Medium Response to Jet Energy Loss and Quantum Simulation of Jet Quenching

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Medium response reference: 2010.01140, on-going work w/ J.Casalderrey-Solana, G.Milhano, D.Pablos, K.Rajagopal

Quantum simulation reference: XY, 2205.07902

INT Workshop: Probing QCD at High Energy and Density with Jets
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InQubator for Quantum Simulation

Jets in High Energy Collisions

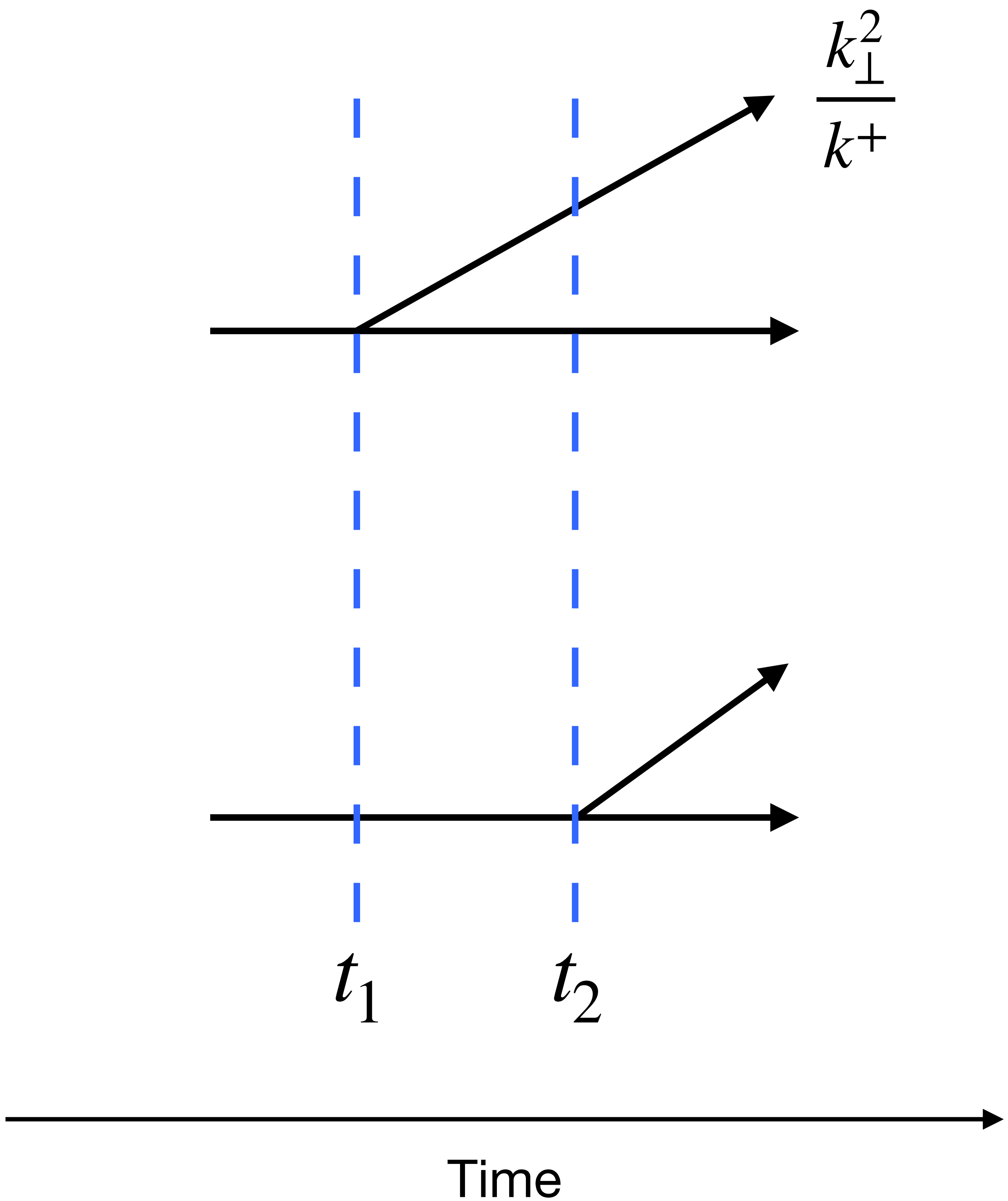
- Jet can be used as probe of QGP
- Measurements on jet quenching/jet substructure modification <— jet energy loss, medium response, selection bias
- Jet energy loss mechanism
  - Perturbative picture: radiation with medium soft kicks —> Landau-Pomeranchuk-Migdal (LPM) effect —> suppression of radiation
  - Nonperturbative picture: AdS/CFT
  - Mixed: perturbative radiation + nonperturbative soft kicks —> LPM

Quantum interference is important

Why Quantum Computing?

Jet Radiation in "Vacuum"

- **Perturbative treatment of jet evolution**



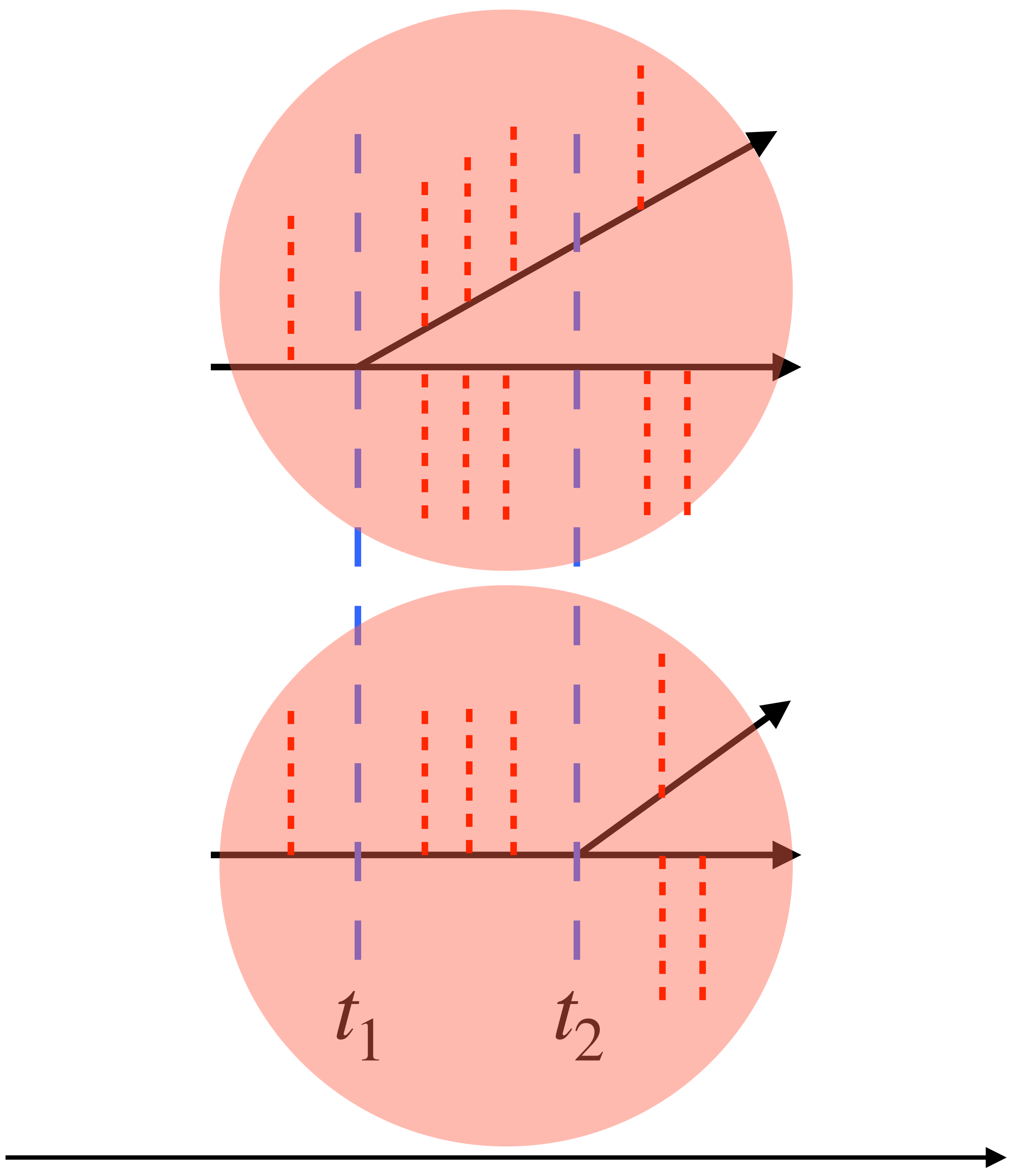
Without LPM: radiations happening at different times contribute coherently

$$\left|1 + e^{i\frac{k_\perp^2}{k^+}(t_2 - t_1)}\right|^2$$

$$= 2 + 2\cos\frac{k_\perp^2}{k^+}(t_2 - t_1)$$

Jet Radiation in Medium: LPM Effect

- **Landau-Pomeranchuk-Migdal (LPM) effect**



In medium: radiations happening at different times lose coherence

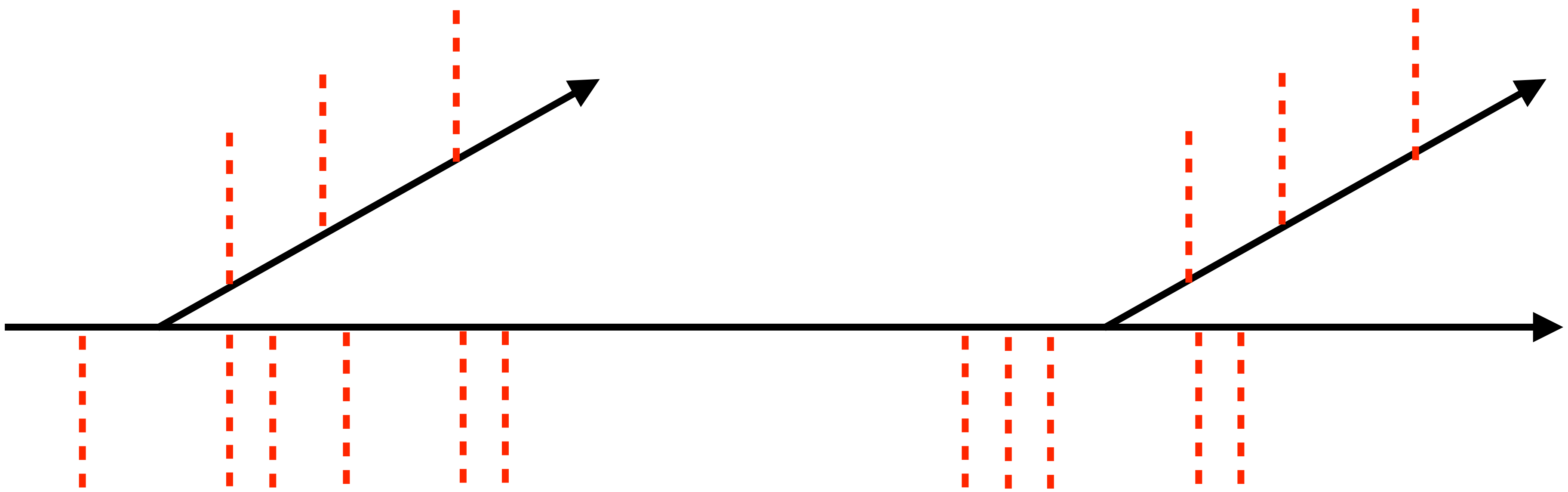
The phase of the radiated parton changes randomly between t_1 and t_2

$$\left|1 + e^{i\frac{k_\perp^2}{k^+}(t_2 - t_1)}\right|^2 \xrightarrow{\text{“average” } k_\perp} 2$$

Gyulassy, Wang
BDMPS-Z
Wiedermann
AMY
and many more

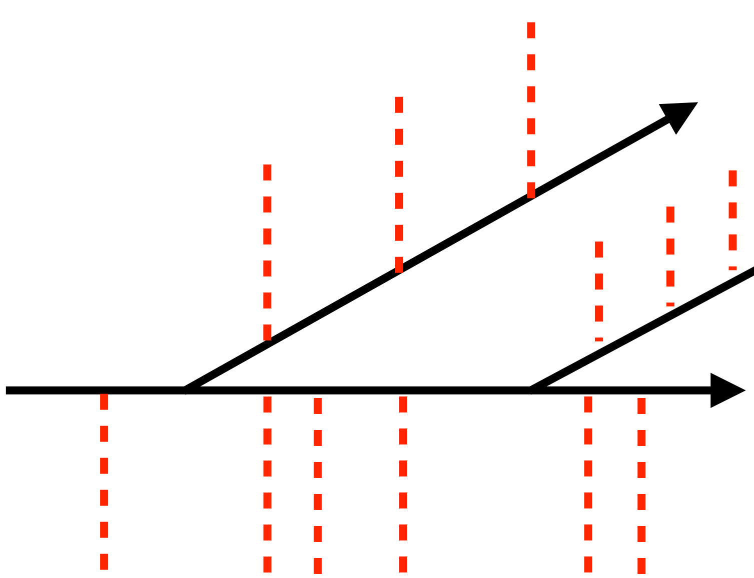
Two Jet Radiations in Medium

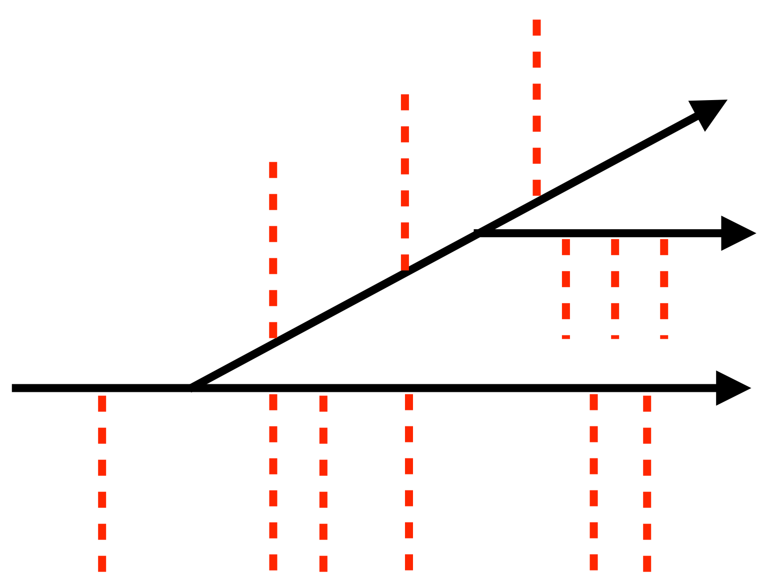
- **Two independent radiations: treat each independently**



Time separation between two radiations >> formation time

- **Two radiations with overlap formation time, coherent treatment**





Mehtar-Tani, et al, 1soft+1hard

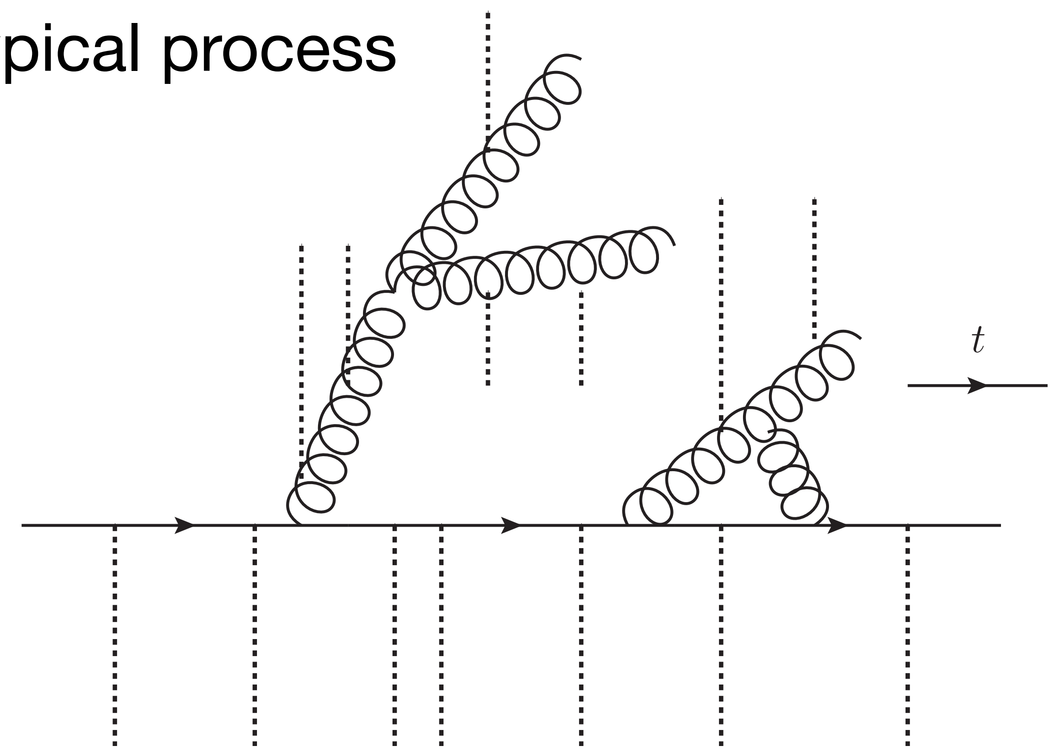
Arnold, Gorda, Iqbal, 2 hard
2007.15018 & follow-up

No studies on three radiations or beyond, dynamical, finite medium

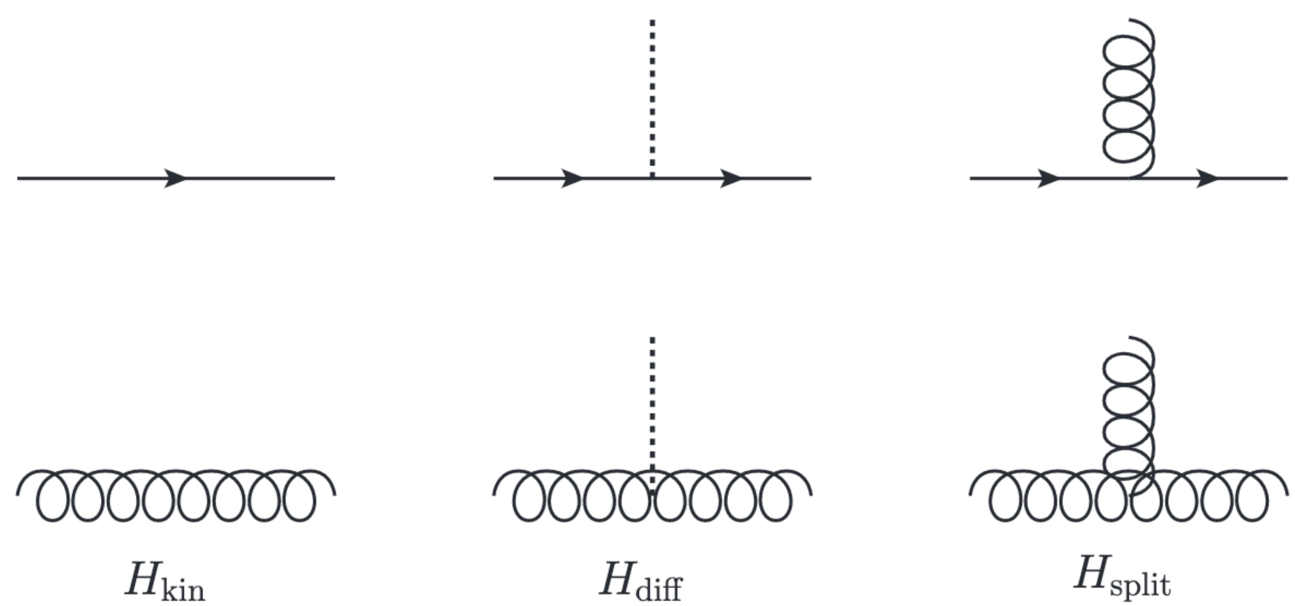
Challenge: Multiple Coherent Amplitudes + Medium Decoherence + Dynamical Medium

- **Numerical approach: Hamiltonian evolution**

A typical process



$$H(x^+) = H_{\rm kin} + H_{\rm diff}(x^+) + H_{\rm split}$$



$$\left(e^{-i(H_{\rm kin}+H_{\rm diff}+H_{\rm split})\Delta t}\right)^{N_t}|\Psi\rangle$$

Light-Front Hamiltonian Dynamics

- **Time evolution**

$$2i\frac{\partial}{\partial x^+}|\Psi\rangle = H|\Psi\rangle$$

- **Light-front Hamiltonian**

$$H = \int \mathrm{d}x^- \, \mathrm{d}^2 x_\perp \bigg(i\psi_+^\dagger\big(-\not{D}_\perp + im\big)\frac{1}{\partial^+}\big(\not{D}_\perp + im\big)\psi_+ - g\psi_+^\dagger A^{-a}T^a\psi_+ + \frac{1}{4}F_\perp^{ija}F_{\perp ij}^a - \frac{1}{8}(\partial^+ A^{-a})^2 + \frac{1}{2}(\partial^+ A_\perp^{ia})(-\partial_i A^{-a} + gf^{abc}A^{-b}A_{\perp i}^c)\bigg)$$

A^- field not dynamical: **Gauss law integrated out**

$$A^{-a} = \frac{2}{\partial^+}\partial^i A_\perp^{ia} - \frac{2g}{\partial^{+2}}\Big(f^{abc}(\partial^+ A_\perp^{ib})A_\perp^{ic} - 2\psi_+^\dagger T^a\psi_+\Big)$$

- **Background field $\bar{A}^-$ to describe medium effects**

$$A^{-a} \to A^{-a} + \bar{A}^{-a}(x^+, x_\perp)$$

$$\big\langle \bar{A}^{-a}(x^+, x_\perp)\bar{A}^{-b}(y^+, y_\perp)\big\rangle = \delta^{ab}\delta(x^+ - y^+)\gamma(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)$$

Hilbert Space

- **Use n-particle state in momentum space (distinguishable)**

$$\bigotimes_{i=1}^{n} \left| q/g,\ k^+ > 0,\ k_x,\ k_y,\ \text{color},\ \text{spin} \right\rangle_i$$

- **Size of Hilbert space with discrete momentum**

$$k^+ \in (0, K^+_{\text{max}}] \,, \quad k_x \in [-K^\perp_{\text{max}}, K^\perp_{\text{max}}] \,, \quad k_y \in [-K^\perp_{\text{max}}, K^\perp_{\text{max}}]$$

Dimension of 1-particle Hilbert space: $2^5 N^+ N_\perp^2$

$$N^+ = \frac{K^+_{\text{max}}}{\Delta k^+} \,, \quad N_\perp = \frac{2K^\perp_{\text{max}}}{\Delta k^\perp} + 1$$

- **To study LPM effect with N particles, need 1-, 2-, … N-particle states**

$$\sum_{n=1}^{N} \left(2^5 N^+ N_\perp^2\right)^n$$

For $N^+ = N_\perp = 100$

Need 100 qubits for N=4

Classical Computing Unable to Simulate

- **Finite evolution time** —> energy conservation up to $\Delta E \sim t^{-1}$
- How to obtain radiation rate from amplitudes with finite t ?

$$\int \frac{\mathrm{d}E_1}{2\pi}\Big(\frac{\sin(\frac{E_1-E_0}{2}t)}{(E_1-E_0)/2}\Big)^2 \to \frac{t}{\pi}\sum_{n=-N}^{N}\Delta z\Big(\frac{\sin^2(n\Delta z)}{n\Delta z}\Big)^2 = t + \mathcal{O}\Big(\frac{1}{N}\Big)$$

We must have enough energy levels within resolution ΔE

- N^+ fixed by $x_{\min}^{-1}$, $K_{\max}^{\perp}$ fixed by $RK_{\max}^{+}$, $\Delta k^{\perp}$ fixed by accuracy

$$d_{\mathcal{H}} \sim (N^+ N_{\perp}^2)^4 \sim 4\Big(\frac{R^2 K_{\max}^{+} t}{x_{\min}^2 \epsilon}\Big)^4$$

- $K_{\max}^{+} = 100$ GeV, $R = 0.1$, $x_{\min} = 0.05$, $t = 10$ fm/c, $\epsilon = 0.1$

$$d_{\mathcal{H}} \approx 6 \times 10^{17}$$ **60 qubits!**

Quantum Simulation of Hamiltonian Evolution

- **Pauli decomposition**

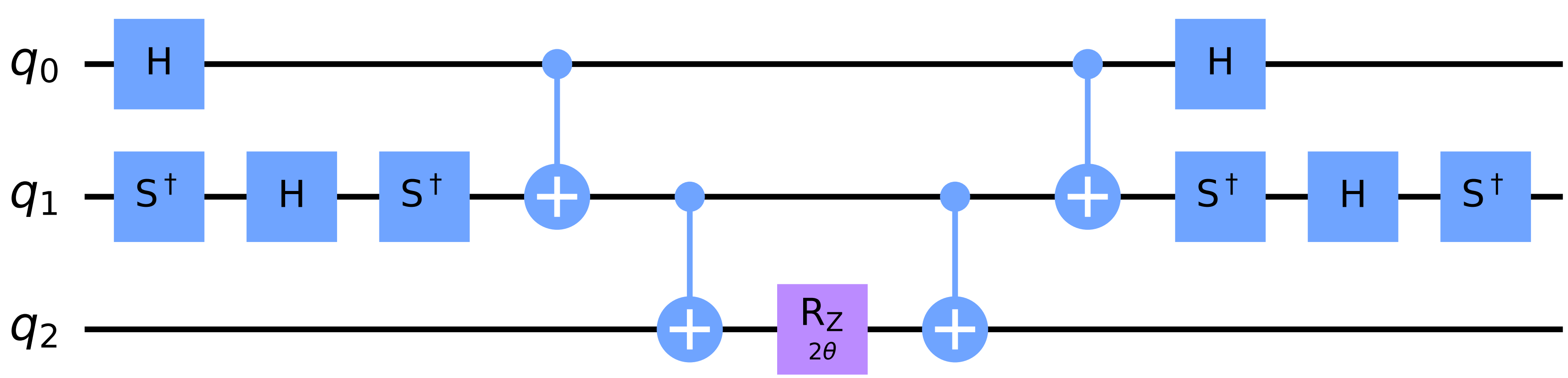
$$H = \sum_{\mu_1, \mu_2, \cdots \mu_n} a_{\mu_1 \mu_2 \cdots \mu_n} \sigma_1^{\mu_1} \otimes \sigma_2^{\mu_2} \otimes \cdots \otimes \sigma_n^{\mu_n}$$

- **Trotterization**

$$e^{-iH\Delta t} = e^{\mathcal{O}((\Delta t)^2)} \prod_{\mu_1, \mu_2, \cdots \mu_n} e^{-i\Delta t \, a_{\mu_1 \mu_2 \cdots \mu_n} \sigma_1^{\mu_1} \otimes \sigma_2^{\mu_2} \otimes \cdots \otimes \sigma_n^{\mu_n}}$$

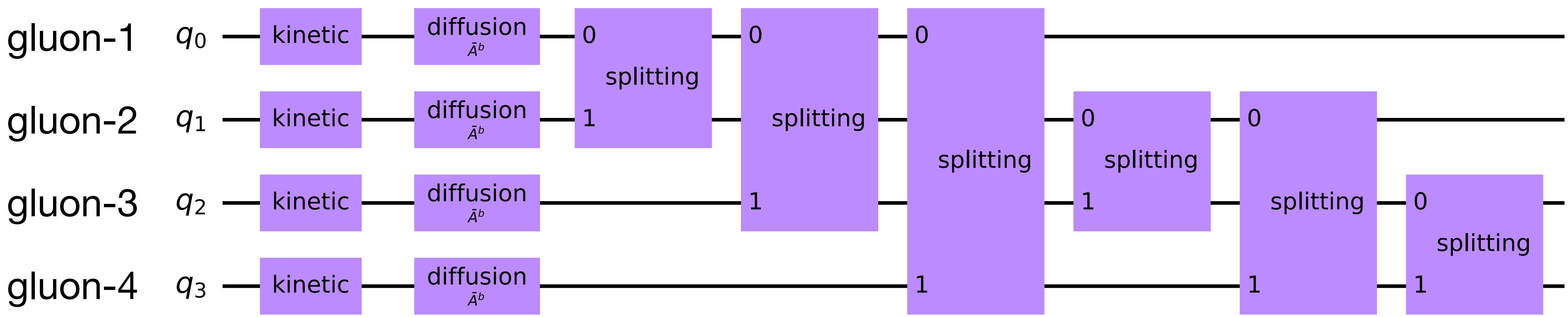
- **Standard circuit**

$$e^{-i\theta \, \sigma_1^x \otimes \sigma_2^y \otimes \sigma_3^z}$$

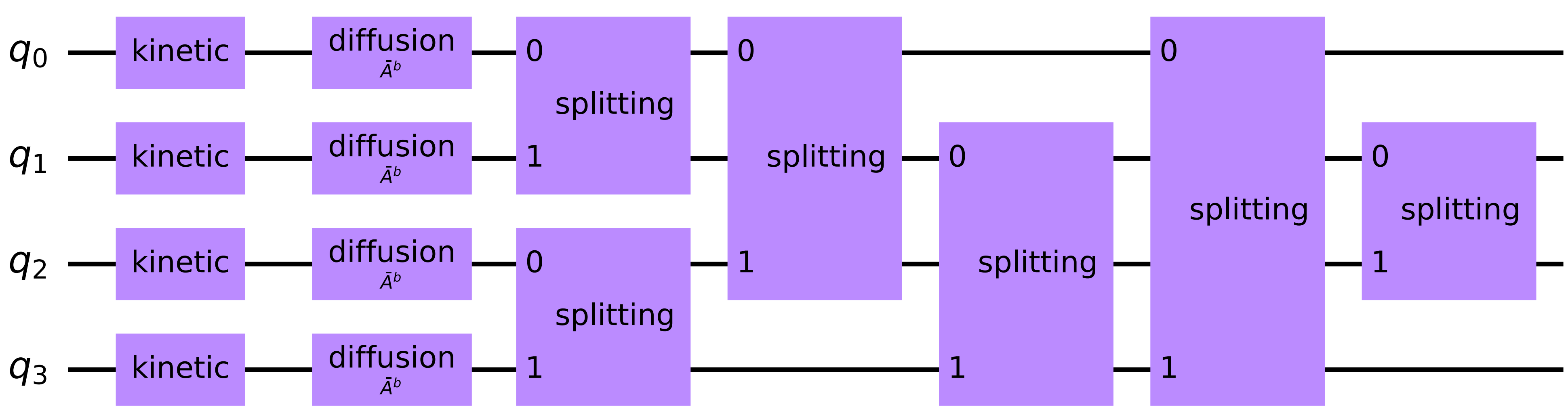


Make Quantum Circuit Physics Aware

- The general method is expensive for large Hilbert space
- Use features of the problem, apply general method to smaller Hilbert space



- Reordering circuits so gates applied simultaneously



Gluon Radiation on Small Momentum Lattice

- **Momentum discretization**

$$k^+ \in K^+_{\max}\{0.5, 1\}\,,\ k_x \in K^\perp_{\max}\{0, 1\}\,,\ k_y \in K^\perp_{\max}\{0, 1\}$$

- **Consider case up to 2 gluons —> 16 qubits**

For each gluon:

$$|\overbrace{q_0}^{\text{describe if gluon exists}}\ \underbrace{q_1 q_2 q_3}_{\text{describe momentum}}\ \overbrace{q_4 q_5 q_6}^{\text{describe color}}\ \underbrace{q_7}_{\text{describe polarization}}\rangle$$

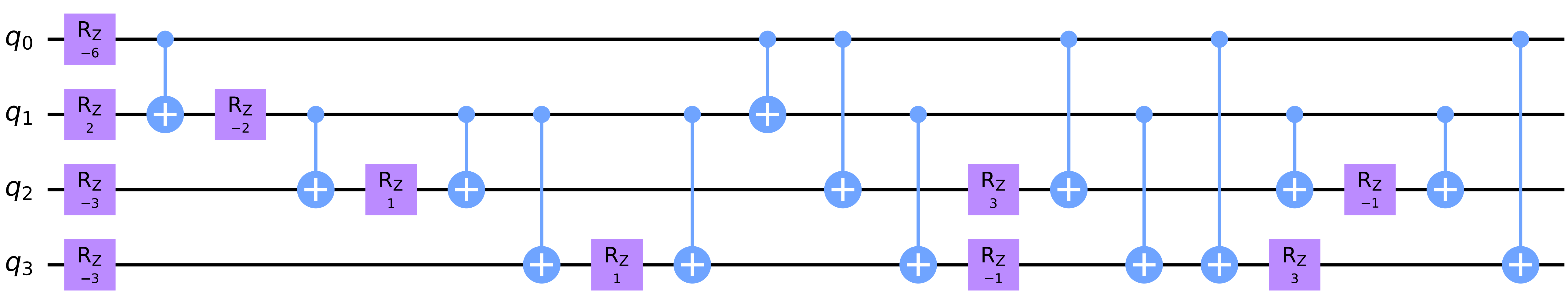
- **Model of background field**

$$\gamma(\boldsymbol{k}_\perp) = g^2 \frac{\pi T m_D^2}{(\boldsymbol{k}_\perp^2 + m_D^2)^2}$$

Make Quantum Circuit Physics Aware

- **For 1-gluon kinetic Hamiltonian, only 4 qubits evolve nontrivially**

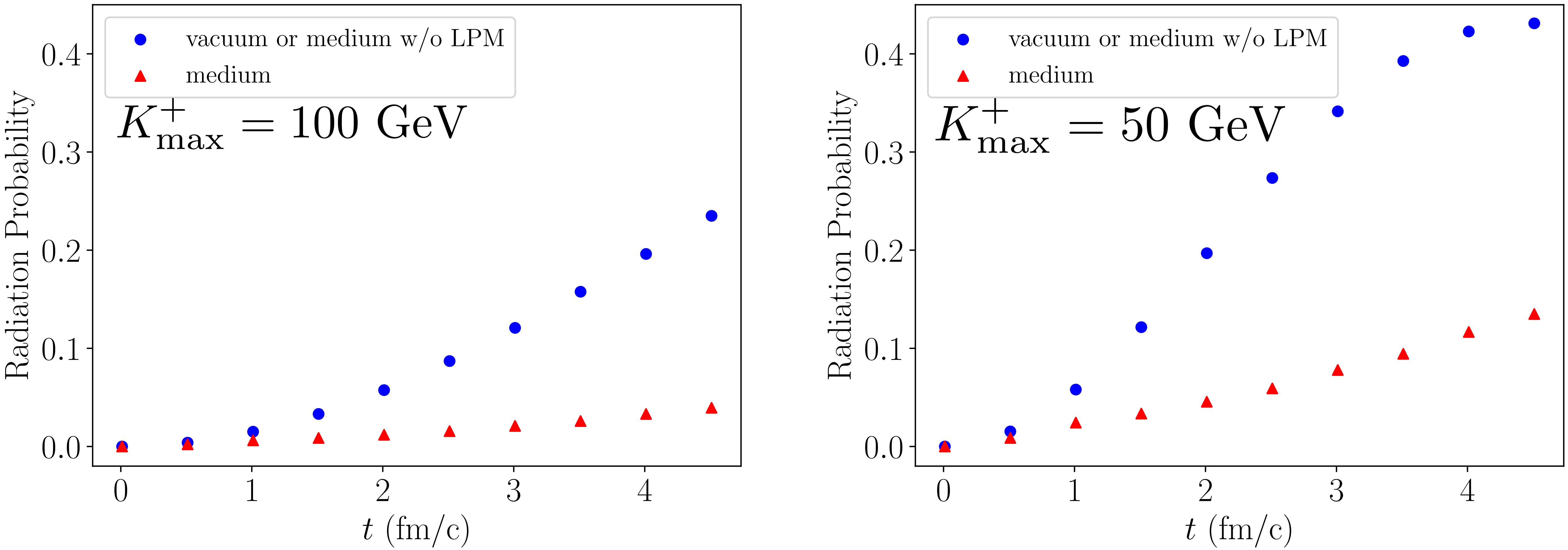
$q_0q_1q_2q_3$	k^+ in units of Δk^+	k_x in units of $\Delta k_\perp$	k_y in units of $\Delta k_\perp$	$\frac{\boldsymbol{k}_\perp^2}{k^+}$ in units of $\frac{\Delta k_\perp^2}{\Delta k^+}$
1000	1	0	0	0
1001	1	0	1	1
1010	1	1	0	1
1011	1	1	1	2
1100	2	0	0	0
1101	2	0	1	0.5
1110	2	1	0	0.5
1111	2	1	1	1



Medium Decoherence Effect in Gluon Radiation

$$K^+_{\max} = 100 \text{ GeV or } 50 \text{ GeV},\ K^\perp_{\max} = 1 \text{ GeV},\ g = 2,\ T = 300 \text{ MeV}$$

- **Radiation probabilities in vacuum v.s. medium**



Vacuum: averaged from 2^{20} shots (number of simulations)

Medium: 500 quantum trajectories averaged, each trajectory has a different set of classical background fields sampled

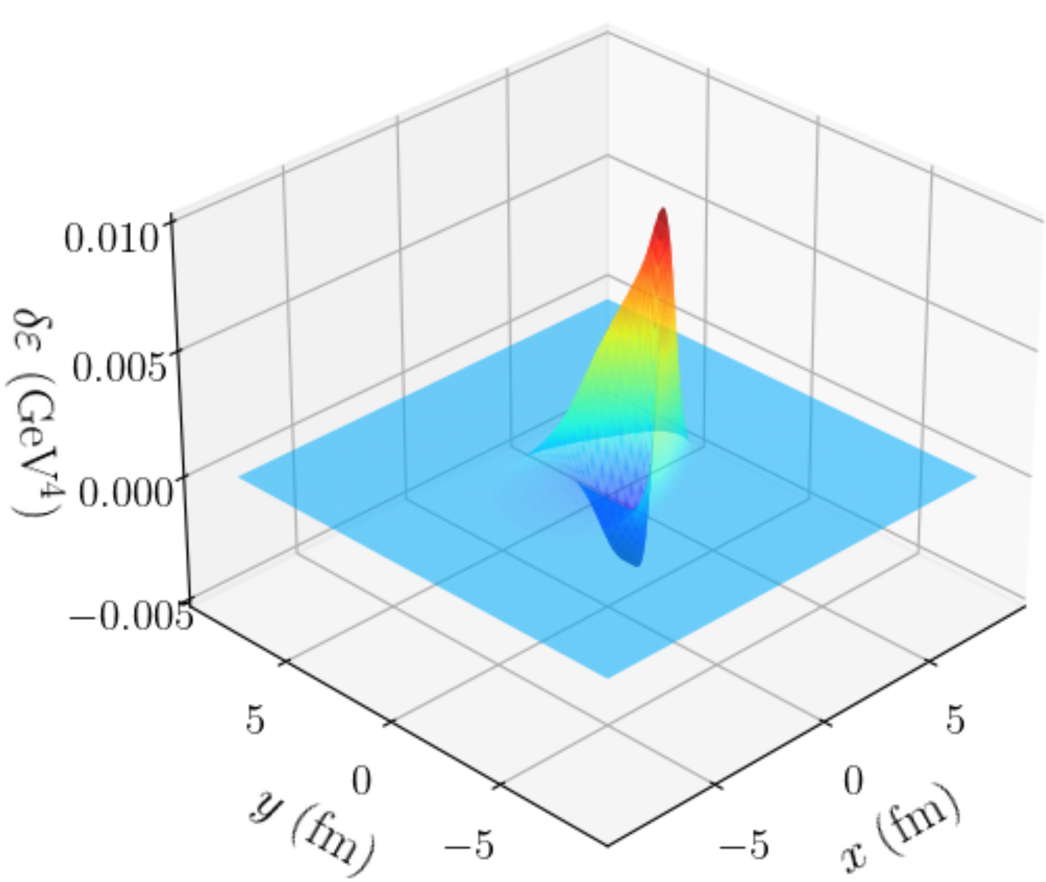
Medium Response

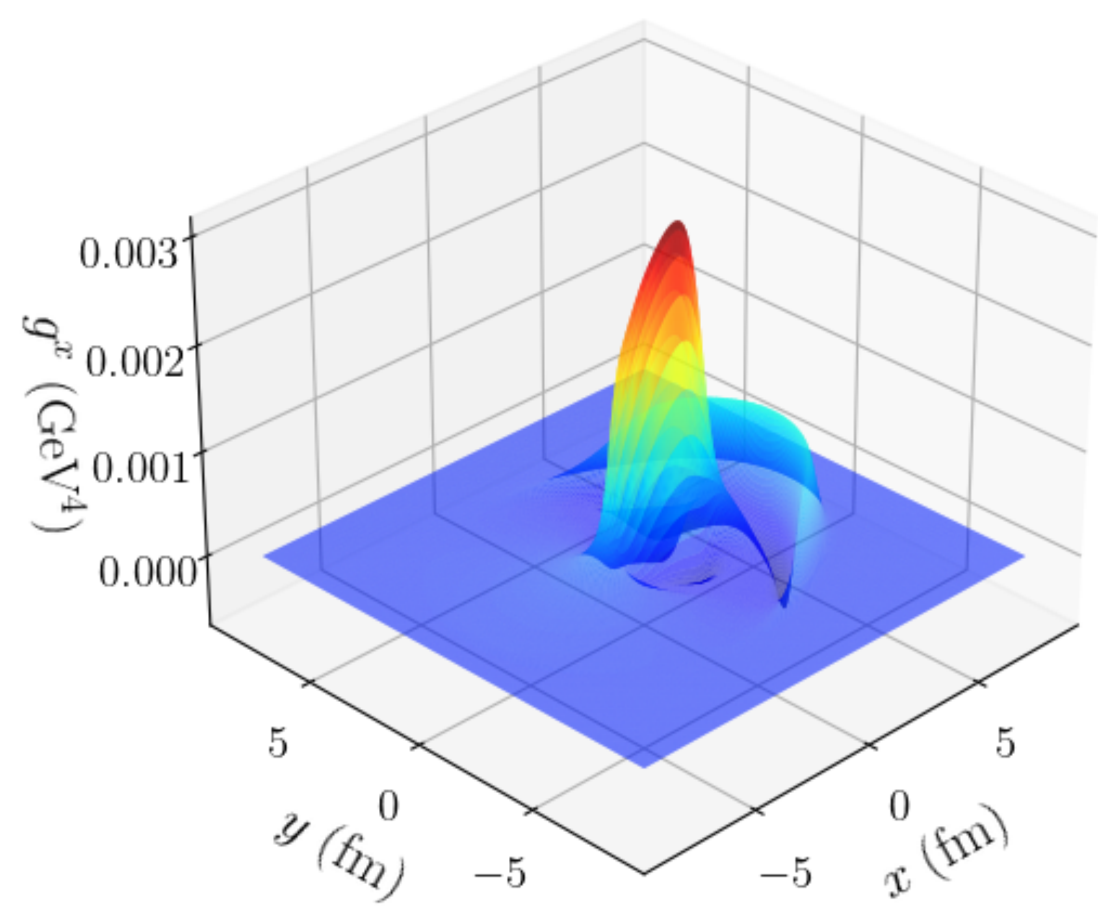
Jet Wake and Hadronization

- Jets lose energy when propagating through QGP
- Energy deposited by jets evolves in QGP, (partially) thermalizes to form jet wake and hadronizes into particles, some of which reconstructed into jets

$$\nabla_\mu T^{\mu\nu} = J^\nu$$

- Important to understand how medium responds to jet energy loss in order to use jets as probes





Efficient Procedure to Study Medium Response

- Consider a blast wave model for the background flow $v\gamma \approx 0.12r$. It well describes transverse flow at late times

- The blast wave model allows us to coordinate transformation into the transversely comoving frame, where we study linearized hydrodynamics on top of Bjorken flow

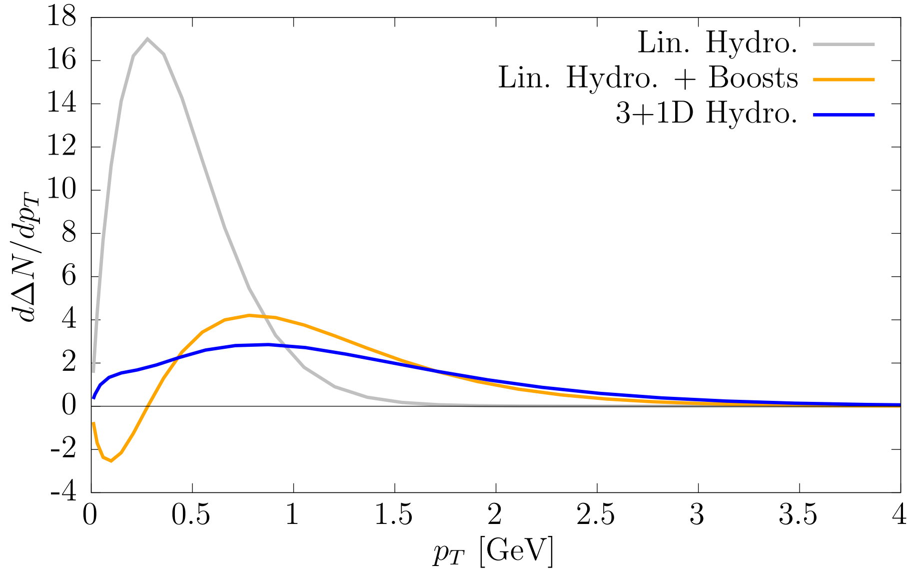
- We discretize the jet trajectory and linearly superpose jet wake solutions on the freezeout hypersurface for each deposition point, **with proper energy weights, coordinate transformation, rotations and boosts**

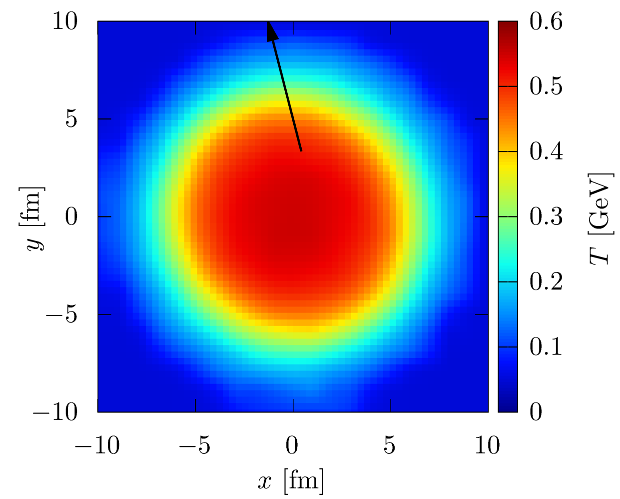
- Cooper-Frye formula for hadronization

$$(2\pi)^3 E \frac{\mathrm{d}\Delta N}{\mathrm{d}^3 p} = \int \mathrm{d}\sigma^\mu p_\mu \, f\left(\frac{u^\mu p_\mu}{T}\right) - \int \mathrm{d}\sigma_0^\mu p_\mu \, f\left(\frac{u_0^\mu p_\mu}{T_0}\right)$$

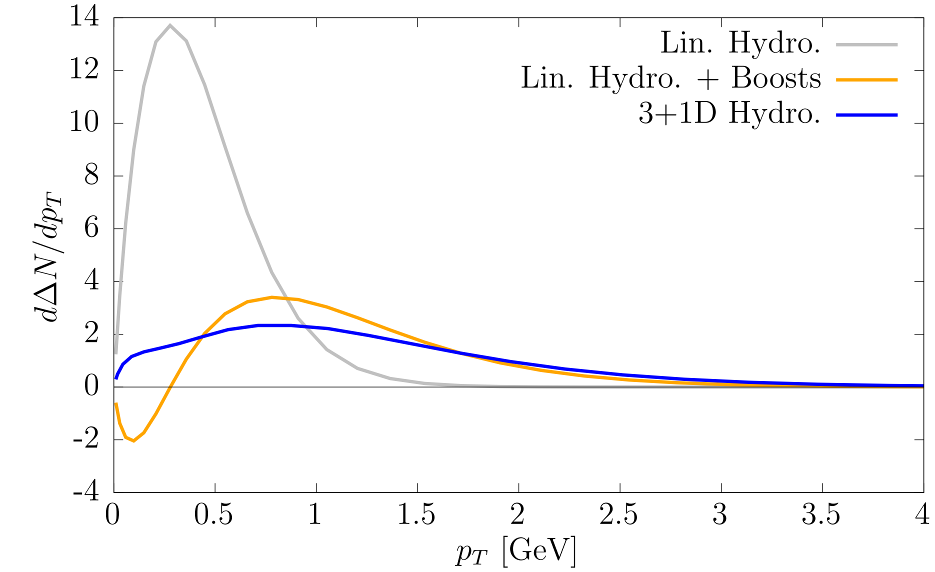
Results for Configuration 1

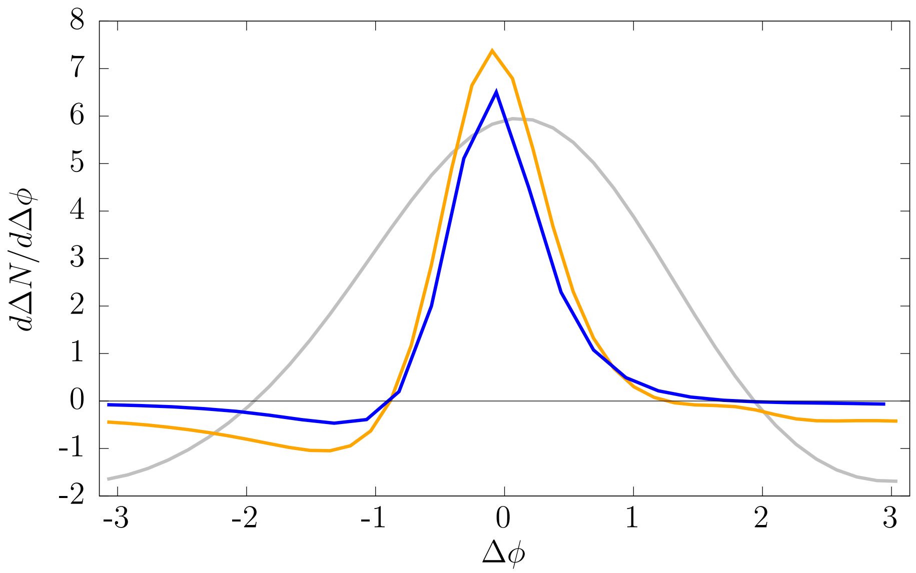
10 GeV

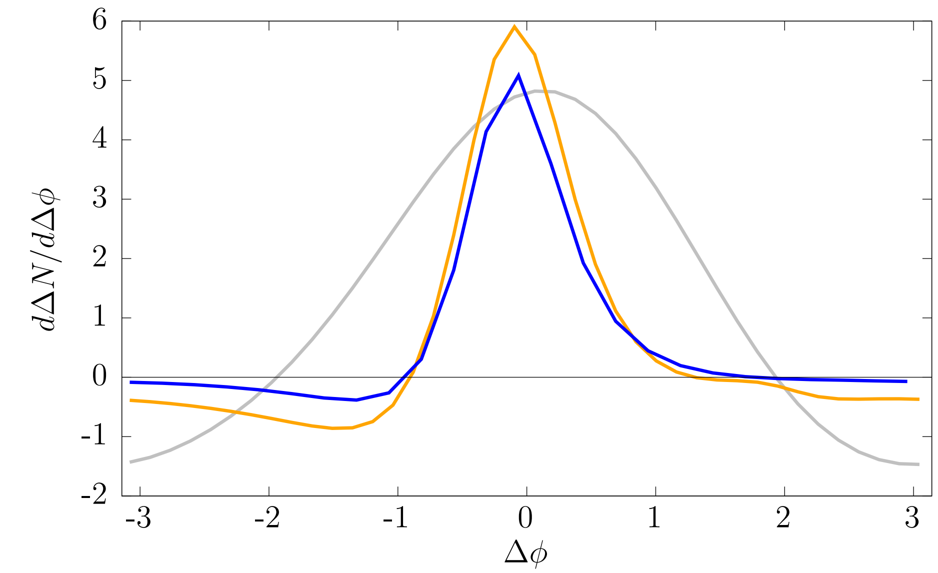


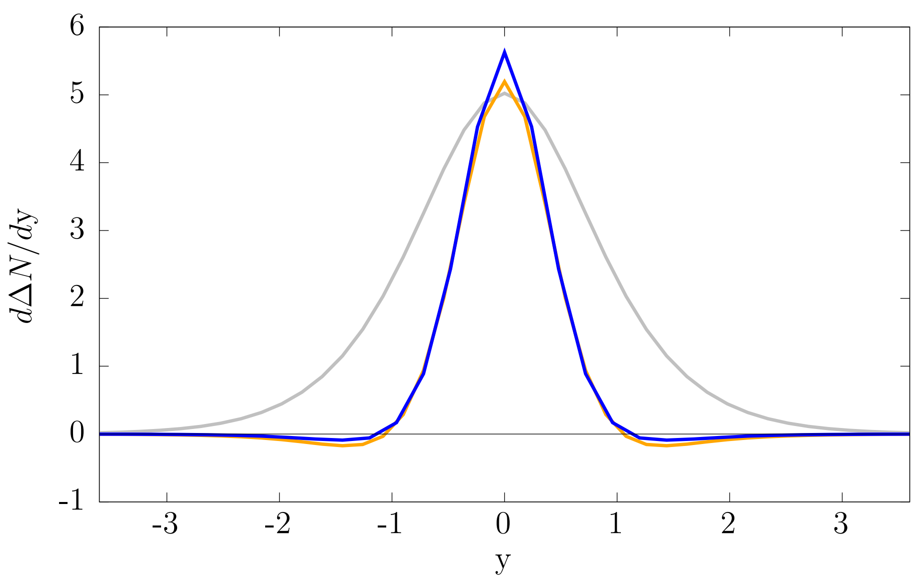


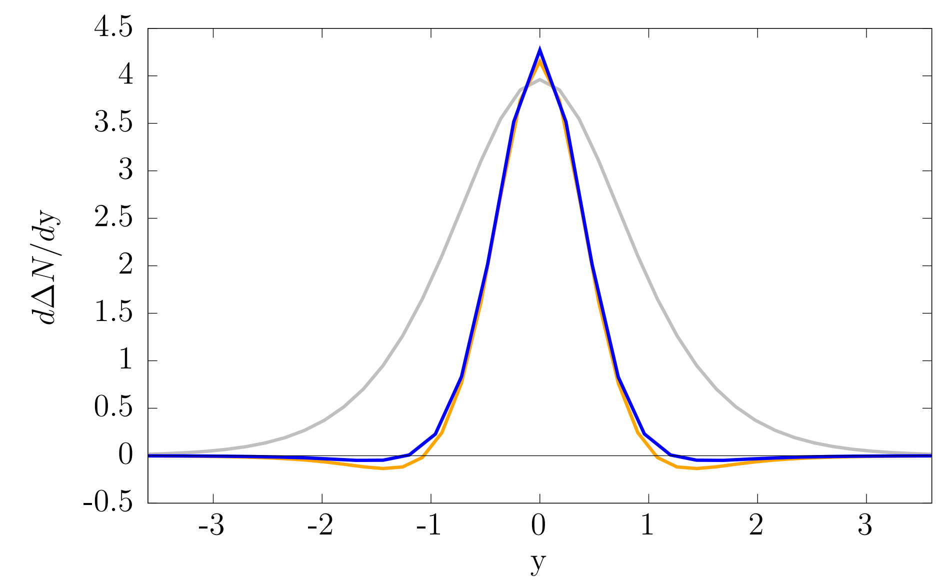
50 GeV





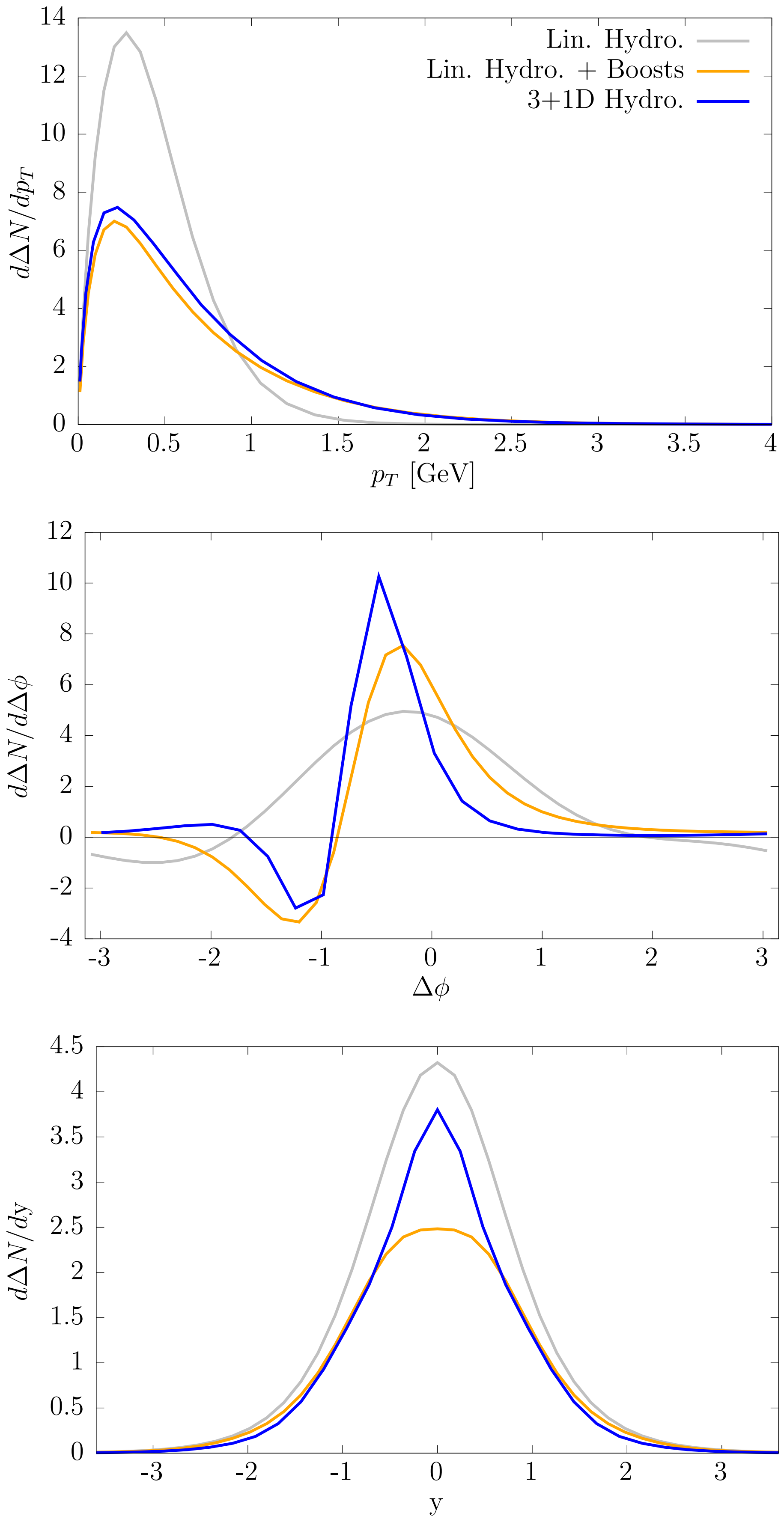


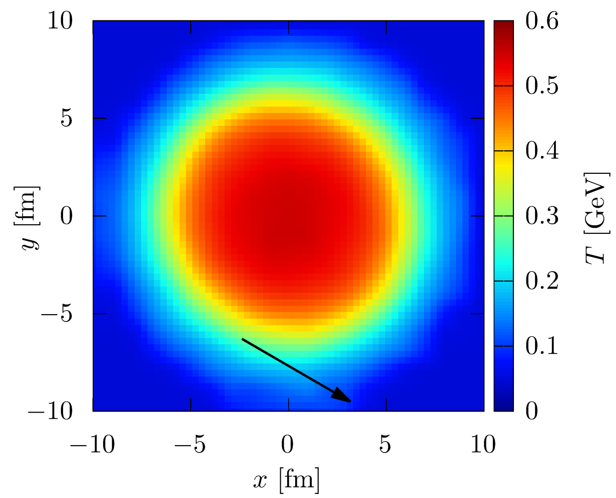


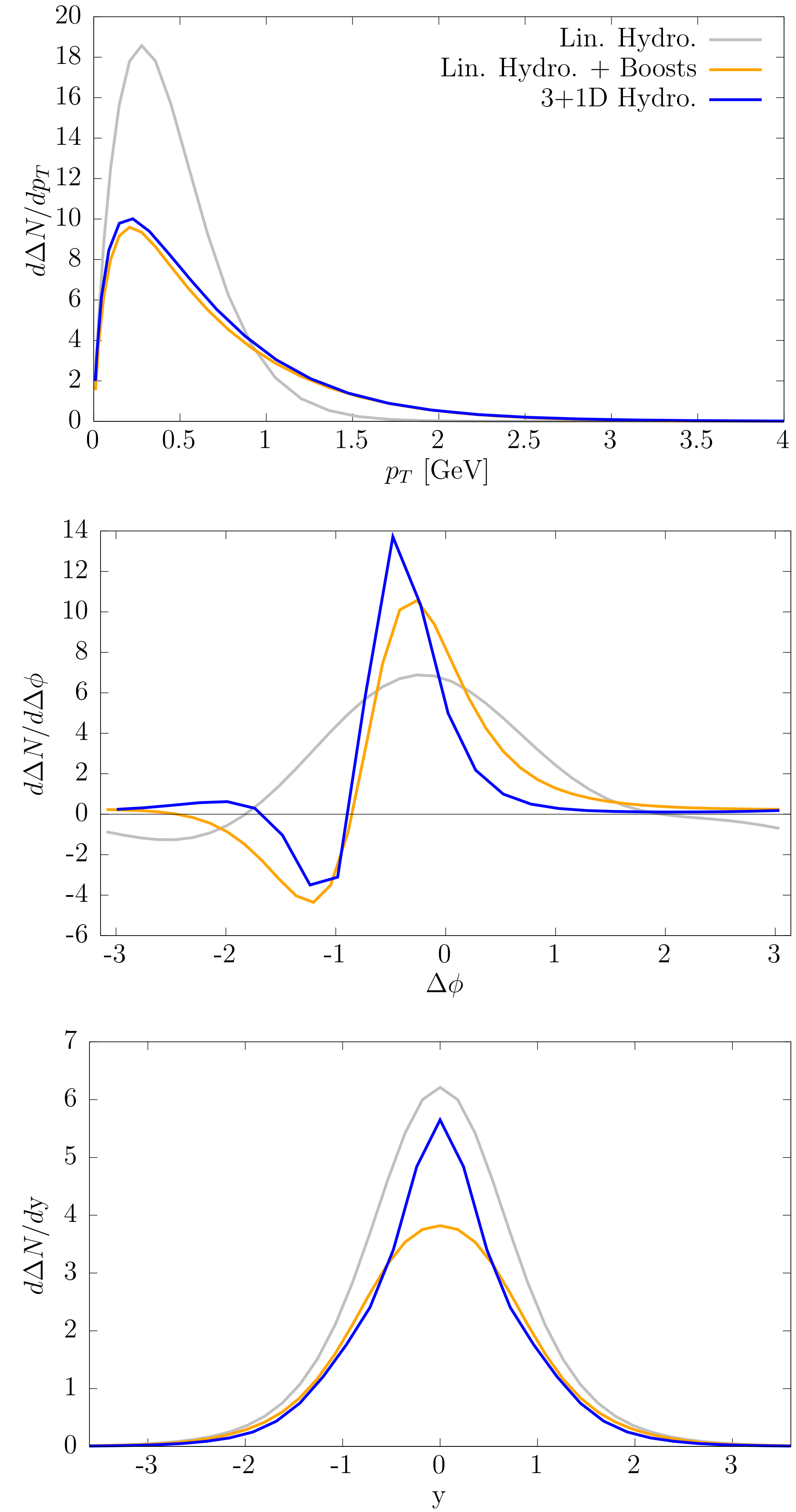


Results for Configuration 2

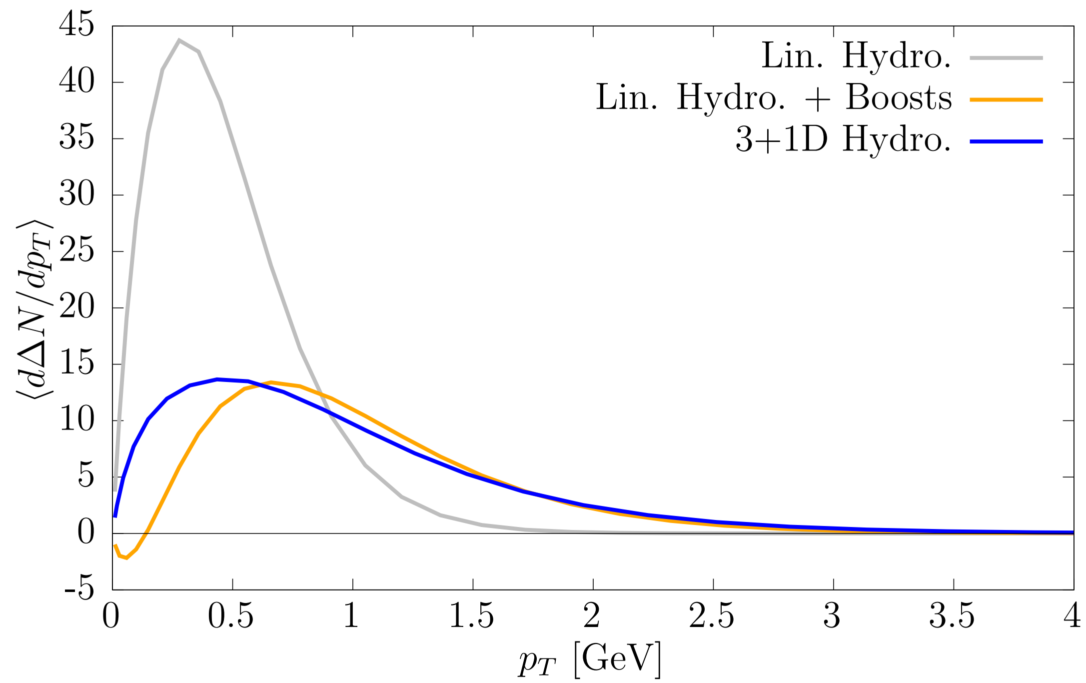
10 GeV **50 GeV**

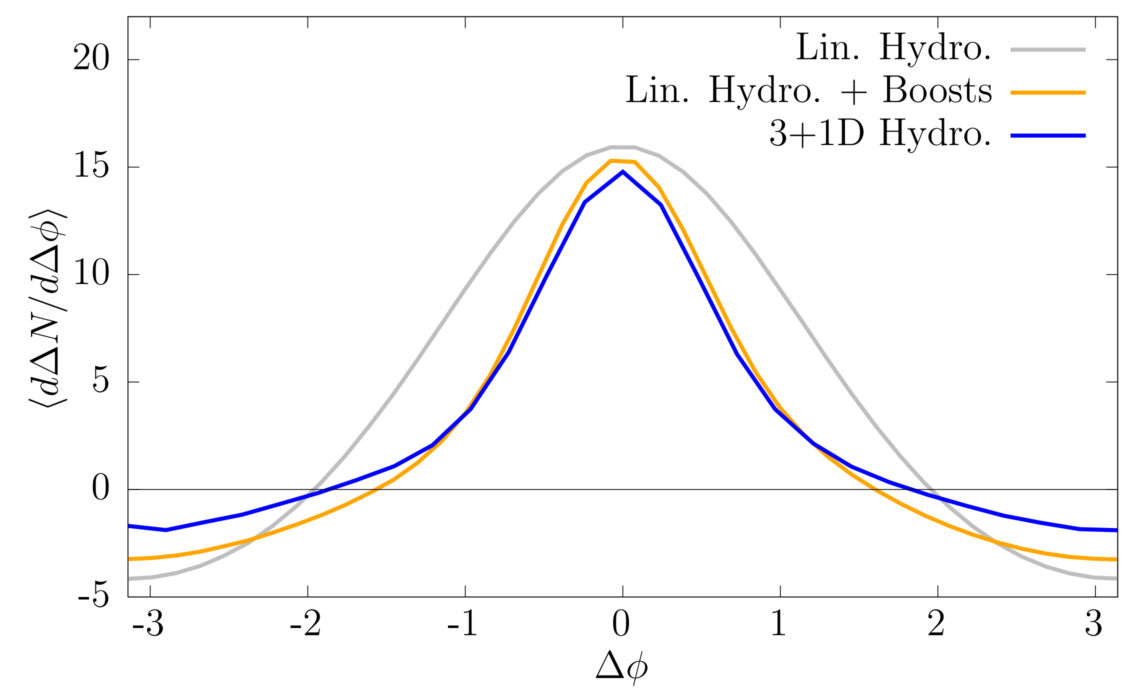


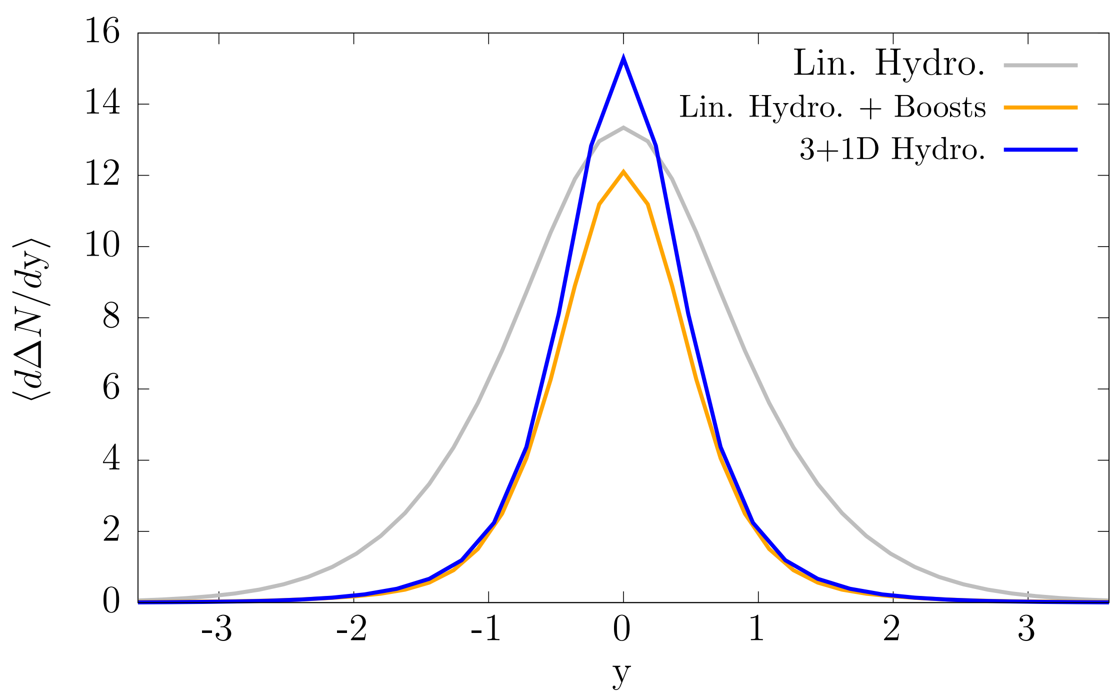




Averaged Results for 50 Configurations







Individual features washed out

Need to describe well event-by-event for good averaged results

Summary

- Quantum simulation for jet quenching in nuclear environments
    - Challenge to study LPM effect in multi-coherent splittings: **interference + decoherence + dynamical medium**
    - Hamiltonian evolution accounts for them automatically
    - Classical computing not enough
    - Design quantum circuits based on the physics problem
- Medium response: efficient setup, future implementation in **Hybrid Model**

Backup: Discretized Light-Front Hamiltonians

- **Kinetic**

$$\left\langle g,\, k_1^+,\, k_{1\perp},\, a_1,\, \lambda_1 \middle| H_{g,\,\mathrm{kin}} \middle| g,\, k_2^+,\, k_{2\perp},\, a_2,\, \lambda_2 \right\rangle = \frac{\boldsymbol{k}_{1\perp}^2}{k_1^+} \delta_{k_1^+ k_2^+} \delta_{k_{1x} k_{2x}} \delta_{k_{1y} k_{2y}} \delta_{a_1 a_2} \delta_{\lambda_1 \lambda_2}$$

- **Diffusion**

$$\left\langle g,\, k_1^+,\, k_{1\perp},\, a_1,\, \lambda_1 \middle| H_{g,\,\mathrm{diff}}(x^+) \middle| g,\, k_2^+,\, k_{2\perp},\, a_2,\, \lambda_2 \right\rangle$$

$$= \frac{ig}{2(2\pi)^2} \Delta k_x \Delta k_y \delta_{k_1^+ k_2^+} \delta_{\lambda_1 \lambda_2} \left(f^{a_2 b a_1} - f^{a_1 b a_2} \right) \bar{A}^{-b}(x^+, \boldsymbol{k}_{1\perp} - \boldsymbol{k}_{2\perp})$$

- **Splitting** $\left\langle g, k_2^+, k_{2\perp}, a_2, \lambda_2; g, k_3^+, k_{3\perp}, a_3, \lambda_3 \middle| H_{g,\,\mathrm{split}} \middle| g, k_1^+, k_{1\perp}, a_1, \lambda_1 \right\rangle$

$$\begin{aligned}
&= -ig\sqrt{\frac{\Delta k^+ \Delta k_x \Delta k_y}{2(2\pi)^3 k_1^+ k_2^+ k_3^+}} f^{a_1 a_2 a_3} \delta_{k_1^+,\, k_2^+ + q^+} \delta_{k_{1x},\, k_{2x}+q_x} \delta_{k_{1y},\, k_{2y}+q_y} \\
&\Bigg(k_1^+ \epsilon_\perp^i(\lambda_1) \left[\frac{k_{2\perp}^j}{k_2^+} \epsilon_\perp^j(\lambda_2) \epsilon_{\perp i}(\lambda_3) - \frac{k_{3\perp}^j}{k_3^+} \epsilon_\perp^j(\lambda_3) \epsilon_{\perp i}(\lambda_2) \right] - k_2^+ \epsilon_\perp^i(\lambda_2) \left[\frac{k_{3\perp}^j}{k_3^+} \epsilon_\perp^j(\lambda_3) \epsilon_{\perp i}(\lambda_1) \right. \\
&\left. - \frac{k_{1\perp}^j}{k_1^+} \epsilon_\perp^j(\lambda_1) \epsilon_{\perp i}(\lambda_3) \right] - k_3^+ \epsilon_\perp^i(\lambda_3) \left[\frac{k_{1\perp}^j}{k_1^+} \epsilon_\perp^j(\lambda_1) \epsilon_{\perp i}(\lambda_2) - \frac{k_{2\perp}^j}{k_2^+} \epsilon_\perp^j(\lambda_2) \epsilon_{\perp i}(\lambda_1) \right] \\
&- k_{1\perp}^i \epsilon_\perp^j(\lambda_1) \Big[\epsilon_{\perp i}(\lambda_2) \epsilon_{\perp j}(\lambda_3) - \epsilon_{\perp i}(\lambda_3) \epsilon_{\perp j}(\lambda_2) \Big] + k_{2\perp}^i \epsilon_\perp^j(\lambda_2) \Big[\epsilon_{\perp i}(\lambda_3) \epsilon_{\perp j}(\lambda_1) \\
&- \epsilon_{\perp i}(\lambda_1) \epsilon_{\perp j}(\lambda_3) \Big] + k_{3\perp}^i \epsilon_\perp^j(\lambda_3) \Big[\epsilon_{\perp i}(\lambda_1) \epsilon_{\perp j}(\lambda_2) - \epsilon_{\perp i}(\lambda_2) \epsilon_{\perp j}(\lambda_1) \Big] \Bigg),
\end{aligned}$$