Topology and Quantum Criticality under Weak-Measurement or Decoherence

Cenke Xu
许岑珂

University of California, Santa Barbara

Topology and Quantum Criticality under Weak-Measurement or Decoherence

Collaborators:

Jong Yeon Lee, KITP

Yi-Zhuang You, UCSD

Chao-Ming Jian, Cornell

Nayan Myerson-Jain, UCSB

Kaixiang Su, UCSB

Topology and Quantum Criticality under Weak-Measurement or Decoherence

Content:

1. Decoherence and weak-measurements, two different symmetry conditions;

2. Quantum critical points (such as Wilson Fisher fixed points) under WM/decoherence, mapping to boundary criticality;

3. “Strange correlator” and symmetry protected topological states/ensembles under WM/decoherence; (related talks by Wang, Turzillo)

4. Summary and Outlook.

Reference:

arXiv:2210.16323, Lee, You, Xu,

arXiv:2301.05238, Lee, Jian, Xu

Topology and Quantum Criticality under Weak-Measurement or Decoherence

Quantum systems become classical, through "decoherence"

⇓

A pure density matrix becomes "mixed", through "decoherence"

$\rho = |\Psi\rangle\langle\Psi|$

$$\mathcal{E}_i[\rho] = \sum_m K_{m,i}\,\rho\, K^\dagger_{m,i}$$

Two symmetry conditions:

"Doubled" symmetry condition: the environment is "measuring" quantities that are symmetric under symmetry G (like energy density), Form of density matrix invariant under G^L and G^R actions separately.

"diagonal" symmetry condition: the environment is weakly measuring quantities that are not symmetric under symmetry G, but the outcomes are averaged over with symmetric probability.

Quantum systems become classical, through "decoherence"

$\Downarrow$

A pure density matrix becomes "mixed", through "decoherence"

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$$

$$\rho = \frac{1}{2}(I + X)$$

$$\rho^D = (1-p)\rho + pZ\rho Z$$

Two symmetry conditions:

The original ket-state is symmetric under Z_2 transformation X;

The original density matrix is symmetric under a doubled symmetry, Z_2^L and Z_2^R

The decohered density matrix is only invariant under the diagonal subgroup of Z_2^L and Z_2^R (simultaneous left and right action X)

For a symmetric quantum many-body state with Hamiltonian *H*, …

$$\hat{\rho}_0 = |\Psi\rangle\langle\Psi| \sim \exp(-\beta\hat{H})$$

$$\begin{aligned}[\hat{\rho}_0]_{\phi_1(\boldsymbol{x}),\phi_2(\boldsymbol{x})} &= \langle\phi_1(\boldsymbol{x})|\Psi\rangle\langle\Psi|\phi_2(\boldsymbol{x})\rangle \\ &\sim \lim_{\beta\to\infty}\int_{\substack{\phi(\boldsymbol{x},0)=\phi_1(\boldsymbol{x})\\ \phi(\boldsymbol{x},\beta)=\phi_2(\boldsymbol{x})}} D\phi(\boldsymbol{x},\tau)\exp(-\mathcal{S}),\end{aligned}$$

$$\mathcal{E}_i[\rho] = \sum_m K_{m,i}\rho K_{m,i}^\dagger$$

$$[\hat{\rho}^D]_{\phi_1(\boldsymbol{x}),\phi_2(\boldsymbol{x})} \sim \lim_{\beta\to\infty}\int_{\substack{\phi(\boldsymbol{x},0)=\phi_1(\boldsymbol{x})\\ \phi(\boldsymbol{x},\beta)=\phi_2(\boldsymbol{x})}} D\phi(\boldsymbol{x},\tau)\exp\left(-\mathcal{S}-\mathcal{S}^{\text{int}}\right);$$

$$\mathcal{S} = \int_0^\beta d\tau d\boldsymbol{x}\ \mathcal{L}(\phi), \quad \mathcal{S}^{\text{int}} = \int d\boldsymbol{x}\ \mathcal{L}^{\text{int}}(\phi(\boldsymbol{x},0),\phi(\boldsymbol{x},\beta)).$$

For a symmetric quantum many-body state with Hamiltonian H, the effect of decoherence is mapped to interactions between two temporal boundaries.

$$[\hat{\rho}^D]_{\phi_1(\boldsymbol{x}),\phi_2(\boldsymbol{x})} \sim \lim_{\beta\to\infty} \int_{\substack{\phi(\boldsymbol{x},0)=\phi_1(\boldsymbol{x}) \\ \phi(\boldsymbol{x},\beta)=\phi_2(\boldsymbol{x})}} D\phi(\boldsymbol{x},\tau) \exp\left(-\mathcal{S} - \mathcal{S}^{\mathrm{int}}\right);$$

$$\mathcal{S} = \int_0^\beta d\tau d\boldsymbol{x}\ \mathcal{L}(\phi), \quad \mathcal{S}^{\mathrm{int}} = \int d\boldsymbol{x}\ \mathcal{L}^{\mathrm{int}}(\phi(\boldsymbol{x},0), \phi(\boldsymbol{x},\beta)).$$

The form of the interaction L^{int}, can be determined by the symmetry conditions, i.e. either "doubled symmetry", or "diagonal symmetry".

If the system has a Lorentz invariance in the IR, the problem of temporal boundary, can be mapped to the problem of spatial boundary. The effect of decoherence is mapped to interactions between two spatial boundaries, restricted by the symmetry conditions.

For a symmetric quantum many-body state with Hamiltonian H, the effect of decoherence is mapped to interactions between two temporal boundaries.

$$[\hat{\rho}^D]_{\phi_1(\boldsymbol{x}),\phi_2(\boldsymbol{x})} \sim \lim_{\beta\to\infty} \int_{\substack{\phi(\boldsymbol{x},0)=\phi_1(\boldsymbol{x}) \\ \phi(\boldsymbol{x},\beta)=\phi_2(\boldsymbol{x})}} D\phi(\boldsymbol{x},\tau) \exp\left(-\mathcal{S} - \mathcal{S}^{\text{int}}\right);$$

$$\mathcal{S} = \int_0^\beta d\tau d\boldsymbol{x}\ \mathcal{L}(\phi), \quad \mathcal{S}^{\text{int}} = \int d\boldsymbol{x}\ \mathcal{L}^{\text{int}}(\phi(\boldsymbol{x},0), \phi(\boldsymbol{x},\beta)).$$

Two classes of problems of nontrivial and universal boundary physics:
(1) symmetry protected topological states with topologically protected boundary; space-time rotation leads to "strange correlator" (You, et.al. arXiv:1312.0626).
(2) Decoherence effects on quantum criticality can be mapped to boundary criticality (Garratt, et.al. arXiv:2207.09476)

Topology and Quantum Criticality under Weak-Measurement or Decoherence

Content:

1. Decoherence and weak-measurements, two different symmetry conditions;

2. Quantum critical points (such as Wilson Fisher fixed points) under WM/decoherence, mapping to boundary criticality;

3. "Strange correlator" and symmetry protected topological states /ensembles under WM/decoherence;

4. Summary and Outlook.

Reference:
arXiv:2210.16323, Lee, You, Xu
arXiv:2301.05238, Lee, Jian, Xu

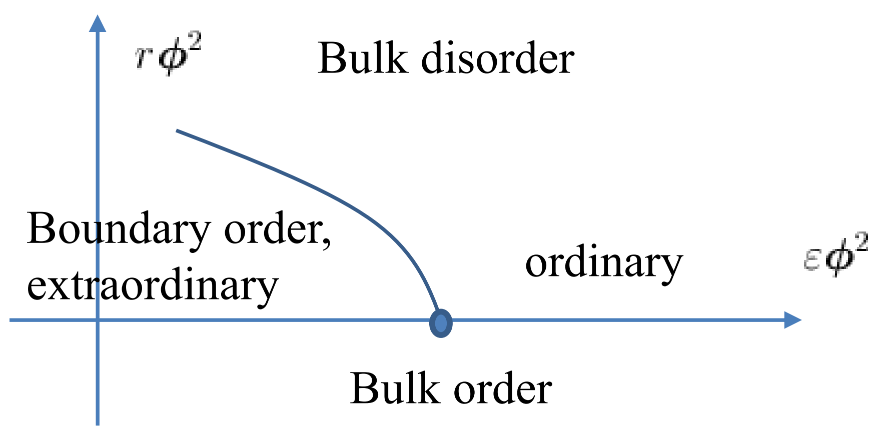
For 1+1d quantum critical points under weak-measurements, please refer to Garratt, et.al. arXiv:2207.09476: decoherence is mapped to the (0+1)d boundary/defect of 1+1d CFT.

$$[\hat{\rho}^D]_{\phi_1(\boldsymbol{x}),\phi_2(\boldsymbol{x})} \sim \lim_{\beta\to\infty} \int_{\substack{\phi(\boldsymbol{x},0)=\phi_1(\boldsymbol{x})\\ \phi(\boldsymbol{x},\beta)=\phi_2(\boldsymbol{x})}} D\phi(\boldsymbol{x},\tau) \exp\left(-\mathcal{S} - \mathcal{S}^{\rm int}\right);$$

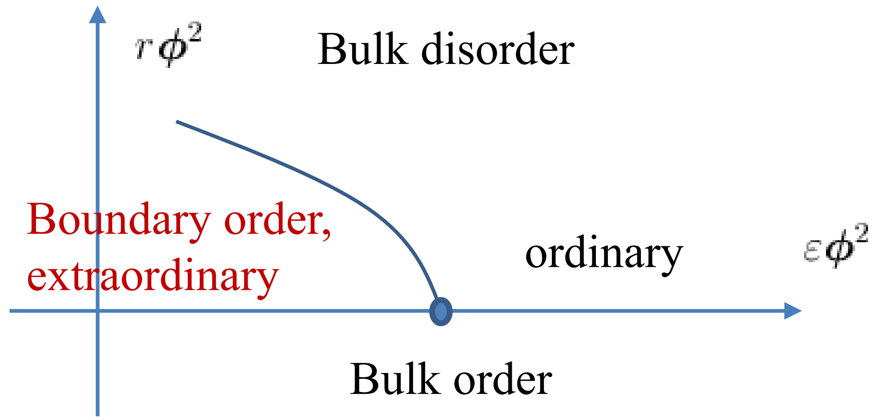
$$\mathcal{S} = \int_0^\beta d\tau d\boldsymbol{x}\ \mathcal{L}(\phi), \quad \mathcal{S}^{\rm int} = \int d\boldsymbol{x}\ \mathcal{L}^{\rm int}(\phi(\boldsymbol{x},0),\phi(\boldsymbol{x},\beta)).$$

We focus on 2+1d critical states under decoherence. This problem is mapped to the boundary criticality of 2+1d CFT.

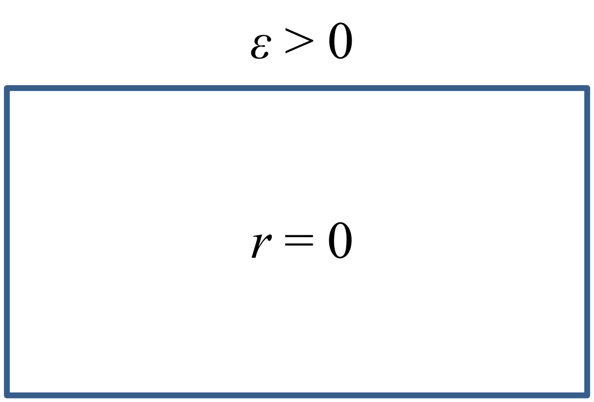
Boundary properties of a *D*-dim (classical) phase transition, assume *D* is large:



Boundary properties of a *D*-dim (classical) phase transition, assume *D* is large:



For the "ordinary" boundary condition, the correlation function at the boundary still decays with a power-law, but the boundary scaling dimension is much bigger than the bulk.



Bulk: $\langle \phi(0)\phi(r) \rangle \sim \frac{1}{r^{2\Delta_D}}$

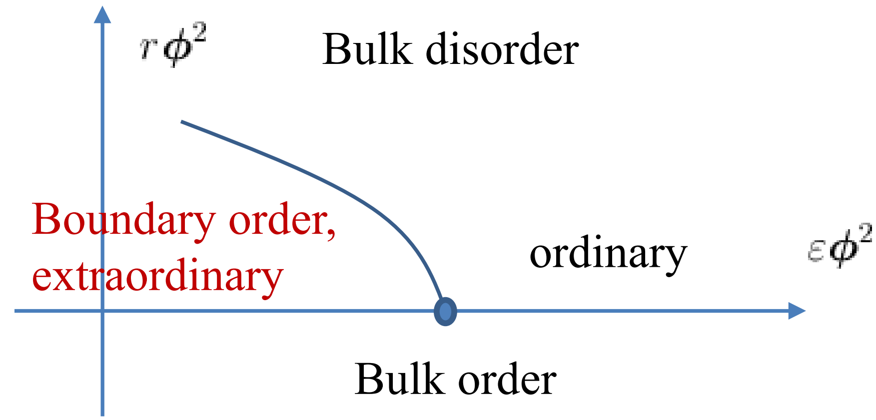
ordinary Boundary: $\langle \phi(0)\phi(x) \rangle \sim \frac{1}{x^{2\Delta_b}}$

For large-*D*

$$\Delta_D = (D-2)/2$$

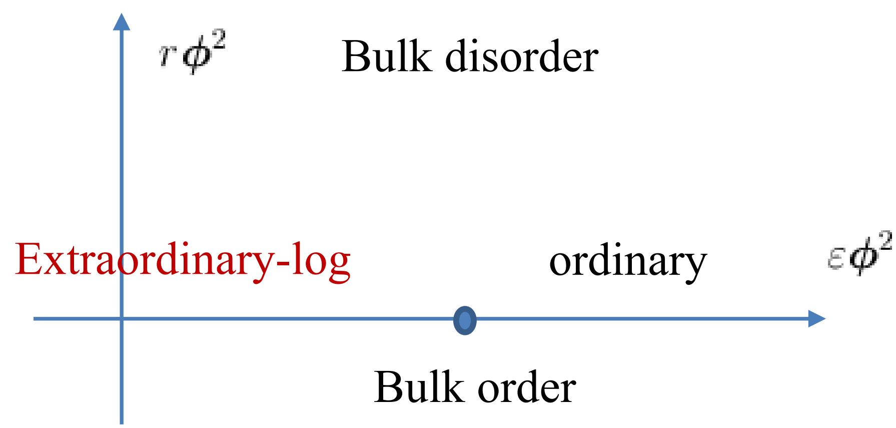
$$\Delta_b \sim D/2$$

Boundary properties of a *D*-dim (classical) phase transition, assume *D* is large:

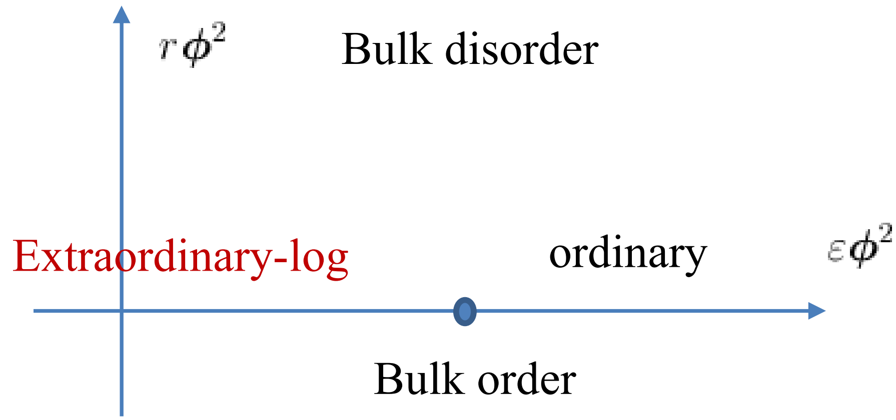


This phase diagram is problematic when D=3 (bulk is 2+1d, boundary is 1+1d): For systems with continuous symmetry, there cannot be a D=2 order without the bulk order (Mermin-Wagner theorem).

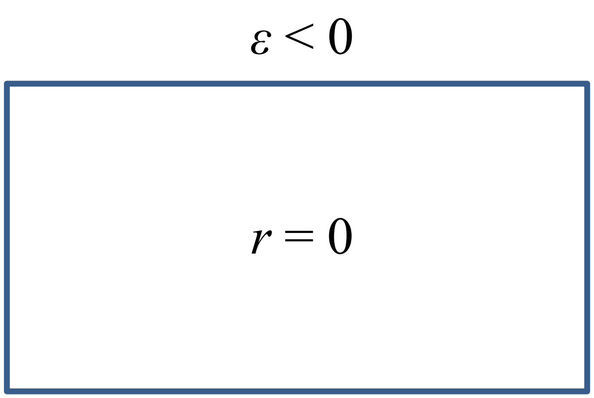
When the bulk is critical, the boundary is not a "local" D=2 system. New result: Extraordinary-log boundary (Metlitski, arXiv:2009.05119)



When the bulk is critical, the boundary is not a "local" D=2 system. New result: Extraordinary-log boundary (Metlitski, arXiv:2009.05119)



In the extraordinary-log boundary phase, the correlation function between the order parameter is "almost long ranged" (even more so when we consider a D=2 defect in the D=3 bulk)



Bulk: $\langle \phi(0) \cdot \phi(r) \rangle \sim \frac{1}{r^{2\Delta_{wf}}}$

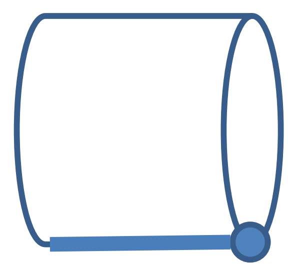
Extraordinary-log boundary $\langle \phi(0) \cdot \phi(x) \rangle \sim \frac{1}{(\log x)^q}$

Back to our problem. We assume the system is a 2+1d O(N) Wilson-Fisher fixed point, and the weak measurement is done on quantities that are fully symmetric, then the density matrix should have the "doubled"

$$[\hat{\rho}^D]_{\phi_1(\boldsymbol{x}),\phi_2(\boldsymbol{x})} \sim \lim_{\beta\to\infty} \int_{\substack{\phi(\boldsymbol{x},0)=\phi_1(\boldsymbol{x}) \\ \phi(\boldsymbol{x},\beta)=\phi_2(\boldsymbol{x})}} D\phi(\boldsymbol{x},\tau) \exp\left(-\mathcal{S} - \mathcal{S}^{\mathrm{int}}\right);$$

$$\mathcal{S} = \int_0^\beta d\tau d\boldsymbol{x}\ \mathcal{L}(\phi), \quad \mathcal{S}^{\mathrm{int}} = \int d\boldsymbol{x}\ \mathcal{L}^{\mathrm{int}}(\phi(\boldsymbol{x},0), \phi(\boldsymbol{x},\beta)).$$

Ordinary observable quantities involves a trace of the density matrix, which glues the two temporal boundaries at $\tau = 0 / \beta$; due to the trace, decoherence becomes a "defect problem" with a slab of defect inserted at $\tau = 0 / \beta$ (subtlety: this requires some post-selection) Extraordinary-log still holds for a D=2 defect inserted in a D=3 bulk (Krishnan, Metlitski, arXiv:2301.05728).



$\tau = 0 / \beta$

Topology and Quantum Criticality under Weak-Measurement or Decoherence

Back to our problem. We assume the system is a 2+1d O(N) Wilson-Fisher fixed point, and the weak measurement is done on quantities that are fully symmetric, then the density matrix should have the "doubled"

$$[\hat{\rho}^D]_{\phi_1(\boldsymbol{x}),\phi_2(\boldsymbol{x})} \sim \lim_{\beta\to\infty} \int_{\substack{\phi(\boldsymbol{x},0)=\phi_1(\boldsymbol{x}) \\ \phi(\boldsymbol{x},\beta)=\phi_2(\boldsymbol{x})}} D\phi(\boldsymbol{x},\tau) \exp\left(-\mathcal{S} - \mathcal{S}^{\rm int}\right);$$

$$\mathcal{S} = \int_0^\beta d\tau d\boldsymbol{x} \; \mathcal{L}(\phi), \quad \mathcal{S}^{\rm int} = \int d\boldsymbol{x} \; \mathcal{L}^{\rm int}(\phi(\boldsymbol{x},0), \phi(\boldsymbol{x},\beta)).$$

The simplest defect term $S^{\rm int}$ that meets the symmetry criteria is just an extra mass term:

$$\mathcal{S}^{\rm int} = \int dxdy \; \varepsilon |\boldsymbol{\phi}(\boldsymbol{x},0)|^2.$$

Then depending on the sign of ε…

$$\mathrm{tr}\{\hat{\rho}^D_P \, \hat{\boldsymbol{\phi}}(\boldsymbol{0}) \cdot \hat{\boldsymbol{\phi}}(\boldsymbol{x})\} \sim \frac{1}{(\ln|\boldsymbol{x}|)^q}.$$

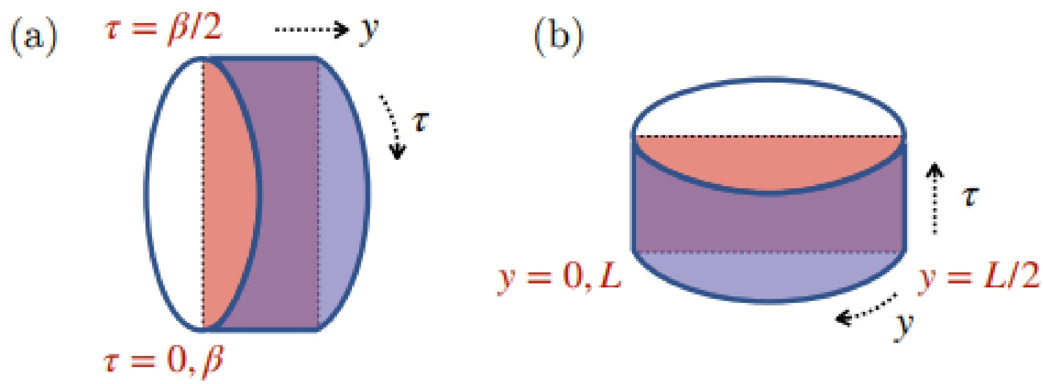
$$\mathrm{tr}\{\hat{\rho}^D_P \; \hat{\boldsymbol{\phi}}(\boldsymbol{0}) \cdot \hat{\boldsymbol{\phi}}(\boldsymbol{x})\} \sim \frac{1}{|\boldsymbol{x}|^{2\Delta^b_\phi}}$$


ε

We can also consider quantities nonlinear with the density matrix, such as the VN entanglement entropy, or Renyi entropy. For example, the second Renyi entropy is mapped to evaluating the following partition function and free energy:

$$S^{(2)} = -\log \mathrm{tr}\{(\hat{\rho}^D)^2\}$$

$$\sim -\log \lim_{\beta\to\infty} \int D\phi(\boldsymbol{x},\tau) \exp\left(-\mathcal{S} - \int d\boldsymbol{x}\ \mathcal{L}^{\mathrm{int}}(\phi(\boldsymbol{x},0), \phi(\boldsymbol{x},\beta/2))\right)$$

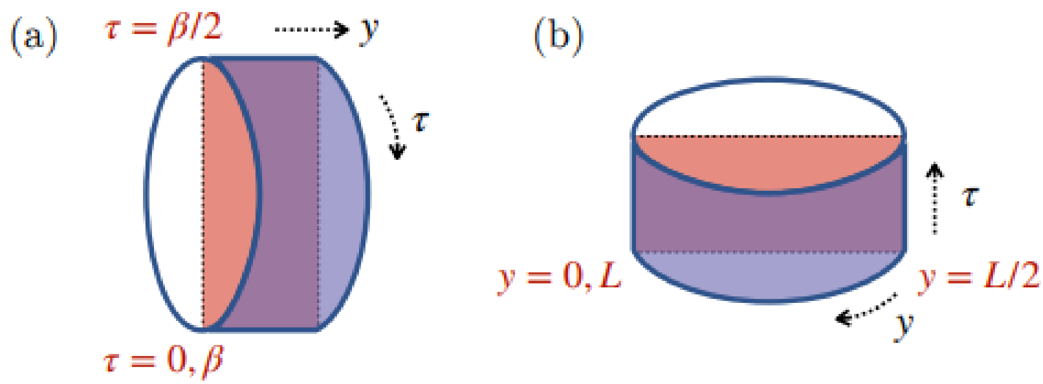


The second Renyi entropy is mapped to evaluating partition function/free energy with nonlocal interaction in space-time between two defect slabs.

We can also consider quantities nonlinear with the density matrix, such as the VN entanglement entropy, or Renyi entropy. For example, the second Renyi entropy is mapped to evaluating the following partition function and free energy:

$$S^{(2)} = -\log \mathrm{tr}\{(\hat{\rho}^D)^2\}$$

$$\sim -\log \lim_{\beta\to\infty} \int D\phi(\boldsymbol{x},\tau) \exp\left(-\mathcal{S} - \int d\boldsymbol{x}\ \mathcal{L}^{\mathrm{int}}(\phi(\boldsymbol{x},0), \phi(\boldsymbol{x},\beta/2))\right)$$



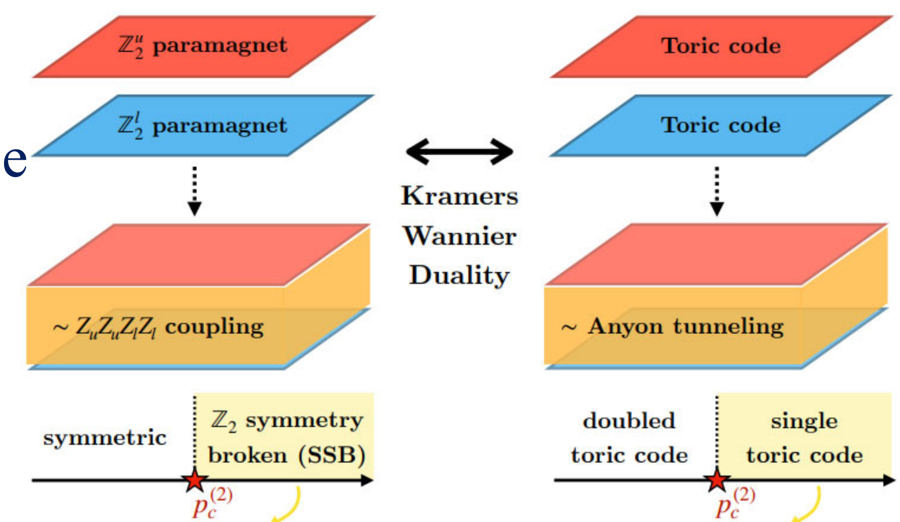
A plethora of possibilities, such as spontaneous breaking of double symmetry to diagonal symmetry, which is a quantum information transition which leads to singularity in the Renyi entropy.

Sometimes it is convenient to literally consider a doubled system, through the "Choi-Jamiolkowski isomorphism"

$$\rho = |\Psi\rangle\langle\Psi| \quad \rightarrow \quad ||\rho\rangle\rangle = |\Psi\rangle \otimes |\Psi\rangle$$

The decoherence is mapped to the "interaction" between the doubled systems.

A lattice example: toric code under decoherence. The decoherence can drive a "pair anyon condense" phase transition in the "doubled" system, dual to spontaneous breaking from the doubled symmetry to diagonal symmetry. See also Bao, et.al arXiv:2301.05687



Topology and Quantum Criticality under Weak-Measurement or Decoherence

Content:

1. Decoherence and weak-measurements, two different symmetry conditions;

2. Quantum critical points (such as Wilson Fisher fixed points) under WM/decoherence, mapping to boundary criticality;

3. "Strange correlator" and symmetry protected topological states/ensembles under WM/decoherence;

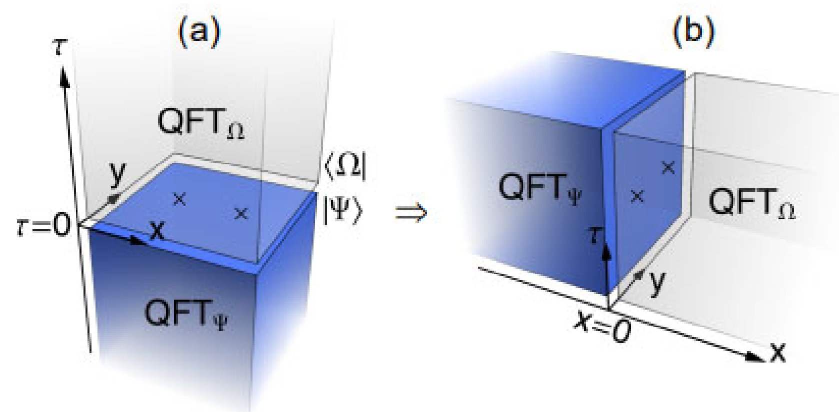
4. Summary and Outlook.

Reference:

arXiv:2210.16323, Lee, You, Xu
arXiv:2301.05238, Lee, Jian, Xu

"Strange correlator" was proposed/conjectured as a simple tool to diagnose a bulk wave function (with periodic boundary condition): is it a trivial insulator or topological insulator/SPT state? (You, et.al. arXiv:1312.0626)

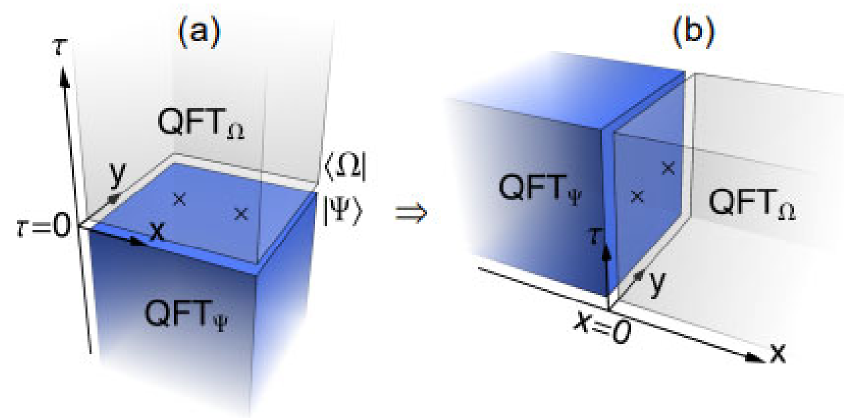
$$C(r) \equiv \frac{\langle \Psi | O(0) O(r) | \Omega \rangle}{\langle \Psi | \Omega \rangle}$$



$|\Omega>$ is a trivial symmetric state, $|\Psi>$ is the wave function we need to diagnose. O is an order parameter that transforms nontrivially under the symmetry;
The prediction is that, the strange correlator $C(r)$ should be "nontrivial" in the limit of large r, i.e. either long range or quasi long range (power-law) correlated, for 1d or 2d nontrivial SPT state $|\Psi>$.

The strange correlator $C(r)$ should be "nontrivial" in the limit of large r, i.e. either long range or quasi long range (power-law) correlated, for 1d or 2d nontrivial SPT state $|\Psi>$. (You, et.al. arXiv:1312.0626)

$$C(r) \equiv \frac{\langle \Psi | O(0) O(r) | \Omega \rangle}{\langle \Psi | \Omega \rangle}$$

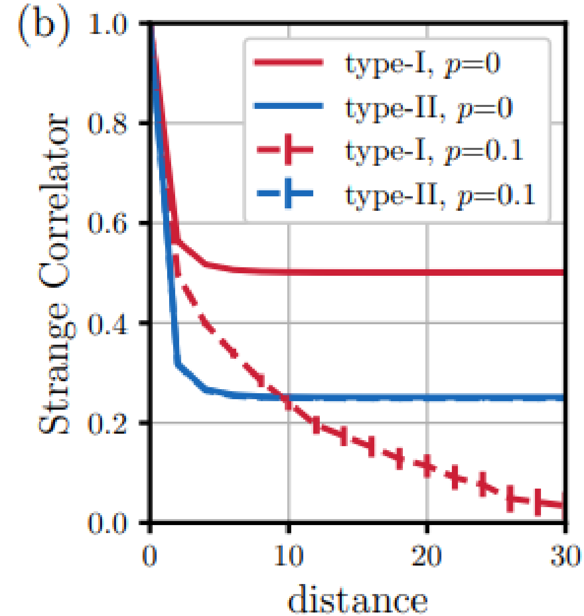


This result can be shown explicitly for noninteracting fermionic TIs, and for a large class of bosonic SPT states (using NLSM description of SPT states). Also tested for various lattice systems (1d and 2d AKLT-like states, interacting TI, etc.), using different numerical methods by other groups.

A simple 1d SPT model protected by the Z_2 x Z_2 symmetry; the two Z_2 symmetries correspond to Ising symmetries on the even and odd sites of the 1d lattice. In order to be more general, we move away from the "commuting" limit by turning on *h*.

$$H = -\sum_n (Z_{n-1} X_n Z_{n+1} + h X_n)$$

$$C_{\text{odd}}(2n) = \frac{\langle \Psi | Z_1 Z_{2n+1} | \Omega \rangle}{\langle \Psi | \Omega \rangle}$$



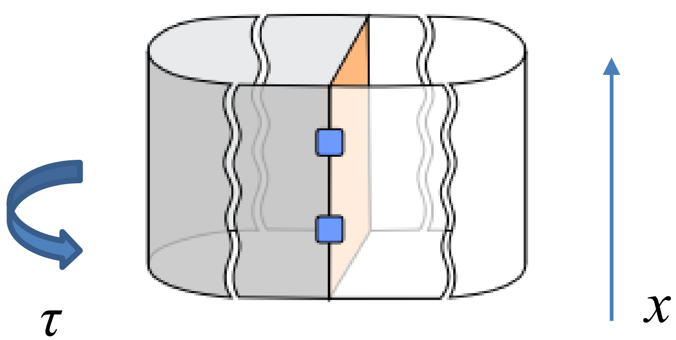
We choose the trivial state $|\Omega\rangle = |+\rangle^{\otimes 2N}$

For the 1d SPT states, the type-I strange correlator is related to the well-known string order operator of the Haldane phase.

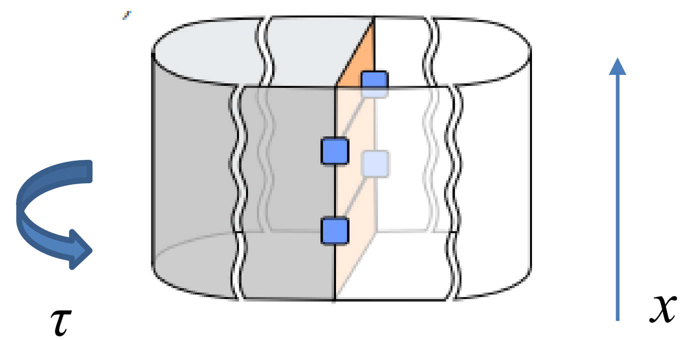
Since we are going to talk about systems under decoherence/WM, we need to generalize the form of the strange correlator to density matrix; there are two natural generalizations:

$$C^{\mathrm{I}}(r) \equiv \frac{\mathrm{tr}(\rho_{\mathrm{spt}} O(0) O(r) \rho_0)}{\mathrm{tr}(\rho_{\mathrm{spt}} \rho_0)}$$

$$C^{\mathrm{II}}(r) \equiv \frac{\mathrm{tr}\left(\rho^{D}_{\mathrm{spt}} O(0) O(r) \rho_0 O(r)^\dagger O(0)^\dagger\right)}{\mathrm{tr}\left(\rho^{D}_{\mathrm{spt}} \rho_0\right)}$$



type-I strange correlator



type-II strange correlator

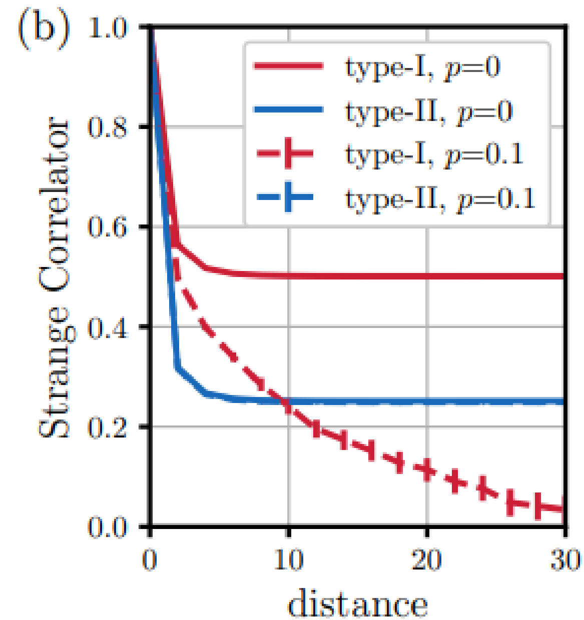
Like the previous section, decoherence is mapped to interactions between two boundaries. We can use either form of generalizations to define a "symmetry protected topological ensemble (SPTE)".

We would like to "weakly" measure the Z_i operators for even sites i. But we consider all measurement outcomes without any pos-selection/bias. This gives the SPE a Z_2 x Z^L_2 x Z^R_2

We still move away from the "commuting" limit, by considering a Hamiltonian

$$H = -\sum_n (Z_{n-1} X_n Z_{n+1} + h X_n)$$

For h = 1/2, and small p ("strength" of measurement), type-I strange correlator is rendered short-ranged, but type-II strange correlator is still robustly long ranged.



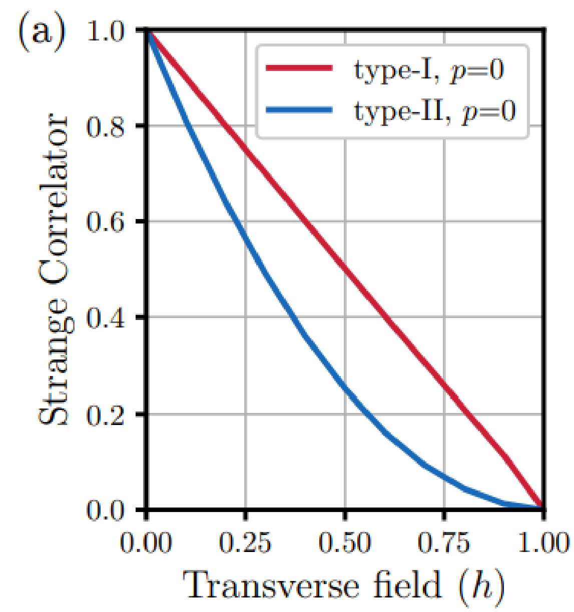
Type-II strange correlator "remembers" that the system was once a pure SPT state.

We would like to "weakly" measure the Z_i operators for even sites i. But we consider all measurement outcomes without any pos-selection/bias. This gives the SPE a $Z_2 \times Z^L_2 \times Z^R_2$

We still move away from the "commuting" limit, by considering a Hamiltonian

$$H = -\sum_n (Z_{n-1} X_n Z_{n+1} + h X_n)$$

For h = 1/2, and small p ("strength" of measurement), type-I strange correlator is rendered short-ranged, but type-II strange correlator is still robustly long ranged.



Type-II strange correlator "remembers" that the system was once a pure SPT state. For large-h, both type-I and II strange correlator will be trivial.

For many bosonic SPT states, we can actually write down their explicit wave functions: (Xu, Senthil, arXiv:1301.6172)
2d example: a 2d SPT state with U(1) x U(1) symmetry.

$$|\Psi\rangle \sim \int D[\boldsymbol{n}(\mathbf{x})] \exp\left(-\mathcal{S}[\boldsymbol{n}(\mathbf{x})]\right) |\boldsymbol{n}(\mathbf{x})\rangle$$

$$\mathcal{S} = \int d^2x \, \frac{1}{g}(\partial \boldsymbol{n}(\mathbf{x}))^2 + \mathrm{WZW}[\boldsymbol{n}(\mathbf{x})]$$

$$\mathrm{WZW}[\boldsymbol{n}(\mathbf{x})] = \int_0^1 du \int d^2x \, \frac{2\pi \mathrm{i}}{\Omega_3} \epsilon_{abcd} \tilde{n}^a \partial_u \tilde{n}^b \partial_x \tilde{n}^c \partial_y \tilde{n}^c.$$

Again, if we rotate space-time, this (2+0)d wave function can be viewed as the action of a (1+1)d Luttinger liquid with $U(1)_\phi$ x $U(1)_\theta$ symmetry:

$$\mathcal{L} = \frac{1}{4\pi K}\left((\partial_\tau \theta)^2 + v^2(\partial_x \theta)^2\right)$$

$$\mathcal{L}_d = \frac{K}{4\pi}\left((\partial_\tau \phi)^2 + v^2(\partial_x \phi)^2\right)$$

$$\boldsymbol{n} \sim (\cos\theta, \sin\theta, \cos\phi, \sin\phi)$$

2d example: a 2d SPT state with U(1) x U(1) symmetry, whose boundary is

$$\mathcal{L} = \frac{1}{4\pi K}\left((\partial_\tau\theta)^2 + v^2(\partial_x\theta)^2\right)$$
$$\mathcal{L}_d = \frac{K}{4\pi}\left((\partial_\tau\phi)^2 + v^2(\partial_x\phi)^2\right)$$

$$\boldsymbol{n} \sim (\cos\theta, \sin\theta, \cos\phi, \sin\phi)$$

Evaluating type-I and type-II strange correlators, reduce to evaluating correlation functions of two interacting copies of (1+1)d Luttinger liquid. For example, if the decohered density matrix has symmetry U(1) x $U(1)^L$ x $U(1)^R$:

$$H^{\text{int}}_{1d,2} \sim \int dx\ \alpha\cos(\phi - \phi')$$

This interaction renders the combinations $\theta_- \sim \theta - \theta'$ and ϕ_- gapped and short-ranged, but leaves θ_+ and ϕ_+ gapless, this leads to trivial type-I strange correlator, but nontrivial type-II strange correlator, which implies mixed anomaly of U(1) x $U(1)^L$ x $U(1)^R$.

Summary and Outlook:

The effects of decoherence and WM can be mapped to the problem of boundaries;
The WM can drastically change the observed correlation functions of a QCP;
Many new phenomena can occur if we compute quantities nonlinear with density matrix;
we also design quantities (type-I and II SC) that can diagnose a mixed state ensemble and define symmetry protected topological ensembles.

Related works:
de Groot et.al. arXiv:2112.04483, Garratt, et.al. arXiv:2207.09476, Bao, et.al arXiv:2301.05687, Ma, Wang, arXiv:2209.02723, Zhang, Qi, Bi, arXiv:2210.17485

NSF

SIMONS FOUNDATION

Summary and Outlook:

(1) Interplay of two boundary effects caused by WM: topological boundary and boundary criticality;
Interplay between bulk critical modes and physical topological boundary (Grover, Vishwanath, 2012; Zhang, Wang, 2018; Jian, et.al. 2020 and many others)

(2) weak-measurement on deconfined QCP and spin liquids with fractionalizations (briefly discussed in arXiv:2301.05238, Lee, Jian, Xu, but a lot more to do).