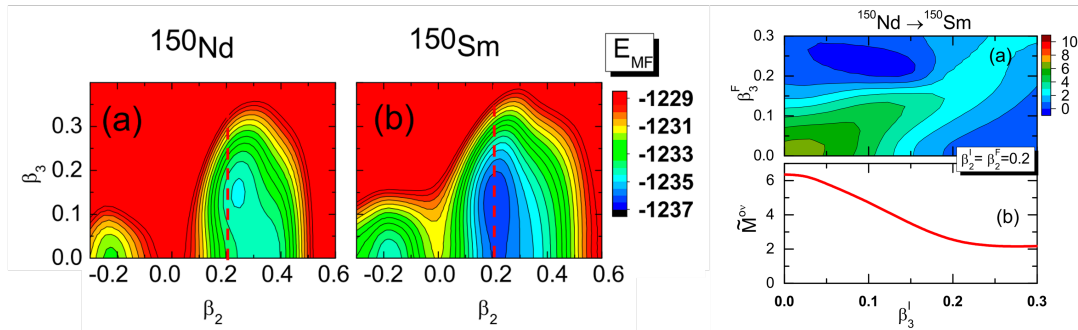


Relativistic effects: about 5% for the $gs2gs$ transition, and 24% for the $gs2ex$ transition in ^{150}Nd

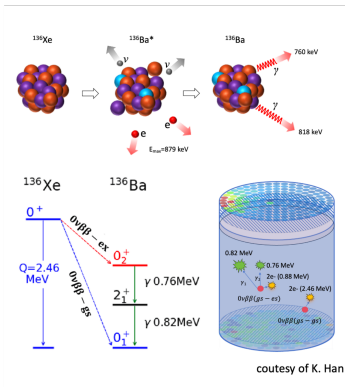


CDFT: JMY, L.S. Song, K. Hagino, P. Ring, J. Meng, *PRC* **91**, 024316 (2015)
EDF(D1S): T. R. Rodriguez and G. Martinez-Pinedo, *PRL* **105**, 252503 (2010)
PHFB: P. K. Rath et al. *PRC* **82**, 064310 (2010); *ibid.* **88**, 064322 (2013).
(R)QRPA: A. Faessler et al., *J. Phys. G: Nucl. Part. Phys.* **39**, 124006 (2012).
IBM: J. Barea, J. Kotila, and F. Iachello, *PRC* **87**, 014315 (2013)
CISM: J. Menendez et al., *Nucl. Phys. A* **818**, 139 (2009).
 + Other calculations

- The NMEs are generally large if both initial and final nuclei are spherical.
- The NMEs are generally small if the deformations of both nuclei are different.



- The $M^{0\nu}(0_1^+ \rightarrow 0_1^+)$ from the PHFB calculation drops from 6.4 to 2.2 as β_3 increases from zero to 0.3.
- The $M^{0\nu}(0_1^+ \rightarrow 0_1^+)$ from the PGCM calculation drops only from 5.6 to 5.2, and $M^{0\nu}(0_1^+ \rightarrow 0_2^+)$ from 1.48 to 0.70 after considering the β_3 fluctuation.



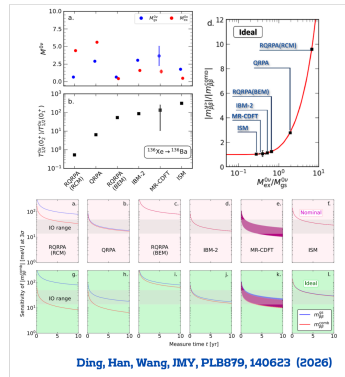
Parameters used in the combined analysis. The values for $0\nu\beta\beta\text{-gs}$ decay are taken from the XLZD [34] experiments.

	$0\nu\beta\beta\text{-gs}$	$0\nu\beta\beta\text{-ex}$ (nominal)	$0\nu\beta\beta\text{-ex}$ (ideal)
Isotopic abundance a	8.86%	8.86%	8.86%
Efficiency ϵ	0.76	0.6	0.6
Fiducial volume M [ton]	8.2	20	60
Background rate B [cts/yr]	0.3150	0.1260	0.0001

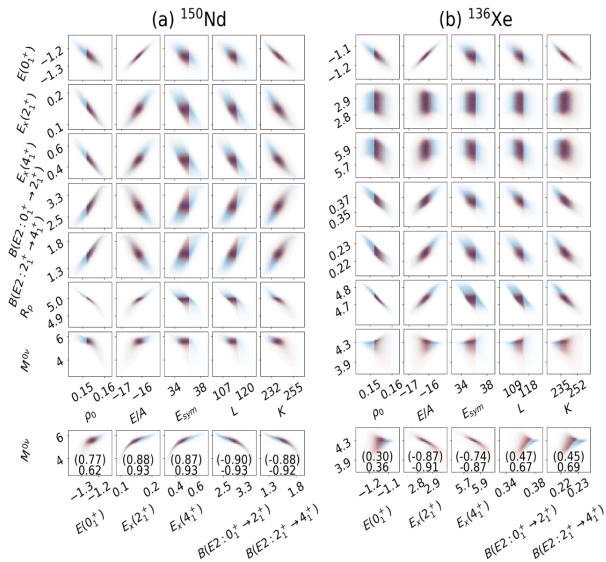
$$\begin{aligned}
 \Delta\chi^2 &= -2[\ln \mathcal{L}(N|B) - \ln \mathcal{L}(N|N)] \\
 &= 2 \sum_i \left[N_i \ln \left(1 + \frac{S_i}{B_i} \right) - S_i \right],
 \end{aligned}$$

where the total events $N_i = S_i + B_i$.

F. Pompa, T. Schwetz, J.Y. Zhu, JHEP 06 (2023) 104.

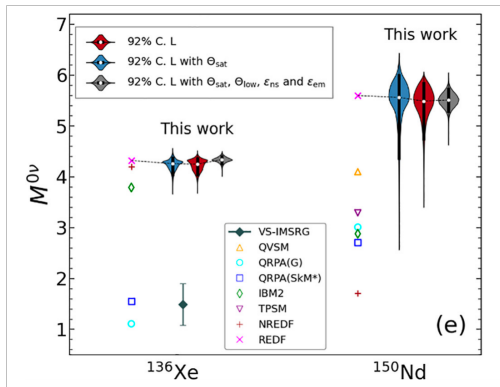


Improving sensitivity of ^{136}Xe from a combined analysis: precise knowledge of the NMEs for different decay channels is key to this strategy.



- 10^6 samples of EDF parameter sets.
- The correlations of $M^{0\nu}$ with nuclear matter properties are generally weak.
- The $M^{0\nu}$ is generally strongly correlated with the $E_x(2_1^+)$ and $B(E2: 0_1^+ \rightarrow 2_1^+)$, even though correlation relations are different for ^{150}Nd and ^{136}Xe . The corresponding Pearson coefficients for ^{150}Nd are 0.93 and -0.93 , respectively. For ^{136}Xe , the Pearson coefficients become -0.91 and 0.67, respectively.

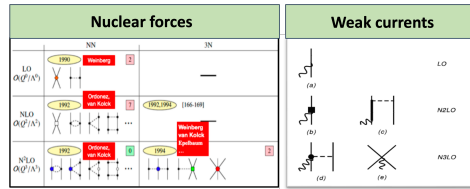
X. Zhang, C.C. Wang, C.R. Ding, JMY, PRC 112, L021302 (2025)



- Around PC-PK1: the $M_L^{0\nu}$ are $4.34^{+0.09}_{-0.10}$ for ^{136}Xe and $5.51^{+0.24}_{-0.26}$ for ^{150}Nd .
- Around PC-F1: the $M_L^{0\nu}$ are $4.21^{+0.07}_{-0.10}$ for ^{136}Xe and $6.07^{+0.30}_{-0.32}$ for ^{150}Nd .
- Statistical uncertainties are much smaller than the potentially dominant systematic uncertainties, which remain challenging to quantify.
- Ab initio nuclear theory is essential for reliably quantifying the systematic uncertainties in $M^{0\nu}$.

X. Zhang, C.C. Wang, C.R. Ding, JMY, PRC 112, L021302 (2025)

- Nuclear forces:** derived fundamentally from QCD or from chiral EFT in terms of $N(\Delta)$ and π degrees of freedom, with the LECs fitted to NN scattering data and few-body systems; the uncertainty is controlled by the power counting $(Q, m_\pi/\Lambda_\chi)^\nu$.
- Many-body solvers:** exact or with systematically improvable errors.
- Uncertainty quantification:** essential for searching new physics ($0\nu\beta\beta$, Schiff moments) at the high-precision frontier.



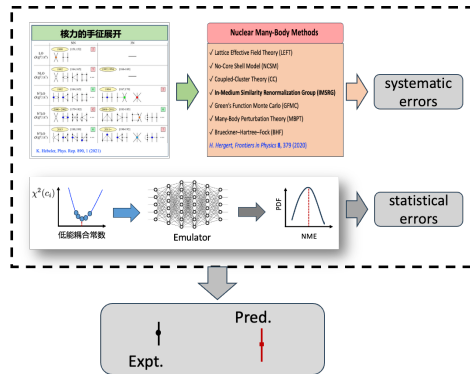
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Nuclear Many-Body Methods

- ✓ Lattice Effective Field Theory (LEFT)
- ✓ No-Core Shell Model (NCSM)
- ✓ Coupled-Cluster Theory (CC)
- ✓ **In-Medium Similarity Renormalization Group (IMSRG)**
- ✓ Green's Function Monte Carlo (GFMC)
- ✓ Many-Body Perturbation Theory (MBPT)
- ✓ Brueckner–Hartree–Fock (BHF)

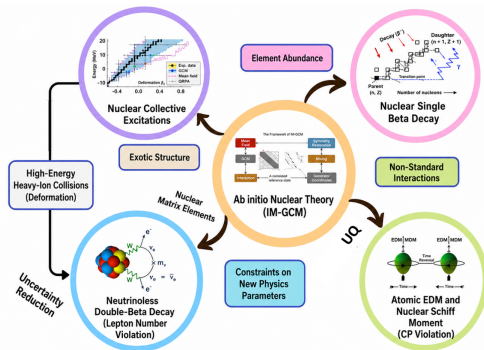
*H. Hergert, [Frontiers in Physics](#) **8**, 379 (2020)*

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Develop ab initio nuclear theory with controllable uncertainties for nuclear structure studies and searches for new physics.

- **Strategy for advancing nuclear theory:** Combine the ab initio **in-medium similarity renormalization group** (IMSRG) with traditional nuclear many-body approaches.
- **Applications:** Benchmark the framework against nuclear-structure observables and apply it to nuclear matrix elements relevant to searches for new physics.



See the talk by Heiko Hergert

IMSRG: embedding particle-hole correlations into the many-body Hamiltonian through a set of continuous unitary transformation.

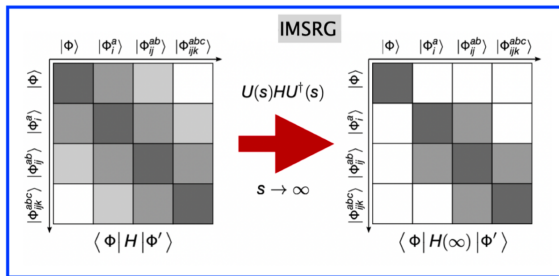
- Apply unitary transformations to H in the configuration space

$$\hat{H}(s) = \hat{U}(s)\hat{H}_0\hat{U}^\dagger(s)$$

Leading to the flow equation

$$\frac{d\hat{H}(s)}{ds} = [\hat{\eta}(s), \hat{H}(s)]$$

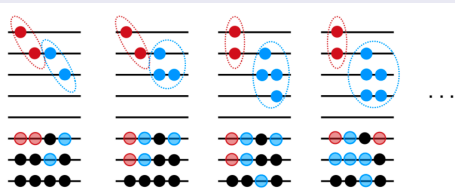
- Generator $\eta(s)$ is chosen in such a way that a given **reference state** is decoupled from its excitations.



If choosing a HF state as the reference state, one has

$$\hat{H}_0 |\psi_{\text{Exact}}^{(0)}\rangle = E_0 |\psi_{\text{Exact}}^{(0)}\rangle \longrightarrow \hat{H}(\infty) |\Phi_{\text{HF}}\rangle = E_0 |\Phi_{\text{HF}}\rangle$$

many-particle many-hole excitations



important for deformed nuclei, but
challenge for most ab initio methods!

IMSRG(3)

- Computational scaling $O(N^9)$
- memory storage N^6

IMSRG(A)

- From a single reference state $|\Phi_{\text{HF}}\rangle$ to exact ground state $|\Psi\rangle$

$$|\Psi\rangle = e^{\hat{\Omega}}|\Phi_{\text{HF}}\rangle,$$

where many-body correlations are
built into the correlation operator $\hat{\Omega}$,

$$\hat{\Omega} = \hat{\Omega}^{(1b)} + \hat{\Omega}^{(2b)} + \hat{\Omega}^{(3b)} + \dots + \hat{\Omega}^{(Ab)}$$

determined from the IMSRG.

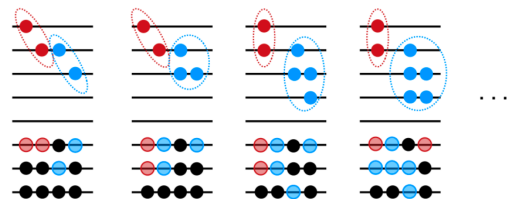
How to overcome this challenge?

Basic idea of MR-IMSRG

Build correlations into the reference state

$$\begin{aligned} |\Psi_{\text{Exact}}\rangle &= e^{\hat{\Omega}^{(1b)} + \hat{\Omega}^{(2b)} + \dots + \hat{\Omega}^{(Ab)}} |\Phi_{\text{HF}}\rangle \\ &\simeq e^{\hat{\Omega}^{(1b)} + \hat{\Omega}^{(2b)} + \dots} |\Phi_{\text{Cor}}\rangle \end{aligned}$$

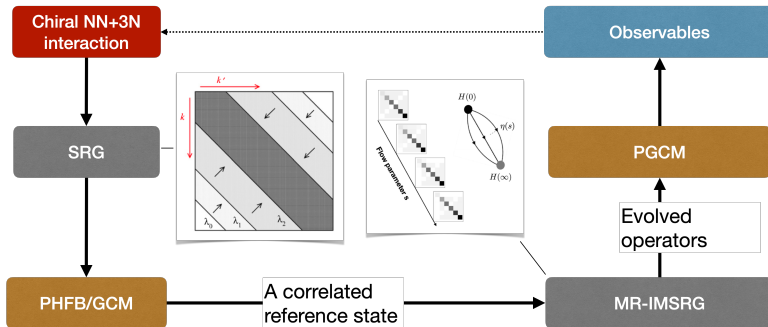
many-particle many-hole configurations



- The correlated reference state $|\Phi_{\text{Cor}}\rangle$ is chosen as a state with many-particle many-hole excitations relevant for nuclear collective excitations.
- Based on this idea, we have developed a novel **IM-GCM** for open-shell deformed nuclei, which enables the ab initio calculations of candidate $0\nu\beta\beta$ decay.

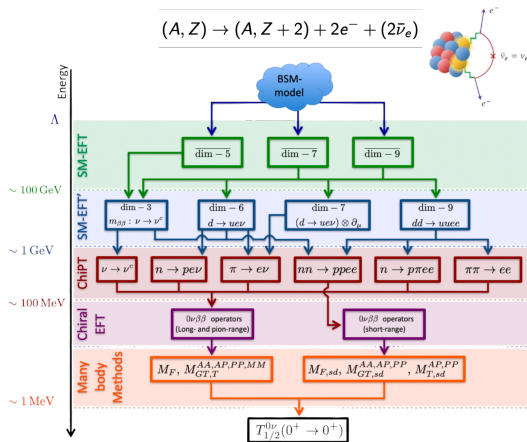
JMY, Bally, Engel, Wirth, Rodríguez, Hergert, PRL124, 232501 (2020)

The Framework of IM-GCM



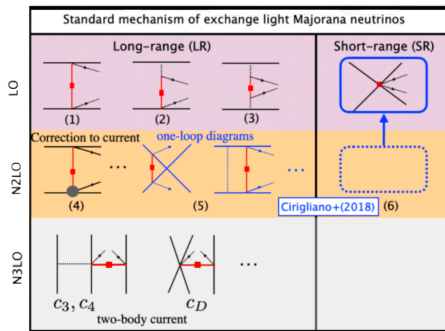
- **In-medium similarity renormalization group (IMSRG)**: capture dynamic correlations associated with high-energy few-particle, few-hole excitations
- **Projected generator coordinate method (PGCM)**: include the collective (static) correlations associated with pairing and deformation.

Lepton-number violating operators for neutrinoless double-beta decay



The $0\nu\beta\beta$ decay operators in the chiral EFT

Power Counting: $\nu = 2A + 2L - 2 + \sum_i \left(\frac{n_f}{2} + d - 2 + n_e \right)_i$



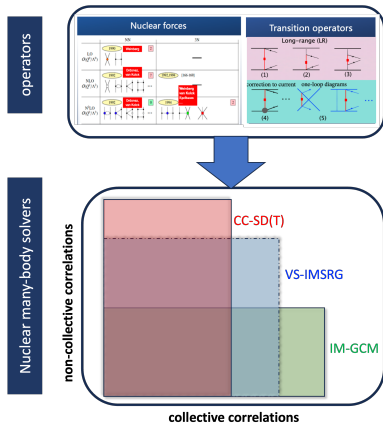
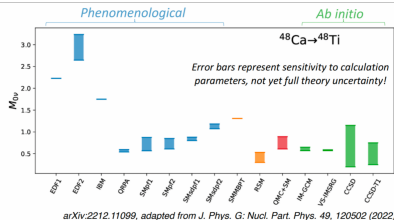
G. Prézeau, M. Ramsey-Musolf, P. Vogel, PRD68, 034016 (2003); V. Cirigliano et al., JHEP 12, 097(2018)

- Multi-reference in-medium generator coordinate method (IM-GCM)
- Valence-space shell model+IMSRG (VS-IMSRG)
- Coupled-cluster with singlets, doublets, and partial triplets (CCSDT1).

JMY et al., PRL124, 232501 (2020)

A. Belley et al., PRL126, 042502 (2021)

S. Novario et al., PRL126, 182502 (2021)



New: cross-checks among different many-body methods.

- A contact transition operator which could either enhance or quench the $0\nu\beta\beta$ decay, is needed to be promoted to LO to ensure renormalizability.

V. Cirigliano et al., PRL120, 202001 (2018)

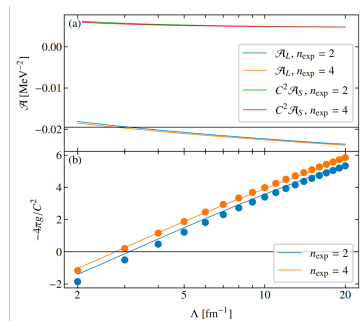
- The synthetic data for the process $2n \rightarrow 2p + 2e^-$ V. Cirigliano et al., PRL126, 172002 (2021); JHEP05, 289(2021)

$$\mathcal{A}_\nu(p=25, p'=30) = -0.0195(5) \text{ MeV}^{-2}.$$

The g_ν^{NN} is determined consistent with the employed chiral interaction, based on the synthetic data

$$\begin{aligned} V_{\nu, S}^{1S_0} &= -2g_\nu^{\text{NN}} V_S(r, r')_{\tau(1)+\tau(2)+} \\ V_S(r, r') &\equiv \left(\frac{m_N g_A^2}{4f_\pi^2} \right)^2 f_\Lambda^{n_{\text{exp}}}(r) f_\Lambda^{n_{\text{exp}}}(r') \\ f_\Lambda^{n_{\text{exp}}}(r) &= \frac{1}{2\pi^2} \int_0^\infty dq q^2 \exp \left[- \left(\frac{q}{\Lambda} \right)^{2n_{\text{exp}}} \right] j_0(qr) \end{aligned}$$

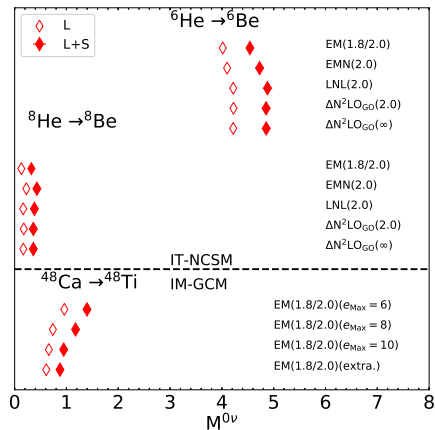
R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)

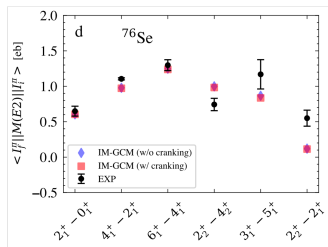
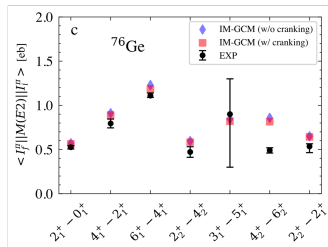
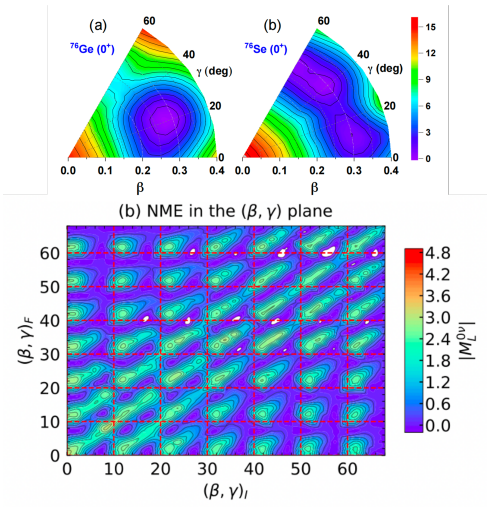


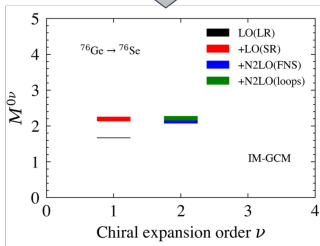
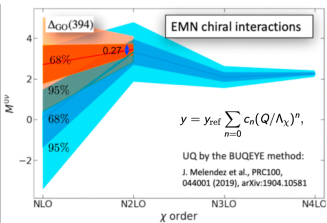
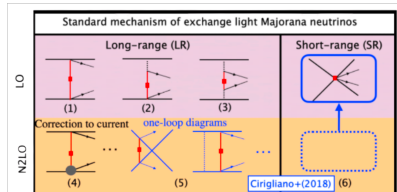
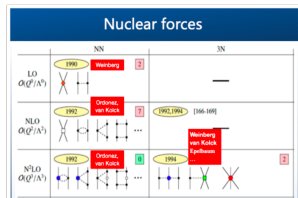
Interaction	λ	$10^3 \tilde{\mathcal{A}}_L$	$10^3 \mathcal{A}_L$	$10^3 \mathcal{A}_S$	$\tilde{g}_\nu^{NN}$	g_ν^{NN}	Δg_ν^{NN}
EM	∞	-15.847	-15.898	3.0152	0.606	0.597	0.083
	2.50	-15.921	-16.024	3.0635	0.584	0.567	0.082
	2.24	-15.923	-16.033	3.0451	0.587	0.569	0.082
	2.20	-15.923	-16.033	3.0408	0.588	0.570	0.082
	2.00	-15.912	-16.025	3.0061	0.597	0.578	0.083
	1.88	-15.898	-16.011	2.9733	0.606	0.587	0.084
	1.80	-15.885	-15.998	2.9446	0.614	0.595	0.085
EMN N ³ LO	∞	-15.857	-15.903	2.3816	0.765	0.755	0.105
	2.00	-15.934	-16.043	2.9031	0.614	0.595	0.086
$\Delta N^2 LO_{GO}$ (394)	∞	-15.846	-15.968	3.1225	0.585	0.566	0.080
	2.00	-15.776	-15.892	2.9610	0.629	0.609	0.084

- The contact term turns out to enhance the NME for ^{48}Ca by 43(7)%.

R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)

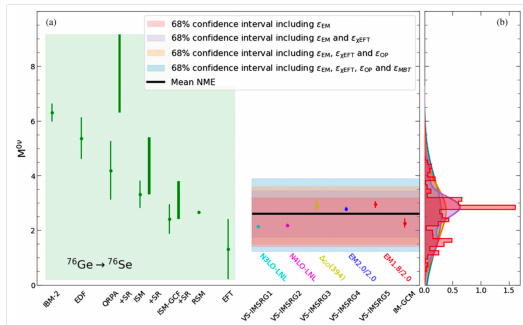
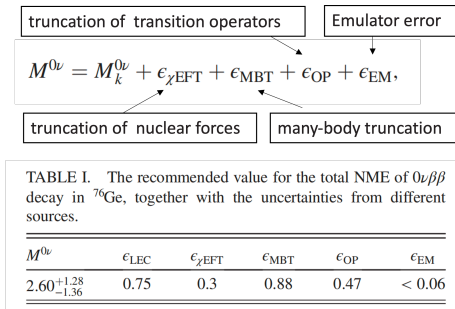




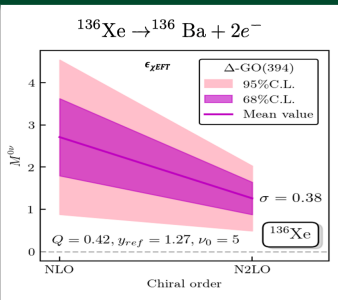


- The NME for ^{76}Ge converges rapidly with respect to the chiral expansion order of transition operators.
- The contact term is about 30%, while the N2LO contributions are about 10%.

See the talks by L. Jokiniemi and A. Belley



- Our recommended value $M^{0\nu} = 2.60^{+1.28}_{-1.36}$.
- Together with the best half-life limit: $> 1.8 \times 10^{26}$ yr, it sets the upper limit $\langle m_{\beta\beta} \rangle = 187^{+205}_{-62}$ meV, and the sensitivity of the next-generation experiment $\langle m_{\beta\beta} \rangle = 22^{+24}_{-7}$ meV, covering almost the entire range of IO hierarchy.



Interaction	LO		N ² LO		N ³ LO	
	long-range	contact	FNS	residual	TBC	Total
EM1.8-2.0	1.07	0.42(8)	-0.07	0.17	-0.32(3)	1.27(11)
N ² LO _{GO} (394)	0.88	0.39(7)	-0.07	0.17	-0.09(1)	1.28(8)

- The LO contact term enhances the NME by more than 30%.
- The net N²LO correction increases the NME by about 10%.
- The two-body current is included with the DD1B current approximation.
- Two different chiral interactions give consistent NMEs, $M^{0\nu} \approx 1.3$.

Results of VS-IMSRG with $e_{\text{Max}} = 14, E_{3\text{Max}} = 24, \hbar\omega = 12$ MeV

EM1.8-2.0: $M_{\text{LR}}^{0\nu} = 1.05, M_{\text{Tot}}^{0\nu} = 1.72$

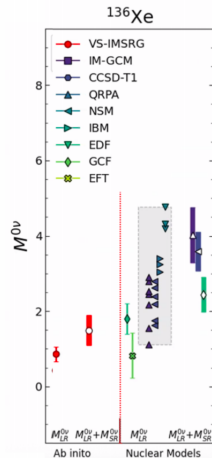
N²LO_{GO}(394): $M_{\text{LR}}^{0\nu} = 1.11, M_{\text{Tot}}^{0\nu} = 1.60$

Table: The recommended value for the total NME of $0\nu\beta\beta$ decay in ^{136}Xe , together with the uncertainties from different sources, based on the $\text{N}^2\text{LO}_{\text{GO}}(394)$ interaction.

$M^{0\nu}$	ϵ_{LEC}	$\epsilon_{\chi\text{EFT}}$	ϵ_{MBT}	ϵ_{OP}	ϵ_{EM}
1.44	—	0.38	0.45	0.10	—

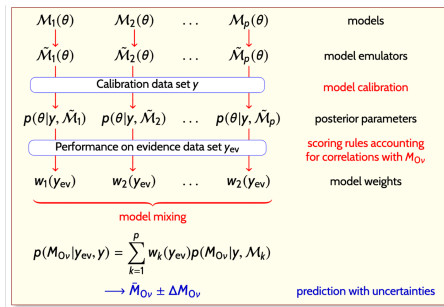
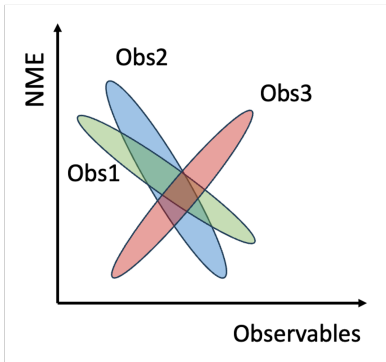
- The many-body truncation error needs to be updated: comparing the results of IM-GCM and VS-IMSRG(3f2)?
- The statistical errors are to be estimated.

The NME by the VS-IMSRG: [1.08-1.90]
a factor of about two uncertainty



A. Beliy et al., arXiv2307.15156 [nucl-th]

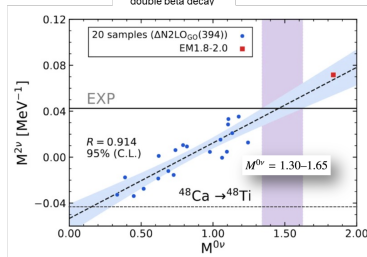
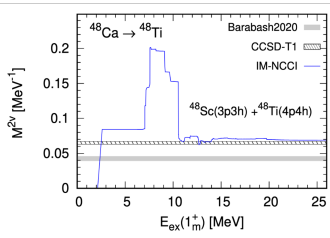
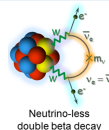
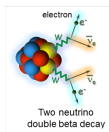
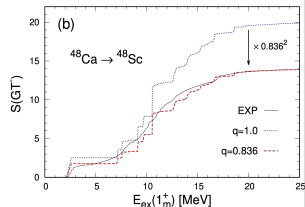
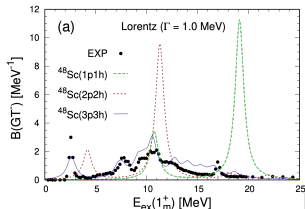
- Identify correlations with other quantities
- Add constraints on the NME using available data
- Bayesian model average (BMA) with weights determined by the performance of each model, better starting from the same inputs.



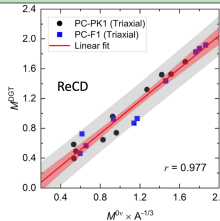
V Cirigliano et al 2022 J. Phys. G: Nucl. Part. Phys. 49 120502

- IM-NCCI: a combination of IMSRG with no-core configuration interaction method
- Applications to GT transition strengths, 2ν and 0ν double-beta decay in ^{48}Ca

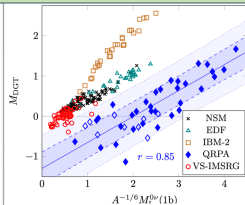
Correlation between the NMEs of neutrinoless and two-neutrino DBD



Effective nuclear forces



Y. K. Wang, P. W. Zhao, J. Meng, PLB 855, 138796 (2024)

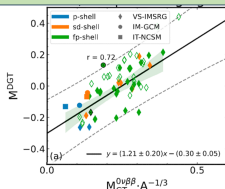


$$M_{\text{DGT}} = -1.295 + 0.645 A^{-1/6} M_L^{0\nu} (1b)$$

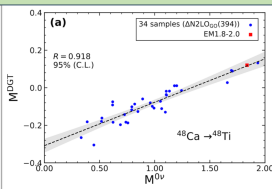
L. Jokiniemi, J. Menéndez, PRC107, 044316 (2023)

- Correlation relation including different isotopes (introducing A dependence)
- Correlation relation including different interactions for the same isotope.

Chiral nuclear forces



JMY et al., PRC106, 014315 (2022)



X. Lian, C. R. Ding, C. L. Bai, JMY, arXiv:2605.19479 (2026)

A strong linear correlation is shown between $M^{0\nu}$ and M^{DGT} in the ab initio calculations for ^{48}Ca .

- angular distribution of heavy-ion collision

$$\frac{dN_{\text{ch}}}{d^2\mathbf{p}} \propto \frac{dN_{\text{ch}}}{p_T dp_T} \left(1 + \sum_{n \geq 1} 2 v_n \cos n(\phi - \phi_n) \right)$$

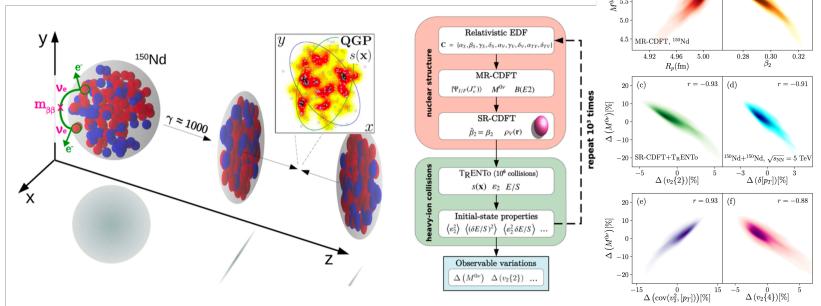
where v_2 is strongly correlated with the initial-state ellipticity parameter, ϵ_2

- observables of interest

$$\begin{aligned} \Delta(v_2\{2\}) &\equiv \frac{1}{2} \Delta(\langle v_2^2 \rangle), \\ \Delta(\delta[p_T]) &\equiv \frac{1}{2} \Delta(\langle (\delta[p_T])^2 \rangle), \\ \Delta(\text{cov}(v_2^2, [p_T])) &\equiv \frac{1}{3} \Delta(\langle v_2^2 \delta[p_T] \rangle), \\ \Delta(v_2\{4\}) &\equiv \frac{1}{4} \Delta(2 \langle v_2^2 \rangle^2 - \langle v_2^4 \rangle) \end{aligned}$$

- Relative variation of an observable

$$\Delta(O) = \frac{O - \langle O \rangle_C}{|\langle O \rangle_C|}$$



Y. Li, X. Zhang, G. Giacalone, JMY, PRL135, 022301 (2025)

URHIC: see the talks by Matthew Luzum and Giuliano Giacalone

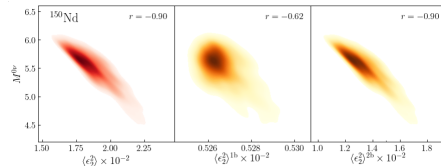
- The anisotropy coefficients in momentum space, v_n (particle flow in final state), are linearly correlated with the anisotropies in coordinate space ε_n (entropy in initial state). [Sousa, Noronha, Luzum, PRC110, 044909 \(2024\)](#)
- The anisotropic flow is related to the expectation value of the *squared eccentricity* (two-body) operator in nuclear ground state [T. Duguet et al., PRL135, 182301 \(2025\)](#)

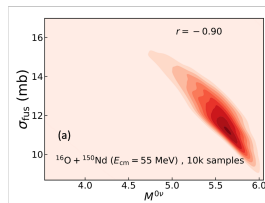
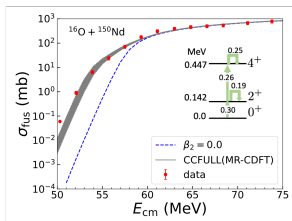
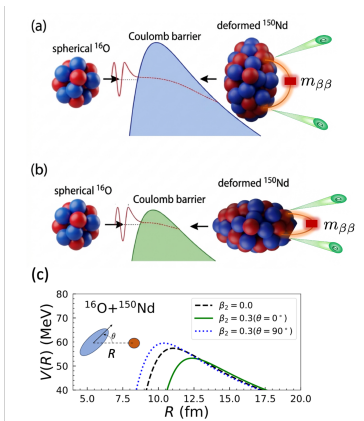
$$\mathcal{E}_2^{(2)} \equiv \mathcal{E}_2^{(1)} \mathcal{E}_{-2}^{(1)}, \quad \mathcal{E}_{\pm 2}^{(1)}(\mathbf{r}_{\perp}) \equiv r_{\perp}^2 e^{\pm i 2 \phi}$$

The normalized variance of the *eccentricity*

$$\begin{aligned} \langle \epsilon_{\ell=2}^2 \rangle &\equiv \frac{1}{2} \frac{\langle \mathcal{E}_2^{(2)} \rangle - \langle \mathcal{E}_2^{(1)} \rangle^2}{\langle R_2^{(1)} \rangle^2} \\ &\equiv \langle \epsilon_2^2 \rangle^{(1b)} + \langle \epsilon_2^2 \rangle^{(2b)} \rightarrow \frac{1}{A} + \frac{3}{4\pi} \beta_2^2 \quad (\text{rigid rotor approx.}) \end{aligned}$$

See the talk by Giuliano Giacalone





- fusion cross section: CCFULL with inputs from MR-CDFT calculation
- correlation between σ_{fusion} of $^{16}\text{O} + ^{150}\text{Nd}$ and the $M^{0\nu}$ of ^{150}Nd

- Phenomenological models (ISM, QRPA, EDF, etc.) face difficulties in quantifying systematic uncertainties in $0\nu\beta\beta$ NMEs, but remain valuable for identifying potential correlations.
- Ab initio nuclear theory based on operators derived from chiral EFT provides a systematically improvable framework with quantifiable uncertainties.
- Combining the IMSRG with established many-body approaches, such as PGCM and NCCI, offers a powerful framework for calculating $0\nu\beta\beta$ NMEs and exploring their correlations with other observables.

Outlook

- Identify robust correlations between $0\nu\beta\beta$ NMEs and other nuclear observables.
- Improve the precision and uncertainty quantification of ab initio calculations.
- Constrain and reduce theoretical uncertainties using available data and Bayesian model averaging (BMA).
- Extend ab initio calculations to NMEs for other decay channels and mechanisms.

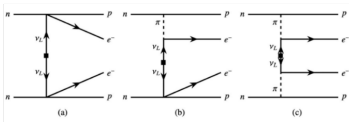
Collaborators

- **SYSU**:
C.R. Ding, C.F. Jiao, Y. Li etc
- **PKU/TUM**: L. S. Song, J. Meng, P. Ring
- **SCU**: Xin Lian, Chunlin Bai
- **SJU**: Ke Han, Shaobo Wang
- **Kyoto Univ.**: X. Zhang, K. Hagino
- **CERN**: Giuliano Giacalone
- **CEA**: Thomas Duguet
- **MSU**: S. Bogner, H. Hergert, R. Wirth
- **UNC**: Jonathan Engel
- **MIT**: Antonie Belly
- **TRIUMF**: Jason Holt
- **Tsukuba Univ.**: Takayuki Miyagi
- **Notre-Dame U**: Ragnar Stroberg
- **UAM**: Benjamin Bally, Tomas Rodriguez

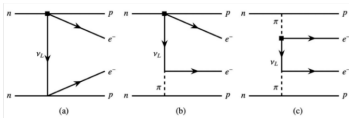
Thank you for your attention!

The NMEs in different mechanisms at the hadronic level

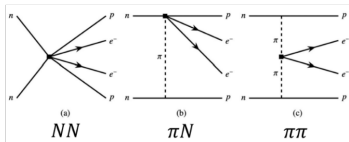
type 1: standard decay mechanism



type 2: momentum-dependent decay mechanism



type 3: short-range decay mechanism



The inverse of half-life with the consideration of different mechanisms, V. Cirigliano et al., JHEP12, 097 (2018)

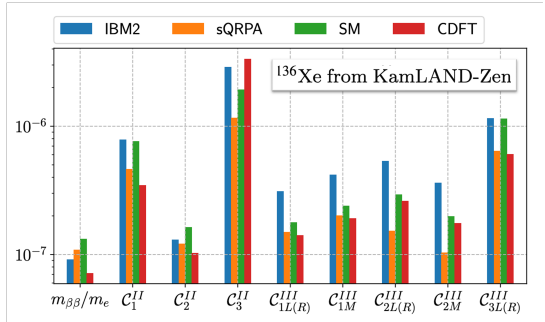
$$\begin{aligned} \left(T_{1/2}^{0\nu}\right)^{-1} = & g_A^4 \left\{ G_{01} \left(|\mathcal{A}_L|^2 + |\mathcal{A}_R|^2 \right) - 2 \left(G_{01} - G_{04} \right) \text{Re} \mathcal{A}_L^* \mathcal{A}_R \right. \\ & + 4 G_{02} |\mathcal{A}_E|^2 + 2 G_{04} \left[|\mathcal{A}_{m_e}|^2 + \text{Re} \left(\mathcal{A}_{m_e}^* \left(\mathcal{A}_L + \mathcal{A}_R \right) \right) \right] \\ & - 2 G_{03} \text{Re} \left[\left(\mathcal{A}_L + \mathcal{A}_R \right) \mathcal{A}_E^* + 2 \mathcal{A}_{m_e} \mathcal{A}_E^* \right] \\ & \left. + G_{09} |\mathcal{A}_M|^2 + G_{06} \text{Re} \left[\left(\mathcal{A}_L - \mathcal{A}_R \right) \mathcal{A}_M^* \right] \right\} \end{aligned}$$

The transition amplitude is defined by

$$\frac{\mathcal{A}_i}{4\pi R_A} = \sum_a \langle 0_F^+ | \sum_{m,n} \int \frac{d^3 \mathbf{q}}{(2\pi)^3} e^{i\mathbf{q} \cdot \mathbf{r}_{mn}} V_i^a(\mathbf{q}^2) (\tau_m^+ \tau_n^+) | 0_I^+ \rangle$$

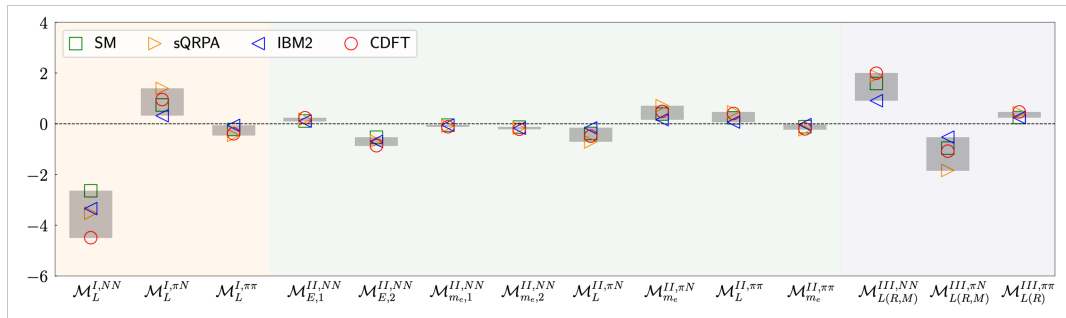
The NMEs in different mechanisms at the hadronic level

$$\begin{aligned}
 \mathcal{A}_L &= \frac{m_{\beta\beta}}{m_e} \left(\mathcal{M}_L^{I,NN} + \mathcal{M}_L^{I,\pi N} + \mathcal{M}_L^{I,\pi\pi} \right) \\
 &\quad + C_3^{II} \left(\mathcal{M}_L^{II,\pi N} + \mathcal{M}_L^{II,\pi\pi} \right) \\
 &\quad + C_{1L}^{III} \mathcal{M}_L^{III,NN} + C_{2L}^{III} \mathcal{M}_L^{III,\pi N} + C_{3L}^{III} \mathcal{M}_L^{III,\pi\pi}, \\
 \mathcal{A}_R &= C_{1R}^{III} \mathcal{M}_R^{III,NN} + C_{2R}^{III} \mathcal{M}_R^{III,\pi N} + C_{3R}^{III} \mathcal{M}_R^{III,\pi\pi}, \\
 \mathcal{A}_E &= C_1^{II} \mathcal{M}_{E,1}^{II,NN} + C_2^{II} \mathcal{M}_{E,2}^{II,NN}, \\
 \mathcal{A}_{m_e} &= C_1^{II} \mathcal{M}_{m_e,1}^{II,NN} + C_2^{II} \left(\mathcal{M}_{m_e,2}^{II,NN} + \mathcal{M}_{m_e}^{II,\pi N} + \mathcal{M}_{m_e}^{II,\pi\pi} \right), \\
 \mathcal{A}_M &= C_{1M}^{III} \mathcal{M}_M^{III,NN} + C_{2M}^{III} \mathcal{M}_M^{III,\pi N}.
 \end{aligned}$$



- The upper limits of the coefficients $\mathcal{C}$ derived from the half-life and the assumption that NMEs depend on only one of the $\mathcal{C}$ s.
- The constraint from the ¹³⁶Xe on $m_{\beta\beta}/m_e$ is the most stringent, while that on C_3^{II} is weakest.

The NMEs in different mechanisms at the hadronic level



- In general, the sizes of the NMEs from the type-I and type-III mechanisms are overall larger than those from the type-II mechanism.

C.R. Ding, G. Li, JMY, PLB856, 138896(2024)