Relativistic Chiral Effective Field Theory for Neutrinoless Double Beta Decay

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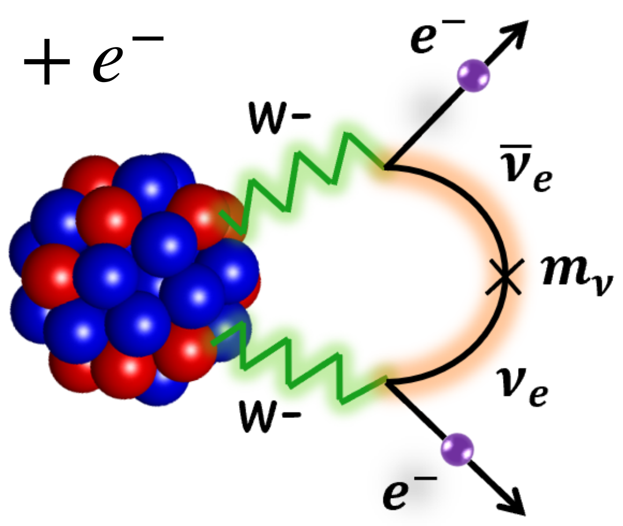
Outline

- **Introduction**
- Relativistic chiral EFT for $0\nu\beta\beta$ decay
  - $nn \to ppee$ amplitudes
  - Nuclear matrix elements
- Summary and outlooks

Nuclear matrix elements (NMEs)

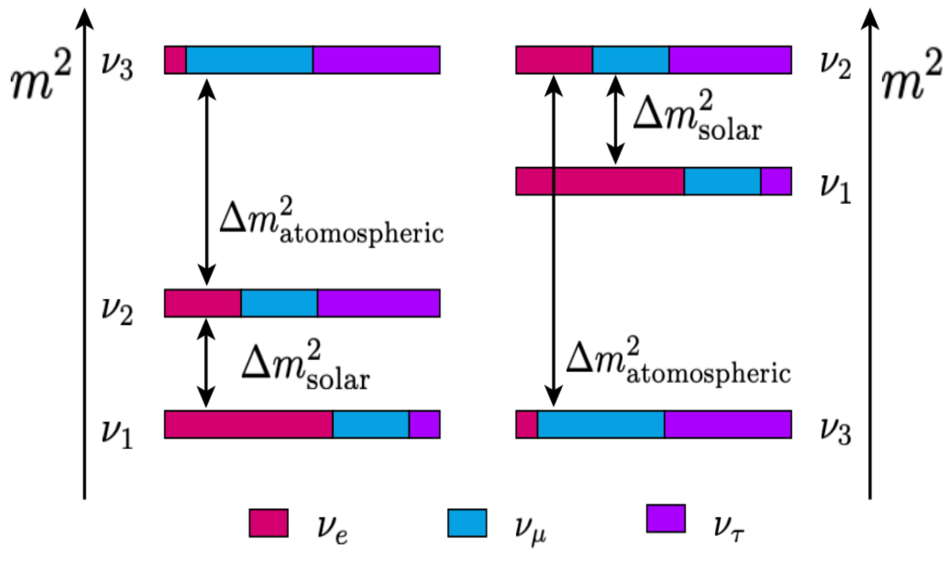
- Neutrinoless $\beta\beta$ decay ($0\nu\beta\beta$): $(A, Z) \rightarrow (A, Z+2) + e^- + e^-$
  - ✓ Lepton number violation
  - ✓ Majorana nature of neutrinos
  - ✓ Neutrino mass scale and hierarchy



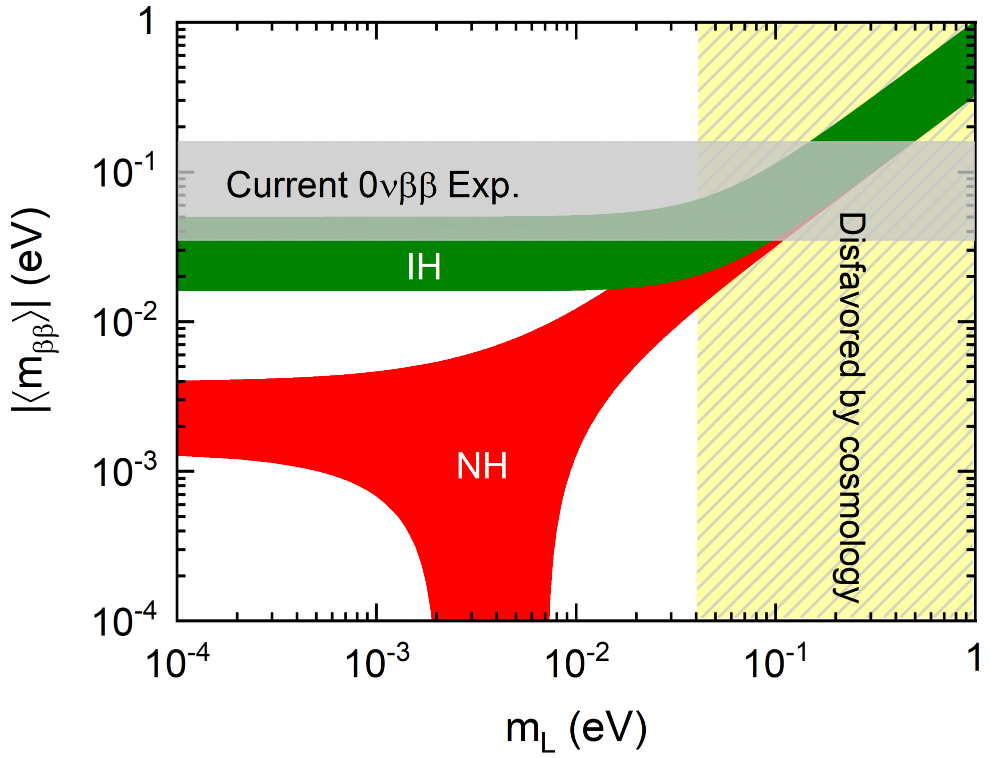
Avignone, Elliott, Engel, Rev. Mod. Phys. 80, 481 (2008)

- Accurate calculation of NMEs is crucial for interpreting the experimental limits

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} |m_{\beta\beta}|^2 |M^{0\nu}|^2 + \ldots$$



Normal Hierarchy (NH) Inverted Hierarchy (IH)



KamLAND-Zen Collaboration, PRL 130, 051801 (2023)

Hiararchies in $0\nu\beta\beta$

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu}\,|m_{\beta\beta}|^2\,|M^{0\nu}|^2 \qquad M_{0\nu} = \langle \Psi_f | V_\nu | \Psi_i \rangle$$

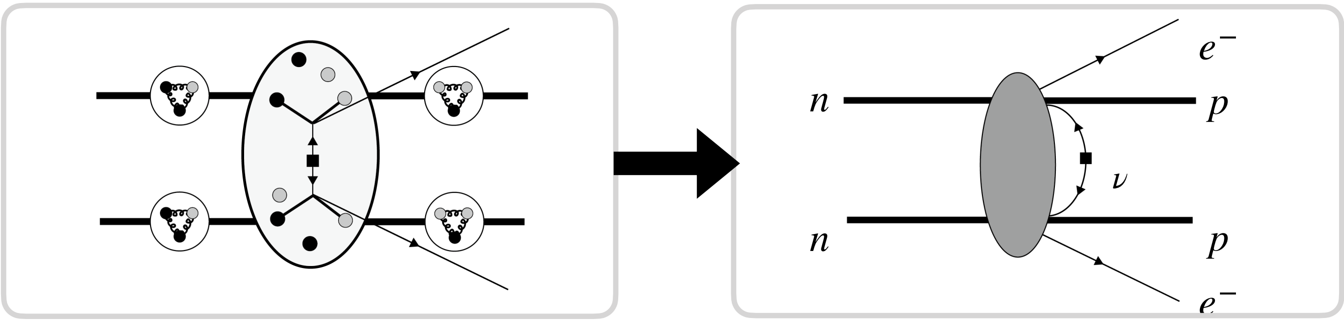
Energy Scale

? $m_{\beta\beta}$ Lepton-number-violating source

$$\mathscr{L}_{\mathrm{BSM}} = \mathscr{L}_{\mathrm{SM}} + \mathscr{L}_5 + \ldots \qquad \mathscr{L}_5 \to -\frac{1}{2} m_{\beta\beta} \nu^T C \nu$$

< 1 GeV V_ν Constructing the $0\nu\beta\beta$ transition operator (neutrino potential V_ν)

Matching quark-level operators to chiral EFT

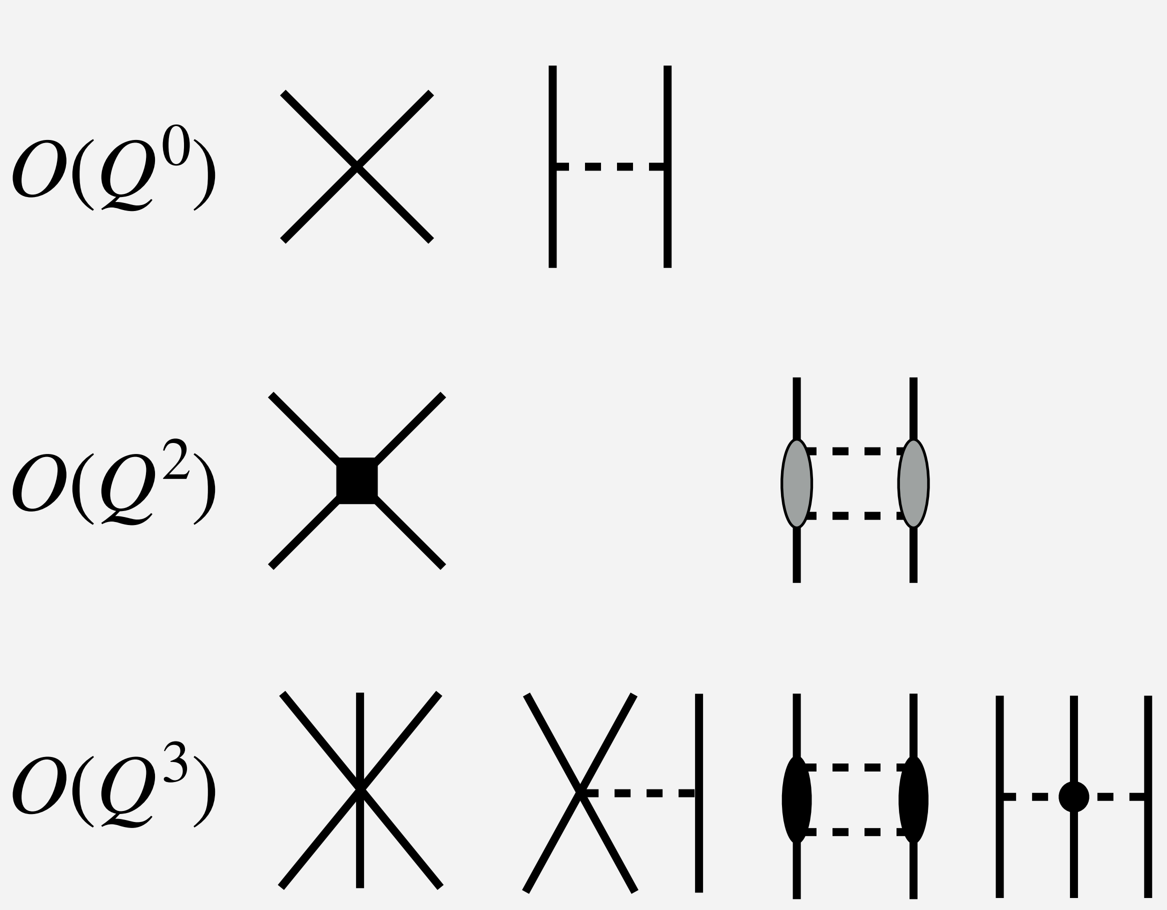


100 MeV $\Psi_{f,i}$ Nuclear many-body calculations with V_ν as inputs

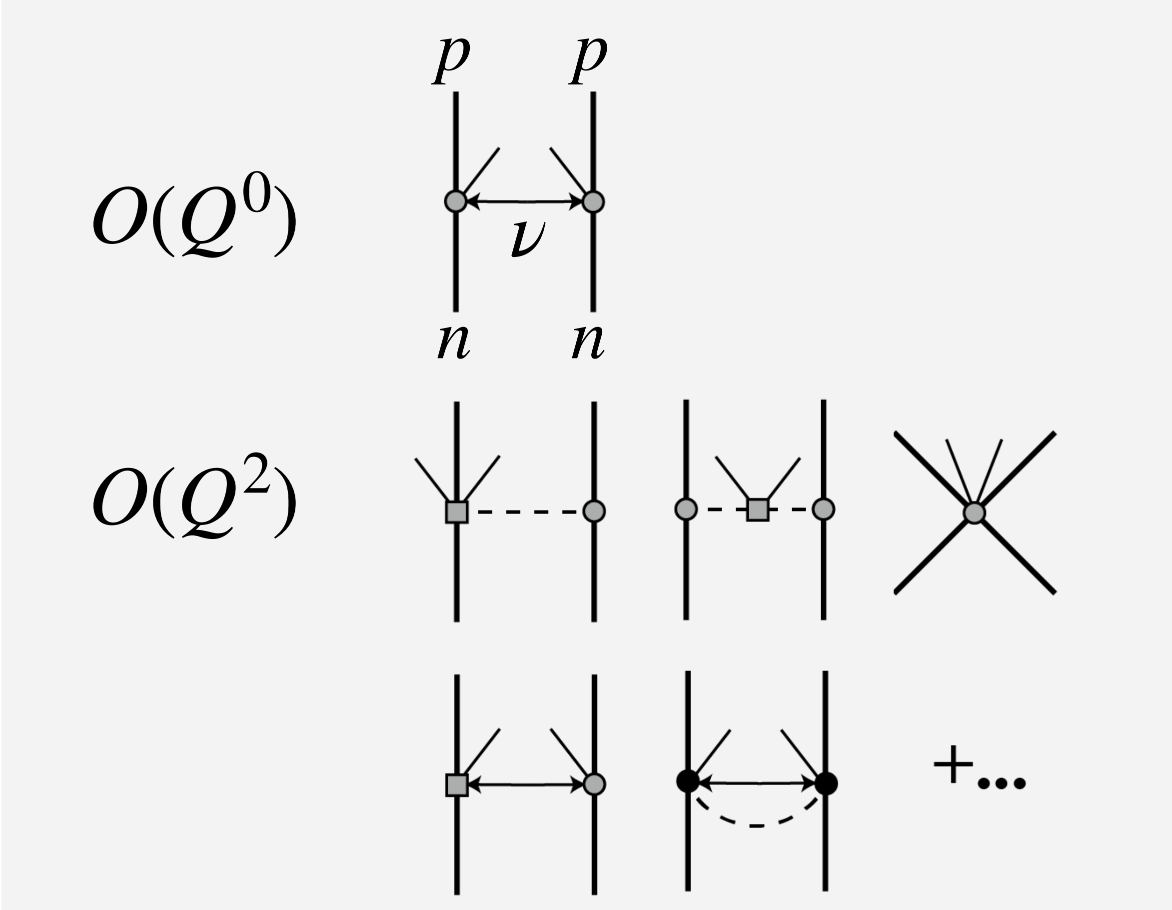
Neutrino potentials from chiral EFT

- Chiral EFT provides a way to derive neutrino potentials **consistently** with the nuclear force and **systematically** based on a **power counting**
- Weinberg power counting (Naive dimensional analysis)?

Nuclear force



Neutrino potential



Epelbaum, Hammer, and Meißner, RMP 81, 1773 (2009)

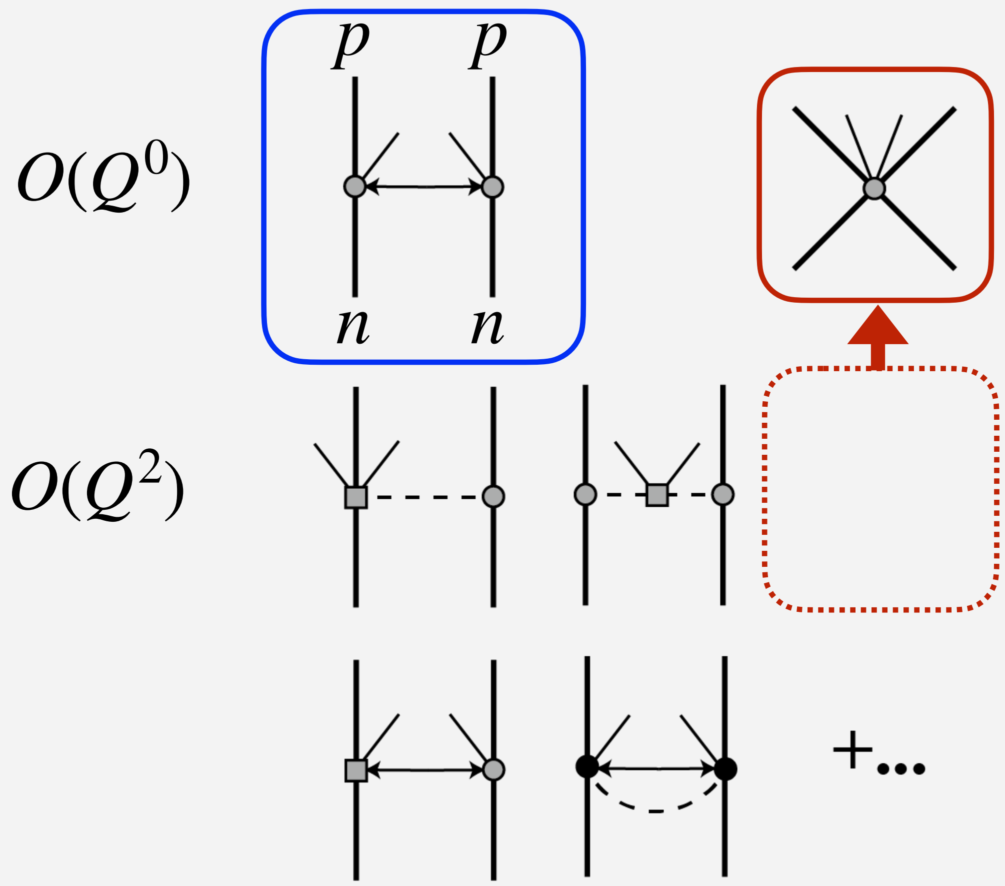
Cirigliano, Dekens, Mereghetti, and Walker-Loud, PRC 97, 065501 (2018)

LO contact term from chiral EFT

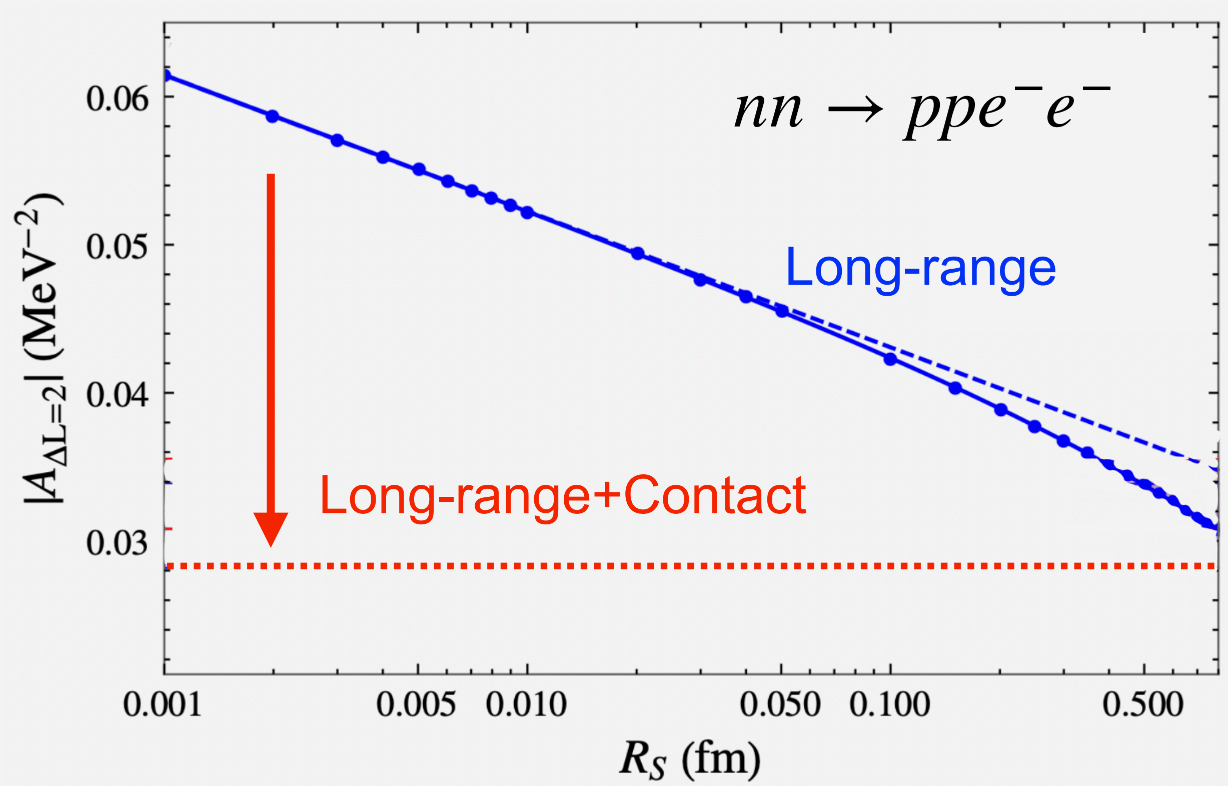
- The LO long-range term does not yield renormalizable amplitudes
- To ensure the renormalizability, a contact term needs to be promoted to LO

$$V_{\mathrm{CT}} = -2g_\nu^{\mathrm{NN}}\tau_1^+\tau_2^+$$

Power counting



Renormalizability

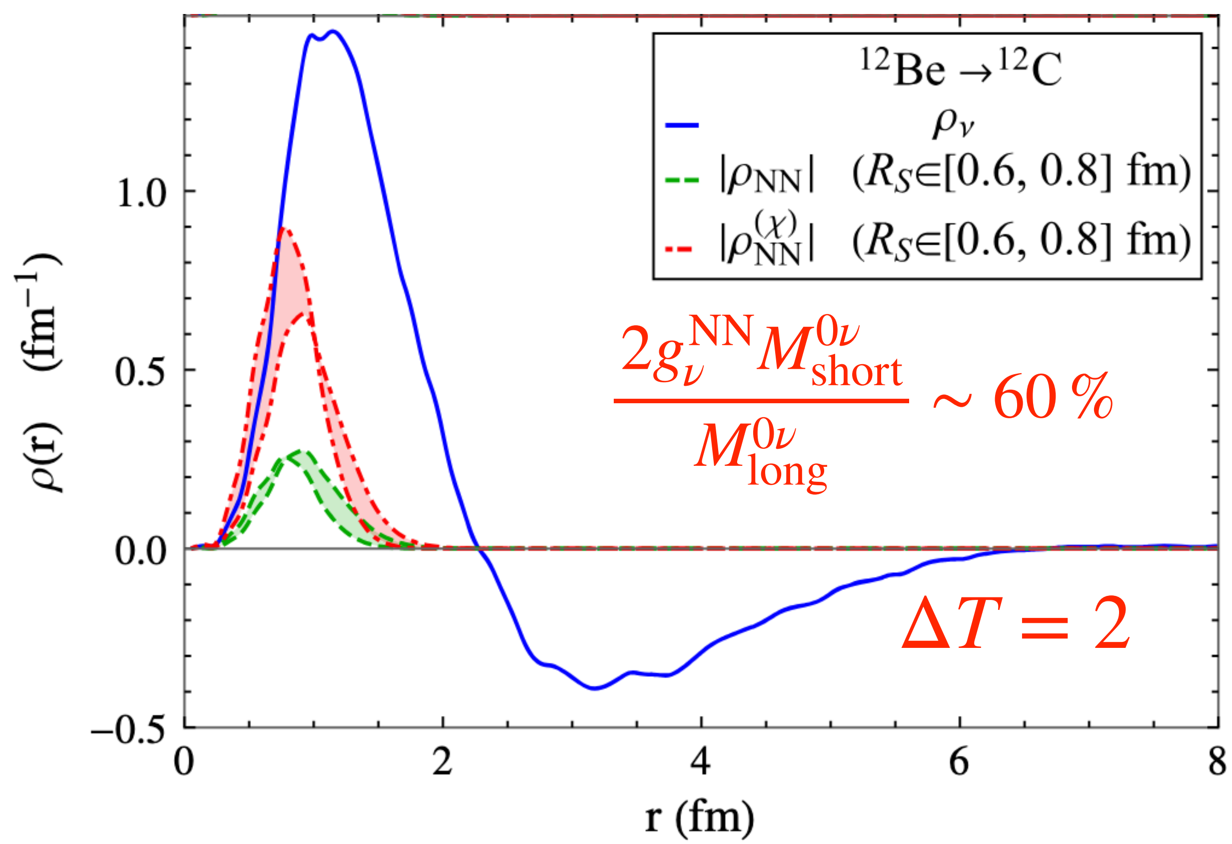


Cirigliano, Dekens, de Vries, Graesser, Mereghetti, Pastore, and van Kolck, PRL 120, 202001 (2018)

Impact of LO contact term

- The LO contact term may significantly change the NME values

$$M^{0\nu} = M^{0\nu}_{\text{long}} - 2g_\nu^{\text{NN}} M^{0\nu}_{\text{short}}$$



Here, it is assumed that $g_\nu^{\rm NN} = (C_1 + C_2)/2$, i.e., fixed by the CIB LEC in NN potential

Cirigliano, Dekens, de Vries, Graesser, Mereghetti, Pastore, and van Kolck, PRL 120, 202001 (2018)

- The LEC $g_\nu^{\rm NN}$ is unknown and largely uncertain—How to determine $g_\nu^{\rm NN}$?

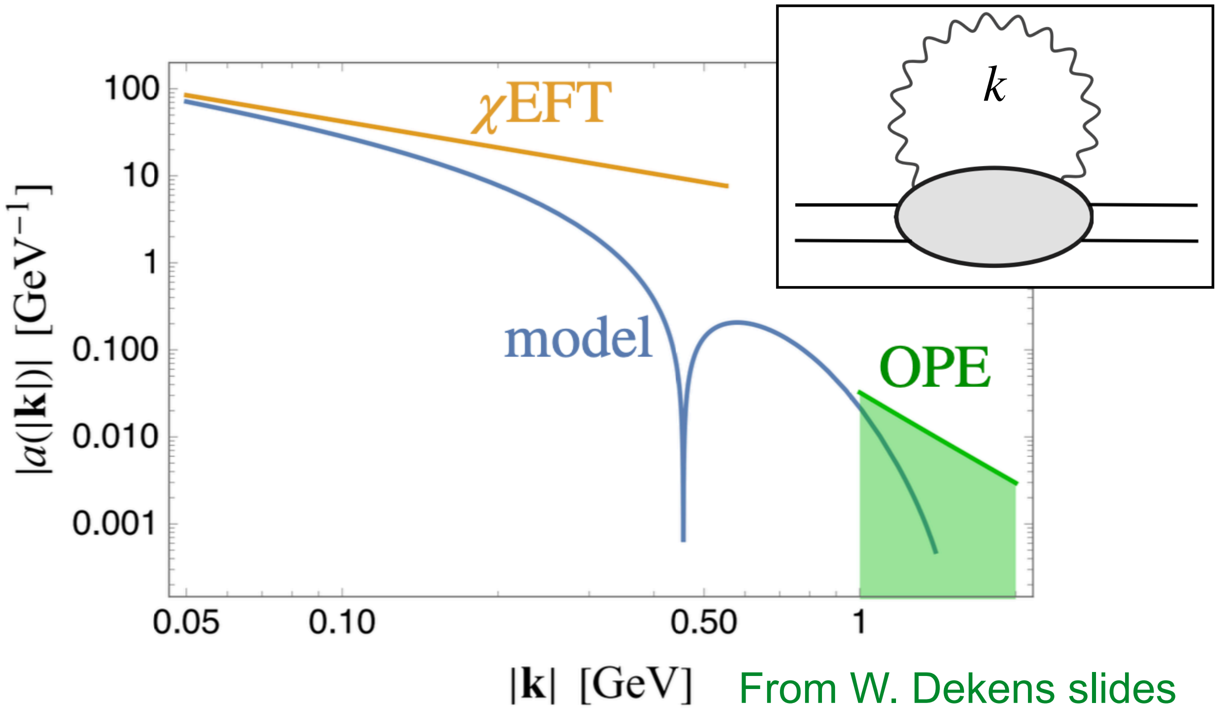
Generalized Cottingham model

- Matching $g_\nu^{\rm NN}$ to a "full-theory" amplitude from a generalized Cottingham model
  - Model dependence
    1. Neglect inelastic intermediate states
    2. Phenomenological off-shell NN amplitudes
    3. Phenomenological weak form factors

    …
  - $nn \to ppee$ amplitude:

$$|\mathcal{A}(p' = 30\ \mathrm{MeV}, p = 25\ \mathrm{MeV})| = 0.0195(5)\ \mathrm{MeV}^{-2}$$

Cirigliano et al., PRL 126, 172002 (2021), Cirigliano et al., JHEP 05, 289 (2021)



From W. Dekens slides

- Serving as a synthetic datum to constrain $g_\nu^{\rm NN}$ in *ab initio* calculations of NMEs

Wirth, Yao, and Hergert, PRL 127, 242502 (2021), …

How to interpret the enhancement of the LO contact term $g_\nu^{\rm NN}$?

How to benchmark the estimations of $nn \to ppee$ amplitude and $g_\nu^{\rm NN}$?

Relativistic EFT formulation?

- The above analyses are all performed in nonrelativstic famework
- Lorentz invariance is a fundamental symmetry—an EFT formulation should be consistent with **Relativity**
  - For kinematics, nonrelativistic expansion is a well controlled approximation

$$\sqrt{p^2 + m_N^2} = m_N + \frac{p^2}{2m_N} + O(p^4) \qquad k_F/m_N \sim 0.25$$

  - In renormalization, relativity could have significant effect

$$T = V + \boxed{\int_0^\infty} VGT$$

Virtual excitations up to high momenta

EFT for $0\nu\beta\beta$ decay can be relativistic!

Studying $0\nu\beta\beta$ decay in a relativistic framework provides a different view of the renormalization and, thus, the contact term in chiral EFT

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Modified Weinberg's approach

- Employ the Lorentz invariant Lagrangian and decompose the fermion propagator

Gegelia and Epelbaum, PLB 716, 338 (2012)

$$\frac{\not p + m}{p^2 - m^2 + i\epsilon} = \frac{2m\,P_+ + (\not p - m\not v)}{p^2 - m^2 + i\epsilon} = \underbrace{\frac{2m\,P_+}{p^2 - m^2 + \mathrm{i}\epsilon}}_{\text{include nonperturbatively}} + \underbrace{\cdots}_{\text{treat as correction}}$$

Keep the pole $p_0 = +\sqrt{p^2 + m_N^2}$ and do NOT perform the nonrelativistic reduction

- Use the Kadyshevsky equation to calculate the scattering *T*-matrix

$$T(\boldsymbol{p}', \boldsymbol{p}; E) = V(\boldsymbol{p}', \boldsymbol{p}) + \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} V(\boldsymbol{p}', \boldsymbol{k}) \frac{m_N^2}{m_N + k^2} \frac{1}{E - 2\sqrt{k^2 + m_N^2} + \mathrm{i}\epsilon} T(\boldsymbol{k}, \boldsymbol{p}; E)$$

Reduces to the usual Lippmann-Schwinger equation in $m_N \to \infty$ Limit

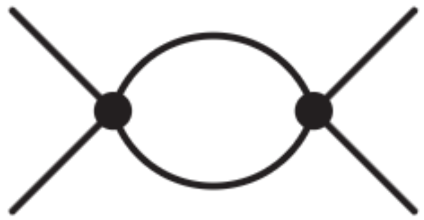
- Apply Weinberg power counting to organize the NN potential

$$V^{(0)} = C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{g_A^2}{4 f_\pi^2} \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{q} \boldsymbol{\sigma}_2 \cdot \boldsymbol{q}}{\boldsymbol{q}^2 + m_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

UV behavior: relativistic v.s. nonrelativistic

- Renormalizability is closely related to the UV behavior of the loop integrals
- Consider the bubble diagram that enters NN scattering

$$I(p) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\,\theta(\Lambda - |\vec{k}|)\,\frac{1}{(k^2+m_N^2)\left[\sqrt{p^2+m_N^2}-\sqrt{k^2+m_N^2}+i\epsilon\right]}$$



- Calculate the loop and then expand over $1/m_N$

$$I = \frac{1}{4\pi^2}\left[\boxed{-\frac{2i\pi p}{m_N}}\boxed{-2\ln\frac{\Lambda}{m_N}} - m_N^2(\pi+\ln 4) + \mathcal{O}\left(\frac{1}{m_N^2},\frac{1}{\Lambda}\right)\right]$$

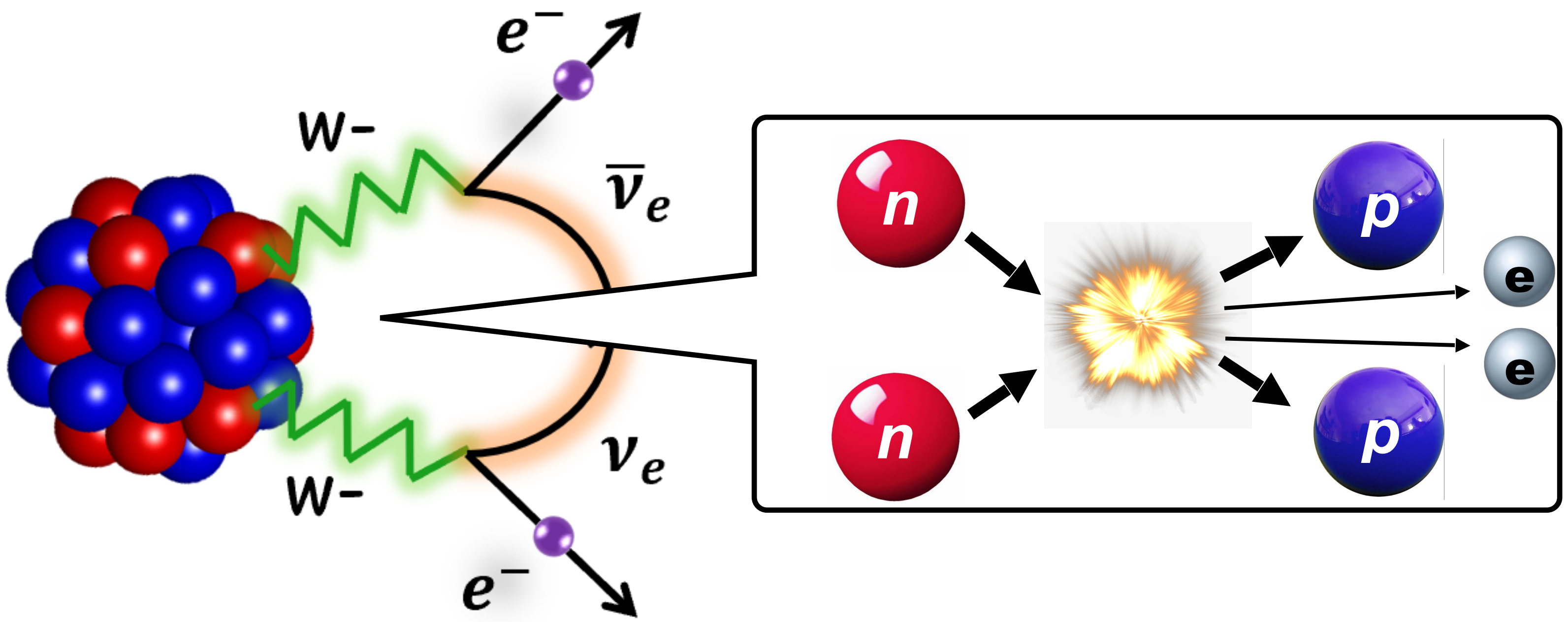
- Expand over $1/m_N$ (Nonrelativistic EFT) and then calculate the loop

$$I = \frac{1}{4\pi^2}\left[\boxed{-\frac{2i\pi p}{m_N}}\boxed{-\frac{4\Lambda}{m_N}} + \mathcal{O}\left(\frac{1}{m_N^2},\frac{1}{\Lambda}\right)\right]$$

Gegelia and Epelbaum, PLB 716, 338 (2012)

- Same low-energy physics; different UV divergence absorbed by contact terms
- Both works in NN scattering using $C_S, C_T, \ldots$; What about $0\nu\beta\beta$ decay g_ν^{NN}?

$nn \to ppee$ process



$$n + n \to p + p + e^- + e^- \qquad A^{0\nu} = \langle nn \,|\, V_\nu \,|\, pp \rangle$$

- Two-body problem can be solved exactly within EFT
- Benchmarks with first-principle lattice QCD calculations

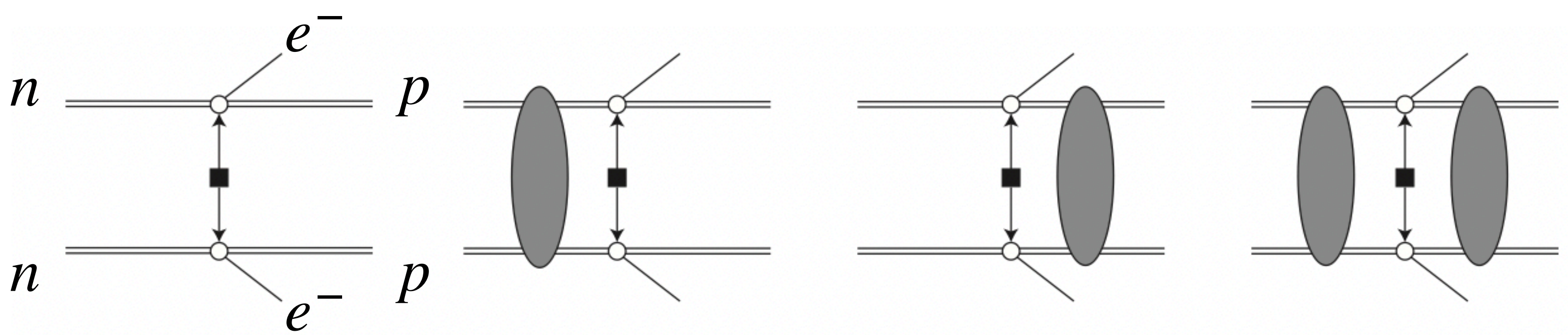
➡ Theoretical laboratory to study and constrain the contact term g_ν^{NN}

$nn \to ppee$ amplitude at LO

- We focus on $nn \to ppee$ in ${}^1S_0$ wave, as it is the only channel requiring a contact term in the nonrelativistic framework。

Cirigliano et al., PRL 120, 202001 (2018)
Cirigliano et al., PRC 97, 065501 (2018)

- The LO $nn \to ppee$ decay amplitude



- Standard LO strong and neutrino potentials

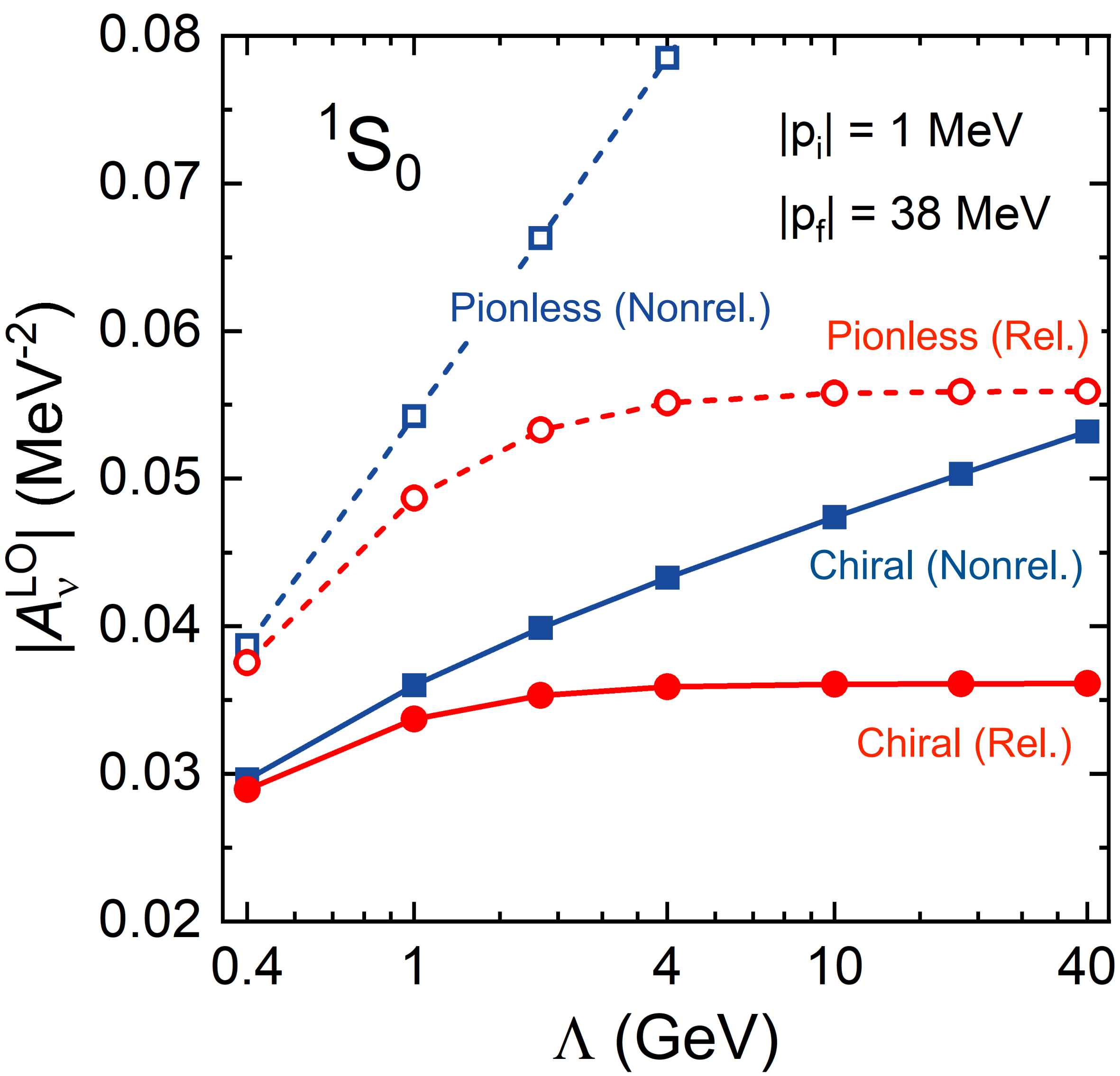
Strong potential: $V_s^{(0)} = C - \frac{g_A^2}{4f_\pi^2}\frac{m_\pi^2}{m_\pi^2+q^2}$ LEC fixed by $a_{np}({}^1S_0) = -23.74(2)$ fm

Neutrino potential: $V_\nu^{(0)} = \frac{\tau_1^+\tau_2^+}{q^2}\left\{1+2g_A^2\left[1+\frac{m_\pi^4}{2(q^2+m_\pi^2)^2}\right]\right\}$ $\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$

Renormalizability at LO

- In a relativistic formulation, renormalizability is achieved without a LO contact term

YLY and P. W. Zhao, PLB 855, 138782 (2024)



$\Lambda \to \infty$

Nonrel.: logarithmic

Rel.: convergent

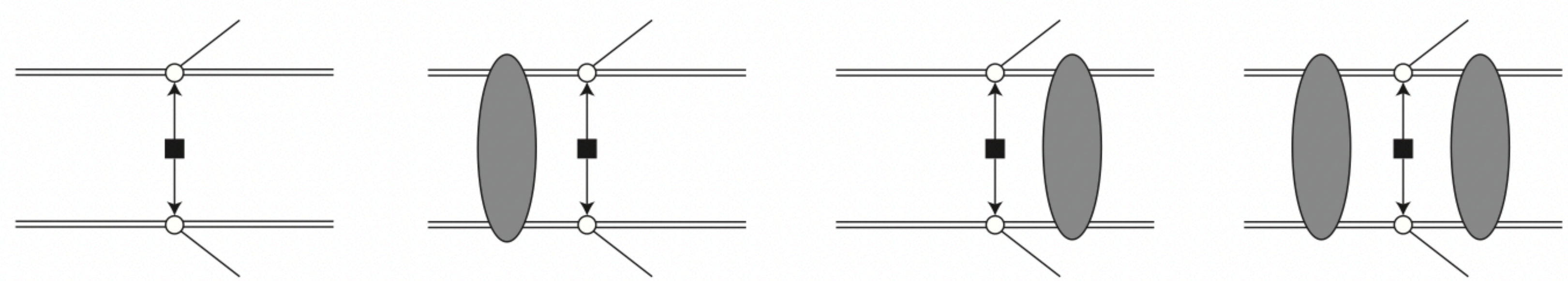
Regularization $V(\boldsymbol{p}', \boldsymbol{p}) \to e^{-p'^4/\Lambda^4} V(\boldsymbol{p}', \boldsymbol{p}) e^{-p^4/\Lambda^4}$

Analysis of UV divergence

- Relativistic scattering equation has a milder ultraviolet (UV) behavior

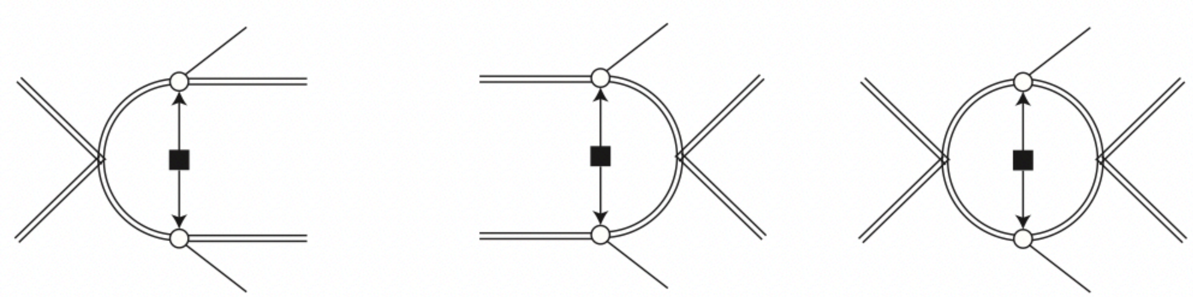
	Relativistic	Nonrelativistic
Propagator:	$\frac{m_N^2}{k^2+m_N^2}\frac{1}{E-2\sqrt{k^2+m_N^2}+\mathrm{i}0^+}$	$\rightarrow \frac{1}{E_{\mathrm{kin}}-k^2/m_N+\mathrm{i}0^+}$
UV behavior ($k\sim\Lambda$):	$O(\Lambda^{-3})$	$O(\Lambda^{-2})$

- The degree of divergence of $nn \rightarrow ppee$ decay amplitude：



UV behavior:

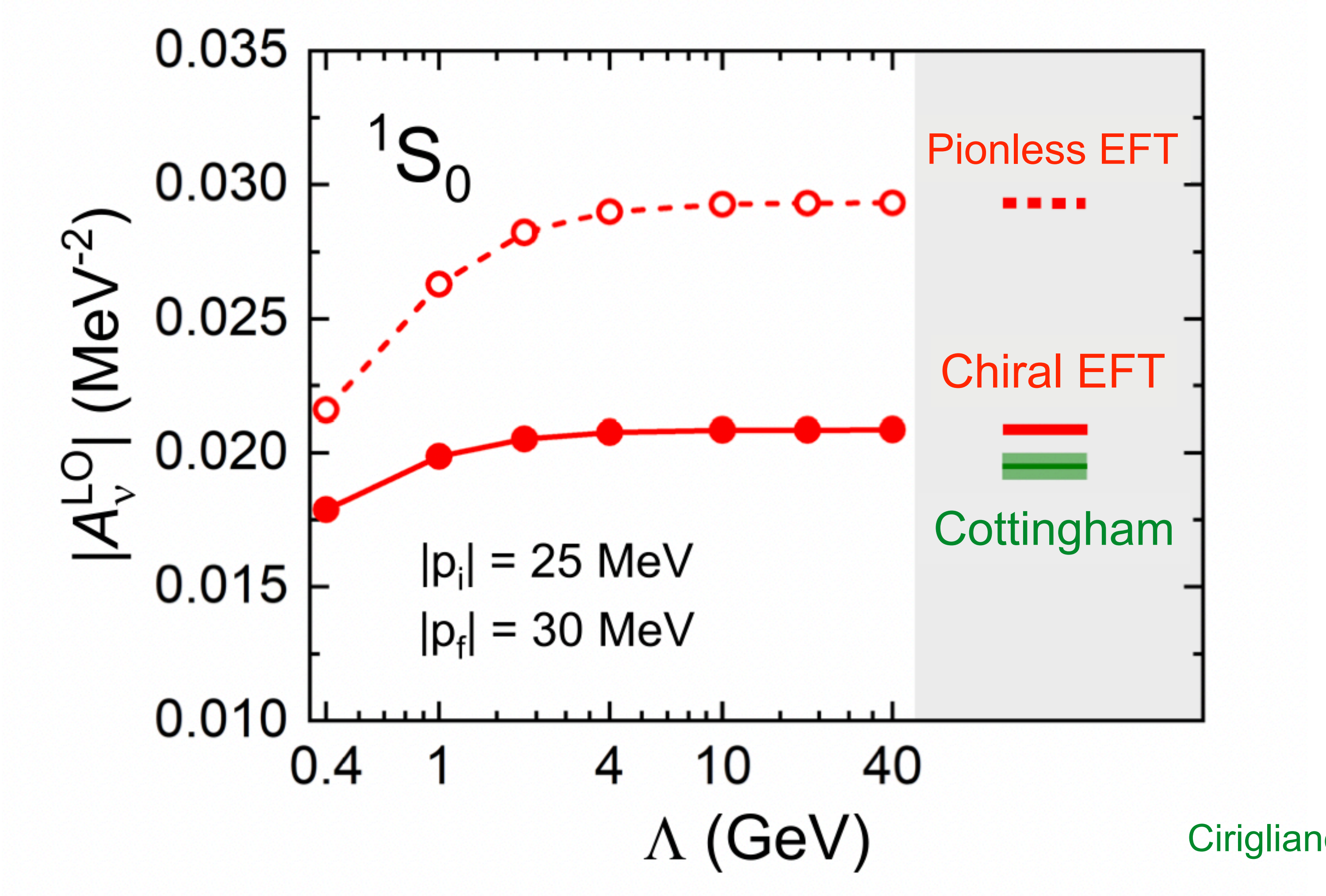
$D = L(3+g)-2$



					Renormalizable?
Relativistic:	–	$O(\Lambda^{-2})$	$O(\Lambda^{-2})$	$O(\Lambda^{-2})$	Yes
Nonrelativistic:	–	$O(\Lambda^{-1})$	$O(\Lambda^{-1})$	$O(\log\Lambda)$	No

LO prediction

- The LO relativistic chiral EFT and the generalized Cottingham model are consistent at 10% level YLY and P. W. Zhao, PLB 855, 138782 (2024)
  - *Input of relativistic EFT: NN $^1S_0$ scattering length (Only one LEC)*
  - *Inputs of generalized Cottingham model: Phenomenological weak form factors, modeling off-shell NN amplitudes, …*



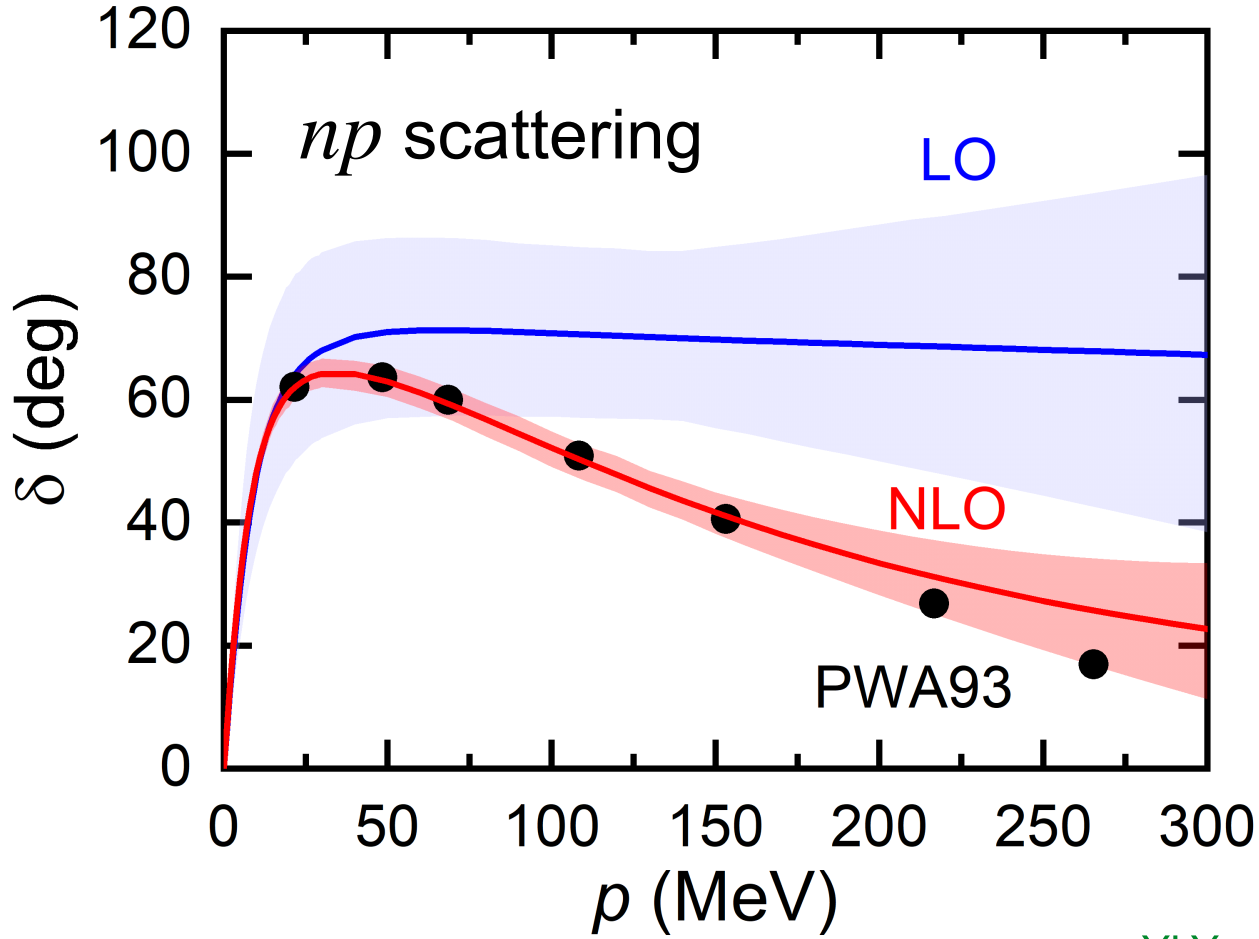
Generalized Cottingham model
Cirigliano et al., PRL 126, 172002 (2021)

Effective range corrections

- Including the effective range corrections in the calculation of the $nn \to ppee$ amplitude to improve its accuracy, while retaining its renormalizability.

$$V_s = \left(C - \frac{g_A^2}{4f_\pi^2} \frac{m_\pi^2}{m_\pi^2 + q^2} \right)_{\rm LO} + \left(D \frac{p'^2 + p^2}{2} \right)_{\rm NLO}$$

- The two LECs are determined by the np phase shifts with a Bayesian approach

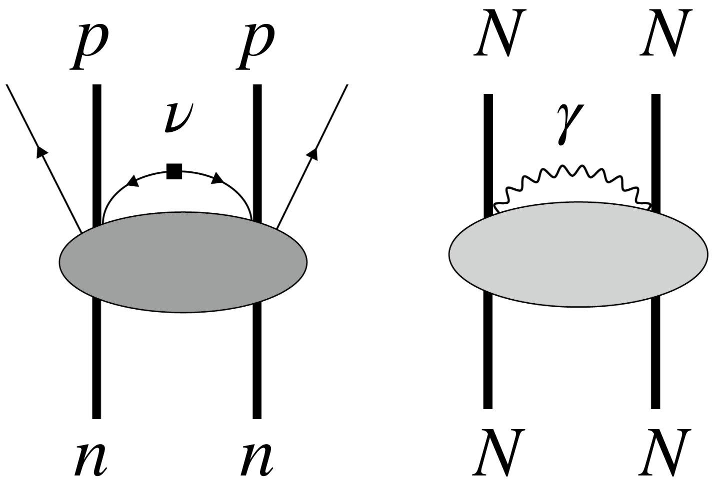


YLY and P. W. Zhao, PRL 134, 242502 (2025)

Validation

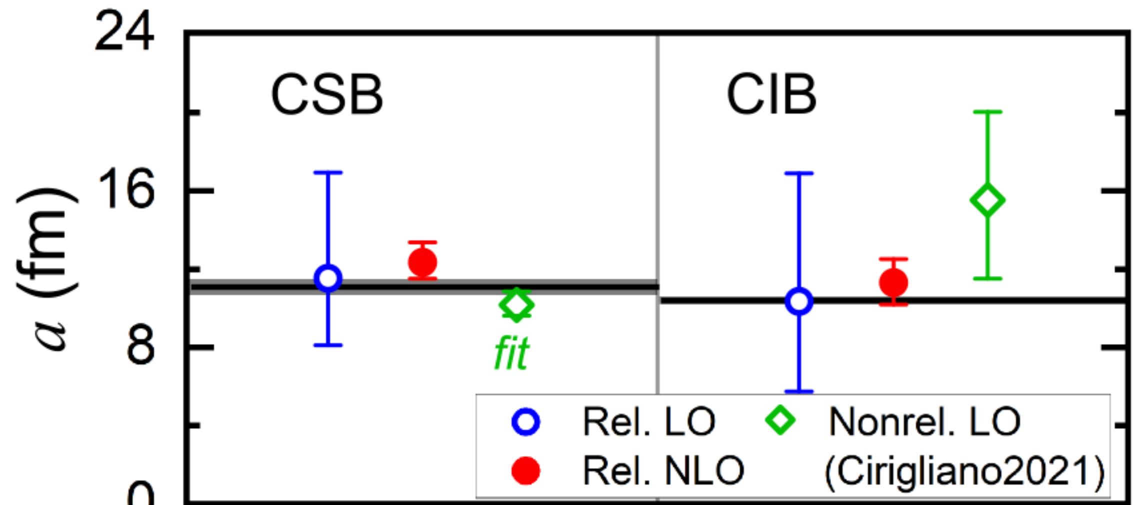
- We first validate our approach on the CIB and CSB of the NN scattering data at low energies, dominated by the one-photon exchange process

$$V_s^{nn,pp} = \left(C - \frac{g_A^2}{4f_\pi^2}\frac{m_{\pi_0}^2}{m_{\pi_0}^2+q^2}\right)_{\rm LO} + \left(D\frac{p'^2+p^2}{2}\right)_{\rm NLO} + V_C^{pp}$$

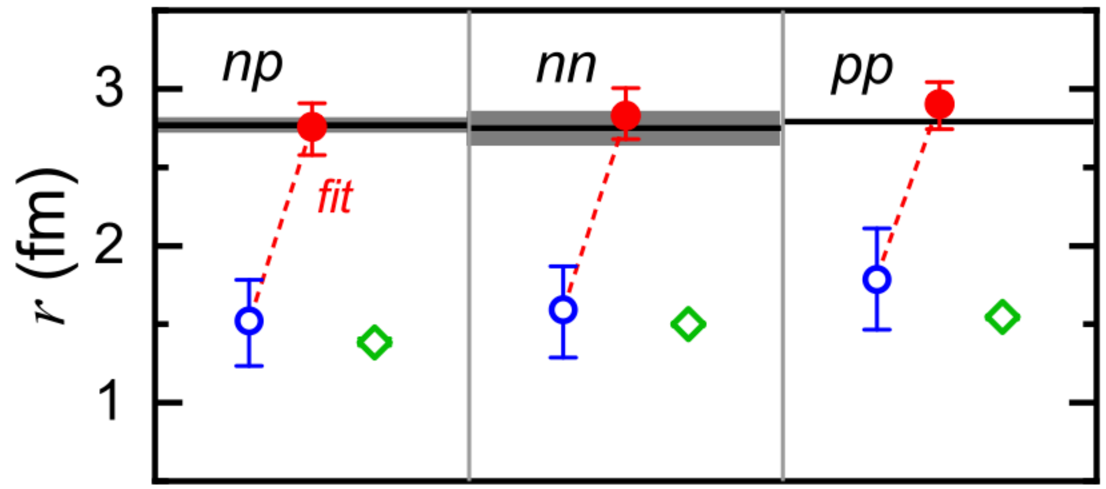


- Predictions agree with experimental data

Scattering length



Effective range



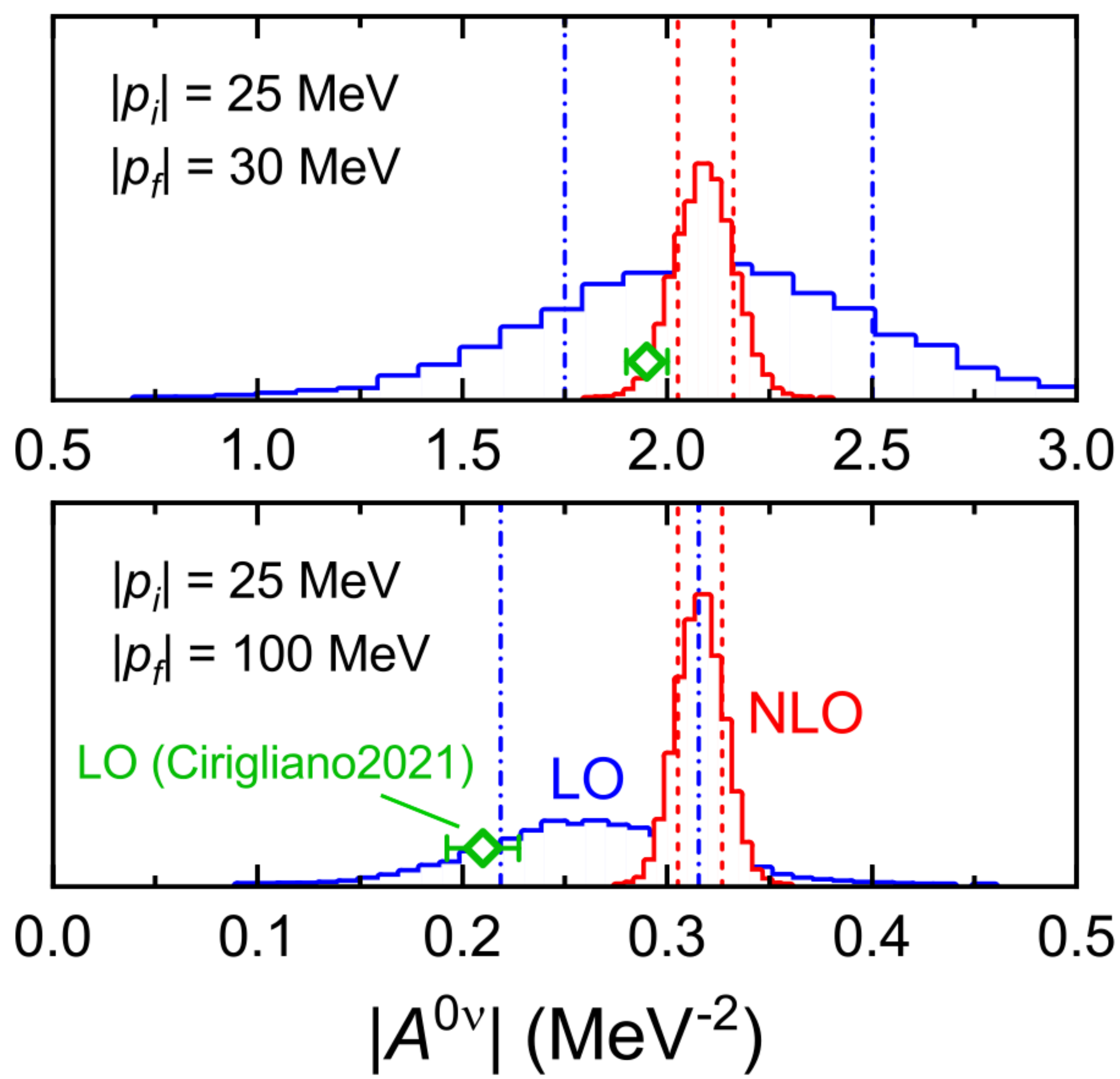
YLY and P. W. Zhao, PRL 134, 242502 (2025)   Cirigliano et al., PRL 126, 172002 (2021)

NLO prediction for $nn \to ppee$

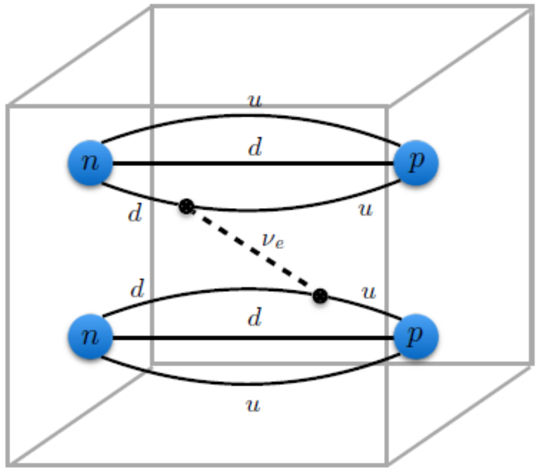
- The NLO corrections significantly reduce the uncertainty of the LO result.
- The NLO prediction is systematically larger than the previous nonrel. LO results.

YLY and P. W. Zhao, PRL 134, 242502 (2025)

$$|\mathcal{A}_{0\nu}| = 0.0209(7)\ \mathrm{MeV}^{-2} \qquad (p, p') = (25,\ 30)\ \mathrm{MeV}$$



Matching EFTs to LQCD



- LQCD can play an important role in constraining g_ν^{NN}
  - ✓ Matching procedure from LQCD to g_ν^{NN}

    Davoudi and Kadam, PRL 126, 152003 (2021); Phys. Rev. D 105, 094502 (2022)
  - ✓ $nn \to ppee$ matrix elements at $m_\pi = 806$ MeV

    Davoudi et al. (NPLQCD), Phys. Rev. D 109, 114514 (2024).

- Matching the LQCD matrix elements in nonrelativistic EFT?

$$M^{0\nu}_{\mathrm{long}}(\Lambda) + 2g_\nu^{\mathrm{NN}}(\Lambda)M^{0\nu}_{\mathrm{short}}(\Lambda) = M^{0\nu}_{\mathrm{LQCD}}$$

One can determine g_ν^{NN} at a given cutoff Λ at an m_π value…

- Matching the LQCD matrix elements in relativistic EFT?

$$M^{0\nu}_{\mathrm{long}}(\Lambda \to \infty) \overset{?}{\longleftrightarrow} M^{0\nu}_{\mathrm{LQCD}}$$

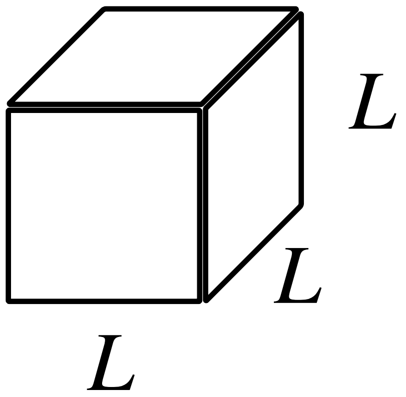
✓ Relativistic EFT is renormalizable without g_ν^{NN}

✓ Can predict $M^{0\nu}$ at an m_π value and benchmark the EFT prediction with LQCD

Relativistic pionless EFT in finite volume

- Motivated by the large pion mass, we employ a pionless EFT
- In a periodic box with size *L*, discretize the momentum mode

$$T(\boldsymbol{n}',\boldsymbol{n}) = V(\boldsymbol{n}',\boldsymbol{n}) + \frac{1}{(2\pi)^3}\sum_{\boldsymbol{k}} V(\boldsymbol{n}',\boldsymbol{k})\,\frac{M^2}{\omega_k^2}\,\frac{1}{E-2\omega_k}\,T(\boldsymbol{k},\boldsymbol{n}) \qquad \boldsymbol{k} \in \frac{2\pi}{L}Z^3$$



- Compute the finite-volume matrix element instead of scattering amplitudes

$$\mathcal{T}_L = \int \mathrm{d}z_0 \int_L \mathrm{d}^3\boldsymbol{z}\,\langle E_n|T[\mathcal{J}(z)\,S_\nu(z)\mathcal{J}(0)]|E_n\rangle$$

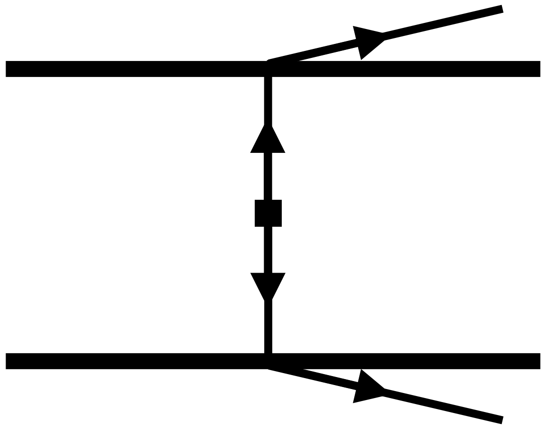
- To obtain the wave function $\phi_n(\boldsymbol{k}) = \langle \boldsymbol{k}\,|\,E_n\rangle$, we solve the eigen equation consistent with the relativistic scattering equation Machleidt and Entem, Phys. Rep. 503, 1 (2011)

$$\hat{H}\phi_n(\boldsymbol{k}) = E_n\phi_n(\boldsymbol{k}), \quad H(\boldsymbol{k}',\boldsymbol{k}) = 2\omega_k\,\delta_{\boldsymbol{k}',\boldsymbol{k}} + \frac{1}{(2\pi)^3}\frac{m_N}{\omega_k}\frac{m_N}{\omega_{k'}}V_S(\boldsymbol{k}',\boldsymbol{k})$$

LO neutrino potential in relativistic pionless EFT

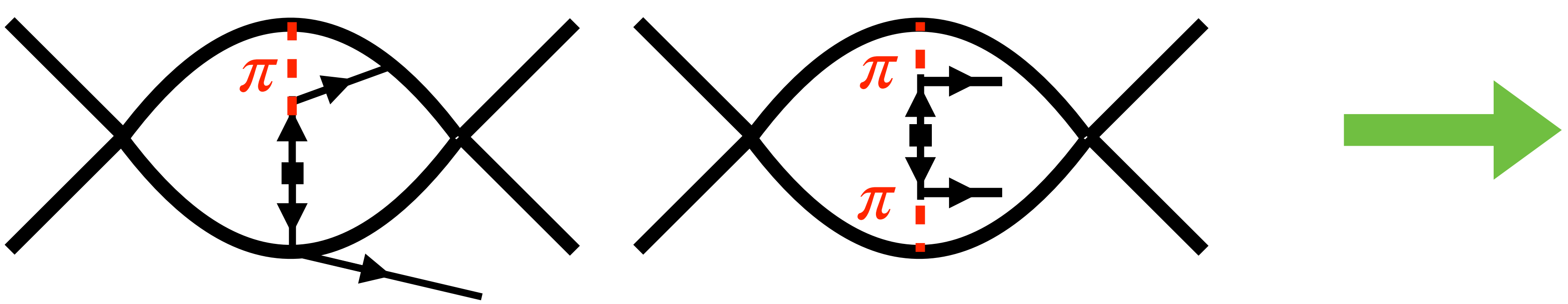
- LO long-range V_ν in pioness EFT

$$V_{\nu L}(\boldsymbol{p}',\boldsymbol{p}) = (1+3g_A^2)\frac{m_N}{\omega_{p'}}\frac{m_N}{\omega_p}\frac{1}{|\boldsymbol{p}'-\boldsymbol{p}|^2}$$

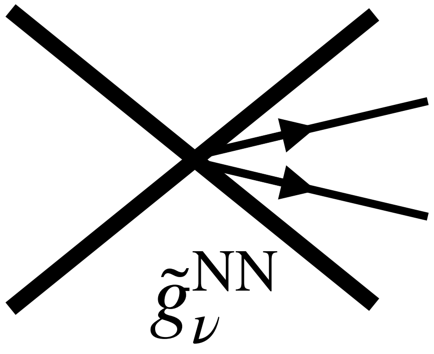


The long-range contribution alone is renormalizable

- However, we find a finite LO short-range contribution from integrating out pions



Chiral EFT



Pionless EFT

- The size of $\tilde{g}_\nu^{\mathrm{NN}}$ is estimated by matching to the pion contributions at threshold

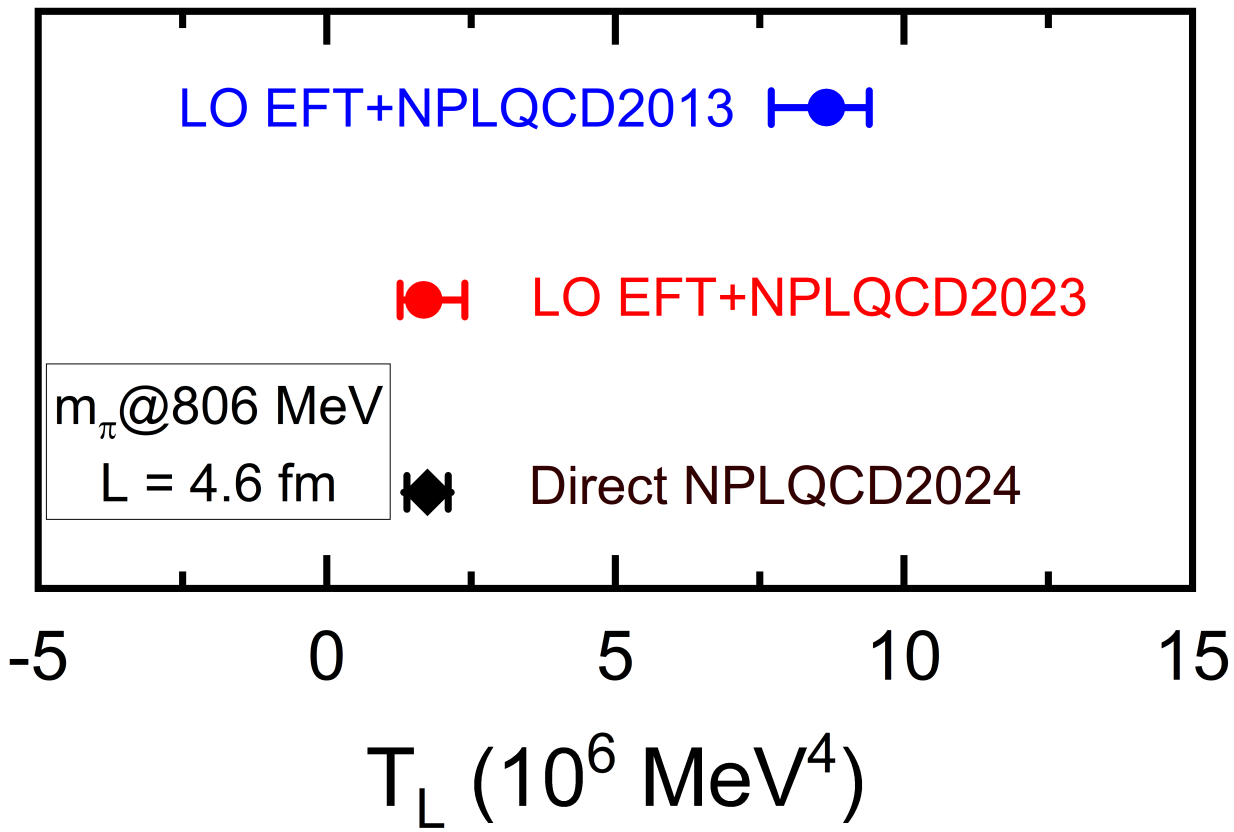
$$\tilde{g}_\nu^{NN} = \frac{(4\pi)^2}{2m_N^2} g_A^2 \delta J^\infty(0,0)$$

YLY and P. W. Zhao, PRD 111, 014507 (2025)

Comparison with LQCD @ $m_\pi = 806$ MeV

- The first benchmark between nuclear EFT and lattice QCD on $0\nu\beta\beta$ decay
- The prediction from LO relativistic pionless EFT is encouraging!

YLY and P. W. Zhao, PRD 111, 014507 (2025)



Inputs of EFT

$g_A = 1.27$

$m_\pi = 0.81$ GeV

$M_N = 1.64$ GeV

$E_{nn} = -17(4)$ MeV

$E_{nn} = -3.3(7)$ MeV

Beane et al. (NPLQCD), PRD 87, 034506 (2013)
Amarasinghe et al. (NPLQCD), PRD 107,094508 (2023)
Davoudi et al. (NPLQCD), PRD 109, 114514 (2024)

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Current status

- The short-range contributions to NMEs may be significantly
- Taking ^{76}Ge as an example:

How to fix $g_\nu^{\rm NN}$	**Many-body method**	$M_{\rm short}/M_{\rm long}$
CIB coupling	QRPA	$32\% - 73\ \%$
CIB coupling	Shell model	$25\% - 40\ \%$
Cottingham model	IM-GCM	$23\% - 30\ \%$

Jokiniemi et al., PLB 823, 136720 (2021); Weiss et al., PRC 106, 065501 (2022); Belley et al., PRL 132, 182502 (2024)

The aim of our study:

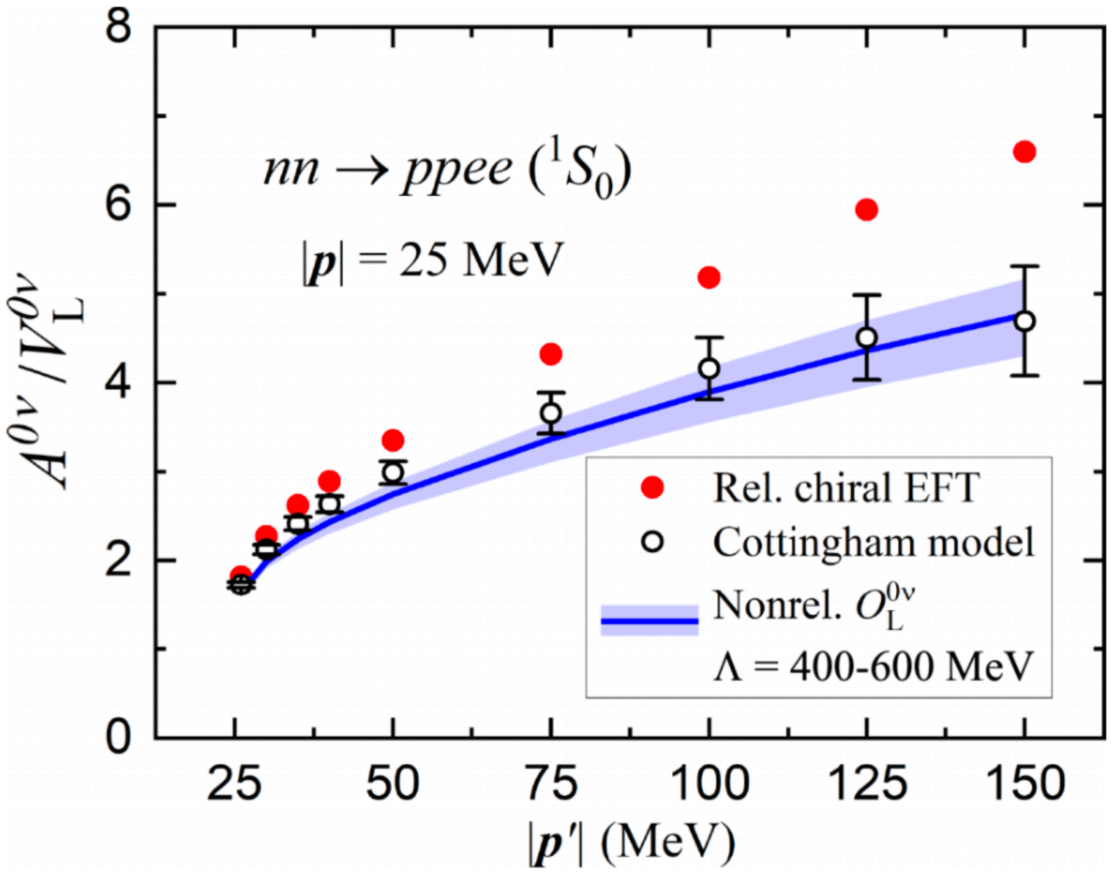
How do the different schemes of fixing $g_\nu^{\rm NN}$ affect NMEs?

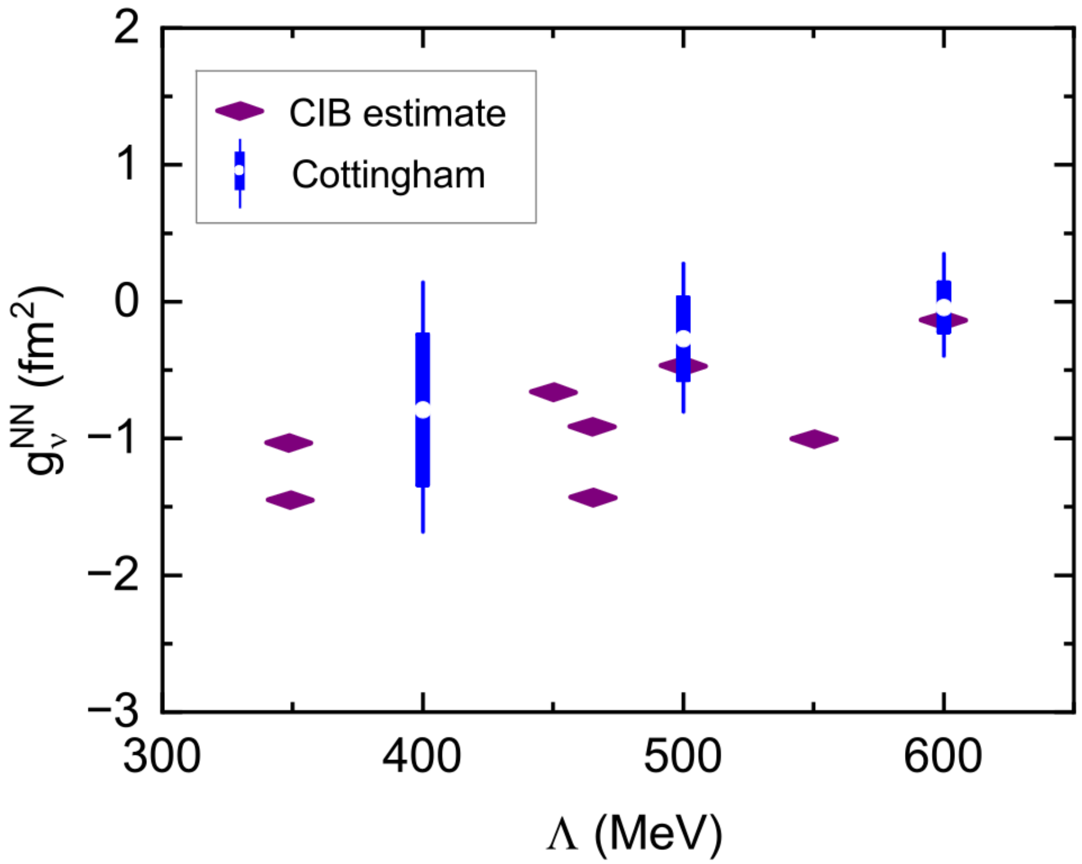
Rel. EFT agrees with Nonrel. EFT(+Cottingham model) at NN level—

What about the Rel. EFT predictions for NMEs?

Uncertainties induced by the determination of $g_\nu^{\rm NN}$

- Variation of momentum cutoffs Λ: usually near/under the breakdown scale Λ_b
- Choice of matching points to the synthetic datum:
  - Why match at $|\boldsymbol{p}| = 25\ \mathrm{MeV}, |\boldsymbol{p}'| = 30\ \mathrm{MeV}$, but not other kinematics?
  - Matching points with $p, p' \sim m_\pi \sim Q$ are, in principle, equivalent at LO
- Taking both aspects into account, we find that $g_\nu^{\rm NN}$ can even change its sign



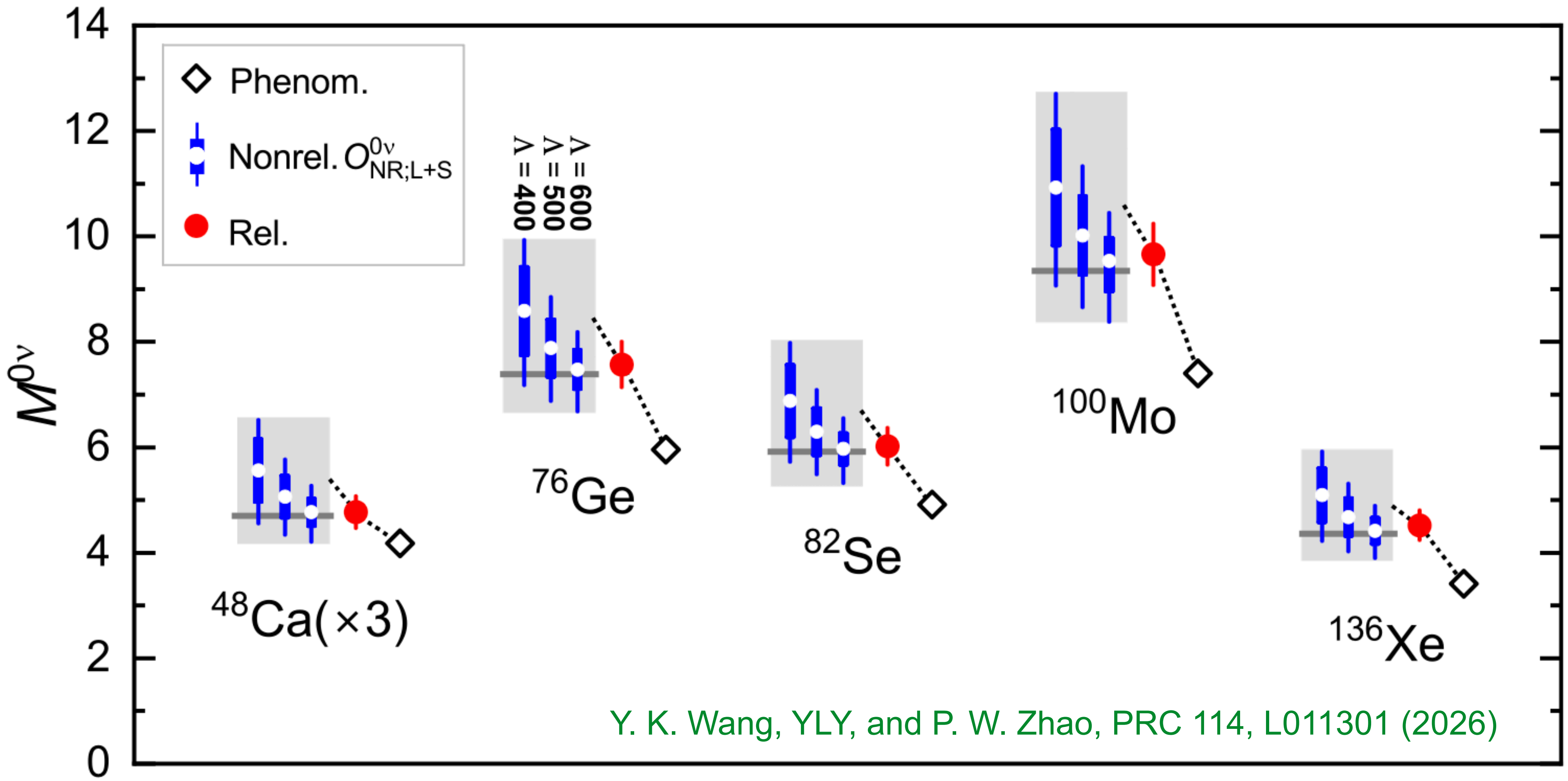


Regulator: $f_\Lambda(\boldsymbol{q}^2) = \exp(-\boldsymbol{q}^2/\Lambda^2)$

Y. K. Wang, YLY, and P. W. Zhao, PRC 114, L011301 (2026)

Relativistic vs Nonrelativistic operators

- The nuclear many-body wave functions for ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, and ^{136}Xe are obtained with Relativistic Configuration-interaction Density Functional Theory
- Relativistic $O_{0\nu,L}$ yields consistent NMEs with Nonrelativistic $O_{0\nu,L+S}$, while circumventing the short-range uncertainties
- **Caveat: The wave functions are not obtained with chiral nuclear forces; not consistent with the neutrino potential**



Summary

Relativistic chiral EFT is developed for studying $0\nu\beta\beta$ decay

the LO contact term is NOT needed for renormalization YLY and P. W. Zhao, PLB 855, 138782 (2024)

✓ NLO predictions on the $nn \to ppee$ amplitude YLY and P. W. Zhao, PRL 134, 242502 (2025)

$$|\mathcal{A}_{0\nu}| = 0.0209(7)\ \mathrm{MeV}^{-2} \qquad (p, p') = (25,\ 30)\ \mathrm{MeV}$$

✓ Validated by reproducing the CSB/CIB contributions in NN scattering and Lattice QCD result at heavy pion mass YLY and P. W. Zhao, PRD 111, 014507 (2025)

✓ NMEs with relativistic configuration-interaction DFT YLY and P. W. Zhao, PRC 114, L011301 (2026)

Outlooks:

Improve the estimated $|\mathcal{A}_{0\nu}|$ by higher-order inelastic corrections

G. Van Goffrier, Phys. Rev. D 111, 055033 (2025)

More benchmarks with future lattice QCD calculations of $0\nu\beta\beta$ decay

In preparation with Xu Feng, Ziyu Wang, and Pengwei Zhao

Relativistic ab initio calculations using chiral nuclear forces

Thank you for your attention!

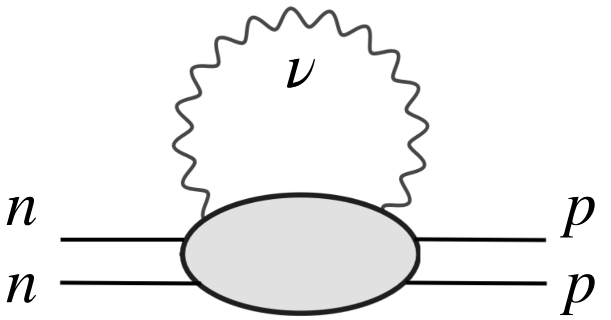
Previous model

- Integral representation

Cirigliano et al., PRL 126, 172002 (2021)

$$\mathcal{A}_\nu \propto \int \frac{d^4k}{(2\pi)^4}\frac{g_{\alpha\beta}}{k^2+i\epsilon}\int d^4x\, e^{ik\cdot x}\langle pp|T\{j_{\rm w}^\alpha(x)j_{\rm w}^\beta(0)\}|nn\rangle$$

$$= \int_0^\Lambda d|\mathbf{k}|\; a_<(|\mathbf{k}|) \;+\; \int_\Lambda^\infty d|\mathbf{k}|\; a_>(|\mathbf{k}|)\,,$$



$$a_<(|\boldsymbol{k}|) = -\frac{r(|\boldsymbol{k}|)}{|\boldsymbol{k}|}\theta(|\boldsymbol{k}|-2|\boldsymbol{p}|)\left[g_V^2(\boldsymbol{k}^2)+2g_A^2(\boldsymbol{k}^2)+\frac{\boldsymbol{k}^2}{2M}g_M^2(\boldsymbol{k}^2)\right]$$

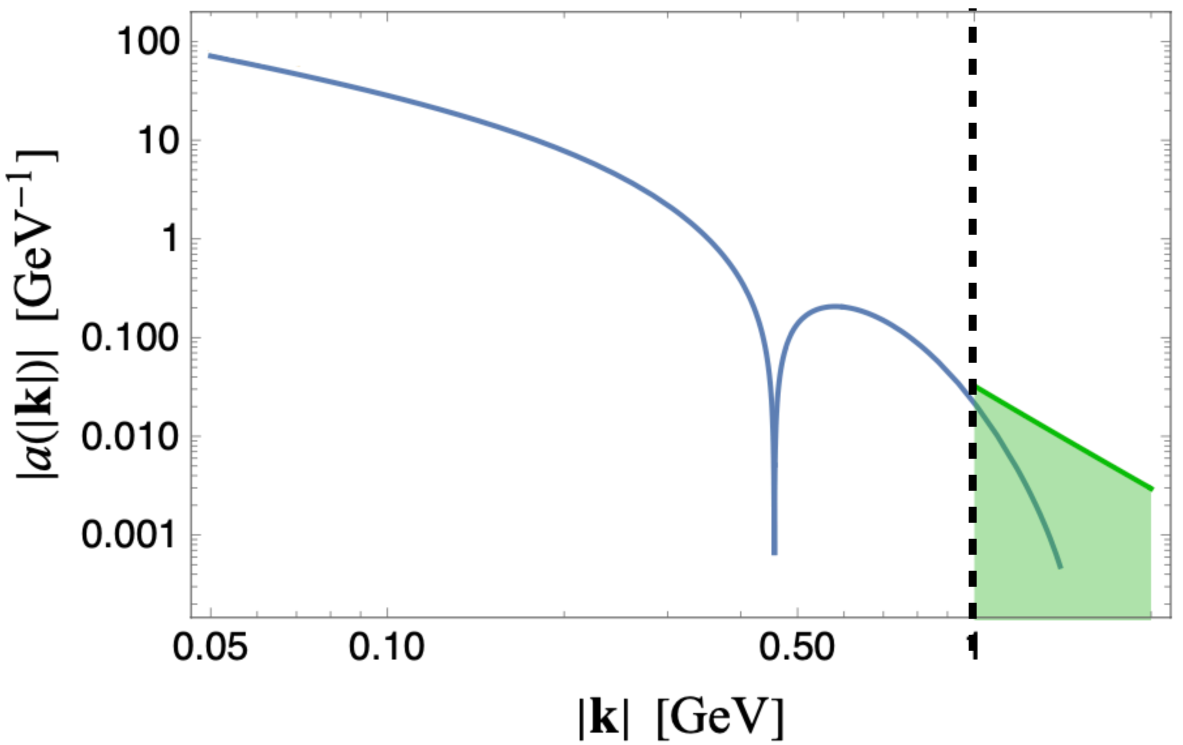
$$a_<(|\boldsymbol{k}|) = \frac{3\alpha_s(\mu)}{\pi}\bar{g}_1^{NN}(\mu)\frac{f_\pi^2}{|\boldsymbol{k}|^3}$$

- Model assumptions and inputs：

Low ← Λ → High

1. Neglect inelastic intermediate states
2. Phenomenological off-shell NN amplitudes
3. Phenomenological weak form factors
4. Separation of low- and high-energy region
5. Unknown matrix element $\bar{g}_1^{NN}(\mu)$

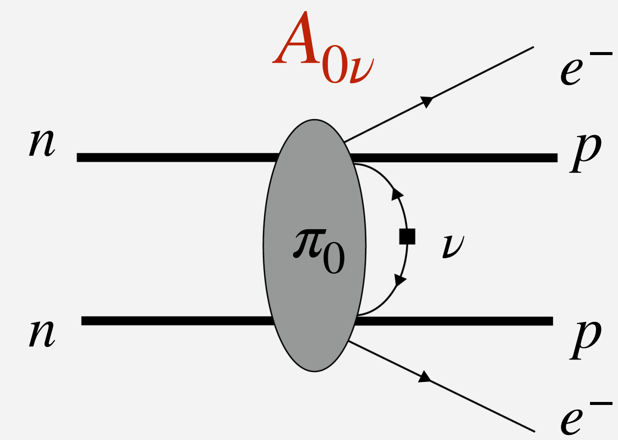
$$\Rightarrow\quad \tilde{\mathcal{C}}_1(\mu_\chi = M_\pi) \simeq 1.32(50)_{\rm inel}(20)_{V_S}(5)_{\rm par}$$



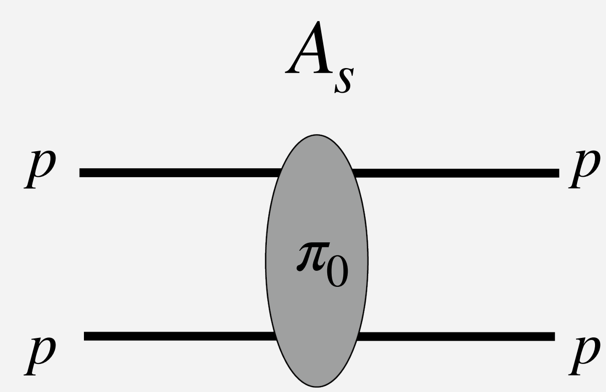
Cirigliano et al., JHEP 05, 289 (2021)

Neutrino and γ exchange

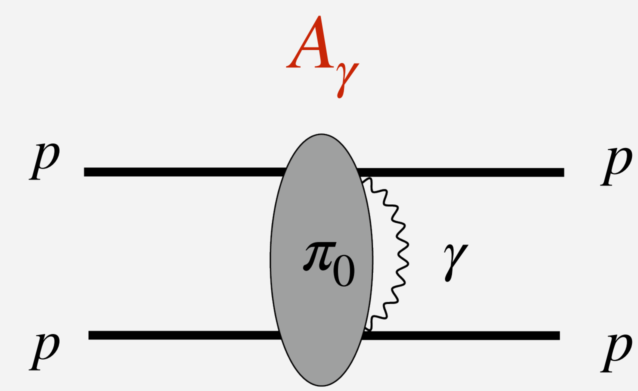
$nn \to ppee$



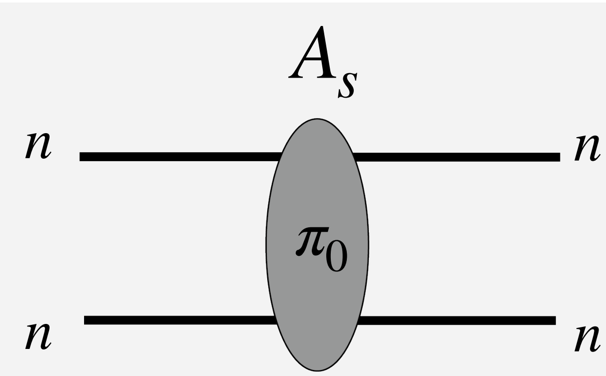
$pp \to pp$



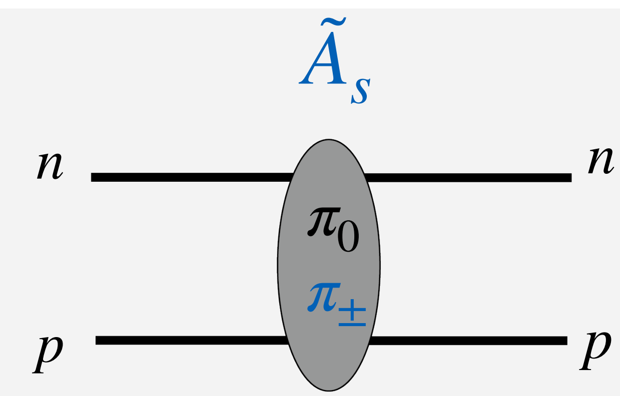
$+$



$nn \to nn$



$np \to np$



CSB: $A_{nn} - A_{pp} \to -A_\gamma$ CIB: $\frac{1}{2}(A_{nn} + A_{pp}) - A_{np} \to \frac{1}{2}A_\gamma + A_s - \tilde{A}_s$

$^1S_0$ Phase shifts