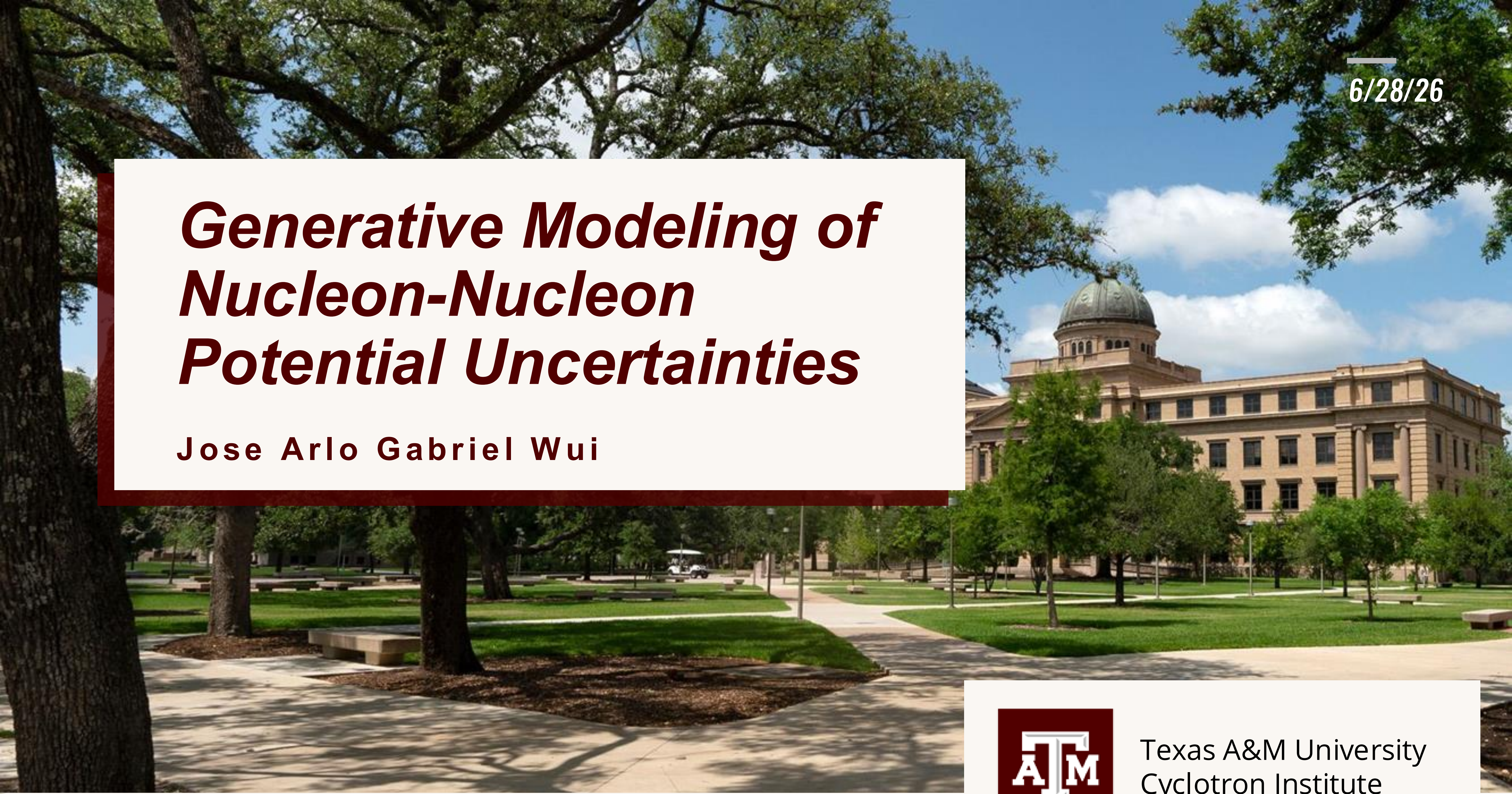

6/28/26

Generative Modeling of Nucleon-Nucleon Potential Uncertainties

Jose Arlo Gabriel Wui

Texas A&M University
Cyclotron Institute

Presentation Outline

- **Motivation:** why and how we study the nuclear force, chiral effective field theory, generative machine learning
- **Project Details:** goal of project, model architecture
- **Preliminary Results:** $^1S_0$ neutron-proton channel
- **Future Extension:** next steps for this project

Motivation

- The nuclear interaction is essential for describing atomic nuclei and matter in astrophysical environments.

Ab initio predictions link the neutron skin of ^{208}Pb to nuclear forces

Baishan Hu[1,*], Weiguang Jiang[2,*], Takayuki Miyagi[1,3,4,*], Zhonghao Sun[5,6,*], Andreas Ekström[2], Christian Forssén[2,✉], Gaute Hagen[6,5,1], Jason D. Holt[1,7], Thomas Papenbrock[5,6], S. Ragnar Stroberg[8,9], Ian Vernon[10]

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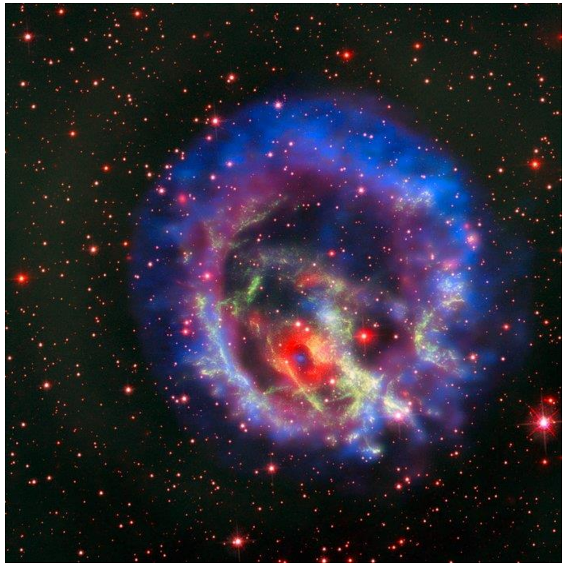


Source: ESO, *An isolated neutron star in the Small Magellanic Cloud*

Motivation

- Chiral effective field theory is an effective tool for studying the nuclear force.

$$V = V_{1\pi} + V'_{2\pi} + V_{ct}$$

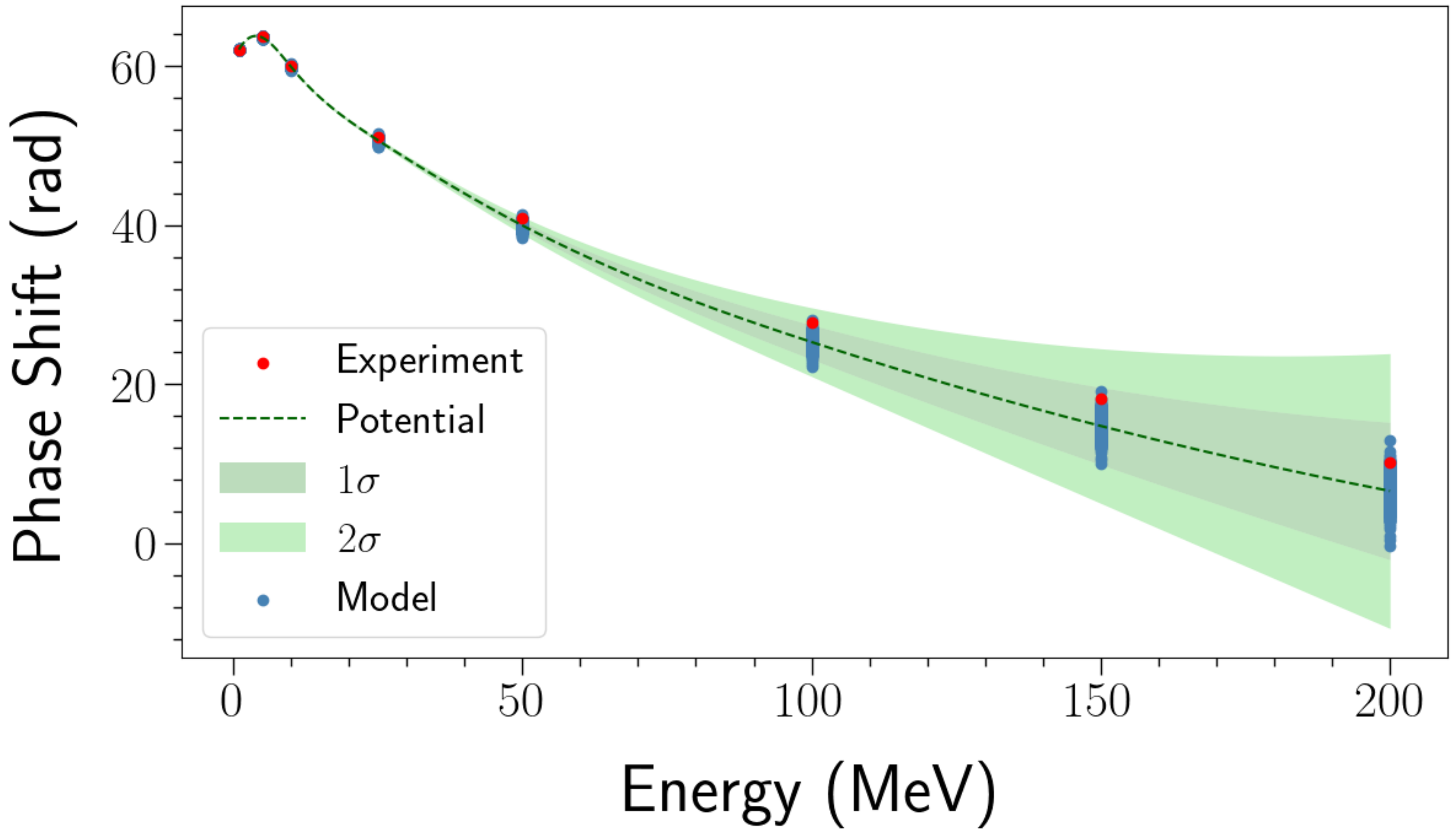
$$V_{ct} = V_{ct}^{(0)} + V_{ct}^{(2)} + V_{ct}^{(4)} + \cdots$$

$$V_{ct}^{(0)}(\mathbf{p}, \mathbf{p}') = C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$$V_{ct}^{(0)}({}^1S_0) = \tilde{C}_{{}^1S_0} = 4\pi(C_S - 3C_T)$$

$$V_{ct}^{(2)}({}^1S_0) = C_{{}^1S_0}(p^2 + p'^2)$$

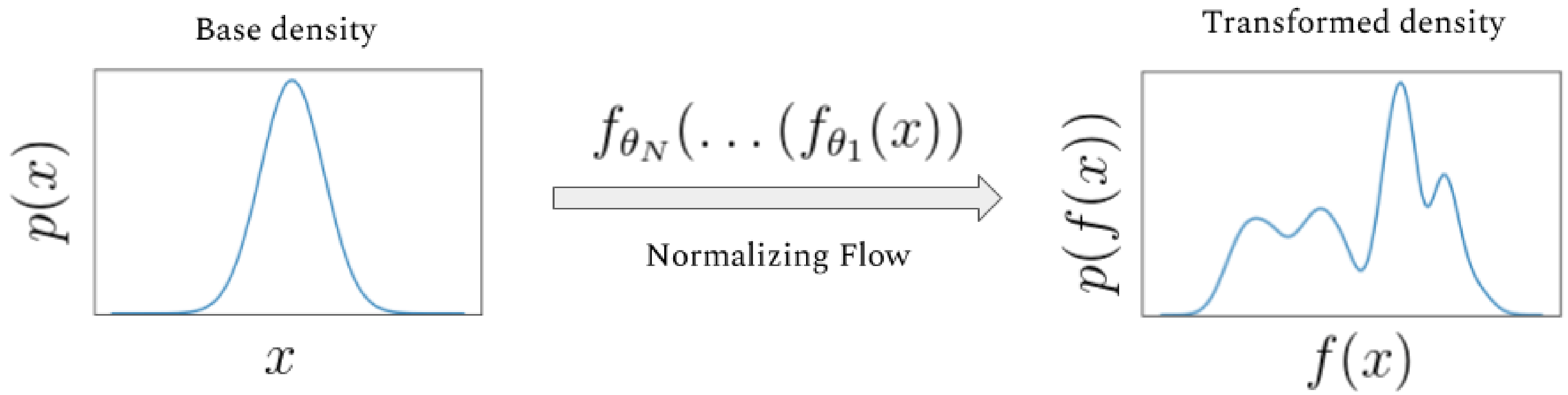
Motivation



Phase Shift (rad)
60
40
20
0
0
50
100
150
200
Energy (MeV)
Experiment
Potential
1σ
2σ
Model

Motivation

- In recent years, advancements in the field of machine learning have led to the development of generative models that can produce new data from existing data distributions.
- One of these are normalizing flows.



Source: Simon Boehm, *The Normalizing Flow Network*

Project Details

Optimized Chiral Nucleon-Nucleon Interaction at Next-to-Next-to-Leading Order

A. Ekström,[1,2] G. Baardsen,[1] C. Forssén,[3] G. Hagen,[4,5] M. Hjorth-Jensen,[1,2,6] G. R. Jansen,[4,5] R. Machleidt,[7] W. Nazarewicz,[5,4,8] T. Papenbrock,[5,4] J. Sarich,[9] and S. M. Wild[9]

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Project Details

TABLE I. Parameters of NNLO$_{opt}$ at $\Lambda = 500$ MeV and $\Lambda_{SFR} = 700$ MeV [3,21]: c_i (in GeV^{-1}), $\tilde{C}$ (in 10^4 GeV^{-2}), and C (in 10^4 GeV^{-4}). The number of decimal digits in the parameters ensure that the phase shifts, in degrees, are computed with a four decimal digit precision.

LEC	Value	LEC	Value	LEC	Value
c_1	−0.918 639 53	c_3	−3.888 687 49	c_4	4.310 327 16
$\tilde{C}^{pp}_{^1S_0}$	−0.151 366 04	$\tilde{C}^{np}_{^1S_0}$	−0.152 141 09	$\tilde{C}^{nn}_{^1S_0}$	−0.151 764 75
$C_{^1S_0}$	2.404 021 94	$C_{^3S_1}$	0.928 384 66	$\tilde{C}_{^3S_1}$	−0.158 434 18
$C_{^1P_1}$	0.417 045 54	$C_{^3P_0}$	1.263 390 76	$C_{^3P_1}$	−0.782 658 50
$C_{^3S_1-^3D_1}$	0.618 141 42	$C_{^3P_2}$	−0.677 808 51		

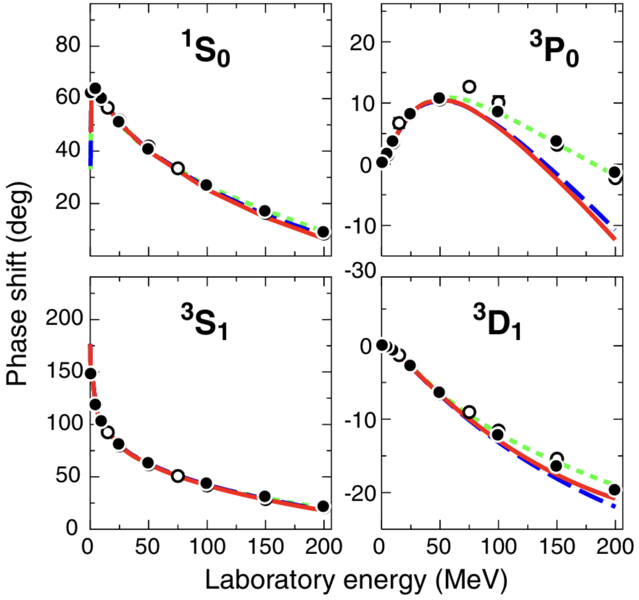
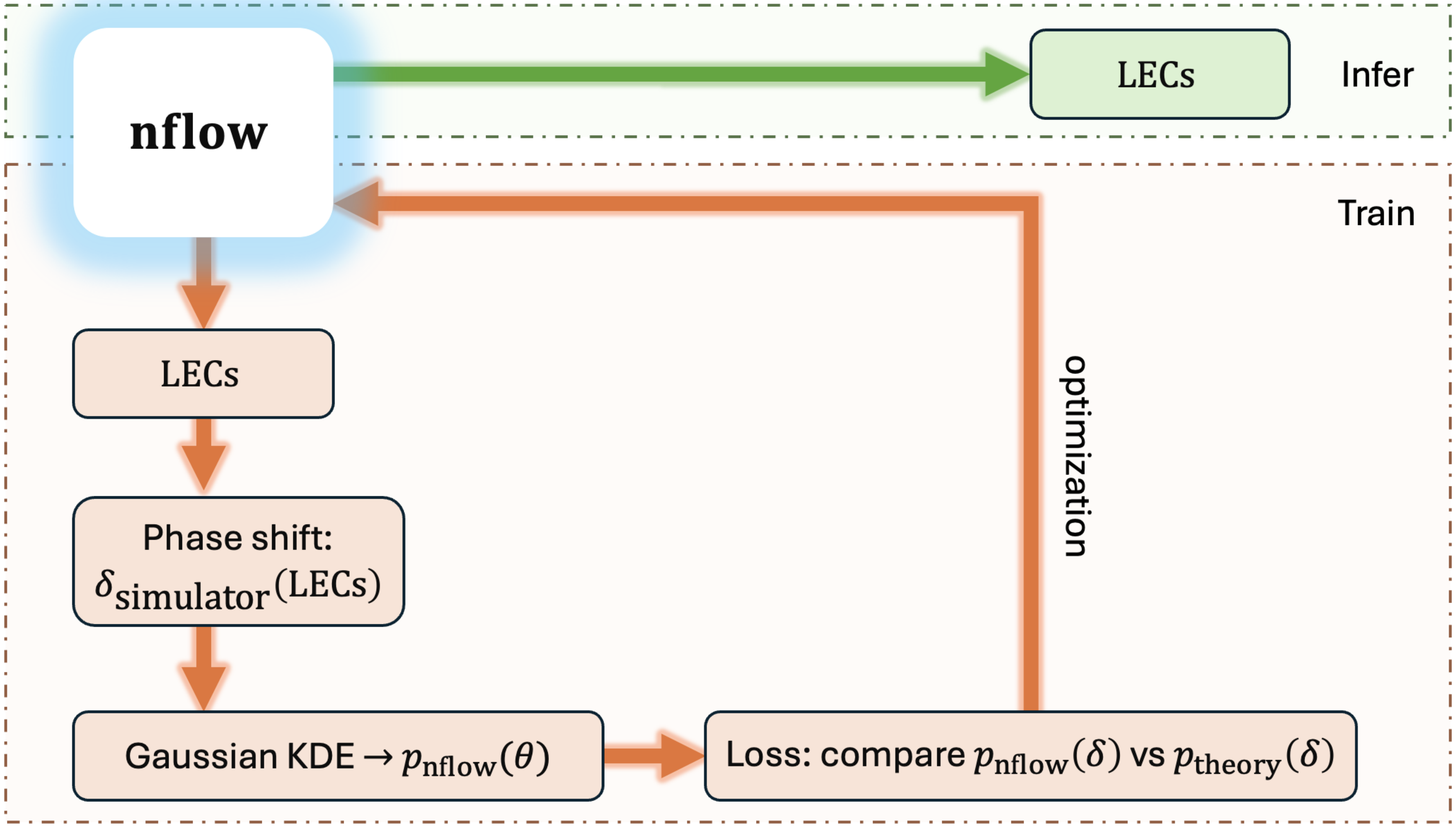
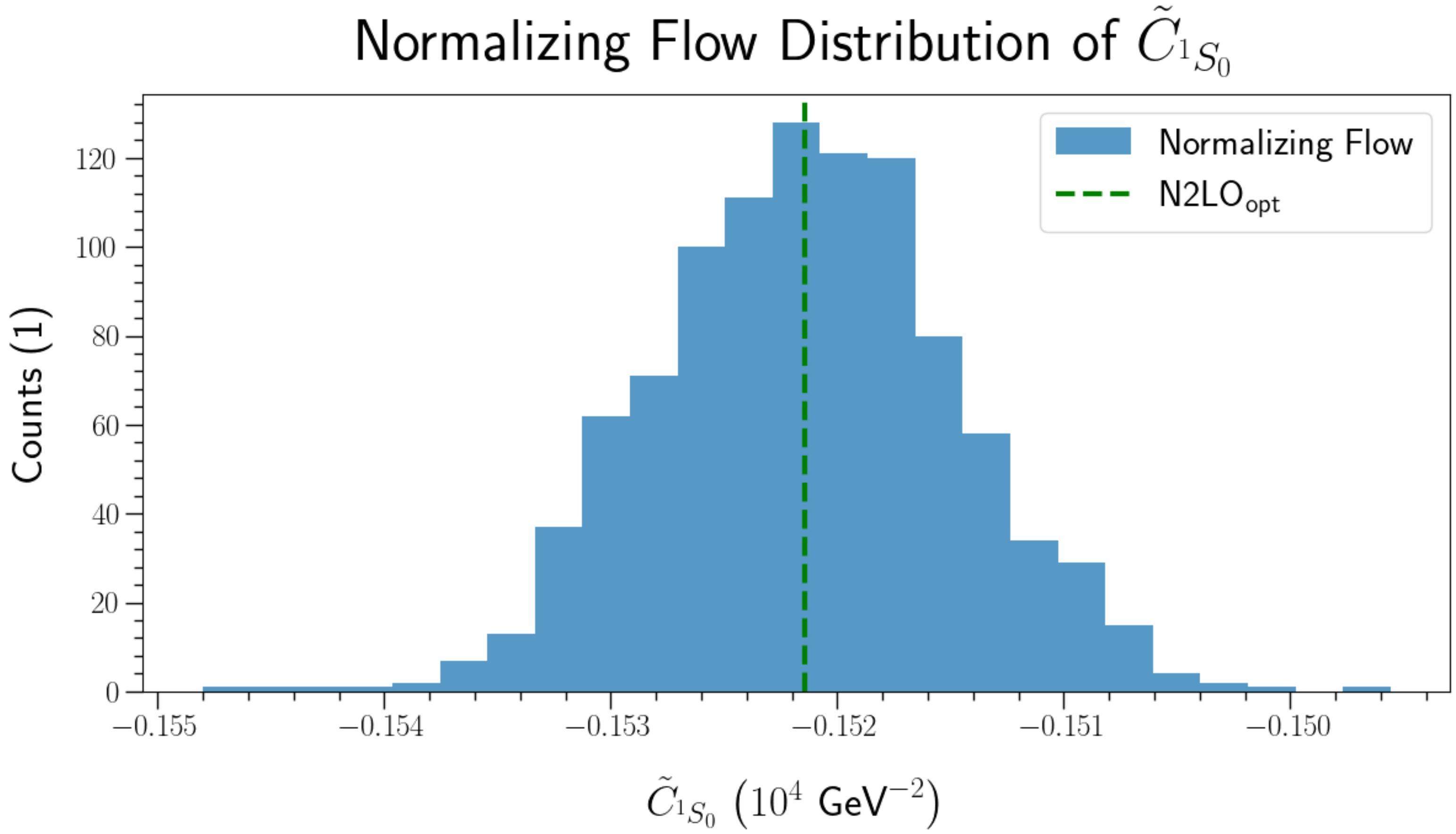


FIG. 1 (color online). Computed np phase shifts of the optimized NNLO potential of this work (solid, red line), the NNLO potential of Ref. [3] (dashed, blue line), and the N^{3}LO potential [4] (green, dotted line) compared with the Nijmegen phase shift analysis [18] (solid dots) and the VPI/GWU analysis SM99 [43] (open circles).

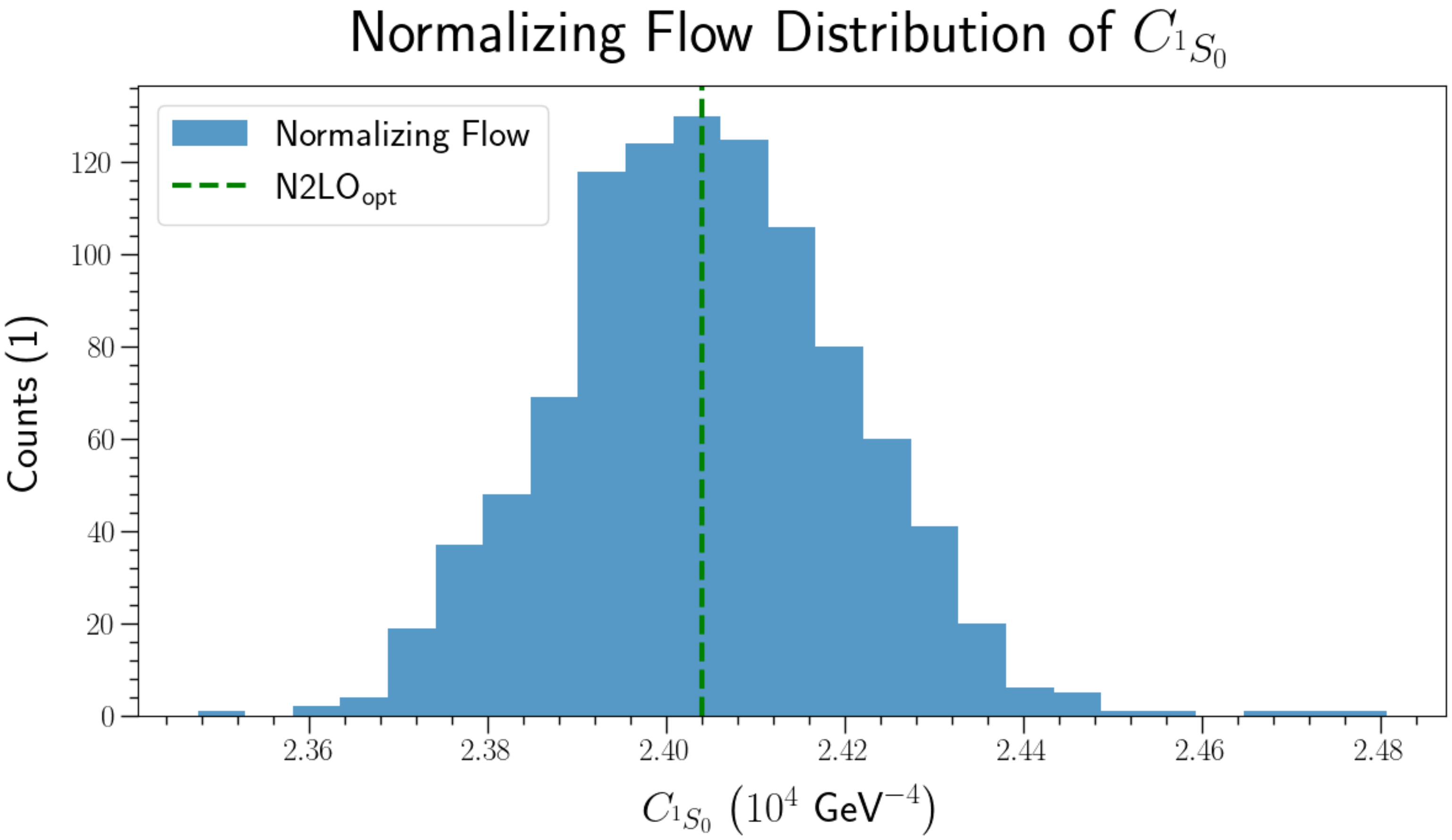
Project Details



Preliminary Results



Preliminary Results



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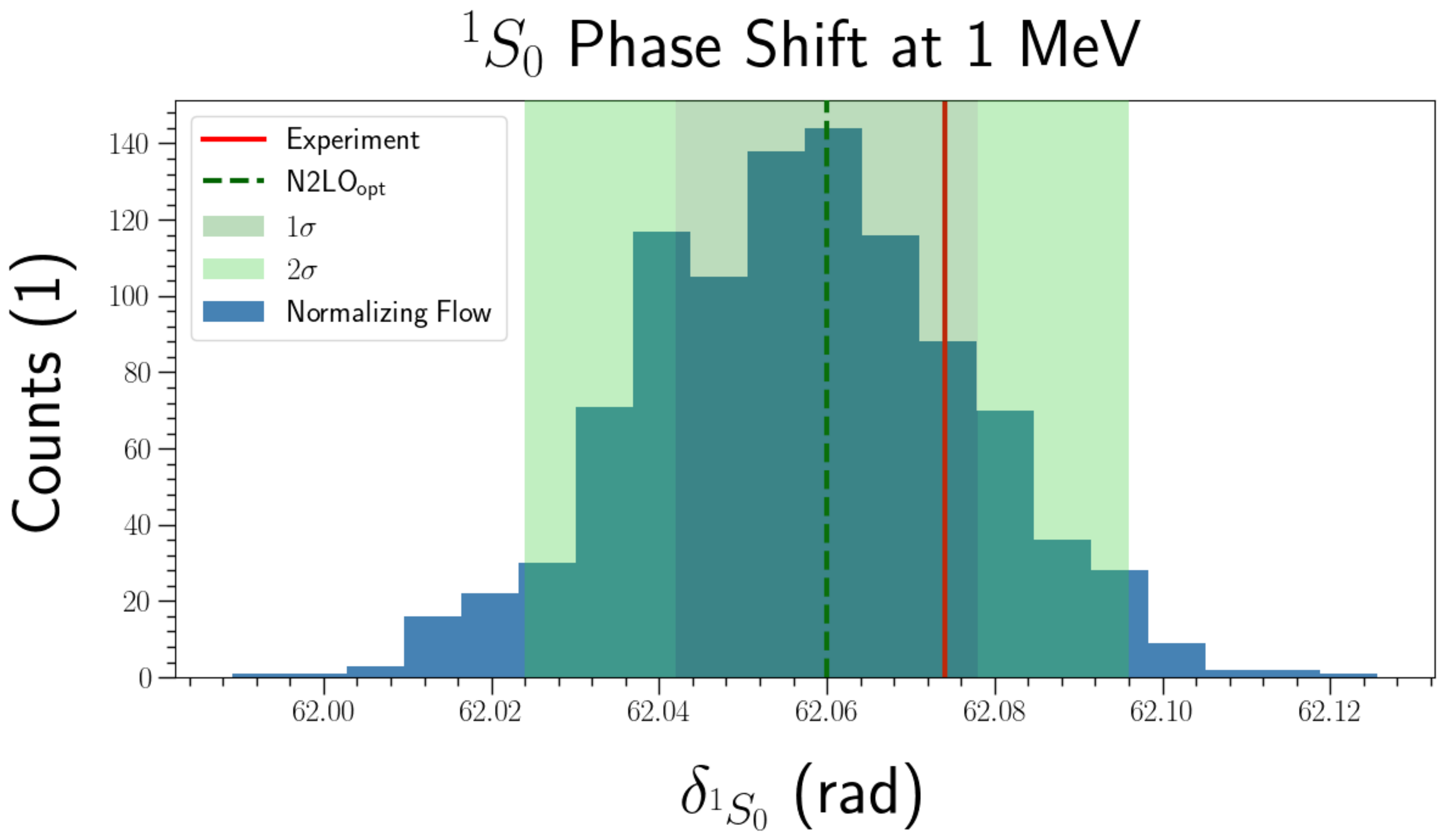
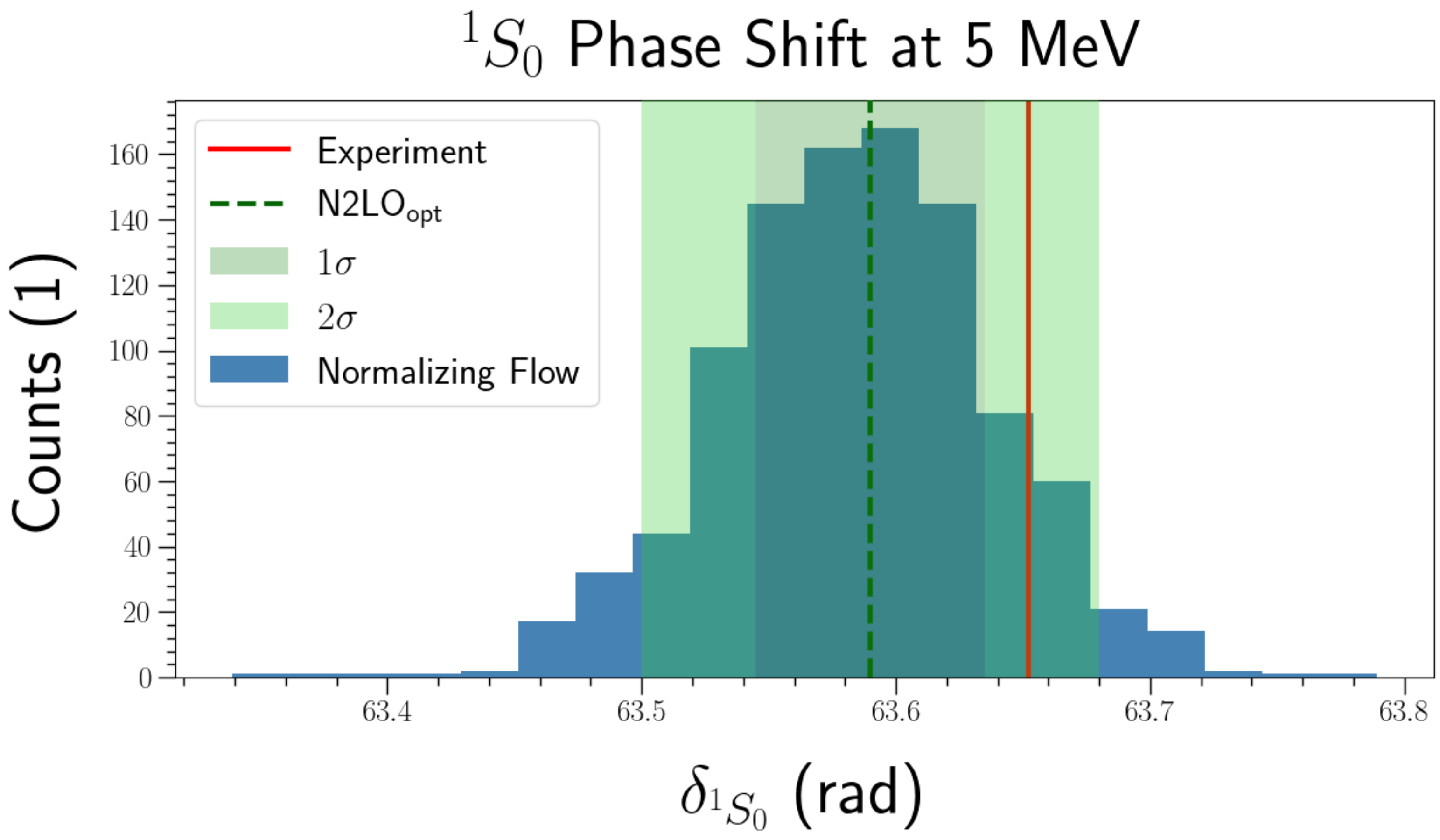
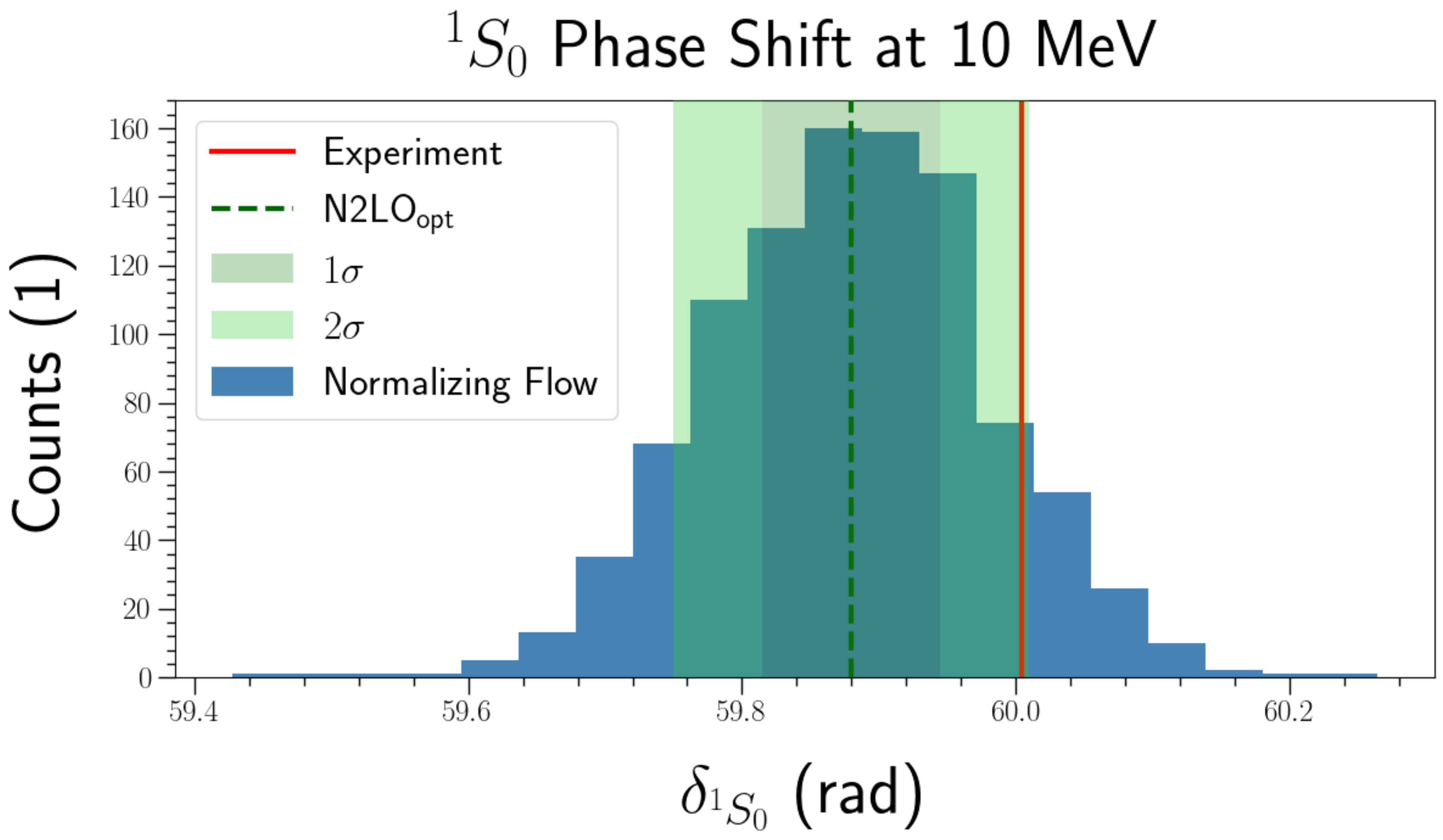


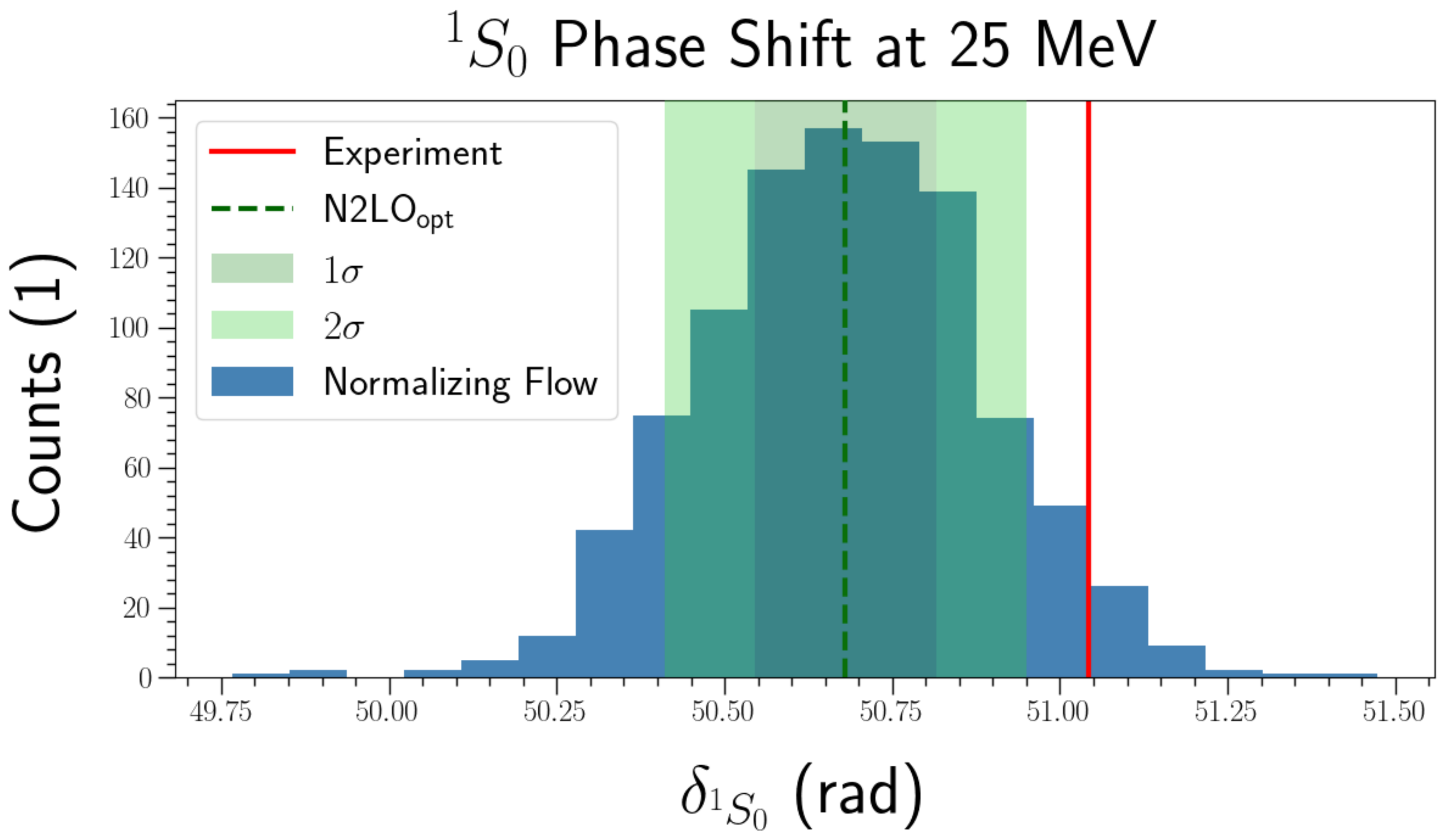
Preliminary Results



Preliminary Results



Preliminary Results



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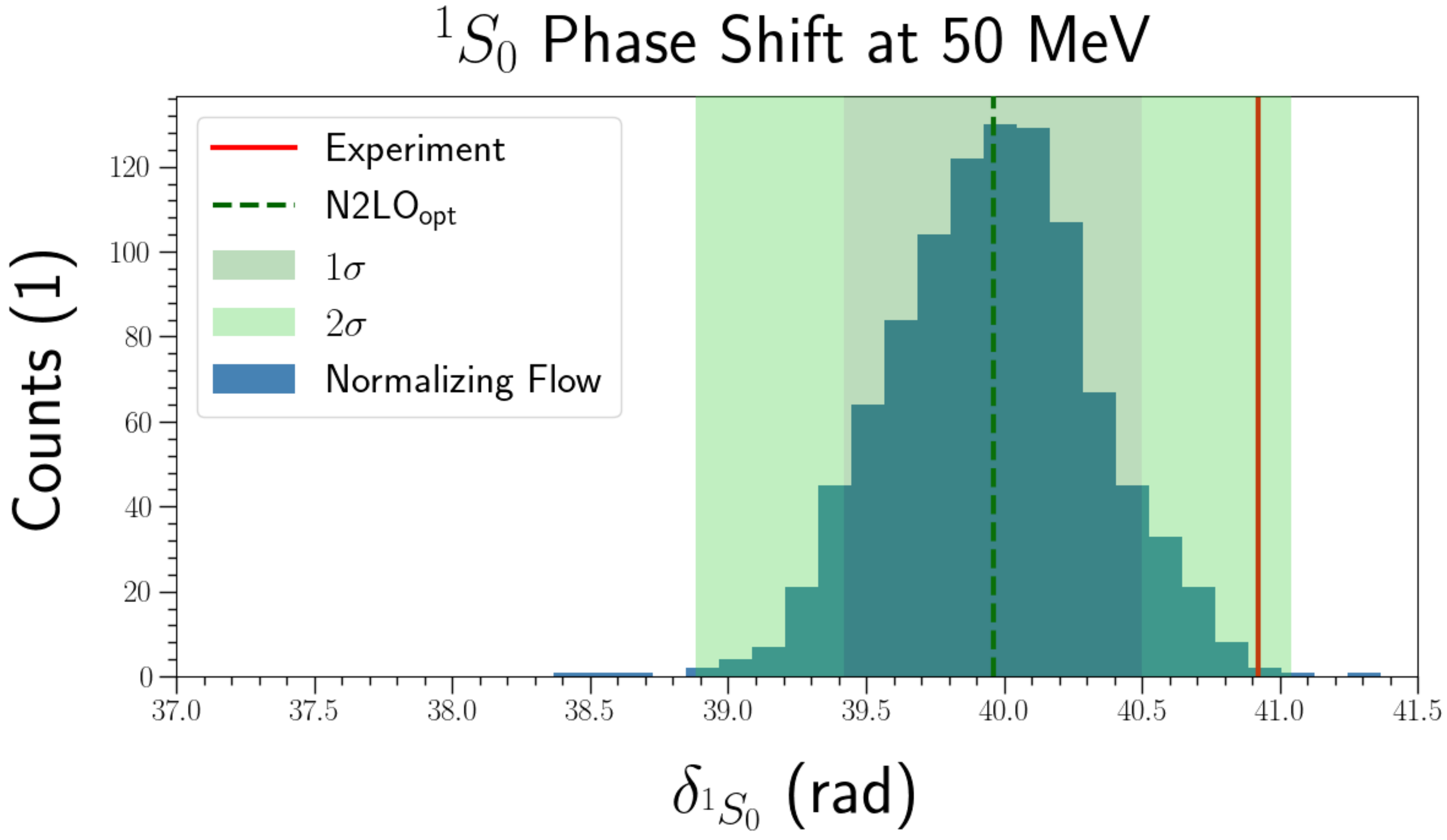
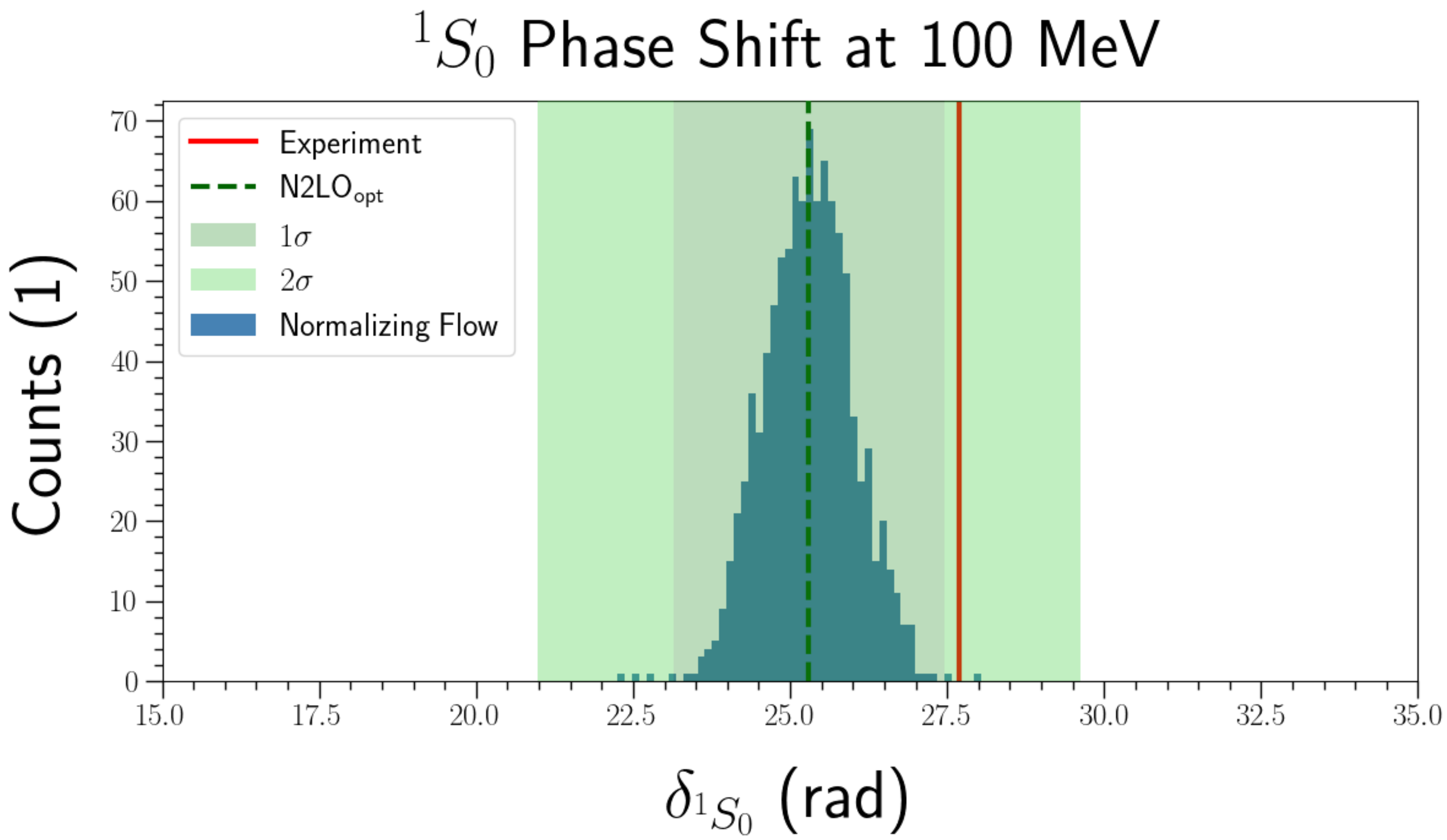
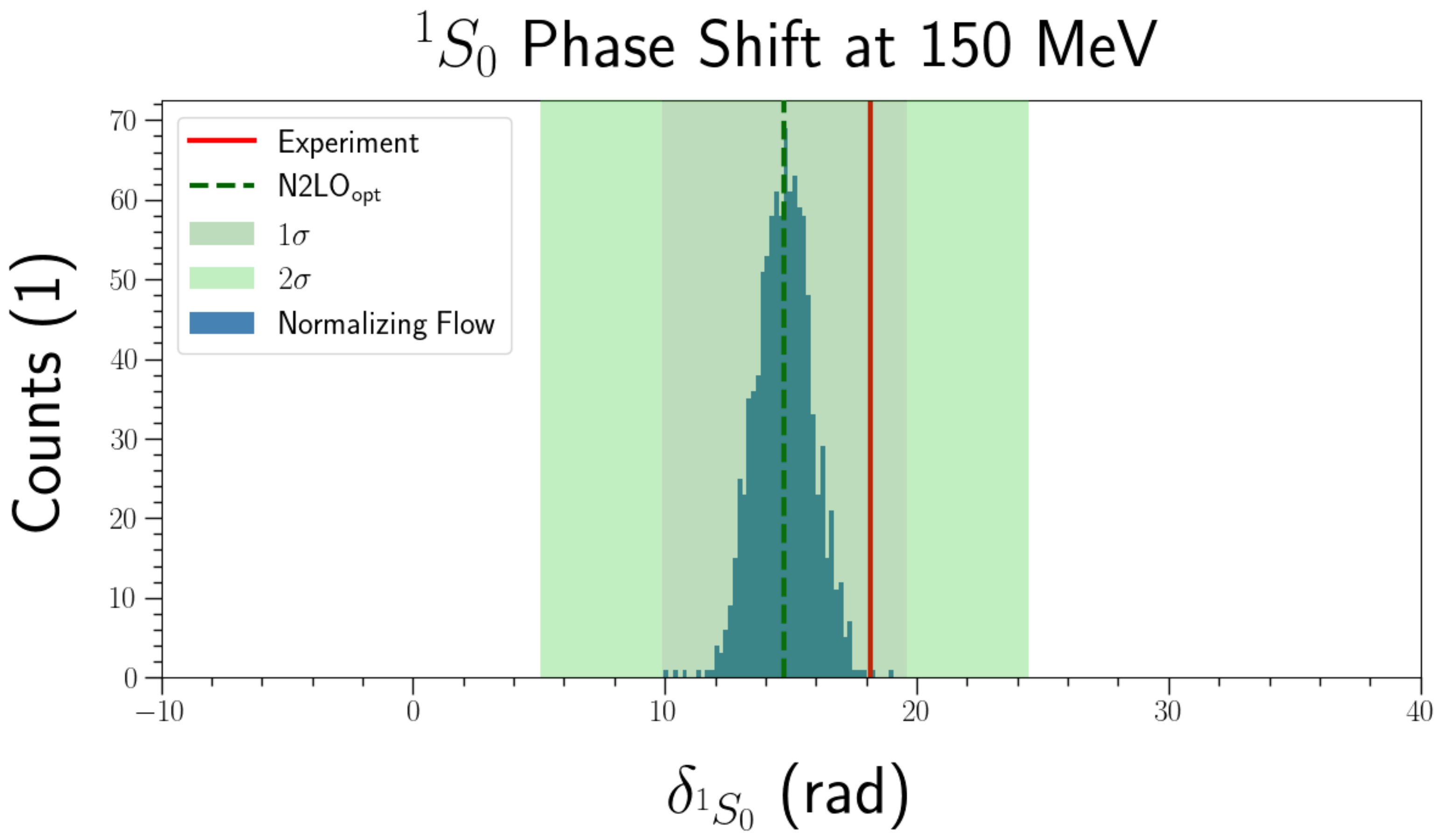


Preliminary Results



Preliminary Results



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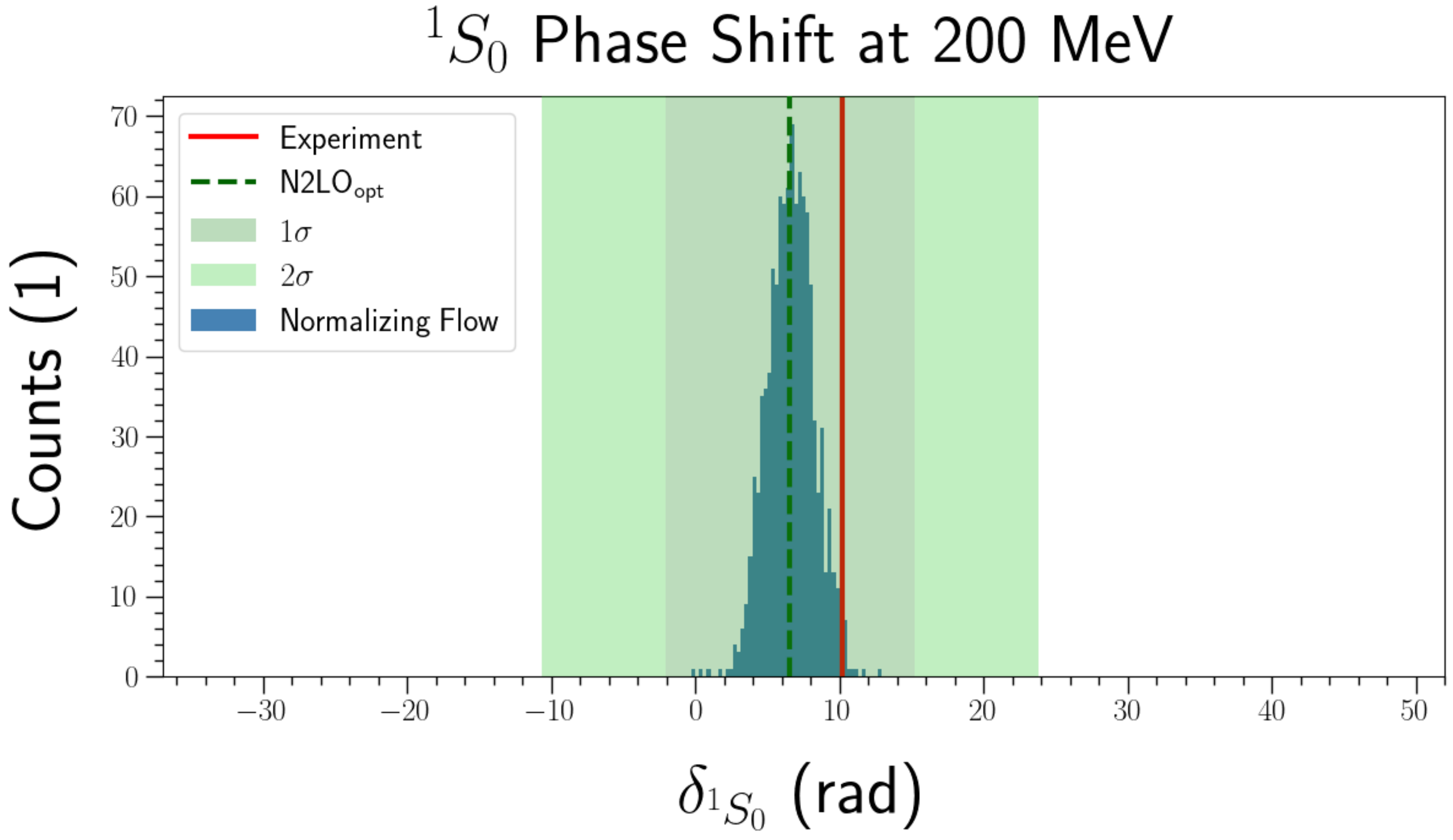
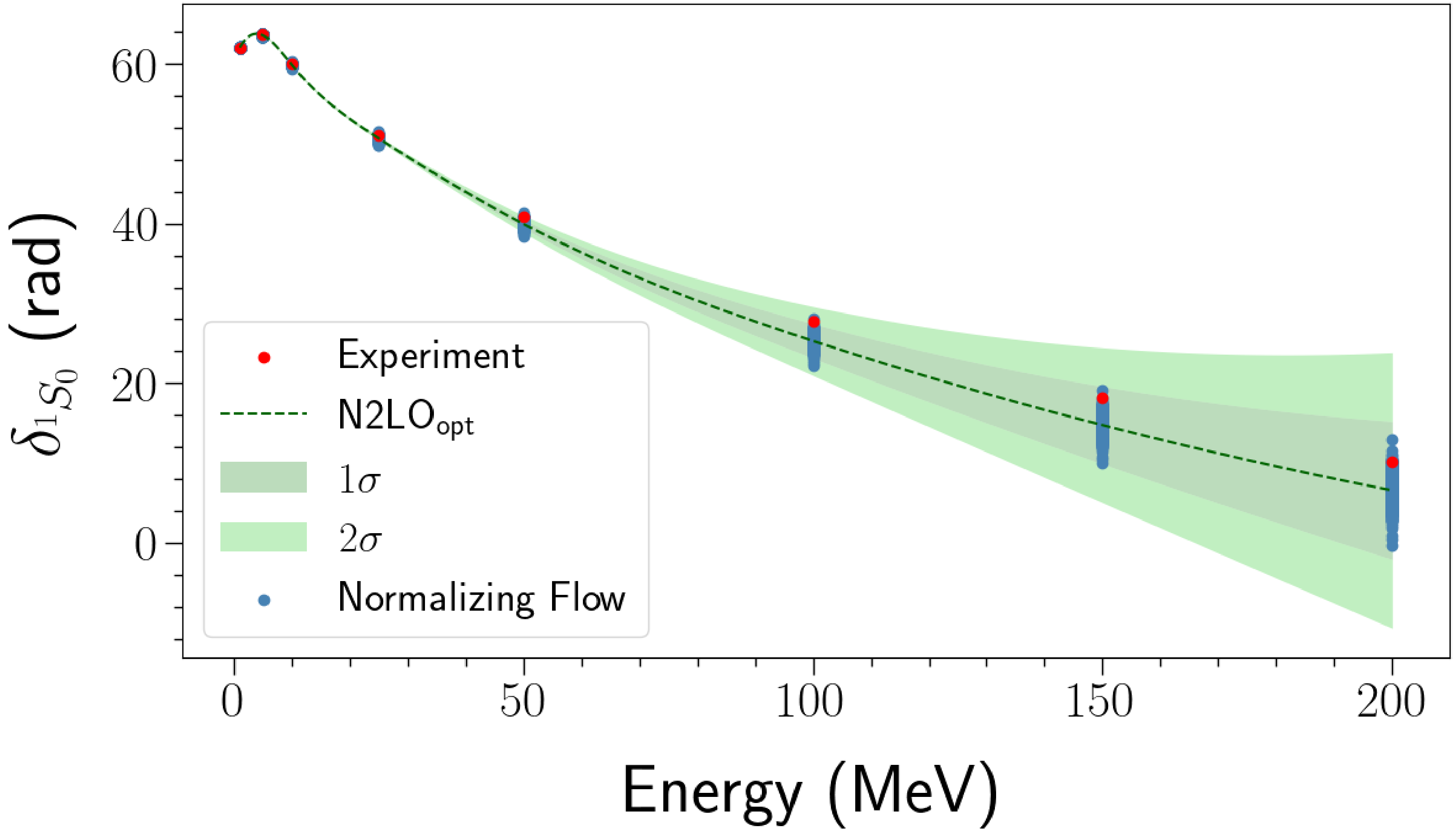
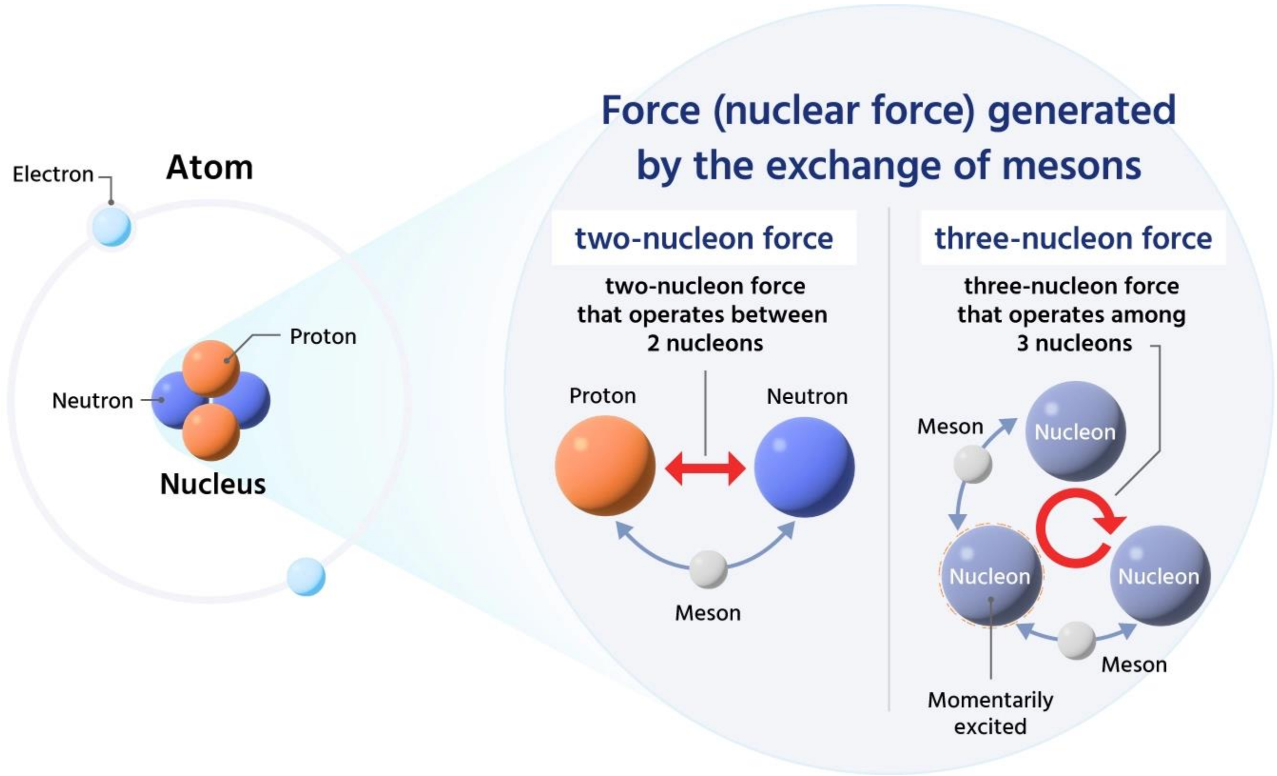


Preliminary Results



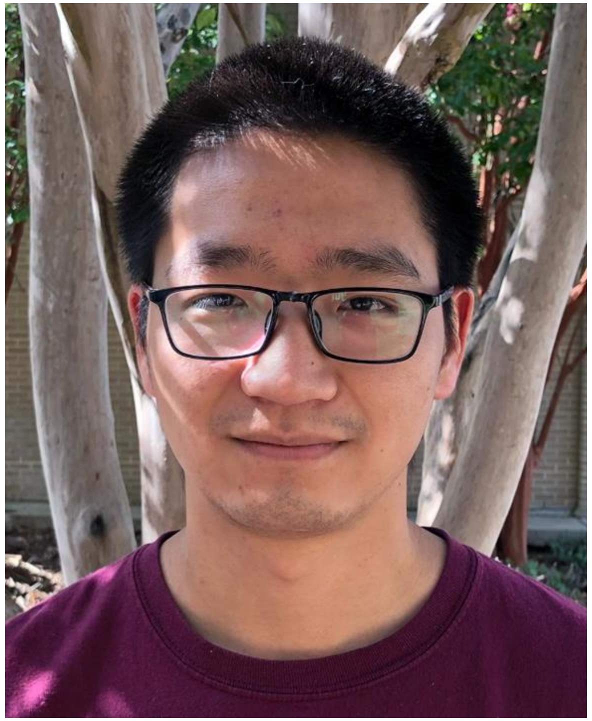
Future Extension

- We hope to apply this technique to nuclear many-body calculations and nuclear three-body forces.



Source: Institute of Science Tokyo, *Unraveling the Origins of the Universe: Physicist Kimiko Sekiguchi Finds Experimental Proof of three-nucleon force*

Acknowledgements



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THANK YOU

Texas A&M University
Department of Physics and Astronomy

Supplementary Material

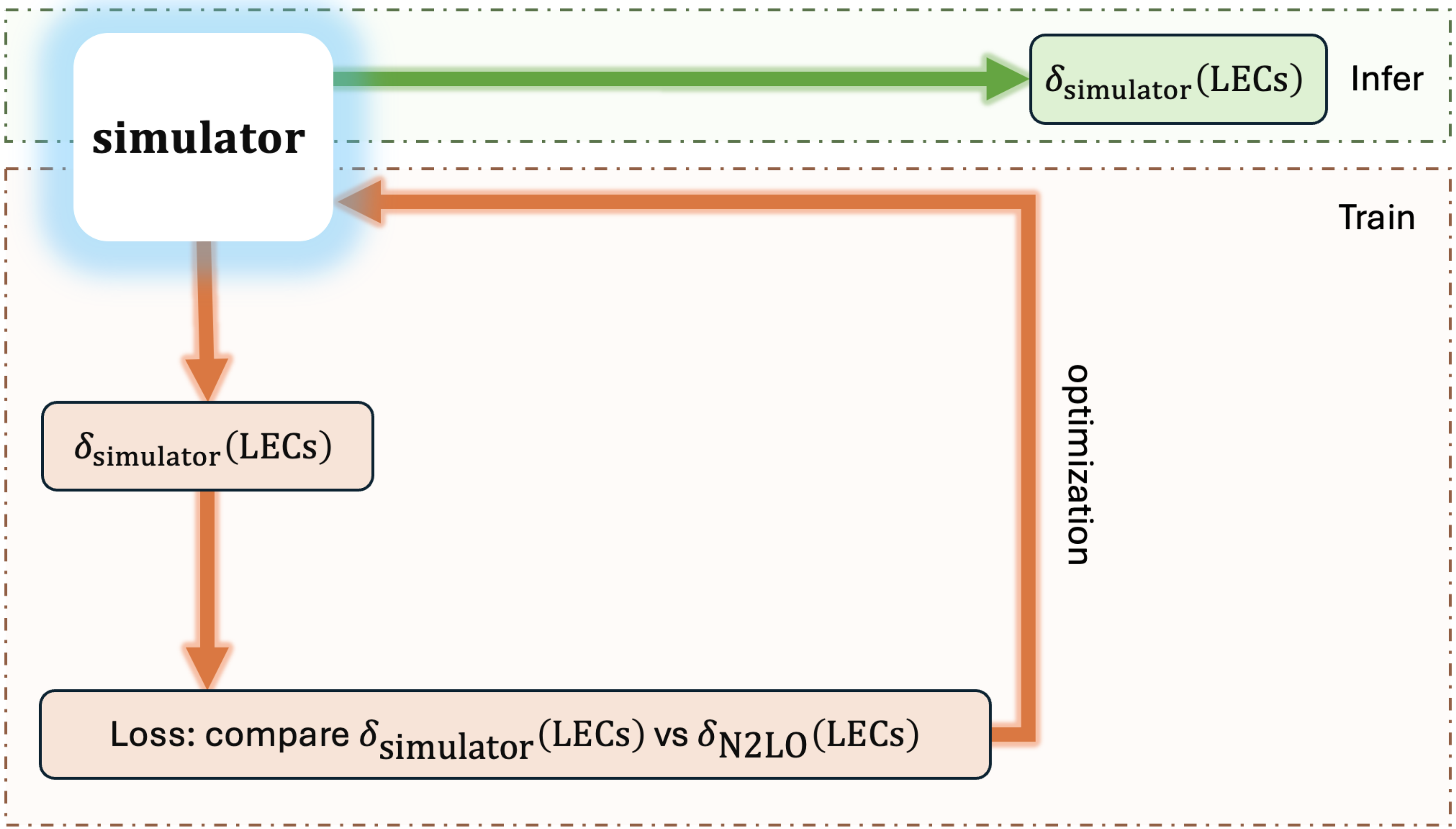
- The primary way of studying the nuclear interaction is by viewing it through a scattering process.

$$-\frac{\hbar^2}{M}\nabla^2\psi_n(\mathbf{r}) + \int d\mathbf{r}' V(\mathbf{r}|\mathbf{r}')\psi_n(\mathbf{r}') = E_n\psi_n(\mathbf{r})$$

$$R\phi_n = V\psi_n$$

$$R(\mathbf{k}|\mathbf{k}_0) = V(\mathbf{k}|\mathbf{k}_0) - \frac{M}{\hbar^2}\,\mathbf{P}\int d\mathbf{k}'\frac{V(\mathbf{k}|\mathbf{k}')R(\mathbf{k}'|\mathbf{k}_0)}{k'^2 - k_0^2}$$

Supplementary Material



Supplementary Material

$$\Delta\delta_{\mathrm{th}} = C_{\mathrm{avg}}(p/\lambda_b)^{\nu+1}$$

$C_{\mathrm{avg}} = (C(50\ \mathrm{MeV}) + C(100\ \mathrm{MeV}) + C(150\ \mathrm{MeV}) + C(200\ \mathrm{MeV}))/4$,
where $C(E) = |\delta_{\mathrm{exp}} - \delta_{\mathrm{th}}|Q^{-(\nu+1)}$ and $Q = \max(p/\lambda_b, M_\pi/\lambda_b)$