Coupled Boltzmann Equations for Quarkonia and Heavy Flavor

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INT Program INT-22-3: Heavy Flavor Production in Heavy-Ion and Elementary Collisions

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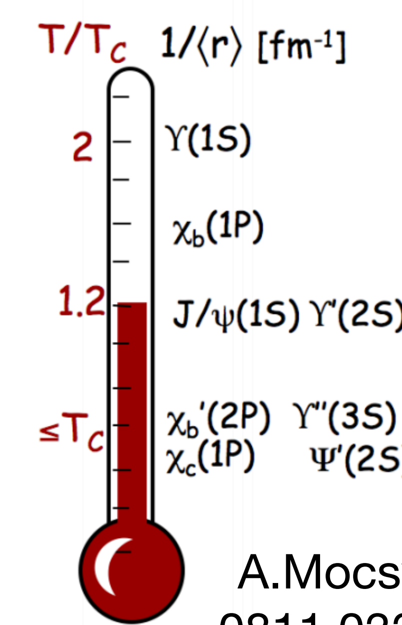
Quarkonium as Probe of Quark-Gluon Plasma

- **Heavy quarkonium as probe of QGP:**
  - **Static screening**: suppression of color attraction —> melting at high T, states of different sizes have different melting T —> thermometer
  - **Dissociation**: induced by in-medium dynamical processes, can happen even below melting T
  - **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting T, crucial for charmonium phenomenology and theory consistency



A.Mocsy 0811.0337

- Cold nuclear matter effect, feed-down contributions

Contents

- What are coupled Boltzmann equations? Inputs?
- Why do we use them?
- How do they work compared with experimental data?

XY, W.Ke, Y.Xu, S.A. Bass, B.Müller 2004.06746

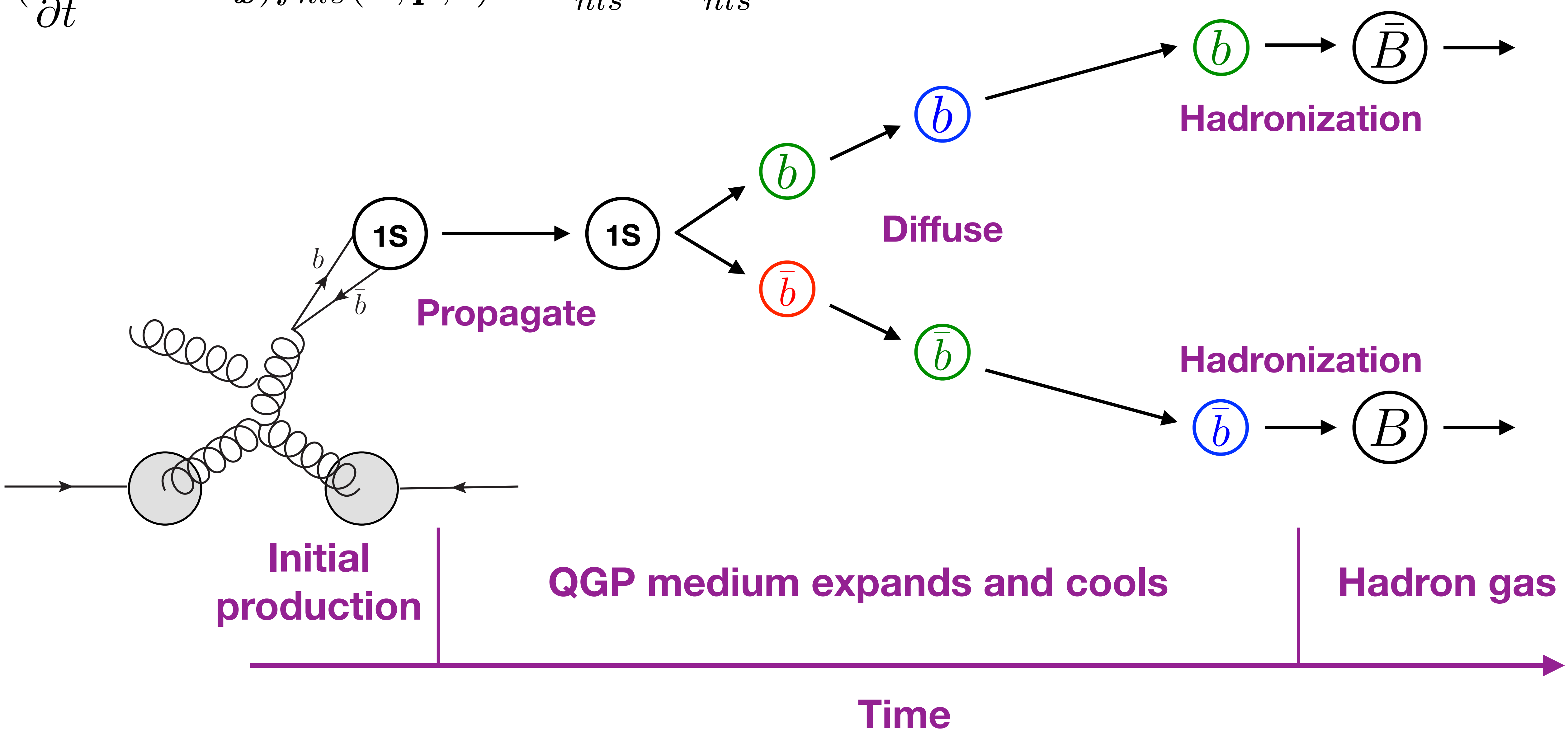
Coupled Transport Equations of Heavy Flavors

Open heavy quark antiquark $C_{Q\bar{Q}}$: HQ scattering; $C^{+}_{Q\bar{Q}}$: recombination; $C^{-}_{Q\bar{Q}}$: dissociation

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}_Q \cdot \nabla_{\boldsymbol{x}_Q} + \dot{\boldsymbol{x}}_{\bar{Q}} \cdot \nabla_{\boldsymbol{x}_{\bar{Q}}}\right) f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) = \mathcal{C}_{Q\bar{Q}} - \mathcal{C}^{+}_{Q\bar{Q}} + \mathcal{C}^{-}_{Q\bar{Q}}$$

Each quarkonium state, nl = 1S, 2S,1P etc.

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}^{+}_{nls} - \mathcal{C}^{-}_{nls}$$



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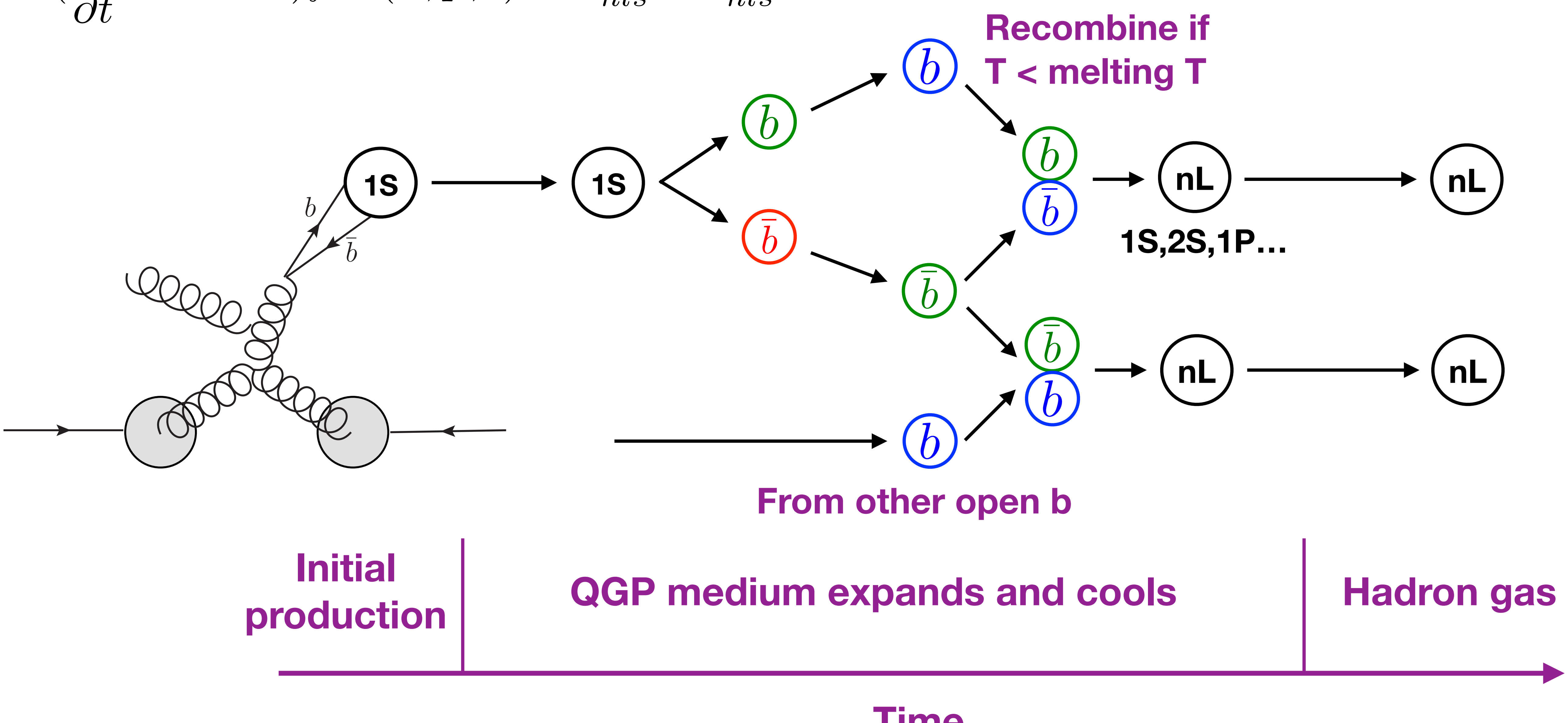

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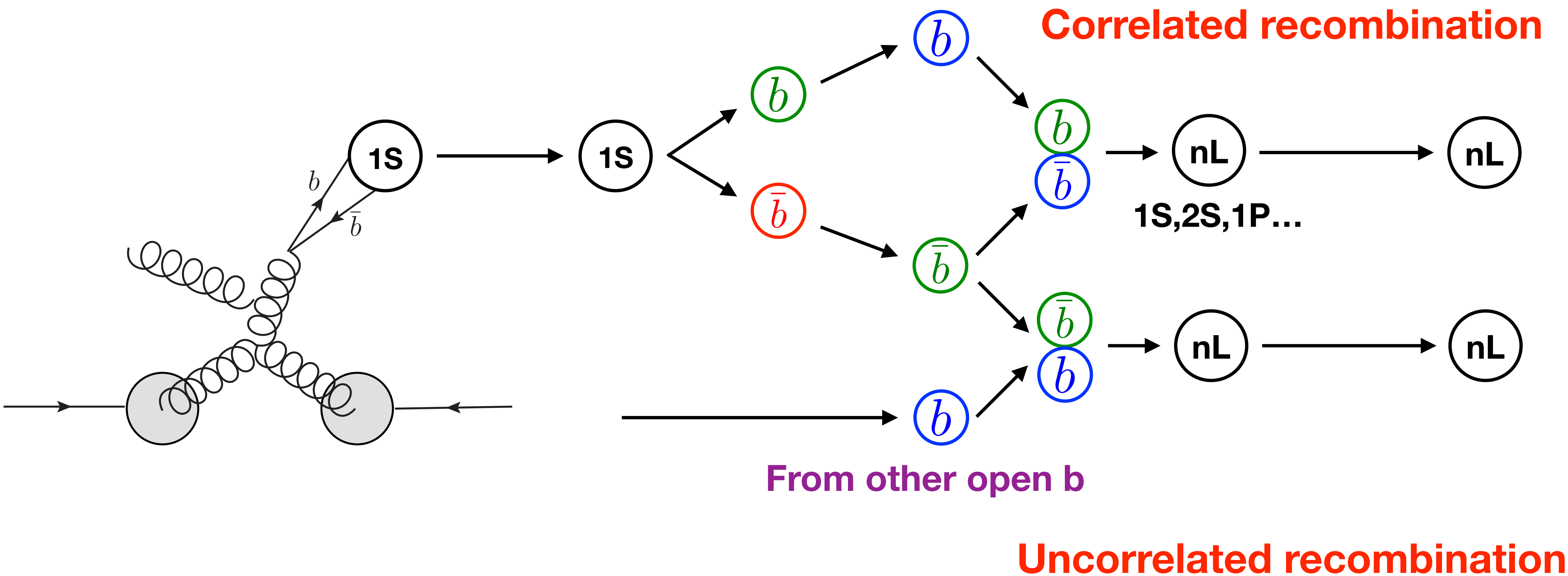

Correlated v.s. Uncorrelated Recombination

- **Correlated recombination:** heavy quark pair from **same** initial hard vertex / dissociation

- **Uncorrelated recombination:** heavy quark pair from **different** initial hard vertices; crucial contribution to charmonium production; important for charmonium but negligible for bottomonium

- Recombination in most transport calculations: uncorrelated $\propto f_Q f_{\bar{Q}}$ $\propto f_{\mathrm{onia}}^{(\mathrm{eq})}$

- **How to incorporate correlated recombination in semiclassical transport? Need 2-particle distribution**

Coupled Transport Equations of Heavy Flavors

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$$f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) \neq f_Q(\boldsymbol{x}_Q, \boldsymbol{p}_Q, t) f_{\bar{Q}}(\boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t)$$

Can handle both correlated and uncorrelated recombination

$$\mathcal{C}_{Q\bar{Q}} = \mathcal{C}_Q + \mathcal{C}_{\bar{Q}}$$

Each independently interact with medium:
(1) Potential between pair screened
(2) Potential depends on color, average over

We use “Lido” for open heavy flavor transport: diffusion + radiation

W.Ke, Y.Xu, S.A.Bass, PRC 98, 064901 (2018)

Derivation of Quarkonium Boltzmann Equation from Open Quantum System

- Heavy $Q\bar{Q}$ pair + QGP evolve together, trace out QGP

$$\mathrm{Tr}_{\mathrm{QGP}}\Big(U(t)(\rho_{Q\bar{Q}}(0)\otimes\rho_{\mathrm{QGP}})U^{\dagger}(t)\Big)$$

- Lindblad-like equation in quantum optical limit (weak coupling between system and environment —> relaxation is longest time scale)

$$\rho_S(t)=\rho_S(0)-i\Big[tH_S+\sum_{a,b}\sigma_{ab}(t)L_{ab},\rho_S(0)\Big]+\sum_{a,b,c,d}\gamma_{ab,cd}\Big(L_{ab}\rho_S(0)L^{\dagger}_{cd}-\frac{1}{2}\{L^{\dagger}_{cd}L_{ab},\rho_S\}\Big)$$

- Use pNRQCD to write down operators, apply Wigner transform and semiclassical expansion **(crucial to justify quantum optical limit, see XY, 2102.01736)**

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Static screening

- Use pNRQCD to write down operators, apply Wigner transform and semiclassical expansion **(crucial to justify quantum optical limit, see XY, 2102.01736)**

$$\frac{\partial}{\partial t}f_{nl}(\boldsymbol{x},\boldsymbol{k},t)+\frac{\boldsymbol{k}}{2M}\cdot\nabla_{\boldsymbol{x}}f_{nl}(\boldsymbol{x},\boldsymbol{k},t)=\mathcal{C}^{+}_{nl}(\boldsymbol{x},\boldsymbol{k},t)-\mathcal{C}^{-}_{nl}(\boldsymbol{x},\boldsymbol{k},t)$$

Recombination **Dissociation**

T.Mehen, XY, 1811.07027, 2009.02408

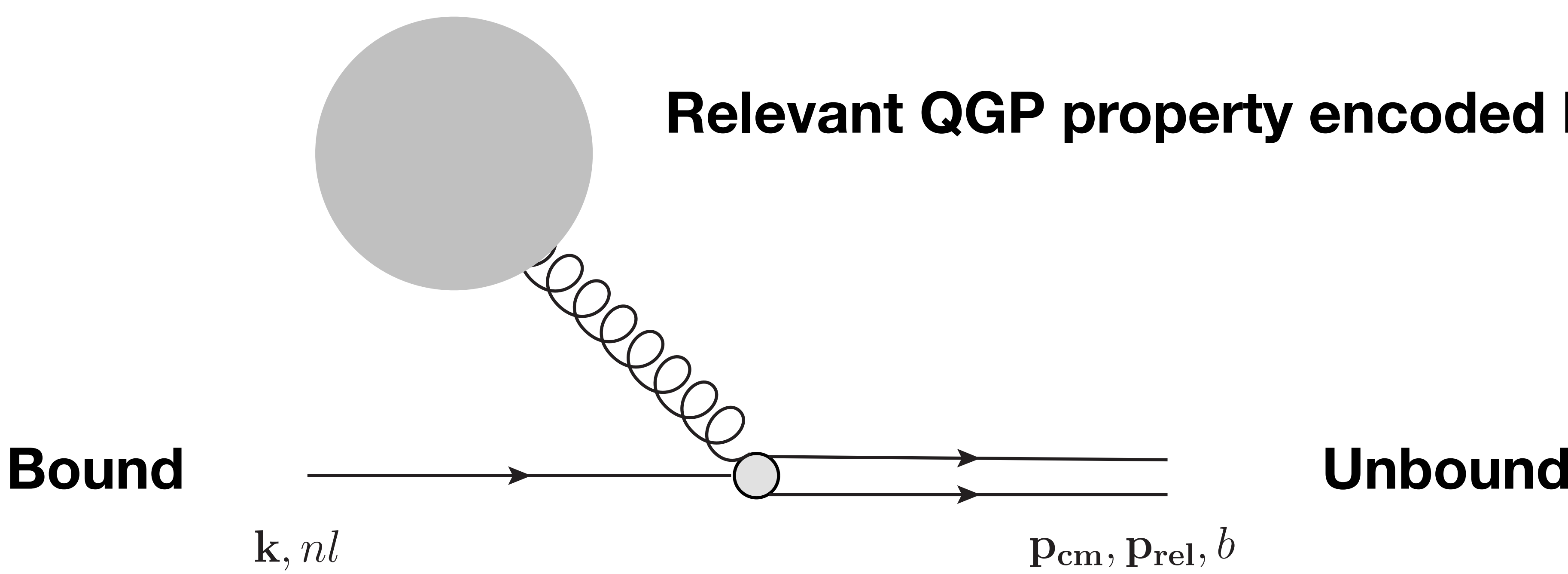
Dissociation & Recombination Terms

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) + \frac{\boldsymbol{k}}{2M} \cdot \nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}_{nl}^{+}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}_{nl}^{-}(\boldsymbol{x}, \boldsymbol{k}, t)$$

$$\mathcal{C}_{nl}^{-}(\boldsymbol{x}, \boldsymbol{k}, t) = g^2 \frac{T_F}{N_c} \sum_{i_1, i_2} \int \frac{d^3 p_{\text{cm}}}{(2\pi)^3} \frac{d^3 p_{\text{rel}}}{(2\pi)^3} \frac{d^4 q}{(2\pi)^4} (2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\text{cm}} + \boldsymbol{q}) \delta\Big(E_{nl} - \frac{p_{\text{rel}}^2}{M} - q_0\Big)$$
$$\times \langle \psi_{nl} | r_{i_1} | \Psi_{\boldsymbol{p}_{\text{rel}}} \rangle \langle \Psi_{\boldsymbol{p}_{\text{rel}}} | r_{i_2} | \psi_{nl} \rangle \, [g_E^{++}]_{i_1 i_2}^{>}(q_0, \boldsymbol{q}) f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t)$$

$$\mathcal{C}_{nl}^{+}(\boldsymbol{x}, \boldsymbol{k}, t) = g^2 \frac{T_F}{N_c} \sum_{i_1, i_2} \int \frac{d^3 p_{\text{cm}}}{(2\pi)^3} \frac{d^3 p_{\text{rel}}}{(2\pi)^3} \frac{d^4 q}{(2\pi)^4} (2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\text{cm}} - \boldsymbol{q}) \delta\Big(E_{nl} - \frac{p_{\text{rel}}^2}{M} + q_0\Big)$$
$$\times \langle \psi_{nl} | r_{i_1} | \Psi_{\boldsymbol{p}_{\text{rel}}} \rangle \langle \Psi_{\boldsymbol{p}_{\text{rel}}} | r_{i_2} | \psi_{nl} \rangle \, [g_E^{--}]_{i_2 i_1}^{>}(q_0, \boldsymbol{q}) f_O(\boldsymbol{x}, \boldsymbol{p}_{\text{cm}}, \boldsymbol{r} = 0, \boldsymbol{p}_{\text{rel}}, t)$$

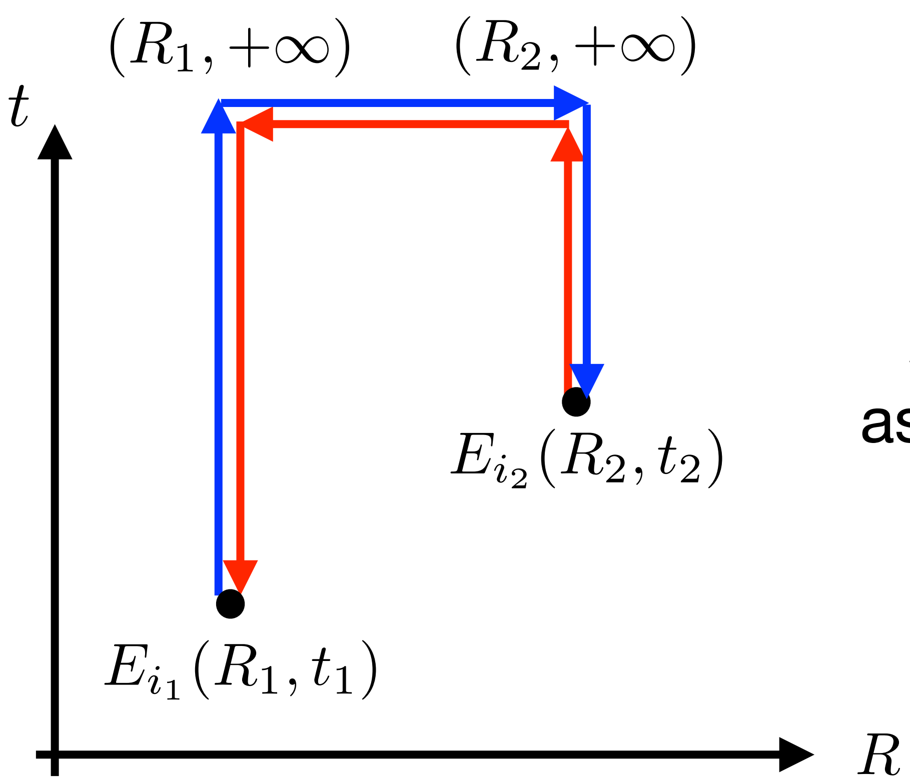
Relevant QGP property encoded by $[g_E^{\pm\pm}]^{>}$



Chromoelectric Field Correlator

$$[g_E^{++}]^{>}_{ji}(y,x) \equiv \Big\langle \big[E_j(y)\mathcal{W}_{[(y^0,\boldsymbol{y}),(+\infty,\boldsymbol{y})]}\mathcal{W}_{[(+\infty,\boldsymbol{y}),(+\infty,\boldsymbol{\infty})]}\big]^a \times \big[\mathcal{W}_{[(+\infty,\boldsymbol{\infty}),(+\infty,\boldsymbol{x})]}\mathcal{W}_{[(+\infty,\boldsymbol{x}),(x^0,\boldsymbol{x})]}E_i(x)\big]^a \Big\rangle_T$$

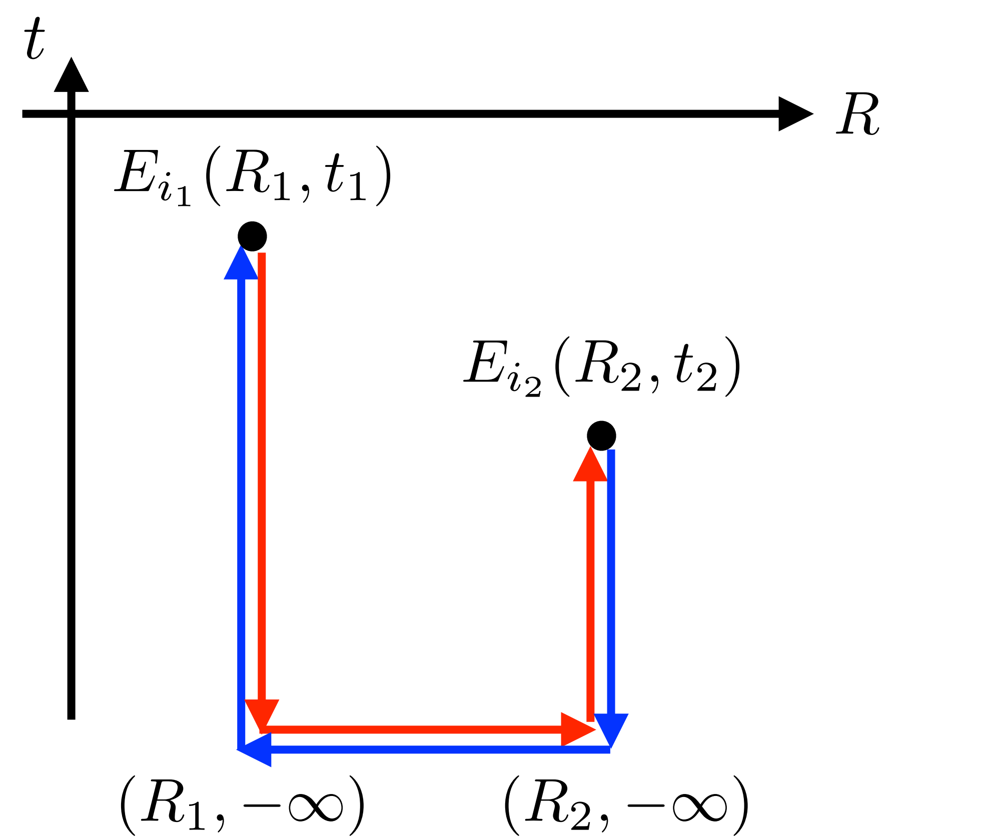
$$[g_E^{--}]^{>}_{ji}(y,x) \equiv \Big\langle \big[\mathcal{W}_{[(-\infty,\boldsymbol{\infty}),(-\infty,\boldsymbol{y})]}\mathcal{W}_{[(-\infty,\boldsymbol{y}),(y^0,\boldsymbol{y})]}E_j(y)\big]^a \times \big[E_i(x)\mathcal{W}_{[(x^0,\boldsymbol{x}),(-\infty,\boldsymbol{x})]}\mathcal{W}_{[(-\infty,\boldsymbol{x}),(-\infty,\boldsymbol{\infty})]}\big]^a \Big\rangle_T$$



PT transformation,
assume state invariant

↔

KMS relation



Dissociation: final-state interaction

Recombination: initial-state interaction

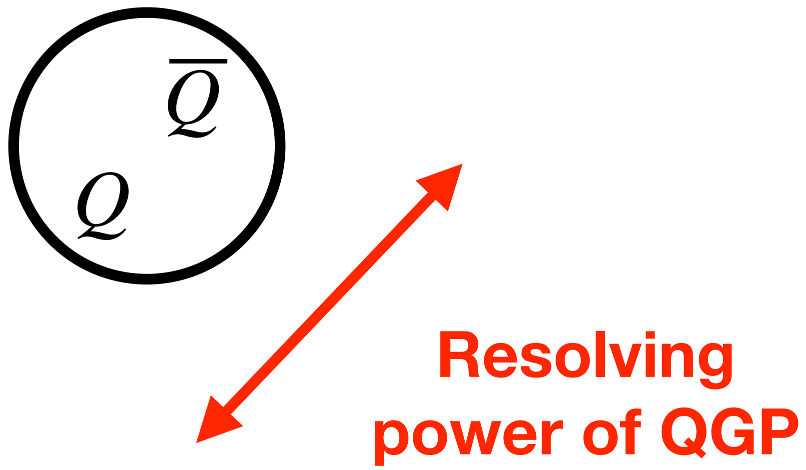
$$[g_E^{\pm\pm}]^{>}(p) = \int \mathrm{d}^4x\, e^{ip\cdot x}[g_E^{\pm\pm}]^{>}(x,0)$$

For total reaction rates, integrating over final momentum leads to setting $R_1 \to R_2$

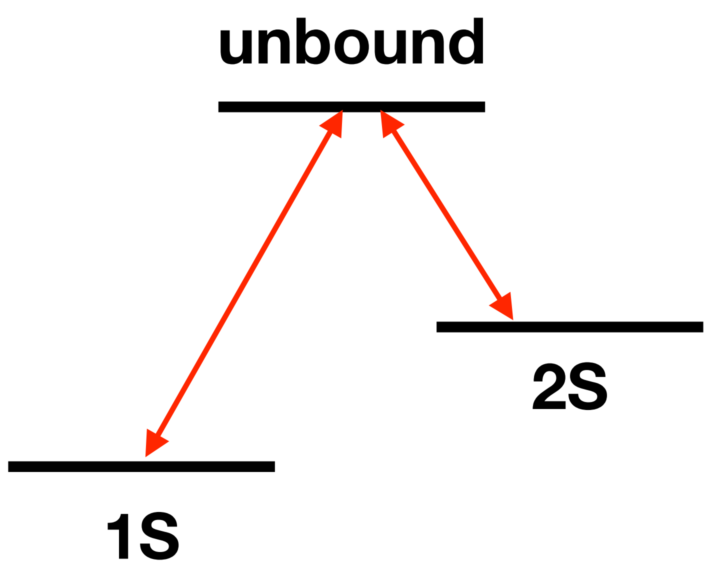
This object is what we learn from experimental data, like HQ diffusion coefficient

Two Temperature Regimes

- **Quantum optical limit (low T)**



Transitions between levels -> Boltzmann

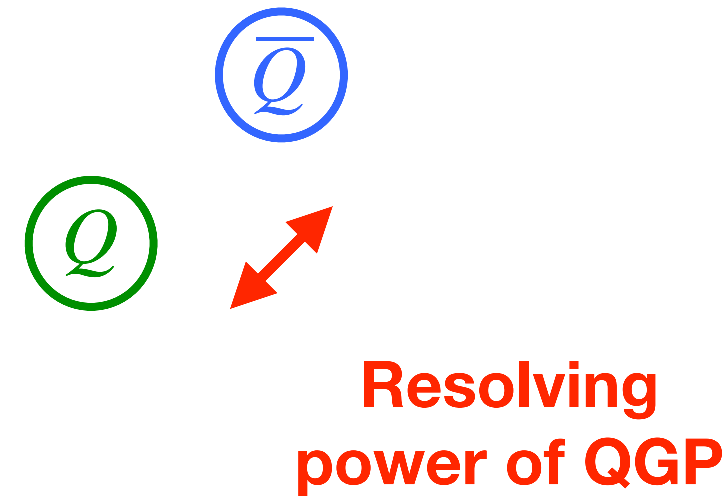


Gapped transition: finite energy transferred

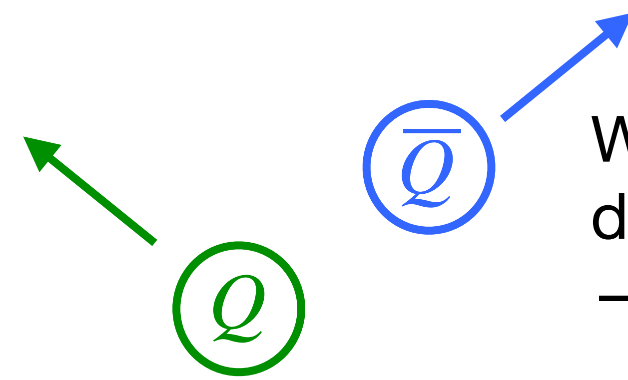
$$[g_E^{\pm\pm}]^>(p_0 > 0)$$

T.Mehen, XY, 1811.07027, 2009.02408

- **Quantum Brownian motion (high T)**



Diffusion of HQ pair -> Langevin



Wavefunction
decoherence
—> dissociation

Gapless transition: transport coefficient

$$[g_E^{\pm\pm}]^>(p_0 = 0) \begin{cases} \kappa_Q & \textbf{NRQCD} \\ \kappa_{Q\bar{Q}} & \textbf{pNRQCD} \end{cases}$$

J.-P. Blaizot, M.A.Escobedo, 1711.10812
T.Miura, Y.Akamatsu, M.Asakawa, A.Rothkopf, 1908.06293
N.Brambilla, et al, 2205.10289

Compare w/ LHC Data on Upsilon at 5.02 TeV

Coulomb potential —> no bottomonium mass change at finite T (see R. Larsen's talk)

Initial conditions: momentum: Pythia + nPDF EPPS16; position: Trento, binary collision

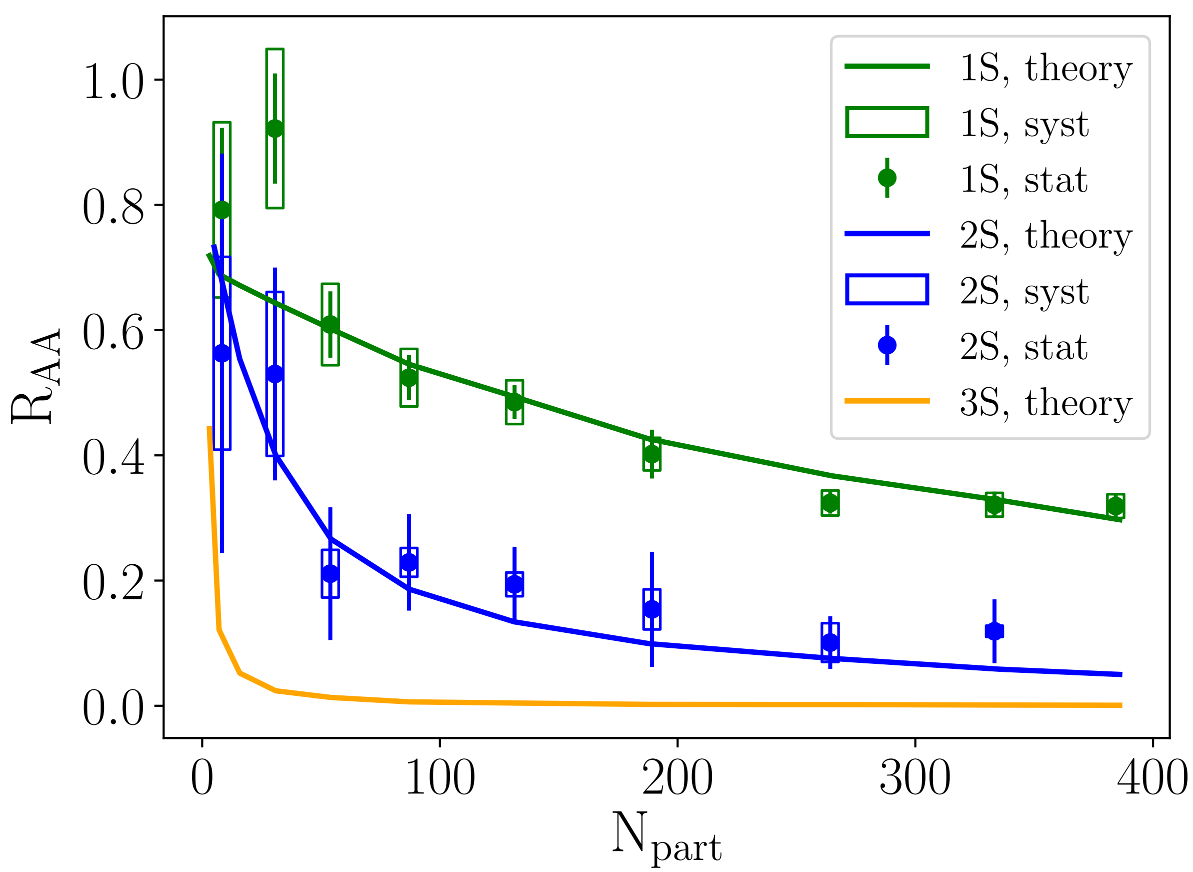
2+1D viscous hydro calibrated; HQ dynamics calibrated

Bottomonium: 1S, 2S, 3S, 1P, 2P; **no recombination for 3S, 2P**

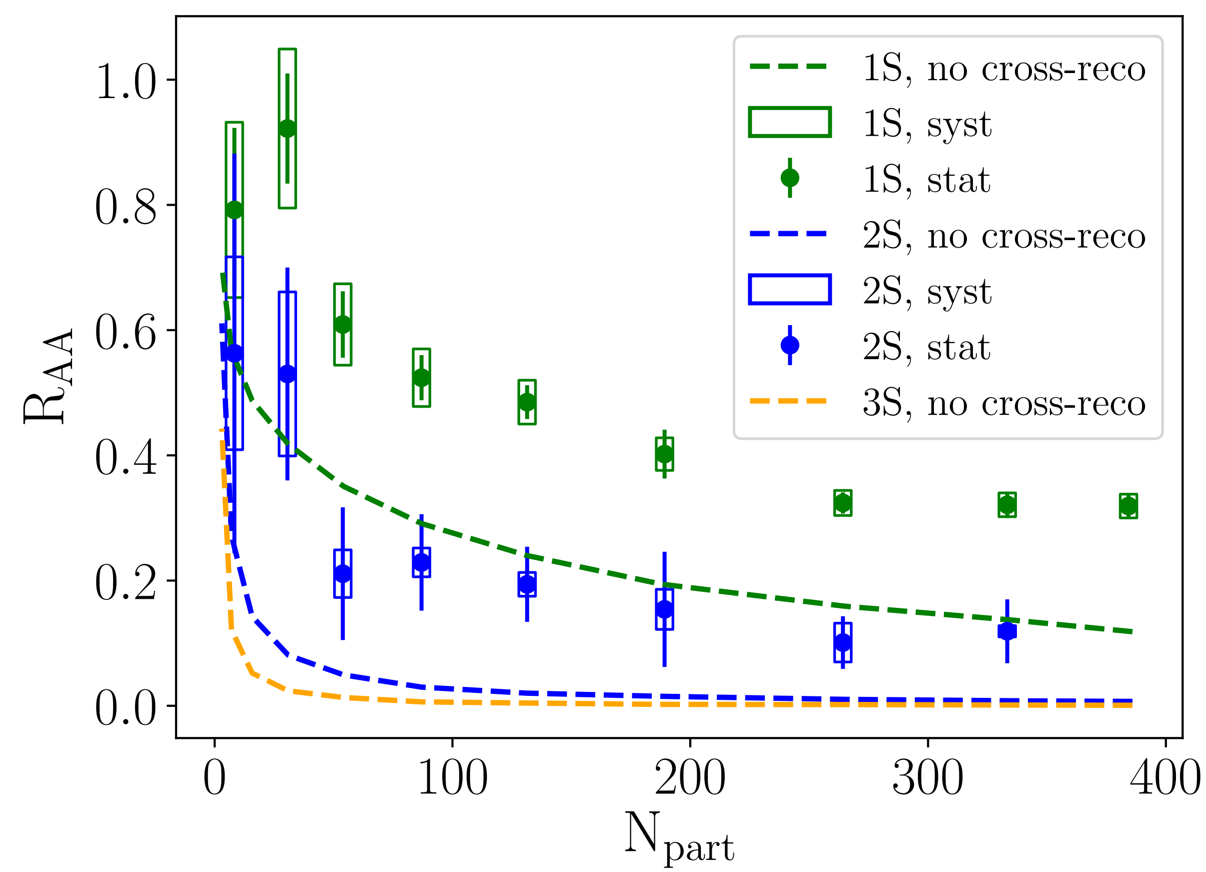
Feed-down networks

e.g. no 2S—>1S, 1S—>1P etc

with cross-talk (correlated) recombination

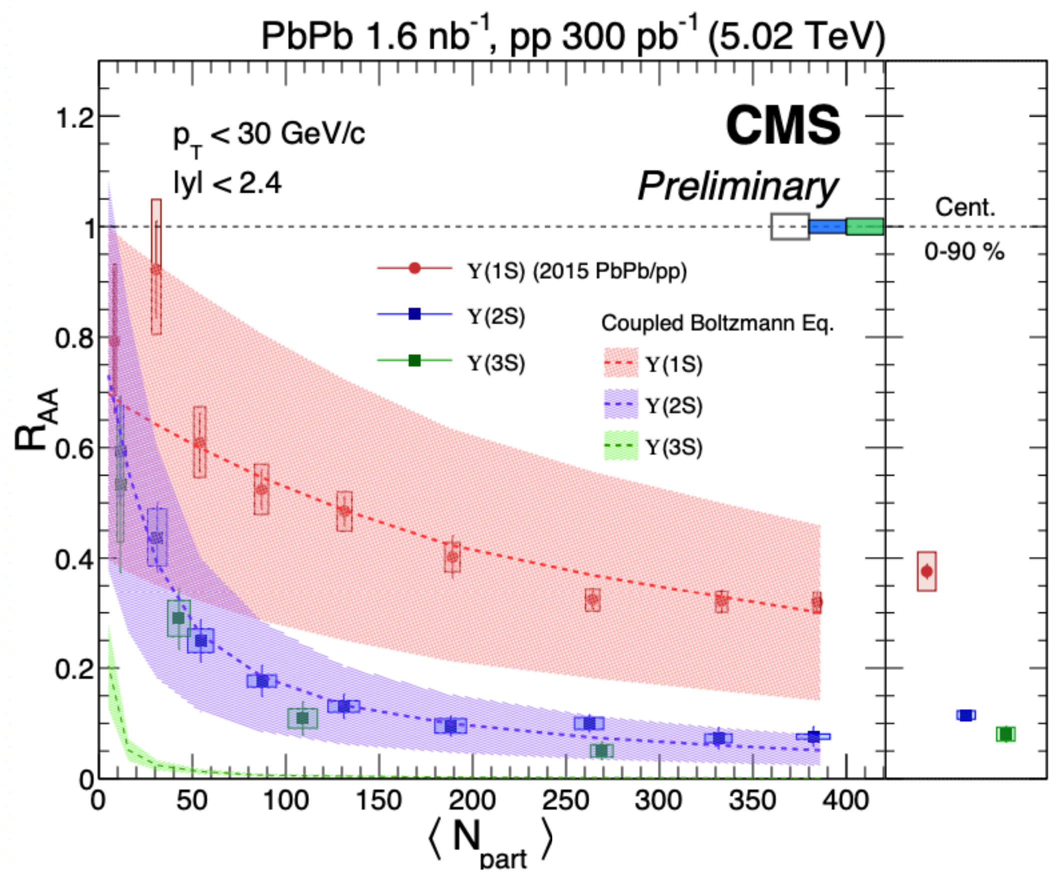


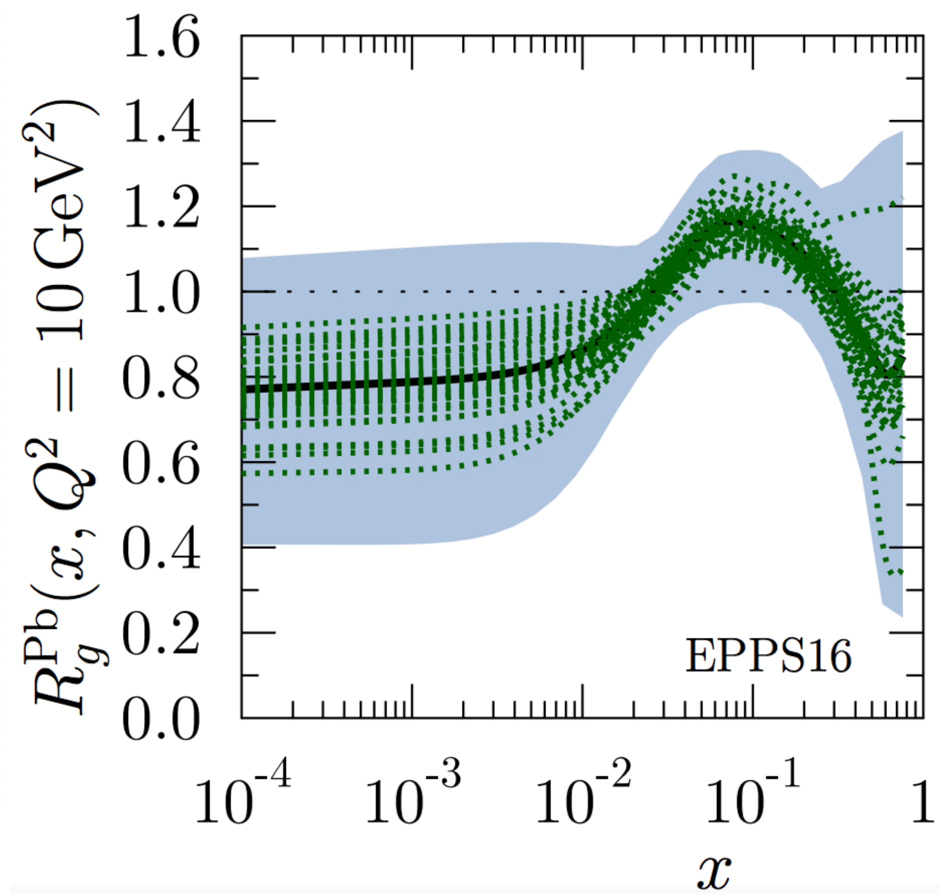
without cross-talk recombination



CMS arXiv:1805.09215

Uncertainty of nPDF and nPDF at RHIC Energy





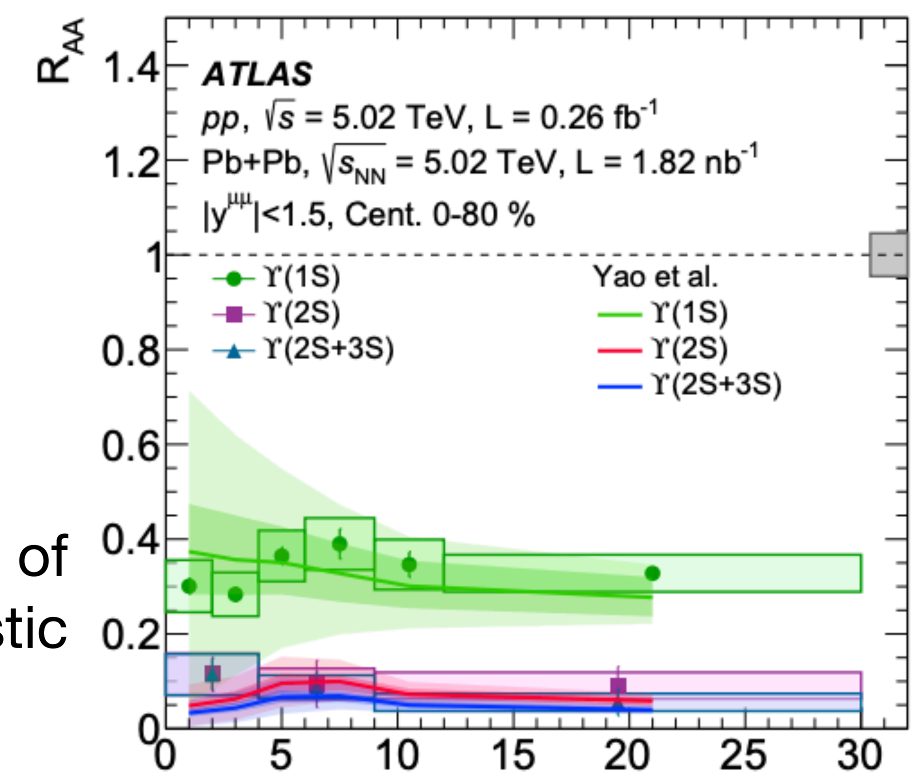
At mid rapidity

$$x \sim \frac{2m_T}{\sqrt{s}}$$

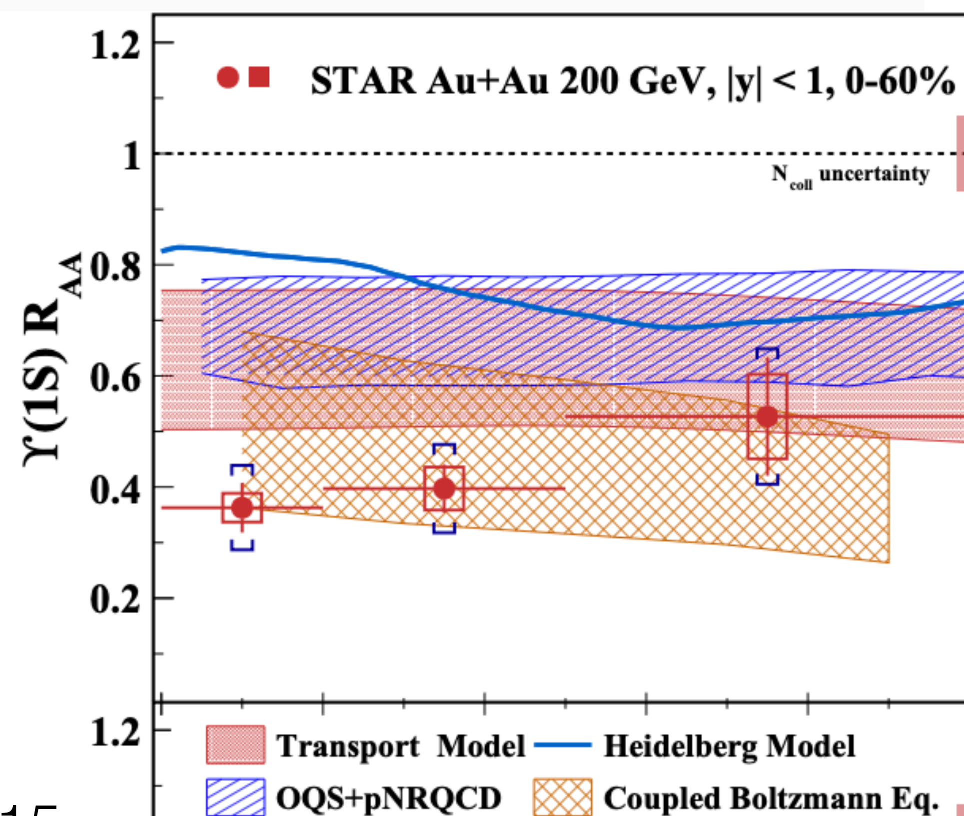
CNM

$$\sim [R_g^{Pb}(x)]^2$$

High p_T: breakdown of nonrelativistic treatment



ATLAS: 2205.03042

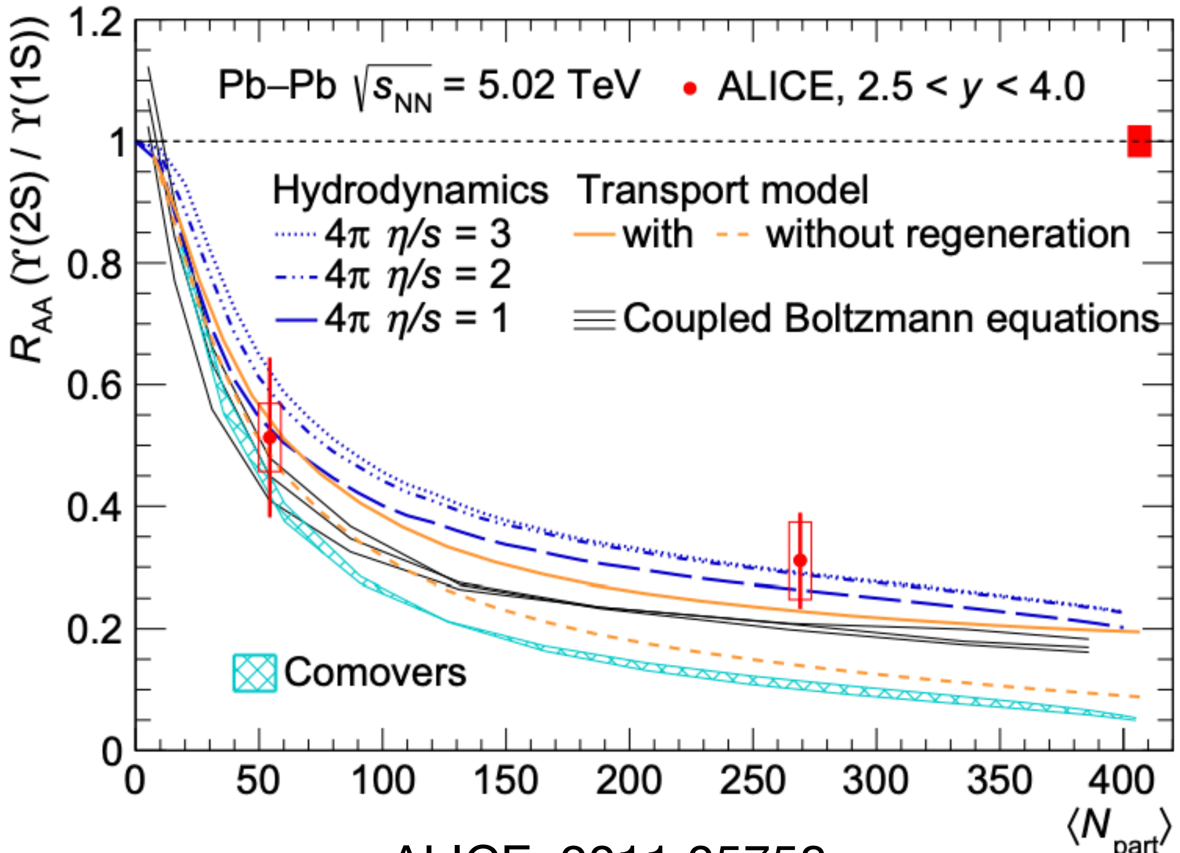


STAR: 2207.06568

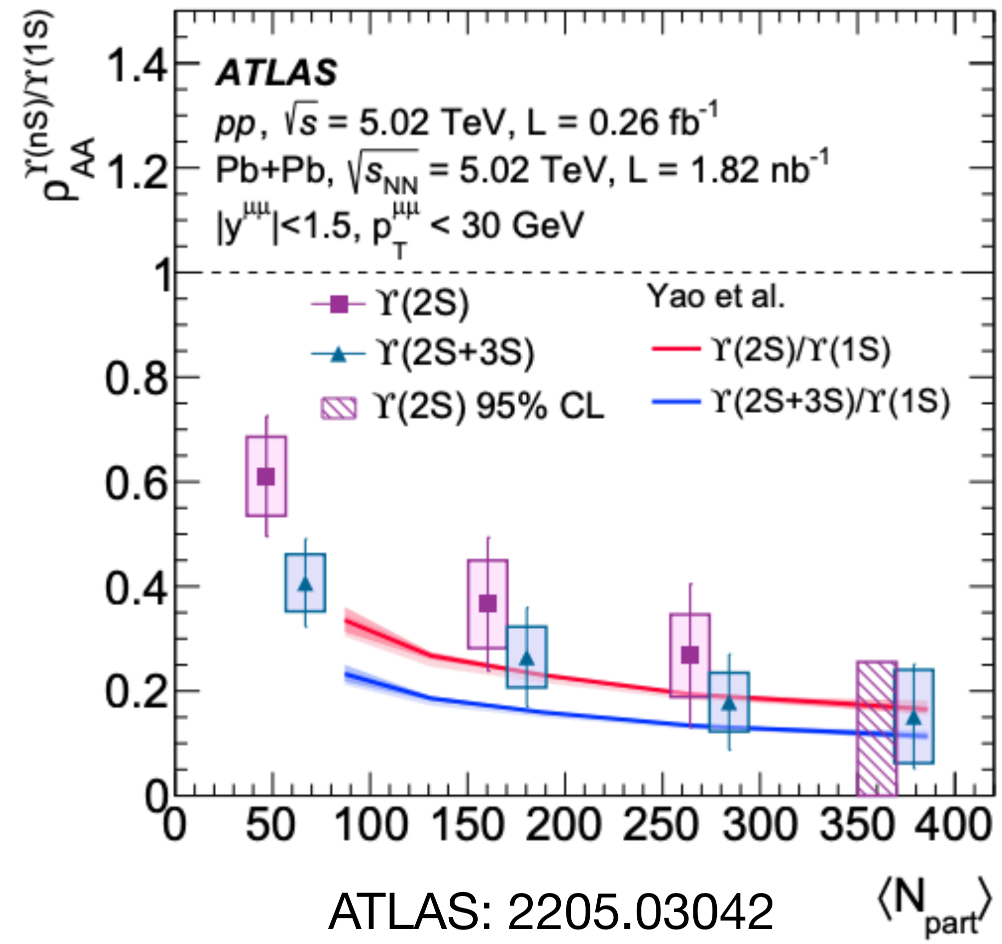
Input of nPDF:

$R_{pAu} = 0.82$
$\pm 0.10(stat)$
$\pm 0.08(syst)$

Double Ratio and Flow Observables

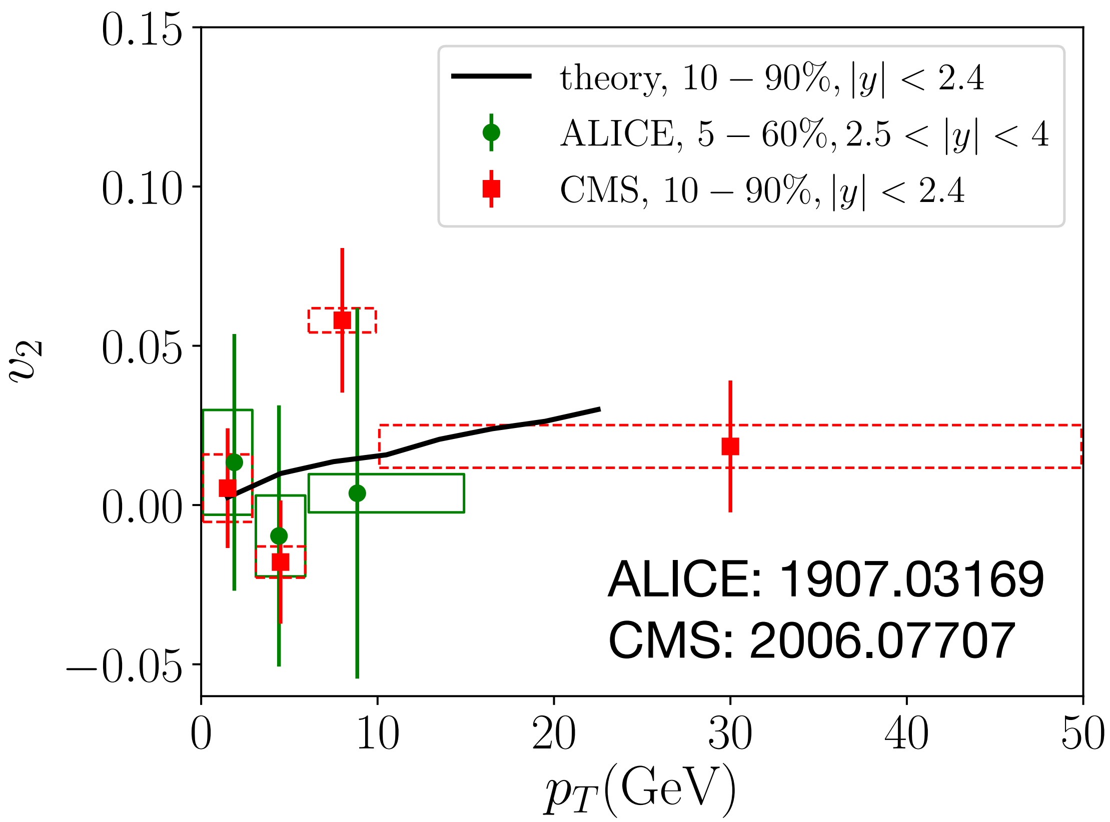


ALICE: 2011.05758



ATLAS: 2205.03042

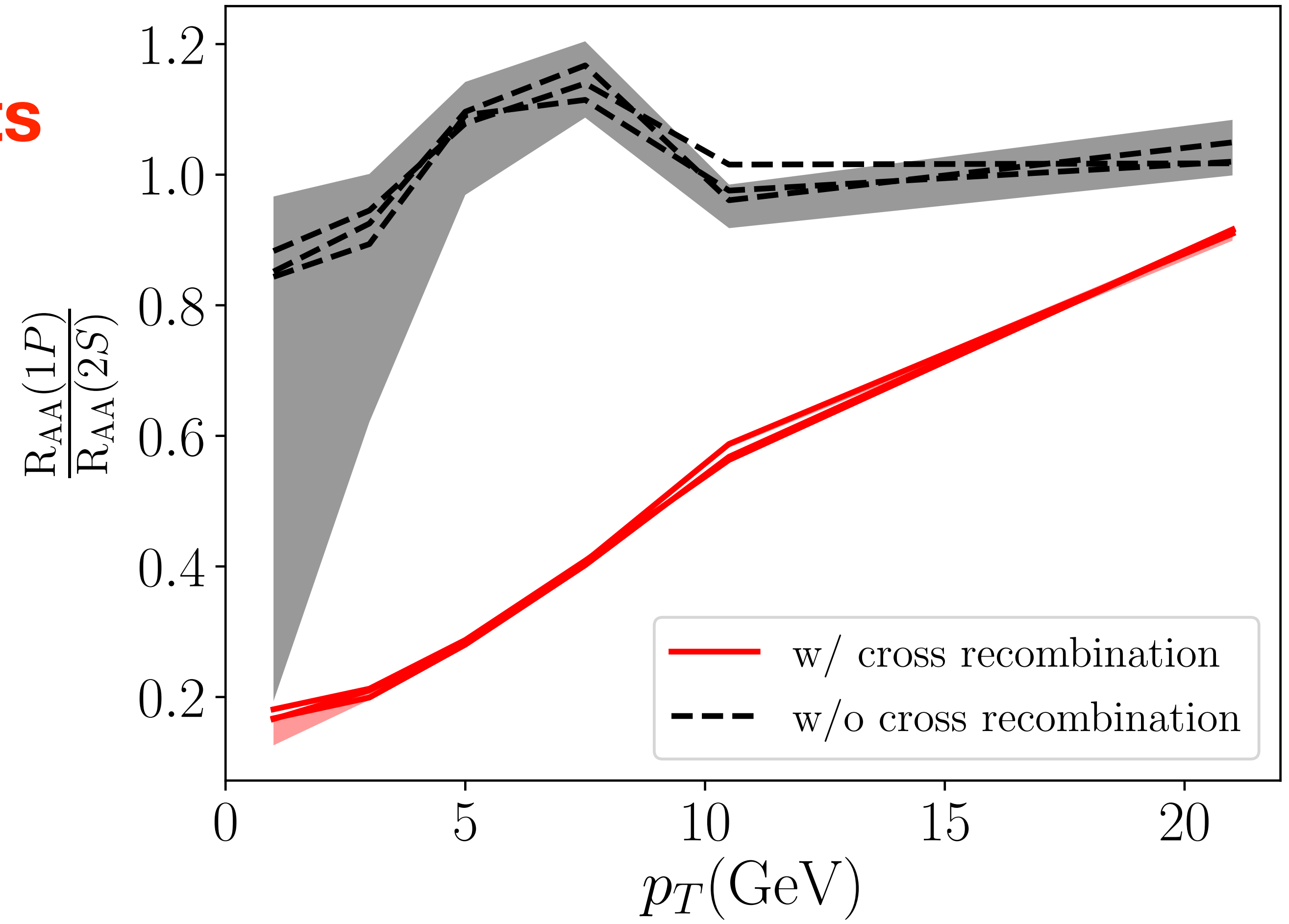
R_{AA} ratios have much smaller nPDF uncertainty



More precise flow observables in LHC Run3

Experimental Test of Correlated Recombination

Correlated recombination predicts 1P more suppressed than 2S



Traditional sequential suppression argument based on hierarchy of binding energy or size —> R_{AA}(2S) ~ R_{AA}(1P), since their binding energies are close

Correlated recombination rates (2S—>unbound—>1P) ~ (1P—>unbound—>2S) because of similar binding energy, but primordial production cross section

$$\frac{\sigma_{1P}}{\sigma_{2S}} \sim 4.5$$

Conclusions

- Coupled Boltzmann equations for open and hidden heavy flavors: Boltzmann equation for quarkonium is derived from open quantum system, valid for both weakly-coupled and strongly-coupled QGPs, see review 2102.01736

- Bottomonium phenomenology: nPDF uncertainty cancels in double ratio observables; importance of correlated recombination —> experimental test; opportunities at RHIC and LHC; include 3S recombination in future

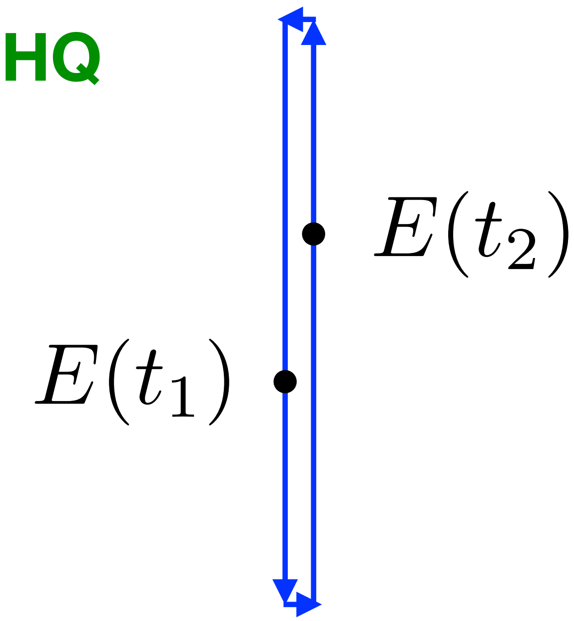
- **Joint experimental, phenomenological and theoretical (lattice) efforts to understand QGP properties from quarkonium production**

  - **Understand chromoelectric field correlators: gauge invariant, encode properties of QGP**

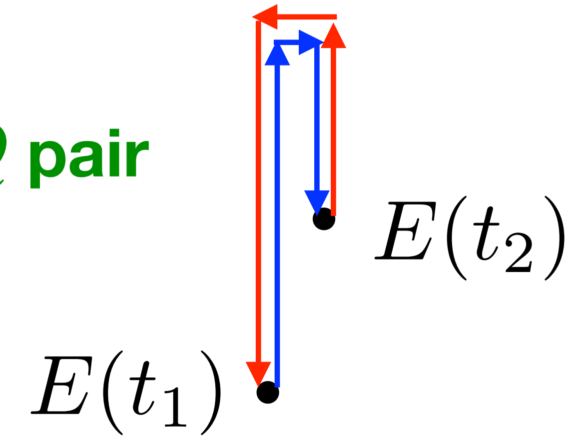
  - **Lattice/analytic calculations, phenomenological Bayesian analysis from experimental data (like what is done for HQ diffusion)**

Conclusions

HQ diffusion: single HQ



Quarkonium transport: $Q\bar{Q}$ pair



- At NLO, they differ by temperature independent constant

Y.Burnier, M.Laine, J.Langelage, L.Mether, 1006.0867
T.Binder, K.Mukaida, B.Scheihing-Hitschfeld, XY, 2107.03945

- In axial gauge, the two would seem to be the same —> problem of axial gauge when infinite Wilson lines are involved

B.Scheihing-Hitschfeld, XY, 2205.04477

- Nonperturbatively, not much is known about the correlator for quarkonium

- Go to Bruno Scheihing-Hitschfeld's talk for details