Jet quenching in anisotropic flowing matter

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17th October 2023, Seattle

Mainly based on 2309.00683
elementary constituents

complex nuclear matter

simplest complex matter
Two main sources of information:

- **Soft particle** coming from the thermal matter
- **Hard particle**: jets, quarkonia...
jets interact with the medium while propagating through it

jet quenching
Jet tomography

- Jet tomography: Jets as differential probes of the spatio-temporal structure of the thermal matter created in HIC

- Modification of jet properties throughout their flight encodes information about the QGP characteristics and evolution

- Most approaches to jet-medium interaction are either empirical or based on multiple simplifying assumptions
Do jets feel the transverse flow and anisotropies of the QGP?
Focus on leading perturbative processes: Two processes that modify jets.

Single particle broadening

Medium induced gluon radiation

Theoretical formulation of jet quenching requires several assumptions to make it tractable. Some of the are

- Eikonal expansion; only sub-eikonal length enhanced terms are kept
- Medium is modeled by a background field
- In the simplest scenario the medium is static and homogeneous
The medium is modeled by a field created by a classical current of sources

\[ gA^{\alpha\mu}(q) = \sum_i g^{\mu 0} e^{-iq \cdot x_i} t_i^\alpha v_i(q) (2\pi) \delta(q_0) \]

The stochastic field can be written as

controls the jet-medium interaction
controls de inhomogeneity
The medium is modeled by a field created by a classical current of sources

\[ \mu_{\mu} = (1, \mathbf{u}, u_z)_{\mu} \]

The stochastic field can be written as

\[ gA^{a\mu}(q) = \sum_{i} u_i^{a} e^{-i \mathbf{q} \cdot \mathbf{x}_i} t_i^{a} v_i(q) (2\pi) \delta(q_0 - \mathbf{q} \cdot \mathbf{u} - q_z u_z) \]

- Green controls the jet-medium interaction
- Orange controls de inhomogeneity
- Blue velocity of the sources
The medium is modeled by a field created by a classical current of sources:

$$gA^\alpha_\mu(q) = \sum_i u_i^\alpha e^{-iq \cdot x_i} t_i^\alpha v_i(q) (2\pi) \delta(q_0 - q \cdot u - q_z u_z)$$

The stochastic field can be written as:

$$u_\mu = (1, u, u_z)_\mu$$

$$v_i(q) = \frac{g^2}{q^2 - \mu^2 + i\epsilon}$$

- Green: controls the jet-medium interaction
- Red: controls de inhomogeneity
- Blue: velocity of the sources
Medium average

Stochastic field \(\rightarrow\) need to specify the average over its configurations \(\rightarrow\) Gaussian statistics

\[
g A^{a\mu}(q) = \sum_i u_i^\mu e^{-iq \cdot x_i} t_i^a v_i(q) (2\pi) \delta(q_0 - \mathbf{q} \cdot \mathbf{u} - q_z u_z)
\]

\[
\langle A^a(q) A^b(\bar{q}) \rangle \sim \langle t^a_i t^b_j \rangle = C \delta_{ij} \delta^{ab}
\]

Colour neutrality

Source averaging
\[
\sum_i = \int d^2 \mathbf{x} \, dz \, \rho(\mathbf{x}, z)
\]
Hydrodynamic variables, $g(\mathbf{x}, z)$, encode the matter structure: $g(\mathbf{x}, z) \equiv \rho(\mathbf{x}, z) \mu^{2}(\mathbf{x}, z) \mathbf{u}(\mathbf{x}, z) \ u_{z}(\mathbf{x}, z)$

Transversely homogeneous matter:

$$g(\mathbf{x}, z) \simeq g(z)$$

Transversely inhomogeneous matter:

$$g(\mathbf{x}, z) \simeq g(z) + \nabla_{\alpha} g(z) \mathbf{x}_{\alpha}$$
Two diagrams to compute

\[ iM_1 = \begin{array}{c|c|c}
J & p - q & p \\
\hline
q & \times & \end{array} \]

Single-Born contribution

\[ iM_2 = \begin{array}{c|c|c|c}
J & p - q - q_i & p - q_i & p \\
\hline
q_i & \sum & q_i & \end{array} \]

Double-Born contribution

\[ \langle |M|^2 \rangle = \langle |M_0^2| \rangle + \langle |M_1|^2 \rangle + \langle M_2 M_0^* \rangle + \langle M_2^* M_0 \rangle \]

Some assumption

- Only first subeikonal corrections are kept
- Dilute and extended medium \( \mu \Delta z \gg 1 \)
The final state parton distribution

\[ E \frac{dN}{d^2pdE} \equiv \frac{1}{2(2\pi)^3} \langle |M|^2 \rangle \]
The final state parton distribution

\[
E \frac{d^2 p}{d^2 p dE} = E \frac{dN^{(0)}}{d^2 p dE} + C \int_0^L dz \int_q \left\{ \left[ 1 - \hat{g}_\alpha \frac{(uE - p + q)\alpha z}{(1 - u_z)E} \right] \left[ 1 + u \cdot \Gamma(q) \right] E \frac{dN^{(0)}}{d^2 (p - q) dE} \right. \\
\left. - \left[ 1 - \hat{g}_\alpha \frac{(uE - p)\alpha z}{(1 - u_z)E} \right] [1 + u \cdot \Gamma_{DB}(q)] E \frac{dN^{(0)}}{d^2 p dE} \right. \\
\left. - \frac{1}{2} \hat{g}_\alpha \frac{\partial}{\partial p_{\alpha}} E \frac{dN^{(0)}}{d^2 p dE} \frac{\pi g^4}{2 \mu (1 - u_z)E} \frac{\delta^{(2)}(q)}{[v(q^2)]^2} \right\} \rho(z) [v(q^2)]^2
\]

Initial distribution

\[
E \frac{dN^0}{d^2pdE} \equiv \frac{1}{2(2\pi)^3} |J(E, p)|^2
\]
The final state parton distribution

\[ E \frac{dN}{d^2p\,dE} = E \frac{dN^{(0)}}{d^2p\,dE} + C \int_0^L dz \int_q \left\{ 1 - \hat{g}_\alpha \frac{(uE - p + q)_\alpha z}{(1 - u_z)E} \right\} \left[ 1 + u \cdot \Gamma(q) \right] E \frac{dN^{(0)}}{d^2(p - q)\,dE} \]

\[ - \left[ 1 - \hat{g}_\alpha \frac{(uE - p)_\alpha z}{(1 - u_z)E} \right] \left[ 1 + u \cdot \Gamma_{DB}(q) \right] E \frac{dN^{(0)}}{d^2p\,dE} \]

\[ \frac{1}{2} \hat{g}_\alpha \frac{\partial}{\partial p_\alpha} E \frac{dN^{(0)}}{d^2p\,dE} \pi g^4 \frac{\sqrt{1 - u^2 - u_z^2}}{2\mu(1 - u_z)E} \frac{\delta^{(2)}(q)}{[v(q^2)]^2} \rho(z) [v(q^2)]^2 \right\} \]

Initial distribution

\[ E \frac{dN^0}{d^2pdE} \equiv \frac{1}{2(2\pi)^3} |J(E, p)|^2 \]
The final state parton distribution

\[ \hat{g}_\alpha = \sum_g \left( \nabla_{\alpha} g \frac{\delta}{\delta g} \right) \]

\[
E \frac{dN}{d^2p \, dE} = E \frac{dN^{(0)}}{d^2p \, dE} + C \int_0^L \, dz \int_q \left\{ 1 - \frac{\hat{g}_\alpha}{(1 - u_z)E} \left( \frac{uE - p + q)_\alpha z}{2(1 - u_z)E} \right) \right\} \\
- \left[ 1 - \frac{\hat{g}_\alpha}{(1 - u_z)E} \right] \left[ 1 + u \cdot \Gamma(q) \right] E \frac{dN^{(0)}}{d^2p \, dE} \\
- \frac{1}{2} \hat{g}_\alpha \frac{\partial}{\partial p_\alpha} E \frac{dN^{(0)}}{d^2p \, dE} \frac{\pi g^4 \sqrt{1 - u^2 - u_z^2}}{2\mu(1 - u_z)E} \frac{\delta^{(2)}(q)}{[v(q^2)]^2} \right\} \rho(z) \left[ v(q^2) \right]^2
\]

Initial distribution

\[
E \frac{dN^0}{d^2pdE} \equiv \frac{1}{2(2\pi)^3} |J(E, p)|^2
\]
Moments of the final distribution

Definition of the moments

\[
\langle p_{\alpha_1} \cdots p_{\alpha_n} \rangle = \frac{\int p (p_{\alpha_1} \cdots p_{\alpha_n}) E \frac{dN}{d^2p \, dE}}{\int p E \frac{dN^{(0)}}{d^2p \, dE}}
\]

and

\[
E \frac{dN^{(0)}}{d^2p \, dE} = \frac{f(E)}{2\pi \omega^2} e^{-\frac{p^2}{2\omega^2}}
\]
Moments of the final distribution

Definition of the moments

\[
\langle \mathbf{p}_{\alpha_1} \cdots \mathbf{p}_{\alpha_n} \rangle = \frac{\int_{\mathbf{p}} \mathbf{p}_{\alpha_1} \cdots \mathbf{p}_{\alpha_n} \, E \frac{dN}{d^2p \, dE}}{\int_{\mathbf{p}} E \frac{dN}{d^2p \, dE}}
\]

and

\[
E \frac{dN(0)}{d^2p \, dE} = \frac{f(E)}{2\pi w^2} e^{-\frac{p^2}{2w^2}}
\]

- Leading odd moments

\[
\langle \mathbf{p}_{\alpha} \rangle = -\frac{1}{2} C \int_0^L dz \left[ 1 - z \hat{\mathbf{g}} \cdot \frac{\mathbf{u}}{1 - u_z} \right] \rho(z) \frac{\mathbf{u}_{\alpha}}{(1 - u_z)E} \int q^2 \left[ E \frac{f'(E)}{f(E)} + q^2 \frac{\partial}{\partial q^2} \right] [v(q^2)]^2
\]

\[
+ C \int_0^L dz \hat{g}_\alpha \frac{\pi g^4 \rho(z) \sqrt{1-u^2-u_z^2}}{4\mu(1-u_z)E}
\]

\[
\langle \mathbf{p}_{\alpha} \mathbf{p}^2 \rangle = C \int_0^L dz \int q \left\{ \hat{g}_\alpha 2w^2 \left[ z \frac{q^2}{(1-u_z)E} + \frac{\delta^{(2)}(q)}{[v(q^2)]^2} \frac{\pi g^4 \sqrt{1-u^2-u_z^2}}{2\mu(1-u_z)E} \right] 
\right.

\left. - \frac{1}{2} \left[ 1 - z \hat{\mathbf{g}} \cdot \frac{\mathbf{u}}{1 - u_z} \right] \frac{\mathbf{u}_{\alpha}}{(1-u_z)E}
\times q^2 \left[ 8w^2 + (10w^2 + q^2) q^2 \frac{\partial}{\partial q^2} + (4w^2 + q^2) E \frac{f'(E)}{f(E)} \right] \right\} \rho(z) [v(q^2)]^2
\]

- Along the flow

- Along the gradients
Moments of the final distribution

Definition of the moments

\[ \langle p_{\alpha_1} \cdots p_{\alpha_n} \rangle \equiv \frac{\int p (p_{\alpha_1} \cdots p_{\alpha_n}) E \frac{dN}{d^2p \, dE}}{\int p E \frac{dN^{(0)}}{d^2p \, dE}} \]

and

\[ E \frac{dN^{(0)}}{d^2p \, dE} = \frac{f(E)}{2\pi w^2} e^{-\frac{p^2}{2w^2}} \]

• Leading odd moments

Jets do feel flow and anisotropies
Moments of the final distribution

Definition of the moments

\[ \langle p_{\alpha_1} \cdots p_{\alpha_n} \rangle = \frac{\int_p (p_{\alpha_1} \cdots p_{\alpha_n}) E \frac{dN}{d^2p \, dE}}{\int_p E \frac{dN}{d^2p \, dE}} \]

and

\[ E \frac{dN^{(0)}}{d^2p \, dE} = \frac{f(E)}{2\pi w^2} e^{-\frac{p^2}{2w^2}} \]

- Leading odd moments

Agreement with previous results

Sadofyev et al. 2104.09513
Barata et al. 2202.08847
Andres et al. 2207.07141
Moments of the final distribution

Definition of the moments

\[
\langle p_{\alpha_1} \cdots p_{\alpha_n} \rangle \equiv \frac{\int_{p} (p_{\alpha_1} \cdots p_{\alpha_n}) \ E \ \frac{dN}{d^2 p \ dE}}{\int_{p} E \ \frac{dN^{(0)}}{d^2 p \ dE}} \quad \text{and} \quad E \ \frac{dN^{(0)}}{d^2 p \ dE} = \frac{f(E)}{2\pi w^2} e^{-\frac{p^2}{2w^2}}
\]

- Quadratic moment of the distribution

\[
\langle p^2 \rangle = 2w^2 + C \int_{0}^{L} dz \left[ 1 - z \ \hat{g} \cdot \frac{u}{1 - u_z} \right] \rho(z) \ \int_{q} q^2 [v(q^2)]^2
\]

\[
\frac{\delta}{\delta L} \int_{q} q^2 [v(q^2)]^2
\]

\[
\hat{q}(z) = \left[ 1 - z \ \hat{g} \cdot \frac{u}{1 - u_z} \right] C \rho(z) \ \int_{q} q^2 [v(q^2)]^2
\]
Moments of the final distribution

Definition of the moments

\[ \langle \mathbf{p}_{\alpha_1} \cdots \mathbf{p}_{\alpha_n} \rangle = \frac{\int_{\mathbf{p}} (\mathbf{p}_{\alpha_1} \cdots \mathbf{p}_{\alpha_n}) E \frac{dN}{d^2 p \, dE}}{\int_{\mathbf{p}} E \frac{dN^{(0)}}{d^2 p \, dE}} \]

and

\[ E \frac{dN^{(0)}}{d^2 p \, dE} = \frac{f(E)}{2\pi w^2} e^{-\frac{p^2}{2w^2}} \]

- Quadratic moment of the distribution

\[ \langle \mathbf{p}^2 \rangle = 2w^2 + C \int_0^L dz \left[ 1 - z \hat{\mathbf{g}} \cdot \frac{\mathbf{u}}{1 - u_z} \right] \rho(z) \int_q q^2 [v(q^2)]^2 \]

\[ \frac{\delta}{\delta L} \]

\[ \hat{q}(z) = \left[ 1 - z \hat{\mathbf{g}} \cdot \frac{\mathbf{u}}{1 - u_z} \right] \hat{q}_0(z) \]
Rough estimation of the effect

\[ \hat{q}(z) = \left[ 1 - z \hat{g} \cdot \frac{\mathbf{u}}{1 - u_z} \right] \hat{q}_0(z) \]

- Only gradients of temperature to leading logarithm
- Everything \( z \) - independent

\[ \hat{q}_L = \left[ 1 - \frac{L}{2} \frac{\nabla \rho}{\rho} \cdot \frac{\mathbf{u}}{1 - u_z} \right] \hat{q}_0 L \]

Chosen parameters

\[ L \simeq 5 \text{ fm} \]
\[ \left| \frac{\nabla T}{T^2} \right| \simeq 0.05 \]

\[ T \simeq 0.3 \text{ GeV} \]
\[ u \simeq 0.7 \] c about \( \frac{\pi}{4} \) to the z-axis
Rough estimation of the effect

\[ \hat{q}(z) = \left[ 1 - z \hat{g} \cdot \frac{u}{1 - u_z} \right] \hat{q}_0(z) \]

- Only gradients of temperature to leading logarithm
- Everything \( z \)-independent

\[ \hat{q}L = \left[ 1 - \frac{L}{2} \frac{3|\nabla T|T|u| \cos(\theta)}{1 - u_z} \right] \hat{q}_0L \]

Chosen parameters

\[ L \simeq 5 \text{ fm} \]
\[ \left| \frac{\nabla T}{T^2} \right| \simeq 0.05 \]

\[ T \simeq 0.3 \text{ GeV} \]
\[ u \simeq 0.7 \text{ c} \quad \text{about} \quad \frac{\pi}{4} \quad \text{to the } z\text{-axis} \]
Estimation of the effect

Rough estimation of the effect

\[ \hat{q}(z) = \left[ 1 - z \hat{g} \cdot \frac{u}{1 - u_z} \right] \hat{q}_0(z) \]

- Only gradients of temperature to leading logarithm
- Everything \( z \) - independent

\[ \hat{q} L = \left[ 1 - \frac{L}{2} \frac{3 |\nabla T| T |u| \cos(\theta)}{1 - u_z} \right] \hat{q}_0 L \]

Chosen parameters

\( L \simeq 5 \, \text{fm} \)

\( T \simeq 0.3 \, \text{GeV} \)

\( u \simeq 0.7 \, c \)

- Full dependence \( \rho \equiv \rho(T) \)
- Other gradients contribute in non trivial way
- \( z \)-dependence must be taken into account

\( u \) about \( \frac{\pi}{4} \) to the \( z \)-axis
Positivity of the jet quenching parameter

The jet quenching parameter is positive

\[ \left| z \hat{g} \cdot \frac{\mathbf{u}}{1 - u_z} \right| < 1 \quad \text{or gradient expansion brakes} \]

\[ \hat{q}(z) = \left[ 1 - z \hat{g} \cdot \frac{\mathbf{u}}{1 - u_z} \right] \hat{q}_0(z) \]

Keeping full dependence \( x \) on in a crude estimate

\[ \rho(x, z) \rightarrow \rho \left( -\frac{\mathbf{u}}{1 - u_z}, z \right) \approx \left[ 1 - z \nabla \rho \cdot \frac{\mathbf{u}}{\rho} \cdot \frac{1}{1 - u_z} \right] \rho(z) \]
Flowing anisotropic medium $\Rightarrow$ anisotropic broadening

Directional effects due to transverse gradient and flow

• Odd moments of the distribution are non-zero and along gradients and flow
  • (potentially lower background but smaller effect)

Novel multiplicative effect on even moments not energy suppressed

• The jet quenching parameter gets a multiplicative correction

$$\hat{q}(z) = \left[ 1 - z \hat{g} \cdot \frac{u}{1 - u_z} \right] \hat{q}_0(z)$$
Focus on leading perturbative processes: Two processes that modify jets.

- Single particle broadening

- Medium induced gluon radiation
Ressummed medium-induced radiation

Ressummed spectrum with transverse gradients

- Asymmetric medium-induced gluon spectrum

\[ \gamma_T = 0.05 \]

\[ \gamma_T = 0.01 \]

Ressummed spectrum with transverse flow

Coming soon
Ressummed spectrum with transverse gradients

• Asymmetric medium-induced gluon spectrum

\[ \gamma_T = 0.05 \]

Ressummed spectrum with transverse flow

Barata et al. 2304.03712

Coming soon
There are 9 possible diagrams

SB and DB diagrams add up to 12 different contributions
The final state parton distribution

\[ E \frac{dN^{(1)}}{d^2 k \, dx \, d^2 p \, dE} \equiv \frac{1}{[2(2\pi)^3]^2} \frac{1}{x(1-x)} \langle |\mathcal{R}_{N=1}|^2 \rangle \]

- Static matter with full gluon kinematics
  - Extending the previous result to hard gluon emissions
  - Without resummation of the interactions
  - Both agree on the correspondent limit
The final state parton distribution

\[ E \frac{dN^{(1)}}{d^2k \, dx \, d^2p \, dE} = \frac{1}{[2(2\pi)^3]^2} \frac{1}{x(1-x)} \langle |R_{N=1}|^2 \rangle \]

- Static matter with full gluon kinematics

Extending the previous result to hard gluon emissions

Without resummation of the interactions

Both agree on the correspondent limit

\[ E \frac{dN^{(1)}}{d^2k \, dx \, d^2p \, dE} = \frac{g^2 C_F}{2(2\pi)^3} x \left( E \frac{dN^{(0)}}{d^2p \, dE} \right) \int_0^L dz \int_q \rho(z) [v(q^2)]^2 \]

\[ \times \left\{ \frac{2 \hat{q} \cdot q}{k^2(k-q)^2} \left( 1 - \cos \left( \frac{(k-q)^2}{2xE} z \right) \right) \left( 1 + \frac{\hat{g} \cdot (k-q)}{2xE z} \right) - \frac{\hat{q} \cdot k}{k^2} \left[ \frac{z}{xE} - \frac{1}{k^2} \sin \left( \frac{k^2}{2xE^2} \right) \right] \right\} + \frac{k \cdot (k-q)}{k^2(k-q)^2} \left[ \hat{g} \cdot (k-q) \right] \left[ \frac{2}{xE} - \frac{k}{k \cdot (k-q)} \right] \sin \left( \frac{(k-q)^2}{2xE} z \right) \]
The final state parton distribution

\[ E \frac{dN^{(1)}}{d^2 k \, dx \, d^2 p \, dE} = \frac{1}{2(2\pi)^3} \frac{1}{x(1-x)} \langle |R_{N=1}|^2 \rangle \]

• Flow-gradient mixture effect

Leading correction to the spectrum

\[ \omega \frac{dI}{d^2 k \, d\omega} = \frac{g^2 C_F}{(2\pi)^2} \int_0^L dz \int_q \left[ 1 - \hat{g} \cdot u \, z \right] \frac{2 k \cdot q}{k^2 (k - q)^2} \left[ 1 - \cos \left( \frac{(k - q)^2}{2xE} z \right) \right] \rho(z) [v(q^2)]^2 \]

Multiplicative modification of the radiation rate \Rightarrow Modification of the induced energy loss
To take home

• Jets do feel the transverse flow and anisotropy, and get bended and distorted

• The transverse flow and anisotropy do affect the medium-induced radiation, modifying the jet substructure

• The interplay between flow and anisotropies modify the amount of quenching of a jet

• These effects can be probed in experiment, leading towards actual jet tomography
To take home

• These effects can be probed in experiment, leading towards actual jet tomography

They may be accessible with different substructure techniques, e.g. EECs

Barata et al. 2308.01294
Thanks