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Illinois Center for Advanced Studies of the Universe
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The Grainger College of Engineering
Physics

# Dependence of the bulk viscosity of neutron star matter on the nuclear symmetry energy

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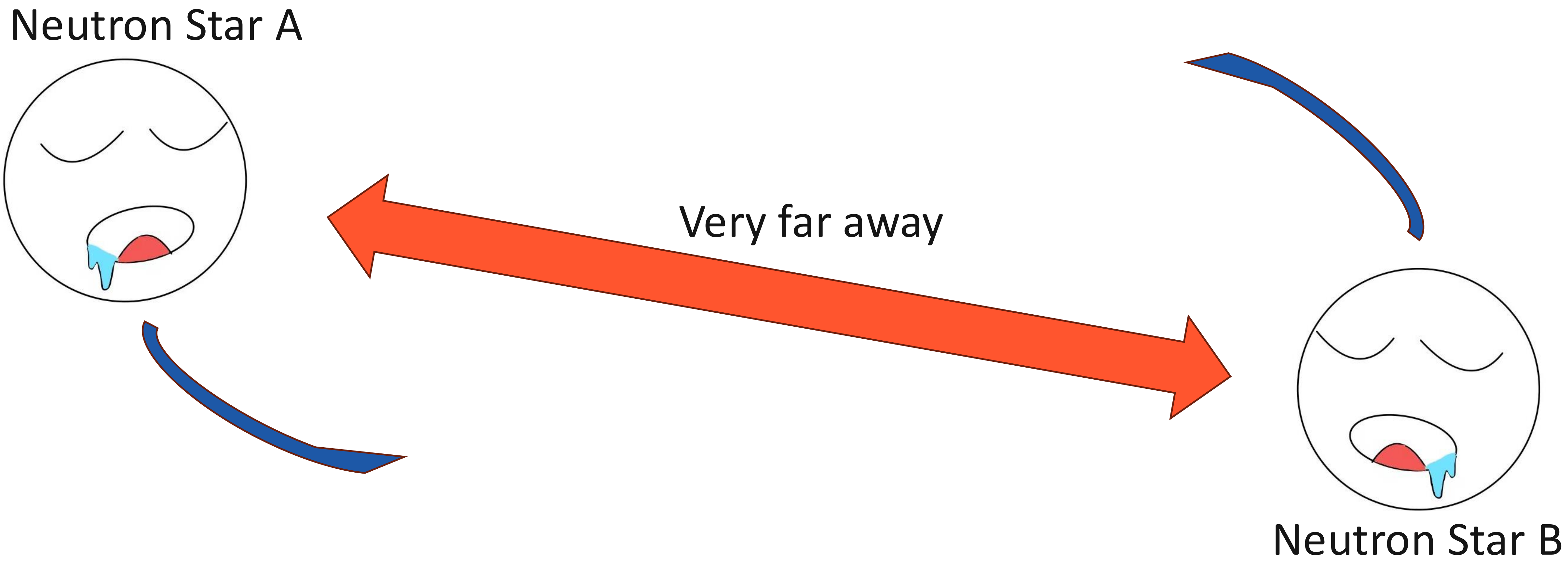
Collaboration with: Mauricio Hippert, Enrico Speranza, Jorge Noronha

INT-25-94W

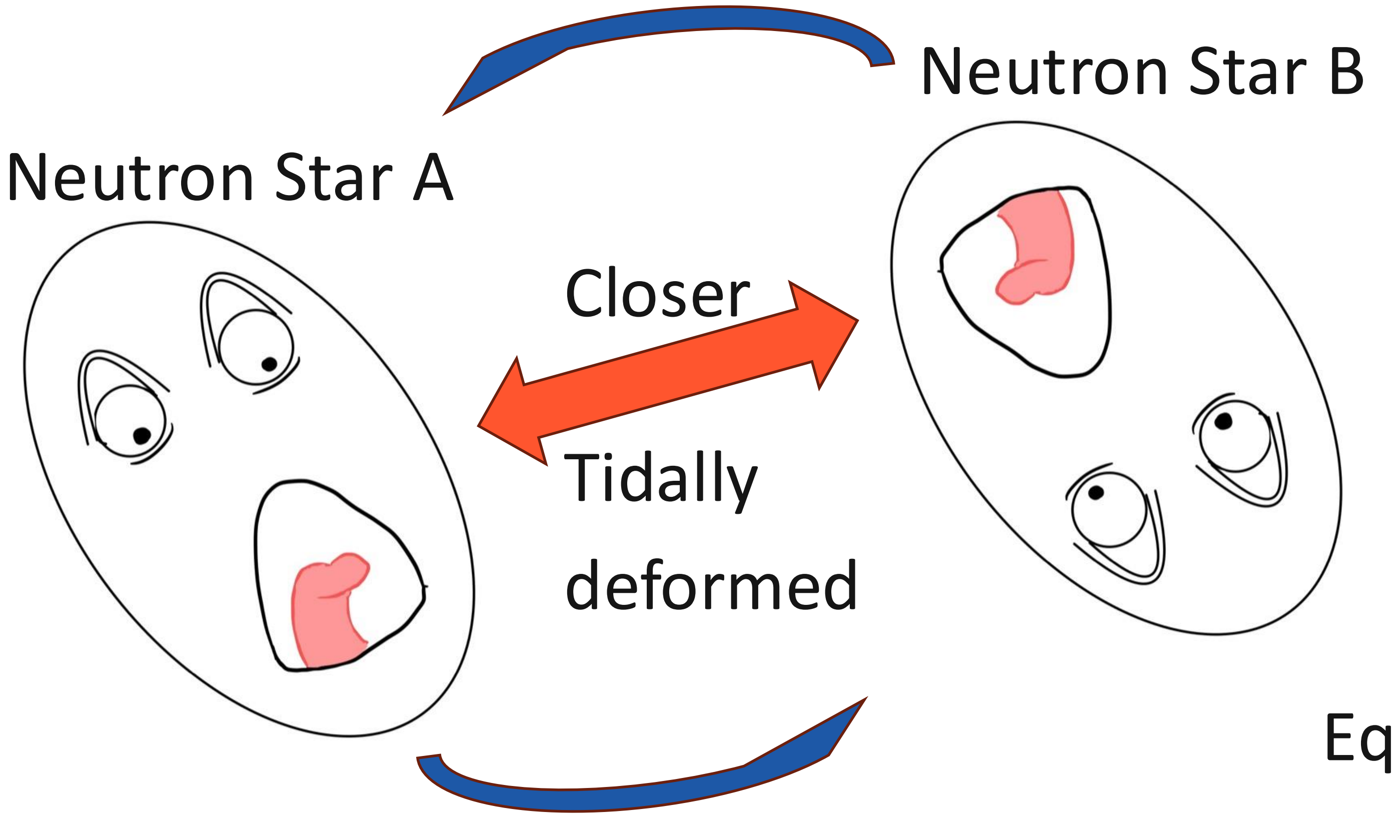
YY, Hippert, Speranza, Noronha, arXiv 2504.07805

National Science Foundation

# Consider a neutron star binary
# Initially, chemical equilibrium…



# Deformed by tidal forces…




Change in density

↓

Out of chemical equilibrium

↓

Equilibrated through weak interactions

# How does the system equilibrate?

Sawyer, PRD (1989)

- Consider neutrino-transparent npe matter: neutron, proton, and electron

- When the density changes, flavor content needs to be adjusted

$$n \to p + e + \bar{\nu}_e \qquad p + e \to n + \nu_e$$

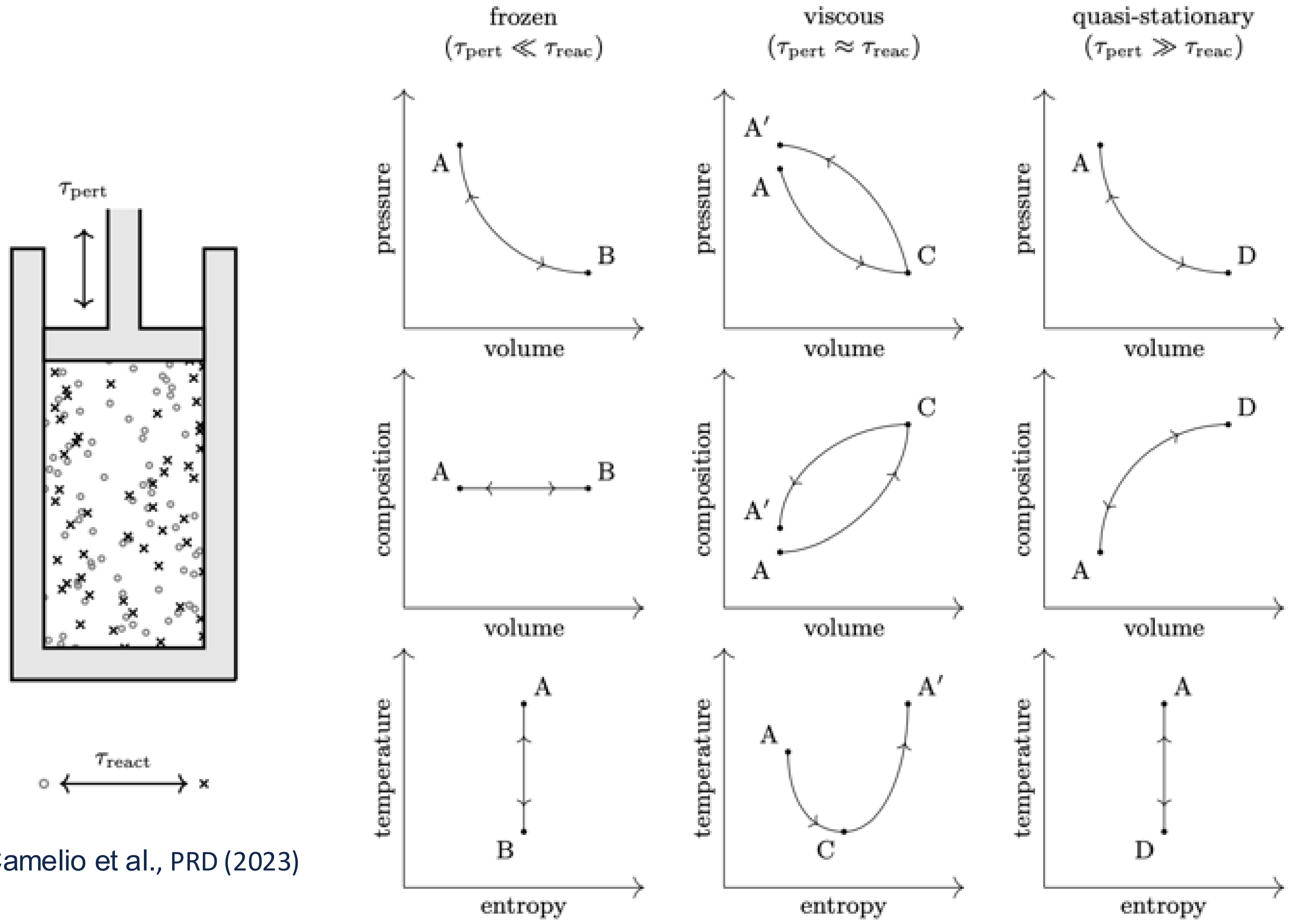
+ modified Urca

Out of equilibrium physics from chemical imbalance

- Beta equilibrium: $\delta\mu = \mu_n + \mu_{\nu_e} - \mu_p - \mu_e = 0$

# How to describe this out-of-equilibrium dynamics?

# Three physical regimes for the system




Camelio et al., PRD (2023)

# What is the mathematical description?

# Reactive fluid

(coupled with Einstein's equations)

- Baryon conservation

$$\nabla_\mu (n_B u^\mu) = 0$$

- Energy-momentum conservation

$$\nabla_\mu \left[ (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu} \right] = 0$$

- Non-conserved electron current

$$\nabla_\mu (n_e u^\mu) = \boxed{\Gamma_e}$$

Reaction rates
(weak interactions)

# Israel-Stewart Equation

- Total pressure is the sum of equilibrium pressure and out-of-equilibrium correction

$$P = P_{eq} + \Pi$$

Gavassino, Noronha, PRD (2024)

- Evolution of $\Pi$ can be described by three variables

$$\Pi = \Pi(\varepsilon, n_B, \delta\mu)$$

With conservation relations

Israel, Stewart, Annals of Physics (1979)
Denicol et al., PRD (2012)

$$u^\mu \nabla_\mu \Pi = -\frac{\Pi}{\tau_\Pi} - \frac{\zeta}{\tau_\Pi} \nabla_\mu u^\mu$$

Resummed bulk-viscous transport coefficients

$$\zeta(\varepsilon, n_B, \delta\mu), \; \tau_\Pi(\varepsilon, n_B, \delta\mu)$$

Gavassino, Noronha, PRD (2024)

# Reactive fluid ⟷ Bulk-viscous fluid

(coupled with Einstein's equations)

**Exact Duality!**

no approximations

Gavassino, Noronha, PRD (2024)
YY, Hippert, Speranza, Noronha, PRC (2024)

**Reactive fluid**

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Reaction rates
(weak interactions)

**Bulk-viscous fluid**

- Baryon conservation

$$\nabla_\mu (n_B u^\mu) = 0$$

- Energy-momentum conservation

$$\nabla_\mu \left[ (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu} \right] = 0$$

Israel-Stewart like Equation!

$$u^\mu \nabla_\mu \Pi = -\frac{\Pi}{\tau_\Pi} - \frac{\zeta}{\tau_\Pi} \nabla_\mu u^\mu$$

Israel, Stewart, Annals of Physics (1979)

# How does the Israel-Stewart equation describe different regimes?

# Consider linear response approximation

- Manifestly the same equation

Gavassino, Noronha, PRD (2024)
YY, Hippert, Speranza, Noronha, PRC (2024)

$$u^\mu \nabla_\mu \Pi = -\frac{\Pi}{\tau_{\Pi,0}} - \frac{\zeta_0}{\tau_{\Pi,0}} \nabla_\mu u^\mu$$

$$\Pi \to \left.\frac{\partial P}{\partial \delta\mu}\right|_{\delta\mu=0} \delta\mu$$

$$\zeta(\varepsilon, n_B, \delta\mu) \to \zeta_0 = \zeta(\varepsilon, n_B, \delta\mu = 0)$$

$$\tau_\Pi(\varepsilon, n_B, \delta\mu) \to \tau_{\Pi,0} = \tau_\Pi(\varepsilon, n_B, \delta\mu = 0)$$

Transport coefficients computed in beta equilibrium

# To introduce a macroscopic scale...

- Introduce frequency dependence by metric perturbation

$$g^{\mu\nu} = \eta^{\mu\nu} + \delta g^{\mu\nu} \qquad \delta g^{\mu\nu} \propto e^{i\omega t} \qquad \delta T^{\mu\nu} \sim G_R \delta g^{\mu\nu}$$

YY, Hippert, Speranza, Noronha, PRC(2024)

- The frequency is the inverse of the perturbation timescale

$$\omega \sim \frac{1}{\tau_{pert}}$$

# To introduce a microscopic scale…

- Introduce frequency dependence by metric perturbation

$$g^{\mu\nu} = \eta^{\mu\nu} + \delta g^{\mu\nu} \qquad \delta g^{\mu\nu} \propto e^{i\omega t} \qquad \delta T^{\mu\nu} \sim G_R \delta g^{\mu\nu}$$

YY, Hippert, Speranza, Noronha, PRC(2024)

$$\Pi = \frac{\zeta_0}{\tau_{\Pi,0}} \frac{i\tau_{\Pi,0}\omega}{1 - i\tau_{\Pi,0}\omega} \eta^{\alpha\beta}\delta g_{\alpha\beta}$$

$$\tau_{\Pi,0} = \tau_{reac} \qquad \omega \sim \frac{1}{\tau_{pert}}$$

# What are the equations for different regimes?

YY, Hippert, Speranza, Noronha, PRC (2024)
YY, Hippert, Speranza, Noronha, arXiv 2504.07805

$$\Pi = \frac{\zeta}{\tau_\Pi} \frac{i\tau_\Pi \omega}{1 - i\tau_\Pi \omega} \eta^{\alpha\beta} \delta g_{\alpha\beta}$$

$\tau_{\Pi,0}\,\omega \gg 1$ ➡ $\tau_{reac} \gg \tau_{pert}$ ➡ Frozen/Elastic $u^\mu \nabla_\mu \Pi = -\frac{\zeta_0}{\tau_{\Pi,0}} \nabla_\mu u^\mu$

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YY, Hippert, Speranza, Noronha, PRC (2024)
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Frozen/Elastic

$\tau_{\Pi,0}\,\omega \gg 1$ → $\tau_{reac} \gg \tau_{pert}$ → $u^\mu \nabla_\mu \Pi = -\frac{\zeta_0}{\tau_{\Pi,0}} \nabla_\mu u^\mu$

Viscous/Resonant

$\tau_{\Pi,0}\,\omega \approx 1$ → $\tau_{reac} \approx \tau_{pert}$ → $u^\mu \nabla_\mu \Pi = -\frac{\Pi}{\tau_{\Pi,0}} - \frac{\zeta_0}{\tau_{\Pi,0}} \nabla_\mu u^\mu$

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YY, Hippert, Speranza, Noronha, PRC (2024)
YY, Hippert, Speranza, Noronha, arXiv 2504.07805

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$$u^\mu \nabla_\mu \Pi = -\frac{\Pi}{\tau_{\Pi,0}} - \frac{\zeta_0}{\tau_{\Pi,0}}\nabla_\mu u^\mu$$

$\tau_{\Pi,0}\,\omega \ll 1$ ➡ $\tau_{reac} \ll \tau_{pert}$ ➡ Navier-Stokes/Quasi-Stationary

$$\Pi = -\zeta_0 \nabla_\mu u^\mu$$

**Reactive fluid** ⟷ **Bulk-viscous fluid**

(coupled with Einstein's equations)

Gavassino, Noronha, PRD (2024)
YY, Hippert, Speranza, Noronha, PRC (2024)

# Neutron star mergers/inspirals are bulk-viscous systems


Given the rates, transport coefficients fully determined by experiments (S, L) at saturation!

# Nuclear Symmetry Energy

- Energy difference between pure neutron matter and symmetric nuclear matter

$$\frac{\varepsilon}{n_B} = \frac{\varepsilon}{n_B}\bigg|_{\delta=0} + E_{sym}(n_B)\delta^2 + \mathcal{O}(\delta^3)$$

$$\delta = 1 - 2Y_e$$

- Can be expanded around nuclear saturation density

$$E_{sym}(n_B) = S + \frac{L}{3}\left(\frac{n_B}{n_{sat}} - 1\right) + \mathcal{O}\left(\left(\frac{n_B}{n_{sat}} - 1\right)^2\right)$$

symmetry energy $S$, and its slope $L$ at $n_{sat}$ ⇐ Experimentally measured!

# How to connect the symmetry energy with the transport coefficients?

# Symmetry energy fixes chemical imbalance

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

- At $T \to 0$ , first law of thermodynamics gives

$$\delta\mu = \frac{2}{n_B}\frac{\partial\varepsilon}{\partial\delta}$$

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YY, Hippert, Speranza, Noronha, arXiv 2504.07805

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YY, Hippert, Speranza, Noronha, arXiv 2504.07805

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- At $T \to 0$ , first law of thermodynamics gives

$$\delta\mu = \frac{2}{n_B}\frac{\partial \varepsilon}{\partial \delta}$$

- Besides the rates and lepton contribution, transport coefficients depend on the symmetry energy

$$\tau_{\Pi,0} \sim \frac{1}{E_{sym}(n_B)} \qquad \zeta_0 \sim \left(\frac{\partial E_{sym}(n_B)}{\partial n_B} \Big/ E_{sym}(n_B)\right)^2$$

Valid for all $n_B$

# Symmetry energy fixes chemical imbalance

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

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- At saturation, simple behavior emerges

$$\tau_{\Pi,0}(n_{sat}) \sim (S + \text{lepton})^{-1} \qquad \zeta_0(n_{sat}) \sim \tau_{\Pi,0}(n_{sat})\,(L + \text{lepton})^2$$

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YY, Hippert, Speranza, Noronha, arXiv 2504.07805

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Experimentally measured observables

# Validation using chiral EFT

- Linearly interpolating between the uncertainty bands Tews et al., Phys. Rev. Res. (2024)

$$(E/A)_\sigma = (1-\sigma)(E/A)_{\rm up} + \sigma(E/A)_{\rm low}$$

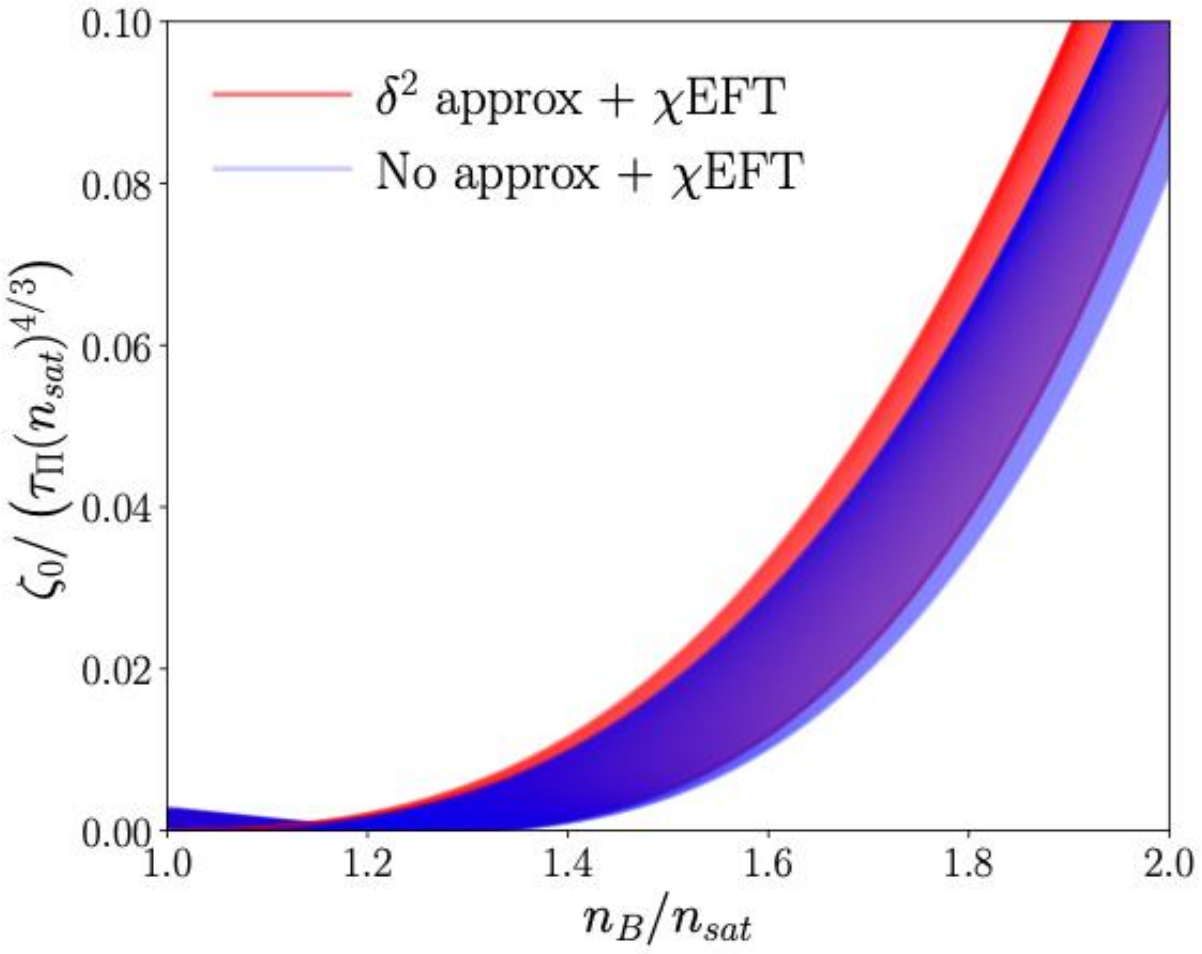
- Fit the curve with the following parametrization

Hebeler et al. PRL (2010)
Bedaque, Steiner, PRL (2015)
Hippert, Noronha, Romatschke, Phys. Lett. B (2025)

$$\frac{E}{A}(n_B, Y_e) - m_B = T_0\left[\frac{3}{5}\left(Y_e^{3/5} + (1-Y_e)^{3/5}\right)\left(\frac{2n_B}{n_{\rm sat}}\right)^{2/3} - [(2\alpha - 4\alpha_L)Y_e(1-Y_e) + \alpha_L]\,\frac{n_B}{n_{\rm sat}} + [(2\eta - 4\eta_L)Y_e(1-Y_e) + \alpha_L]\left(\frac{2n_B}{n_{\rm sat}}\right)^{\gamma}\right]$$

# Validation using chiral EFT

YY, Hippert, Speranza, Noronha, arXiv 2504.07805




- Blue: Calculated directly from chiral EFT parameterizations

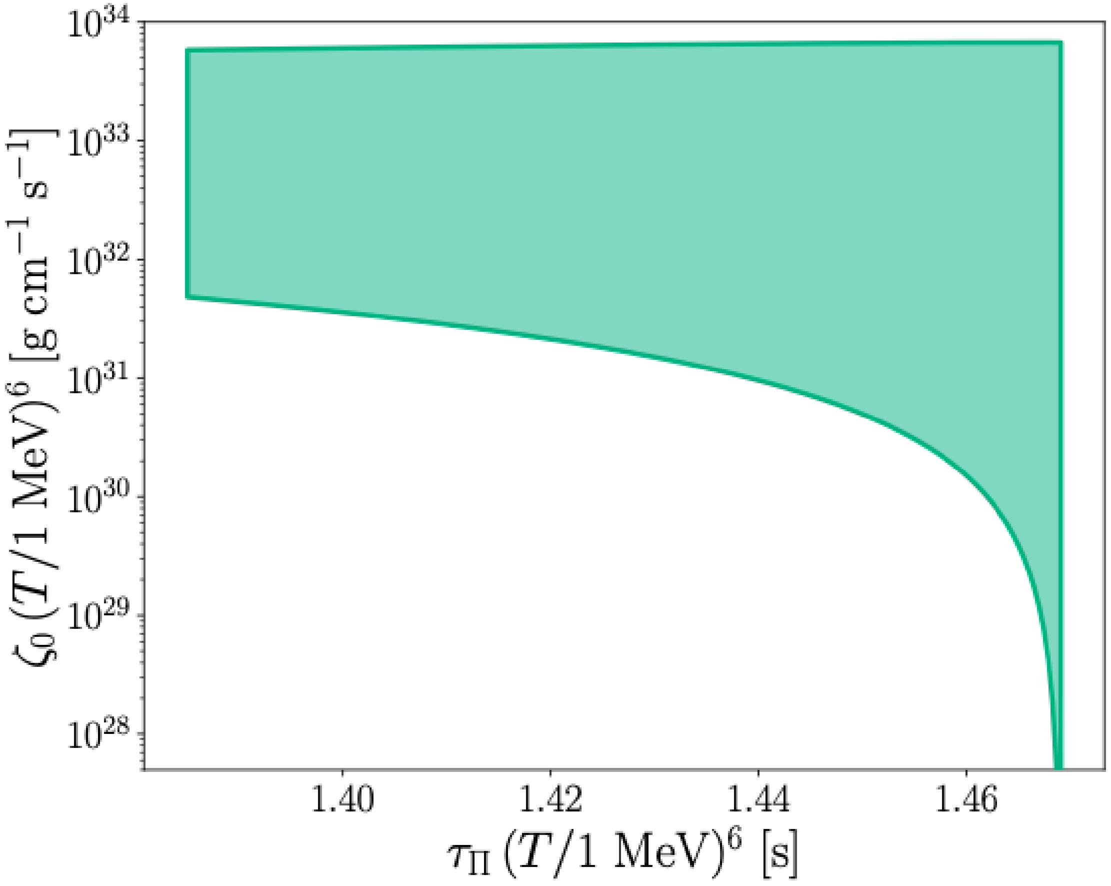
Tews et al., Phys. Rev. Res. (2024)
Hippert, Noronha, Romatschke, Phys. Lett. B (2025)
Hebeler, Lattimer, Pethick, Schwenk, PRL (2010)
Bedaque, Steiner, PRL (2015)

- Red: Calculated using the respective symmetry energy of the chiral EFT parametrizations

Calculations from $E_{sym}$ work very well

# Dependence on S & L
YY, Hippert, Speranza, Noronha, arXiv 2504.07805

$S \in [30, 40]$ MeV, $L \in [30, 150]$ MeV




- Most Estimates

$$L \sim 50\ \mathrm{MeV}$$

Lattimer, Particles (2023)

- PREX I+II:

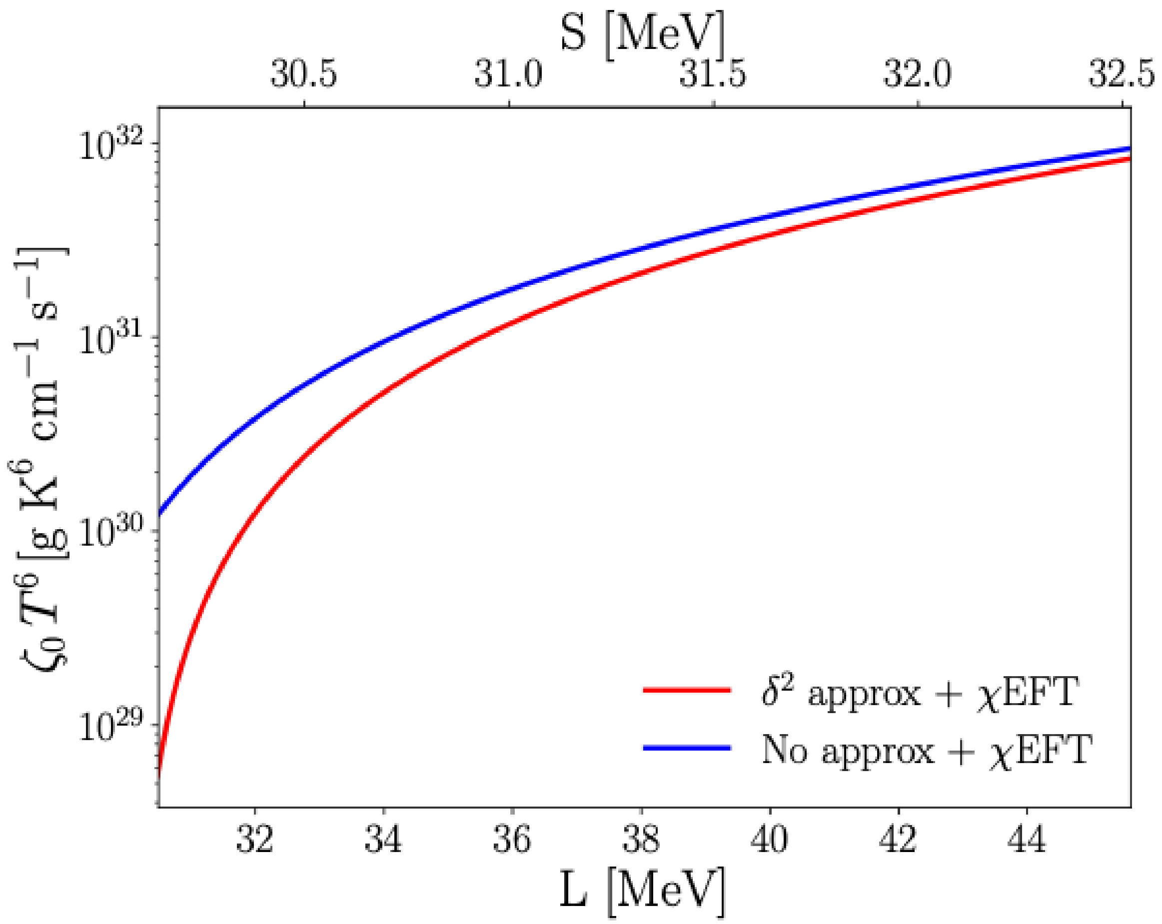
$$L = 106 \pm 37\ \mathrm{MeV}$$

Adhikari et al., PRL (2019)

Varies by orders of magnitude

# Tight constraint on S & L

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

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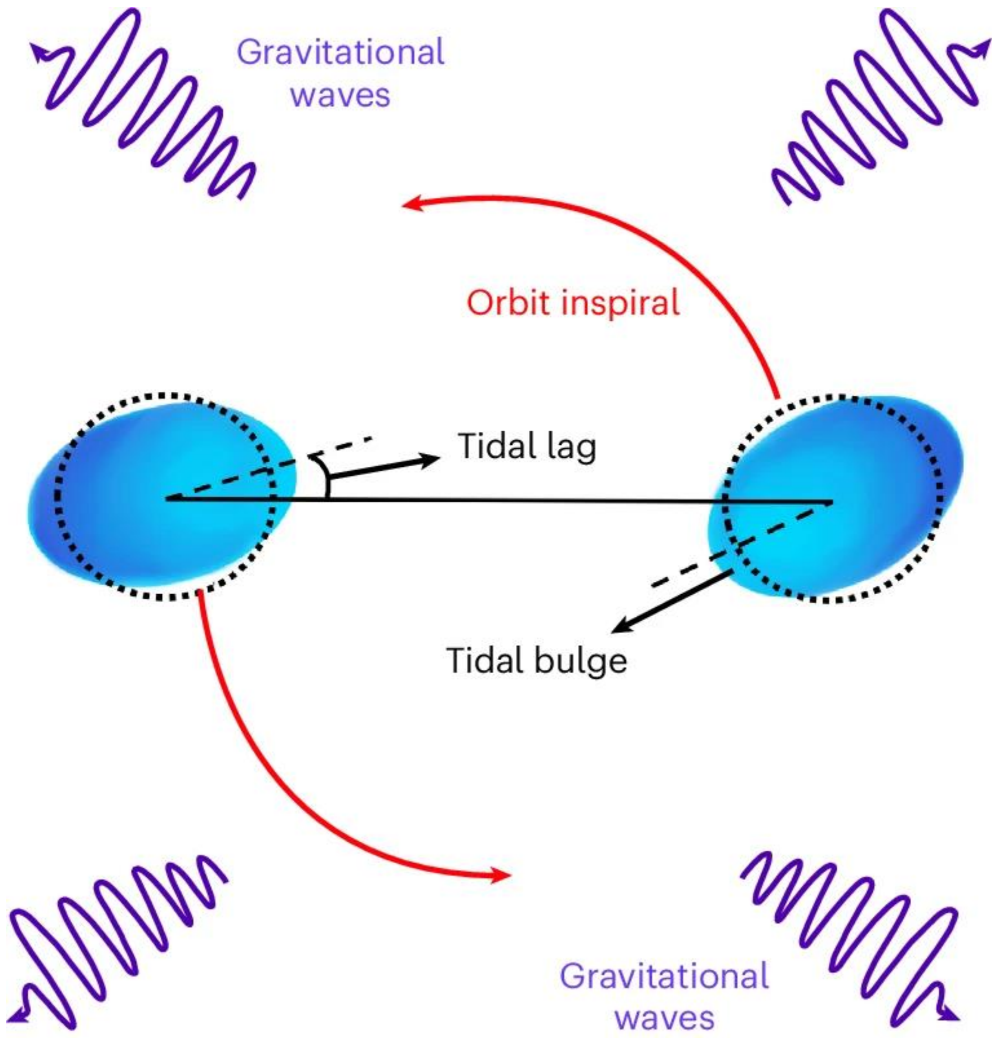
Hippert, Noronha, Romatschke, Phys. Lett. B (2025)
Hebeler, Lattimer, Pethick, Schwenk, PRL (2010)
Bedaque, Steiner, PRL (2015)

- Red: Calculated using the respective symmetry energy of the chiral EFT parametrizations

Still varies by orders of magnitude

# How are dissipative effects encoded in the inspiral phase?

Baoitti et al., PRL (2010)
Bernuzzi, Nagar, Dietrich, Damour, PRL (2015)
Arras, Weinberg, Mon. Not. Roy. Astron. Soc. (2019)
Radice, Bernuzzi, Perego, Ann. Rev. (2020)
Passamonti, Andersson, Pnigouras, Mon. Not. Roy. Astron. Soc. (2021)
Andersson, Mon. Not. Roy. Astron. Soc. (2021)
Most et al., Mon. Not. Roy. Astron. Soc. (2021)
Ghosh, Sieniawska, Builik, Özel, PRD (2024)
Saketh et al., PRD (2024)
Williams, Pratten, Schmidt, PRD (2024)
Most et al., Astrophys .J. Lett. (2024)




Ripley, Hegade, Chandramoui, Yunes, Nature Astronomy (2024)

# For npe matter near saturation

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

- As $T \to 0$

$$\tau_\Pi \to \infty \qquad \zeta \to \infty \qquad \frac{\zeta}{\tau_\Pi} \sim \mathrm{const}$$

- What are the consequences?

- How does npe matter under these conditions react to compression/expansion?

# For npe matter near saturation

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

- As $T \to 0$

$$\tau_\Pi \to \infty \qquad \zeta \to \infty \qquad \frac{\zeta}{\tau_\Pi} \sim \text{const}$$

- Resulting EoM respects time-reversal symmetry

$$u^\mu \nabla_\mu \Pi = -\frac{\Pi}{\tau_\Pi} - \frac{\zeta}{\tau_\Pi} \nabla_\mu u^\mu \quad \Rightarrow \quad u^\mu \nabla_\mu \Pi = -\frac{\zeta}{\tau_\Pi} \nabla_\mu u^\mu$$

# For npe matter near saturation

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

- As $T \to 0$

$$\tau_\Pi \to \infty \qquad \zeta \to \infty \qquad \frac{\zeta}{\tau_\Pi} \sim \text{const}$$

- Resulting EoM respects time-reversal symmetry

$$u^\mu \nabla_\mu \Pi = -\frac{\zeta}{\tau_\Pi} \nabla_\mu u^\mu$$

→

No entropy production!

Frozen regime: npe matter would behave like a “relativistic elastic fluid”!

# Frozen regime in the inspiral phase

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

$$\tau_\pi \omega \gg 1$$

Frozen

→

Early Inspiral $T \approx 10^5$ K $\omega \approx 2\pi \times 400$ Hz

$$\tau_\Pi \omega \big|_{n_{\rm sat}} \approx 3.8 \times 10^{26} \gg 1$$

- npe matter at saturation in the inspiral phase is in the frozen regime during compression/expansion

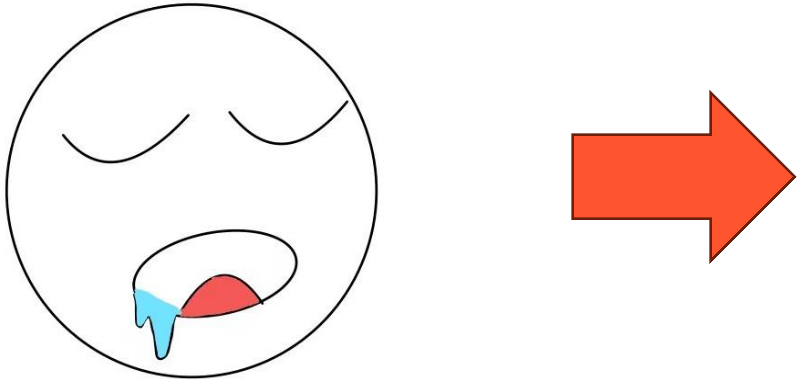
- system not in beta equilibrium, but no entropy production

# Can we connect these microscopic physics to macroscopic observables?

# Connection to macroscopic properties

Ripley, Hegade, Chandramoui, Yunes, Nature Astronomy (2024)
Hegade, Ripley, Yunes, PRD (2024)

Neutron Star



Tidally deformed

- Multipole moment: $I_{ij}$
- External tidal field: $\mathcal{E}_{ij}$

- Assume linear response

$$I^{ij}(\omega) = m^5 \tilde{K}(\omega)\mathcal{E}_{ij}(\omega)$$

# Connection to macroscopic properties

Ripley, Hegade, Chandramoui, Yunes, Nature Astronomy (2024)
Hegade, Ripley, Yunes, PRD (2024)

- Like a harmonic oscillator

$$\tilde{K}(\omega) = -\frac{\Lambda\omega_f^2}{\omega^2 + i\gamma\omega - \omega_f^2} = \Lambda + i\omega\Xi + \Lambda_2\omega^2 + \mathcal{O}(\omega^3)$$

- Connect dissipative tidal deformability to the average bulk viscosity

$$\Xi = \frac{2}{3}k_2\frac{1}{C^6}\frac{c\,\tau_d}{R} \qquad \Xi = \frac{c^3}{G}\frac{p_2\,\Lambda}{C}\frac{\langle\zeta\rangle}{\langle\varepsilon\rangle m}$$

# New questions

Hegade, YY, et al., to appear

- How accurate is the "pocket formula"?
- Can we constrain S and L through dissipative tidal deformability in the inspirals?
- Strangeness? YY, Cruz Camacho, Hippert, Noronha-Hostler, arXiv 2504.18764

# Conclusions

- npe bulk viscous transport coefficients at saturation universally given by S and L (and the rates)

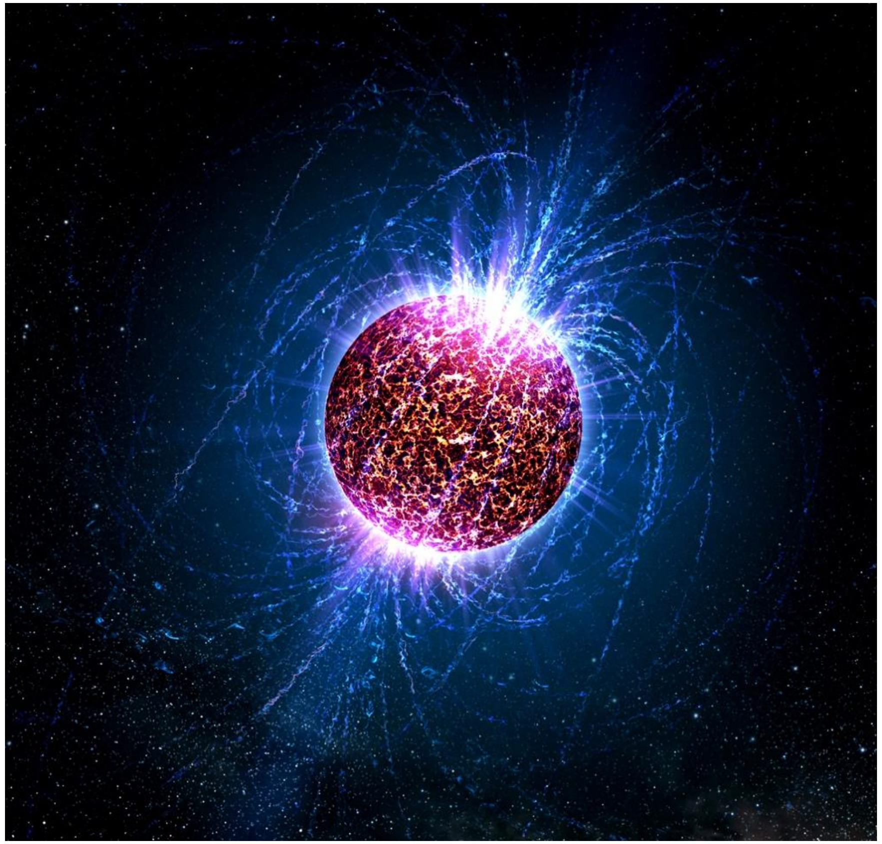
- Small changes in L leads to orders of magnitude changes in bulk viscosity at saturation

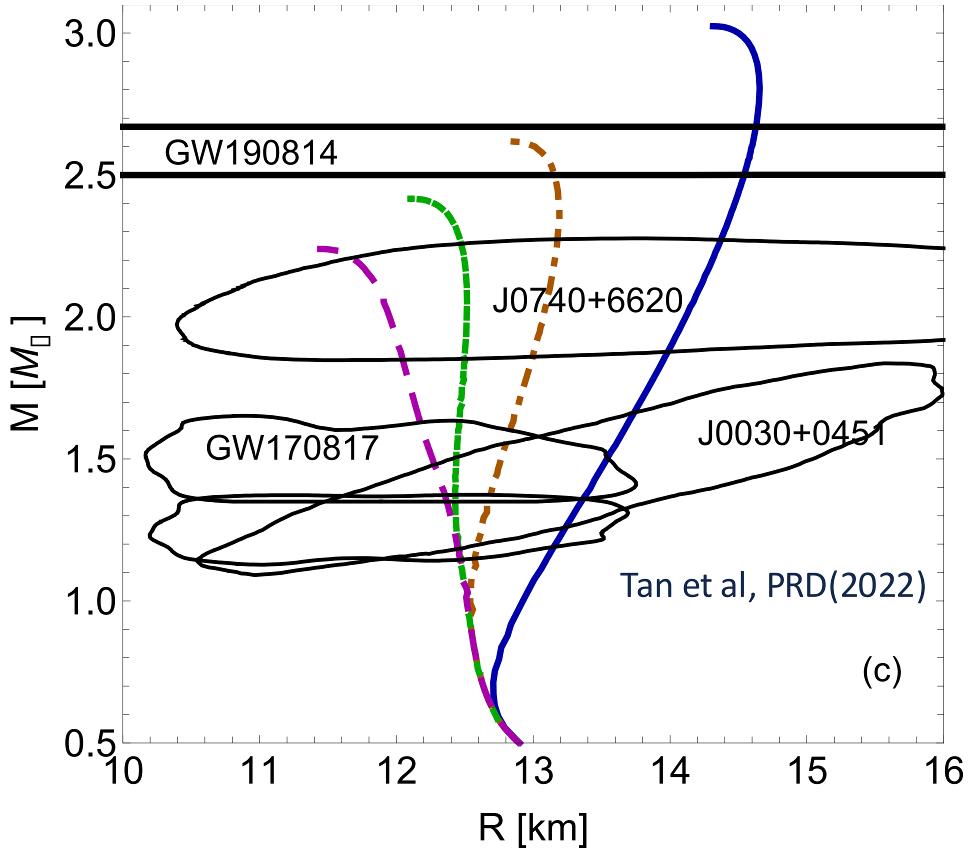
- Possible way to constrain properties of QCD EOS using astrophysical out-of-equilibrium observables such as dissipative tidal deformability

# EXTRA SLIDES

# Equation of state of neutron stars

- Observations constrain EoS, $p = p(\varepsilon, n_B)$

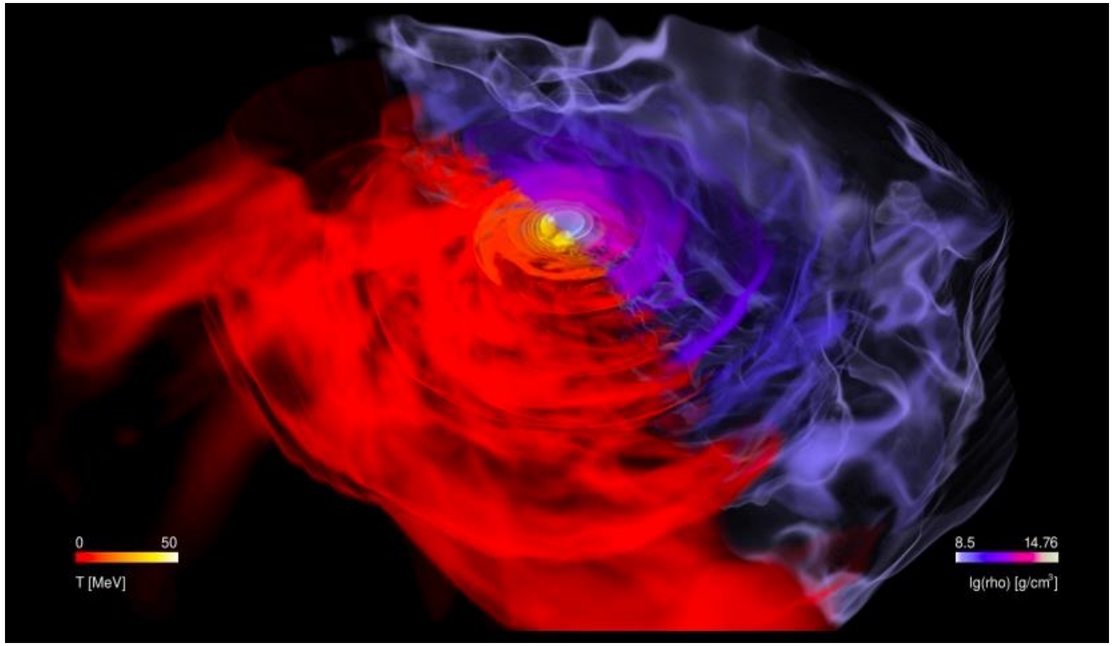





Tan et al, PRD(2022)

# What we need for neutron star mergers?

In order to accurately simulate neutron star mergers




Most et al., PRL (2019)

- Need more than just the finite-T EoS
- Solve fluid dynamics coupled to GR
- Dissipative processes (neutrino physics)
- Deviations from beta equilibrium

How does it get out of equilibrium?

## Reactive fluid ⟷ Bulk-viscous fluid

(coupled with Einstein's equations)

Gavassino, Noronha, PRD (2024)
YY, Hippert, Speranza, Noronha, PRC (2024)

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$$\nabla_\mu \left[ (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu} \right] = -Q u^\nu$$

- Non-conserved electron current

$$\nabla_\mu (n_e u^\mu) = \boxed{\Gamma_e}$$

Reaction rates
(weak interactions)

- Total pressure

$$P = P_{eq} + \Pi$$

**Israel-Stewart like Equation!**

$$u^\mu \nabla_\mu \Pi = -\frac{\Pi}{\tau_\Pi} - \frac{\zeta}{\tau_\Pi} \nabla_\mu u^\mu - \frac{\partial \Pi}{\partial \varepsilon} u^\mu Q_\mu$$

Israel, Stewart, Annals of Physics (1979)

# Elastic regime in the inspiral phase?

- Introduce frequency dependence by metric perturbation

$$g^{\mu\nu} = \eta^{\mu\nu} + \delta g^{\mu\nu} \qquad \delta g^{\mu\nu} \propto e^{i\omega t} \qquad \delta T^{\mu\nu} \sim G_R \delta g^{\mu\nu}$$

YY, Hippert, Speranza, Noronha, PRC(2024)

$$\Pi = \frac{\zeta}{\tau_\Pi} \frac{i\tau_\Pi \omega}{1 - i\tau_\Pi \omega} \eta^{\alpha\beta} \delta g_{\alpha\beta}$$

Frequency-dependent bulk viscosity

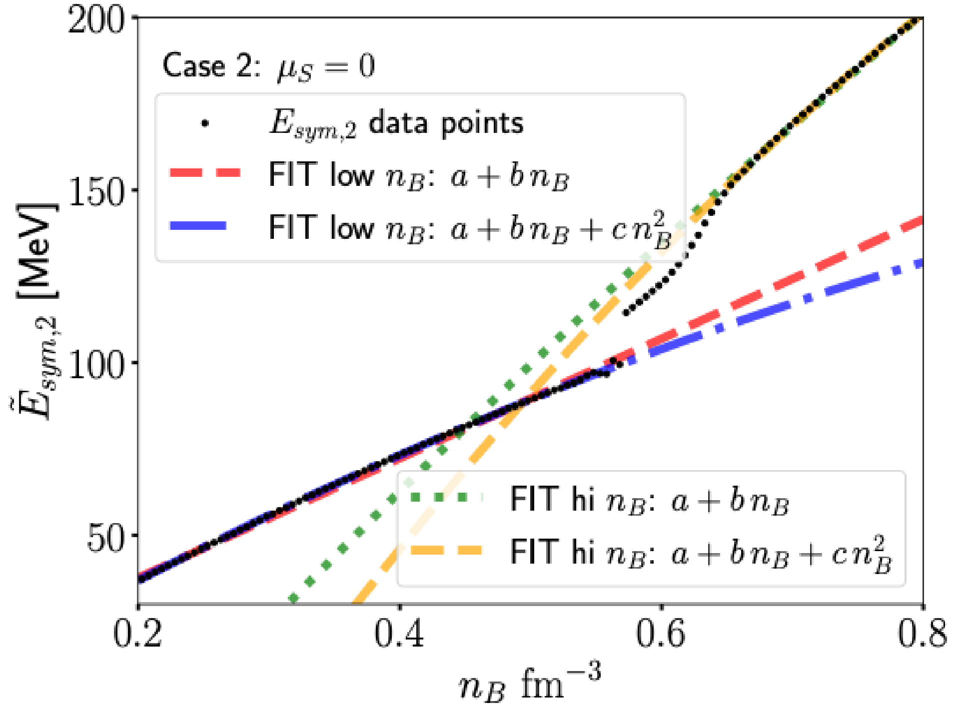
$$\zeta(\omega) = \frac{\zeta_0}{1 + \tau_\Pi^2 \omega^2}$$

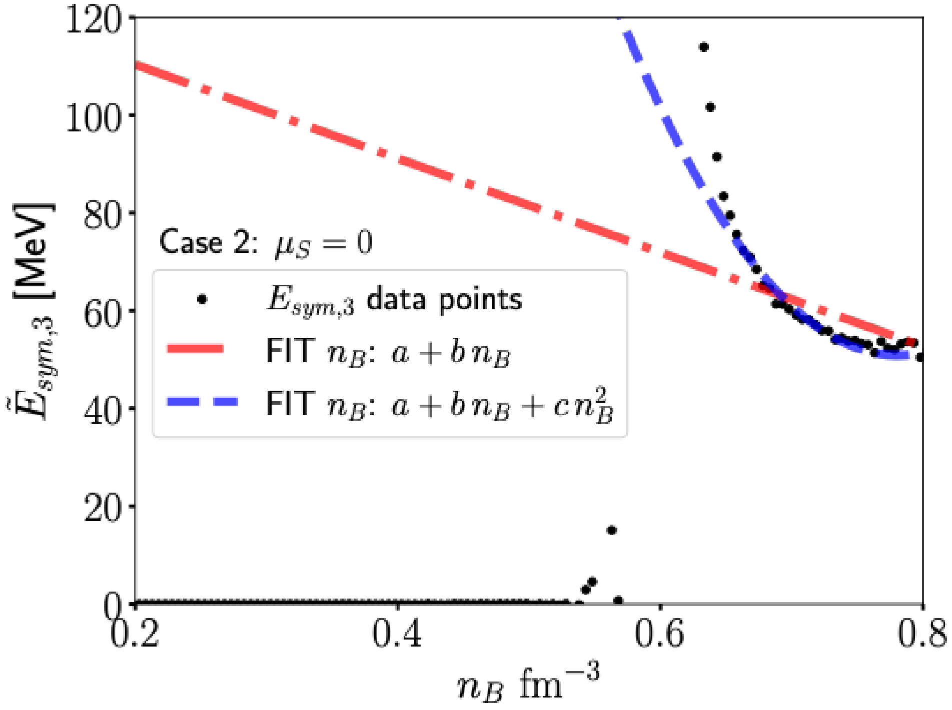
# How good is symmetry energy expansion?

- For a chiral mean field model, symmetry energy expansion works even at high baryon density

YY, Cruz Camacho, Hippert, Noronha-Hostler, arXiv 2504.18764

$$\frac{\varepsilon}{n_B} = \frac{\varepsilon}{n_B}\bigg|_{\delta=0} + E_{sym,2}(n_B)\delta^2 + E_{sym,3}(n_B)\delta^3 + \mathcal{O}(\delta^4)$$



# Transport coefficients at $n_{sat}$

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

$$\tau_\Pi(n_{\rm sat}) = \frac{n_{\rm sat}}{\lambda}\left[8S + \frac{\partial^2 E_e}{\partial Y_e^2}\right]^{-1}$$

$$\zeta_0(n_{\rm sat}) = \lambda\,\tau_\Pi^2\left[4\frac{L}{3n_{\rm sat}}(2Y_e - 1) + \frac{\partial^2 E_e}{\partial n_B \partial Y_e} - \frac{1}{n_{\rm sat}}\frac{\partial E_e}{\partial Y_e}\right]^2$$

# How dissipative tidal deformability enters the waveform

Ripley, Hegade, Chandramoui, Yunes, Nature Astronomy (2024)

- Fourier transform of the GW strain

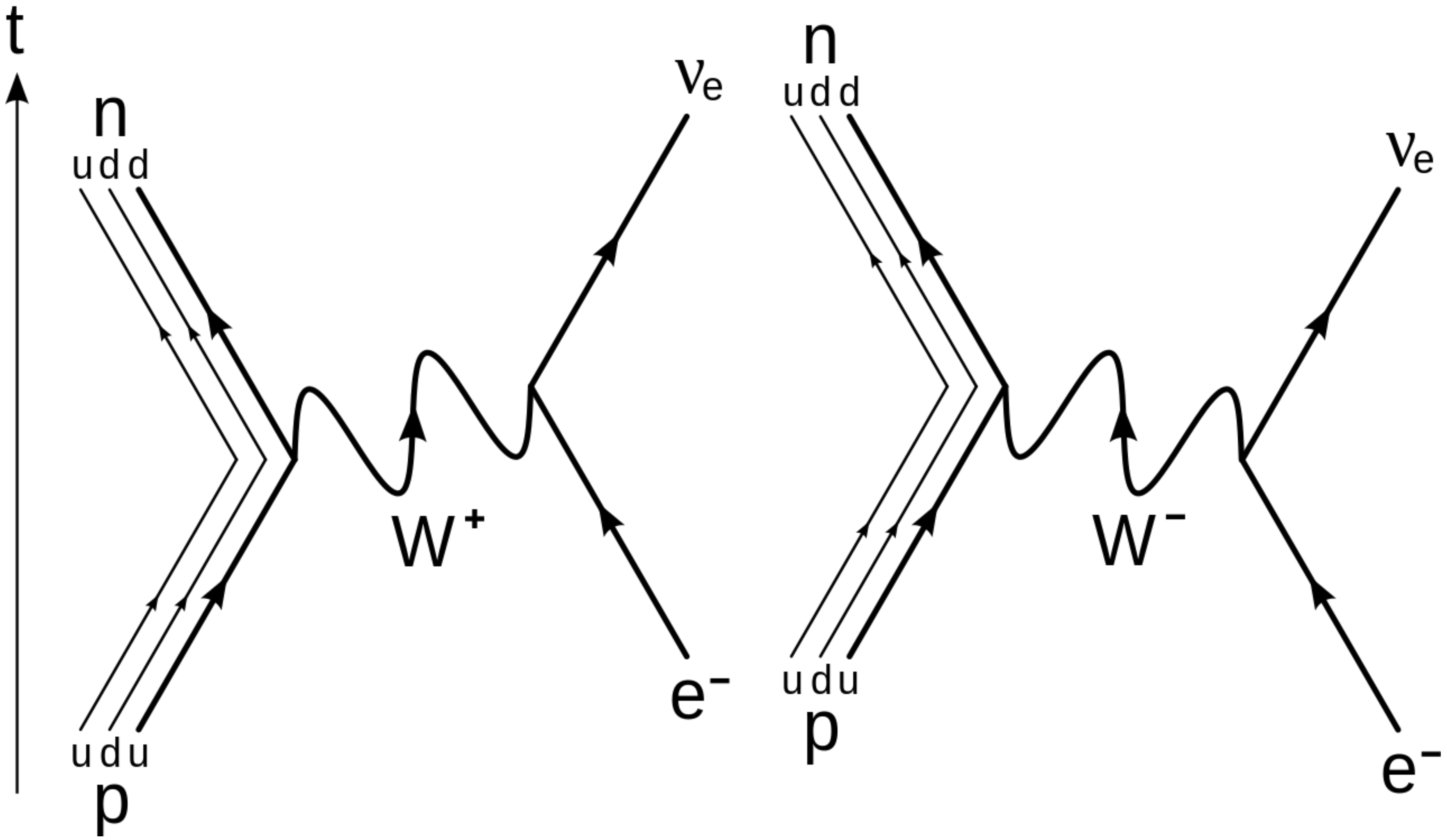
$$\tilde{h}(f) = A(f;\theta)e^{i\Psi(f;\theta)}$$

- Enhance the Fourier GW phase from the IMRPhenomPv2_NRTidal model

$$\Psi(f;\theta) = \Psi_{\mathrm{Pv2NRT}}(f;\theta) - \frac{225}{4096}\frac{(m_A+m_B)^2}{m_A m_B}\tilde{\Xi}\,u^3\log(u)$$

$$u = (G\pi M f/c^3)^{1/3}$$

# Direct Urca



# Modified Urca

- Chemical reactions with a spectator X

$$n + X \rightarrow p + X + e + \overline{\nu}_e$$

$$p + e + X \rightarrow n + X + \nu_e$$

# How does it get out of equilibrium?

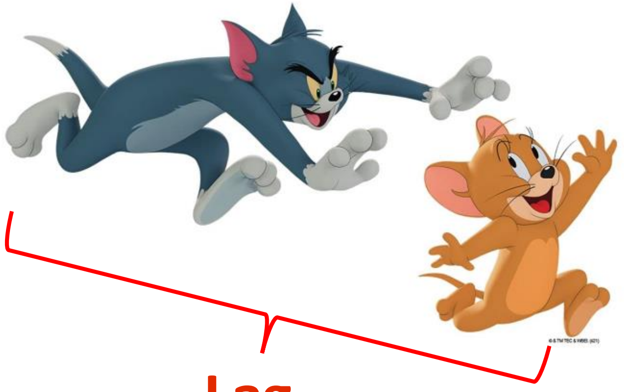
- Change in electron fraction is related to the Urca rates

$$\Gamma_{\bar{\nu}} - \Gamma_{\nu} = n_B \frac{d(\delta Y_e)}{dt} = \Gamma_e$$

- Urca processes & density oscillations: same timescale

↓

Deviation from beta equilibrium



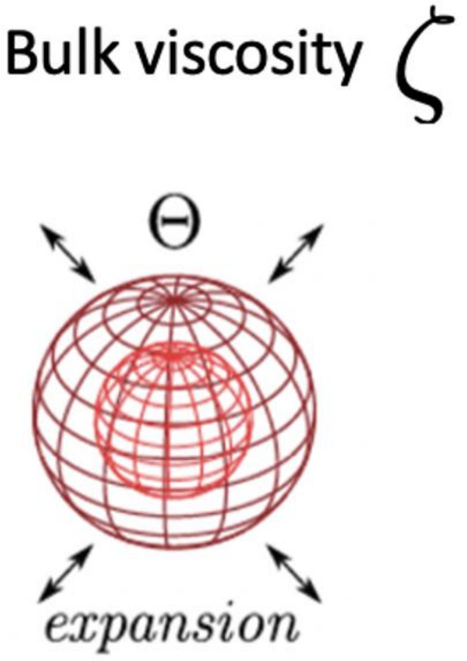

- Energy loss from compression/expansion: bulk viscosity?

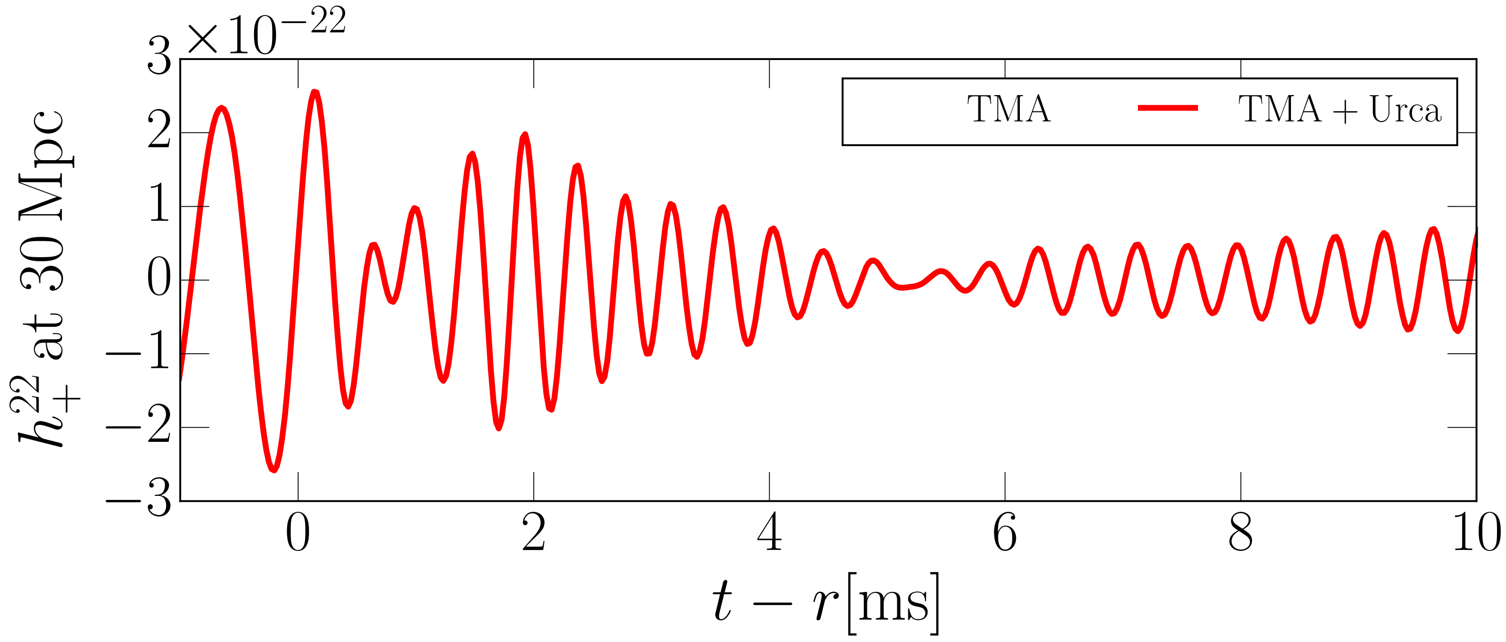
# Bulk viscosity in neutron star mergers?

Alford et al., PRL (2018)

- Density oscillations + weak interactions: deviations from beta equilibrium

➡ realistic simulations with URCA rates



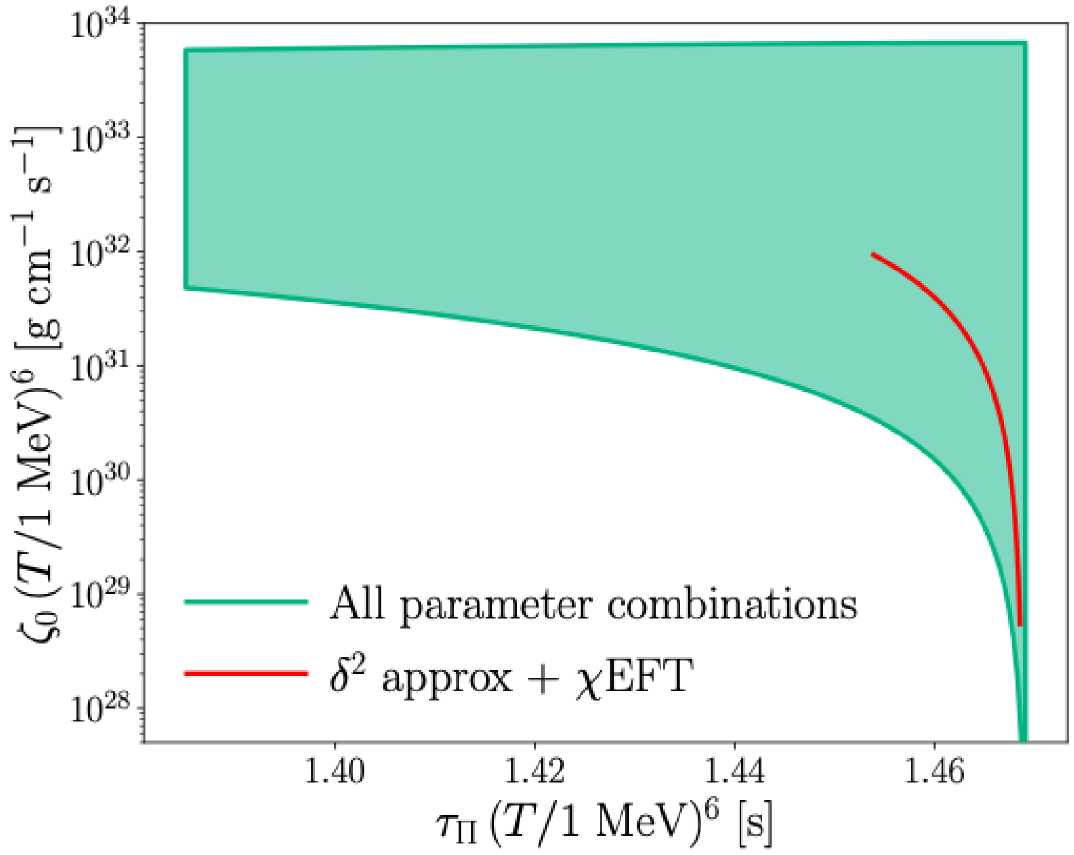




Most et al, arXiv:2207.00442

# Dependence on S & L

YY, Hippert, Speranza, Noronha, arXiv 2504.07805

$$S \in [30, 40]\ \mathrm{MeV},\ L \in [30, 150]\ \mathrm{MeV}$$




- Most Estimates

$$L \sim 50\ \mathrm{MeV}$$

Lattimer, Particles (2023)

- PREX I+II:

$$L = 106 \pm 37\ \mathrm{MeV}$$

Adhikari et al., PRL (2019)

# Constraints on bulk viscosity transport coefficients

$$\left[\frac{\zeta}{\tau_\Pi} + n_B \left.\frac{\partial P}{\partial n_B}\right|_{\varepsilon, Y_e = Y_e^{eq}}\right] \frac{1}{\varepsilon + P} \leq 1 - \left.\frac{\partial P}{\partial \varepsilon}\right|_{n_B, Y_e = Y_e^{eq}}$$

Bemfica, Disconzi, Noronha, PRL (2019)

# Bulk viscosity from the phase lag

Sawyer, PRD (1989)

- Assume

$$n_i = n_i(\mu_i)$$

$$Y_e = Y_{e,0} + Re(\delta Y_{e,0} e^{i\omega t})$$

- Energy density dissipation

$$\langle \dot{\mathcal{E}}_{diss} \rangle = -\frac{\zeta}{\tau}\int_0^\tau dt(\nabla \cdot \vec{v})^2 = \frac{n_{B,0}}{\tau}\int_0^\tau (P+\delta P)\frac{d}{dt}(V+\delta V)dt$$

Bulk viscosity

$$\zeta = \frac{-\gamma C^2}{\omega^2 + (\gamma B/n_B)^2}$$

# Effects of frequency

Camelio et al., PRD (2023)

| $\tau_\pi \omega \gg 1$ | $\tau_\pi \omega \approx 1$ | $\tau_\pi \omega \ll 1$ |
| --- | --- | --- |
| Elastic | Resonant | Navier-Stokes |

Slow equilibration:

- Fixed composition
- Reversible process

Dissipation:

- Macro timescale comparable to micro timescale

Fast equilibration:

- Instantaneously equilibrated mixture