Shell-model calculation of the radial overlap correction to superallowed $0^+ \to 0^+$ nuclear β decays

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Outline

Introduction

- β decay is a low-energy process, then the 4-fermion contact interaction should be valid
- General form of weak Hamiltonian for β decay

$$\begin{aligned}\mathcal{H}_\beta &= (\bar{\psi}_p\psi_n)[\bar{\psi}_e(C_S + C'_S\gamma_5)\psi_\nu] \\ &+(\bar{\psi}_p\gamma_\mu\psi_n)[\bar{\psi}_e\gamma_\mu(C_V + C'_V\gamma_5)\psi_\nu] \\ &+\tfrac{1}{2}(\bar{\psi}_p\sigma_{\lambda\mu}\psi_n)[\bar{\psi}_e\sigma^{\lambda\mu}(C_T + C'_T\gamma_5)\psi_\nu] \\ &-(\bar{\psi}_p\gamma_\mu\gamma_5\psi_n)[\bar{\psi}_e\gamma^\mu\gamma_5(C_A + C'_A\gamma_5)\psi_\nu] \\ &+(\bar{\psi}_p\gamma_5\psi_n)[\bar{\psi}_e\gamma_5(C_P + C'_P\gamma_5)\psi_\nu] \\ &+h.c.\end{aligned}$$

where C_i and C'_i must be experimentally determined

Introduction

- Standard model :

$$
\begin{array}{ll}
\Im(C_i) = 0,\ \Im(C_i') = 0, & \mathcal{T} \text{ symmetry} \\
\Im(C_i) = 0,\ \Re(C_i') = 0, & \mathcal{C} \text{ symmetry} \\
C_i = C_i', & \text{Maximum } \mathcal{P} \text{ violation} \\
C_V = 1,\ C_S = 0, & \text{CVC} \\
C_A \approx -1.276, & \text{Free neutron decay} \\
C_S = C_T = C_P = 0, & V - A \text{ theory}
\end{array}
$$

These properties must be experimentally tested !

- Superallowed $0^+ \to 0^+$ Fermi β decay : $|C_S/C_V| \leq 0.0017$ and $|V_{ud}| = 0.97373(31)$

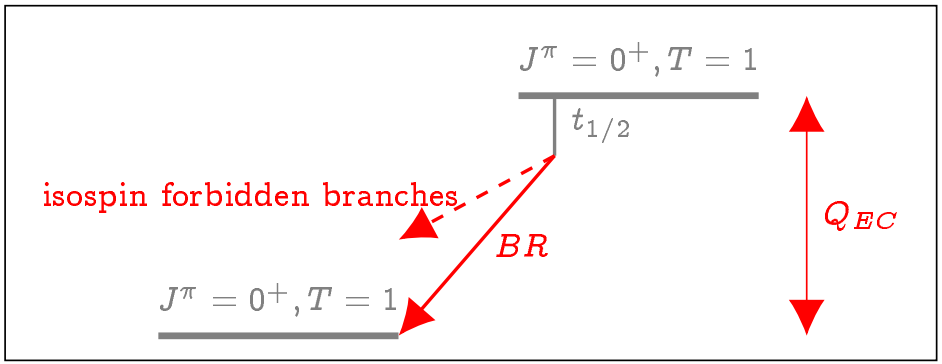
Hardy & Towner, PRC 102, 045501 (2020)

Introduction

- The superallowed $0^+ \rightarrow 0^+$ Fermi β decay of $T = 1$ nuclei,

$$Ft = ft(1 + \delta'_R)(1 - \delta_C + \delta_{NS}) = \frac{K}{2G_F^2 V_{ud}^2 (1 + \Delta_R)}$$

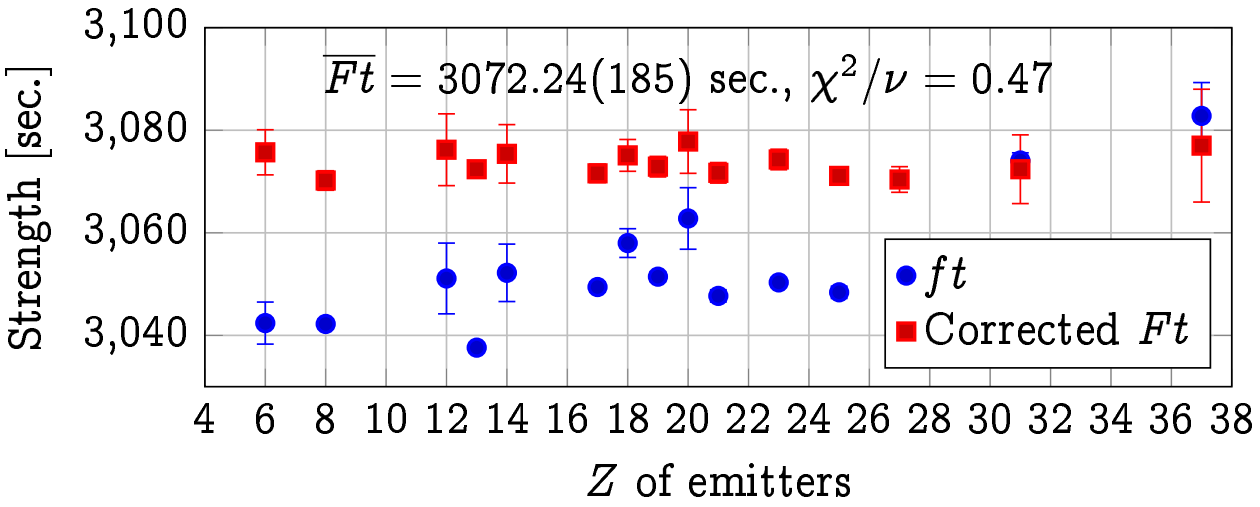
where $K = 2\pi^3 \hbar^7 c^6 \ln(2)/(m_e c^2)^5$, ft=statistical rate function $\times$ partial half life, δ_C=isospin-symmetry breaking (ISB) correction, (δ'_R, δ_{NS}, Δ_R)=radiative corrections



15 cases: ^{10}C, ^{14}O, ^{22}Mg, ^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, ^{74}Rb

Introduction

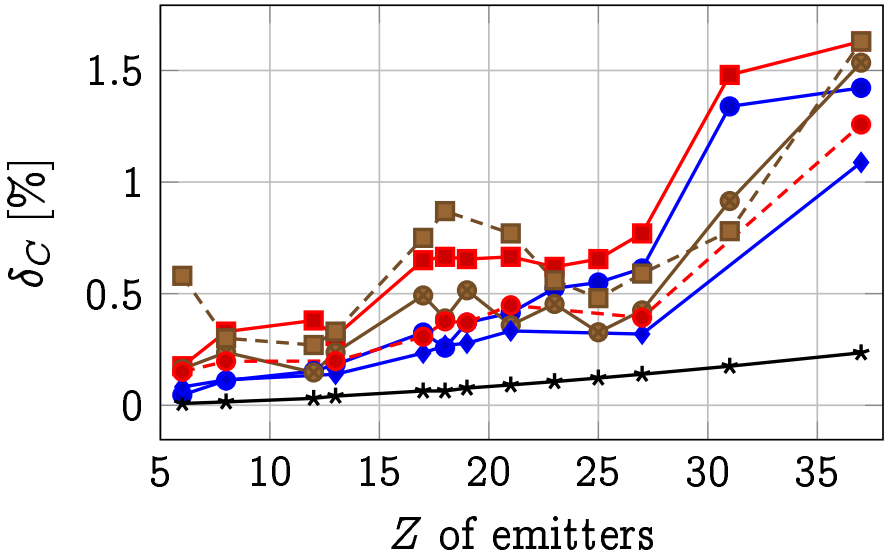
- Current status of the superallowed $0^+ \rightarrow 0^+$ studies [*Hardy & Towner, PRC 102, 045501 (2020)*]



The extracted $|V_{ud}| = 0.97373(31)$, disagrees with the CKM unitarity by $\sim 2\sigma$

δ_C correction

- Existing calculations



- Damgaard (1969)
- SM-WS (2020)
- SM-HF (1985)
- IVMR (2009)
- RHF-RPA (2009)
- RH-RPA (2009)
- JT-proj. DFT (2012)

The shell model is the best Standard-Model supporter. However, there remains a considerable discrepancy between SM-WS and SM-HF.

Hardy & Towner, PRC 102, 045501 (2020)
Ormand & Brown, NPA 440,274 (1985)

Aims

- Search for the origins of discrepancies between WS and HF radial wave functions:
  - Updated fitting procedure of WS potential parameters
  - use modern Skyrme functionals (inclusion of CIB, CSB, treat the Coulomb exchange with the GGA, ..., suppress spurious effect)
- Enlarge configuration spaces, e.i., ^{50}Mn, ^{54}Co, ^{62}Ga in full pf-shell
- Investigate the higher order terms
- Use the Standard Model as a filter

Damgaard model

- Damgaard evaluated δ_C for superallowed $0^+ \to 0^+$ Fermi β decay within the harmonic oscillator potential

$$U^q(r) = \frac{1}{2}m\omega^2 r^2 + \delta_{qp}\frac{Ze^2}{2R_C}\left(3 - \frac{r^2}{R_C^2}\right)$$

the Coulomb term causes an isospin mixing,

$$u^{proton}(r) = (1-\alpha^2)^{\frac{1}{2}}u_{n,l}(r) + \alpha u_{n+1,l}(r) + ...$$

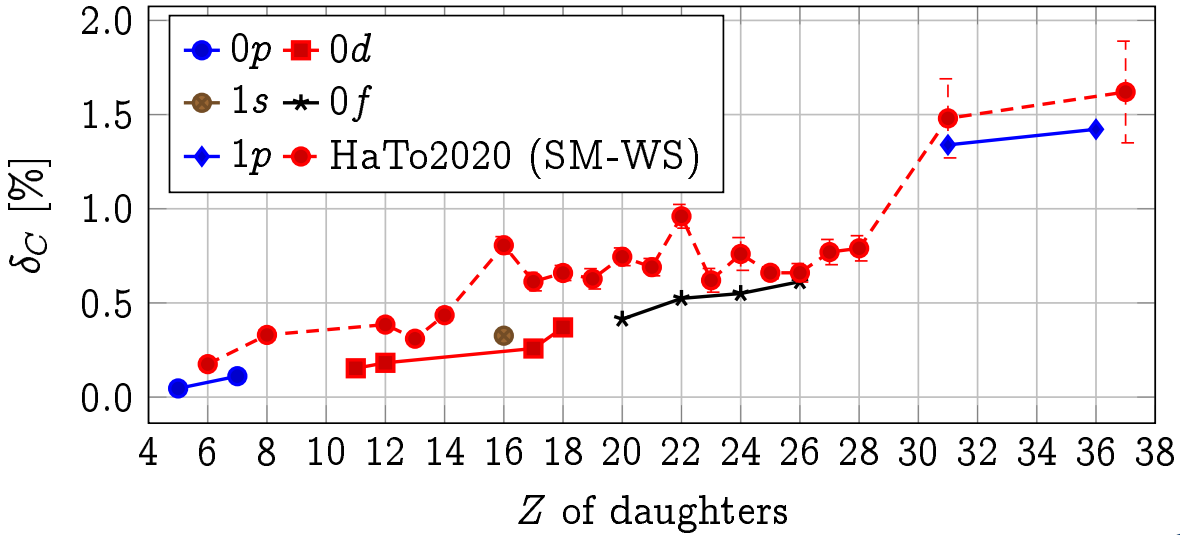
Using the first-order perturbation theory, he derived

$$\begin{aligned} \alpha &= \frac{\langle u_{n+1,l}|V_C|u_{n,l}\rangle}{2\hbar\omega} = -\frac{Ze^2b^2}{4R_C^3\hbar\omega}\sqrt{(n+1)(n+l+3/2)} \\ \delta_C \approx \alpha^2 &= \frac{Z^2e^4b^4}{16R_C^6(\hbar\omega)^2}(n+1)(n+l+3/2) \end{aligned}$$

Damgaard, NPA 130, 233 (1969)

Damgaard model

- Damgaard model describes the characteristic features of δ_C
  - Mass dependence: $\delta_C \propto A^{\frac{4}{3}}$ (for $N \approx Z$)
  - Shell structure effect: $\delta_C \propto (n+1)(n+l+\frac{3}{2})$

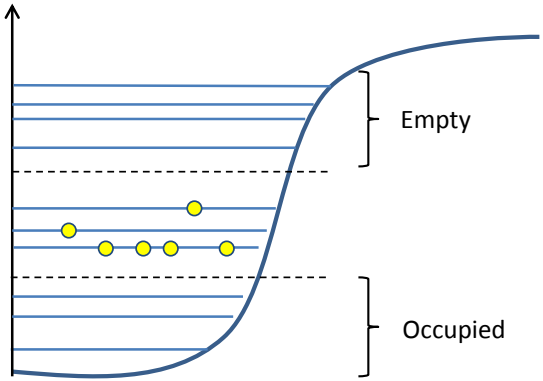


This indicates that δ_C is dominated by the radial mismatch effect !

Shell-model formalism

- Inclusion of residual interaction

$$H = \sum_{i=1}^{A} \left(\frac{\boldsymbol{p}_i^2}{2m} + U(\boldsymbol{r}_i) \right) + \sum_{i<j=1}^{A} V(\boldsymbol{r}_i - \boldsymbol{r}_j) - \sum_{i=1}^{A} U(\boldsymbol{r}_i)$$



Solution of the eigenproblem by diagonalization of the Hamiltonian matrix:

- $\Psi_p = \sum_k c_{kp} \Phi_k$
- $\sum_{k=1}^{n} H_{lk} c_{kp} = E_p c_{lp}$
- Input: ϵ_j, $\langle ij|V_{res}|kl\rangle_{JT}$

Microscopic: MPBT, IMSRG, Coupled-Cluster, OLS NCSM, etc.
Phenomenological: USD (sd-shell), KB3G, GXPF1A (pf-shell), => excellent spectroscopy

Shell-model formalism

- Fermi matrix element in realistic basis consists of a nodal mixing part

$$M_F = \sum_{k_a} \langle f||a^\dagger_{k_a n} a_{k_a p}||i\rangle \langle k_{an}||\tau_+||k_{ap}\rangle + \sum_{k_a k_b}^{n_a \neq n_b} \langle f||a^\dagger_{k_a n} a_{k_b p}||i\rangle \langle k_{an}||\tau_+||k_{bp}\rangle$$

  where $k = nlj$.

- The n mixing part is too heavy for the shell model. However, its exclusion has been criticized by Miller-Schwenk [*PRC 80, 064319 (2009), PRC 78, 035501 (2008)*]

- For the present work, we stay in the conventional framework of Towner-Hardy and Ormand-Brown

Hardy & Towner, PRC 102, 045501 (2020)
Ormand & Brown, NPA 440,274 (1985)

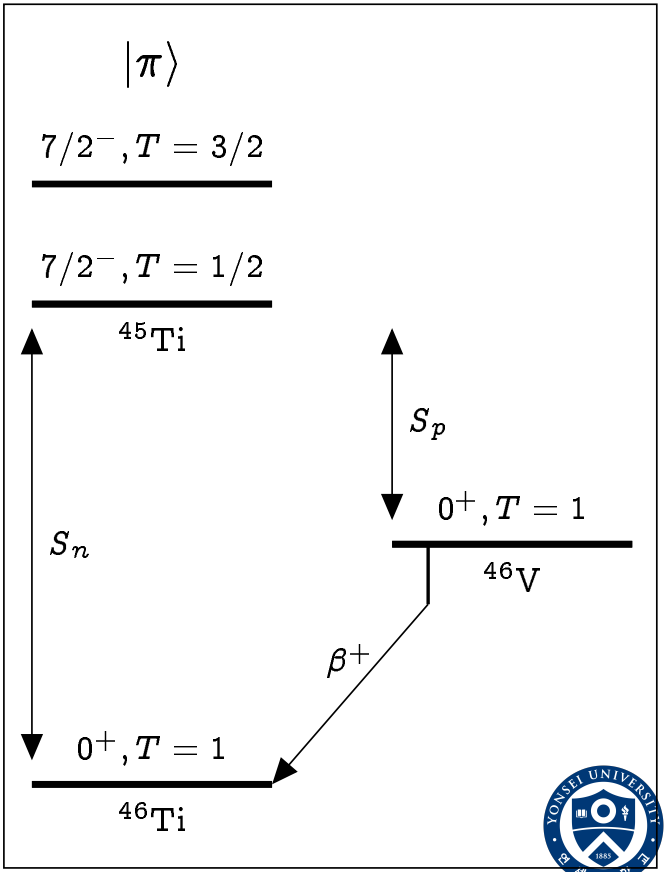
Shell-model formalism

- Following Towner-Hardy [*PRC 102, 045501 (2020)*], we expand M_F in terms of intermediate states

$$M_F \approx \sum_{k\pi} \langle f||a^\dagger_{kn}||\pi\rangle \langle \pi||a_{kp}||i\rangle \times \int_0^\infty u^\pi_{kn}(r) u^\pi_{kp}(r) dr,$$

  so that the radial wfs can be evaluated with excitation energy of the intermediate nucleus

- This leads to a considerable and systematic improvement



Shell-model calculation

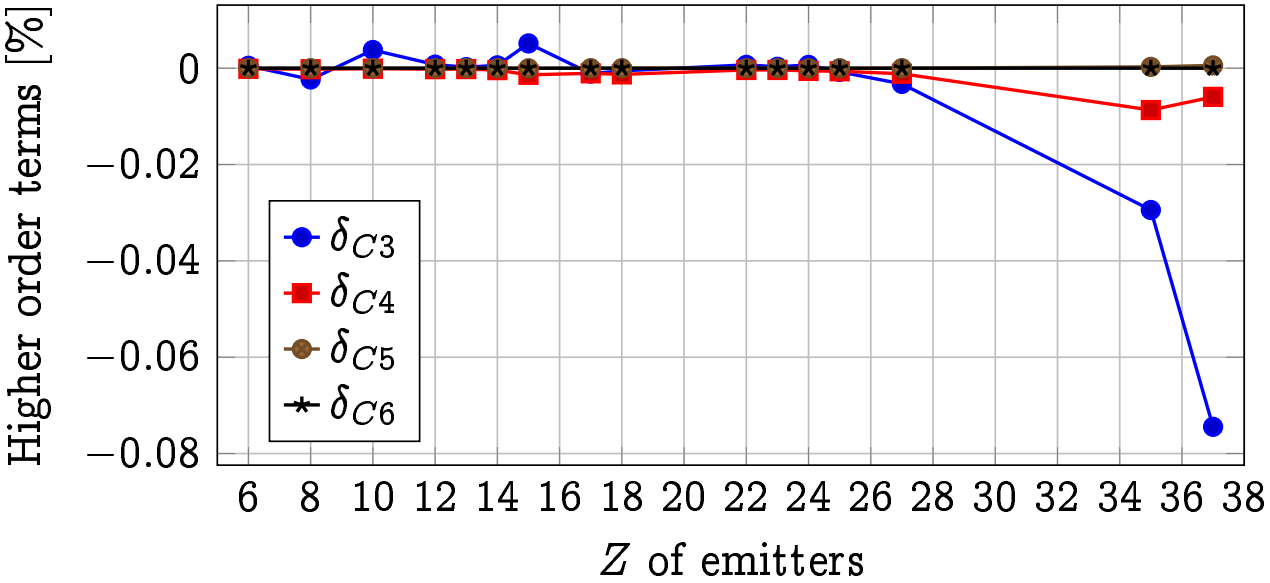
- We derived that $M_F^2 \approx 2\left(1 - \sum_{i=1}^{6} \delta_{Ci}\right)$ where

$$
\begin{aligned}
\delta_{C1} &= 2 - \sqrt{2}\sum_k \sqrt{2j+1}\,\langle f||[a^\dagger_{kn} \otimes \tilde{a}_{kp}]^{(0)}||i\rangle\,, && \text{LO}\\
\delta_{C2} &= \sqrt{2}\sum_{k\pi} \Lambda_k^{np\pi}\,\langle f||a^\dagger_{kn}||\pi\rangle^T\,\langle i||a^\dagger_{kp}||\pi\rangle^T\,, && \text{LO}\\
\delta_{C3} &= -\delta_{C2} + \sqrt{2}\sum_{k\pi} \Lambda_k^{np\pi}\,\langle f||a^\dagger_{kn}||\pi\rangle\,\langle i||a^\dagger_{kp}||\pi\rangle\,, && \text{NLO}\\
\delta_{C4} &= -(\delta_{C1} + \delta_{C2})^2/4, && \text{NLO}\\
\delta_{C5} &= -\delta_{C3}\sqrt{|\delta_{C4}|}, && \text{NNLO}\\
\delta_{C6} &= -(\delta_{C3})^2/4, && \text{NNNLO}
\end{aligned}
$$

with $\Lambda_k^{np\pi} = 1 - \int_0^\infty u^\pi_{kn}(r)u^\pi_{kp}(r)dr$. T indicates isospin invariant quantities

Higher-order terms

- The higher order terms are negligible for light and medium nuclei



- Only δ_{C3} is significant for ^{74}Rb.

Xayavong and Smirnova, arXiv:2201.01035 [nucl-th]

Model spaces and effective interactions

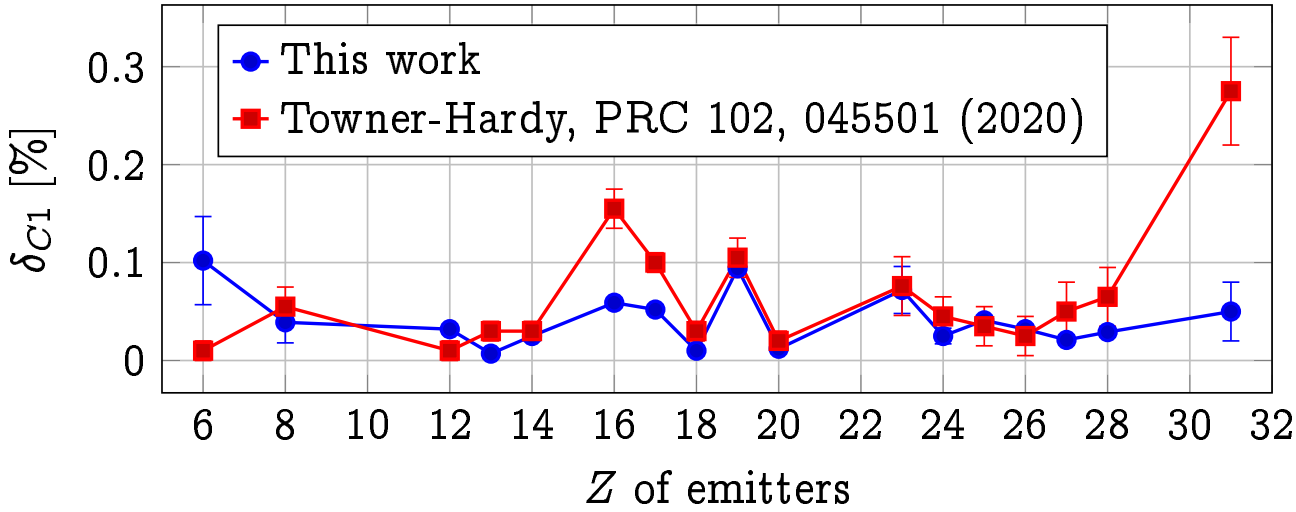
- Well-established effective interactions in full model spaces

Mass	model space	interaction
9-14	p shell	CKP/CKI/CKII
17-22	$1p_{1/2}1d_{5/2}2s_{1/2}$	REWIL/ZBMI/ZBMII
25-34	sd shell	USD/USDA/USDB
37-46	$2s_{1/2}1d_{3/2}1f_{7/2}2p_{3/2}$	ZBM2-MOD
49-54	pf	GXPF1A/KB3G/FPD6
61-74	$2p_{3/2}2p_{1/2}1f_{5/2}1g_{9/2}$	JUN45/MRG

- The isospin nonconserving component is taken from Ormand and Brown, NPA 440 (1985) 174-300. For the cross-shell model spaces, we add only the Coulomb interaction and isovector s.p. energies.

δ_{C1} correction

Our preliminary result for δ_{C1} (http://www.ntse.khb.ru/files/uploads/2018/proceedings/Smirnova.pdf)



- Our values are too large for ^{10}C and too small for ^{30}S, ^{34}Cl, ^{62}Ga because of the error in ΔE.
- A further refinement of the INC interactions is required.

Towner-Hardy protocol

- Phenomenological Woods-Saxon potential.

$$U^q_{ws}(r) = -V^q_0 f(r, a_0, r_0) - V^q_0 V_{ls} \frac{r_s^2}{\hbar^2} \frac{1}{r} \frac{d}{dr} f(r, a_s, r_s) \langle \boldsymbol{l} \cdot \boldsymbol{\sigma} \rangle + \delta_{qp} V_c(r, R_C) + V^q_{surf} \frac{1}{r} \frac{d}{dr} f(r, a_s, r_s)$$

where $a_0 = a_s = 0.662$ fm, $V_{ls} = 0.22$, $r_s = 1.16$ fm. The depth V^q_0 and V^q_{surf} are constrained with separation energies, whereas r_0 and R_C with charge radii.

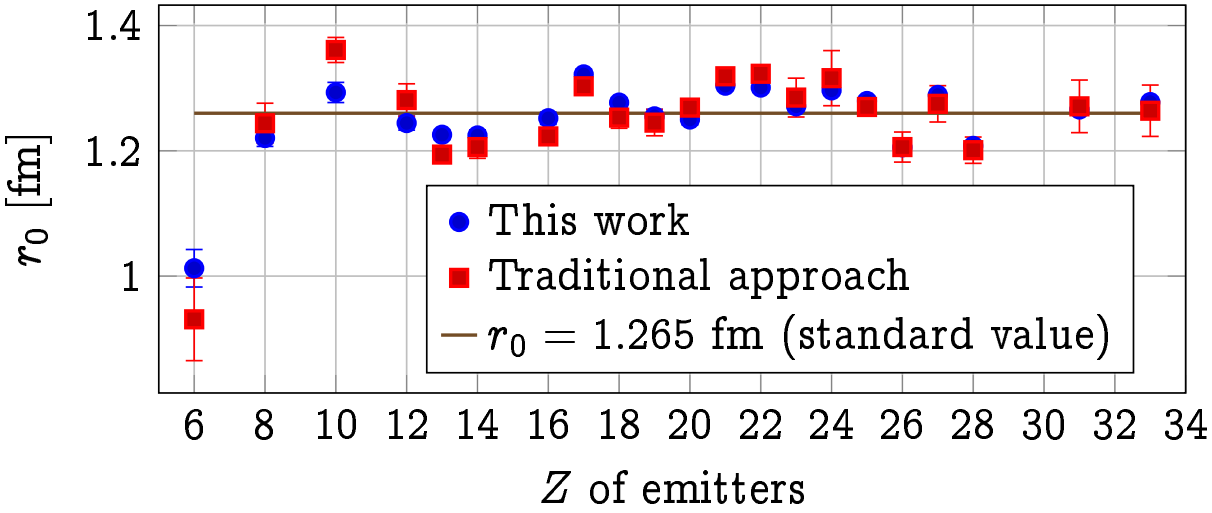
- As an improvement, we made the following generalization

$$\langle r^2_{ch} \rangle = \frac{1}{Z} \sum_{k\pi} | \langle i || a^\dagger_{kp} || \pi \rangle |^2 \int_0^\infty |u^\pi_{kp}(r)|^2 r^2 dr + \text{corr. terms}$$

so that the depth and length parameter can be adjusted self-consistently.

Towner-Hardy protocol

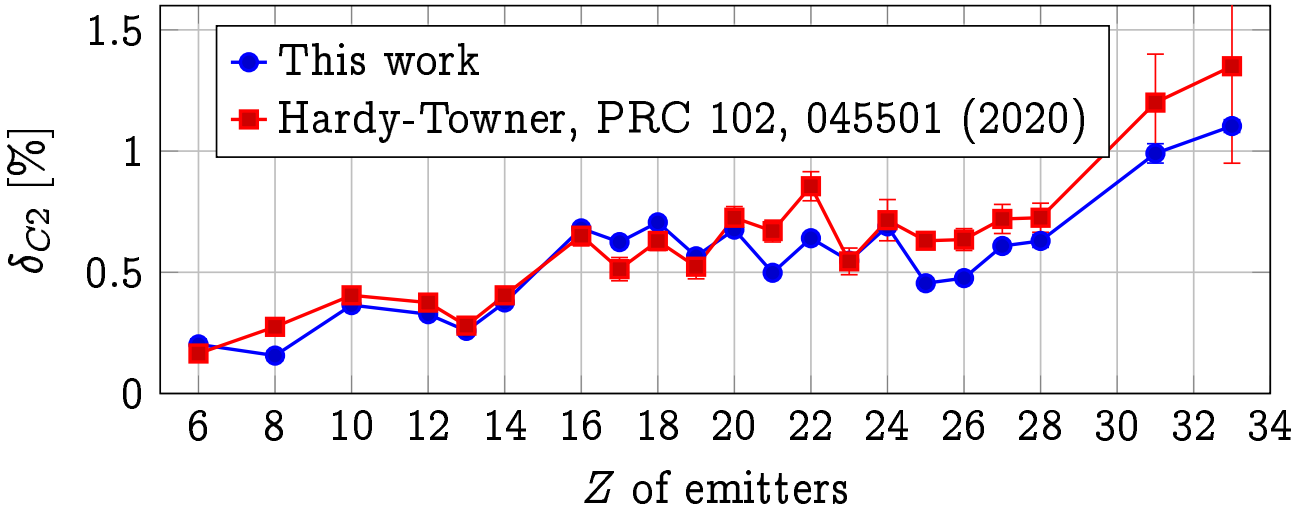
- Resulting values of the length parameter



- The generalization has a considerable impact on charge radii
- The major contribution to uncertainty is the exp. error on the charge radii
- r_0 suddenly decreases at $A \leq 10$ (Failure of the center-of-mass correction in the WS potential?)

Towner-Hardy protocol

- Result for δ_{C2} [*partially published in PRC 97, 024324 (2018)*]



- We obtain smaller values for ^{14}O, ^{42}Sc, ^{42}Ti, ^{50}Mn, ^{50}Fe, ^{54}Co and ^{54}Ni due to difference in configuration spaces and fitting procedure
- Uncertainty is greatly reduced because of the generalization of charge radii
- Calculation for ^{70}Br and ^{74}Rb is in progress

Ormand-Brown protocol

- Use local equivalent potential constructed from Skyrme-HF solution :

$$U^L_{kq}(r,\epsilon_{kq}) = \frac{m^*_q(r)}{m}\Big\{U_q(r) + \frac{d^2}{dr^2}\frac{\hbar^2}{4m^*_q(r)} - \frac{m^*_q(r)}{2\hbar^2}\Big[\frac{d}{dr}\frac{\hbar^2}{m^*_q(r)}\Big]^2 + \frac{1}{2}W_q(r)\langle\boldsymbol{\sigma}\cdot\boldsymbol{l}\rangle + \delta_{qp}V_{coul}(r)\Big\} + \Big[1 - \frac{m^*_q(r)}{m}\Big]\epsilon_{kq}$$

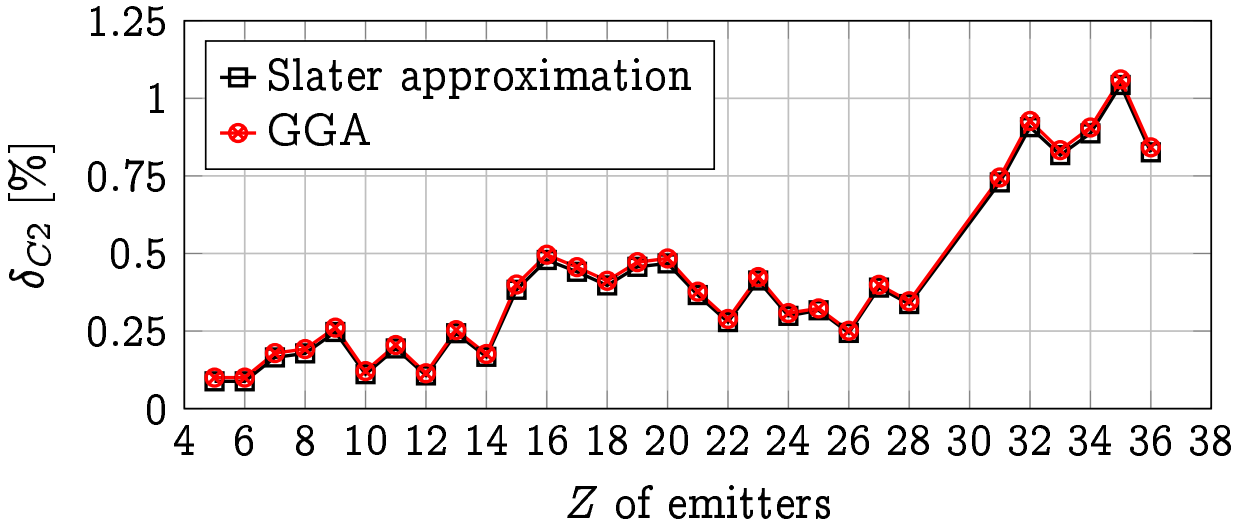
  where $U_q(r)$, $W_q(r)$ and $V_{coul}(r)$ are functionals of various densities. Following Ormand-Brown, we scale $U_q(r)$ to reproduce separation energies.

- As improvement, i) we treat the Coulomb exchange with the GGA instead of the Slater approximation, ii) include CIB and CSB forces, and iii) suppress the spurious isospin mixing

Xayavong and Smirnova, PRC 105, 044308 (2022)

Ormand-Brown protocol

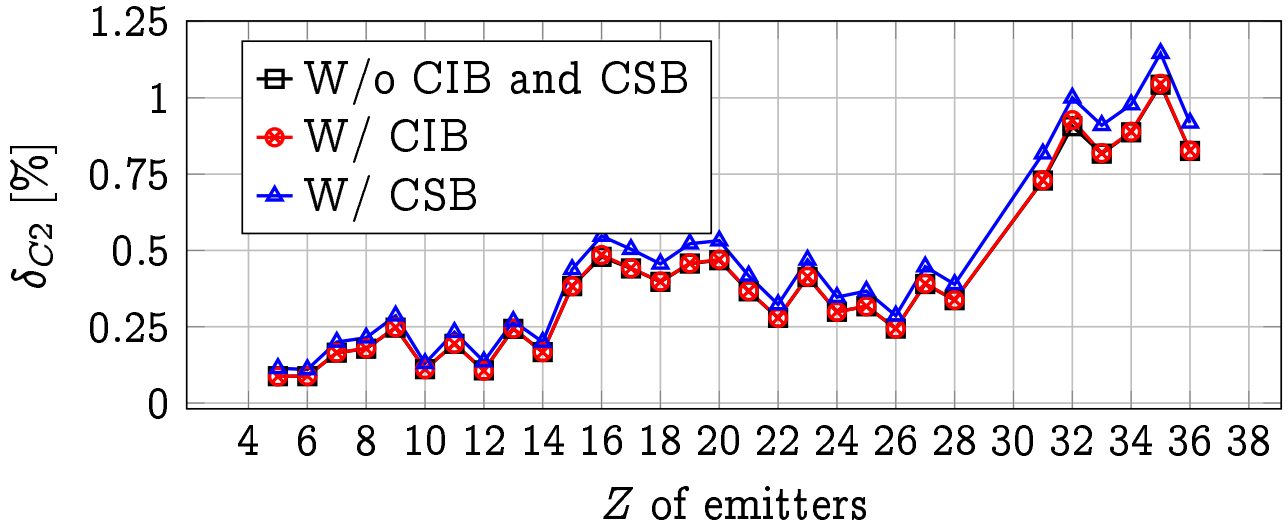
- The exact Coulomb exchange is nonlocal in coordinate space
- We employ the GGA functional developed by Naito *et al.*, PRC 99, 024309 (2019)



- The correction increases by 2-14 % relative to the Slater approximation

Ormand-Brown protocol

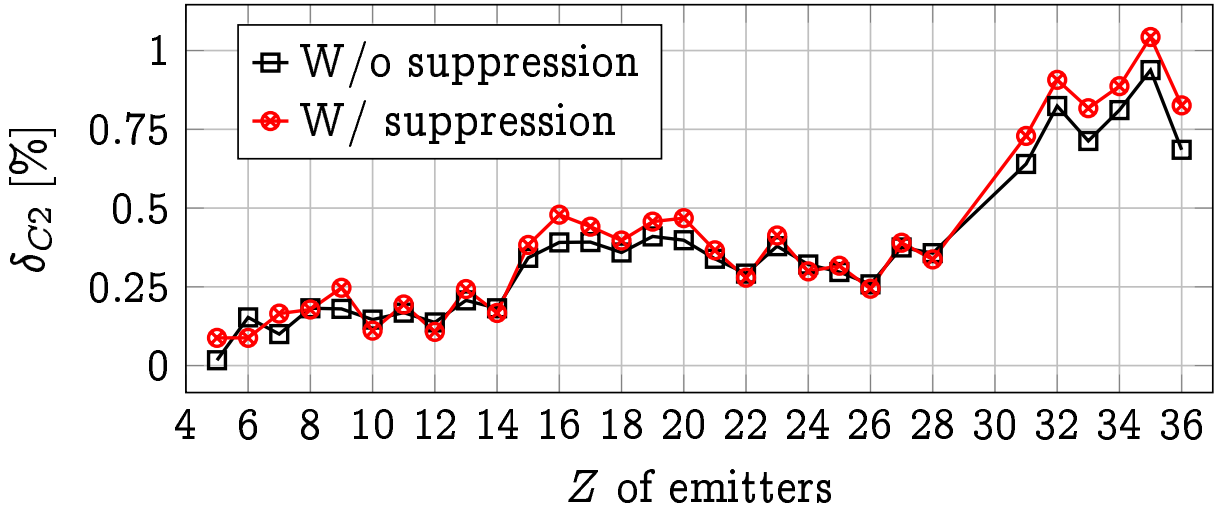
- We also included the CIB and CSB forces of Suzuki *et al.*, PRL 112, 102502 (1995)



- The CIB effect is completely negligible, whereas the CSB contributes 10-30 %.

Ormand-Brown protocol

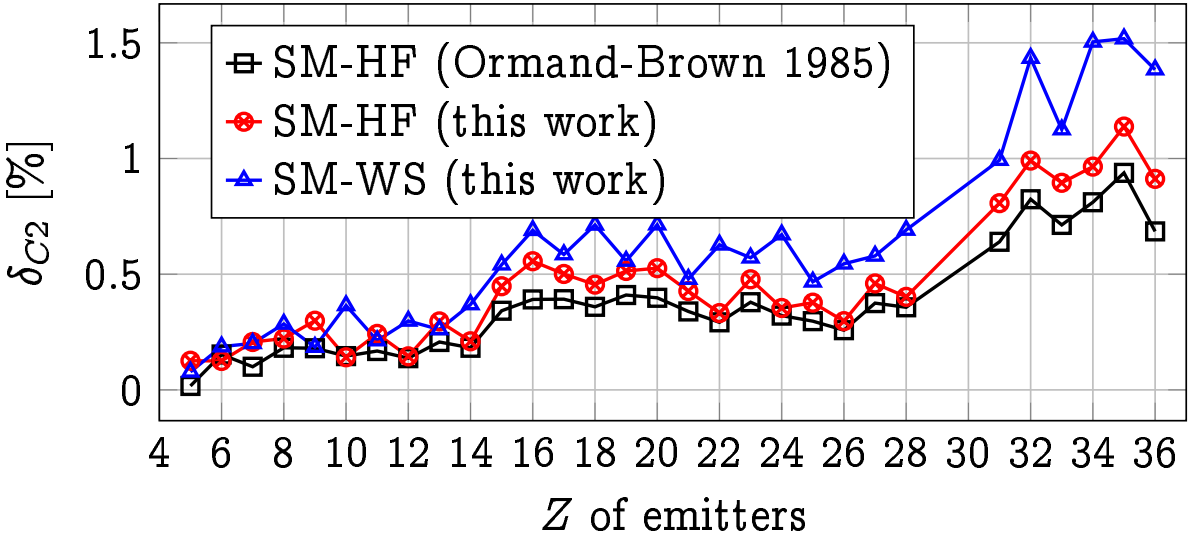
- Suppression of spurious isospin mixing



- The suppression leads to a considerable increase for $16 \leq Z \leq 20$ and $Z \geq 33$
- The emitters with $21 \leq Z \leq 28$ are mostly unaffected
- Complicated effect in light nuclei where nuclear isovector term is dominated

Ormand-Brown protocol

- Our SM-HF result is better than that of Ormand-Brown

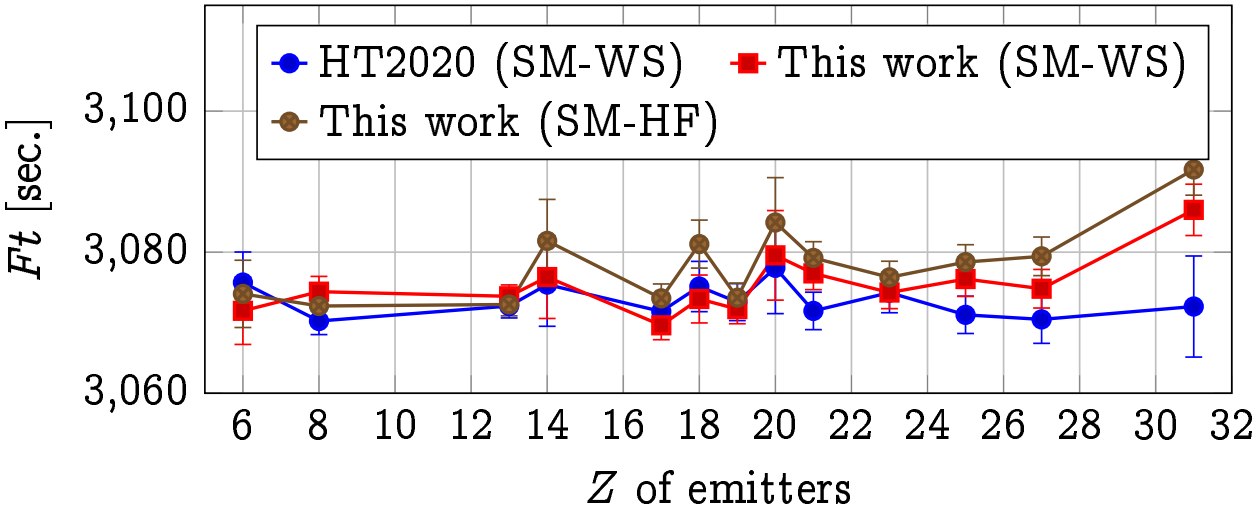


- A considerable gap between SM-WS and SM-HF remains
- Deformation/correlation should be removed from the data before scaling the HF field. However, this is practically impossible.

Ormand & Brown, NPA 440,274 (1985)

Standard-Model implication

- Constancy of the corrected Ft values (only 13 cases !)



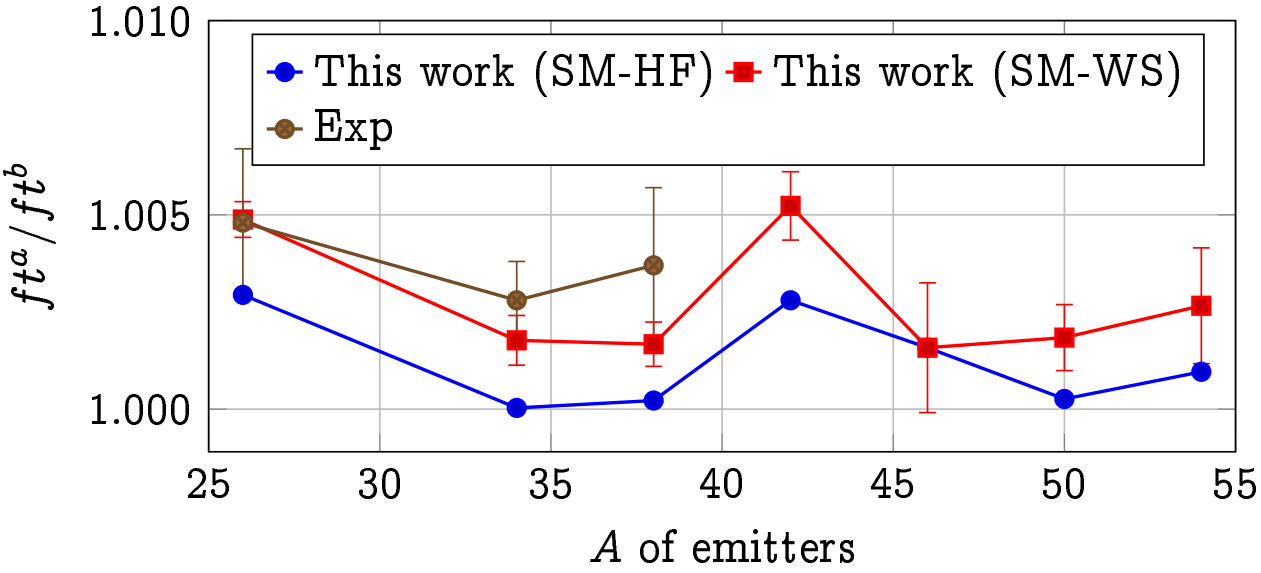
- CVC filter :

	HT2020 (SM-WS)	This work (SM-WS)	This work (SM-HF)
$\overline{Ft}$	3073.148 ± 0.748	3075.310 ± 0.706	3078.332 ± 0.706
χ^2/ν	0.493	1.869	4.040

- Our calculation of δ_{C1} must be improved => Construction of new effective INC interactions. SM-HF doesn't agree with CVC

Mirror test for δ_C correction

- This test is particularly sensitive to the local variation of δ_C.



- SM-HF underestimates the experimental data

Xayavong and Smirnova, PRC 105, 044308 (2022)

Summary

- Improved SM-WS and SM-HF calculations for superallowed $0^+ \rightarrow 0^+$ Fermi β decays
- Full model-space calculations using well-established effective interactions
- New method for charge radius calculation, allowing us to fit the depth and length parameter self-consistently
- A number of deficiencies in the HF calculation have been investigated and some of them have been resolved. The new result agrees better with CVC, but still poorer than does the WS result.

Outlook

- For future study of δ_{C1}, it is necessary to develop new INC interactions especially for the cross-shell model spaces
- Use an effective Fermi operator
- Center-of-mass correction for ^{10}C and ^{14}O
- Replace the uniform charged approximation with a realistic Coulomb potential
- Our approach is readily extended to the studies of mirror β decay and Gamow-Teller β decays, which are also important to the fundamental issues of electroweak theory.