All-order multiple scattering at high energy in small systems

(In preparation)

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In memory of Ricardo Vazquez

X. Feal

IGFAE, University of Santiago de Compostela

Multiple scattering central role in jet/parton processes calculations:

Momentum broadening effects in jet energy correlators

→ Andres et. al. 2303.03413, see Fabio's talk.

Moliere scattering in jet quenching and jet substructure

→ See Raymond Ehlers talk and Barata et. al. 2009.13667

Interplay between classical & quantum corrections to $\hat{q}$

→ See Eamonn Weitz talk.

Quantum partonic transport

→ Barata et. al. 2210.06519

Momentum broadening effects in $c\bar{c}$ production

→ See Jasmine Brewer's talk.

Collisional and radiative energy-loss in chiral magnetic current

→ See Kirill Tuchin's talk

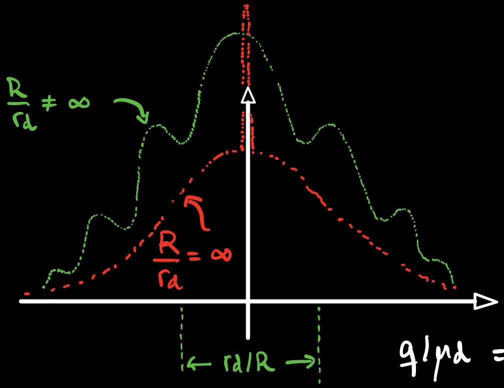
* THIS TALK: study in more detail finite system-size effects in multiple scattering and the emergence of optical diffraction.

Infinite medium / nucleus approximation

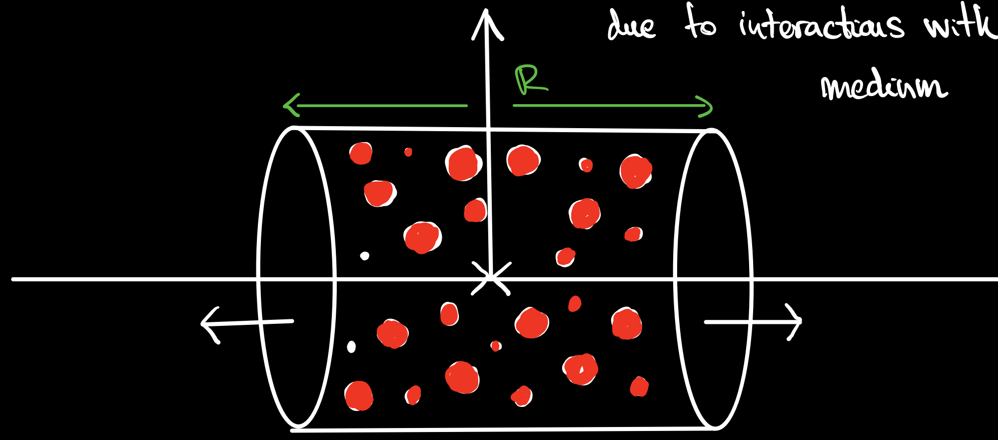
$R/r_d \to \infty$ where

$R \sim$ transverse size medium

$r_d \sim$ size medium constituent



q/μ_d = momentum transfer due to interactions with the medium



* For r_d I use LO HTL:

$r_d \sim \mu_d^{-1}$ where

$$\mu_d^2(T) = 4\pi\alpha_s(T)\left(1+\frac{N_f}{6}\right)T^2+\dots$$

$N_f = 3$, $\alpha_s(M_Z^2) = 0.12$

* For R, I use averaged parton path length, limited up from above by freeze-out time

		T_{ef}	μ_d	r_d	R	R/r_d
Pb-Pb $\sqrt{s_{NN}} = 13$ TeV	0-5%	557 MeV	1.24 GeV	0.16 fm	5 fm	31
	50-60%	401 MeV	0.95 GeV	0.20 fm	3.3 fm	16
Pb-Pb $\sqrt{s_{NN}} = 2.76$ TeV	0-5%	470 MeV	1.08 GeV	0.18 fm	5 fm	27
	50-60%	338 MeV	0.84 GeV	0.23 fm	3.1 fm	13
Xe-Xe $\sqrt{s_{NN}} = 5.44$ TeV	0-5%	462 MeV	1.07 GeV	0.18 fm	4.3 fm	24
	50-60%	334 MeV	0.82 GeV	0.24 fm	2.6 fm	11
Au Au $\sqrt{s_{NN}} = 200$ GeV	0-5%	362 MeV	0.88 GeV	0.22 fm	4.6 fm	21
	50-60%	254 MeV	0.67 GeV	0.3 fm	2.5 fm	8
Cu-Cu $\sqrt{s_{NN}} = 200$ GeV	0-5%	300 MeV	0.76 GeV	0.26 fm	2.9 fm	11
	30-40%	252 MeV	0.67 GeV	0.3 fm	2.2 fm	7

Quantum electrodynamics case : ordinary matter

Particle spin-$\frac{1}{2}$ in $A_\mu(x)$

$$(i\not{D}-m)\psi = 0 \qquad D_\mu = \partial_\mu + ieA_\mu \qquad \not{D} = D_\mu \gamma^\mu$$

At HE use $(-i\not{D}-m)(+i\not{D}-m)\psi = 0$

$$\left[m^2 - (\hat{p}_\mu - eA_\mu)^2 + e\sigma_{\mu\nu}F^{\mu\nu}\right]\psi = 0 \qquad \hat{p}_\mu \equiv i\frac{\partial}{\partial x^\mu}$$

Assume particle's energy greatly exceeds magnitude of interaction with A_μ, and A_μ varies slowly particle's wavelength:

$$\boxed{\psi^\pm(x) = \psi_0^\pm(x)\, U_A^\pm(x) \qquad ip^\mu \frac{\partial U_A^\pm}{\partial x^\mu} = ep_\mu A^\mu(x)\, U_A^\pm(x)}$$

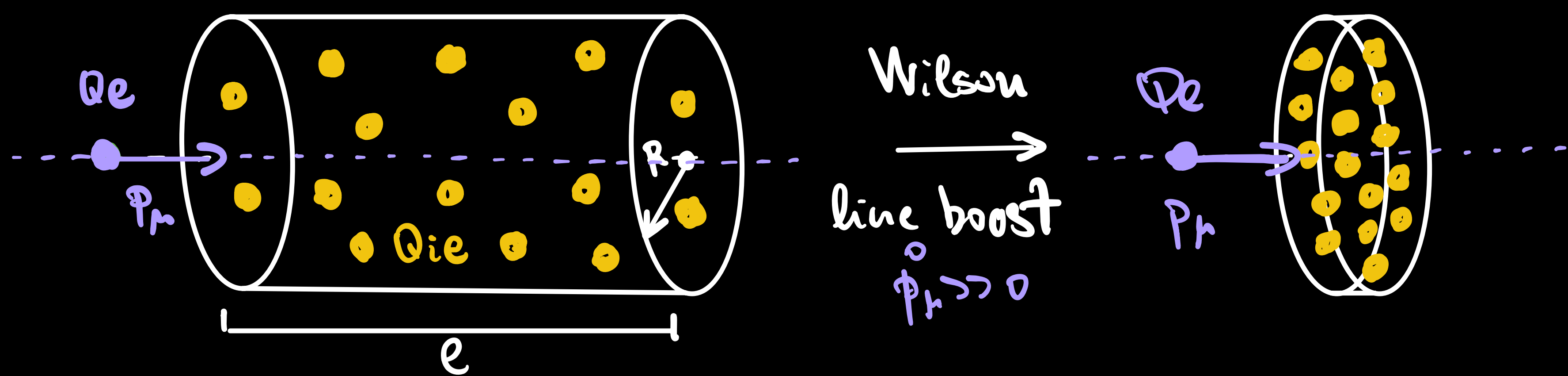
SOLUTION:

$$\underbrace{U_A^{\pm}(x) = \exp\left\{ i\,\frac{Qe}{m}\int_{-\infty}^{0} dz\; p^{\mu} A_{\mu}\left(x + \frac{p}{m} z\right) \right\}}_{\text{Wilson line propagator in } A_\mu \text{ till space-time point } x}$$

$A_\mu(x) \equiv$ n classical and static sources with worldlines

$x^i_\mu = (t', \vec{x}^i)$ $\quad \dot{x}^i_\mu = (1, \vec{0})$ $\quad$ randomly distributed in a finite volume

$$J^{(n)}_\mu(x) = \sum_{i=1}^{n} Q_i e \int_{-\infty}^{+\infty} dt'\, \dot{x}^i_\mu(t')\, \delta^4(x - x_i(t')) = g_{\mu 0} \sum_{i=1}^{n} Q_i e\, \delta^3(\vec{x} - \vec{x}_i)$$



$$A^{(n)}_\mu(x) = \int d^4y\, \underbrace{D^B_{\mu\nu}(x-y)}_{\text{Photon propagator}}\, J^\nu_{(n)}(y) = g_{\mu 0}\,\frac{1}{4\pi}\underbrace{\sum_{i=1}^{n} \frac{Q_i e}{|\vec{x}-\vec{x}^i|}\, e^{-\mu_d|\vec{x}-\vec{x}^i|}}_{n \text{ screened Coulomb fields}}$$

Scattering amplitude to all-orders in $A_\mu(x)$ at HE:

$$\langle p_f s_f | \hat{M} | p_i s_i \rangle = 2\pi\, \delta(E_f - E_i)\, \beta\, \delta_{s_f s_i}\; i\, T^{(n)}_{fi}(\vec{q}_t)$$

$$\boxed{T^{(n)}_{fi}(\vec{q}_t) = \frac{1}{i}\int d^2x_t\, e^{-i q_t \cdot x_t}\left\{\prod_{i=1}^{n} W_i(\vec{x}_t) - 1\right\}}$$

$$\underbrace{W_i(\vec{x}_t) \equiv W(\vec{x}_t - \vec{x}_t^{\,i})}_{\text{Wilson line in the field of the i-th source}} = \exp\left\{ i\,\frac{Qe}{v}\int_{-\infty}^{+\infty} dx_3\, A_0(\vec{x}_t - \vec{x}_t^{\,i},\, x_3 - x_3^i) \right\} = \exp\left\{ i\,\frac{Qe}{v}\, 2K_0(\mu_d |\vec{x}_t - \vec{x}_t^{\,i}|) \right\}$$

DIFF. ELASTIC CROSS SECTION : probability for the particle to go from $\vec{p}_i$ to $\vec{p}_f = \vec{p}_i + \vec{q}_t$ after interacting to all-orders at HE with the n sources:

$$\boxed{\left\langle \frac{d^2\sigma_{el}^{(n)}}{d\vec{q}_t^{\,2}} \right\rangle_v = \left\langle T_{fi}^{(n)}(\vec{q}_t)\, T_{fi}^{(n)*}(\vec{q}_t) \right\rangle_v}$$

Medium average:

$$\left\langle F^{(n)}(\vec{x}_1,\dots,\vec{x}_n) \right\rangle_v = \int d^3\vec{x}_1\, f(\vec{x}_1) \cdots \int d^3\vec{x}_n\, f(\vec{x}_n)\, F^{(n)}(\vec{x}_1,\dots,\vec{x}_n)$$

$f(\vec{x})$: probability of a source to be found at $\vec{x}$

Noticing $T_{fi}^{(n)} = S_{fi}^{(n)} - \mathbb{1}$ and $\langle x^2 \rangle = \langle x \rangle^2 + \sigma^2$

$$\left\langle \frac{d^2\sigma_{el}^{(n)}}{d\vec{q}_t^{\,2}} \right\rangle_V = \underbrace{\Pi_2^{(n)}(\vec{q}_t)}_{\substack{\text{takes the average} \\ \text{at amplitude level}}} + \underbrace{\Sigma_2^{(n)}(\vec{q}_t)}_{\substack{\text{takes the average} \\ \text{at cross-section level}}}$$

$\downarrow$ COHERENT

$\downarrow$ INCOHERENT

$$\Pi_2^{(n)}(\vec{q}_t) = \left| \left\langle T_{fi}^{(n)}(\vec{q}_t) \right\rangle_V \right|^2$$

Identity valid for any averaged (traced-over) cross-section.

$$\Sigma_2^{(n)}(\vec{q}_t) = \left\langle S_{fi}^{(n)}(\vec{q}_t)\, S_{fi}^{(n)*}(\vec{q}_t) \right\rangle_V - \left\langle S_{fi}^{(n)}(\vec{q}_t) \right\rangle_V \left\langle S_{fi}^{(n)*}(\vec{q}_t) \right\rangle_V$$

Coherent differential cross-section

$$\left\langle T_{fi}^{(n)}(\vec{q}_t) \right\rangle_V = \frac{1}{i} \int d^2x_t \, e^{-i\vec{q}_t\cdot\vec{x}_t} \left\{ \left\langle \prod_{i=1}^{n} W_i(\vec{x}_t) \right\rangle_V - 1 \right\}$$

Then:

$$\left\langle \prod_{i=1}^{n} W_i(\vec{x}_t) \right\rangle_V = \left\{ \int d^3\vec{x}_1 \, f(x_1) \, W_1(\vec{x}_t) \right\}^n = \left\{ 1 + \frac{1}{n} \int d^3\vec{x}_1 \, \rho(\vec{x}_1) \left(W_1(\vec{x}_t) - 1 \right) \right\}^n$$

$$\underset{n \gg 1}{\sim} \underbrace{\exp\left\{ \int d^3\vec{x}_1 \, \rho(\vec{x}_1) \left(W_1(\vec{x}_t) - 1 \right) \right\}}_{\text{exponential of a Wilson line}} \qquad \underbrace{\rho(\vec{x}) \equiv n f(\vec{x})}_{\text{number density}}$$

Write this in terms of the single elastic amplitude using

$$W_1(\vec{x}_t) - 1 \equiv W(\vec{x}_t - \vec{x}_t') - 1 = i \int \frac{d^2\vec{q}_t}{(2\pi)^2} \, e^{+i\vec{q}_t\cdot(\vec{x}_t - \vec{x}_t')} \, T_{fi}^{(1)}(\vec{q}_t)$$

and define the "window" function of the medium as

$$\tilde{\rho}(\vec{k}_t, 0) = \int d^3\vec{x} \; e^{-i\vec{k}_t \cdot \vec{x}_t} \, \rho(\vec{x}_t, x_3)$$

Then

valid to all-orders in α and ρ

$$\Pi_2^{(n)}(\vec{q}_t) = \left| \frac{1}{i} \int d^2x_t \; e^{-i\vec{q}_t \cdot \vec{x}_t} \left[\exp\left\{ i \int \frac{d^2k_t}{(2\pi)^2} \, e^{+i\vec{k}_t \cdot \vec{x}_t} \, \tilde{\rho}(\vec{k}_t, 0) \, T_{fi}^{(1)}(\vec{k}_t) \right\} - 1 \right] \right|^2$$

$$= \underbrace{\left| \rho(\vec{k}_t, 0) \, T_{fi}^{(1)}(\vec{k}_t) \right|^2}_{\text{LO in } \rho \text{ , all orders in } \alpha} + \mathcal{O}(\rho^2)$$

<u>Example</u>: homogeneous brick of matter with cylindrical geom.

$$\rho(\vec{x}) = \rho_0\, \Theta(\ell/2 - |x_3|)\, \Theta\left(R - \sqrt{x_1^2 + x_2^2}\right) = \rho_0\, \Theta(\ell/2 - |x_3|)\, \Theta\left(R - \sqrt{x_t^2}\right)$$

$$\Rightarrow \rho(\vec{k}_t) = \rho_0\, \ell\, \frac{2\pi R}{|\vec{k}_t|} J_1(|\vec{k}_t| R) \qquad \rho_0 = \frac{n}{\pi R^2 \ell}$$

$$\Pi_{fi}^{(n)}(\vec{q}_t) = \underbrace{\frac{4n^2}{(|q_t|R)^2} J_1^2(|\vec{q}_t| R)} \times \underbrace{\left| T_{fi}^{(1)}(\vec{q}_t) \right|^2} \qquad \left| \frac{(Qe)^2}{\vec{q}_t^{\,2} + \mu_d^2} \right|^2$$

HE particle being diffracted <u>about the disk</u> with osc. $|q_t| \sim \frac{1}{R}$

single elastic diff. cross section

$n=1$

$|q_t| \sim \frac{1}{r_d}$

Cross-section for the scattering with n centers equal to n^2 times the cross-section with a single center

$\Rightarrow$ <u>coherent scattering with whole medium</u>

Incoherent differential cross-section

$\left(R \gg r_d = \mu_d^{-1} \right)$

$$\Sigma_2^{(n)}(\vec{q}_t) = \pi R^2 \, e^{-\rho_0 \ell \, \sigma_{el}^{(1)}(\vec{0})}$$

$$\times \int d^2x_t \, e^{-i q_t \cdot x_t} \left[e^{\rho_0 \ell \, \sigma_{el}^{(1)}(\vec{x})} - 1 \right]$$

MOLIERE EQUATION FOR FINITE ℓ

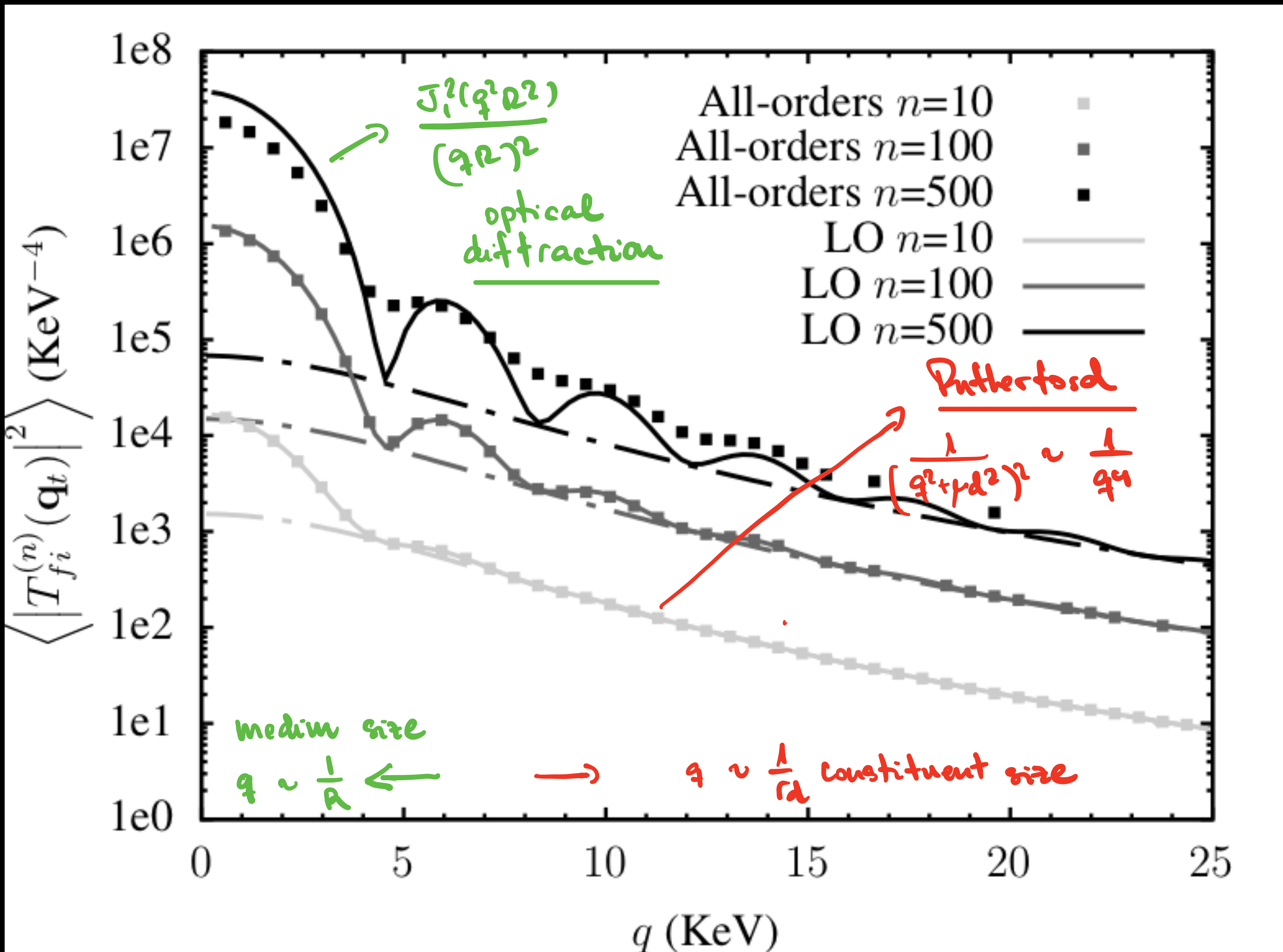
Where the F.T. of the single $n=1$ diff. elastic cross-section

$$\sigma_{el}^{(1)}(\vec{x}_t) = \int \frac{d^2k_t}{(2\pi)^2} \, e^{+i \vec{k}_t \cdot \vec{x}_t} \left| T_{fi}^{(1)}(\vec{k}_t) \right|^2$$

and I used optical theorem to all-orders in α

$$2 \, \mathrm{Im} \, T_{fi}^{(1)}(\vec{0}) = \sigma_{el}^{(1)} \equiv \sigma_{el}^{(1)}(\vec{0})$$

Example: $Z = 7$ (Nitrogen) $\mu_d \sim \alpha m Z^{1/3} \sim 7$ keV $R \sim 6 r_d$



TOTAL ELASTIC CROSS-SECTION OF MULTIPLE SCATTERING

$$\sigma_{el}^{(n)} = \int \frac{d^2 q_t}{(2\pi)^2} \left\{ \Sigma_2^{(n)}(\vec{q}_t) + \Pi_2^{(n)}(\vec{q}_t) \right\} = 2\pi R^2 \, \mathrm{Re} \left\{ 1 - e^{i \rho_0 \ell T_{fi}^{(1)}(\vec{0})} \right\}$$

$$\boxed{= 2\pi R^2 \left\{ 1 - e^{-\frac{\rho_0 \ell \sigma_{el}^{(1)}}{2}} \cos\left[\frac{\rho_0 \ell \sigma_{osc}^{(1)}}{2}\right]\right\} \quad \text{all orders in } \alpha \text{ and } \rho_0}$$

$$\sigma_{el}^{(1)} = 2 \, \mathrm{Im} \, T_{fi}^{(1)}(\vec{0})$$

$$\sigma_{osc}^{(1)} = 2 \, \mathrm{Re} \, T_{fi}^{(1)}(\vec{0})$$

single $n=1$ elastic cross-section

At low medium densities $n \to 0$

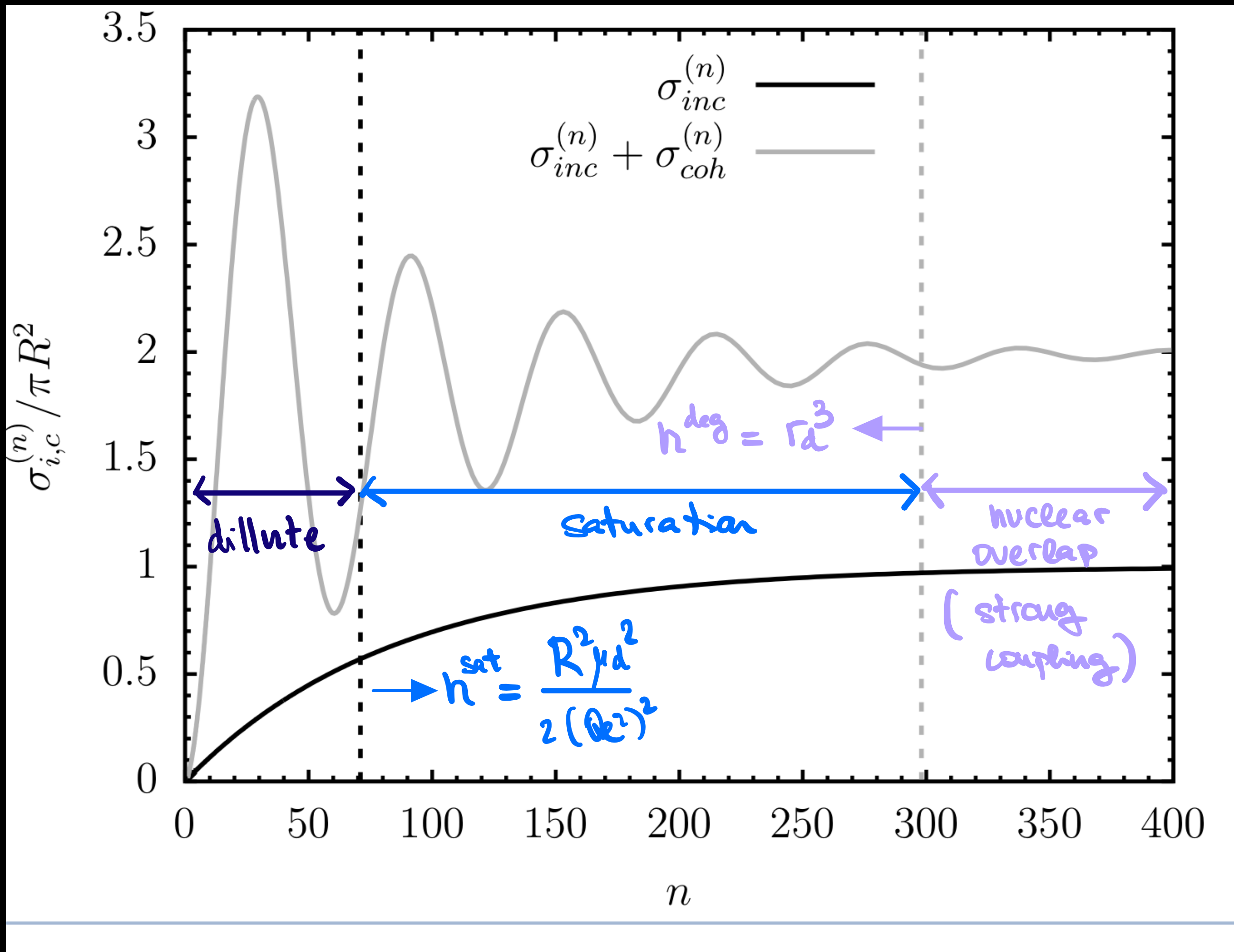
$$\sigma_{el}^{(n)} = 2\pi R^2 \left\{ \frac{\rho_0 \ell \sigma_{el}^{(1)}}{2} + \cdots \right\} = n \sigma_{el}^{(1)} + \cdots$$

Saturation $\rho_0^{sat} \ell \gg \dfrac{2}{\sigma_{el}^{(1)}} \approx \dfrac{\mu_d^2}{2\pi (Qe^2)^2}$ $\qquad n \to \infty$

$$\underline{\underline{\sigma_{el}^{(n)} = 2\pi R^2}}$$

CORRECT BLACK DISK LIMIT!

Half the scattering events are coherent diffraction, even if $R \to \infty$



$\Rightarrow$ sizable contributions to $\langle q_t^2 \rangle$ and hence $\hat{q}$

HE ELASTIC SCATTERING IN QUANTUM CHROMODYNAMICS

$$T^{(n)}_{fi}(\vec{q}_t) = \frac{1}{i}\int d^2\vec{x}_t\, e^{-i\vec{q}_t\cdot\vec{x}_t}\, \mathcal{P}\exp\left\{-i\frac{g_s}{\beta}\int_{-\infty}^{x_3} dx_3'\, t_a A^0_a(x_t, x_3')\right\}$$

$$\underbrace{\frac{1}{N_c}\mathrm{Tr}\left\langle T^{(n)\dagger}_{fi}(\vec{q}_t)\, T^{(n)}_{fi}(\vec{q}_t)\right\rangle}_{\text{Diff. cross-section}} = \underbrace{\Pi^{(n)}_2(\vec{q}_t)}_{\text{Coherent}} + \underbrace{\Sigma^{(n)}_2(\vec{q}_t)}_{\text{incoherent}}$$

$$\Sigma^{(n)}_2(\vec{q}_t) = \pi R^2\, e^{-\rho_0 L\,\sigma^{(1)}_{ee}(\vec{0})}\int d^2x_t\, e^{-i\vec{q}_t\cdot\vec{x}_t}\left(e^{\rho_0 L\,\sigma^{(1)}_{ee}(\vec{x}_t)} - 1\right)$$

$$\sigma^{(1)}_{ee}(\vec{x}_t) = \int\frac{d^2q_t}{(2\pi)^2}\, e^{+i\vec{q}_t\cdot\vec{x}_t}\,\frac{d^2\sigma^{(1)}_{ee}}{d^2q_t} = \frac{g_s^2}{\mu^2}\,\frac{N_c^2-1}{4N_c^2}\,\mu|\vec{x}_t|\,K_1(\mu|\vec{x}_t|) + \dots$$

Single $n=1$ diff. elastic cross section

$$\frac{d^2\sigma^{(1)}_{ee}}{d^2q_t} = \frac{1}{N_c}\mathrm{Tr}\left(T^{(1)\dagger}_{fi}(\vec{q}_t)\, T^{(1)}_{fi}(\vec{q}_t)\right) = g_s^2\,\frac{N_c^2-1}{4N_c^2}\,\frac{1}{(\vec{q}_t^{\,2}+\mu^2)^2} + \dots$$

Incoherent cross-section satisfies the Moliere equation

$$\frac{\partial \Sigma_2^{(n)}}{\partial \ell}(\vec{q}_t) = \rho_0 \int \frac{d^2\vec{k}_t}{(2\pi)^2} \frac{d\sigma_{el}^{(1)}}{d^2\vec{k}_t} \left\{ \Sigma_2^{(n)}(\vec{q}_t - \vec{k}_t) - \Sigma_2^{(n)}(\vec{q}_t) \right\} + \rho_0 n \Omega^2 \frac{d^2\sigma_{el}^{(1)}}{d^2 q_t} \cdot e^{-\rho_0 \ell \sigma_{el}^{(1)}}$$

Transport equation at cross-section level

Penetrating dist. "ℓ" without yet interacting

Coherent cross-section does not :

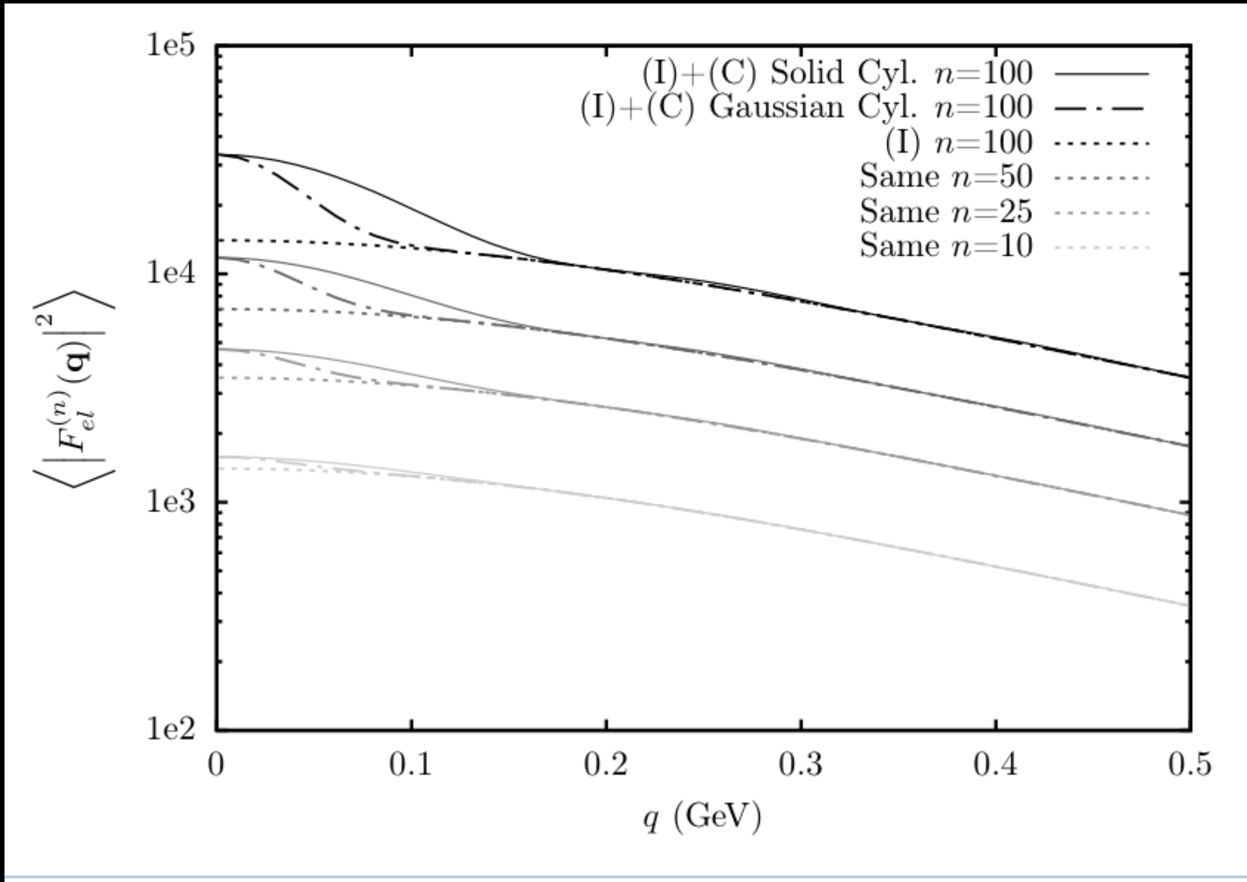
$$\Pi_2^{(n)}(\vec{q}_t) = \left| \int d^2x_t \, e^{-i\vec{q}_t \cdot \vec{x}_t} \left\{ e^{\rho_0 \ell \Pi_{el}^{(1)}(\vec{x}_t)} - 1 \right\} \right|^2 = \left| \Pi_1^{(n)}(q_t) \right|^2$$

$$\Pi_{el}^{(1)}(\vec{x}_t) = \int \frac{d^2\vec{q}_t}{(2\pi)^2} \, e^{+i\vec{q}_t \cdot \vec{x}_t} \, \tilde{\rho}(\vec{q}_t, 0) \, \frac{1}{N_c} \mathrm{Tr}\, T_{fi}^{(1)}(\vec{q}_t)$$

Quantum transport equation at amplitude level

$$\frac{\partial \Pi_1^{(n)}}{\partial \ell}(\vec{q}_t) = \rho_0 \int \frac{d^2\vec{k}_t}{(2\pi)^2} \tilde{\rho}(\vec{k}_t, 0) \frac{1}{N_c} \mathrm{Tr}\left(T_{fi}^{(1)}(\vec{k}_t) \right) \Pi_1^{(n)}(\vec{q}_t - \vec{k}_t) + \tilde{\rho}(\vec{k}_t, 0) \frac{1}{N_c} \mathrm{Tr}\, T_{fi}^{(1)}(\vec{q}_t)$$

Examples: typical QCD medium for Cu-Cu $\sqrt{s_{NN}} = 200$ GeV collisions



($\mu_d \approx 0.5$ GeV $R = 10\, r_d$)

LO term diffraction in classical Non-Abelian background $\mathcal{O}(\alpha_s^4)$

Summary & possible new directions

→ Diffraction is a non-classical effect: cannot be reduced to the form of a stochastic equation.

→ Necessary for a precise tomography of color and electromagnetic charge distributions in small systems

* effects in di-lepton accoplanarity distributions?
* related phenomena: transition radiation and dielectric suppression

→ Procedure can be extended to study bound systems and hence effects of charge correlations near phase transition

* imprints of critical opalescence phenomena?

Thanks!