Eletroweak Parton Distribution Functions & Applications at High-Energy Muon Colliders

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INT WORKSHOP INT-22-83W
Parton Distributions and Nucleon Structure
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Based on work with T. Han and Y. Ma
2007.14300, 2103.09844, 2106.01393

Why muon colliders?

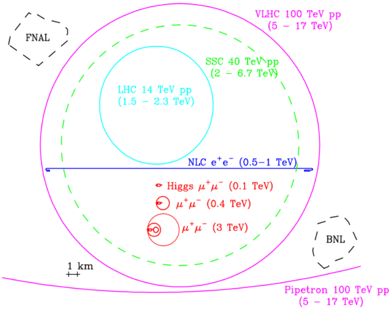
- **Leptons** are the ideal probes of short-distance physics
  - Cleaner background comparing to hadron colliders
  - High-energy physics probed with much smaller collider energy
- **Electron colliders**
  - A glorious past: discovery of charm, τ, and gluon
  - Important future: Precision EW constraints on BSM physicss, Higgs physics
- **Muon colliders**
  - A s-channel Higgs factory: Higgs production enhanced by $m_\mu^2/m_e^2 \sim 40000$
    - Direct measurements on y_μ and Γ_H
  - Multi-TeV muon colliders: Less radiations then electron
    - Center of mass energy $3-15$ TeV and the more speculative $E_{\rm cm} = 30$ TeV
    - New particle mass coverage $M \sim (0.5-1)E_{\rm cm}$
    - Great accuracies for WWH, $WWHH$, H^3, H^4
    - [See Snowmass WPs, 2203.08033, 2203.07964, Report 2209.01318.]

Muon Collider Physics Potential Pillars

Direct search of heavy particles	High rate indirect probes	High energy probes
SUSY-inspired, WIMP, VBF production, 2->1	Higgs single and self-couplings, rare Higgs decays, exotic decays	difermion, diboson, EFT, Higgs compositeness

A possible high-energy muon collider

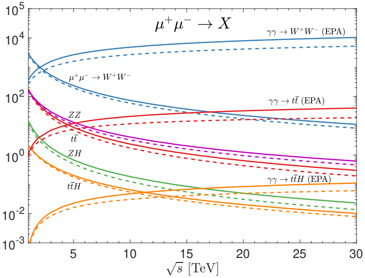
Size and Benchmarks [Ankenbrandt et al., arXiv:physics/9901022]

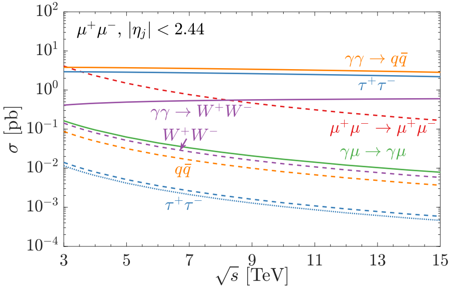


Integrated luminosity: $\mathscr{L} = (E_{\rm cm}/10~{\rm TeV})^2 \times 10~{\rm ab}^{-1}$ [The Muon Smasher's Guide, 2103.14043]

$\sqrt{s}$ [TeV]	1	3	6	10	14	30	50	100
$\mathcal{L}_{\rm int}^{\rm opt}$ [ab^{-1}]	0.2	1	4	10	20	90	250	1000
$\mathcal{L}_{\rm int}^{\rm con}$ [ab^{-1}]	0.2	1	4	10	10	10	10	10

Vector boson fusions vs. annihilations





[Han, Ma, KX, 2007.14300] [Han, Ma, KX, 2103.09844]

General features:

- The annihilations decrease as $1/s$.
- ISR needs to be considered, which can give over 10% enhancement.
- The fusions increase as $\ln(s)$, which take over at high energies.
- The large collinear logarithm $\ln\left(Q^2/m_\ell^2\right)$ needs to be resummed.
- W^+W^- as a reference to separate high-energy EW and low-energy QED/QCD

Q: How to treat parton properly at high energies when W/Z become active?

EW physics at high energies

- At high energies, every particle become massless

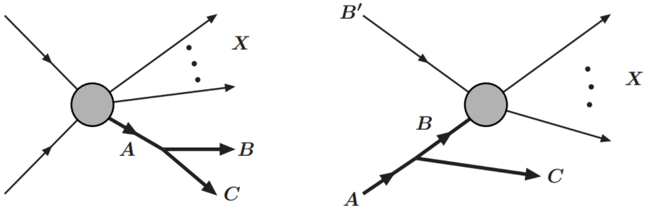
$$\frac{v}{E} : \frac{v}{100\ \text{TeV}} \sim \frac{\Lambda_{\text{QCD}}}{100\ \text{GeV}},\ \frac{v}{E}, \frac{m_t}{E}, \frac{M_W}{E} \to 0!$$

- The splitting phenomena dominate due to large log enhancement
- The EW symmetry is restored: $SU(2)_L \times U(1)_Y$ unbroken
- Goldstone Boson Equivalence:

$$\varepsilon_L^\mu(k) = \frac{E}{M_W}(\beta_W, \hat{k}) \simeq \frac{k^\mu}{M_W} + \mathscr{O}(\frac{M_W}{E})$$

  The violation terms is power counted as $v/E \to$ QCD higher twist effects Λ_{QCD}/Q [Cuomo, Wulzer, 1703.08562; 1911.12366].
- We mainly focus on the splitting phenomena, which can be factorized and resummed as the EW PDFs in the ISR, and the Fragementaions/Parton Shower in the FRS.
- Other interesting aspects: the polarized EW boson scattering, top-Yukawa coupling effect

Factorization of the EW splittings



$$\mathrm{d}\sigma \simeq \mathrm{d}\sigma_X \times \mathrm{d}\mathscr{P}_{A\to B+C}\,, \quad E_B \approx zE_A, \quad E_C \approx \bar{z}E_A, \quad k_T \approx z\bar{z}E_A\theta_{BC}$$

$$\frac{\mathrm{d}\mathscr{P}_{A\to B+C}}{\mathrm{d}z\mathrm{d}k_T^2} \simeq \frac{1}{16\pi^2}\frac{z\bar{z}|\mathscr{M}^{(\mathrm{split})}|^2}{(k_T^2+\bar{z}m_B^2+zm_C^2-z\bar{z}m_A^2)^2}, \quad \bar{z}=1-z$$

- The dimensional counting: $|\mathscr{M}^{(\mathrm{split})}|^2 \sim k_T^2$ or m^2
- To validate the fractorization formalism
  - The observable σ should be infra-red safe
  - Leading behavior comes from the collinear splitting

[Ciafaloni et al., hep-ph/0004071; 0007096; Bauer, Webber et al., 1703.08562; 1808.08831]

[Manohar et al., 1803.06347; Han, Chen, Tweedie, 1611.00788]

PDFs and Fragmentations (parton showers)

- Initial state radiation (ISR): PDFs [Bauer et al., 1703.08562; 1808.08831, Manohar et al., 1808.08831, Han, Ma, KX, 2007.14300],

$$f_B(z,\mu^2) = \sum_A \int_z^1 \frac{\mathrm{d}\xi}{\xi} f_A(\xi) \int_{m^2}^{\mu^2} \mathrm{d}\mathscr{P}_{A\to B+C}(z/\xi, k_T^2)$$

$$\frac{\mathrm{d}f_B(z,\mu^2)}{\mathrm{d}\mu^2} = \sum_A \int_z^1 \frac{\mathrm{d}\xi}{\xi} \frac{\mathrm{d}\mathscr{P}_{A\to B+C}(z/\xi,\mu^2)}{\mathrm{d}z\mathrm{d}k_T^2} f_A(\xi,\mu^2)$$

  - The leading order splitting gives the effective W approximation (EWA) [Kane, Repko, Rolnick, PLB1984, Dawson, NPB1985, Chanowitz, Gaillard, NPB1985]
  - $W_L(Z_L)$ capture the remnants of EWSB, governed by power correction $\mathscr{O}(M_Z^2/Q^2)$ to the Goldstone Equivalence.
- Final state radiation (FSR): Fragmentations [Bauer et al., 1806.10157; Han, Ma, KX, 2203.11129] or parton showers [Han et al., 1611.00788]

$$\Delta_A(t) = \exp\left[-\sum_B \int_{t_0}^t \mathrm{d}t' \int \mathrm{d}z \frac{\mathrm{d}\mathscr{P}_{A\to B+C}(z,t')}{\mathrm{d}z\mathrm{d}t'}\right]$$

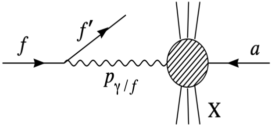
Parton inside of a lepton

Equivalent photon approximation (EPA) [Fermi, Z. Phys. 29, 315 (1924), von Weizsacker, Z. Phys. 88, 612 (1934)]

Treat photon as a parton constituent in the lepton [Williams, Phys. Rev. 45, 729 (1934)]

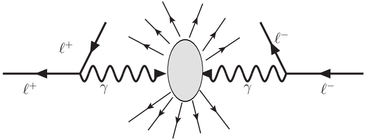
$$\sigma(\ell^- + a \to \ell^- + X) = \int \mathrm{d}x\, f_{\gamma/\ell}\hat{\sigma}(\gamma a \to X)$$

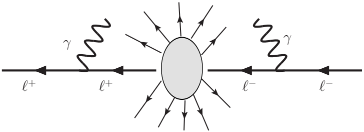
$$f_{\gamma/\ell,\mathrm{EPA}}(x_\gamma, Q^2) = \frac{\alpha}{2\pi}\frac{1+(1-x_\gamma)^2}{x_\gamma}\ln\frac{Q^2}{m_\ell^2}$$



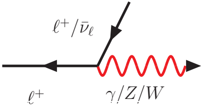
Extra terms to Improve: [Budnev, Ginzburg, Meledin, Serbo, Phys. Rept. (1975)], [Frixione, Mangano, Nason, Ridolfi, 9310350]

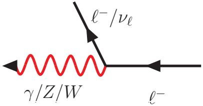
Photon fusions and annihilations with initial state radiations





Effective W approximation (EWA) [Kane, Repko, Rolnick, PLB1984, Dawson, NPB1985, Chanowitz, Gaillard, NPB1985]





The novel features of the EWA

- The EW PDFs must be polarized due to the chiral nature of the EW theory

$$f_{V_+/A_+} \neq f_{V_-/A_-}, \qquad f_{V_+/A_-} \neq f_{V_-/A_+},$$
$$\hat{\sigma}(V_+B_+) \neq \hat{\sigma}(V_-B_-), \quad \hat{\sigma}(V_+B_-) \neq \hat{\sigma}(V_-B_+)$$

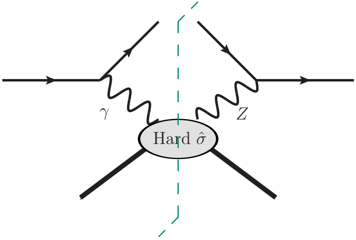
We are not able to factorize the cross sections in an unporlarized form.

$$\sigma \neq f_{V/A}\hat{\sigma}(VB),\ f_{V/A} = \frac{1}{2}\sum_{\lambda,s_1} f_{V_\lambda/A_{s_1}},\ \hat{\sigma}(VB) = \frac{1}{4}\sum_{\lambda,s_2}\hat{\sigma}(V_\lambda B_{s_2})$$

- The interference gives the mixed PDFs

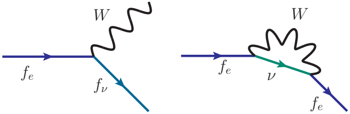
$$f_{\gamma Z} \sim \langle\Omega|A^{\mu\nu}Z_{\mu\nu}|\Omega\rangle + \text{h.c.},$$

similarly for f_{hZ_L}.



- Bloch-Nordsieck theorem violation due to the non-cancelled divergence in $f \to f'V$: cutoff M_V/Q [Han, Ma, KX, 2007.14300] or fully inclusive observables [Fornal, Manohar, Waalewijn, 1802.08687]

$$f_1 = \frac{1}{2}(f_\nu + f_e) \sim \frac{\alpha_W}{2\pi}\log,$$
$$f_3 = \frac{1}{2}(f_\nu - f_e) \sim \frac{\alpha_W}{2\pi}\log^2.$$

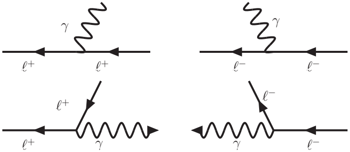


Go beyond the EPA/EWA

We have been doing:

- $\ell^+\ell^-$ annihilation

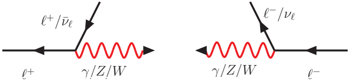
- EPA and ISR



- "Effective W Approx." (EWA)

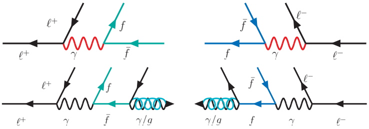
[Kane, Repko, Rolnick, PLB 148 (1984) 367]

[Dawson, NPB 249 (1985) 42]

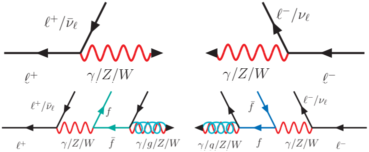


We complete:

- Above $\mu_{\rm QCD}$: QED$\otimes$QCD
  q, g become active [Han, Ma, KX, 2103.09844]



- Above $\mu_{\rm EW} = M_Z$: EW$\otimes$QCD
  EW partons emerge [Han, Ma, KX, 2007.14300]



In the end, every content is a parton, i.e. the full SM PDFs.

The PDF evolution: DGLAP

- The DGLAP equations

$$\frac{\mathrm{d}f_i}{\mathrm{d}\log Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P_{ij}^I \otimes f_j$$

- The initial conditions

$$f_{\ell/\ell}(x, m_\ell^2) = \delta(1-x)$$

- Three regions and two matchings
  - $m_\ell < Q < \mu_{\rm QCD}$: QED
  - $Q = \mu_{\rm QCD} \lesssim 1$ GeV: $f_q \propto P_{q\gamma} \otimes f_\gamma, f_g = 0$ [Simplified. Non-pert. parameterization.]
  - $\mu_{\rm QCD} < Q < \mu_{\rm EW}$: QED$\otimes$QCD
  - $Q = \mu_{\rm EW} = M_Z$: $f_\nu = f_t = f_W = f_Z = f_{\gamma Z} = 0$
  - $Q > \mu_{\rm EW}$: EW$\otimes$QCD.

$$\begin{pmatrix} f_B \\ f_{W^3} \\ f_{BW^3} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & -2c_W s_W \\ s_W^2 & c_W^2 & 2c_W s_W \\ c_W s_W & -c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_\gamma \\ f_Z \\ f_{\gamma Z} \end{pmatrix}$$

- We work in the (B, W) basis.
- Double logs are retained through [Bauer, Ferland, Webber, 1703.08562.]

$$f_3 = \frac{\alpha_W}{2\pi} \log \int_x^{1-M_V/Q} \mathrm{d}z P_{ff} \otimes (f_\nu - f_e) \sim \frac{\alpha_W}{2\pi} \log^2$$

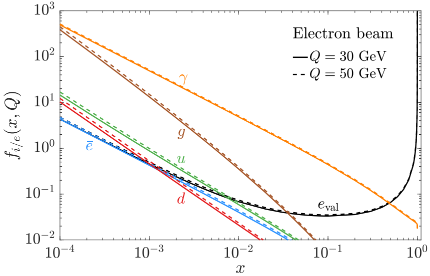
Same physics as the Rapidity RGE [Manohar, Waalewijn 1802.08687]

The QED⊗QCD PDFs for lepton colliders

Electron beam:

- Scale unc. 10% for $f_{g/e}$ [2103.09844]
- $\mu_{\rm QCD}$ unc. 15%
- The averaged momentum fractions $\langle x_i \rangle = \int x f_i(x) \mathrm{d}x$ [%]

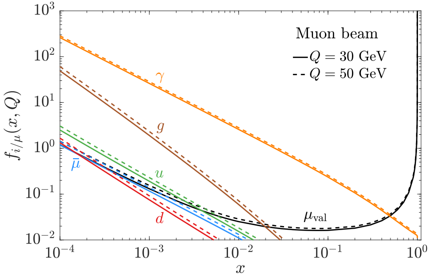
$Q(e^{\pm})$	$e_{\rm val}$	γ	ℓsea	q	g
30 GeV	96.6	3.20	0.069	0.080	0.023
50 GeV	96.5	3.34	0.077	0.087	0.026
M_Z	96.3	3.51	0.085	0.097	0.028



Muon beam:

- Scale unc. 20% for $f_{g/\mu}$ [2103.09844]
- $\mu_{\rm QCD}$ unc. 5% [2106.01393]

$Q(\mu^{\pm})$	$\mu_{\rm val}$	γ	ℓsea	q	g
30 GeV	98.2	1.72	0.019	0.024	0.0043
50 GeV	98.0	1.87	0.023	0.029	0.0051
M_Z	97.9	2.06	0.028	0.035	0.0062

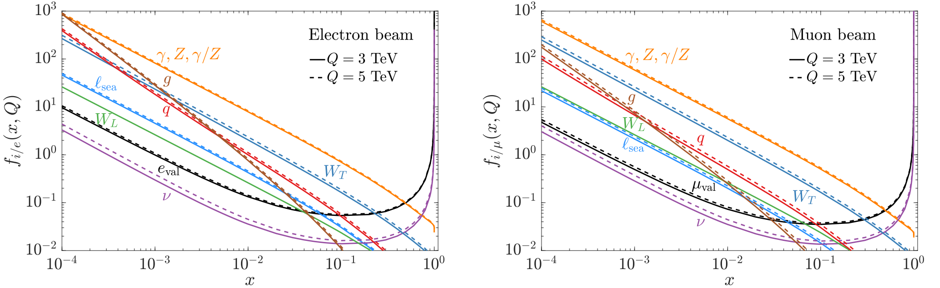


EWPDFs of a lepton

- The sea leptonic and quark PDFs

$$\nu = \sum_i (\nu_i + \bar{\nu}_i), \ \ell\text{sea} = \bar{\ell}_{\text{val}} + \sum_{i \neq \ell_{\text{val}}} (\ell_i + \bar{\ell}_i), \ q = \sum_{i=d}^{t} (q_i + \bar{q}_i)$$

Even neutrino becomes active.



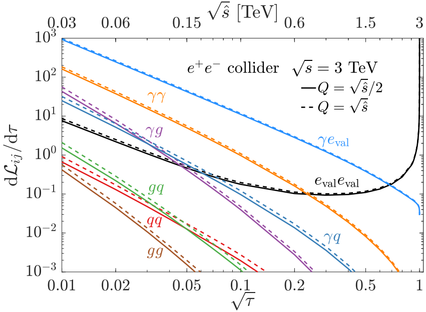
- **All SM particles are partons** [Han, Ma, KX, 2007.14300]
- $W_L(Z_L)$ does not evolve: **Bjorken-scaling restoration**: $f_{W_L}(x) = \frac{\alpha_2}{4\pi}\frac{1-x}{x}$.
- The EW correction can be large: $\sim 50\%$ (100%) for $f_{d/e}$ $(f_{d/\mu})$ due to the relatively **large SU(2) gauge coupling**. [Han, Ma, KX et. al, 2106.01393]
- Scale uncertainty: $\sim 15\%$ (20%) between $Q = 3$ TeV and $Q = 5$ TeV

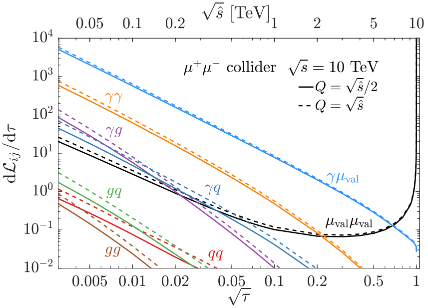
Parton luminosities at high-energy lepton colliders

Consider a 3 TeV e^+e^- machine and a 10 TeV $\mu^+\mu^-$ machine

- Partonic luminosities for

$$\ell^+\ell^-, \gamma\ell, \gamma\gamma, qq, \gamma q, \gamma g, gq, \text{ and } gg$$

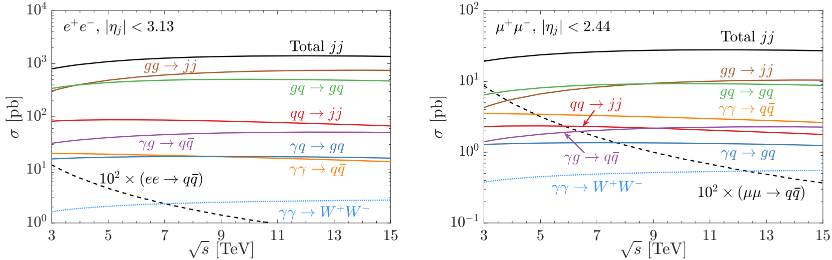




- The partonic luminosity of $\gamma g + \gamma q$ is $\sim 50\%$ (20%) of the $\gamma\gamma$ one
- The partonic luminosities of qq, gq, and gg are $\sim 2\%$ (0.5%) of the $\gamma\gamma$ one
- Given the large strong coupling, **sizable QCD cross sections are expected**.
- Scale unc. are $\sim 20\%$ (50%) for photon (gluon) luminosities

Di-jet production at possible lepton colliders

- Low-p_T range is dominated by non-perturbative hadron production [backup for details]
  High-p_T range $p_T > (4 + \sqrt{s}/3\,\mathrm{TeV})$ GeV: perturbatively computable
- Threshold cut: $\hat{s} > 20$ GeV
- Detector angle: $\theta_{\mathrm{cut}} = 5^\circ(10^\circ) \Leftrightarrow |\eta| < 3.13(2.44)$



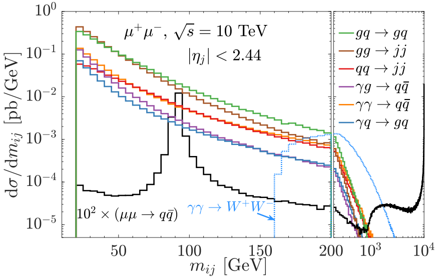
- Including the QCD contribution leads to much larger total cross section.
- gg initiated cross sections are large for its large multiplicity
- gq initiated cross sections are large for its large luminosity.
- $\gamma\gamma$ initiated cross sections are slightly smaller than the EPA estimations.
- scale variation $Q : \sqrt{\hat{s}}/2 \to \sqrt{\hat{s}}$ brings a $6\% \sim 15\%$ $(30\% \sim 40\%)$ enhancement

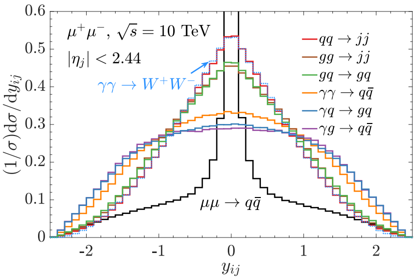
Kinematic distributions

A conservative acceptance cut: $10^\circ < \theta < 170^\circ \Leftrightarrow |\eta| < 2.44$

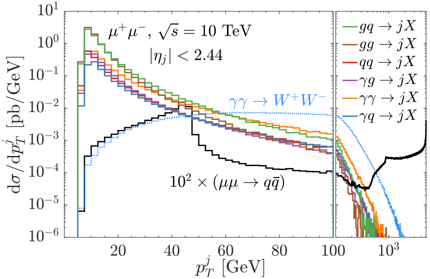
Two different mechanisms: $\mu^+\mu^-$ **annihilation** v.s. **fusion processes**

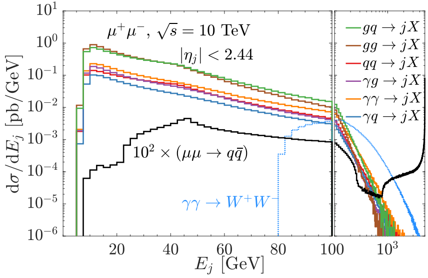
- Annihilation is more than 2 orders of magnitude smaller than fusion process.
- Annihilation peaks at $m_{ij} \sim \sqrt{s}$;
- Fusion processes peak near m_{ij} threshold.
- Annihilation sharply peaked around $y_{ij} \sim 0$, spread out due to ISR;
- Fusion processes spread out, especially for γq and γg initiated ones.

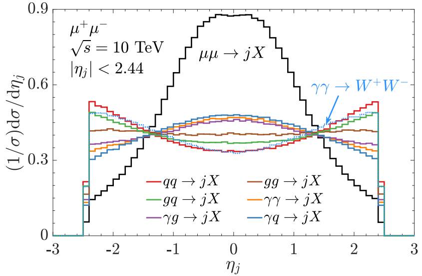




Inclusive jet distributions at a muon collider







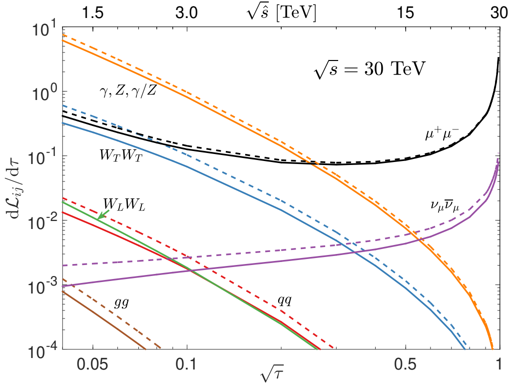
- Jet production dominates over WW production until $p_T \gtrsim 60$ GeV or $E_j \gtrsim 200$ GeV.
- QCD contributions are mostly forward-backward; $\gamma\gamma$, γq, and γg initiated processes are more isotropic.

A high-energy muon collider

- All SM particles are partons at high energies: $\langle x_i \rangle = \int x f_i(x) \mathrm{d}x$ [%]

Q	$\mu_{\rm val}$	$\gamma, Z, \gamma Z$	$W^\pm$	ν	ℓsea	q	g
M_Z	97.9	2.06	0	0	0.028	0.035	0.0062
3 TeV	91.5	3.61	1.10	3.59	0.069	0.13	0.019
5 TeV	89.9	3.82	1.24	4.82	0.077	0.16	0.022

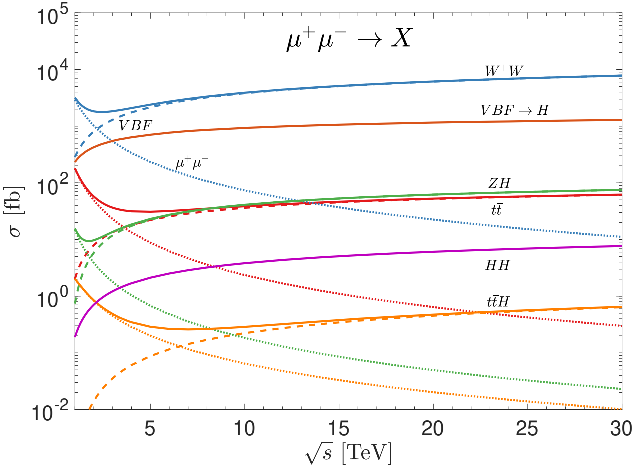
- We need polarized PDFs due to the chiral nature of EW theory
- The EW parton luminosities [Han, Ma, KX, 2007.14300]



EW semi-inclusive processes

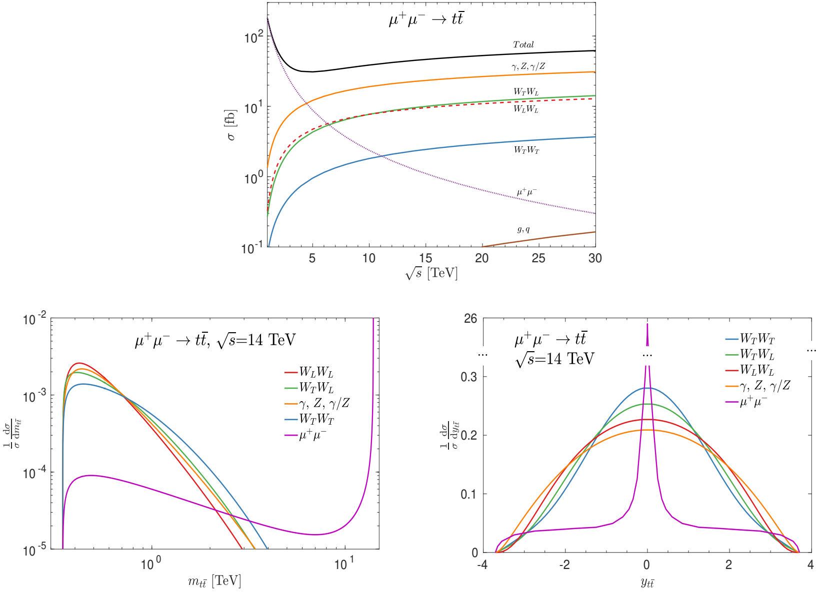
Just like in hadronic collisions:

$\mu^+\mu^- \to$ exclusive particles + remnants



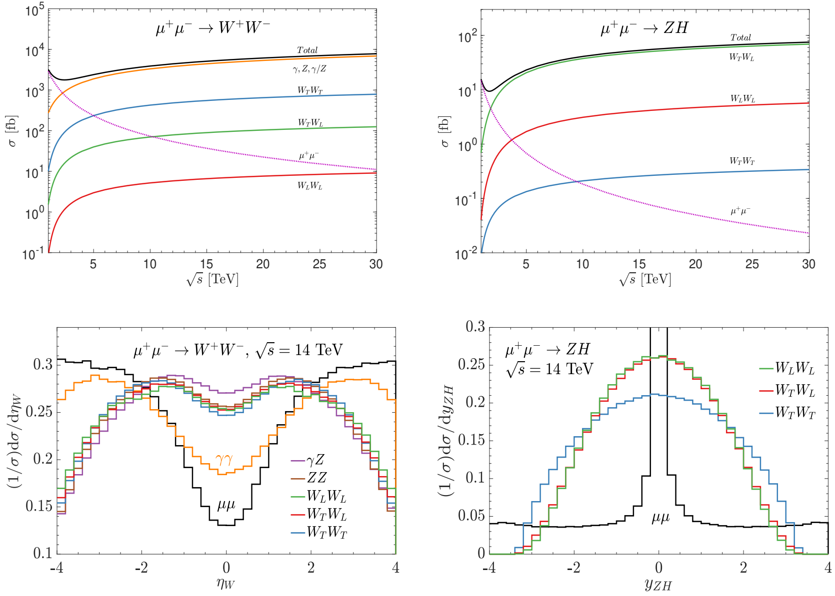
[Han, Ma, KX, 2007.14300]

$t\bar{t}$ production at a muon collider



[Han, Ma, KX, 2007.14300]

W^+W^-, ZH production



[Han, Ma, KX et al., 2106.01393]

Summary and prospects

- A high-energy muon collider is a dream machine for new physics search, both for energy and precision frontiers
- **The parton picture play an important role**
  - At very high energies, the collinear splittings dominate. **All SM particles should be treated as partons that described by EW PDFs.**
  - The large collinear logarithm needs to be resummed via solving the DGLAP equations, so the **QCD partons (quarks and gluons) emerge**.
  - When $Q > \mu_{\rm EW}$, the EW splittings are activated: the EW partons appear, and the existing QED$\otimes$QCD PDFs may receive big corrections.

A high-energy muon collider is an EW version HE LHC

  - Two classes of processes: $\mu^+\mu^-$ annihilation v.s. VBF [Han, Ma, KX, 2007.14300]
  - Quark and gluon initiated jet production dominates [Han, Ma, KX, 2103.09844]
  - EW PDFs are essential for high-energy muon colliders [Han, Ma, KX, 2007.14300, 2106.01393]

$\gamma\gamma \rightarrow$hadrons at CLIC

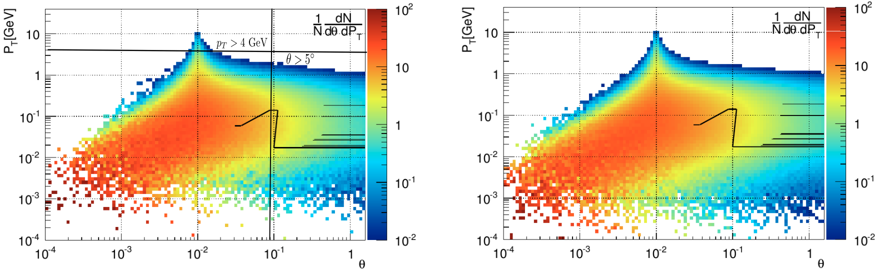
- Large photon induced non-perturbative hadronic production

  [Drees and Godbole, PRL 67 (1991) 1189, hep-ph/9203219]

  [Chen, Barklow, and Peskin, hep-ph/9305247; Godbole et al., Nuovo Cim. C 034S1 (2011)]

  - $\sigma_{\gamma\gamma\rightarrow \text{ hadrons}}$ may reach micro-barns level at TeV c.m. energies
  - $\sigma_{\ell\ell\rightarrow \text{ hadrons}}$ may reach nano-barns, after folding in the $\gamma\gamma$ luminosity

- The events populate at low p_T regime
  So we can separate from this non-perturbative range via a p_T cut.



(a) Pythia sample

(b) SLAC sample

[T. Barklow, D. Dannheim, M. O. Sahin, and D. Schulte, LCD-2011-020]